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Scattering of radio frequency waves by edge density blobs in tokamak plasmas

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Scattering of radio frequency waves by edge density blobs in tokamak plasmas'

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Abstract

The density blobs present in the plasma edge in magnetic fusion devices can scatter radio frequency (RF) waves through refraction. Using the geometric optics approximation for the waves, a Fokker-Planck equation for the scattering of rays by a random distribution of blobs is derived. It is found that the scattering can diffuse the rays in space and in wave-vector space. The diffusion in space can make the rays miss their intended target region, and the diffusion in wave-vector space can broaden the wave spectrum and modify the wave damping profile. For ITER-type plasmas the wave scattering can lead to the electron cyclotron beams missing their intended target region of growing neoclassical tearing modes.

Introduction

In fusion plasmas RF waves in the electron cyclotron (EC) and lower hybrid (LH) range of frequencies are commonly used to modify the current profile. In ITER, EC waves are expected to stabilize the neoclassical tearing mode (NTM) by providing current in the island region. LH waves could be used in ITER to modify the current profile closer to the edge of the plasma. These RF waves propagate from the excitation structures to the core of the plasma through an edge region which is characterized by turbulence – in particular, density fluctuations [1]. These fluctuations, in the form of blobs [2], can modify the propagation properties of the waves by refraction and diffraction [3]. In this paper we formulate the theory for scattering of geometric optic rays by randomly distributed density blobs in the edge region. In an ITER-type of plasma environment, we find that for EC waves spatial diffusion is important. Since the distance of propagation from the edge to the core of the plasma is of the order of a meter, the EC waves could be deflected away from the intended target region. The diffusion of EC waves in wave vector space could lead to a reduction in the current drive efficiency.

Fokker-Planck model for RF interaction with density blobs

The Hamilton-Jacobi ray equations for the wave-vector $\mathbf{k}$ and spatial position $\mathbf{r}$ at time t are [4]

$$\frac{d\mathbf{k}}{dt} = -\nabla_{\mathbf{r}}\omega, \qquad \frac{d\mathbf{r}}{dt} = \nabla_{\mathbf{k}}\omega, \qquad \frac{d\omega}{dt} = \frac{\partial\omega}{\partial t} \tag{1}$$

where $\omega(\mathbf{r},\mathbf{k},t)$ is the frequency of the RF wave. If $\eta = ck/\omega$ is the refractive index with $k = |\mathbf{k}|$, and we allow for η to be function of the density alone, then

$$\eta = \eta\Big(\Big\{\omega_{p\alpha}^2(n_\alpha(\mathbf{r}))\Big\},\theta\Big), \qquad \omega_{p\alpha}^2(n_\alpha(\mathbf{r})) = \frac{q_\alpha^2 n_\alpha(\mathbf{r})}{\epsilon_0 m_\alpha}, \qquad \theta = \cos^{-1}\left(\frac{\mathbf{k}\cdot\mathbf{i_z}}{k}\right) \tag{2}$$

where q_α and m_α are the charge and mass, respectively, of species α, and θ is the angle of propagation relative to the direction of the magnetic field (along the z-direction $\mathbf{i_z}$). We have assumed that the density blobs are localized in the edge region over radial distances that are small compared to the scale length of the magnetic field. So the magnetic field across the blobs is assumed to be uniform. The spatial dependence is only through the density fluctuations $n(\mathbf{r})$ in the plasma frequency $\omega_{p\alpha}$. The density in the edge region is assumed to be a sum of a uniform background density and a spatially fluctuating density corresponding to the blobs. The magnitude of the fluctuating density is taken to be small compared to the background density and used as an expansion parameter for the refractive index. Then, to first order

$$n_\alpha(\mathbf{r}) \approx n_{\alpha 0} + \delta n_\alpha(\mathbf{r}), \qquad \eta \approx \eta_0 + \frac{1}{\epsilon_0}\sum_\alpha \frac{q_\alpha^2}{m_\alpha}\frac{\partial \eta_0}{\partial \omega_{p\alpha 0}^2}\delta n_\alpha(\mathbf{r}) \tag{3}$$

The Hamilton-Jacobi ray equations become

$$\frac{d\mathbf{k}}{dl} \equiv \dot{\mathbf{k}} \approx \frac{k}{\epsilon_0\eta_0^2}\sum_\alpha \frac{q_\alpha^2}{m_\alpha}\left(\frac{\partial\eta_0}{\partial\omega_{p\alpha 0}^2}\right)\nabla_{\mathbf{r}}(\delta n_\alpha) \tag{4}$$

$$\frac{d\mathbf{r}}{dl} \equiv \dot{\mathbf{r}} \approx \frac{\mathbf{i_k}}{\eta_0} + \frac{1}{\eta_0^2\sin\theta}(\mathbf{i_z}\cdot\mathbf{I_T})\left[\frac{d\eta_0}{d\theta} + \frac{1}{\epsilon_0}\sum_\alpha \delta n_\alpha \frac{q_\alpha^2}{m_\alpha}\frac{d}{d\theta}\left(\frac{\partial\eta_0}{\partial\omega_{p\alpha 0}^2}\right)\right] \tag{5}$$

where $l = ct$ is a measure of the distance along the ray, c is the speed of light, $\mathbf{I_T} = \mathbf{I} - \mathbf{i_k}\mathbf{i_k}$ is a dyadic with $\mathbf{I}$ being the unit tensor and $\mathbf{i_k}$ being the unit vector along $\mathbf{k}$. From these equations it is easy to derive the correlation functions $\left\langle \dot{\mathbf{k}}(\mathbf{r};\mathbf{k})\dot{\mathbf{k}}(\mathbf{r}+\mathbf{i_s}\sigma;\mathbf{k})\right\rangle$, $\left\langle \dot{\mathbf{r}}(\mathbf{r};k)\dot{\mathbf{r}}(\mathbf{r}+\mathbf{i_s}\sigma;\mathbf{k})\right\rangle$, $\left\langle \dot{\mathbf{k}}(\mathbf{r};k)\dot{\mathbf{r}}(\mathbf{r}+\mathbf{i_s}\sigma;\mathbf{k})\right\rangle$, and $\left\langle \dot{\mathbf{r}}(\mathbf{r};k)\dot{\mathbf{k}}(\mathbf{r}+\mathbf{i_s}\sigma;\mathbf{k})\right\rangle$, along the ray path for a statistical ensemble of blobs. Here $\mathbf{i_s}$ is the unit vector along the direction of the group velocity, and $<$ ..... $>$ is an ensemble average over a distribution of blobs. We can then define diffusion tensor

$$\mathbf{D}(\mathbf{X}) = \frac{1}{\Delta s}\int_9^{\Delta s} ds \int_{-s}^{\Delta s - s} d\sigma \left\langle \dot{\mathbf{X}}'[\mathbf{r}(s);\mathbf{k}]\ \dot{\mathbf{X}}'[\mathbf{r}(s)+\mathbf{i_s}\sigma;\mathbf{k}]\right\rangle \tag{6}$$

$$\approx \int_{-\infty}^{\infty} d\sigma \left\langle \dot{\mathbf{X}}'[\mathbf{r}(s);\mathbf{k}]\ \dot{\mathbf{X}}'[\mathbf{r}(s)+\mathbf{i_s}\sigma;\mathbf{k}]\right\rangle \tag{7}$$

where $\mathbf{X} = (\mathbf{r},\mathbf{k})$ is a six-dimensional phase space vector and Δs is the correlation length for the blobs. The prime refers to the rate of change of the fluctuations and is given by

$$\dot{\mathbf{X}}' = \left[\frac{1}{\epsilon_0\eta_0^2\sin\theta}(\mathbf{i_z}\cdot\mathbf{I_T})\sum_\alpha \delta n_\alpha \frac{q_\alpha^2}{m_\alpha}\frac{d}{d\theta}\left(\frac{\partial\eta_0}{\partial\omega_{p\alpha 0}^2}\right) + \frac{k}{\epsilon_0\eta_0^2}\sum_\alpha \frac{q_\alpha^2}{m_\alpha}\left(\frac{\partial\eta_0}{\partial\omega_{p\alpha 0}^2}\right)\nabla_{\mathbf{r}}\delta n_\alpha\right] \tag{8}$$

The evolution equation for a distribution of rays $f(\mathbf{X}, l)$ is

$$\frac{\partial f}{\partial l} + \left[\mathbf{i_k} + \frac{1}{\eta_0^2 \sin\theta}(\mathbf{i_z}\cdot\mathbf{I_T})\frac{d\eta_0}{d\theta}\right]\cdot\nabla_{\mathbf{r}} f = \frac{\partial}{\partial \mathbf{X}}\cdot\left[\mathbf{D}(\mathbf{X})\cdot\frac{\partial f}{\partial \mathbf{X}}\right] \tag{9}$$

Ray scattering in the cold plasma approximation

The angle of propagation of an electromagnetic wave in a cold plasma is [5]

$$\tan^2\theta = \frac{P\left(\eta_0^2 - L\right)\left(\eta_0^2 - R\right)}{\left(LR - S\eta_0^2\right)\left(\eta_0^2 - P\right)} \tag{10}$$

where

$$P = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}, \qquad R = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega(\omega + \omega_{c\alpha})}, \qquad L = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega(\omega - \omega_{c\alpha})} \tag{11}$$

$S = (R+L)/2$, $D = (R-L)/2$, and $\omega_{c\alpha}$ is the cyclotron frequency.

For a ray defined initially as a delta function in $\mathbf{k}$ space, the RMS broadening of the three components of $\mathbf{k}$ can be determined analytically for an isotropic turbulence in randomly distributed spherical blobs with random radii [3]. We find

$$\begin{bmatrix} \Delta k_x^{RMS} \\ \Delta k_y^{RMS} \\ \Delta k_z^{RMS} \end{bmatrix} = \frac{2\sqrt{6}}{9}\sqrt{L_0 D_0}\begin{bmatrix} \sqrt{3} \\ \sqrt{3 - \sin^2\alpha} \\ \sqrt{3 - \cos^2\alpha} \end{bmatrix} \tag{12}$$

where the x and y directions are perpendicular to the z direction along the magnetic field, L_0 is the distance of propagation along the ray,

$$D_0 = \frac{1}{4\sqrt{3}\eta_0^4 V_0}\left(\frac{\pi \Delta r \omega}{c}\right)^2 \left(\nu_0^2 + \delta_0^2\right)\sum_{\alpha,\beta}\omega_{p\alpha 0}^2\omega_{p\beta 0}^2\left(\frac{\partial \eta_0^2}{\partial \omega_{p\alpha 0}^2}\right)\left(\frac{\partial \eta_0^2}{\partial \omega_{p\beta 0}^2}\right) \tag{13}$$

is the scalar diffusion coefficient, ν_0 is a dimensionless random variable which is a measure of the density fluctuation in a blob, δ_0 is the standard deviation of the random density fluctuations, V_0 is a measure of the volume occupied by the blobs, Δr is the characteristic size of a blob, and

$$\alpha = \cos^{-1}\left[\frac{\eta_0^2\cos\theta + \frac{d\eta_0}{d\theta}\sin\theta}{\sqrt{\eta_0^4 + \left(\frac{d\eta_0}{d\theta}\right)^2}}\right], \tag{14}$$

$$\frac{d\eta_0^2}{d\theta} = \frac{2\eta_0^2\left(\eta_0^2 - L\right)\left(\eta_0^2 - R\right)\left[\eta_0^2(P-S) + LR - PS\right]}{\left[\eta_0^4(PS - 2S^2 + LR) - 2\eta_0^2(P-S)LR + PS\,LR - L^2R^2\right]\tan\theta} \tag{15}$$

From Eq. (12) we note that there is an effective transverse (to the magnetic field) broadening of the wave vector. This leads to an angular deflection of the ray by an angle $\Delta\alpha$ relative to the

path of the unperturbed ray,

$$|\Delta\alpha| \;=\; \left| 1+\eta_0 \frac{\eta_0 \frac{\partial^2\eta_0}{\partial\theta^2} - 2\left(\frac{\partial\eta_0}{\partial\theta}\right)^2}{\eta_0^4+\left(\frac{\partial\eta_0}{\partial\theta}\right)^2} \right| |\Delta\theta|. \tag{16}$$

For the extraordinary X-mode propagating across the magnetic field, $\eta^2 = RL/S$, while for the ordinary O-mode, $\eta^2 = P$. Their respective deflections are

$$\left|\frac{\Delta\alpha}{\Delta\theta}\right|_X \;=\; \left|1-\frac{D^2P}{16S\,(RL-PS)}\left(\frac{S}{RL}\right)^{1/2}\right|, \qquad \left|\frac{\Delta\alpha}{\Delta\theta}\right|_O \;=\; \left|1-\frac{(P-R)(P-L)}{\sqrt{P}\,(RL-PS)}\right| \tag{17}$$

We use the above results to study the diffusive effect of blob scattering on RF waves in ITER-like plasmas. For an electron-deuterium plasma we assume an edge magnetic field of 4.13 T, an edge electron density of 10^{19} m^{-3}, and the ECRF frequency to be 170 GHz. We further assume that the relative edge fluctuations are 20% of the edge density and the edge temperature is 200 eV. Then the ion Larmor radius ρ_i is 0.5 mm. Experimental evidence [6] suggests that the fluctuation spectrum peaks around $\Delta r/\rho_i \approx 15-30$, so that, at the edge, Δr can be as large as 1.5 cm. The number of blobs in the flux surface along the poloidal direction is $N_b = L_p/(2\Delta r)$ where L_p is the poloidal arc length. In the radial direction the number of blobs is $N_r = \Delta_b/(2\Delta r)$ where Δ_b is a measure of the radial distance travelled by the blobs. Estimates based on experiments suggest that Δ_b is between 3 cm and 15 cm. There are roughly $N_p = N_b \times N_r$ blobs in a poloidal plane occupying a volume $V_0 = 4\pi(\Delta r)^2 N_p/3$. Assuming $L_0 \approx 0.2-0.3$ m, $L_p \approx 0.6$ m, and that 20-30% of the radial propagation distance in ITER is populated by blobs, we find that the O-mode beam will be deflected between 5 mm to 15 mm per meter of ray propagation. The X-mode beam will be deflected between 1.5 cm and 7 cm per meter of ray propagation. The deflection increases as the edge density increases. It is possible that the deflection could be large enough so that EC waves miss the NTM island.

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