An Energy Release Mechanism of Current Loops in the Solar Atmosphere: Solar Flares and Corona Heating

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ABSTRACT

A mechanism is proposed to account for the energy release by a bipolar current loop in the solar atmosphere. This mechanism is electromagnetic in nature and utilizes the non-zero $\underline{J} \times \underline{B}$ forces. No dissipation of the magnetic energy by resistivity, magnetic reconnection, or circuit interruption is required. From the observed fact that bipolar current loops do exist in quasi-equilibrium in the solar corona, a class of equilibria is deduced. It is shown that these configurations cannot be force-free. A simple model current loop is used to illustrate this mechanism. It is found that some equilibrium loops are unstable to major radius perturbations, resulting in expansion of the loops. The condition for instability is given in terms of a circuit parameter ϵ. A critical current $I_{cr} \approx 10^{11}$A is found to exist such that a current loop with $I \gg I_{cr}$ can attain high supersonic velocities producing strong shocks while a current loop with $I < I_{cr}$ expands at slower subsonic velocities. As the loop expands, the $\underline{J} \times \underline{B}$ force converts the magnetic energy into thermal and particle kinetic energy in the regions immediately outside the loop. The time scale of the energy release is found to be tens of minutes for the supersonic case, corresponding to solar flares, and much longer for the subsonic case, corresponding to corona heating.

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I. INTRODUCTION

It is generally believed that the energy released in solar flares and corona heating is derived from the magnetic fields associated with currents in the corona. The fundamental unanswered question is the nature of the mechanisms which convert the magnetic energy into other forms of energy such as thermal and kinetic energies. For solar flares, the energy release takes place in tens of minutes. while for corona heating, the time scales may be much longer.

The recent Skylab observations show that a large fraction (up to 80%) of solar flares are associated with bipolar current loops. A typical loop has a major radius of $10^4 - 10^5$ km (solar radius $R \approx 7 \times 10^5$ km), minor radius $10^3 - 10^4$ km, and is immersed in the corona with temperature $\approx 2 \times 10^6$ K and densities $10^{10} - 10^{12}$ particles/cm^3. The energy released is $10^{28} - 10^{32}$ erg. One important observed property of such toroidal structures is that they may be stable for up to a day.

Previous flare theories have invoked current interruptions (Alfven and Carlqvist,1967), magnetic reconnection (Vasyliunus,1975), and resistive dissipation (anomalous resistivity) (DeJager and Svestka,1969) of the magnetic fields. For recent reviews on solar flares, see Sturrock (1980) and Svestka (1976). The need of anomalous resistivity arises from the fact that the classical conductivity in the corona is $\sigma \approx 10^{16}\,\mathrm{sec}^{-1}$, corresponding to a collision frequency of $\nu/\omega_{pe} \approx 10^{-7}$. However, due to the large inductance of the loop, the effective conductivity must be as low as $\sigma \approx 10^5\,\mathrm{sec}^{-1}$ to allow the loop to dissipate its energy on a time scale of 10 minutes, requiring resistivity (collision frequency) enhancement of eleven orders of magnitude.

Another frequently invoked asusmption is that of force–free equilibrium configurations ($\underline{J} \times \underline{B} = 0$), neglecting the gas pressure. Recently, a number of investigators (Giachetti, Van Hoven, and Chiuderi, 1977) have pointed out, on the basis of observation, the possible existence of non–zero pressure gradient across the minor radius. However, the non–zero pressure gradient is still considered negligible in considering the stability of the current loop and the energy conversion mechanism. In this paper, we point out that the positive pressure gradient along the minor radius is a necessary requirement for the existence of quasi–equilibrium toroidal current distribution. Based on this realization, we propose a specific mechanism in which the non–zero $\underline{J} \times \underline{B}$ force plays the central role in converting the magnetic energy into thermal and particle kinetic energies. Furthermore, this model does not require resistive heating (anomalous resistivity), current interruption and magnetic reconnection.

A related phenomenon of importance in the solar atmosphere is the heating of the corona to high temperatures ($\sim 2 \times 10^6$ K). Recently, a model for corona heating has been proposed in which resistive heating, magnetic reconnection, and wave heating are neglected (Book, 1980). This model, however, also uses a force-free configuration and buoyancy force. In the context of our present model, corona heating can be understood as due to current loops evolving on much slower time scales than those of flare-causing loops, while dissipating the magnetic energy by the non-zero $\underline{J} \times \underline{B}$ force. In this regard, important parameters I_{cr} and ϵ will be introduced. In particular, these two parameters determine the stability properties against major radius perturbations and the time scales of energy conversion.

II. THE STRUCTURE OF THE SOLAR CORONA.

In this section, we give a brief description of the solar corona and a typical current loop. Throughout this paper, we will refer to the values given below. The current loop has a major radius R, minor radius a, temperature $\overline{T}$, number density $\overline{n}$, and pressure $\overline{p}$. It is assumed to be at a height $\sim R$ from the photosphere. The ambient corona has a density n_a, temperature T_a and pressure p_a. The values of these parameters are as follows:

$$
\begin{aligned}
R &= 10^{10}\,\text{cm},\\
a &= 10^{9}\,\text{cm},\\
\overline{p} &= \text{average internal pressure},\\
n_a &= 5 \times 10^{11}\,\text{cm},^{-3}\\
T_a &= 1.5 \times 10^{6}\,{}^\circ\text{K},\\
I_t &= 2 \times 10^{11}\text{A}.
\end{aligned}
$$

The corona pressure is thus $p_a = 2n_a kT_a = 207\,\text{dyn}/\,\text{cm}^2$. The characteristic scale height H of the corona is

$$H = \frac{2kT_a}{m_i g} = 1.4 \times 10^{10}\,\text{cm},$$

where $g = 2.4 \times 10^4\,\text{cm}\,/\,\text{sec}^2$ is the gravitational acceleration in the region of the current loop. Thus, the corona pressure takes the form

$$p_a(\delta R) = p_0 \exp(-\delta R/H), \tag{1}$$

where p_0 is the pressure at the top of the current loop and δR is the distance from the loop measured along the major radius.

III. EQUILIBRIUM STRUCTURE OF A CURRENT LOOP.

We start our analysis from the fact that observation shows the existence of quasi-equilibrium bipolar current loops in the corona. In our analysis, we consider a single isolated loop as shown in Fig. 1. The electromagnetic and presure forces acting on a toroidal plasma is given by the following quation, well known in the study of fusion plasmas(Shafranov, 1966);

$$M\frac{dV}{dt} = \frac{\pi I_t^2}{c^2}\left[\ln\left(\frac{8R}{a}\right) + \beta_p - \frac{3}{2} + \frac{\ell_i}{2}\right] - F_d, \tag{2}$$

where

$$\begin{aligned}
M &= (\pi^2 a^2 R\overline{\rho}) = \text{total mass of the semi-circular loop,}\\
\overline{\rho} &= \text{average density of the loop,}\\
V &= \text{velocity of the loop along the major radius,}\\
I_t &= \text{total toroidal current,}\\
\ell_i &= \text{internal inductance}\left[\ell_i \equiv \frac{2\pi\int dr\, rB_p^2}{\pi a^2 B_p^2(a)}\right],\\
F_d &= \text{drag force.}
\end{aligned}$$

In the above expression, only the force acting on the loop above the photosphere is included. The quantity β_p is defined as follows in terms of the average internal pressure $\overline{p}$ and the poloidal magnetic field B_p measured at the edge of the minor radius;

$$\beta_p = \frac{\overline{p} - p_a}{B_p^2/8\pi}. \tag{3}$$

The equilibrium condition is obtained by setting and $dV/dt = 0$. Thus,

$$\beta_p = -\ln(8\frac{R}{a}) - \frac{\ell_i}{2} + \frac{3}{2}. \tag{4}$$

Since ℓ_i is of order unity, we see that β_p is negative. Equation (3) shows that a necessary consequence of the existence of a toroidal current loop is that the average internal pressure be less than the corona pressure;

$$\overline{p} - p_a < 0. \tag{5}$$

Then, pressure gradients inside the current loop must be mostly positive along the minor radius and the frequently used force-free ($J \times B = 0$) assumption is inconsistent with an equilibrium toroidal loop in the corona. As an example, we take a surface current distribution for which $\ell_i = 0$. Then,

$$\beta_p = -2.9.$$

We deduce another important property of the current loop from the inequality (5). The equation for the equilibrium force balance along the minor radius is

$$\frac{dp}{dr} = \frac{1}{c}(J_p B_t - J_t B_p), \tag{6}$$

where J_p and J_t refer to the poloidal and toroidal current densities, and B_t and B_p refer to the toroidal and poloidal components of the magnetic field, respectively, as shown in Fig. 2. For a bipolar current loop with the toroidal current flowing in one direction, Eq. (6) shows that there must be a sufficiently strong poloidal current distribution in order for dp/dr to be positive inside the loop. In general, a relationship between B_t and B_p can be determined from Eq. (4) for a given equilibrium.

The negative β_p has two important consequences. One is that toroidal equilibria with sufficiently large $dp/dr > 0$ are MHD stable (Xue,1980). Secondly, $\beta_p < 0$ implies that the total electromagnetic force ($\underline{J} \times \underline{B}$) is radially outward along both the major and minor radii. The role this fact plays in energy conversion will be investigated in the following sections.

IV. THE EVOLUTION OF A CURRENT LOOP.

A. A Model Current Loop.

In this section, we use a simple model current distribution to describe the evolution of a current loop initially in quasi–equilibrium in the corona. In order to illustrate the basic mechanism unambiguously, we choose a simple current profile consistent with the requirements obtained in the preceding section.

Let the current be distributed over a thin layer of thickness δ at the minor radial boundary $r = a$. We define I_t and I_p by the following expressions.

$$I_t \equiv 2\pi \int_0^a dr\, r J_t,$$

and

$$I_p \equiv 2\pi R \int_0^a dr\, J_p.$$

In determining the equilibrium pressure profile, we approximate the large aspect ratio ($R/a \approx 10$) loop by a straight cylinder. The equilibrium force balance is given by

$$\frac{d}{dr}(P + \frac{B_t^2 + B_p^2}{8\pi}) = -\frac{1}{4\pi}\frac{B_p^2}{r}. \tag{7}$$

Inside the current loop, we have

$$\begin{cases} B_t = (\frac{2I_p}{cR}), \\ B_p = 0, \qquad\qquad r < a \\ p = \overline{p}, \end{cases} \tag{8}$$

and outside the current loop, we have

$$\begin{cases} B_t = 0, \\ B_p = (\frac{2I_t}{ca})\frac{a}{r}, \qquad\qquad r > a \\ p = p_a, \end{cases} \tag{9}$$

where p_a is matched to the corona pressure. Integrating Eq. (7) across the boundary at $r = a$, we can relate $\overline{p}$ and p_a. Expressing this jump condition in terms of β_p [Eq. (3)], we find

$$\beta_p = 1 - \frac{B_t^2(0)}{B_p^2(a)}. \tag{10}$$

In this surface current model, we have $\ell_i = 0$. From Eq. (4), we determine the equilibrium value of β_p

$$\beta_p = -2.9,$$

where $R/a = 10$ is used. Thus,

$$\overline{p} - p_a = -2.9(\frac{B_p^2(a)}{8\pi}),$$

$$\frac{B_t(0)}{B_p(a)} = 1.97,$$

and

$$\frac{I_p}{I_t} = \frac{R}{a}(1 - \beta_p)^{1/2} = 19.7.$$

From the estimated value of $I_t \sim 2 \times 10^{11}$A, we find

$$B_p(a) \sim 40\text{gauss},$$

$$B_t(0) \sim 78.8\text{gauss},$$

$$\overline{p} = p_a - (\frac{B_t^2}{8\pi} - \frac{B_p^2}{8\pi}) = 22.4\text{dyn}/\text{cm}^2,$$

$$p_a = 207\text{dyn}/\text{cm}^2,$$

and

$$I_p = 3.9 \times 10^{12}\text{A}.$$

From these values, we estimate the total magnetic energy of the system.

$$\epsilon_t = \frac{B_t^2}{8\pi}(\pi^2 a^2 R) = 2.4 \times 10^{31}\,\mathbf{erg},$$
$$\epsilon_p = \frac{1}{4}L_t I_t^2 = 3.0 \times 10^{31}\,\mathbf{erg},$$
$$\epsilon_T = \epsilon_t + \epsilon_p = 5.4 \times 10^{31}\,\mathbf{erg}\,. \tag{11}$$

where L is the self inductance of the circular current loop given by

$$L = \frac{4\pi R}{c^2}[\ln(8\frac{R}{a}) - 2] = 3.33 \times 10^{-10}\,\mathrm{cm}^{-1}\,\mathrm{sec}^{2}\,. \tag{12}$$

Here, only the energy associated with the loop above the photosphere is included. It is of interest to note that Eq. (2) can be reproduced from Eq. (11) using the principle of virtual work.

From Eq. (11), we find

$$F_{EM} = \frac{\partial \epsilon_T}{\partial R} = \frac{\pi I_t^2}{c^2}[\ln(\frac{8R}{a}) + \frac{1}{2}\beta_p - \frac{3}{2}].$$

This is the electromagnetic force acting along the major radius. The partial differentiation with respect to R is carried out holding the currents constant so that the magnetic field is allowed to vary according to the changes in the circuit geometry. Note that the electromagnetic force depends on β_p. This simply means that the pressure difference $(\overline{p} - p_a)$ affects the current distribution which determines the electromagnetic force. If we consider the toroidal and poloidal energy content, we have

$$\frac{\partial \epsilon_t}{\partial R} = \frac{\pi I_t^2}{2c^2}(\beta_p - 1)$$

and

$$\frac{\partial \epsilon_p}{\partial R} = \frac{\pi I_t^2}{c^2}[\ln(\frac{8R}{a}) - 1].$$

This shows that during the major radius expansion, the poloidal magnetic field component B_p does work $(J_t B_p)$ on the current loop. Thus, B_P loses energy to the loop. On the other hand, the loop does work $(J_p B_t)$ on the toroidal magnetic field component so that B_t gains energy from the loop motion. The net result is a loss of energy from the magnetic field.

During the minor radius expansion associated with the major radius expansion, the B_p component gains energy while the B_t component loses energy to the fluid elements moving radially outward. The net loss by the magnetic field is

$$-\beta_p \frac{\pi I_t^2}{c^2}(\frac{R}{a})\delta a$$

where a small displacement δa is considered.

If we calculate the radially inward pressure force acting on the current layer, we find the total pressure force F_p is given by

$$F_p = \frac{\pi I_t^2}{2c^2} \beta_p.$$

We see that Eq. (2) contains the net electromagnetic force and pressure force acting along the major radius. During the major radius expansion, the loop does work against the pressure force (Fig. 2).

B. Evolution of the Model Current Loop.

For the evolution of the model current loop, we consider the forces acting on the current loop by perturbing the major radius by a small amount from the initial equilibrium position. Then, Eq. (1) shows that the corona pressure $p_a(R)$ changes by

$$\delta p_a = -\frac{\delta R}{H} p_a \tag{13}$$

where p_a refers to thte coronal pressure at the equilibrium position and H is the scale height given in Sec. II. It should be noted, however, that although the small displacment δR is applied uniformly to the major radius, the feet of the current loop are fixed in the photosphere as shown in Fig. 1. The electromagnetic force acting on this loop is calculated from Eq. (2);

$$M\frac{dV}{dt} = \frac{\pi I_t^2}{c^2}(\frac{\delta R}{R} - \frac{\delta a}{a} + \delta\beta_p) \tag{14}$$

where $M = \pi^2 a^2 R\overline{p}$, $V = d(\delta R)/dt$, and $\delta a =$ change in the minor radius. The quantity $\delta\beta_p$ is [Eq. (3)]

$$\delta\beta_p = \frac{\delta\overline{p} - \delta p_a}{B_p^2/8\pi} - 2\beta_p\frac{\delta B_p}{B_p} \tag{15}$$

where $\delta\overline{p}$ is the change in the average internal pressure.

Due to the small resistive dissipation, there are a number of conservation relationships. One is

$$B_t a^2 \approx \text{const} \tag{16}$$

and another is

$$L_T I_t \approx \text{const} \tag{17}$$

where L_T is the self inductance of the entire circuit including the part below the photosphere. The change in I_t actually depends on the geometry of the entire circuit. If the bipolar loop is only a small part of the total circuit, then I_t remains nearly unchanged. In this case, we set

$$I_t \simeq \text{const.} \tag{18}$$

Moreover, we assume that the current loop is thermally well insulated from the corona on the relevant time scale. This gives the adiabatic expansion law

$$\overline{p}\overline{V}^{5/3} = \text{const},$$

and

$$\delta\overline{p} = -\frac{10}{3}\overline{p}\frac{\delta a}{a} - \frac{5}{3}\overline{p}\frac{\delta R}{R}. \tag{19}$$

where $\overline{V} = \pi^2 a^2 R$ is the volume of the loop.

Using the definition of β_p,

$$\begin{aligned}\delta\overline{p} - \delta p_a &= -\frac{5}{3}(2\frac{\delta a}{a} + \frac{\delta R}{R}) + (p_a\frac{R}{H})\frac{\delta R}{R} \\ &= 2[\frac{B_p^2}{8\pi}(\frac{\delta B_p}{B_p}) - \frac{B_t^2}{8\pi}(\frac{\delta B_t}{B_t})].\end{aligned}$$

Thus, we obtain

$$\frac{\delta a}{\delta R} = \frac{a}{R}\frac{p_a R/H - (5/3)\overline{p}}{4(B_p^2/8\pi)(1/2 - \beta_p) + (10/3)\overline{p}}. \tag{20}$$

For the loop described in Sec. II,

$$\frac{\delta a}{\delta R} \simeq 0.12(\frac{a}{R}).$$

This shows that the minor radius expands at approximately $\mathbf{0.12}$ of the major radius expansion rate. Thus, all the fluid elements inside the current loop have radially outward velocities ($\delta R > 0$ for the time being) along the major radius.

Using Eq. (18), we obtain

$$\frac{\delta B_p}{B_p} = -\frac{\delta a}{a},$$

and we find

$$\delta\beta_p = 2(1 - \beta_p)(\frac{\delta a}{a}), \tag{21}$$

where $\overline{p}$ is the average internal pressure and p_a is the corona pressure in the equilibrium position. Using Eq. (20), we find

$$\delta\beta_p = 2(1 - \beta_p)\left(\frac{p_a R/H - (5/3)\overline{p}}{4(B_p^2/8\pi)(1/2 - \beta_p) + (10/3)\overline{p}}\right)\frac{\delta R}{R}. \tag{22}$$

Thus, Eq. (14) becomes

$$M\,\frac{dV}{dt} \simeq \frac{\pi I_t^2}{c^2}\left[1+(1-2\beta_p)\,\frac{p_a R/H-(5/3)\overline{p}}{4(B_p^2/8\pi)(1/2-\beta_p)+(10/3)\overline{p}}\right]\left(\frac{\delta R}{R}\right), \tag{23}$$

where $V = d(\delta R)/dt$. For the current loop described in Sec. II, we have

$$\frac{dV}{dt} \approx 1.8\left(\frac{\pi I_t^2}{c^2}\right)\left(\frac{\delta R}{R}\right).$$

and the current loop is unstable to a small displacement along the major radius. Since the quantity in the square brackets in Eq. (23) is $(\delta R/R - \delta a/a + \delta\beta_p)$, it is always positive [Eq. (20)].

Integrating Eq. (23) with respect to time, we obtain

$$\delta R = \delta R_0 \exp(\frac{t}{\tau}),$$

and

$$V = \frac{\delta R_0}{\tau}\exp(\frac{t}{\tau}),$$

where

$$\tau = \left(\frac{Mc^2 R}{\pi I_t^2}\right)^{1/2}\left[1+(1-2\beta_p)\,\frac{p_a R/H-(5/3)\overline{p}}{4(B_p^2/8\pi)(1/2-\beta_p)+(10/3)\overline{p}}\right]^{-1/2}. \tag{24}$$

Thus, the major radius expands with the e–folding time τ. For the model current loop describe in this paper

$$\tau \simeq 270\,\text{sec}\,.$$

Here, the interior number density $\overline{n}$ is taken to be 10^{11} cm^{-3}. It is of interest to note that $\tau \sim I_t^{-1}$. A current loop with smaller I_t e–folds more slowly.

In the above analysis, we set $I_t =$ constant. This is equivalent to stating that the observed current loop in the corona is a small section of a much larger circuit imbedded below the photosphere. If, however, the structure of the entire circuit is such that the current I_t can change significantly, then Eq. (17) leads to

$$\frac{\delta I_t}{I_t} = \epsilon\left[-\frac{\ln(8R/a)-1}{\ln(8R/a)-2}\frac{\delta R}{R} + \frac{1}{\ln(8R/a)-2}\frac{\delta a}{a}\right], \tag{25}$$

where the important quantity ϵ is a function of the ratio of the magnetic flux encircled by the loop above the photosphere and the flux enclosed by the entire circuit. The value of ϵ ranges from zero to unity. If we set $\epsilon = 1$ in Eq. (25), then

$$\frac{\delta a}{\delta R} = \frac{a}{R}\left[2\left(\frac{B_p^2}{8\pi}\right)\frac{\ln(8R/a)-1}{\ln(8R/a)-2} + p_a R/H - (5/3)\overline{p}\right]\left[2(\frac{B_p^2}{8\pi})\left(\frac{1}{\ln(8R/a)-2}+1-2\beta_p\right)+\frac{10}{3}\overline{p}\right]^{-1},$$

and

$$\delta\beta_p = 2(1-\beta_p)\Big[-\frac{\ln(8R/a)-1}{\ln(8R/a)-2} + \Big(1+\frac{1}{\ln(8R/a)-2}\Big)\frac{R}{a}\Big(\frac{\delta a}{8R}\Big)\Big]\frac{\delta R}{R}.$$

This shows that in the limit $\epsilon = 1$, $\delta\beta_p$ is negative. Substituting the above expression into Eq. (14), we find that $(\delta R/R - \delta a/a + \delta\beta_p) < 0$, indicating that the force tends to restore the displacement. Therefore, for $\epsilon = 1$, the current loop is in stable equilibrium against major radius perturbations. Such a loop may remain in quasi-equilibrium for an extended period of time. We conclude that a critical value ϵ_{cr} must exist such that

$$0 \leq \epsilon_{cr} \leq 1, \tag{26}$$

corresponding to marginal stability. For a typical loop described in Sec. II, $\epsilon_{cr} \approx 0.2$.

We deduced the characteristic time τ [Eq. (24)] for the major radius of a current loop to e–fold from the basic laws of physics (for $\epsilon = 0$ case). The exponential expansion is valid for δR such that $(\delta R/H) \ll 1$, where $H \simeq 1.4 \times 10^{10}$ cm. So far, the drag force F_d has not been specified. From a simple consideration, we obtain

$$F_d = c_d(2\pi a R V^2 n_a m_i),$$

where c_d is the drag coefficient and n_a is the local corona density. This expression shows that the drag force is determined by $n_a V^2$. At the same time, the parameter ϵ increasees as the major radius increases, which tends to reduce the electromagnetic force. In conjunction with Eq. (2), we see that in some cases, the loop velocity may attain a quasi–saturation level. Thus, the evolution of a current loop is determined by the competing effects of $n_a V^2$ and ϵ. In particular, if a quasi–saturation level is reached in a region, then we can estimate the order of magnitude of the saturation velocity V_* by ($c_d \approx 0.5$ from aerodynamic considerations)

$$V_* \approx \left[\frac{I_t^2}{m_i c^2 n_a a R}\right]^{1/2}. \tag{27}$$

For example, $I_t = 2 \times 10^{11}$A, and we have, with $n_a \sim 10^{11}$ cm^{-3},

$$V_* \sim 10^8 \,\mathrm{cm\,/\,sec}.$$

The speed of sound in the corona is

$$C_s \simeq (\frac{5}{3} \times \frac{2nkT}{nm_i})^{1/2} \approx 2 \times 10^7 \,\mathrm{cm\,/\,sec}.$$

Thus, it is possible for the current loop to attain and exceed the sonic velocity. Note that

$$V_* \sim \frac{I_t}{n_a^{1/2}}$$

This shows that loops with low toroidal currents may not attain the speed of sound and loops with high currents may significantly exceed the speed of sound. We see that there must be a critical current

$$I_{cr} \approx 10^{11}\text{A} \tag{28}$$

which separates the subsonic and supersonic current loops. Moreover, the saturation velocity V_* decreases for increasing n_a. Thus, the loops tend to rise more slowly in the lower corona and the chromosphere.

C. Shock Waves.

If a current loop attains supersonic velocities, shocks are formed in the corona in front of the expanding loop. The shock waves propagate with velocity V_* with considerable heating in the region behind the shock front. Thus, heating takes place mainly outside the current loop. In particular, the region near the apex of the loop undergoes the most heating.

If a shock wave is formed with a velocity V_s, then we can define

$$M_s = \frac{V_s}{C_s},$$

where M_s is the Mach number and C_s is the sound velocity in front of the shock. Since the shock frontmoves with the current loop ($V_s = V_*$), the temperature T_* behind the shock front can be determined by (Landau and Lifshitz, 1959)

$$\frac{T_*}{T_a} = \frac{[2\gamma M_s^2 - (\gamma - 1)][(\gamma - 1)M_s^2 + 2]}{(\gamma + 1)^2 M_s^2}, \tag{29}$$

where the adiabatic index γ is $5/3$. In this example, $M_s \simeq 5$ and $T_*/T_a \simeq M_1^2/3 \approx 8$. With $T_a \simeq 1.5 \times 10^6$ K, we obtain

$$T_* \simeq 1.2 \times 10^7 \text{ K}.$$

D. Energy Conversion.

It is of interest to estimate the rate at which the magnetic energy is released. For the current loops which eventually attain high supersonic velocities ($M_s \sim 5$) ($\epsilon \ll 1$ and $I_t \gg I_{cr}$), the magnetic energy is converted into thermal and kinetic energy largely by the shock waves. When the saturation velocity is attained, the electromagnetic force is equal to the drag force. Thus, the rate at which the magnetic energy is dissipated by the electromagnetic force is given by Eq. (2) [with the quantity in the square brackets of order unity] multiplied by the saturation velocity V_*,

$$\frac{\pi I_t^2}{c^2} V_* \sim 1.3 \times 10^{29}\,\mathrm{erg}\,/\,\mathrm{sec}\,.$$

Here, $V_* \sim 1 \times 10^8\,\mathrm{cm}\,/\,\mathrm{sec}$ and $I_t \sim 2 \times 10^{11}\mathrm{A}$. We see that a large fraction of the magnetic energy can be dissipated through shocks on the time scale of hundreds of seconds during the supersonic expansion stage.

For the slower subsonic expansion, we note that the rate of magnetic energy dissipation is proportional to V_*^3 (Sec. IV.B) and that the saturation velocity V_* for the subsonic expansion is one order of magnitude lower. Thus, the energy dissipation rate is over three orders of magnitude lower than that of the supersonic case. We attribute corona heating and solar wind acceleration to energy released by slowly expanding current loops.

V. DISCUSSIONS

In the preceding sections, we investigated the forces acting on a current loop immersed in the solar atmosphere (Fig. 1). It was shown that a class of toroidal equilibria exists in the corona and that these configurations are *not* force–free, characterized by $\beta_p < 0$, corresponding to the fact that the average pressure gradient dp/dr is positive inside the loop. This means that the electromagnetic force is radially outward along both the major and minor radii. In equilibrium, the electromagnetic force is balanced by the pressure force. If the major radius expands, then the magnetic field does work to the current loop.

Using a simple model current loop, it was shown that the geometry of the *entire* circuit, including the part below the photosphere, is critically important in determining the evolution of the loop. In this regard, we introduced a parameter ϵ which is a measure of the ratio of the magnetic flux enclosed by the loop above the photosphere to the flux through the entire circuit. The value of ϵ ranges from zero to one. For $\epsilon \lesssim 1$, the exposed flux is comparable to the total flux of the circuit and for $\epsilon \ll 1$, the exposed flux is a small fraction of the total flux. There exists a critical value ϵ_{cr} [Eq. (26)] such that for $\epsilon > \epsilon_{cr}$, the current loop is a stable equilibrium against major radius perturbations. Such a loop may remain in quasi–static equilibrium for an extended period of time. For $\epsilon < \epsilon_{cr}$, the current is an unstable equilibrium. Such unstable current loops can be further divided into two categories. If the toroidal current I_t of a loop is less than a critical current $I_{cr} \sim 10^{11}$A [Eq. (28)], then the loop expands at subsonic velocities. In this case, the rate of energy conversion from the magnetic field is relatively slow. We attribute corona heating and solar wind acceleration to the energy released by such slowly expanding current loops.

If $I_t \gg I_{cr}$, then the loop can attain high supersonic velocities, developing shock waves. In this case, the corona gas in front of the expanding loop can be heated to more than $10^7\,^oK$, dissipating the magnetic energy in tens of minutes (Sec. IV.C). We attribute solar flares to these supersonic current loops. In the context of this model, the dissipated energy manifests itself mainly outside the current loop.

In summary, we have proposed a new model for the evolution of bipolar current loops in the solar atmosphere. The model can account for the behavior of current loops with a wide range of parameters; stable current loops, unstable subsonic loops with low rates of energy release and supersonic loops with high rates of energy conversion. Furthermore, we have seen that the evolution of a current is not only influenced by what is above the photosphere but also by what is below. Detailed numerical calculations are being carried out and the results will be reported in a future publication.

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FIGURE CAPTIONS

Fig. 1. A typical bipolar current loop in the solar atmosphere. $R \approx 10^{10}$ cm and $a \approx 10^{9}$ cm. V is the expansion velocity of the loop and V_s is the shock velocity. The components of the magnetic field (B_t, B_p) and the components of the current (J_t, J_p) are indicated.

Fig. 2. The forces acting on the current loop. V is the expansion velocity of the loop, F_p is the pressure force and F_{EM} is the electromagnetic force.

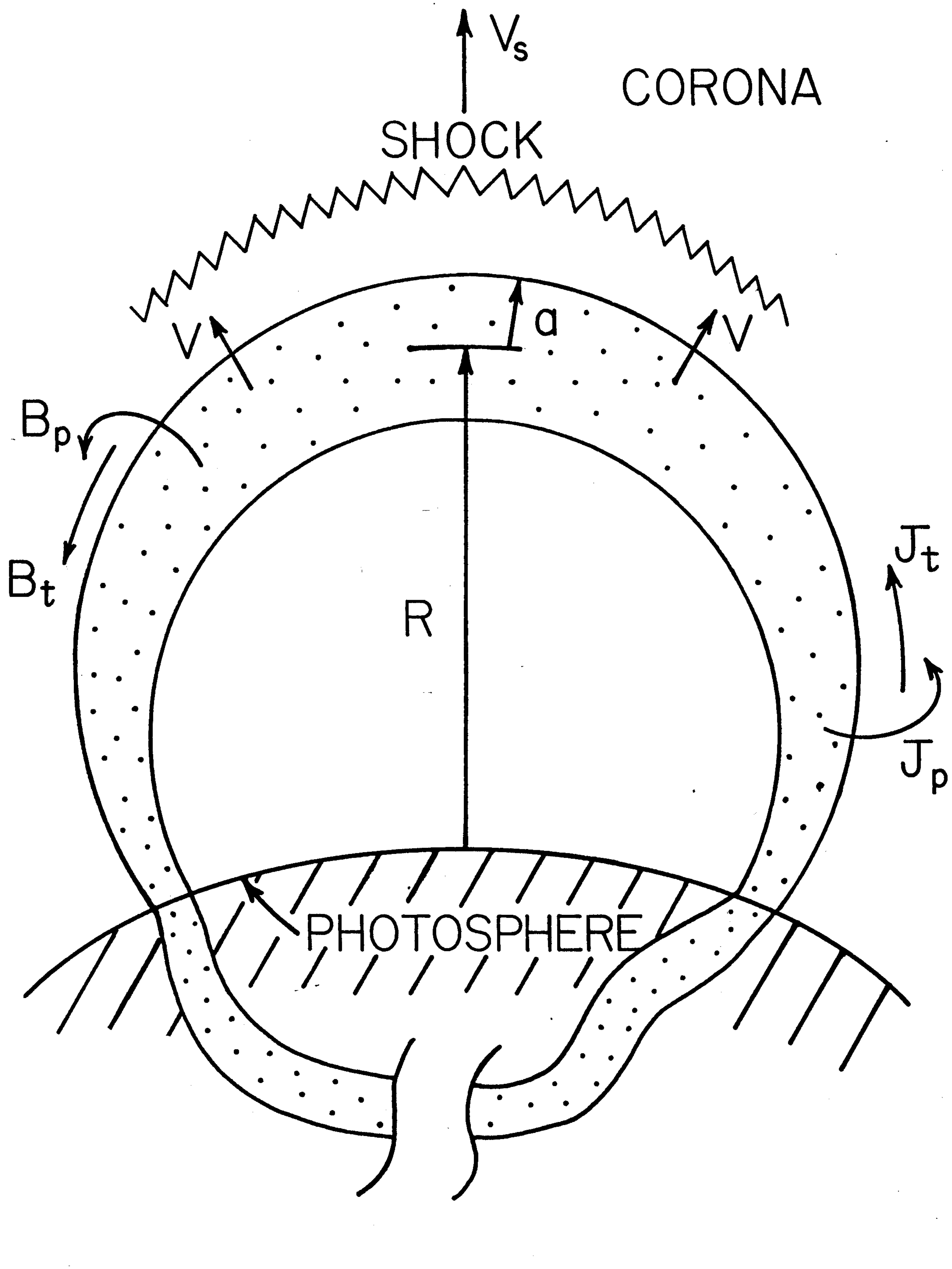

Fig. 1

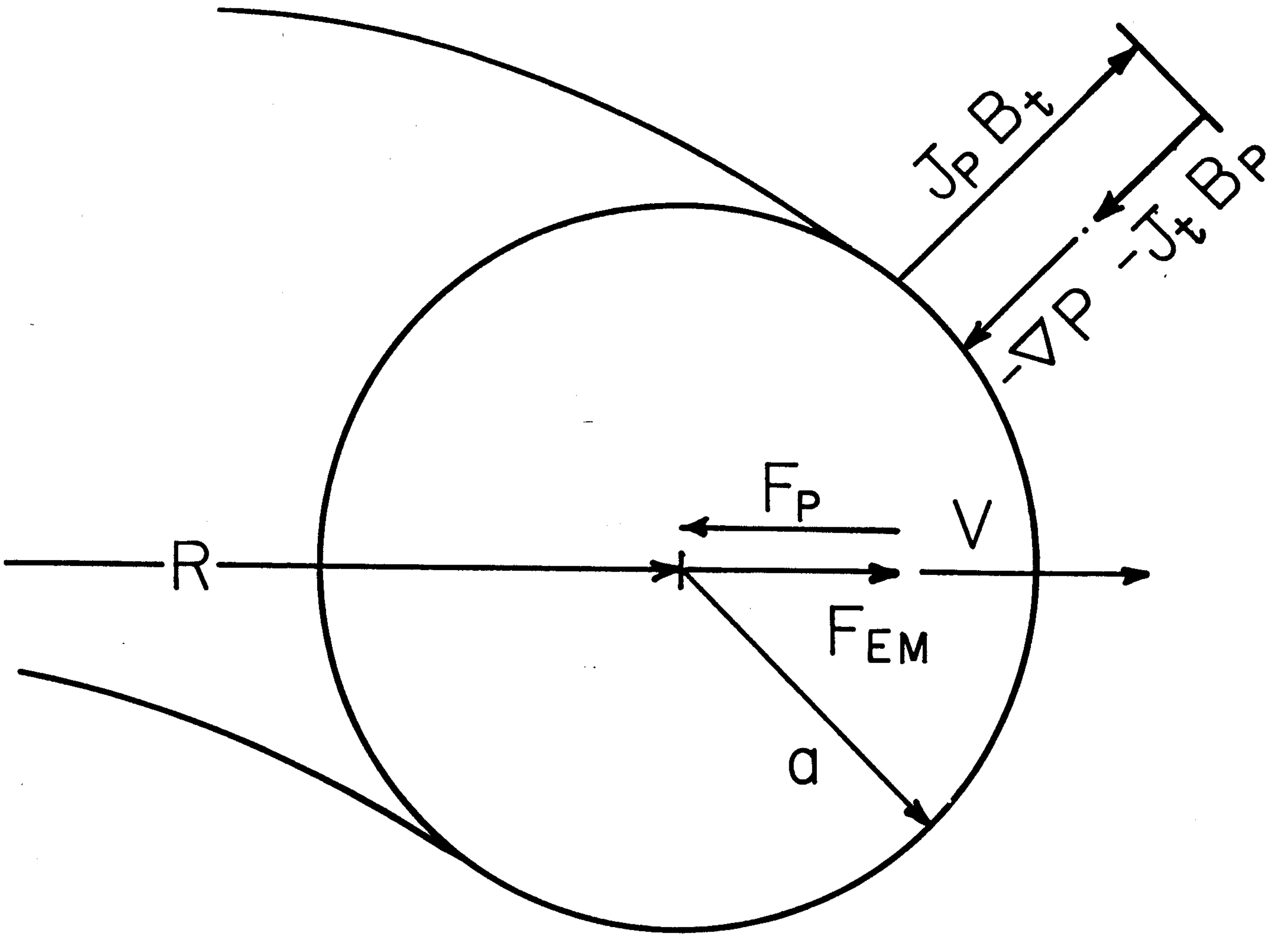

Fig. 2