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TANDEM MIRROR
HOT ELECTRON ANCHOR

J. Kesner, R.S. Post, and D.K. Smith
Massachusetts Institute of Technology
Plasma Fusion Center
Cambridge, MA 02139

D.E. Baldwin
Lawrence Livermore National Laboratory
Livermore, CA

Y.C. Lee
University of Maryland
College Park, MD

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Abstract

An ECRH generated hot-electron plasma is shown to provide an attractive alternative to a neutral-beam formed plasma for the MHD "anchor" of a tandem mirror. This mirror cell would provide a Finite-Larmor - Radius-like stabilization for $m = 1$ modes. For a significant MHD fluid-like response, a small ion core beta is seen to be required and stabilization results from the response of the core beta in the hot-electron depressed magnetic field. At a higher core-beta the hot electrons behave in a fluid-like manner. Utilization is most promising in an outboard anchor tandem mirror arrangement.

I. Introduction

As originally conceived, the tandem mirror confines a long solenoid of plasma in a uniform magnetic field [1,2]. The solenoid called the central cell is bounded at each end by mirror "plugs" that run at a relatively high density, whose purpose is to provide electrostatic plugging of the central cell as well as MHD stability. The latter function results from the plugs being "minimum-B" cells which create an average minimum-B overall confinement system.

More recently proposed tandem mirror configurations separated the plug into two separate mirror cells [3-5], one providing electrostatic central cell ion confinement as well as an electron "thermal barrier" [6]; the other serving as the MHD anchor. The MHD anchor was usually envisioned to be a neutral beam driven cell producing a high ion beta plasma which is localized in the good field curvature of the minimum-B "anchor."

In this paper, we propose a new method for producing the tandem mirror MHD anchor; creation of an ECRH heated high-beta hot electron plasma in the minimum-B anchor cells. This scenario will be seen to provide a considerable reduction in anchor cost, technology and power requirements. Since the gas source and power are much reduced from the neutral beam driven anchor case, ECRH anchors in present day experiments may be run steady state, aiding in tandem mirror startup. The ability to produce high-Beta ECRH mirror plasmas has been demonstrated experimentally in the INTEREM series experiments [7] which used 10.6 GC (X-band) ECRH heating to produce $\beta \sim 30\text{to}50\%$ plasmas at densities in the 10^{11}cm^{-3} range and hot electron energies above 100 KeV.

The TARA tandem mirror magnetic geometry [4,5] is particularly appropriate for use of an ECRH anchor since in this configuration the anchors are exterior to the plugs and can be run at the low densities required for ECRH accessibility. Additionally, in this configuration the quadrupole fields of the "outboard" anchors do not upset the axisymmetry of the central cell and plug provided the anchor is properly designed so that equilibrium does not require axial currents to flow through the central cell.

Stability of the hot electron plasma as an MHD anchor is akin to the EBT ring problem [8,9]. At low core beta (the core beta is the beta that results from ions and unheated electrons) the hot electrons are not expected to respond to central cell drift frequency MHD fluctuations but enhance the core beta generated stabilization due to the magnetic field depression that they generate. When this stabilization is insufficient an increase in core beta resulting from, for example, application of ion cyclotron resonance frequency (ICRF) heating will enhance stability by causing the hot electrons to behave in an MHD fluid-like manner.

In section II we will describe the physics of the hot electron anchor using an electrostatic "local" analysis.

Section III will detail the integration of the anchor cell within the tandem mirror. Section IV summarizes the conclusions of this study.

II. Physics Considerations

A number of the physics issues can be illuminated through the use of an electrostatic (low-beta) local plasma description.

In cylindrical (r, z) coordinates assuming derivatives along field lines vanish ($\frac{\partial}{\partial z} \rightarrow 0$), we can write the gyrophase averaged, fourier analyzed distribution function fluctuation as

$$\hat{f}_{e\omega}(r, z) = \frac{-ef_0}{T} \left\{ 1 - \frac{\omega - \omega_*}{\omega - \omega_D} J_0^2(k_\perp \rho) \left(1 + \frac{\mu B}{\omega_{ci}^2} \frac{\partial^2}{\partial r^2} \right) \right\} \hat{\phi} \quad (1)$$

For T the temperature, $\hat{\phi}$ the fluctuating potential, J_0 the Bessel function of order zero, k_\perp the azimuthal wave number ($k_\perp = m/r$), ω_* the drift frequency, $\omega_* = \frac{mcT}{erB} \frac{\partial \ell n n}{\partial r}$, ω_D the magnetic drift frequency, $\omega_D = \frac{mcT}{erB} \frac{\partial \ell n B}{\partial r}$, μ is the magnetic moment and ρ the ion gyroradius. For a low beta plasma $\frac{\partial \ell n B}{\partial r} = \frac{1}{R_c}$, with R_c the field line radius of curvature.

For the central cell and plug, we can take $\omega \gg \omega_D$ to perform the velocity space integration which yields the density fluctuation, whereas for the hot electron component in the anchor we will make the opposite approximation, namely $\omega \ll \omega_D$. In this way we obtain the following dispersion relation

$$\int \frac{\omega_p^2}{T} \frac{d\ell}{B} \left\{ \left(-1 + \frac{\omega_{*i}}{\omega} \right) \rho^2 \frac{\partial^2 \hat{\phi}}{\partial r^2} + 2 \frac{\omega_* \omega_{Di}}{\omega^2 I_\alpha} \hat{\phi} \right. \\ \left. + \left(1 - \frac{\omega_{*i}}{\omega} \right) (k_\perp \rho)^2 \hat{\phi} \right\} + \int_\alpha \frac{d\ell}{B} \frac{\omega_{ph}^2 \omega_*}{T} \left(\frac{1}{\omega} - \frac{1}{\omega_{Dh}} \right) \hat{\phi} = 0 \quad (2)$$

with $\omega_{p\alpha}$ the plasma frequency and T_α the temperature for species α . We assume ω_*/T is constant through the device. ω_{ph} and ω_{Dh} are the respective plasma and curvature drift frequencies for the hot electrons. The first term represents the integral over the central cell, plug and anchor "core" plasma and contains the well known FLR response terms. The last term is the anchor term which contains the core plasma term ($\frac{1}{\omega}$) which result from the imbalance between the anchor ion density, n_{ia} and the core electron density, n_{ca} ($n_{ca} = n_{ia} - n_{ch}$ for n_{ch} the hot electron density) and the anchor hot electron response term ($1/\omega_{Dh}$) which may be ignored. The anchor core plasma thus enters in two ways, first in the MHD response term $\omega_* \omega_{Di}$; second, through the charge imbalance term.

The local approximation $\frac{\partial^2}{\partial r^2} = 0$ will now give a resulting "local" quadratic dispersion relation

$$a\omega^2 + b\omega + c = 0 \quad (3)$$

with

$$a = \left[n_i k_{\perp}^2 \rho^2 \right]_c / T_c + 2 \left[n_i k_{\perp}^2 \rho^2 \right]_p / T_p \quad (3a)$$

$$b = 2(\omega_*/T) \left\{ n_{ch} - \left[n_i k_{\perp}^2 \rho^2 \right]_a - \left[n_i k_{\perp}^2 \rho^2 \right]_p - \left[n_i k_{\perp}^2 \rho^2 \right]_c \right\} \quad (3b)$$

$$c = (\omega_*/T) [D_c + 2D_p + 2(D_{ia} + D_{ea})] \quad (3c)$$

for

$$\left[n_i k_{\perp}^2 \rho^2 \right]_{\alpha} = \int_{\alpha} \frac{dl}{B} (k_{\perp} \rho)^2 n_i, [n_i]_{\alpha} = \int_{\alpha} lpha \frac{dl}{B} n_i$$

and

$$D_{\alpha} = \int_{\alpha} \frac{dl}{B} n_i \omega_D = \frac{mc}{8\pi e} \int_{\alpha} \frac{\beta}{r R_c} dl \quad (3d)$$

D_c, D_p, D_{ia} and D_{ea} represent the appropriate drive terms from the central cell, plug and thermal ions and electrons of the anchor respectively. The stability criterion is then $b^2 > 4ac$. The inertial term, $3a$, and the MHD drive term, $3c$, come principally from the plug and central cell. The term $3b$ contains the anchor charge imbalance term plus the finite Larmor radius stabilization terms (to be referred to as FLR terms). These terms enter with opposite signs and therefore could cancel for some mode number, m . This reflects the fact that in the hot electron anchor the ions drift with the wave uncovering electrons whereas in the rest of the device the electrons uncover ions which cannot repond fully to the wave since they see a finite gyroradius averaged wave field.

Avoidance of this cancellation requires

$$\int \frac{dl}{B} n_i (k_{\perp} \rho)^2 > \int \frac{dl}{B} n_{ch} \quad (4)$$

When this condition is met, the net response is FLR-like and an FLR stabilization is possible. Since $k_{\perp} \sim m/r$ stability of the $m = 2$ mode implies stability of all higher modes. This effect would be enhanced

by ICRF heating in the anchor which will be seen to be desirable for other reasons. For $m = 1$, a proper radial treatment would eliminate the FLR terms leaving an "FLR-like" stabilization term that results from the charge imbalance term.

Stability is then determined by the balance of the FLR terms with the drive terms and a necessary condition for instability is $c > 0$. If inequality 4 is not met, a cancellation in the FLR term can occur and for some mode number $b_m \rightarrow 0$. (This cancellation is not seen for a "hot-ion" anchor because of a reversal in signs). In this situation the real frequency of the mode would drop to near zero (recall $\omega \sim b/2a$) and even the core plasma would have to be treated as "rigid." However, this event would probably only result in a small increase in plasma radius which would serve to decrease k_{\perp} . Thus the cancellation would be upset and the wave frequency would rise to the range of the central cell drift frequency.

An electromagnetic treatment in slab geometry has recently been carried out by Baldwin and Pearlstein [10] and yields similar results. Additionally they find that for sufficiently low core beta, and provided that the core plasma gradient-B drift frequency is much less than the wave frequency ($\omega \sim \omega_{*c}$), stabilization results from the core plasma drifting in the well created by the hot "rigid" electrons, that is $3e$ should be replaced by $D_a \rightarrow \int \frac{\beta_c}{r\Delta} dl$ with $\Delta \equiv (B/\Delta_{\perp}B)$. For sufficiently high core beta, the hot electrons can be treated as an MHD fluid and $D_a \rightarrow \int \beta_{II} dl / rR_c$. The core beta at which this change of behavior occurs is the so-called Van Damm-Lee condition [9] $\beta_{\omega} = 2(1 + T_{\parallel}/T_{\perp})(r_p/R_c)_a / (1 + \beta_h)$.

The fluid response of the core beta can be elucidated by reference to recent work on kinetic energy principles. Antonsen and Lee have derived a fully electromagnetic energy principle which, if we ignore potential variations and assume an eikonal approximation (high mode number approximation), reduces to [11]:

$$\begin{aligned} \Delta W = & \frac{1}{2} \int d\alpha d\beta \int \frac{dl}{B} \left\{ \frac{\sigma |\nabla S|^2}{4\pi} (\hat{b} \cdot \nabla(XB))^2 + \frac{\tau}{4\pi} |\hat{Q}_L|^2 \right. \\ & \left. - |XB|^2 (\underline{e} \cdot \underline{K}) \left[\frac{\sigma}{\tau} \underline{e} \cdot \hat{\nabla} P_{\perp} + \underline{e} \cdot \hat{\nabla} P_{\parallel} \right] \right\} + \Delta W_K \end{aligned} \quad (6)$$

with α, β the Clebsch co-ordinates $\underline{B} = B\hat{b} = \nabla\alpha \times \nabla\beta$, $\hat{\nabla}$ the gradient on α and β ($\hat{\nabla} = \nabla\alpha \frac{\partial}{\partial\alpha} + \nabla\beta \frac{\partial}{\partial\beta}$) and K the field line curvature ($K \equiv \hat{b} \cdot \nabla b$). S is the eikonal and we require the field line displacement $\underline{\xi}$ to be $\underline{\xi} = \hat{\xi}(x) \exp(iS(x))$. We decompose $\hat{\xi}$ as $\hat{\xi} = X\hat{b} \times \nabla S + Y\nabla S$. $\sigma \equiv 1 + 4\pi(P_{\perp} - P_{\parallel})/B^2$ and $\tau = 1 + \frac{4\pi}{B} \frac{\partial P_{\perp}}{\partial B}$. ΔW_K is the kinetic term which contains the perturbed distribution function appropriately averaged over bounce and drift motion [11]. For $P(B)$ hot electron equilibrium ($P_{\perp H} = P_{\perp H}(B)$) $\hat{\nabla} P_{\perp H} = 0$ and for rapidly drifting hot electrons ($\omega \ll \omega_D$) the kinetic term can be shown to be small by the ratio Kr_n with $r_n \equiv (d \ln P_c / dr)^{-1}$. For $\sigma, \tau > 0$ a necessary condition for instability then becomes

$$\int \frac{d\ell}{B} |XB|^2 (\underline{e} \cdot \underline{K}) [(\underline{e} \cdot \hat{\nabla} P_{\perp c}) \frac{\sigma}{\tau} + \underline{e} \cdot \hat{\nabla} P_{\parallel}] > 0 \quad (7)$$

with $\underline{e} = B^{-1} \hat{b} \times \nabla S$.

For interchange modes ($|XB| = \text{constant}$), Eq. 7 takes the approximate form

$$\int d\ell \frac{\beta_c}{R_c r_p} (\sigma/\tau) < 0 \quad (7a)$$

with $R_c \equiv (\underline{e} \cdot \underline{K}/|e|)^{-1}$. Eq. 7a exhibits the form of the MHD drive term of Eq. 3e, except for the additional factor (σ/τ) . Using the MHD magnetostatic force balance and assuming a P(B) equilibrium from hot electrons, we find

$$\tau = \Delta \left(\frac{1}{R_c} + \frac{\beta_c}{2r_p} \right)$$

with

$$\Delta \equiv \frac{B}{\nabla_{\perp} B} \text{ and } r_p \equiv \frac{P_c}{\nabla_{\perp} P_c}$$

The drive then becomes

$$D = \int \frac{d\ell \beta_c / \Delta}{r_p (1 + \beta_c R_c / 2r_p)} \sim \int \frac{d\ell \beta_H / R_c}{r_p (1 + \beta_H \Delta / R_c \beta_c)} \quad (8)$$

where we have set $\sigma \sim 1$ and estimated $\beta_H \sim 2r_p/\Delta$. (A similar form can also be shown to result from a non-P(B) hot electron distribution [12].) Eq. 8 reproduces the already mentioned response in the limit of high and low core beta as well as indicating the response in the intermediate core beta regime. For small core beta ($\beta_c \ll r_p/R_c$) the core plasma responds in an MHD fluid manner except that the relevant scale length becomes that of the field depression brought about by the hot electrons rather than the vacuum radius of curvature ($D \rightarrow \int d\ell \beta_c / r_p \Delta$); for high core beta, the hot electron response is similar to that of an MHD fluid ($D \rightarrow \int d\ell \beta_H / R_c r_p$). In the intermediate case when $\beta^* \sim \Delta / R_c \beta_H = 2r_p / R_c$ (the Van Damm-Lee condition for loss of hot electron ring rigidity in the EBT case), we find $D \sim \int d\ell \beta_H / 2R_c r_p$. Note that if the hot electrons were in a maximum rather than a minimum-B cell, the sign in the denominator of 8 would be reversed. In the minimum-B cell situation presently being considered, nothing dramatic occurs at the Van Damm-Lee beta condition. β^* represents the beta value at which optimum stabilization will occur. Clearly, a strong stabilizing effect implies small Δ (large hot electron field depression), small vacuum radius of curvature and core beta in the range of $2r_p/R_c$. At this core beta, the hot electrons weight the vacuum curvature as if they were an MHD fluid.

III. Tandem Mirror with Hot Electron Anchor

One tandem mirror arrangement that could utilize the hot electron anchor is embodied in the TARA experiment [4,5]. Here the central cell is terminated by axisymmetric plugs and the flux tube then maps through a transition region into a quadrupole anchor. The outboard location of the anchor permits the anchor density to be low enough for easy ECRH access. The plugs are formed by neutral beam injection utilizing sloshing-ions for microstability and a thermal barrier can be formed at the midplane of the plugs. The schematic axial magnetic field strength profile is shown in fig. 1a and the potential profile in fig. 1b.

Following the results of the INTEREM [7] experiments, the anchor electrons could be heated at X-band frequency (10.6 GC for an anchor resonance at 3.8 kG) with a density in the low to mid 10^{11} cm^{-3} range to attain an electron beta of 30 to 50%. The requirement for a significant core fluid response requires a core beta $\sim 2\%$ for $\tau = 0.2$, requiring ions in the 5-10 keV range ($\beta^* = 2\%$ for $r_p/R_c = .01$). These ion energies would be readily attainable through the use of ICRF heating by evanescent fields, as demonstrated in the Phaedrus experiment [13].

With no ion heating, the neutralizing ions in the anchor are cold ($T_i \sim 2 - 6 \text{ eV}$ due to Frank-Condon ionization). Baldwin [14] has shown that in this situation the potential shown schematically in fig. 2b would be expected to peak at the mirror throats and have a small midplane depression so as to provide electrostatic confinement for the cold neutralizing ions. The potential well that forms in the transition region would fill up to close to the peak ion density. When ICRF is applied, ions would be heated along velocity space trajectories defined by the resonance field $\Theta_{RF} = \sin^{-1} \sqrt{B_o/B_r}$ with B_o the minimum field and B_r the resonance field. For diffusive ion heating, the potential depression would then deepen so that the loss cone boundary permits the core ions to be fuelled and heated with sufficient confinement for neutralizing the hot electrons. As indicated in Fig. 2d by the solid curve, the loss cone boundary will remain close to the velocity space RF heating trajectory. For low ion energies, the single pass kick an ion receives in passing through resonance with the RF field can be several hundred eV and in this case the cell potential could rise positively without cutting off the fuel source (dashed curve, fig. 2d). The exact potential level depends on the microstability of the trapped ion population but we can expect the potential to rise to a level such that the ion density will just neutralize the electrons.

At the projected densities (mid 10^{11} cm^{-3} range) little ICRF power will be required and good penetration of antenna near fields is assured [13]. An ion density of $5 \times 10^{11} \text{ cm}^{-3}$, energy of 10 KeV and estimated confinement time of 5 ms requires $\lesssim 2 \text{ kW}$ of power. Even a significant degradation of confinement would still only result in modest ICRF power levels.

IV. Conclusions

We have presented here a new method for creating a tandem mirror MHD anchor, utilizing electron and ion RF heating. In this scenario, the ECRH heated hot electron population digs a diamagnetic well in the anchor which enhances the fluid-like stabilization of the core ion (and electron) beta. Additionally, a FLR-like charge imbalance stabilizing term appears to be operative for $m = 1$ flute perturbations. For the next generation of tandem mirror experiments, such as TARA, this innovation offers the possibility of replacing several Megawatts of neutral beam power with ~ 1 KW of ECRH in X-band and < 10 kW of ICRF power. Thus a substantial reduction in power requirements, as well as technology, would result.

In a reactor scenario use of the hot electron anchor is particularly promising. In the reactor parameter range, the disparity in drift frequencies between hot electrons and central cell plasma drifts is not large and the hot electrons would be expected to behave in a MHD fluid-like manner [15]. The anchor could still consist of a relatively low-field, large volume cell and therefore not require the development of very high field quadrupole coils.

An interesting arrangement suggested by this work would be an axisymmetric maximum-B outboard hot-electron anchor creating a totally symmetric geometry. With the hot electrons behaving in a rigid fashion ($\omega_{Dh} \gg \omega_{*c}$ low core beta) the system could be FLR stabilized for modes with $m \geq 2$ if Eq. 4 is satisfied and the hot electron anchor could stabilize the $m = 1$ mode via the charge imbalance term. Unlike the EBT mode of stabilization, this method would not require the presence of core beta in the ring-produced field depression. However, an additional EBT-like stabilization would occur in the presence of some core beta (provided the Van Damm-Lee limitation on core beta is not violated).

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Figure Captions

1. Schematic of axial variation of *a*) magnetic field and *b*) electrostatic potential for an outboard anchor tandem mirror arrangement. Only right half of machine is shown.
2. *a*) Schematic magnetic field in a mirror cell. *b*) Potential variation in the presence of a hot electron component with a cold ion background ($T_i \lesssim 10\text{eV}$). *c*) Expected potential when ion heating is used. *d*) Velocity space boundaries at cell midplane indicating the loss cone and RF heating trajectory. Dashed line is expected loss cone boundary if the cell develops a midplane potential maximum.

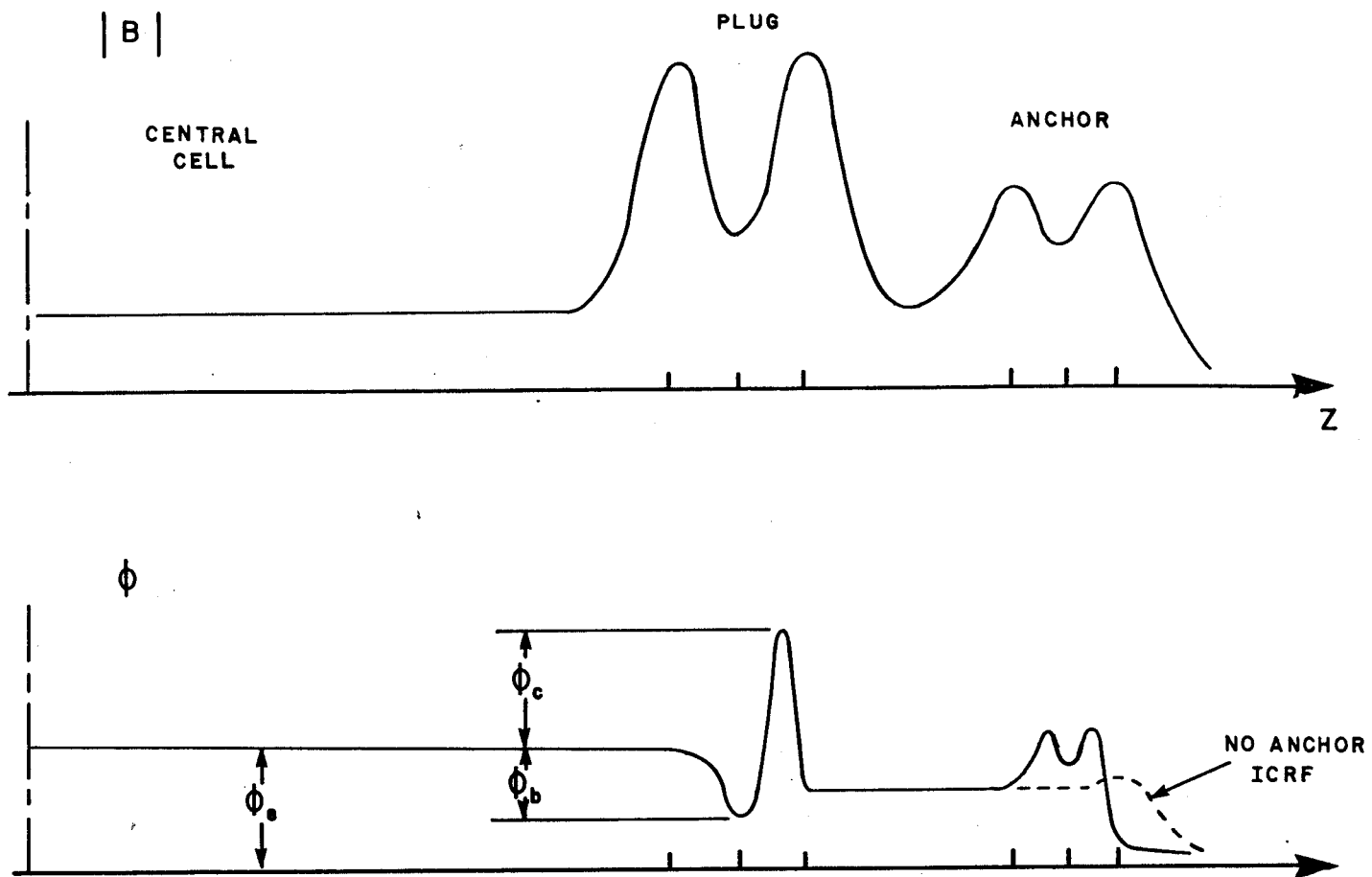


FIG 1.

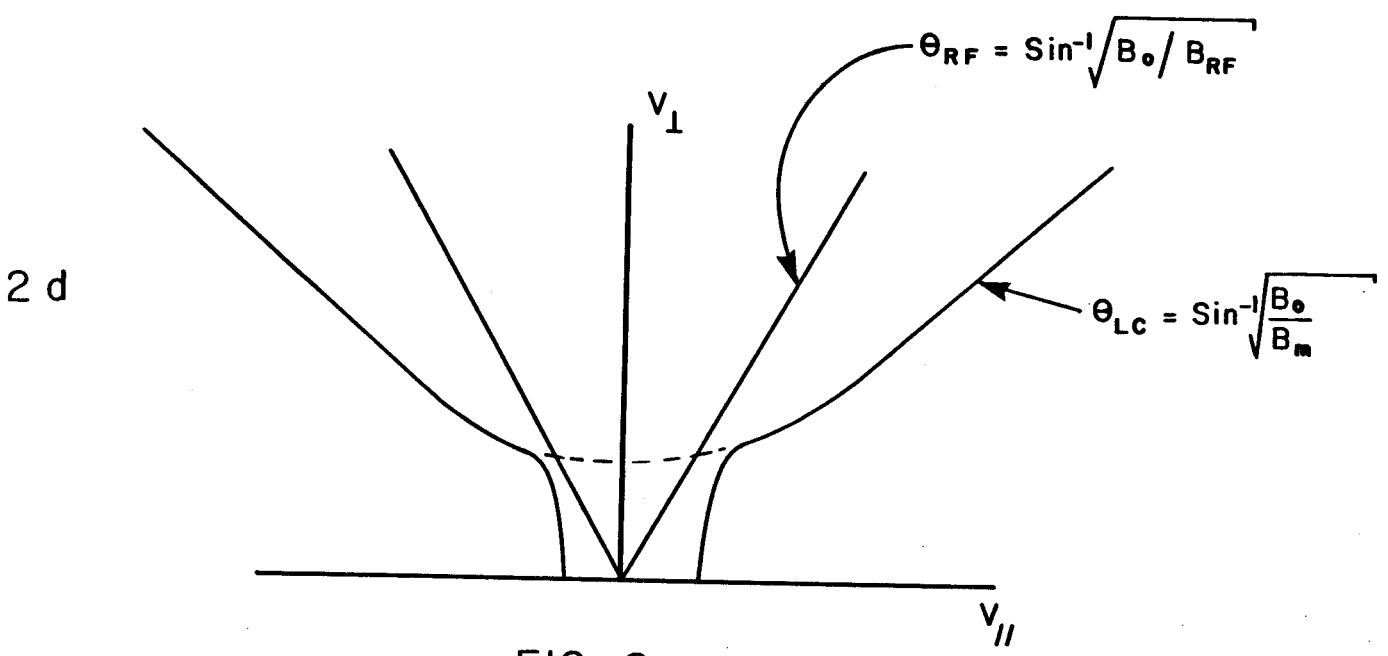
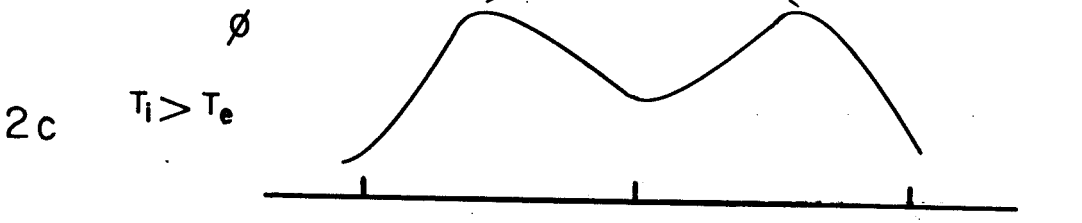
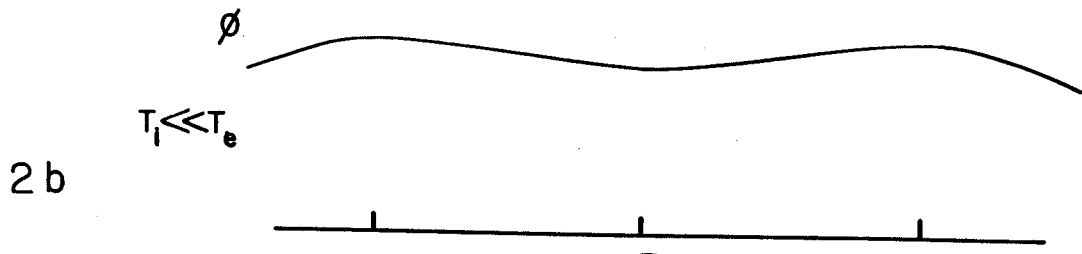
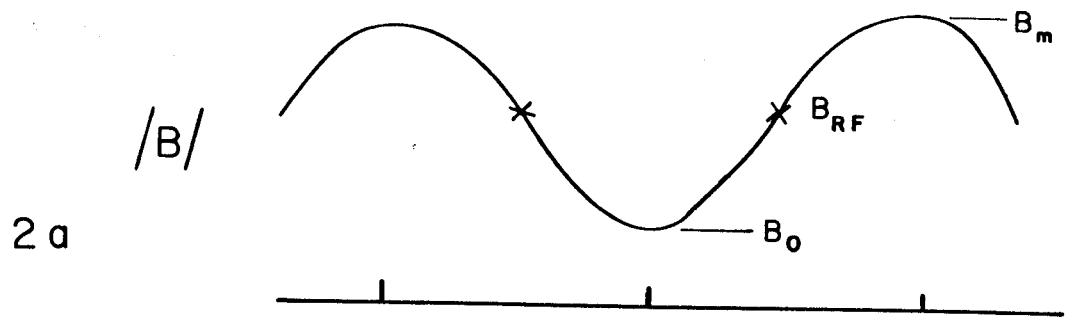


FIG. 2