ON MHD EQUILIBRIUM AND STABILITY PROPERTIES

OF SOLAR MAGNETIC LOOPS

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PFC/JA-80-29 July, 1980

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Abstract

Based on the MHD analysis, it is proved that the solar magnetic loops without applied toroidal field and vertical field can only exist in the case of $\beta_p \left(= \frac{< \Delta p >}{B_p^2/2\mu_o} \right) < 0$, the core pressure of the loop is smaller than that of the environment. The current and magnetic structure corresponding to the $\beta_p < 0$ magnetic loops lead it to be stable as observed on the Skylab.

It is interesting, from plasma physics point of view, to understand why the solar magnetic loops have so much long-lived time as compared with the Alfven transit time for MHD instability to grow. In the laboratory, scientists had fought with the stability problems of plasma torus for more than three decades, and we know now that a large enough applied toroidal field and a vertical equilibrium field are necessary for quasi-steady equilibrium and stability of toroidal plasmas. All these two fields are absent in the solar magnetic loops, and we may ask why they are stable?

After analysis of the parameters for equilibrium of toroidal magnetic loops, we concluded that the βp must be smaller than zero if no vertical field is applied. The core pressure of the torus is lower than that of the environment and a positive pressure gradient exists here. It is the main difference between toroidal plasma and cylindrical plasma, and also indicated that the force-free field can't be used anywhere without restriction even for $\beta \ll 1$.

We shall start from the analysis of equilibrium of a solar magnetic loop, the basic data was observed on Skylab several years ago [1], [2], [3], [4]. The majority part of the loop is near a torus with circular cross section, then [5]

$$\ell n 8 \frac{R}{a} + \beta_p - \frac{3}{2} + \frac{\ell_i}{2} = \frac{4\pi R B_{\perp}}{\mu_0 I_t} = 0 \qquad (1)$$

must be satisfied if no vertical field was applied.

Here

$$\beta_p = \frac{\int_v \Delta p d\underline{v}}{\underline{v} \cdot \frac{B_p^2}{2\mu_0}}$$

$\frac{R}{a}$ - aspect ratio of the torus with the circular cross section

ℓ_i - internal inductance, determined by the distribution of the electric currents, for surface current model $\ell_i = 0$

I_t - torodial current

$B_\perp$ - vertical equilibrium magnetic field

from (1)

$$\beta_p = - \ell n 8\frac{R}{a} + \frac{3}{2} - \frac{\ell_i}{2} < 0 \qquad (2)$$

$\beta_p < 0$ means the pressure is larger outside than inside of the torus. This tendancy will not change if cross section or loops are not exactly circular.

Observations indicated that the core temperature of the loop is one order of magnitude smaller than that of the environment, so we can use surface current model without much inaccuracy, from the definition of β_p and use the surface current model approximation of larger aspect ratio torus,

$$\beta_p = 1 - \frac{I_p^2}{I_t^2} \left(\frac{a}{R} \right)^2 \qquad (3)$$

I_p total poloidal current

$$B_t = \frac{\mu_o I_p}{2\pi a}$$

so $$\frac{B_t}{B_p} = \frac{I_p}{I_t}\frac{a}{R} = (1 - \beta_p)^{\frac{1}{2}} \qquad (4)$$

When a loop exists without applied vertical field and applied toroidal field, then from (2), $\beta_p < 0$, and consequently, the induced torodial field is larger than the induced poloidal field, otherwise the loop can't reach the quasi-equilibrium state.

A typical magnetic loop had the following observed data:

$$2R \simeq 10^{4.8} km$$

$$2a \simeq 10^{3.9} km$$

$$T_{core} \simeq 10^{4.9}\ ^0K$$

$$T_{environment} \simeq 10^{6.3}\ ^0K$$

from (2)

$$\beta_p = - \ell n\ 8 \frac{R}{a} + \frac{3}{2}$$

$$= - \ell n\ 8 \times \frac{10^{4.8}}{10^{3.9}} + \frac{3}{2} \simeq - 2.66$$

If we estimated $I_t \simeq 10^{10}$ A, then the poloidal magnetic field at the edge of the loop is

$$\beta_p = \frac{\mu_o I_t}{2\pi a} = \frac{4 \times 10^{-7} \times 10^7}{\pi \times 10^{3.9}} \simeq 5 \times 10^{-4} \ \text{Weber}/_{m^2}$$

$$= 5 \text{ gauss}$$

the pressure difference across the loop is

$$\Delta p = \beta_p \cdot \frac{B_p^2}{2\mu_o} = - 2.66 \times \frac{(5 \times 10^{-4})^2}{8\pi \times 10^{-7}} = - 2.64 \times 10^{-1} \ \text{Newton}/_{m^2}$$

$$= - 2.64 \ {}^{\text{dyne}}/_{cm^2}$$

the induced toroidal field

$$B_t = B_p \, (1 - \beta_p)^{\frac{1}{2}} = 5 \times (\, 1 + 2.66)^{\frac{1}{2}}$$

$$\simeq 9.55 \text{ gauss}$$

all the results are consistent within the region of obversational data. But actually once the total toroidal current is determined; the other parameters are dependent upon it and can't be variated arbitrarily.

The safety factor

$$q_a = \frac{a}{R}\frac{B_t}{B_p} = \frac{1}{8} \times \frac{9.55}{5} = 0.24$$

In the laboratory, it was already recognized that only a very few types of plasma configurations can be stablized under very serious conditions, so it is interesting to analyze the stability problems of solar magnetic loops based on their equilibrium parameters and show what causes it to be stabilized.

We shall check three cases: (1) sausage stability, (2) the local stability and (3) the global stability.

The local stability criterion for cylindrical plasma is Suydam criterion, it is a balance between negative pressure gradient as the only destabilization effect and shear as a stabilization effect. For toroidal configuration with circular cross section, the Suydam condition is modified to

$$\frac{1}{4}\left(\frac{\frac{dq}{dr}}{q}\right)^2 + \frac{\frac{1}{r}\frac{dp}{dr}}{B_t^2/2\,\mu_o}\left(1 - q^2\right) > 0$$

Since we have positive pressure gradient and $q_a < 1$ in toroidal equilibrium without applied vertical and torodial field, the local stability criterion is definitely satisfied, the only destabilization effect of local instability is cancelled now.

For global stability [6], the loop is stable against the sausage instability (m = 0) if $\frac{B_t}{B_p} > \frac{1}{\sqrt{2}}$ in zero applied toroidal magnetic field case. Now B_t is larger than B_p so m = 0 is stable.

For long wavelength m = 1 perturbation, the possible longest wavelength is $2\pi R$. So $ka = \frac{a}{R}$, and the growth rate

$$\omega^2 = \frac{B_p^2\, k^2}{\mu_o \rho_o} \left[1 - \left(\frac{B_p}{B_t}\right)^2 \ell n \ \frac{1}{ka}\right]$$

$$= \frac{B_p^2\, k^2}{\mu_o\, \rho_o} \left[1 - \left(\frac{5}{9.55}\right)^2 \ \ell n \ 8\right] > 0$$

$\omega^2 > 0$ means stable • so it is stable for m = 1 perturbation. The situation is even better if we consider part of the torus is immersed in the high density region and tied here in real case.

Finally, we can see $q_a \geq 1$ is not necessary for stability in this type of magnetic structure.

All the formulas shown above are familiar with fusion plasma physicists. We used them to calculate the equilibrium and stability properties of solar magnetic loops as a special case of zero applied toroidal field and zero vertical field. It is shown there are some intrinsic relationships exist between the fundamental parameters. The most important result is the torus with $\beta_p < 0$ can only reach equilibrium, and consequently, the current and magnetic structure for $\beta_p < 0$ torus lead the loop stable as we observed on the Skylab.

ACKNOWLEDGEMENT

The author wishes to thank Professor Bruno Coppi for his helpful discussions.

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