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Electron Transport by Radio Frequency Waves in Tokamak Plasmas

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Abstract. A relativistic kinetic description for momentum and spatial diffusion of electrons by radio frequency (RF) waves and non-axisymmetric magnetic field perturbations in a tokamak is formulated. The Lie perturbation technique is used to obtain a non-singular, time dependent evolution equation for resonant and non-resonant electron diffusion in momentum space and diffusion in configuration space. The kinetic equation for the electron distribution function is different from the usual quasilinear equations as it includes interactions that are non-Markovian. It is suitable for studying wave-particle interaction in present tokamaks and in ITER. A primary goal of RF waves, and, in particular, of electron cyclotron waves, in ITER is to control instabilities like the neoclassical tearing mode (NTM). Non-axisymmetric effects due to NTMs are included in the kinetic formalism.

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INTRODUCTION

The interaction of electrons with RF waves in tokamaks encompasses interesting and complex physics. The externally launched RF fields are localized, in space, and do not uniformly fill the toroidal plasma. So the dynamical phase space of the electrons, in general, is a composite of chaotic motion, motion in and around islands, and regular motion. Taking all of these aspects into account, we have derived the relativistic operator for momentum and spatial diffusion of electrons by RF waves and due to NTMs in a tokamak [1]. The operator is suitable for numerical implementation. It is not plagued by the resonant delta function which appears in previous formalisms [2, 3] and gets treated in an *ad hoc* fashion in numerical codes. The momentum diffusion leads to plasma current and the spatial diffusion affects the radial current profile.

DIFFUSION AND FOKKER-PLANCK DESCRIPTION

The evolution of the electron distribution function in the presence of RF waves is given by Fick's second law or the Fokker-Planck equation

$$\frac{\partial f(I)}{\partial t} = \frac{\partial}{\partial I}\left\{ D(I)\frac{\partial}{\partial I} f(I) \right\} \tag{1}$$

where I is a vector of the dynamical actions (or variables describing the electron dynamics), f is the distribution function, and D is the diffusion tensor. D contains all the necessary information about the interaction of electrons with waves. The time step that appears on the left hand side of (1) and the form of $D(I)$ are inputs that have to be determined independently. For this it is instructive to consider the standard map [4] which describes, in one-dimensional space, the interaction of a charged particle with an infinite set of plane electrostatic waves. The waves have the same wave number but their frequencies are integer multiples of a fundamental frequency.

$$\frac{dx}{dt} = v, \quad \frac{dv}{dt} = \frac{qE}{m} \sum_{n=-\infty}^{\infty} \sin(kx - n\omega t) \tag{2}$$

Here x and v are the position and velocity of a particle of charge q and mass m, respectively, k and ω are the wave vector and angular frequency, respectively, and E is electric field amplitude of each wave. The change in particle velocity and wave phase after every time step $T = 2\pi/\omega$ is given by

$$I_{n+1} = I_n + K \sin\theta_n \pmod{2\pi} \tag{3}$$
$$\theta_{n+1} = \theta_n + I_{n+1} \pmod{2\pi} \tag{4}$$

where $\theta = kx$, $I = kTv$ and $K = (2\pi/\omega)^2(qkE/m)$ is the normalized amplitude.

It is well known that as K increases the dynamical phase space becomes chaotic [4] and diffusion of the electrons in action space I ensues. If we make the Markovian assumption (Brownian motion) that the particle motion randomizes in a single step $I_0 \to I_1$, then we obtain the quasilinear diffusion

$$D_{QL} \equiv D_1 = \frac{\left\langle (I_1 - I_0)^2 \right\rangle}{2} = \frac{1}{4\pi} \int_0^{2\pi} d\theta_0 \ (I_1 - I_0)^2 = \frac{K^2}{4} \tag{5}$$

where $< .. >$ denotes an ensemble average. An important advantage of the Markovian assumption is that the long time diffusion ($n \to \infty$) is the same as for a single step. Thus, the time step on the left hand side of (1) corresponds to a single interaction time. On the right hand side of (1), $D = D_{QL}$ is independent of I. This is a property of the standard map for which the entire phase space is affected as K increases.

It turns out that the Markovian assumption is not valid for the standard map. As Rechester and White pointed out in [5], the diffusion coefficient is markedly different from D_{QL}. Only for very large, and impractical, K does the actual diffusion approach D_{QL}. In [5] the diffusion is defined by

$$D \equiv D_n = \frac{\left\langle (I_n - I_0)^2 \right\rangle}{2n} \tag{6}$$

for $n \gg 1$. The particle motion is not de-correlated in one interaction time but takes significantly larger number of interactions. Then the evolution time step on the left hand side of (1) has to be large enough for decorrelation of the particle orbit in its interaction

with electrostatic waves. The correlation function

$$C_n = \left\langle \left(I_n^{(p)} - I_{n-1}^{(p)} \right) \left(I_1^{(p)} - I_0^{(p)} \right) \right\rangle_p \tag{7}$$

where $< .. >_p$ is an ensemble average for a set of randomly distributed particles. The correlation time n_c is such that for $n > n_c$ we have $C_n \approx 0$.

While the standard map is quite idealistic in terms of representing wave particle interactions it exhibits interesting physical properties which can be well analyzed. One idealization is that the entire particle phase space is affected. This is not the case when RF waves are used to heat or drive currents in laboratory plasmas. The waves are localized in space and only part of the particle distribution function is affected by the waves. Some aspects of particle interaction with spatially localized fields has been studied in [6]. There are several points that become evident from this study: the diffusion defined by (6) is dependent on the number of interactions n and the initial energy (or velocity) of the particles I_0. The primary reason for these dependencies is that the affected particle phase space is bounded. In the limit $n \to \infty$ it is evident from (6) that $D_n \to 0$. For the standard map this is not the case. Also, particles near the boundaries of the RF affected phase space behave differently from those away from the boundaries. Thus, for a proper description of the interaction of electrons with RF waves in tokamaks, we need a different approach.

KINETIC FORMULATION OF RF INDUCED TRANSPORT

We make use of the Hamiltonian formalism and the Lie perturbation method to derive a kinetic equation for the evolution of the electron distribution function in the presence of RF waves in a tokamak. Since the RF waves can be poloidally and toroidally localized, an electron interacting with RF waves during one toroidal transit may not do so during its next transit. The drifts induced by toroidal and poloidal magnetic fields and the magnetic trapping of electrons are properly taken into account in our formalism. The effect of magnetic field perturbations, e.g., due to NTMs, are also included in the dynamics of electrons.

The dynamics of electrons in a tokamak and their interaction with RF waves is given by the Hamiltonian

$$H(\mathbf{J}, \theta, t) = H_0(\mathbf{J}) + \varepsilon H_1(\mathbf{J}, \theta, t) \tag{8}$$

where $H_0(\mathbf{J})$ is the unperturbed Hamiltonian that describes the electron motion in an axisymmetric tokamak. $\mathbf{J}$ are the constant actions – the magnetic moment, the toroidal flux, and the canonical angular momentum. The corresponding canonical angles θ are the gyrophase of the particle, poloidal angle, and the toroidal angle. The effect of the RF waves is assumed to be perturbative and given by H_1. In (8) ε is assumed to be less than one and is an ordering parameter. We carry out a Lie transform canonical perturbation theory using ε as the expansion parameter. There exists a Lie operator $\mathscr{O}_{\mathscr{L}}$ such that it evolves, using the dynamical equations, phase space variables $(\mathbf{J}, \theta)$ from their given values at time t to those at time $t + \Delta t$ [1]

$$\mathscr{O}_{\mathscr{L}} : \ (\mathbf{J}, \theta)_t \ \longrightarrow (\mathbf{J}, \theta)_{t+\Delta t} \tag{9}$$

A distinct advantage of the Lie operator is the way it transforms any function of the canonical variables

$$\mathscr{O}_{\mathscr{L}}{}^{-1}\, f(\mathbf{J},\theta) \;=\; f(\mathscr{O}_{\mathscr{L}}\{\mathbf{J},\theta\}) \tag{10}$$

If $f(\mathbf{J},\theta)$ is assumed to be the distribution then, from (10),

$$f(\mathbf{J},\theta)_{t+\Delta t} - f(\mathbf{J},\theta)_t \;=\; \left(\mathscr{O}_{\mathscr{L}}{}^{-1} - \mathbf{I}\right)\cdot f(\mathbf{J},\theta)_t \tag{11}$$

where $\mathbf{I}$ is the identity operator. Dividing by Δt and taking the limit $\Delta t \to 0$ gives

$$\frac{\partial}{\partial t} f(\mathbf{J},\theta,t) \;=\; \left[\frac{\partial}{\partial t}\left(\mathscr{O}_{\mathscr{L}}{}^{-1} - \mathbf{I}\right)\right]\cdot f(\mathbf{J},\theta,t) \tag{12}$$

which is the evolution equation for the distribution function. The Lie operator $\mathscr{O}_{\mathscr{L}}$ is time dependent and contains the dynamics of the entire electron population and its interaction with RF waves. Equation (12) is completely reversible in time. By averaging over the angles θ we destroy this reversibility and obtain the diffusion equation

$$\frac{\partial}{\partial t} f(\mathbf{J},t) \;=\; \left[\frac{\partial}{\partial \mathbf{J}}\cdot \mathbf{D}(\mathbf{J},t)\cdot \frac{\partial}{\partial \mathbf{J}}\right] f(\mathbf{J},t) \tag{13}$$

The diffusion operator $\mathbf{D}(\mathbf{J},t)$ is not only a function of the actions but also of time. In (13) the evolution of f occurs on the same time scale as the evolution of $\mathbf{D}$ through the dynamical equations. The operator $\mathbf{D}$ is non-singular. In prior studies on quasilinear theory [2, 3], the diffusion operator contains a singular Dirac δ function. The singularity does not lend itself to any sensible numerical implementation.

We refer the reader to [1] for the derivation leading to (13). A more detailed description of the basic evolution equation will be included in an upcoming paper.

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