LOWBITRON—A SOURCE OF COHERENT

ELECTROMAGNETIC RADIATION

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ABSTRACT

The LOWBITRON—a longitudinal wiggler, beam interaction device—is a novel source of submillimeter wave radiation. It comprises a relativistic electron beam gyrating in a longitudinally rippled, periodic magnetic field of the form $\vec{B}=\hat{z}[B_0+B_1\sin(k_0 z)]$, where $k_0 = 2\pi/\ell$ and ℓ is the period. Transverse electron density bunching gives rise to a convectively unstable wave that propagates along the guiding magnetic field $\hat{z}B_0$. Its growth rate has been computed for the case of a cold beam. The device exploits the large growth rate of gyrotrons on the one hand; and the double Doppler upshift in frequency characteristic of free electron lasers, on the other hand.

During the past ten years, relativistic electron beams have been used with the view of generating intense coherent electromagnetic radiation in the centimeter, millimeter, and submillimeter wavelengths ranges. For this purpose two main types of electron beam instabilities have been used: the cyclotron maser instability[1] characterized by azimuthal electron bunching, and emission frequencies associated with the electron gyrofrequency or one of its harmonics; and the free electron laser instability[2] characterized by axial electron bunching, and emission frequencies associated with the Doppler upshifted period of an imposed, transverse, periodic (wiggler) magnetic field.

Each mechanism has its advantages (and disadvantages). The cyclotron maser (gyrotron) has a large instability growth rate and good efficiency, but is limited in frequency to the millimeter wavelength range due to technical difficulties of constructing practical tubes with very large magnetic fields. The free electron laser has smaller growth rates and efficiencies; in addition, there are very strong requirements on beam quality. However, because of the double Doppler upshift experienced by the waves, very high frequencies can be achieved with the free electron laser.

The LOWBITRON—a longitudinal wiggler beam interaction device—, is a hybrid system intended to exploit the good characteristics of both of the above mechanisms. It is illustrated schematically in Fig. 1. A relativistic electron beam in the form of a thin pencil propagates on axis of an evacuated drift tube, which also acts as the waveguide for the electromagnetic radiation. The beam electrons gyrate in the combined, uniform magnetic field of a solenoid, and a longitudinally rippled (wiggler) field, such that the total

imposed field on the axis has the form,

$$\vec{B} = \hat{z}[B_0 + B_1 \sin(k_0 z)] \tag{1}$$

where $k_0 = 2\pi/\ell$ is the wavenumber and ℓ the periodicity of the wiggler. Such a field can be generated in one of several ways, as for example by driving current azimuthally in alternate directions through a periodic assembly of copper rings[3] as is illustrated in Fig. 1; or by making the rings from samarium-cobalt[4] or other magnetic material and magnetizing the rings in the axial direction as is done in systems employing periodic focusing;[5] or, in pulsed systems, by using the technique of magnetic diffusion.[6] In all of these methods, the magnetic field at a distance r from the axis is approximately

$$\vec{B} \simeq \hat{z}[B_0 + B_1 I_0(k_0 r) \sin(k_0 z)] - \hat{r} B_1 I_1(k_0 r) \cos(k_0 z) \tag{2}$$

where I_0 and I_1 are modified Bessel functions. Near the axis, such that $k_0 r < 1$, Eq. (2) reduces to Eq. (1). We point out that in the lowbitron the periodic field modulation is longitudinal rather than transverse, as in the case of free electron lasers. This is advantageous because longitudinal modulations can be more easily produced and at considerably larger amplitudes.

Before entering the interaction region shown in Fig. 1, each electron has a transverse velocity $v_\perp$ which it acquires at the electron gun situated to the left of the figure. To this purpose one can use a form of magnetron injection gun[7] placed in the fringing field of the solenoid. This yields an annular beam, which is then adiabatically compressed to its paraxial position by means of the spatially rising, solenoidal field. Alternately, one can envision using a magnetically shielded Pierce type of gun in con-

junction with transverse magnetic wiggler excitation, to impart[3] the necessary $v_\perp$ to the beam electrons.

In traversing the interaction region of Fig. 1, the beam electrons become bunched in the perpendicular direction and as a result, a convectively unstable right circularly polarized wave propagates along the guiding magnetic field. The radiation is coupled out of the system by conventional waveguide or quasi-optical techniques. The spent electrons are collected on a beam dump situated in the fringing solenoidal magnetic field. The radiation frequency is found (approximately) by requiring that the phase velocity ω/k of the Doppler shifted cyclotron mode excited in the rippled magnetic field of Eq. (1),

$$\omega = \Omega_0/\gamma + (k + k_0)v_\| , \tag{3}$$

equals the phase velocity of an electromagnetic mode of the waveguide

$$k^2 c^2 = \omega^2 - \omega_c^2 \tag{4}$$

Here ω and k are the frequency and parallel wavenumber of the excitation, respectively; $\Omega_0 = eB_0/m_0$ is the nonrelativistic cyclotron frequency in the guiding magnetic field; $\gamma = (1-\beta_\|^2-\beta_\perp^2)^{-1/2} = (1+eV/m_0 c^2)$ with V as the beam voltage; $\beta = v/c$; and ω_c is the cutoff frequency of a waveguide mode. Solving Eqs. (3) and (4) subject to the assumption that $\omega \gg \omega_c$, and thus neglecting effects due to finite transverse geometry, yields

$$\omega = (1 + \beta_\|)\gamma_\|^2 [k_0 c\beta_\| + \Omega_0/\gamma] \tag{5}$$

or, alternately,

$$\omega = (1 + \beta_\|)\gamma^2(1 + \gamma^2\beta_\perp^2)^{-1} [k_0 c\beta_\| + \Omega_0/\gamma] \tag{6}$$

where $\gamma_\shortparallel = (1-\beta_\shortparallel^2)^{-1/2}$. Equations (5) and (6) show that in the limit of no wiggler $(k_0 \to 0)$ one recovers the results obtained for gyrotrons, whereas in the limit $\Omega_0 \to 0$ one has the same frequency as for free electron laser interactions.

As an example, let us consider an electron beam with $\gamma=3$ (V=1.02MV), and perpendicular velocity $v_\perp=0.3c$, traveling in an axial guiding magnetic field $B_0=10kG$ and a wiggler field of wavenumber $k_0=6$ (periodicity $\ell=1.05cm$). It then follows from Eq. (6) that the interaction frequency $\omega/2\pi=329GHz$, (or a wavelength of 0.91mm). This is to be compared with the frequency 472GHz of a free electron laser using a beam of the same total energy, $\gamma=3$, but operating with $\beta_\perp << 1$ which is normally the case. Thus, we see that the lowbitron has a somewhat smaller operating frequency than the free electron laser. This smaller operating frequency comes about because $\beta_\perp$ is relatively large, a necessary requirement to achieve a good growth rate in the lowbitron, and $\gamma_\shortparallel$ is thereby reduced. On the other hand, a gyrotron in a 10kG magnetic field and at a typical operating energy with $\gamma \approx 1$, has an output frequency $\sim 28GHz$, which is an order of magnitude below that of the lowbitron. We point out, however, that in the lowbitron, the values of magnetic field B_0, γ, and k_0 cannot be chosen at will, but are related through the inequality

$$k_0 c\beta_\perp\gamma/\Omega_0 \lesssim 1 \qquad (7)$$

This comes from the fact that the electron gyroradius must be sufficiently small so that the transverse field modulation $|\hat{r}B_1 I_1(k_0 r)|$ of Eq. (2) felt by the electron can be neglected compared with the longitudinal modulation $|\hat{z}B_1 I_0(k_0 r)|$.

The growth rate of the instability and the small frequency

mismatch (relative to the frequency given by Eqs. (5) and (6))
were computed from the linearized, relativistic Vlasov equation
under the assumption that the unneutralized electron beam is suf-
ficiently tenuous so that static space charge forces can be ne-
glected. Also, the electron beam was assumed to be cold, and its
zeroth order distribution function was taken to be

$$f_0 = \delta(P_\perp - p_\perp)\delta(P_{\shortparallel} - p_{\shortparallel})/2\pi P_\perp \tag{8}$$

where $P_{\shortparallel}$ is the momentum along the axis, $P_\perp$ is the momentum per-
penducular to the axis and δ is the Dirac delta function. We take
an rf electric field of the form $E_k \exp(ikz-i\omega t)$, and apply the method of
characteristics. Keeping terms to second order in B_1, and terms
having resonant denominators of the form $(\omega-(k+k_0)v_{\shortparallel}-\Omega_0/\gamma)$, we
arrive at the dispersion equation for the right-hand, circularly
polarized wave propagating along the rippled magnetic field given
by Eq. (1). Keeping only the largest terms in the coefficients of
each power of x, we obtain,[9] in the high gain regime[2]

$$x^4 - (a/2)x^3 + bx^2 + (k_0 cbQ^2/8\omega_p)x + Q^2 b^2/16 = 0 \tag{9}$$

for the complex mismatch parameter $x=(kc-\omega)/\omega_p$. Here $\omega_p = (Ne^2/m\varepsilon_0)^{1/2}$ is the nonrelativistic plasma frequency; N is the
electron number density; $a=(\omega_p/\omega)(\Omega_0/k_0 v_{\shortparallel}\gamma)$; $b\equiv(v_\perp^2/2v_{\shortparallel}^2\gamma)$; and $Q\equiv (eB_1/k_0 p_{\shortparallel})<1$. The equation is readily solved in two regimes.

(a) The "low density" regime

Under the condition that $\omega_p << (3k_0 cQ^2/4a)$ the x^3 term of Eq.
(9) is negligible. We furthermore assume that $x^4 >> Q^2 b^2/16$ which
allows us to neglect the last term of the equation. The resulting
cubic is then easily evaluated with the result that

$$x = -(1/2)(A + B) \pm i(\sqrt{3}/2)(A - B) \tag{10}$$

where

$$A = \left\{ -\frac{k_0 cQ^2 b}{16\omega_p} + \left[\left(\frac{b}{3}\right)^3 + \frac{k_0^2 c^2 b^2 Q^4}{256\omega_p^2} \right]^{1/2} \right\}^{1/3}$$

$$B = -\left\{ \frac{k_0 cQ^2 b}{16\omega_p} + \left[\left(\frac{b}{3}\right)^3 + \frac{k_0^2 c^2 b^2 Q^4}{256\omega_p^2} \right]^{1/2} \right\}^{1/3}$$

In the limit $(b/3)^{3/2} \gg (k_0 cQ^2 b)/16\omega_p \gg (b^{5/2}/2)(\omega_p/k_0 c)^2$, which requires sufficiently large transverse beam velocities $v_\perp$, Eq. (10) reduces to

$$k - (\omega/c) \simeq (k_0 Q^2)/16 \pm i(\omega_p v_\perp)/(\sqrt{2}\, v_{\shortparallel}\gamma^{1/2}c). \qquad (11)$$

Here the growth rate of the instability given by the second term of Eq. (11) is similar to that for the gyrotron,[10] and is independent of the wiggler field amplitude B_1.

On the other hand, in the limit $(b/3)^{3/2} \ll (k_0 cQ^2 b)/16\omega_p$ of large wiggler field amplitudes, Eq. (10) becomes

$$k - (\omega/c) \simeq (1/4)(\omega_p^2 k_0 bQ^2/c^2)^{1/3} \pm i(\sqrt{3}/4)(\omega_p^2 k_0 bQ^2/c^2)^{1/3}$$
$$(12)$$

from which we see that now the growth rate depends on both the wiggler amplitude B_1 and transverse beam veloctiy $v_\perp$.

(b) The "high density" limit

Here we neglect the terms x^4 and $(a/2)x^3$ of Eq. (9) and obtain a quadratic equation for x. In the limit that $\omega_p^2 \gg (k_0^2 c^2 Q^2/32b)$, we find that,

$$k - (\omega/c) \simeq -(k_0 Q^2/16) \pm i(\omega_p Qb^{1/2})/4c \qquad (13)$$

We now take two examples. In the first example, we set $\gamma=3$, $\beta_\perp=0.3$, $B_0=10kG$, $B_1=2kG$, $k_0=6$, and a beam current $I=1A$. We also assume a beam radius of 1.54mm which equals one Larmor radius.

Substituting these values in Eq. (10) which is appropriate to the "low density" regime, we find the spatial growth rate of the instability, $\Gamma \equiv 2 \text{Imk} = 0.035 \text{cm}^{-1}$, or 14.8dB per meter of interaction length. In our second example, we change the current from 1A to 10kA, but leave all other parameters unchanged. Substitution into Eq. (13), appropriate to the "high density" regime, yields $\Gamma = 0.060 \text{cm}^{-1}$, or 26dB per meter. In both cases the radiation frequency is ~330GHz. We note, however, that, with the chosen parameters, relatively little is gained by going to high currents.

In conclusion, then, we have described the basic concepts of a novel source of coherent electromagnetic radiation capable of generating (or amplifying) waves in the millimeter and submillimeter wavelength ranges. It employs a longitudinal wiggler field which has the effect of doubly Doppler upshifting the right circularly polarized electromagnetic wave propagating along the guiding magnetic field. The computations of the growth rate were carried out for a cold electron beam traveling in free space (transverse beam and waveguide dimensions were neglected). To assure oneself that one indeed operates in the cold beam regime one must satisfy the inequalities.

$$\frac{\Gamma c}{2\omega} > \left[\frac{(1 + \beta_{\shortparallel})k_0 v_{\shortparallel} + \beta_{\shortparallel}\Omega_0/\gamma}{\omega}\right]\left[\frac{\Delta p_{\shortparallel}}{\beta_{\shortparallel} p_{\shortparallel}}\right]$$

and

$$\frac{\Gamma c}{2\omega} > \left[\frac{v_{\perp}}{v_{\shortparallel}}\right]\left[\frac{\Delta p_{\perp}}{p_{\shortparallel}}\right] \tag{14}$$

where $\Delta p_{\shortparallel}$ and $\Delta p_{\perp}$ are the small axial and transverse spreads in particle momenta, respectively.

REFERENCES

1. J.L. Hirshfield and V.L. Granatstein, IEEE Trans. Microwave Theory and Techniques MTT-25, 522, (1977), and references therein.

2. N.M. Kroll and W.A. McMullin, Phys. Rev. A17, 300 (1978); P. Sprangle, R.A. Smith, and V.L. Granatstein, Naval Research Laboratory, Washington, DC Memorandum Report No. 3911, December 1978 and references therein.

3. D.B. McDermott, T.C. Marshall, S.P. Schlesinger, R.K. Parker, and V.L. Granatstein, Phys. Rev. Lett. 41, 1368 (1978).

4. K. Halbach, Lawrence Berkeley Laboratory, University of California, Accelerator and Fusion Research Division Report No. LBL 11393, August 1980.

5. G.R. Brewer article in "Focusing of Ion Beams" edited by A. Septier, (Academic Press) p. 73.

6. K.D. Jacobs, R.E. Shefer, and G. Bekefi, Appl. Phys. Lett. 37, 583 (1980). Also K.D. Jacobs, G. Bekefi, and J.R. Freeman, M.I.T. Plasma Fusion Center Report No. PFC/JA-81, January 1981.

7. S. Ahn, V. Granatstein, and J.L. Hirshfield editors "Collection of Papers on the Electron Cyclotron Maser (Gyrotron)" Vol. I. Soviet Papers, Naval Research Laboratory Report, Washington, DC, NRL Memorandum No. 3937 October 1979.

8. P. Ferguson, S. Hegji, H. Jory, A. Nordquist, and R. Symons, Varian Associates, Inc. Palo Alto, California, Final Technical Report RADC-TR-80-132, 1980; also K.D. Jacobs and G. Bekefi, Proc. IEEE International Conference on Plasma Science, Santa Fe, N.M., 1981 p. 47.

9. W. A. McMullin and G. Bekefi, M.I.T. Plasma Fusion Report (to be published).

10. J. L. Hirshfield, K. R. Chen, and S. Kainer, Appl. Phys. Lett. $\underline{33}$, 847 (1978).

CAPTIONS TO FIGURE

Fig. 1. Schematic drawing of the LOWBITRON interaction region. The current-carrying rings which generate the required longitudinal magnetic field modulation can be replaced either by permanent ring magnets,[4,5] or by a diffusive wiggler.[6]
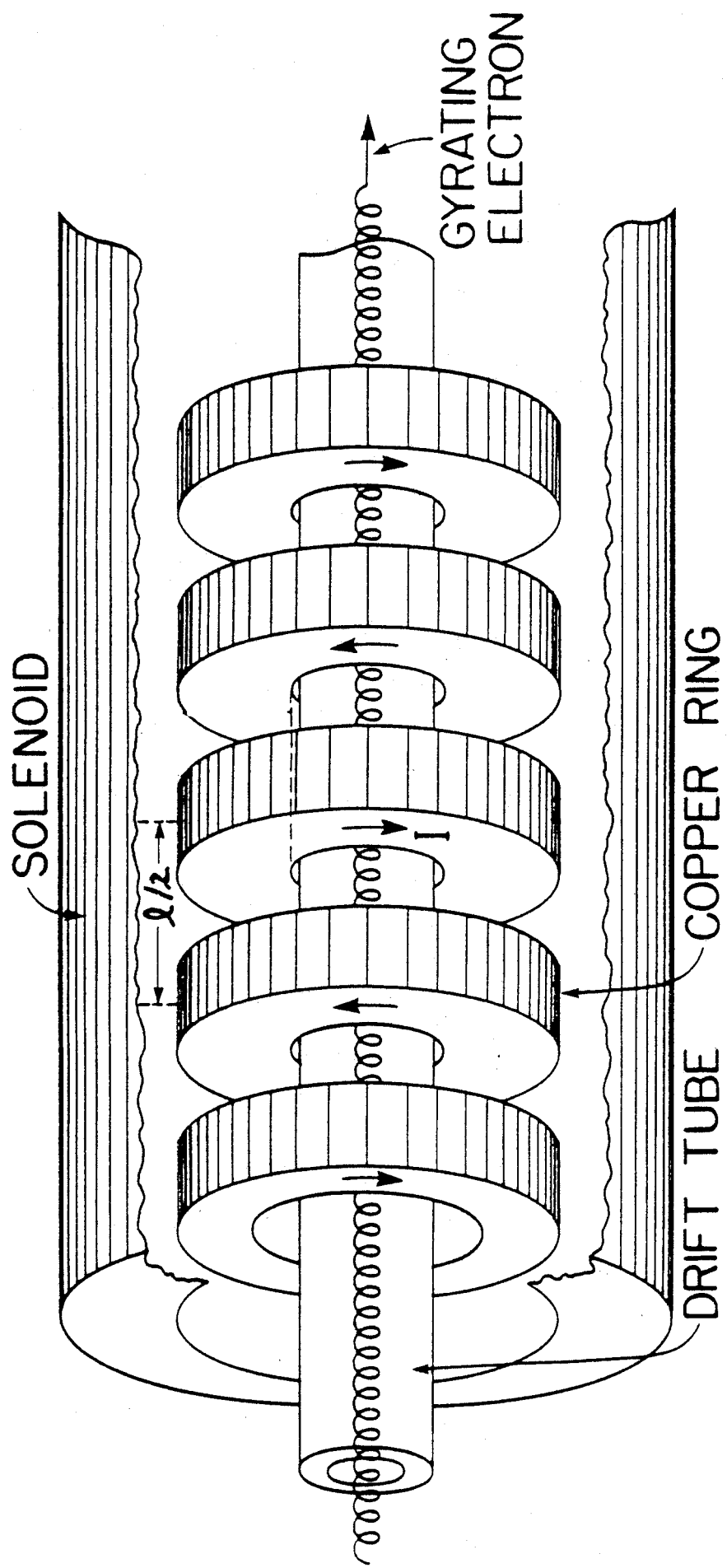
SOLENOID

GYRATING ELECTRON

COPPER RING

DRIFT TUBE

$\ell/2$

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Fig.1 McMullin and Bekefi