STEADY STATE RF-CURRENT DRIVE BASED
UPON THE RELATIVISTIC FOKKER-PLANCK EQUATION
WITH QUASILINEAR DIFFUSION

by

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ABSTRACT

A simple analytic treatment based upon the moments of the relativistic two-dimensional Fokker-Planck equation combined with resonant quasilinear diffusion is used to describe steady-state RF current drive by waves. For RF diffusion parallel to the magnetic field (as in lower-hybrid current drive) it is shown that current carried by high parallel momenta must have an associated high averaged perpendicular energy. The model is also applied to unidirectional perpendicular RF diffusion which is relevant to electron cyclotron current drive.

I. INTRODUCTION

Recent current drive experiments on large tokamak plasmas[1,2] have demonstrated the maintenance of considerable currents by the RF in the absence of the ohmic electric field. An important feature of these experiments is that the excited RF spectra have velocities that are resonant with very energetic electrons, twenty to a two hundred times the thermal energy of the bulk plasma which is about 1 keV in both experiments. Both experiments show that the RF generated current is carried by electrons effectively in the 50-100 keV range and is characterized by a perpendicular temperature which is of the same order in energy[3,4]. Apart from recent numerical work[5] the large perpendicular temperature, with relativistic effects taken fully into account, cannot be understood or successfully predicted by previous available numerical or theoretical work.

In this paper an analytic treatment based on the method of moments is presented. In Sec. II the relativistic Fokker-Planck equation for energetic electrons colliding with a thermal background of electrons and ions is derived as the Landau limit of the relativistic Balescu-Lenard collision operator. In Sec. III the energy, velocity, and momentum moments of the relativistic Fokker-Planck equation are derived in the presence of RF diffusion. Since our main interest is in determining the average parallel and perpendicular momenta of the current carrying electrons in steady-state the simplest possible momentum distribution which conveys this information is employed, namely a displaced delta function in both momenta parallel and perpendicular to the externally applied magnetic field. This procedure provides us with the evolution equation of the average energy, velocity, and current of the current carrying electrons. In Sec. IV, the evolution equations are solved for the steady state with parallel diffusion (as relevant to lower-hybrid, LH, current drive). This gives us the relations among the power dissipated, average energy, and current carried by the energetic electrons. The figure of merit as well as the average perpendicular energy are then readily calculated. In Sec. V the model is applied to a steady state current drive situation with perpendicular RF diffusion which is relevant when electron cyclotron waves are used. Finally, in Sec. VI the difference of the analysis and results presented here and previous work is discussed.

II. THE RELATIVISTIC FOKKER-PLANCK EQUATION

For the collisional model we use the Landau limit of the relativistic Balescu-Lenard collision operator[6-7]. The collisional flux in momentum space is given by (see Appendix A)

$$\vec{S}_{\alpha\beta} = -2q_\alpha^2 q_\beta^2 n_\beta \int d^3p_\beta \int d^3k \,\delta(\vec{k}\cdot\vec{v}_\alpha - \vec{k}\cdot\vec{v}_\beta) \frac{\vec{k}\vec{k}}{k^4} \frac{(1-\vec{\beta}_\alpha\cdot\vec{\beta}_\beta)^2}{\left[1-\left(\frac{\vec{k}\cdot\vec{\beta}_\alpha}{k}\right)^2\right]^2}$$
$$\cdot \left(\frac{\partial}{\partial\vec{p}_\alpha} - \frac{\partial}{\partial\vec{p}_\beta}\right) f_\alpha(\vec{p}_\alpha) f_\beta(\vec{p}_\beta) \tag{1}$$

where the labels α and β refer to the test and the field species respectively, and the vector $\vec{\beta}$ is the relativistic beta: $\vec{\beta} = \frac{\vec{v}}{c} = \frac{\vec{p}}{mc\gamma}$ with $m, \gamma, \vec{v}$ and $\vec{p}$ being the rest mass, the relativistic gamma: $\gamma = (1 + p^2/m^2c^2)^{1/2}$, velocity and momentum, respectively. Furthermore q is the charge and n_β the density of the field species. It can be shown[8] that for non-relativistic field species the collisional flux $\vec{S}_{\alpha\beta}$ reduces to,

$$\vec{S}_{\alpha\beta} = -\int d^3p_\beta \overline{\overline{\tau}}_{\alpha\beta} \cdot \left(\frac{\partial}{\partial\vec{p}_\alpha} - \frac{\partial}{\partial\vec{p}_\beta}\right) f_\alpha(\vec{p}_\alpha) f_\beta(\vec{p}_\beta) \tag{2}$$

where the tensor $\overline{\overline{\tau}}_{\alpha\beta}$ is defined as

$$\overline{\overline{\tau}}_{\alpha\beta} = A_{\alpha\beta} \frac{|\vec{v}_\alpha - \vec{v}_\beta|^2 \overline{\overline{I}} - (\vec{v}_\alpha - \vec{v}_\beta)(\vec{v}_\alpha - \vec{v}_\beta)}{|\vec{v}_\alpha - \vec{v}_\beta|^3} = A_{\alpha\beta} \frac{\partial^2}{\partial\vec{v}_\alpha\partial\vec{v}_\alpha} |\vec{v}_\alpha - \vec{v}_\beta| \tag{3}$$

with

$$A_{\alpha\beta} = 2\pi q_\alpha^2 q_\beta^2 n_\beta \ln\Lambda_{\alpha\beta} . \tag{4}$$

$\ln\Lambda_{\alpha\beta}$ is the Coulomb logarithm which corresponds to a relativistic particle (α) colliding with a non-relativistic one (β).

The collisional flux given by Eq.(2) can also be written in the following form,

$$\vec{S}_{\alpha\beta} = -\overline{\overline{D}}_{\alpha\beta} \cdot \frac{\partial f_\alpha}{\partial\vec{p}_\alpha} + \vec{F}_{\alpha\beta} f_\alpha \tag{5}$$

where the collisional diffusion tensor $\overline{\overline{D}}_{\alpha\beta}$ and frictional force vector $\vec{F}_{\alpha\beta}$ are defined by,

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$$\overline{\overline{D}}_{\alpha\beta} = \int d^3p_\beta \overline{\overline{\tau}}_{\alpha\beta} \ , \quad \vec{F}_{\alpha\beta} = -\int d^3p_\beta f_\beta \vec{\nabla}_{\vec{p}_\beta} \cdot \overline{\overline{\tau}}_{\alpha\beta} \tag{6}$$

with $\vec{\nabla}_{\vec{p}_\beta}$ is the gradient operator $\frac{\partial}{\partial \vec{p}_\beta}$.

As long as one considers fast test particles interacting with a thermal background of field particles the magnitude of the relative velocity $|\vec{v}_\alpha - \vec{v}_\beta|$ can be expanded around $v_\alpha = |\vec{v}_\alpha|$:

$$|\vec{v}_\alpha - \vec{v}_\beta| \simeq \vec{v}_\alpha - \vec{v}_\beta \cdot \frac{\partial v_\alpha}{\partial \vec{v}_\alpha} + \frac{1}{2}\vec{v}_\beta \vec{v}_\beta : \frac{\partial^2 v_\alpha}{\partial \vec{v}_\alpha \partial \vec{v}_\alpha} \tag{7}$$

where we dropped terms of order $(v_\beta/v_\alpha)^3$ and higher. Introducing the notation $<>$ for averaging over the field particle distribution ($< \ldots > = \int d^3p_\beta (\ldots) f_\beta$ and assuming that the thermal background does not carry a current (i.e. $< \vec{v}_\beta > = 0$) yields for the diffusion tensor $\overline{\overline{D}}_{\alpha\beta}$:

$$\frac{\overline{\overline{D}}_{\alpha\beta}}{A_{\alpha\beta}} = \frac{\partial^2 v_\alpha}{\partial \vec{v}_\alpha \partial \vec{v}_\alpha} + \frac{1}{2}\frac{\partial^2}{\partial \vec{v}_\alpha \partial \vec{v}_\alpha}\left[< \vec{v}_\beta \vec{v}_\beta > : \frac{\partial^2 v_\alpha}{\partial \vec{v}_\alpha \partial \vec{v}_\alpha}\right] \tag{8}$$

and for the friction tensor $\vec{F}_{\alpha\beta}$:

$$m_\beta \frac{\vec{F}_{\alpha\beta}}{A_{\alpha\beta}} = \frac{\partial}{\partial \vec{v}_\alpha} \cdot \frac{\partial^2 v_\alpha}{\partial \vec{v}_\alpha \partial \vec{v}_\alpha}\left(v_\alpha + \frac{1}{2} < \vec{v}_\beta \vec{v}_\beta > : \frac{\partial^2 v_\alpha}{\partial \vec{v}_\alpha \partial \vec{v}_\alpha}\right) \tag{9}$$

where the identify $\vec{\nabla}_{\vec{p}_\beta} \cdot \overline{\overline{\tau}}_{\alpha\beta} = -\frac{1}{m_\beta}\vec{\nabla}_{\vec{v}_\alpha} \cdot \overline{\overline{\tau}}_{\alpha\beta}$, valid for nonrelativistic field particles, has been used.

After some algebra Eqs. (8) and (9) become,

$$\frac{\overline{\overline{D}}_{\alpha\beta}}{A_{\alpha\beta}} = \frac{\overline{\overline{I}}v_\alpha^2 - \vec{v}_\alpha \vec{v}_\alpha}{v_\alpha^3} - \frac{< v_\beta^2 >}{v_\alpha^2}\frac{\overline{\overline{I}}v_\alpha^2 - 3\vec{v}_\alpha \vec{v}_\alpha}{v_\alpha^3} \tag{10}$$

and

$$\frac{\vec{F}_{\alpha\beta}}{A_{\alpha\beta}} = -\vec{v}_\alpha \frac{1}{m_\beta v_\alpha^3} \tag{11}$$

Expressing now the velocities of the test particles in terms of their momenta in Eqs. (10) and (11) yields for the collisional flux $\vec{S}_{\alpha\beta}$,

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$$\frac{\vec{S}_{\alpha\beta}}{m_\alpha A_{\alpha\beta}} = \frac{\gamma_\alpha}{p_\alpha^3}\left\{\left[\left(1 - \gamma_\alpha^2 \frac{<m_\alpha^2 v_\beta^2>}{p_\alpha^2}\right)p_\alpha^2 \bar{\bar{I}}\right.\right.$$
$$\left.\left. - \left(1 - \gamma_\alpha^2\frac{<m_\alpha^2 v_\beta^2>}{p_\alpha^2}\right)\vec{p}_\alpha\vec{p}_\alpha\right]\cdot\frac{\partial f_\alpha}{\partial\vec{p}_\alpha} + 2\frac{m_\alpha}{m_\beta}\gamma_\alpha\vec{p}_\alpha f_\alpha\right\} \tag{12}$$

This formula for the colllisional flux $\vec{S}_{\alpha\beta}$ coincides with the one derived (by very different means) by Mosher[9] in the limit $<v_\beta^2> \rightarrow 0$. In Appendix B the cylindrical form of Eq. (12) is also derived; this form is very useful for computational purposes[5].

III. MOMENTS OF THE FOKKER-PLANCK EQUATION COMBINED WITH RF DIFFUSION

The continuity equation in momentum space, if only collisions are present, is simply,

$$\frac{\partial f_\alpha}{\partial t} + \sum_\beta \vec{\nabla}_{\vec{p}_\alpha}\cdot\vec{S}_{\alpha\beta} = 0 \tag{13}$$

where the collisional flux $\vec{S}_{\alpha\beta}$ is given by Eq. (12).

The kinetic energy of the relativistic test particle α is $m_\alpha c^2(\gamma_\alpha - 1)$. Multiplying Eq. (13) by this energy and integrating over $\vec{p}_\alpha$ yields,

$$\int d^3p_\alpha \frac{\partial f_\alpha}{\partial t}m_\alpha c^2(\gamma_\alpha - 1) = \frac{d^c}{dt}<m_\alpha c^2\gamma_\alpha>_\alpha$$
$$= -\sum_\beta 2m_\alpha A_{\alpha\beta}\int d^3p_\alpha\frac{1}{p_\alpha}\left(\frac{<v_\beta^2>}{v_\alpha^2}\vec{p}_\alpha\cdot\frac{\partial f_\alpha}{\partial\vec{p}_\alpha} + \frac{m_\alpha}{m_\beta}\gamma_\alpha f_\alpha\right) \tag{14}$$

where $\frac{d^c}{dt}$ refers to the collisional rate of change and $<>_\alpha$ denotes the averaging over the test particle distribution function f_α. After partially integrating, the last equation yields,

$$\frac{d^c}{dt}<m_\alpha c^2\gamma_\alpha>_\alpha = -\sum_\beta 2m_\alpha A_{\alpha\beta}\int d^3p_\alpha\frac{f_\alpha}{p_\alpha}\left(\frac{m_\alpha}{m_\beta}\gamma_\alpha - 2\frac{<v_\beta^2>}{c^2}\right) \tag{15a}$$

Since the field particles are treated non-relativistically, this simplifies to

$$\frac{d^c}{dt} < m_\alpha c^2 \gamma_\alpha >_\alpha = -\sum_\beta 2 m_\alpha A_{\alpha\beta} \frac{m_\alpha}{m_\beta} (\frac{\gamma_\alpha}{p_\alpha})_\alpha \tag{15b}$$

Multiplying now the continuity equation by $\vec{p}_\alpha$ and integrating over $\vec{p}_\alpha$ one obtains the momentum conservation equation

$$\int d^3 p_\alpha \frac{\partial f_\alpha}{\partial t} \vec{p}_\alpha = \frac{d^c}{dt} < \vec{p}_\alpha >_\alpha = \int d^3 p_\alpha \sum_\beta \vec{S}_{\alpha\beta} . \tag{16}$$

Performing the integration finally yields

$$\frac{d^c}{dt} < \vec{p}_\alpha >_\alpha = -\sum_\beta 2 m_\alpha A_{\alpha\beta} \langle \frac{\gamma_\alpha}{p_\alpha^3} \left(\frac{m_\alpha}{m_\beta} \gamma_\alpha + 1 \right) \vec{p}_\alpha \rangle_\alpha \tag{17}$$

The momentum conservation equation does not coincide with the velocity conservation equation if relativistic effects are taken into consideration. The velocity conservation equation, on the other hand, is related to the electric current conservation, and it will be a useful equation to have if current is to be externally driven. Taking the velocity moment of Eq.(13) yields, after partially integrating once and using the identity $\frac{\partial}{\partial \vec{p}_\alpha} \left(\frac{\vec{p}_\alpha}{\gamma_\alpha} \right) = \frac{\overline{\overline{I}} - \vec{\beta}_\alpha \vec{\beta}_\alpha}{\gamma_\alpha}$,

$$\int d^3 p_\alpha \frac{\vec{p}_\alpha}{\gamma_\alpha m_\alpha} \frac{\partial f_\alpha}{\partial t} = \frac{d^c}{dt} < \vec{v}_\alpha >_\alpha = \int d^3 p_\alpha \frac{\overline{\overline{I}} - \vec{\beta}_\alpha \vec{\beta}_\alpha}{\gamma_\alpha m_\alpha} \cdot \sum_\beta \vec{S}_{\alpha\beta} \tag{18}$$

Evaluating the integral gives,

$$\frac{d^c}{dt} < \vec{v}_\alpha >_\alpha = -\sum_\beta 2 A_{\alpha\beta} \int d^3 p_\alpha \frac{\vec{p}_\alpha}{p_\alpha^3} \left(1 - \frac{< v_\beta^2 >}{c^2} + \frac{m_\alpha}{m_\beta \gamma_\alpha} \right) \tag{19}$$

or since we approximate $\frac{< v_\beta^2 >}{c^2}$ by zero,

$$\frac{d^c}{dt} < \vec{v}_\alpha >_\alpha = -\sum_\beta 2 A_{\alpha\beta} \langle \frac{\vec{p}_\alpha}{p_\alpha^3} \left(1 + \frac{1}{\gamma_\alpha} \frac{m_\alpha}{m_\beta} \right) \rangle_\alpha \tag{20}$$

Equations (15b), (17) and (20) express the collisional energy, momentum, and velocity relaxation of a relativistic test species interacting with a thermal bath. In our particular case the thermal bath consists of thermal electrons and thermal ions of densities n_e and n_i respectively. To the extent that the energetic species (namely the relativisticly

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treated electron) is a minority species, i.e. $n_{e'} \ll n_e$ where the prime refers to the minority species, quasineutrality implies

$$q_e n_e + q_i n_i = 0 \tag{21}$$

or

$$n_e = Z_i n_i \tag{22}$$

where Z_i is the ionic charge number. Therefore in our particular case Eqs. (15), (17) and (20) take the form,

$$\frac{d^c}{dt} < \gamma > = -\nu_c \langle \frac{\gamma}{(\gamma^2 - 1)^{1/2}} \rangle \tag{23}$$

$$\frac{d^c}{dt} < \gamma\vec{\beta} > = -\nu_c \langle \frac{aZ_i + 1 + \gamma}{(\gamma^2 - 1)^{3/2}} \gamma^2 \vec{\beta} \rangle \tag{24}$$

and

$$\frac{d^c}{dt} < \vec{\beta} > = -\nu_c \langle \frac{aZ_i + 1 + 1/\gamma}{(\gamma^2 - 1)^{3/2}} \gamma\vec{\beta} \rangle \tag{25}$$

where we have suppressed the energetic electron index, and ν_c and a are defined as follows,

$$\nu_c = \frac{4\pi e^4 n_e \ln \Lambda_{e'e}}{m_e^2 c^3} \quad , \quad a = \frac{\ln \Lambda_{e'i}}{\ln \Lambda_{e'e}} \tag{26}$$

In deriving Eqs. (23), (24) and (25) we have ignored terms of order (m_e/m_i). The Coulomb logarithms are given by,[9]

$$\ln \Lambda_{e'e} = \ln \left[\frac{\lambda_{De} m_e c^2 < \gamma > < \beta >^2}{2(< \gamma > + 1)^{1/2} e^2} \right], \ln \Lambda_{e'i} = \ln \left(\frac{\lambda_{De} m_e c^2 < \gamma > < \beta >^2 \sqrt{2}}{2 Z_i e^2} \right) \tag{27}$$

where λ_{De} is the electron-Debye length. For moderately relativistic electrons and $Z_i \simeq 1$ they are approximately equal.

In the presence of an externally imposed driving mechanism (RF waves in particular) the evolution equations (23) to (25) take the form,

$$\frac{d}{dt} < \gamma > = -\langle \frac{\gamma}{(\gamma^2 - 1)^{1/2}} \rangle + P_d \qquad (28)$$

$$\frac{d}{dt} < \gamma\vec{\beta} > = -\langle \frac{Z_i + 1 + \gamma}{(\gamma^2 - 1)^{3/2}} \gamma^2\vec{\beta} \rangle + \vec{F}_d \qquad (29)$$

and

$$\frac{d}{dt} < \vec{\beta} > = -\langle \frac{Z_i + 1 + \gamma + 1/\gamma}{(\gamma^2 - 1)^{3/2}} \gamma\vec{\beta} \rangle + \vec{A}_d \qquad (30)$$

where we set $a = 1$. The quantities P_d, $\vec{F}_d$ and $\vec{A}_d$ are defined as the driven RF power (P_d) and the macroscopic manifestation of the associated force and acceleration $(\vec{F}_d, \vec{A}_d)$ in the ensuing wave-particle interactions. They are normalized to $\nu_c mc^2$, $\nu_c mc$ and $\nu_c c$ respectively. The time t here is normalized to ν_c^{-1}. The simplest evaluation of Eqs. (28)–(30) is for an effective distribution function $f(p_\parallel, p_\perp)$ given by,

$$f(p_\parallel, p_\perp) = \frac{\delta(p_\parallel - \bar{p}_\parallel)\delta(p_\perp - \bar{p}_\perp)}{2\pi\bar{p}_\perp} \qquad (31)$$

where $\bar{p}_\parallel$ and $\bar{p}_\perp$ correspond to "average", effective values of the parallel and perpendicular momenta; the average perpendicular momentum vector is randomly oriented on the plane perpendicular to the magnetic field. The distribution function $f(p_\parallel, p_\perp)$ depends implicitly on time through the time dependence of $\bar{p}_\parallel$ and $\bar{p}_\perp$ The average value $\bar{p}_\parallel$ is generally related to the location of the spectrum and, in a sense, provides an estimate of the average momentum of the resonant electrons if the resonant region is centered around $p_{\parallel 0} = \bar{p}_\parallel$. On the other hand, the average value $\bar{p}_\perp$ is related to the average perpendicular kinetic energy these electons are carrying (see Sec. IV).

Using the distribution function of Eq. (31) in Eqs. (28)–(30) we obtain

$$\frac{d\gamma}{dt} = -\frac{\gamma}{(\gamma^2 - 1)^{1/2}} + P_0 \qquad (32)$$

$$\frac{d}{dt}(\gamma\vec{\beta}) = -\frac{Z_i + 1 + \gamma}{(\gamma^2 - 1)^{3/2}}\gamma^2\vec{\beta} + \vec{F}_0 \qquad (33)$$

$$\frac{d\vec{\beta}}{dt} = -\frac{Z_i + 1 + 1/\gamma}{(\gamma^2 - 1)^{3/2}}\gamma\vec{\beta} + \vec{A}_0 \tag{34}$$

where the quantities P_0, $\vec{F}_0$, $\vec{A}_0$, γ and $\vec{\beta}$ refer to the RF quantities P_d, $\vec{F}_d$, $\vec{A}_d$, γ and $\vec{\beta}$ respectively, for the distribution function in Eq. (26) that is they are functions of $\bar{p}_\parallel$ and $\bar{p}_\perp$. Consistency of these three equations, namely Eq.(33) being derivable from Eqs.(32) and (34) implies that,

$$\vec{A}_0 = \frac{\vec{F}_0}{\gamma} - \vec{\beta}\frac{P_0}{\gamma} \tag{35}$$

IV. STEADY STATE CURRENT DRIVE WITH PARALLEL RF DIFFUSION

In the presence of a unidirectional RF spectrum acting along the magnetic field direction (e.g. a lower-hybrid wave spectrum), only the parallel components of $\vec{A}_0$ and $\vec{F}_0$ are present. Furthermore, the macroscopic manifestations of the well known relationship[10] between the densities of power dissipated (normalized to $n\nu_c mc^2$) and force dissipated (normalized to $n\nu_c mc$), namely,[11]

$$P_0 = \vec{F}_0 \cdot \vec{\beta} \tag{36}$$

implies that, in this case,

$$F_{0\parallel} = \frac{P_0}{\beta_\parallel} \tag{37}$$

Solution of Eqs. (32), (33), utilizing Eq. (37) yields,

$$P_0 = \frac{\gamma}{(\gamma^2 - 1)^{1/2}} \tag{38}$$

$$\beta_\parallel = \left[\frac{\gamma^2 - 1}{\gamma(Z_i + \gamma + 1)}\right]^{1/2} \tag{39}$$

The first equation provides the relationship between the normalized RF driven power density and the average kinetic energy ϵ (in normalized units $\epsilon = \gamma - 1$) of the energetic electrons. The second equation, on the other hand, provides the (normalized) current density these electrons are carrying as a function of their average kinetic energy.

From Eqs. (38) and (39) we obtain

$$\frac{\beta_{\parallel}}{P_0} = \frac{(\epsilon+1)^2 - 1}{(\epsilon+1)^{3/2}(\epsilon+2+Z_i)^{1/2}} \equiv g(\epsilon, Z_i) \tag{40}$$

In unnormalized form this gives

$$\frac{J}{p_d} = \frac{e}{mc\nu_c} g(\epsilon, Z_i) \tag{41}$$

Thus the figure of merit is found to be

$$\left(\frac{I}{P_D}\right)_{\frac{Amp}{Watt}} = \frac{31.2}{\ln\Lambda} \frac{g(\epsilon, Z_i)}{R_m n_{20}} \tag{42}$$

where R_m is the major radius of the tokamak in meters and n_{20} is the plasma density in units of $10^{20}/m^3$. In the nonrelativistic limit, $\epsilon \ll 1$, $g \sim \epsilon$ and the figure of merit increases for current carried by more energetic electrons. In the ultrarelativistic limit, $\epsilon \gg 1$, $g \to 1$ and Eq.(42) gives an upper bound on the figure of merit. Recent experiments on PLT and Alcator C can be considered to be effectively in the range $\epsilon \approx 0.1 - 0.2$. Considerable improvement in the figure of merit is therefore possible by RF current drive with more energetic electrons. This should be possible with the use of the fast wave in the lower-hybrid range of frequencies[12]. The limit will most likely be dictated by how well energetic electrons can be confined in the plasma.

Finally, taking into account Eq. (39) and the identity $\gamma^2 = 1 + q_{\parallel}^2 + q_{\perp}^2$, with $q_{\parallel}$ and $q_{\perp}$ being the normalized (to mc) parallel and perpendicular momentum respectively (note that $q_{\parallel} = \gamma\beta_{\parallel}$), yields for $Z_i = 1$,

$$q_{\parallel} = \frac{q_{\perp}}{2}\left[\frac{q_{\perp}^2 + (q_{\perp}^4 + 16q_{\perp}^2 + 16)^{1/2}}{2}\right]^{1/2} . \tag{43}$$

For a given $q_{\parallel}$ (or $p_{\parallel 0}/p_{th}$) this equation will provide the value of $q_{\perp}$ and therefore γ can be determined; knowing $q_{\perp}$ and γ one readily obtains the corresponding value of $v_{\perp}/v_{th}$ from $v_{\perp}/v_{th} = p_{\perp}/(\gamma p_{th}) = q_{\perp}/(\gamma\beta_{th})$ (with $\beta_{th} = v_{th}/c$) and, subsequently, the value of $v_{\perp}^2/(2v_{th}^2)$ which is denoted by "$T_{\perp}$"$/T_B$ ($T_B = m_e v_{th}^2$). The latter is plotted vs. $p_{\parallel 0}/p_{th}$ in Fig. 1 and provides an estimate of what mean square perpendicular velocity one can expect for various locations of the resonant region ($p_{\parallel 0}/p_{th}$). This estimate, which in the nonrelativistic limit is the measure of the perpendicular temperature, is

in very good agreement with the numerical results of a two-dimensional Fokker-Planck code, as reported elsewhere[5].

The figure of merit normalized to bulk (B) thermal quantities (J in units of $en_e v_{th}$ and p_d in units of $n_e m_e v_{th}^2 \nu$, with $\nu = \nu_c/\beta_{th}^3$) is given by $(J/p_d)_B = (\beta_\parallel/P_0)/\beta_{th}^2$ and is plotted versus the average kinetic energy in KeV of the current carrying electrons, $mc^2 \epsilon$, in Fig. 2. The relativisticly normalized figure of merit, Eq. (39), $\beta_\parallel/P_0$, on the other hand, is plotted versus ϵ in Fig. 3 for various values of Z_i.

V. STEADY STATE OPERATION WITH PERPENDICULAR RF DIFFUSION

In the presence of an RF spectrum acting unidirectionally but in the direction perpendicular to the magnetic field (e.g., as in electron cyclotron current drive) one has from Eq. (36),

$$F_{0\perp} = \frac{P_0}{\beta_\perp} \tag{44}$$

In the steady state (32) and (33), utilizing Eq. (44) yield,

$$\beta_\perp = \left[\frac{\gamma^2 - 1}{\gamma(Z_i + \gamma + 1)} \right]^{1/2} \tag{45}$$

as well as Eq. (38).

The finite value of $\beta_\perp$ will induce a finite $\beta_\parallel$ or, in other words, the imposed electron cyclotron heating will drive a current in the parallel direction. Equation (45) and the identity $\beta^2 = \beta_\parallel^2 + \beta_\perp^2 = 1 - 1/\gamma^2$ readily yield,

$$\beta_\parallel = \left(\frac{Z_i + 1}{\gamma} \right)^{1/2} \beta_\perp \tag{46}$$

Comparing this result with the one of Eq. (37) yields,

$$(\beta_\parallel)_\perp = \left(\frac{Z_i + 1}{\gamma} \right)^{1/2} (\beta_\parallel)_\parallel \tag{47}$$

and similary for the normalized figure of merit,

$$\left(\frac{\beta_\parallel}{P_0}\right)_\perp = \left(\frac{Z_i + 1}{\gamma}\right)^{1/2} \left(\frac{\beta_\parallel}{P_0}\right)_\parallel \qquad (48)$$

where the external subscripts $\perp$, $\parallel$ refer to the perpendicular and parallel RF diffusion respectively.

VI. SUMMARY AND DISCUSSION

Based upon the 2D relativistic Fokker-Planck equation with parallel quasilinear diffusion due to RF fields we have formulated a set of moment equations that give a global description of steady-state current drive. The steady-state regime we describe is the one in which current generation is achieved with strong quasilinear diffusion on a time scale which is short compared to the heating of the bulk distribution function. Thus the temperature evolution of the bulk distribution function was ignored and the relativistic Fokker-Planck equation was simplified to describe electron collisions for $p > p_{th}$, including in particular the high energy tail electrons interacting with the applied RF fields. The set of moment equations that we use for describing the steady state of current drive include the global effects of the RF fields. Characterizing the distribution function of the current carrying electrons in steady state by an effective parallel momentum and an effective perpendicular momentum and imposing the relationshiop between force density and power density dissipated, the moment equations in steady state give the figure of merit as a function of the effective energy of these electrons as well as a constraining relation between the parallel and perpendicular effective momenta. This constraining relation shows that current drive produced with electrons effectiely at high parallel momenta (i.e. with RF spectra at high phase velocities) must also have high perpendicular momenta. This is consistent with observations in recent, intense current drive experiments with lower-hybrid waves. A comparison of our analytical results with results from a numerical integration of the relativistic 2D equations, for parameters of Alcator C current drive experiments, has been reported elsewhere.[13] The enhanced perpendicular temperatures are very well predicted by our moment equations constraint.

Previous work[14] on relativistic current drive has only focused towards the figure of merit and was based on an ad-hoc physical model suitable only for the weak RF regime.

In Ref. 14 the calculation of RF current generation is based upon a physical model in which the "RF kicks" of electrons in momentum space are accumulated in a time-asymptotic fashion. This model is combined with the energy and momentum moments of only the collisional part of the relativistic Fokker-Planck equation. Therefore, the slowing down equation for energy and momentum are the same as our Eqs. (32) and (33) with the RF terms $(P_0, \vec{F}_0, \vec{A}_0)$ set to zero. In the nonrelativistic limit the results of that model have been confirmed from an approximate solution of the Fokker-Planck equation only in the weak RF regime.[15]. Indeed, in that regime it may be justifiable to ignore the effect of the RF diffusion on the slowing down equation. However, for moderate or strong RF diffusion the slowing down equation for energy and momentum must account for the presence of the RF fields, as is done in the present paper. Furthermore the present analysis is not based on any ad-hoc physical model (as the time-asymptotic accumulation of "RF kicks of Ref. 14) but instead utilizes the energy and momentum slowing down equations including the effects of the RF.

In Fig. 4 the normalized figure of merit is plotted as a function of the normalized momentum of the current carrying electrons for $Z_i = 1$ and $q \simeq q_{\parallel}$; the curves shown are based upon Eqs. (40) and (48) [note that for $q \simeq q_{\parallel}$ one has $\epsilon \simeq (1 + q_{\parallel}^2)^{1/2} - 1$] and are to be compared with Fig. 1 in Ref. 14. For example, at $q_{\parallel} = 5$ the analysis of the present paper gives for the figure of merit a value of about .85 and .45 for parallel and perpendicular diffusion, respectively, as opposed to .75 and .25 in Ref. 14. [Note that ν in Ref. 14 is twice the one we use, Eq. (26).] Therefore the prediction of the present analyses are more optimistic for both the parallel and perpendicular diffusion, especially for the perpendicular diffusion. For values of $q_{\parallel} \lesssim 2$ the perpendicular diffusion is slightly more efficient in driving current parallel to the magnetic field than the parallel one. This is also shown in Fig. 2. In Ref. 14 the perpendicular diffusion is less efficient than the parallel one for all values at $q_{\parallel}$. However the $q_{\parallel} \to \infty$ limit is the same for both analyses, namely 1 and 0 for parallel and perpendicular diffusion respectively. The asymptotic behavior of the figure of merit for large ϵ (and so $q_{\parallel}$) is also shown in Fig. 3 for various values of Z_i.

APPENDIX A–THE LANDAU FORM OF THE COLLISIONAL FLUX

The relativistic generalization of the Balescu-Lénard collisional flux of the test species α interacting with a field species labeled by β is:[6-7]:

$$\vec{S}_{\alpha\beta} = -2q_\alpha^2 q_\beta^2 n_\beta \int d^3 p_\beta \int d^3 k \delta(\vec{k} \cdot \vec{v}_\alpha - \vec{k} \cdot \vec{v}_\beta) \frac{\vec{k}\vec{k}}{(\vec{k} \cdot \vec{v}_\alpha)^4} |\vec{v}_\beta \cdot \overline{\overline{Z}}_{\vec{k}, \vec{k} \cdot \vec{v}_\alpha} \cdot \vec{v}_\alpha|^2$$
$$\cdot \left(\frac{\partial}{\partial \vec{p}_\alpha} - \frac{\partial}{\partial \vec{p}_\beta} \right) f_\alpha(\vec{p}_\alpha) f_\beta(\vec{p}_\beta) \tag{A1}$$

where q, n, $\vec{p}$, $\vec{v}$ and $f(\vec{p})$ refer to the charge, density, momentum, velocity and momentum space distribution function of the particles respectively (α or β-species). The tensor $\overline{\overline{Z}}_{\vec{k}, \vec{k} \cdot \vec{v}_\alpha} \equiv \overline{\overline{Z}}(\vec{k}, \omega = \vec{k} \cdot \vec{v}_\alpha)$ is the dispersion tensor defined as

$$\overline{\overline{Z}} = \frac{\overline{\overline{I}}_L}{\epsilon_L} + \frac{\overline{\overline{I}}_T}{\epsilon_T - \frac{\omega^2}{k^2 c^2}} \tag{A2}$$

where the scalars ϵ_L and ϵ_T are the longitudinal and transpose dielectric functions (parts of the dielectric tensor $\overline{\overline{\epsilon}}$), respectively

$$\epsilon_L = \frac{\vec{k} \cdot \overline{\overline{\epsilon}} \cdot \vec{k}}{k^2} \;,\; \epsilon_T \overline{\overline{I}}_T = \overline{\overline{\epsilon}} - \epsilon_L \overline{\overline{I}}_L \tag{A3}$$

with the projectors $\overline{\overline{I}}_L$ and $\overline{\overline{I}}_T$ defined in terms of the unit dyadic $\overline{\overline{I}}$ and the wave vectors $\vec{k}$ as

$$\overline{\overline{I}}_L = \frac{\vec{k}\vec{k}}{k^2} \;,\; \overline{\overline{I}}_T = \overline{\overline{I}} - \frac{\vec{k}\vec{k}}{k^2} \tag{A4}$$

When the shielding is absent $\epsilon_L = \epsilon_T = 1$ and the colllisional flux in Eq. (1) is reduced to the one for the Lorentz gas

$$\vec{S}_{\alpha\beta} = -2q_\alpha^2 q_\beta^2 n_\beta \int d^3 p_\beta \int d^3 k \delta(\vec{k} \cdot \vec{v}_\alpha - \vec{k} \cdot \vec{v}_\alpha) \frac{\vec{k}\vec{k}}{k^4} \frac{(1 - \vec{\beta}_\alpha \cdot \vec{\beta}_\beta)^2}{\left[1 - \left(\frac{\vec{k} \cdot \vec{\beta}_\alpha}{k} \right)^2 \right]^2}$$
$$\cdot \left(\frac{\partial}{\partial \vec{p}_\alpha} - \frac{\partial}{\partial \vec{p}_\beta} \right) f_\alpha(\vec{p}_\alpha) f_\beta(\vec{p}_\beta) \tag{A5}$$

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APPENDIX B–THE FOKKER-PLANCK EQUATION IN CYLINDRICAL COORDINATES

In the presence of a unidirectional RF wave and especially for a LH-wave acting parallelly to the magnetic field (unit vectors parallel and perpendicular to the magnetic field denoted by $\vec{e}_\parallel$ and $\vec{e}_\perp$ respectively), the diffusion of the fast electrons (species α) is characterized by a flux $\vec{S}_{\alpha,T}$ given by,

$$\vec{S}_{\alpha,T} = (\sum_\beta \overline{\overline{D}}_{\alpha\beta} + D_{QL}\vec{e}_\parallel\vec{e}_\parallel) \cdot \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} + \sum_\beta \vec{F}_{\alpha\beta}f_\alpha \qquad (B1)$$

where D_{QL} is the quasilinear diffusion coefficient, and $\overline{\overline{D}}_{\alpha\beta}$, $\vec{F}_{\alpha\beta}$ are given by Eqs. (10)-(11). One has in cylindrical coordinates,

$$S_{\alpha,T_\parallel} = -D_\parallel \frac{\partial f_\alpha}{\partial p_{\alpha\parallel}} - D_X \frac{\partial f_\alpha}{\partial p_{\alpha\perp}} - F_\parallel f_\alpha \qquad (B2)$$

and

$$S_{\alpha,T_\perp} = -D_X \frac{\partial f_\alpha}{\partial p_{\alpha\parallel}} - D_\perp \frac{\partial f_\alpha}{\partial p_{\alpha\perp}} - F_\perp f_\alpha \qquad (B3)$$

where the diffusion coefficients $D_\parallel$, D_X, $D_\perp$ and the friction coefficients $F_\parallel$ and $F_\perp$ are,

$$D_\parallel = \sum_\beta \vec{e}_\parallel \cdot \overline{\overline{D}}_{\alpha\beta} \cdot \vec{e}_\parallel + D_{QL} \qquad (B4)$$

$$D_X = \sum_\beta \vec{e}_\parallel \cdot \overline{\overline{D}}_{\alpha\beta} \cdot \vec{e}_\perp = \sum_\beta \vec{e}_\perp \cdot \overline{\overline{D}}_{\alpha\beta} \cdot \vec{e}_\parallel \qquad (B5)$$

$$D_\perp = \sum_\beta \vec{e}_\perp \cdot \overline{\overline{D}}_{\alpha\beta} \cdot \vec{e}_\perp \qquad (B6)$$

$$F_\parallel = \sum_\beta \vec{e}_\parallel \cdot \vec{F}_{\alpha\beta} \, , \, F_\perp = \sum_\beta \vec{e}_\perp \cdot \vec{F}_{\alpha\beta} \qquad (B7)$$

Applying now Eqs. (10-11) and taking into account that the field species are the thermal electrons and ions yields,

$$D_\parallel/2A = \frac{\gamma}{p^3}\left[\left(\varsigma - \gamma^2 \frac{<m_e^2 v_e^2>}{2p^2}\right)p_\perp^2 + \gamma^2 \frac{<m_e^2 v_e^2>}{2p^2}p_\parallel^2\right] + D_{QL} \qquad (B8)$$

$$D_X/2A = -\frac{\gamma}{p^3}\left(\varsigma - \frac{3}{2}\gamma^2\frac{<m_e^2 v_e^2>}{p^2}\right)p_\parallel p_\perp \qquad (B9)$$

$$D_\perp/2A = \frac{\gamma}{p^3}\left[\left(\varsigma - \gamma^2\frac{<m_e^2 v_e^2>}{2p^2}\right)p_\parallel^2 + \gamma^2\frac{<m_e^2 v_e^2>}{2p^2}p_\perp^2\right] \qquad (B10)$$

$$F_\parallel/2A = \frac{\gamma^2}{p^3}p_\parallel \,, F_\perp/2A = \frac{\gamma^2}{p^3}p_\perp \qquad (B11)$$

with,

$$2A = 4\pi e^4 n_e m_e \ln \Gamma_{ee'} \qquad (B12)$$

and

$$\varsigma = (Z_i + 1)/2 \qquad (B13)$$

Subsequently, normalizing the time $t \to t\nu_c$, the momenta $p \to p/m_e\sqrt{<v_e^2>}$ ($<v_e^2> \equiv v_{th}^2$), all the diffusion coefficients $D \to D/\nu_c m_e^2 <v_e^2>$ and the friction coefficients $F \to F/\nu_c m_e\sqrt{<v_e^2>}$ yields,

$$D_\parallel = \frac{\gamma}{p^3}\left[\left(\varsigma - \frac{\gamma^2}{2p^2}\right)p_\perp^2 + \frac{\gamma^2}{p^2}p_\parallel^2\right] + D_{QL} \qquad (B14)$$

$$D_X = -\frac{\gamma}{p^3}\left(\varsigma - \frac{3\gamma^2}{2p^2}\right)p_\parallel p_\perp \qquad (B15)$$

$$D_\perp = \frac{\gamma}{p^3}\left[\left(\varsigma - \frac{\gamma^2}{2p^2}\right)p_\parallel^2 + \frac{\gamma^2}{p^2}p_\perp^2\right] \qquad (B16)$$

$$F_\parallel = \frac{\gamma^2}{p^3}p_\parallel \,, F_\perp = \frac{\gamma^2}{p^3}p_\perp \qquad (B17)$$

where all the quantities involved are the normalized ones. The normalized fluxes are given by the same, formally, Eqs. (B2-B3) and the normalized continuity equation for the fast electron distribution f is

$$\frac{\partial f}{\partial t} = -\frac{\partial S_\parallel}{\partial p_\parallel} - \frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}(p_\perp S_\perp) \qquad (B18)$$

Transforming Eq. (B18) partially into (μ, p) coordinates with $\mu = \tan^{-1} \frac{p_\perp}{p_\parallel}$ and $p = (p_\parallel^2 + p_\perp^2)^{1/2}$ yields,

$$\frac{\partial f}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left(\frac{\gamma^3}{p} \frac{\partial f}{\partial p} + \gamma^2 f \right) + \frac{\gamma}{p^3} \left(\varsigma - \frac{\gamma^2}{2p^2} \right) \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial f}{\partial \mu} \right] + \frac{\partial}{\partial p_\parallel} \left(D_{QL} \frac{\partial f}{\partial p_\parallel} \right) \quad (B19)$$

It is easy to show that, with $D_{QL} = 0$, a distribution of the form $\exp\left(-\frac{p^2}{\gamma+1} \right)$ is a steady state solution of Eq. (B19) as expected; in the nonrelativistic limit this solution goes to the Maxwellian distribution $\exp(-\frac{p^2}{2})$.

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FIGURE CAPTIONS

Figure 1. Effective perpendicular temperature, "$T_\perp$", (normalized to the bulk temperature T_B), defined from $(v_{\perp 0}^2/2v_{th}^2) \equiv ("T_\perp"/T_B)$ with $m_e v_{th}^2 \equiv T_B$, is plotted as a function of the effective parallel momentum $p_{\parallel 0}$ (normalized to the bulk thermal momentum $m_e v_{th}$). Here $Z_i = 1$ is assumed.

Figure 2. The thermally normalized (to $nev_{th}/nmv_{th}^2\nu$) figures of merit $(J/p_d)_B$ for parallel ($\parallel$) and perpendicular ($\perp$) RF diffusion are plotted as functions of the energy in keV carried by the energetic electrons. Here $Z_i = 1$ is assumed. The dashed curves correspond to the nonrelativistic limit.

Figure 3. The relativisticly normalized figure of merit $\beta_\parallel/P_0$ for parallel RF diffusion is plotted as a function of the normalized kinetic energy $\epsilon = \gamma - 1$ carried by the energetic electrons. Here $Z_i = 1, 2, 4$ for the a, b, c-labeled curves, respectively.

Figure 4. The relativistically normalized (to $nec/nmc^2\nu$) figures of merit $(J/p_d)_R$ for parallel ($\parallel$) and perpendicular ($\perp$) RF diffusion are plotted as functions of the average normalized momentum of energetic electrons $q(= p/mc)$. Here $q \simeq q_\parallel$ and $Z_i = 1$ are assumed. The dashed curves correspond to the nonrelativistic limit.
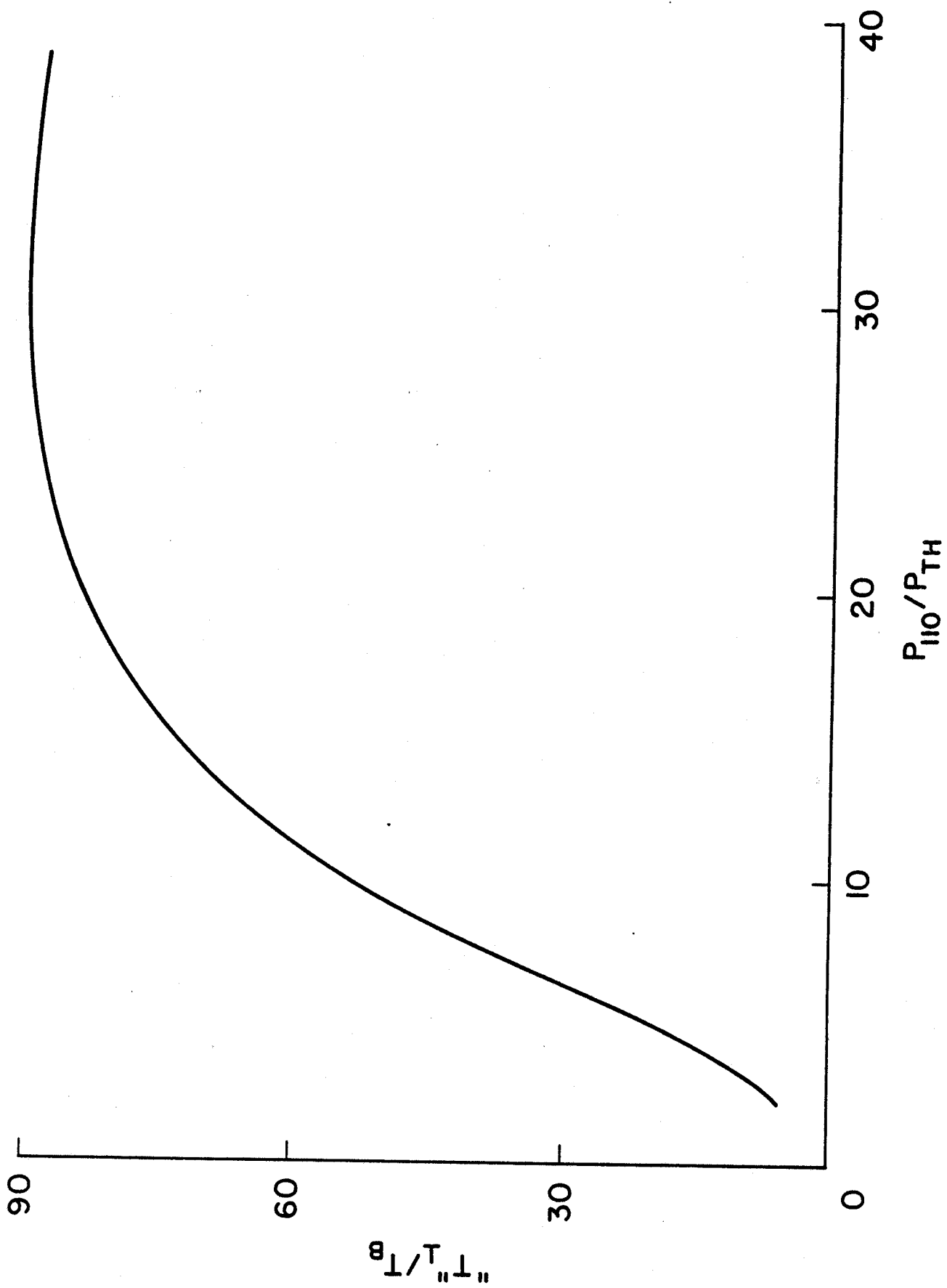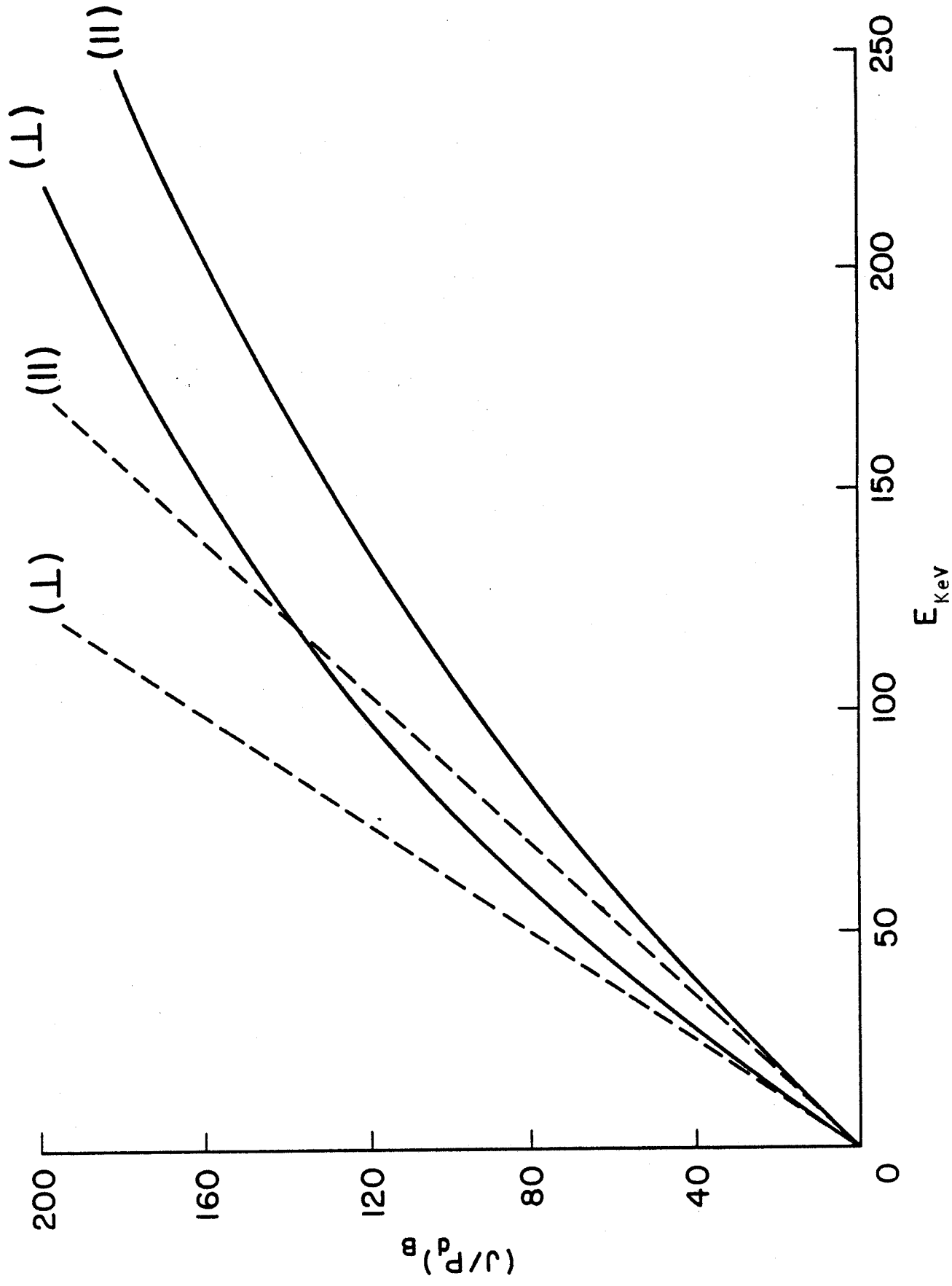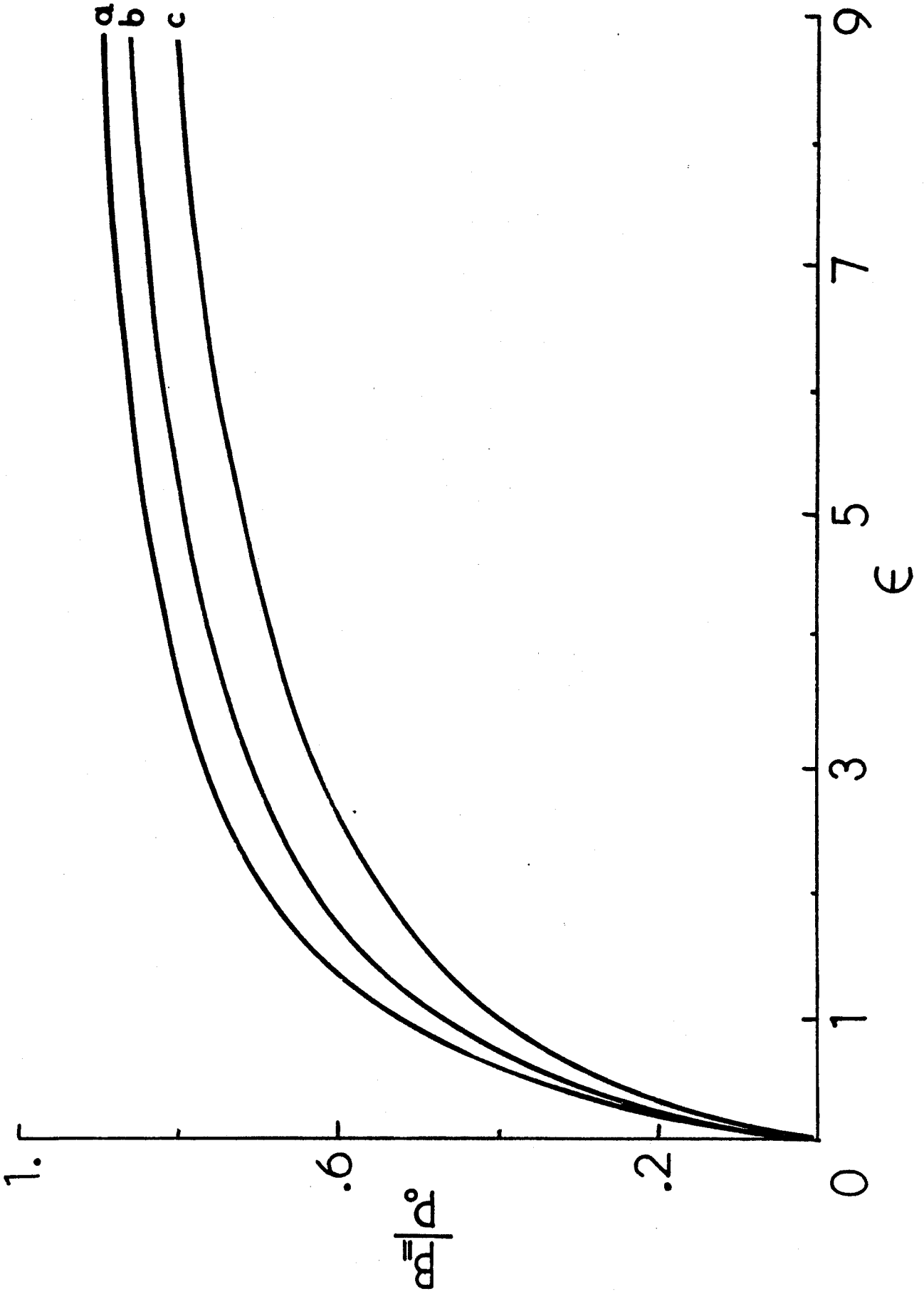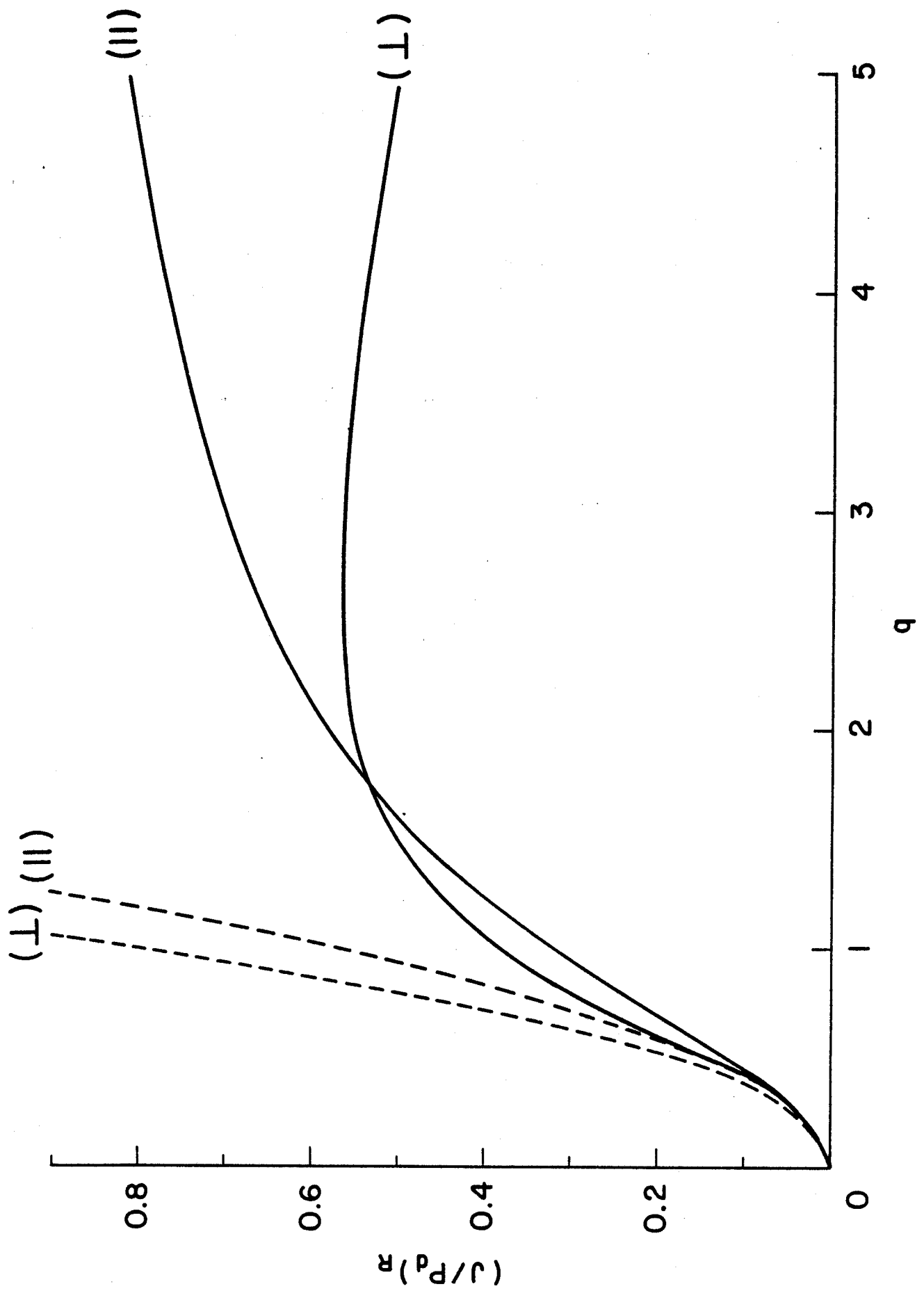
F I G. 1

F I G. 2

F I G. 3

FIG. 4