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STABILIZATION OF THE TEARING MODE IN LOW DENSITY TOKAMAK PLASMAS BY TURBULENT ELECTRON DIFFUSION*

E. ESAREY, J.P. FREIDBERG, K. MOLVIG
Plasma Fusion Center
and
Dept. of Nuclear Engineering
Massachusetts Institute of Technology

C.O. BEASLEY, JR. and W.I. VAN RIJ
Oak Ridge National Laboratory
PFC/JA-82-30

ABSTRACT

The effect of turbulent electron diffusion from stochastic electron orbits on the stability of low beta fluctuations is considered. A set of coupled self-adjoint equations is derived for the fluctuation potentials $\tilde{\phi}$ and $\tilde{A}_{\|}$. For the tearing mode, it is shown that stability is obtained for sufficiently large values of the diffusion coefficient. Provided $D_\epsilon \sim 1/n$, this implies that a density threshold must be surpassed before the tearing mode is observed. Numerical calculations also support this conclusion.

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One of the main concerns of tokamak research is the prevention of major plasma disruptions. It is generally considered that such major disruptions must be eliminated in an actual fusion reactor to prevent prohibitive damage to the first wall. The prevailing theoretical picture of major disruptions features low poloidal mode number (low m) tearing modes which saturate to produce magnetic islands. It is possible for such magnetic islands overlap to form large stochastic magnetic regions which enhance particle diffusion, and in the case of major disruptions, lead to catastrophic plasma confinement loss [1,3,4]. Control of such disruptions requires elimination or suppression of these tearing mode magnetic islands.

Traditionally, the tearing mode is analyzed using resistive MHD (magnetohydrodynamic) theory which predicts instability for $\Delta' > 0$, a condition which is generally satisfied by experimental profiles when $m = 2$ [5]. Here Δ' is the jump in the logarithmic radial derivative of the perturbed magnetic potential, $\tilde{A}_{||}$, across the rational surface. Recent experimental results from ALCATOR C, however, show that a threshold in plasma density must be surpassed before the $m = 2$ tearing mode is observed, even though $\Delta' > 0$ [5]. This observation, which is in qualitative disagreement with resistive MHD theory, motivated the present work.

In this paper, a fully kinetic approach to the tearing mode is used which includes the effects of turbulent electron diffusion resulting from stochastic electron orbits [6]. A system of coupled self-adjoint equations is derived for the perturbed potentials $\tilde{A}_{||}$ and $\tilde{\phi}$. This system follows from Ampere's Law and quasi-neutrality applied to the linear ion response and the nonlinear electron response resulting from the normal stochastic approximation (NSA) [6]. The NSA includes the effects of electron diffusion in the electron response and is valid in regions where the electrons experience stochastic orbits. In this limit the NSA is essentially equivalent to the direct interaction approximation (DIA) [7]. The resulting system of coupled equations is globally valid and includes the effects of collisions, equilibrium current, diffusion and shear. In the appropriate limit this system yields both the finite β drift wave [8] and the tearing mode. Since the system is self-adjoint, a variational principle can be formed. In this problem the tearing mode exists in a background of microturbulence such as that due to drift waves. This same set of coupled equations yields unstable finite β drift waves when analyzed for high m modes, from which a turbulent diffusion coefficient is calculated [8]. This microturbulence is the source of the electron diffusion, and hence the diffusion coefficient is a known quantity independent of the tearing mode. This system of coupled equations for $\tilde{A}_{||}$ and $\tilde{\phi}$ reduce to ideal MHD at large x ($\tilde{E}_{||} < 0$ and marginal stability for $\tilde{A}_{||}$). For the tearing mode, the equations decouple to leading order in the small parameter ω/ω_c, $\omega_c^3 = (k_{||}' v_e)^2 D_e/3$, (typically, $\omega/\omega_c \approx \omega_{*e}/\omega_c \approx 10^{-1}$ for the $m = 2$ tearing mode), yielding independent equations for $\tilde{\phi}$ and $\tilde{A}_{||}$. This allows the tearing mode to be analyzed entirely in terms of the magnetic potential $\tilde{A}_{||}$.

Two analytical results are derived. First, it is proved from the full integral equations that a system unstable in resistive MHD, $\Delta'(0) > 0$, becomes stable at sufficiently large D_e. Second, an approximate expression for the dispersion relation is derived. Stabilization at sufficiently high values of D_e is shown by identifying the appropriate terms in the variational integral as the energy drive for the tearing mode. In the ideal MHD limit, this drive yields the quantity $\Delta'(0)$. However, using the full kinetic operators, the energy drive goes to zero for large values of the diffusion coefficient. The dispersion relation indicates that the tearing mode has a real frequency equal to the electron diamagnetic frequency, ω_{*e}, and a linear growth rate proportional to $\Delta'(x_c)$, where $x_c = \omega_c/k'_{\|}v_e$. This result is similar to the classic collisionless tearing mode as calculated by Laval et al., [9], where they find $\gamma \sim \Delta'(0)$. Numerical calculations indicate $\Delta'(x)$ to be a decreasing function of x; hence, stability is given by $x_c > x_0$, where $\Delta'(x_0) = 0$. This condition for stability, $\Delta'(x_c) = 0$, indicates that the magnetic energy involved in $\tilde{A}_{\|}$ and $\tilde{J}_{\|e}$ outside the tearing region, $|x| > x_c$, is zero and thus there is no available free energy to drive the tearing mode. This is analogous to nonlinear estimates of the magnetic island saturation width, w, given by the value where $\Delta'(w) = 0$[2,3]. Physically, electron diffusion prohibits the tearing region from becoming too small (limited to a width x_c), whereas in resistive MHD the layer thickness is limited only by dissipation, which alters the growth rate without affecting stability.

In this problem, slab geometry with a sheared magnetic field is used; $\boldsymbol{B} = B_0(\hat{e}_z + x/L_s\hat{e}_y)$ where $L_s = Rq^2/rq'$. In the final dispersion relation, the effects of the region outside the tearing layer, $|x| > x_c$, only appear through the term $\Delta'(x_c)$. Hence, generalization to cylindrical geometry involves primarily the appropriate numerical determination of the function $\Delta'(x)$.

The nonlinear electron response is calculated by applying the NSA to the drift kinetic equation and extracting the adiabatic piece of the response in the renormalization [6]. This process yields the perturbed electron response, $\tilde{f}_e = (e\tilde{\phi}/T_e)f_0 + \tilde{h}_e$, where

$$\left(\frac{\partial}{\partial t} + v_{\|}\boldsymbol{b}\cdot\nabla - D\frac{\partial^2}{\partial x^2}\right)\tilde{h}_e = S(x) \equiv \frac{c}{B}\nabla(\tilde{\phi} - \frac{v_{\|}}{c}\tilde{A}_{\|}) \times \boldsymbol{b}\cdot\nabla(f_0 + f_1) - \frac{ef_0}{T_e}\frac{\partial}{\partial t}(\tilde{\phi} - \frac{v_{\|}}{c}\tilde{A}_{\|}). \quad (1)$$

Here f_1 represents the current-carrying piece of the equilibrium distribution function.

Using the classical linear ion response and applying quasi-neutrality and Ampere's Law yield the coupled system

$$\left[\frac{d^2}{dx^2} - \left(\Lambda + \chi^2 + \frac{i}{d}(\omega - \omega_{*e})R_0 + \frac{i}{d}\omega_{*e}\eta_J R_1\right)\right]\tilde{\phi}$$
$$+ \left[\frac{i}{d}(\omega - \omega_{*e})R_1 - \frac{i}{d}\omega_{*e}\eta_J R_2 + x_e\chi^2\right]\frac{v_e}{c}\tilde{A}_{\|} = 0 \quad (2)$$

$$-\frac{\tau v_A^2}{dc^2}\left[\frac{d^2}{dx^2} - b + x_e^2 a^2 + \frac{v_e^2}{\tau v_A^2} i(\omega - \omega_{*e})R_2 - \frac{v_e^2}{\tau v_A^2} i\omega_{*e}\eta_J R_3\right]\tilde{A}_\parallel$$

$$+\left[\frac{i}{d}(\omega - \omega_{*e})R_1 - \frac{i}{d}\omega_{*e}\eta_J R_2 + x_e\chi^2\right]\frac{v_e}{c}\tilde{\phi} = 0 \qquad (3)$$

where

$$\Lambda = \frac{1}{d}\left(1 + \tau - \Gamma_0\left(\tau + \frac{\omega_{*e}}{\omega}\right)\right); \quad d = (\Gamma_0 - \Gamma_1)\left(\tau + \frac{\omega_{*e}}{\omega}\right)$$

$$\chi^2 = \frac{\Gamma_0}{d}\left(\tau + \frac{\omega_{*e}}{\omega}\right)\left(1 + \frac{\omega}{k_\parallel v_i} Z\left(\frac{\omega}{k_\parallel v_i}\right)\right), \qquad a^2 = \frac{v_e^2}{\tau v_A^2}\Gamma_0\left(1 + \frac{\omega}{k_\parallel v_i} Z\left(\frac{\omega}{k_\parallel v_i}\right)\right)$$

where $\eta_J = -(2L_n\partial J_\parallel/\partial x)/env_e, b = \rho_i^2 k_y^2, \tau = T_e/T_i, x_e = \omega/k_\parallel v_i$ and where $Z(x) =$ Plasma Dispersion Function and the resonant operators, $R_\parallel$, are given by

$$R_n[\psi] = \int_{-\infty}^{\infty} dv_\parallel \int_{-\infty}^{\infty} dx' \int_0^{\infty} d\tau G(x, x'; v_\parallel, \tau)\left(\frac{v_\parallel}{v_e}\right)^n f_{0\parallel}(v_\parallel)\psi(x')$$

with the kernel,

$$G(x, x'; v_\parallel, \tau) = \frac{1}{\sqrt{4\pi D\tau}}\exp[i(\omega - k'_\parallel v_\parallel x)\tau - \frac{1}{3}(k'_\parallel v_\parallel)^2 D\tau^3 - (x - x' - iDk'_\parallel v_\parallel \tau^2)^2/4D\tau].$$

It can be shown that the above system is self-adjoint. Denoting equations (2) and (3) by $L_1\tilde{\phi} + L_x\tilde{A}_\parallel = 0$ and $L_2\tilde{A}_\parallel + L_x\tilde{\phi} = 0$, where L_x represents the coupling operator, a quadratic form, S, can be constructed

$$S = \int_{-a}^{a} dx[\tilde{\phi}L_1\tilde{\phi} + \tilde{A}_\parallel L_2\tilde{A}_\parallel + 2\tilde{\phi}L_x\tilde{A}_\parallel] \qquad (4)$$

which upon variation, $\delta S = 0$, yields equations (2) and (3), assuming the boundary conditions $\tilde{\phi}, \tilde{A}_\parallel = 0$ at the edge of the plasma, $\pm a$.

In the case of the tearing mode, the above operators scale with respect to $\epsilon \equiv \omega_{*e}/\omega_c \approx 10^{-1}$ as $L_1 \sim 1$, $L_2 \sim \epsilon, L_x \sim \epsilon$. Hence, the contributions of the coupling terms to the dispersion relation are subdominant by order ϵ. Thus, in effect, equations (2) and (3) decouple to leading order, leaving $L_1\tilde{\phi} = 0, L_2\tilde{A}_\parallel = 0$. It is possible to show in more detail that for $|x| \gg x_c$ and $|x| \ll x_c$, this decoupling occurs to higher order in ϵ. Thus, defining the tearing mode as primarily a magnetic fluctuation reduces the problem to that of solving $L_2\tilde{A}_\parallel = 0$. Also, at large x, $|x| > x_c$, equations (2) and (3) reduce to the ideal MHD equations

$$\tilde{E}_{\|} = 0, \qquad \left[\frac{d^2}{dx^2} - b - \frac{4\pi}{c}\rho_i^2 \frac{k_y J'_{\|}}{k'_{\|} x B_0}\right]\tilde{A}_{\|} = 0. \tag{5}$$

Likewise, the corresponding terms in the quadratic form, S, [eq. (4)], reduce to the ideal MHD energy principle. As a side note, the standard dispersion relation for the resistive tearing mode is recovered by replacing the spatial diffusion operator, $-D\frac{\partial^2}{\partial x^2}$, with a krook collision operator, ν_{ei}, in equation (1). Evaluating the variational form, S, in this limit yields the well known resistive MHD dispersion relation [10].

A proof is now given stating that the tearing mode is stabilized for sufficiently large values of the diffusion coefficient. The energy drive for the tearing mode is identified as the negative energy resulting from the following terms in the variational form.

$$S' = \frac{\tau v_A^2}{dc^2}\int_{-a}^{a} dx\left[\left(\frac{d\tilde{A}_{\|}}{dx}\right)^2 + b\tilde{A}_{\|}^2 + \tilde{A}_{\|}\frac{4\pi}{c}\tilde{J}_{\|e}[\tilde{A}_{\|}]\right], \tag{6}$$

$$\frac{4\pi}{c}\tilde{J}_{\|e}[\tilde{A}_{\|}] = \frac{v_e^2}{\tau v_A^2} i\eta_J \omega_{*e} R_3[\tilde{A}_{\|}]$$

where it is assumed that $\tilde{A}_{\|}[\pm a] = 0$.

It is helpful, as a source of motivation, to note that the above expression for the negative energy drive, S', reduces to a term proportional to $\Delta'(0)$ in the ideal limit. In the limit where $x > x_c, x_e$, outside the tearing layer, the perturbed current operator , $\tilde{J}_{\|_e}$, reduces to the third term in the ideal MHD equation for $\tilde{A}_{\|}$, [equation (5)]. Notice that in this limit the operator $\tilde{J}_{\|e}$ is singular at the rational surface, $x = 0$. The integration in equation (6) is then redefined up to $\pm\epsilon_0$ about $x = 0$, and the limit as $x \to 0$ is taken. Doing this, and requiring $\tilde{A}_{\|}$ to satisfy equation (5), yields the following expression for the negative energy drive, S'

$$S' = \frac{-\tau v_A^2}{dc^2} A_{\|0}^2 \Delta'_0; \quad \text{where} \quad \Delta'_0 = \lim_{\epsilon\to 0}\left[\frac{A'_{\|}}{A_{\|}}\right]_{-\epsilon_0}^{\epsilon_0}. \tag{7}$$

Hence, Δ'_0 represents the available negative energy necessary to drive the tearing mode unstable. A detailed study of the magnetic driving energy is given by E. A. Adler et al. [11]

Notice in equation (6) that negative energy can only result from the term involving the perturbed current operator $\tilde{J}_{\|e}[\tilde{A}_{\|}]$(the other two terms are stabilizing). In the ideal MHD limit, this operator is unbounded as $x \to 0$, whereas the full kinetic operator is finite at the rational surface. Retaining this full kinetic operator, a bounded integral, S_0, is defined over the perturbed current term in equation (6), $A_{\|}\tilde{J}_{\|_e}[A_{\|}]$, which is a well behaved function of D_e. In fact, it can be shown analytically that $S_0 \to 0$ as $D_e \to \infty$,which then justifies the conclusion that the tearing mode is stabilized for sufficiently large values of the diffusion coefficient.

To determine a dispersion relation for the kinetic tearing mode, the equation $L_2\tilde{A}_{\parallel} = 0$ is solved for the fluctuation vector potential $\tilde{A}_{\parallel}$. Using the assumption $|x_c/x_T| < 1$, $x_T^{-1} \equiv [d\ln\tilde{A}_{\parallel}/dx]$, the resonant operators, R_n, can be approximated numerically to yield multiplicative operators [12]. The equation $L_2\tilde{A}_{\parallel} = 0$ reduces to the approximate leading order relations

$$\left[\frac{d^2}{dx^2} - b - \frac{4\pi}{c}\rho_i^2\frac{k_y J'_{\parallel}}{k_{\parallel}B_0}\right]\tilde{A}_{\parallel} = 0, \quad |x| > x_c \tag{8}$$

$$\left[\frac{d^2}{dx^2} + \frac{v_e^2}{\tau v_A^2}i(\omega - \omega_{*e})\frac{.47}{\omega_c}\right]\tilde{A}_{\parallel} = 0, \quad |x| < x_c \tag{9}$$

Equation (8) for the region $|x| > x_c$ is again the marginal stability equation for $\tilde{A}_{\parallel}$ from ideal MHD. The solution to this equation is denoted as the ideal MHD solution (A_+ for $x > x_c$ and A_- for $x < -x_c$).

Solving equation (9) for the region $|x| < x_c$ and requiring $\tilde{A}_{\parallel}$ and $\tilde{A}'_{\parallel}$ to be continuous at $x = \pm x_c$ yields the leading order dispersion relation

$$\Delta'(x_c) \simeq -i\frac{v_e^2}{\tau v_A^2}\frac{(\omega - \omega_{*e})}{k'_{\parallel}v_e\rho_i} \tag{10}$$

where $\Delta'(x_c) = \Delta'_+ - \Delta'_-$, $\Delta'_+ = (A'_+/A_+)\big|_{x_c}$ and $\Delta'_- = (A'_-/A_-)\big|_{-x_c}$.

Numerical calculations of $\Delta'(x)$ for ALCATOR C profiles indicate that $\Delta'(x)$ is a monotonically decreasing function of x, with $\Delta'(0) > 0$. Hence, stability is obtained for $x_c > x_0$, where $\Delta'(x_0) = 0$. This stability criterion can be written as $D_e > 3k'_{\parallel}v_e x_0^3$; $\quad k'_{\parallel} = mq'/Rq^2$, where the functional dependence of x_0 on various quantities must be determined numerically. This equation indicates that increased turbulent electron diffusion stabilizes the tearing mode. Consequently, if $D_e \approx 1/n$, then there exists some critical density below which the tearing mode is stabilized.

A related analysis reported by Meiss et al. [13], arrived at a much different conclusion, namely that diffusion had virtually no effect on the tearing mode. The results of the present work differ only by the inclusion of the additional physical effect of turbulent smearing of the perturbed current which thus reduces the available energy, $\Delta'(x_c)$. The analysis of reference [13], by asymptotically matching an inner solution to an ideal MHD solution at large x, intrinsically contained the full MHD energy, $\Delta'(0)$, and could not consider this effect. In all other respects, although different in approach, the present results agree with those of reference [13].

In conclusion, the tearing mode is stabilized for sufficiently large values of turbulent electron diffusion. Provided $D_e \sim 1/n$ implies that a density threshold must be surpassed before $m = 2$ tearing modes are observed. Physically, turbulent electron diffusion prevents a perturbed current from forming within a correlation distance, x_c, of the rational surface. Hence, turbulent diffusion cuts into the available magnetic driving energy,

Δ'. Theoretically predicted values of the critical density are in approximate agreement with experimental values; however, the experimental scaling of $n_c \sim B^2$ [5] has not been explicitly derived unless $D_e \sim B^2$. Numerical solutions of the basic equations (2) and (3) also demonstrate stabilization for sufficiently large D_e and support the analytical expression for stabilization given by $x_c \sim x_0$, $\Delta'(x_0) = 0$.

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