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# Limitations, Insights and Improvements to Gyrokinetics

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# Limitations, Insights and Improvements to Gyrokinetics

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**Abstract:** For a tokamak, we consider gyrokinetic quasineutrality limitations when evaluating the axisymmetric radial electric field; provide an insight by considering the gyrokinetic entropy production restriction on an ion temperature pedestal like that of ITER; and an present an improved hybrid gyrokinetic-fluid treatment valid on slowly evolving transport time scales.

## 1. Introduction

In our recent work we consider the limitations of gyrokinetic quasineutrality in evaluating the axisymmetric radial electric field in a tokamak [1]; derive a gyrokinetic entropy production restriction on the ITER ion temperature pedestal and zonal flow behavior in the pedestal [2]; and formulate a hybrid gyrokinetic-fluid treatment of turbulence valid on slowly evolving transport time scales [3]. Here we attempt to provide more insight into the detailed calculations published in [1-3].

Standard gyrokinetics, which is only accurate to first order in the gyroradius over major radius expansion, incorrectly determines the axisymmetric, long wavelength electrostatic potential. We explicitly demonstrate this feature of gyrokinetics by considering a steady-state theta pinch with a distribution function correct to second order in the gyroradius expansion [1]. Here, we present the argument as to why gyrokinetic quasineutrality improperly determines the potential in the long wavelength, axisymmetric limit for a tokamak. The basic conceptual error is due to an inconsistent treatment of quasineutrality and must be corrected to recover agreement with the required intrinsic ambipolarity of tokamaks [4,5].

Using canonical angular momentum as the radial variable allows strong radial gradients (as in the pedestal) to be treated gyrokinetically [2] while retaining all the other features of standard gyrokinetics. Entropy production is then found to require a physical lowest order banana regime ion distribution function to be nearly an isothermal Maxwellian with the ion temperature scale much greater than the poloidal ion gyroradius. Thus, the background ion temperature profile in ITER cannot have a pedestal width as short as the poloidal ion gyroradius width of the density pedestal. Weak ion temperature variation with subsonic pedestal flow requires electrostatically restrained ions and magnetically confined electrons. These features result in finite orbit modifications to the zonal flow residual [6].

Simulating tokamaks on transport time scales requires evolving drift wave turbulence with axisymmetric neoclassical and zonal flow radial electric field effects retained. However, full electric field effects are difficult to keep since they require evaluating the ion distribution function to higher order in the gyroradius expansion than in standard gyrokinetics. A hybrid gyrokinetic-fluid treatment of electrostatic turbulence that takes advantage of moments of the full Fokker-Planck equation removes the need to go to third order in the gyroradius expansion [3]. This hybrid description self-consistently evolves potential as well as density and temperature profiles and flows, and models all electrostatic turbulence effects with wavelengths much longer than an electron gyroradius.

## 2. Limitations of the Gyrokinetic Determination of the Radial Electric Field

A new recursive procedure is used to derive the electrostatic gyrokinetic equation for the full distribution function (a "full f" description) accurate to first order in an expansion of gyroradius over magnetic field characteristic length scale [1]. The procedure employs new, nonlinear gyrokinetic variables that are constructed to higher order than is typically the case by generalizing the linear procedure of [7]. The results of [1] are fully consistent with the constant magnetic field limit of Dubin *et al.* [8] to second order in $\rho_i/L$, where $\rho_i$ is the ion gyroradius and L the perpendicular scale length of the background profiles. They are also consistent with standard gyrokinetic results [9]. Our higher order gyrokinetic variables are required in the hybrid description of section 4 to retain neoclassical viscous effects. The gyrokinetic procedure employed provides fresh insights into the limitations of the gyrokinetic quasineutrality equation that in the long wavelength limit must not determine the axisymmetric electrostatic potential because of intrinsic ambipolarity [4,5].

The axisymmetric radial electric field in a tokamak consists of two components that give rise to **E**×**B** drifts comparable to diamagnetic flows and magnetic drifts (this situation is normally referred to as the drift ordering). The relatively small amplitude, but rapidly radially varying zonal flow component of the electrostatic potential is generated by the turbulence associated with ion temperature gradient (ITG) modes, trapped electron modes (TEMs), and other tokamak instabilities. It is superimposed on a large amplitude component with a slow global structure on the scale of the background radial gradients. Gyrokinetic quasineutrality properly determines temporally varying electrostatic potential $\Phi$ in the short wavelength limit, but it would violate intrinsic ambipolarity if it determined the steady state, axisymmetric, global long wavelength radial electric field component that impacted the evolution of the turbulence. This error occurs because standard gyrokinetics is only accurate to order $\rho_i/L$, whereas corrections of higher order are required to evaluate the long wavelength potential.

In a steady state, axisymmetric tokamak, intrinsic ambipolarity [4,5] requires the heat and particle fluxes to be independent of electrostatic potential to second order in the expansion in ion gyroradius $\rho_i$ divided by the local scale length L. This property is most easily seen to order $\rho_i/L$ by considering the axisymmetric drift kinetic equation for the leading correction $f_{1i}$ to the lowest order ion Maxwellian $f_{0i}(\psi,E)$ found by solving

$$v_\parallel \vec{n}\cdot\nabla f_{1i} - C_{1ii}\{f_{1i}\} = -\vec{v}_{di}\cdot\nabla\psi\frac{\partial f_{0i}}{\partial\psi} = -v_\parallel \vec{n}\cdot\nabla\left(\frac{Iv_\parallel}{\Omega_i}\frac{\partial f_{0i}}{\partial\psi}\right), \quad (1)$$

where $E = v^2/2+e\Phi/M$ is the total energy, $\vec{v}_{di}$ is the magnetic plus electric drifts, $C_{1ii}$ is the linearized ion-ion collision operator with $C_{1ii}\{v_\parallel f_0\} = 0$, and $\vec{B} = I(\psi)\nabla\zeta + \nabla\zeta\times\nabla\psi = B\vec{n}$ is the tokamak magnetic field with $\zeta$ the toroidal angle and $\psi$ the poloidal flux function. Letting $g_i = f_{1i} + (Iv_\parallel/\Omega_i)(\partial f_{0i}/\partial\psi)$ gives

$$v_\parallel \vec{n}\cdot\nabla g_i = C_{1ii}\{g_i - \frac{Iv_\parallel}{\Omega_i}\frac{\partial f_{0i}}{\partial\psi}\} = C_{1ii}\{g_i - \frac{If_{0i}v_\parallel}{\Omega_i T}\left(\frac{Mv^2}{2T_i} - \frac{5}{2}\right)\frac{\partial T_i}{\partial\psi}\}, \quad (2)$$

showing the only drive for $g_i$ is $\partial T_i/\partial\psi$, and giving a vanishing ion particle flux since $\langle n\vec{V}_i\cdot\nabla\psi\rangle_\psi = \langle\int d^3v f_{1i}\vec{v}_{di}\cdot\nabla\psi\rangle_\psi = -\langle(I/\Omega_i)\int d^3v v_\parallel^2 \vec{n}\cdot\nabla f_{1i}\rangle_\psi = 0$, where $\langle...\rangle_\psi$ denotes flux surface average and $Iv_\parallel/\Omega_i$ moment of (1) is employed. A moment procedure for the electron particle flux using $C_{1e}\{f_{1e}\} = C_{1ee}\{f_{1e}\} + C_{ei}\{f_{1e}\}$ with $C_{1ee}$ the electron-electron operator and $C_{ei}\{f_{1e}\} = L_{ei}\{f_{1e} - (m/T_e)V_{\parallel i}v_\parallel f_{0e}\}$ the unlike electron-ion Lorentz operator, gives the electron radial particle flux as $\langle n\vec{V}_e\cdot\nabla\psi\rangle_\psi = (mcI/e)\langle B^{-1}\int d^3v v_\parallel C_{1e}\{f_{1e} - (m/T_e)V_{\parallel i}v_\parallel f_{oe}\}\rangle_\psi$. The electron drift kinetic equation can be written as $v_\parallel \vec{n}\cdot\nabla g_e = C_{1e}\{g_e + (Iv_\parallel/\Omega_e)(\partial f_{0e}/\partial\psi) - (m/T_e)V_{\parallel i}v_\parallel f_{0e}\}$ with $g_e = f_{1e} - (Iv_\parallel/\Omega_e)(\partial f_{0e}/\partial\psi)$. The $\partial\Phi/\partial\psi$

drives in the collision operator cancel, making $g_e$ independent of the radial electric field so that $\langle n\vec{V}_e\cdot\nabla\psi\rangle_\psi = (mcI/e)\langle B^{-1}\int d^3v v_\parallel C_{1e}\{g_e + (Iv_\parallel/\Omega_e)(\partial f_{0e}/\partial\psi) - (m/T_e)V_{\parallel i}v_\parallel f_{oe}\}\rangle_\psi$ cannot depend on the radial electric field to order $(\rho_i/L)^2$ since $C_{ii}/C_{ee} \sim \nu_{ii}/\nu_{\varepsilon\varepsilon} \sim (m/M)^{1/2} \sim \rho_i/L$ is normally assumed, with and the ion-ion $\nu_{ii}$ and $\nu_{\varepsilon\varepsilon}$ electron-electron collision frequencies.

Alternately, a moment description can be used to further demonstrate that intrinsic ambipolarity must be satisfied to order $\rho_i^2/L^2$ and demonstrate that it is the flux surface average of conservation of toroidal angular momentum that must give the radial electric field (parallel viscosity does not enter this constraint). To order $\rho_i^2/L^2$ the cross field viscosity is diamagnetic (and so collisionless to lowest order) and the radial flux of toroidal angular momentum may be written in terms of the ion gyroviscosity $\vec{\pi}_{ig}$ within small up-down asymmetric contributions as [10] $\langle R^2\nabla\zeta\cdot\vec{\pi}_{ig}\cdot\nabla\psi\rangle_\psi = \langle (MI/B)\int d^3v v_\parallel f_{1i}\vec{v}_{di}\cdot\nabla\psi\rangle_\psi = 0$. Inserting $f_{1i} = g_i - (Iv_\parallel/\Omega_i)(\partial f_{0i}/\partial\psi)$, using $\langle \int d^3v f_{0i}(v_\parallel/B)^2 v_\parallel \vec{n}\cdot\nabla(v_\parallel/B)\rangle_\psi = 0$, and recalling $g_i$ depends only on $\partial T_i/\partial\psi$ gives a $\partial\Phi/\partial\psi$ independent result. Hence, the correct neoclassical radial electric field must be determined from toroidal angular momentum conservation in next order.

By considering a steady state $\theta$ pinch using a model collision operator, we have explicitly shown that gyrokinetic quasineutrality cannot determine the axisymmetric, long radial wavelength electrostatic potential to order $\rho_i^2/L^2$ [1]. As argued in the preceding paragraphs, a similar situation occurs in axisymmetric tokamaks. In standard gyrokinetic treatments intrinsic ambipolarity is violated when the ion distribution function is retained to order $\rho_i/L$ in the guiding center density and to order $\rho_i^2/L^2$ in the finite orbit polarization term. However, when $f_i$ is kept to order $\rho_i^2/L^2$ in both places, the radial electric does not enter and therefore cannot be determined, and no inconsistency arises. To determine this axisymmetric radial electric field higher order effects must be retained.

These results indicate that the gyrokinetic quasineutrality equation is not an effective tool for finding the electrostatic potential if the long wavelength components are to be properly retained in the analysis. In section 4 we discuss how second order accurate gyrokinetic variables can be employed [1,2] in a hybrid gyrokinetic-fluid moment description to insure the required accuracy in the gyroradius expansion.

## 3. Gyrokinetics in the Pedestal and Internal Barriers

A new gyrokinetic technique has been developed and applied to analyzing pedestal and internal transport barrier (ITB) regions [2]. In contrast to typical gyrokinetic treatments [7-9], canonical angular momentum $\psi_* \equiv \psi - (Mc/e)R^2\vec{v}\cdot\nabla\zeta = \psi + \Omega_i^{-1}\vec{v}\times\vec{n}\cdot\nabla\psi - (Iv_\parallel/\Omega_i)$ is taken as the gyrokinetic radial variable rather than the radial guiding center location $\Psi \equiv \psi + \Omega_i^{-1}\vec{v}\times\vec{n}\cdot\nabla\psi$. Such an approach allows strong radial plasma gradients to be treated, while retaining zonal flow and neoclassical behavior and the effects of turbulence. The nonlinear gyrokinetic equation obtained is capable of handling such problems as large poloidal **E**×**B** drift and orbit squeezing effects on zonal flow, collisional zonal flow damping, as well as neoclassical transport in the pedestal or ITB. This choice of gyrokinetic variables allows the toroidally rotating Maxwellian solution of the isothermal tokamak limit to be exactly recovered [11].

More importantly, we can prove that a physically acceptable solution for the lowest order ion distribution function in the banana regime anywhere in a tokamak (including the pedestal, internal transport barriers, and on axis) must be nearly this same isothermal Maxwellian solution in the sense that the ion temperature variation radial scale must be much greater than poloidal ion gyroradius $\rho_{pi}$. Consequently, in the banana regime the background radial ion temperature profile in ITER cannot have a pedestal similar to that of plasma density or electron temperature if they vary on the scale of a poloidal ion gyroradius. To understand this insight first recall that the vanishing of the entropy production on a flux surface, $\langle \int d^3v \ell n f_{0i} C_{1ii}\{f_{0i}\} \rangle_\psi = 0$, requires the lowest order axisymmetric ion distribution function $f_{0i}$ to be a local Maxwellian, with $f_{0i}$ independent of poloidal angle in the banana regime. However, in the pedestal or an internal barrier (or on axis), drift departures from flux surfaces can become comparable to the local scale length ($\rho_{pi} \nabla \ell n n \sim 1$ with n the plasma density) and the entropy production argument has to be modified to account for the loss of locality due to finite poloidal ion gyroradius effects requiring an equilibrium to be established over the entire pedestal (or barrier). Using the new gyrokinetic variables, we find that entropy production must vanish in the pedestal [6]:

$$\int_{\Delta V} d^3r \int d^3v \ell n f_{0i} C_{1ii}\{f_{0i}\} = 0, \qquad (3)$$

where $\Delta V$ is the volume of the pedestal (between the top of the pedestal where $\rho_{pi} \nabla \ell n n << 1$ and the separatrix) or the internal transport barrier (between inner and outer bounding flux surfaces having $\rho_{pi} \nabla \ell n n << 1$) or an on axis region bounded by the magnetic axis and a flux surface with $\rho_{pi} \nabla \ell n n << 1$. As a result, $f_{0i}$ must be drifting Maxwellian at most, giving $C_{1ii}\{f_{0i}\} = 0$. In the banana regime $f_{0i}$ is independent of poloidal angle as well. Consequently, to make the Vlasov operator vanish $f_{0i} = f_{0i}(\psi_*, E, \mu)$ must be Maxwellian, where $E = v^2/2 + e\Phi/M$ is the total energy and $\mu$ the magnetic moment. It is only possible to make a drifting Maxwellian out of these variables by ignoring the $\mu$ dependence and assuming the drift is nearly a rigid toroidal rotation of frequency $\omega_i$ with the ion temperature variation slow compared to the poloidal ion gyroradius ($\rho_{pi} \nabla \ell n T_i << 1$, $\rho_{pi} \nabla \ell n \omega_i << 1$) as for an isothermal Maxwellian [2,11]:

$f_{0i}(\psi_*, E) = n(M/2\pi T_i)^{3/2} \exp[-M(\vec{v} - \omega_i R^2 \nabla\zeta)^2/2T_i] = \eta (M/2\pi T_i)^{3/2} \exp(-ME/T_i - e\omega_i \psi_*/cT_i)$, where $\eta = n \exp[(e\Phi/T_i) + (e\omega_i \psi/cT_i) - (M\omega_i^2 R^2/2T_i)]$ must also be nearly constant ($\rho_{pi} \nabla \ell n \eta << 1$). Thus, for a density pedestal having a scale length $L \sim \rho_{pi}$, the background ion temperature profile must have a much larger scale length than the pedestal - a restriction that will need to be satisfied by the ITER pedestal. As a result, the ion temperature pedestal must be somewhat broader than the poloidal ion gyroradius variation allowed for the density pedestal, and the peak ion temperature in the core of ITER as set by ballooning-peeling calculations in the presence of bootstrap current may be reduced [12].

In addition, for a density scale length of $\rho_{pi}$, lowest order perpendicular momentum balance gives $\omega_i = -c[d\Phi/d\psi + (en)^{-1} d(nT_i)/d\psi]$ with $cR(en)^{-1} d(nT_i)/d\psi \sim v_i$ = ion thermal speed and $\Phi(\psi)$ the axisymmetric electrostatic potential. Consequently, in a subsonic pedestal in the banana regime it must be that to lowest order the ions are electrostatically confined [2] with $e d\Phi/d\psi \approx -(T_i/n) dn/d\psi$. This behavior is observed in the banana regime H mode pedestal of DIII-D [13] and to some extent in the slightly more collisional Alcator C-Mod H mode pedestal [14]. Using total pressure balance we then see that the electrons must be magnetically confined with a mean flow $\vec{V}_e$ comparable to the ion thermal speed ($\vec{V}_e \sim v_i$). To form and sustain an H mode pedestal, presumably electron heating and weak electron radial heat transport will keep the electron temperature higher than that of the ions, and temperature equilibration will be prevented by strong radial ion heat loss in the pedestal. In this simple limit the electrons carry essentially all the pedestal current.

Next, we address the issue of zonal flow in the pedestal. Rosenbluth and Hinton [15] demonstrated that plasma polarization is the key factor affecting the linear stage of zonal flow dynamics in a tokamak. Namely, the tokamak plasma "shields" the original zonal flow potential so that only some fraction of it, the residual, survives. Physically, this shielding is provided by the dipole moment induced in plasmas by the zonal flow. Accordingly, two components of polarization are distinguished in tokamak plasmas. The classical one originates from the dipole moment associated with ion gyrocenters. This plasma response is relevant on the time scale greater than the cyclotron period. The other polarization is neoclassical in origin and due to shifting the center of the banana or passing orbits of the ions in a tokamak. This mechanism takes affect on the time scale greater than the bounce period. The residual is the ratio of the final perturbed potential $\delta\Phi(t\to\infty)$ to its initial value $\delta\Phi(t{=}0)$ for a step function change in the perturbed density:

$$\frac{\delta\Phi(t\to\infty)}{\delta\Phi(t=0)}=\frac{\varepsilon_{pol-cl}}{\varepsilon_{pol-cl}+\varepsilon_{pol-nc}}\ , \tag{4}$$

where $\varepsilon_{pol\text{-}cl}$ and $\varepsilon_{pol\text{-}nc}$ are the classical and neoclassical plasma polarizations relating the perturbed polarization densities $\delta n_{pol}$ and potentials $\delta\Phi$ in $\varepsilon_{pol}\kappa^2\delta\Phi = -\,4\pi e\delta n_{pol}$.

The strong localized axisymmetric radial electric field that arises in the pedestal modifies the collisionless zonal flow residual of Rosenbluth and Hinton [15] due to the strong poloidal **E**×**B** drift and its associated finite orbit effects as well as orbit squeezing [16]. For example, in the banana regime the axisymmetric radial electric field of ion poloidal gyroradius pedestal must approximately satisfy $e d\Phi_0/d\psi = -(T_i/n)dn/d\psi$. Then the residual associated with the small amplitude, shorter wavelength, axisymmetric zonal flow potential $\delta\Phi$ will differ substantially from that of reference [15] as shown in reference [6] and discussed in the following paragraphs.

Retaining the poloidal **E**×**B** drift as well as parallel streaming the ion poloidal drift frequency becomes

$$\dot{\theta} = [v_{||} + cIB^{-1}\Phi'(\psi)]\vec{n}\cdot\nabla\theta\ . \tag{5}$$

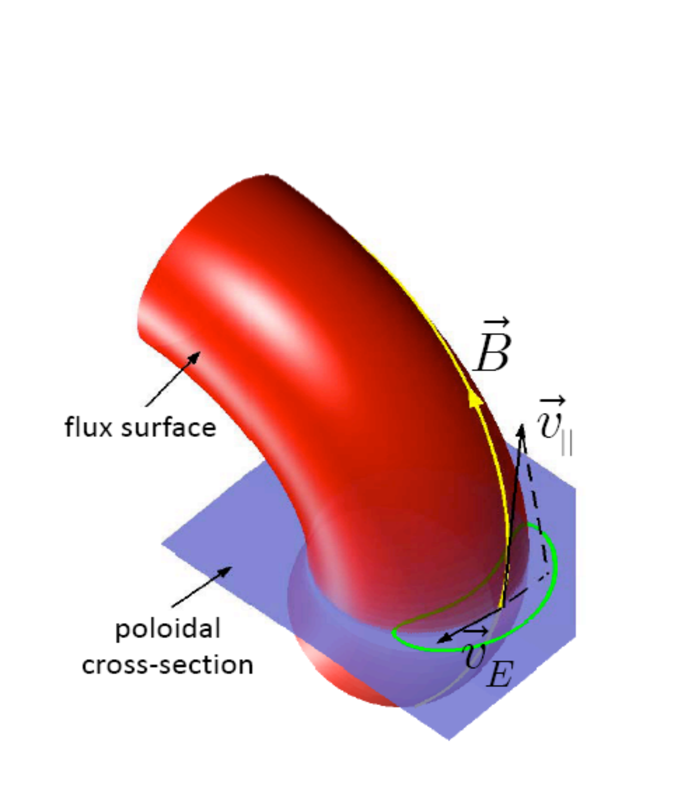


*Figure 1: Projection of the parallel streaming into the poloidal plane.*

In the tokamak core, the second term on the right side of (5) is much less than the first one, whereas in the pedestal these terms are comparable, thereby modifying the poloidal motion of particles. We remark that the **E**×**B** drift, $\vec{v}_E \sim v_i\rho_i/\rho_{pi}$, remains much less than the ion thermal speed as required by our gyrokinetic ordering. However, as sketched in figure 1, $\vec{v}_E$ is nearly parallel to the poloidal plane while $v_{||}\vec{n}$ is almost perpendicular to it with only a small poloidal component of $v_{||}\rho_i/\rho_{pi}$. As a result, these two velocities can compete in the poloidal cross-section of a tokamak.

In the conventional core case, a particle is trapped if its $v_{||}$ is small enough. However, as equation (5) suggests, for a particle to be trapped in the pedestal its $v_{||}$ must be rather close to $-cIB^{-1}\Phi'(\psi)$. Then, as a trapped particle in the pedestal undergoes its banana motion projected onto the poloidal cross-section, its parallel velocity oscillates around the value of $-cIB^{-1}\Phi'(\psi)$ (rather than zero) as illustrated in figure 2. Accordingly, as banana particles play a key role in neoclassical phenomena such as radial ion heat flux or polarization, the evaluation of these effects has to be revisited in the pedestal where they will differ from the core.

It is interesting to notice that due to modifications of the trapping condition, banana particles acquire rather complicated toroidal behavior. Indeed, toroidal motion is still dominated by $v_{||}$ but now it has a bounce average value of $-cIB^{-1}\Phi'(\psi)$. Of course, even in the conventional case banana particles drift toroidally because of toroidal components of the magnetic drift. However, these toroidal drifts are much less than $v_i$, while within the pedestal ordering $-cIB^{-1}\Phi'(\psi) \sim v_i$. As a result, the toroidal orbits of particles trapped poloidally in the pedestal dramatically differ from those in the core as shown in figure 2.

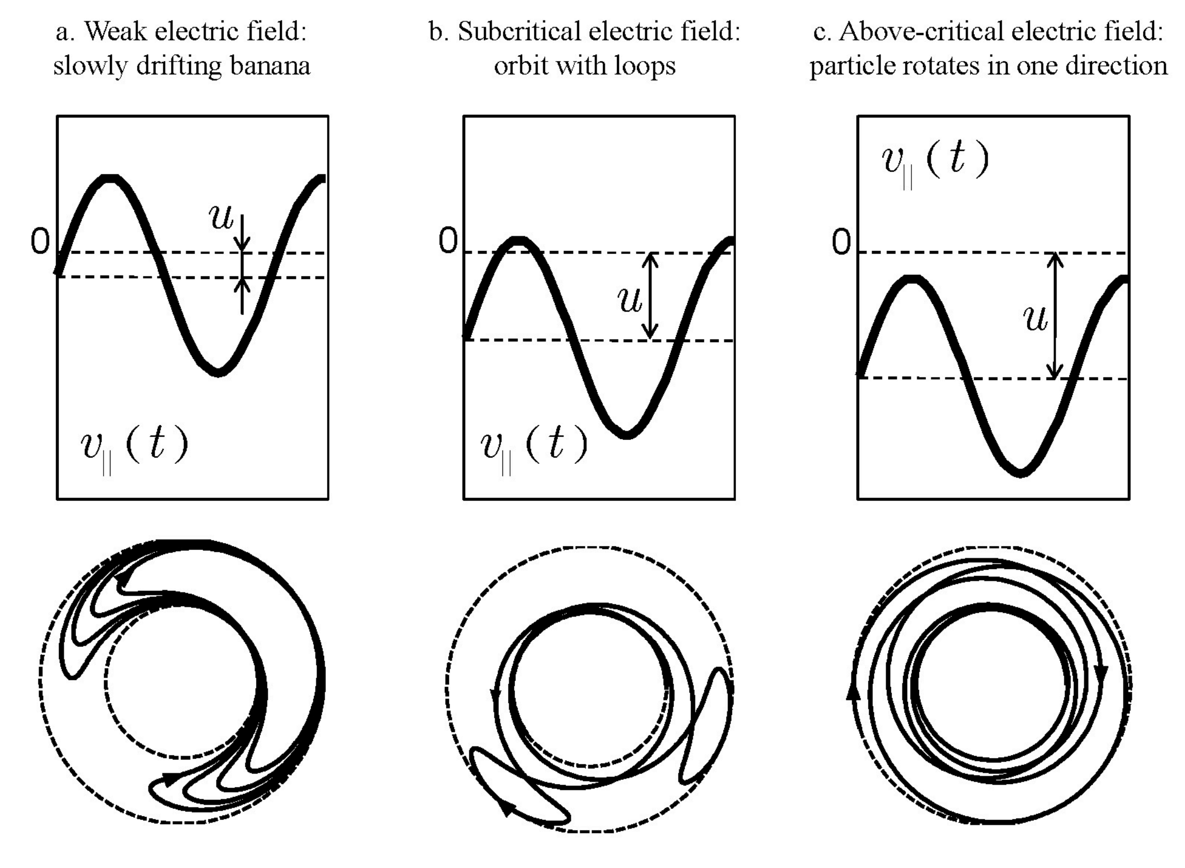


*Figure 2: Toroidal projections of trapped particle orbits for weak, subcritical ($v_{||}+u < 0$), and above critical ($v_{||}+u > 0$) **E×B** drift.*

We can see this in more detail in the large aspect ratio limit where the effect of a strong electric field with shear on particle orbits can be treated by considering a quadratic potential well in $\psi$. Expanding $\Phi$ around the flux surface that the trapped and barely passing particles are localized about, namely $\psi_* - \Delta$ with $\Delta = Iu/\Omega$ and $u = cI\Phi'_*/B$, we obtain

$$\Phi(\psi) = \Phi_* + [\psi - (\psi_* - \Delta)]\Phi'_* + (1/2)[\psi - (\psi_* - \Delta)]^2\Phi''_* , \qquad (6)$$

with $\Phi_* \equiv \Phi(\psi_* - \Delta)$, $\Phi'_* \equiv \Phi'(\psi_* - \Delta)$, and $\Phi''_* \equiv \Phi''(\psi_* - \Delta)$. Assuming the radial variation of B and I are weak, equation (5) can be written as

$$qR_o\dot{\theta} = S(v_{||} + u) , \qquad (7)$$

with $R_0$ the major radius and $S = 1+cI^2\Phi''_*/\Omega B$ the orbit squeezing coefficient that becomes unity for $\Phi''_* = 0$. This result displays the u shift in $v_{||}$ due to large **E**×**B** drift as well as the effect of orbit squeezing.

A detailed evaluation of zonal flow residual is presented in [6]. The two main qualitative features are as follows. First, as the shift u becomes comparable to the ion thermal speed, the trapped particle region moves toward the tail of the Maxwellian centered about v = 0, dramatically reducing the number of trapped ions and the neoclassical polarization, and thereby enhancing the residual to make it closer to unity, as shown in figure 3.

The second, is that in the pedestal the neoclassical polarization and therefore the residual become complex; a feature requiring a bit more discussion. Consider the polarization $\boldsymbol{\delta P}$ caused by a radial dipole moment induced in a plasma that is linearly proportional to the dipole inducing electric field $\boldsymbol{\delta E} = -\nabla\delta\Phi = -\nabla\psi(\partial\delta\Phi/\partial\psi)$. For scalar flux function susceptibility $\alpha=\alpha(\psi)$, then $\boldsymbol{\delta P} = \alpha\boldsymbol{\delta E}$. Allowing for a strong gradient in the susceptibility and assuming $\nabla\psi$ is weakly varying, the polarization density $\delta n_{pol}$ given by $\delta n_{pol} = -4\pi\nabla\cdot\boldsymbol{\delta P}$ for $\delta\Phi$ proportional to $\exp(-i\kappa\psi)$ gives

$$\delta n_{pol} = -4\pi|\nabla\psi|^2(\kappa^2\alpha+i\kappa\delta\alpha/\partial\psi)\delta\Phi. \quad (8)$$

Recalling that the polarization density and potential are related by $\varepsilon_{pol}\kappa^2\delta\Phi = -4\pi e\delta n_{pol}$, we see that the neoclassical plasma polarization $\varepsilon_{pol\text{-}nc}$ in equation (4) becomes complex due to the strong radial dependence of the susceptibility that can vary on the scale of a poloidal ion gyroradius. The imaginary term corresponds to shifting the entire plasma pedestal as a whole in response to the applied electric field.

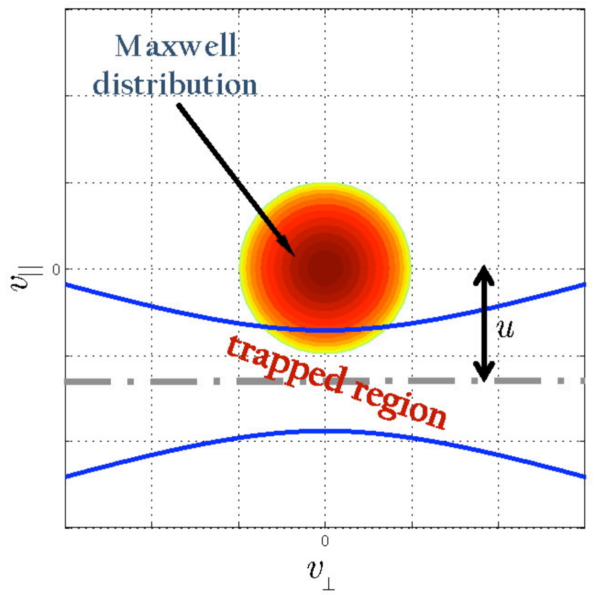


*Figure 3: In the presence of a strong radial electric field the trapped region moves toward the tail of the Maxwellian distribution.*

## 4. Gyrokinetic Closure and Radial Electric Field on Transport Time Scales

Simulating electrostatic turbulence in tokamaks on transport time scales requires retaining and evolving a complete turbulence modified neoclassical transport description, including all the axisymmetric neoclassical and zonal flow radial electric field effects, as well as the turbulent transport normally associated with drift instabilities. Full electric field effects and their evolution are more difficult to retain than density and temperature evolution effects since the need to satisfy intrinsic ambipolarity in the axisymmetric, long wavelength limit requires evaluating the ion distribution function to higher order in gyroradius over background scale length than standard gyrokinetic treatments as already noted earlier. To avoid having to derive and solve a gyrokinetic equation valid to order $(\nu_{ii}/\Omega_1)(\rho_i/L)^2$, an alternate hybrid gyrokinetic-fluid treatment is formulated that employs moments of the full Fokker-Planck equation to remove the need to solve for a very high order gyrokinetic distribution function. The description is an extension to gyrokinetics of drift kinetic treatments that yield expressions for the ion perpendicular viscosity as well as for the electron and ion parallel viscosities, gyroviscosities, and heat fluxes for arbitrary mean-free path plasmas, in which the lowest order distribution function is a Maxwellian [17].

This hybrid description evolves electrostatic potential, plasma density, ion and electron temperatures, and ion and electron flows using conservation of charge, number, ion and electron energy, and total and electron momentum, respectively [3]. All electrostatic effects with wavelengths much longer than an electron gyroradius are retained so that ion temperature gradient (ITG) and trapped electron mode (TEM) turbulence and the associated zonal flow as well as all neoclassical behavior are treated. Closure for the electrons is obtained by solving the electron drift kinetic equation to find the leading order correction to the Maxwellian electron distribution function $f_{0e}$ needed to evaluate the parallel electron viscosity (or pressure anisotropy) as well as the momentum and energy exchange terms with the ions. In addition, the $\vec{v}v^2/2$ moment of the exact electron Fokker-Planck equation is used, along with this first order correction to $f_{0e}$, to evaluate the electron heat flux (collisional plus diamagnetic), thereby achieving closure for the electrons. Ion closure is achieved similarly by solving the ion gyrokinetic equation to leading order in $\rho_i/L$. However, ion closure is somewhat more complicated because the $\vec{v}\vec{v}$ as well as the $\vec{v}v^2/2$ moments of the ion Fokker-Planck equation must be used to evaluate the ion gyroviscosity and perpendicular viscosity, along with the ion heat flux. Moreover, to recover the correct results in the axisymmetric, long wavelength limit, the gyrokinetic variables must be determined to one order higher than normal [1,2]. This complication is a result of toroidal angular momentum attempting to flow in flux surfaces to some approximation, thereby reducing the size of the fast time and flux surface averages of the radial flux of toroidal angular momentum. Once this is done complete closure is obtained and a description valid on transport time scales is recovered that properly evolves the electrostatic potential and flows, as well as density and temperatures [3].

To simplify the expression for the ion viscosity we may expand the ion distribution function in powers of $\rho_i/\rho_{pi}$. To order $\rho_i\rho_{pi}/L^2$ the Taylor expanded form of the gyrophase dependent portion of f from equation (80) of reference [1] is given by

$$f_i - \langle f_i \rangle = \Omega_i^{-1}\vec{v}\times\vec{n}\cdot[\nabla f_{0i} + (Zef_{0i}/T_i)\nabla\Phi] \\ -[\vec{v}\cdot\vec{v}_d + (v_\parallel/4\Omega_i)(\vec{v}_\perp\vec{v}\times\vec{n} + \vec{v}\times\vec{n}\vec{v}_\perp):\nabla\vec{n}]B^{-1}\partial f_{1i}/\partial\mu_0 \ . \tag{9}$$

where order $(\rho_i/L)^2$ corrections are ignored. Here $f_{0i}$ is a stationary Maxwellian, $f_{1i} \sim f_{0i}\rho_{pi}/L$ is the leading order correction, $\langle f_i \rangle$ is the gyroaverage of $f_i$, $\vec{v}_d$ is the sum of the magnetic and **E**×**B** drifts, $\mu_0$ is the lowest order magnetic moment, and $\nabla$ is performed holding v and $\mu_0$ fixed. By using this $\rho_{pi} >> \rho_i$ form rather than the full result from equation (80) the evaluation of the ion viscosity in reference [3] can be considerably simplified.

Moreover, in reference [3] explicit expressions were given using the gyrokinetic results of reference [1]; however, the gyrokinetic description of reference [2] that is specifically formulated to conveniently treat the pedestal can be employed instead. Rather than using the typical gyrokinetic variables and gyrokinetic equation of [1] we can use the alternate gyrokinetic variables and gyrokinetic equation of [2]. Of course, the $\rho_i/\rho_{pi} << 1$ expansion can again be employed to simplify the treatment of the ion viscosity.

In this hybrid description distribution functions are only used to evaluate moments needed for closure and collisional exchange [3] - they are not used to evaluate density, temperature and flows since these are found from the moment equations. The results are given in terms of a few velocity space integrals of the gyrokinetic distribution function and make possible a turbulent hybrid fluid-gyrokinetic description that includes the neoclassical radial electric field as well as long wavelength turbulence and zonal flow effects. Moment equations evolve all other quantities such as density, temperatures, flows, and potential. As a result, either PIC or continuum, lowest order gyrokinetic and drift kinetic solvers may be employed, and the kinetic equations need not be solved in conservative form. In addition, the flux surface average of conservation of toroidal angular momentum contains axisymmetric radial electric field terms from both the Reynolds stress and the collisional perpendicular ion viscosity whose respective coefficients compare as

$$\left\langle \frac{\tilde{n}}{n} \frac{\partial}{\partial \zeta} \left( \frac{e\tilde{\Phi}}{T_i} \right) \right\rangle_\psi \text{ vs } \frac{q^2 R_0}{L_\perp} \frac{\nu_{ii}}{\Omega_i} , \qquad (10)$$

with tildes denoting fluctuating quantities, $L_\perp$ the local perpendicular scale length, R the major radius, and q the safety factor. For $\tilde{n}/n \sim e\tilde{\Phi}/T_i \sim 10^{-2}$ with 0.1 de-phasing, both quantities are of order $10^{-5}$ in the pedestal for $L_\perp \sim q^2\rho_{pi}$ and ITER like numbers of B = 5.3 T, $T_i$ = 8 keV, n = $10^{19}$ m$^{-3}$, and $R_0$ = 6 m. Consequently, even though momentum relaxation is expected to be anomalous, the axisymmetric steady state radial electric field in the pedestal may be determined by a competition between the turbulent Reynolds stress and collisional perpendicular ion viscosity for some parameter regimes.

## 5. Discussion

In an axisymmetric, single ion species tokamak, intrinsic ambipolarity requires the distribution functions, and heat and particle fluxes to be independent of electrostatic potential to leading order in gyroradius. Moreover, a moment description can be used to demonstrate that intrinsic ambipolarity must be satisfied to second order in the ion gyroradius expansion. We have shown that standard gyrokinetics incorrectly determines the axisymmetric, long wavelength electrostatic potential to leading order in gyroradius over major radius by considering a steady-state theta pinch with a distribution function correct to second order [1]. A similar problem arises in tokamaks as stressed herein. In both cases the correct radial electric field is determined from toroidal angular momentum conservation.

Using canonical angular momentum as the radial variable allows strong gradients to be treated gyrokinetically. Entropy production then requires a physical lowest order banana regime ion distribution function to be nearly an isothermal Maxwellian with the ion temperature scale much greater than the poloidal ion gyroradius [2]. Thus, the background ion temperature profile must have a pedestal with a scale much larger than that of any density pedestal varying on a ion poloidal gyroradius scale. In addition, weak ion temperature variation with subsonic flow in such a pedestal requires electrostatically restrained ions and magnetically confined electrons thereby impacting zonal flow behavior and altering the neoclassical polarization [6].

Simulating tokamaks on transport time scales requires evolving drift turbulence with axisymmetric neoclassical and zonal flow radial electric field effects retained. Full electric field effects are far more difficult to retain than density and temperature effects since they require evaluating the ion distribution function to higher order than standard gyrokinetics. An electrostatic hybrid gyrokinetic-fluid treatment using moments of the full Fokker-Planck equation removes the need to go to higher order. This hybrid description evolves potential, density, temperatures, and flows, and includes all electrostatic neoclassical and turbulent effects with wavelengths much longer than an electron gyroradius.

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BLACK & WHITE FIGURES

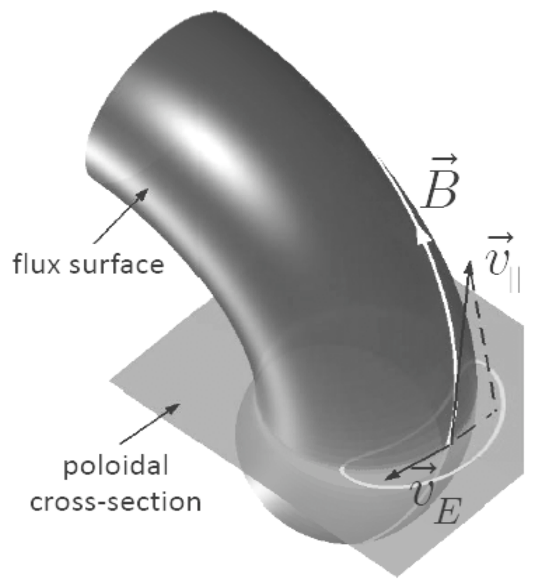


*Figure 1: Projection of the parallel streaming into the poloidal plane.*

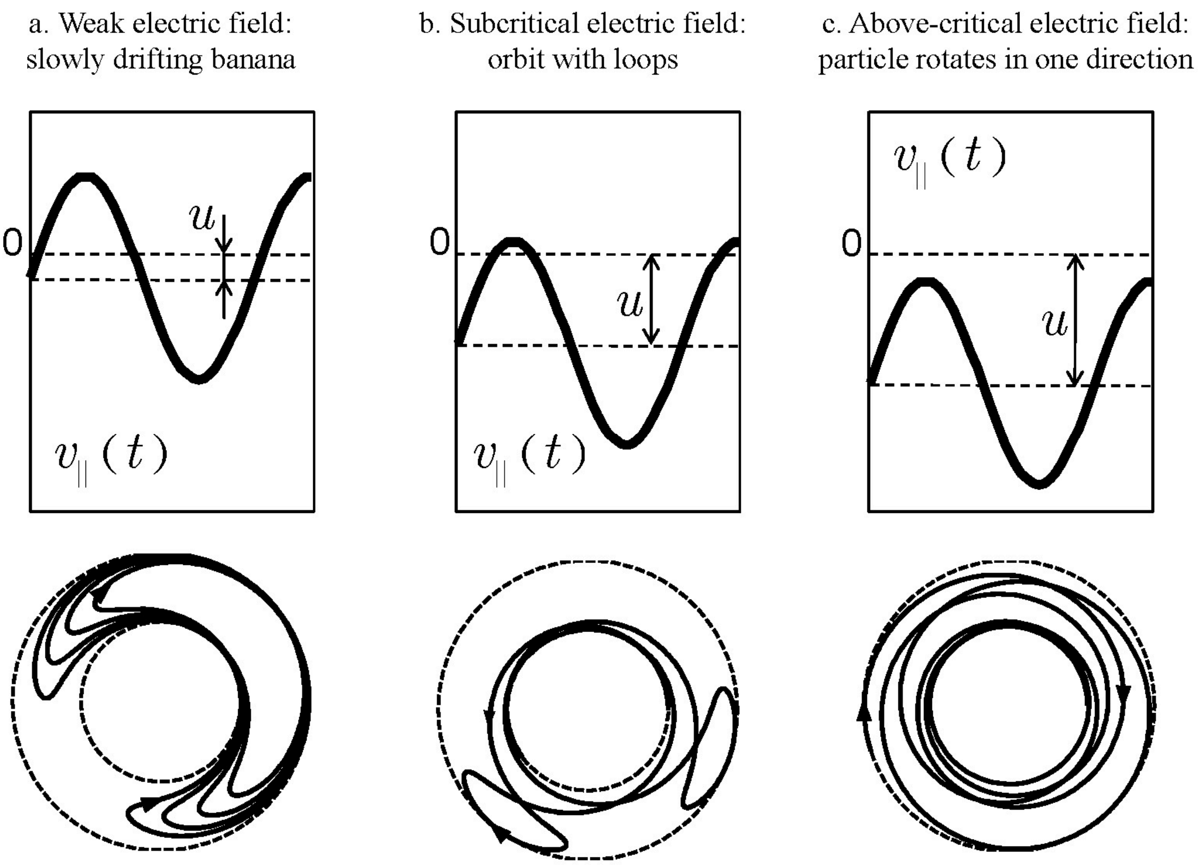


*Figure 2: Toroidal projections of trapped particle orbits for weak, subcritical ($v_{||}+u < 0$), and above critical ($v_{||}+u > 0$) **E**×**B** drift.*

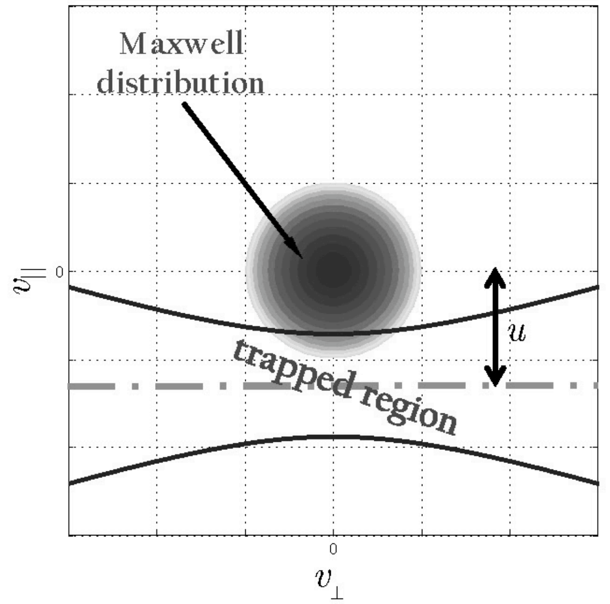


*Figure 3: In the presence of a strong radial electric field the trapped region moves toward the tail of the Maxwellian distribution.*