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Fully self-consistent ion-drag force calculations for dust in collisional plasmas with external electric field

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Fully self-consistent ion-drag force calculations for dust in collisional plasmas with external electric field

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The ion-drag force on a spherical dust particle immersed in a flowing plasma with external electric field is self-consistently calculated using the Particle In Cell code SCEPTIC in the entire range of charge-exchange collisionality. Our results, not based on questionable approximations, extend prior analytic calculations valid only in a few limiting regimes. Particular attention is given to the force direction, shown never to be directed opposite to the flow except in the continuum limit, where other forces are of much stronger magnitude.

Understanding the behaviour of a single dust particle embedded in a flowing plasma is a physical problem of practical relevance to the field of dusty plasmas, with concrete applications in fusion-grade, astrophysical, laboratory and technological plasmas [1]. The most important observable effect is perhaps momentum transfer between the plasma and the dust grain —the ion-drag force [2, 3, 4]— responsible for the dust dynamics as well as the formation of static configurations. Force calculations have very recently received renewed attention in the context of weakly ionized plasmas, where effects of fundamental physical importance transcending the field of dusty plasmas have been proposed. It has for example been proposed that two positively charged dust particles can attract each other by a process analogous to cooper pairing in superconductors [5].

More surprisingly, it is found that in the same strongly collisional limit the ion-drag on a single dust grain could be negative [6, 7, 8], implying that those particles see the surrounding plasma as a superfluid [9]. Several published calculations have been done by treating the plasma as a linear dielectric medium, and are therefore only valid for large shielding lengths $\lambda_s \gg b_{90}$, where b_{90} is the 90^o scattering parameter of the dust particle. Only the weak and moderate [3] as well as strongly collisional [6] regimes have been investigated with this approach, and the results of Ref. [3] are not directly quantifiable because they depend on the dust floating charge Q_f, not known *a priori*. In the highly collisional limit, full non-linear calculations have been performed under the assumption of negligible ion diffusivity (mobility regime) [7, 8], predicting that ion-drag reversal only occurs at strong collisionality. Some Monte-carlo simulations [10] claim a negative ion-drag occurs also in the weak collisionality regime.

The purpose of this letter is to calculate the ion-drag force on a conducting spherical dust particle (radius r_p) in the entire range of collisionality, by solving the full self-consistent non linear problem; this bridges the gaps between all available partial results. For this purpose we use the Particle In Cell (PIC) code SCEPTIC [4], recently upgraded to account for constant collision-frequency (ν_c) charge-exchange events [11]. More complex collision models could be implemented, but the present approach limits the number of parameters while only negligibly affecting the physics [12]. SCEPTIC is a hybrid kinetic code where the collisionless electrons have the Boltzmann density $n_e = n_\infty \exp(\phi)$ ($\phi = eV/T_e$ and n_∞ is the electron density at infinity) and the ions are advanced according to Newton's equation between two collisions ($m_i \mathrm{d}\mathbf{v}/dt = -Ze\nabla V$) up to convergence of the simulation. The electrostatic potential is solved on a two-dimensional spherical mesh centered on the dust particle and extending several electron Debye lengths $\lambda_{De} = \sqrt{\epsilon_0 T_e/n_\infty e^2}$. The dust potential can be prespecified or floating, i.e. self-consistently set to a value such as to equate the ion current with the analytic collisionless electron current $I_e = 4\pi r_p^2 v_{te}/2\sqrt{\pi}$ ($v_{te} = \sqrt{2T_e/m_e}$ is the electron thermal speed). At the outer boundary we impose $\mathrm{d}\ln\phi/\mathrm{d}r = -r_p(1/(\lambda_s r) + 1/r^2)$, a condition valid at weak and moderate collision frequency as well as in the continuum limit if $\lambda_s \gg r_p$.

The ions are reinjected at the outer boundary according to their distribution function at infinity ("SCEPTIC1" convention in Ref. [11]). We focus on the situation where the neutral background is stationary and the ion flow driven by an external electric field $\mathbf{E}_{\text{ext}} = \mathbf{v_d} m_i \nu_c / Ze$ (Z is the ion charge, usually Z=+1), whose role is to compensate the ion-neutral friction at infinity. In this case, provided $\nu_{ii} \ll \nu_c$ (ν_{ii} is the ion-ion Coulomb collision frequency), the ion distribution at infinity is given by [13]:

$$f_i^\infty(\mathbf{v}) = \frac{1}{(v_{ti}\sqrt{\pi})^2}\frac{1}{2v_d}\exp(-\frac{\mathbf{v}^2}{v_{ti}^2})\mathrm{erfcx}\left(\frac{v_{ti}}{2v_d} - \frac{v_z}{v_{ti}}\right), \quad (1)$$

where $\mathrm{erfcx}(x) = \exp(x^2)\mathrm{erfc}(x)$. f_i^∞ does not depend on ν_c, and tends to a drifting Maxwellian f_M with temperature T_i and thermal speed $v_{ti} = \sqrt{2T_i/m_i}$ as $v_d/v_{ti} \to 0$. Here T_i refers to the temperature of the neutral background; the effective ion temperature is $T_{i,z}^\infty = T_i + m_i v_d^2 \geq T_i$ and $T_{i,\perp}^\infty = T_i$.

The ion-drag $\mathbf{F_i}$ is the sum of the electrostatic force on the dust particle surface $\mathbf{F_E^p}$ arising from the interaction of its usually negative charge with the flow-induced anisotropy of the plasma (calculated by integration of the Maxwell stress tensor at the particle surface), the momentum collected by direct ion impact $\mathbf{F_{im}^p}$, and the electron-pressure force $\mathbf{F_e^p} = 0$ (averaging to zero on a conducting body [4]): $\mathbf{F_i} = \mathbf{F_{im}^p} + \mathbf{F_E^p}$. Momentum conservation implies that in steady state, the ion-drag be also equal to the rate of momentum flux across any control surface surrounding the dust particle, in particular the outer boundary of the computational domain [4]. In a collisionless plasma: $\mathbf{F_i} = \mathbf{F^o_{im}} + \mathbf{F^o_E} + \mathbf{F^o_e}$ ($\mathbf{F^o_{im}}$: net ion momentum flux into the computational domain; $\mathbf{F^o_E}$ and $\mathbf{F^o_e}$: integrals of the electrostatic stress and electron pressure on the boundary). Of course because the SCEPTIC domain is not infinite, the electrostatic stress and electron pressures at the outer boundary are not negligible. In the presence of ion-neutral collisions, one must also consider ion friction with the neutrals and the momentum provided by the external electric field inside the control volume:

$$\mathbf{F^o_n} = m_i \nu_c \int_{\text{Domain}} n_i(\mathbf{x})(\mathbf{v_d} - \mathbf{v}(\mathbf{x}))\mathrm{d}^3\mathbf{x}, \qquad (2)$$

where $\mathbf{v}(\mathbf{x})$ is the ion fluid (local average) velocity. F^o_n can be either positive or negative; the integral in Eq. (2) is however convergent and F^o_n tends to a limit as the domain size is increased. We will refer to $\mathbf{F_{im}}$, $\mathbf{F_E}$, $\mathbf{F_e}$, $\mathbf{F_n}$ respectively as the "Ion", "E-field", "Electrons" and "Collisions" forces.

We begin by comparing our code with the results of Schweigert and coauthors [10], who propose a weak collisionality regime ($\lambda_{mfp} = \sqrt{8T_i/\pi m_i}/\nu_c \gg \lambda_s$) for which they find a negative ion-drag. The dimensional parameters corresponding to their Fig. (2) are $r_p = 4.7\mu m$, $\lambda_s = 20 - 100\mu m$, $Q_p = 3.6 \cdot 10^4 e$ (dust charge), $T_e = 6eV$, $T_i = 0.026eV$, $P = 75 - 150Pa$ (neutral pressure), $\sigma_c = 3.53 \cdot 10^{-15} cm^2$ (constant collision cross-section), $n_\infty = 3.6 \cdot 10^9 cm^{-3}$. Helium ions are used, but this information is irrelevant because their work is based on a direct orbit-integration approach, where the dust charge as well as the potential distribution are prespecified. If we use the usual linearized formula for the sphere capacitance: $C = 4\pi\epsilon_0 r_p(1 + r_p/\lambda_s)$, for simplicity set $\lambda_s^{-2} = \lambda_D^{-2} = \lambda_{De}^{-2} + \lambda_{Di}^{-2}$, and convert constant cross-section into constant collision-frequency according to $\nu_c = \sigma_c\sqrt{8T_i/\pi m_i}P/T_i$, we find the corresponding parameters to use for our SCEPTIC simulations. Fig. (1) shows the total ion-drag as well as its different components for the case corresponding to the curve $\lambda_s = 20\mu m$ and $P = 150Pa$ in Fig. (2) of Schweigert's publication [10]. The ion-drag calculated by SCEPTIC at the dust surface and the outer boundary of the computational domain agree (their balance is of course different), which gives us strong confidence that our code performs properly and the runs are well converged. More important the total ion-drag is positive, and does not agree with Schweigert's according to which it can reverse for low drift velocities.

Our hypothesis to explain Schweigert's erroneous conclusion is as follows. Fig. (2) shows the two ion-drag curves from Fig. (2) of Ref. [10] with $\lambda_s = 20\mu m$ (lines), as well as the ion-drag computed by SCEPTIC at the outer boundary of the domain without accounting for F^o_n (lines with circle-markers), and the *correct* ion-drag (lines with square-markers).

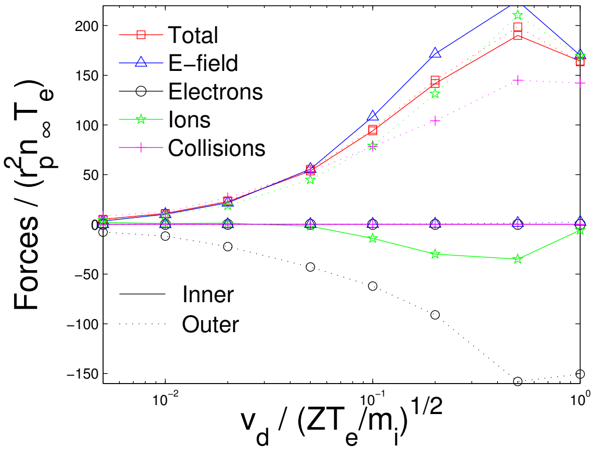

FIG. 1: Ion-drag force as a function of drift velocity computed by SCEPTIC for $T_i = 4.33 \cdot 10^{-3} ZT_e$, $\lambda_{De} = 64.8r_p$, $\phi_p = -1.49$ (Here imposed rather than floating), and $\nu_c = 6.27 \cdot 10^{-3}\sqrt{ZT_e/m_i}/r_p$; corresponding to the curve $\lambda_s = 20\mu m$ and $P = 150Pa$ in Fig. (2) from Ref. [10]. "Inner" (solid lines) and "Outer" (dotted lines) refer to the forces calculated at the dust surface and outer boundary of the computational domain; of course the total force (squares) does not depend on where it is evaluated.

The strong similarity between Schweigert's results and SCEPTIC-computed drags not accounting for F^o_n suggests that the far-from-negligible F^o_n has been neglected in Ref. [10].

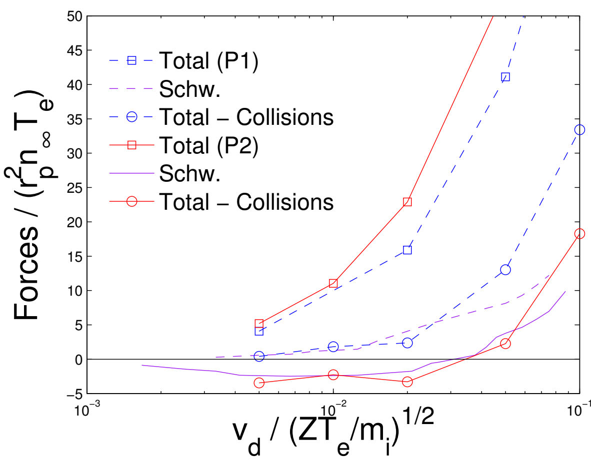

FIG. 2: Ion-drags from Fig. (2) in Ref. [10] at $\lambda_s = 20\mu m$ ("Schw."), ion-drags computed by SCEPTIC at the outer boundary of the computational domain ("Total"), and ion-drags computed by SCEPTIC without accounting for F^o_n ("Total−Collisions"). P_1: $P = 75Pa$ i.e. $\nu_c = 3.14 \cdot 10^{-3}\sqrt{ZT_e/m_i}/r_p$; P_2: $P = 150Pa$ i.e. $\nu_c = 6.27 \cdot 10^{-3}\sqrt{ZT_e/m_i}/r_p$ (P_2 curves correspond to the parameters of Fig. (1)). The *correct* ion-drag is is always *positive* (lines with square-markers).

The only regime where we have observed a negative ion-drag is when $\nu_c \gg \sqrt{ZT_e/m_i}/r_p$. In this highly collisional limit a fluid treatment is appropriate, and analytic calculations assuming negligible diffusivity (i.e. mobility-dominated physics) and $\lambda_s \gg r_p$ have recently been published [7, 8]. According to this model the ion density is uniform on the ion fluid streamlines, hence the ion current (required for an analytic calculation of the floating potential) and the electrostatic stress at the particle surface only depend on the stream-lines' topology [7, 8]:

$$I_i = \pi n_\infty T_e r_p \frac{|\phi_p|}{m_i \nu_c}\left[\frac{1}{\tilde{a}_0^2} + 2 + \tilde{a}_0^2\right] \qquad (3)$$

$$F_E^p = \pi r_p^2 n_\infty T_e |\phi_p| \left[(1-\tilde{a}_0^{-4}) - 2\tilde{a}_0^{-1}\int_0^{\mathrm{acos}(\tilde{a}_0^{-2})} \sqrt{\tilde{a}_0^2 + \tilde{a}_0^{-2} - 2\cos\theta}\cos\theta \mathrm{d}\theta\right], \qquad (4)$$

where $\tilde{a}_0 = a_0/r_p = \max\left(1, \sqrt{r_p m_i \nu_c v_d/(Ze|V_p|)}\right)$. F_E^p is always *negative* or null, because in the considered regime the downstream shadow of the dust particle is fully ion-depleted [6]. The ion impact part of the ion-drag, depending on both the stream-lines topology and the ion dynamics along those lines, can be decomposed in two parts. The first corresponds to the flux of **z**-momentum to the dust if the ions were indeed cold [7]:

$$F_{im,1}^p = \pi(r_p^2 n_\infty T_e) v_d \frac{|\phi_p|}{r_p \nu_c}\left(\tilde{a}_0^2 + \frac{8}{3} + \frac{2}{\tilde{a}_0^2} - \frac{1}{3\tilde{a}_0^6}\right) \qquad (5)$$

The second accounts for the ion pressure at the dust surface, due to the effective ion temperature along the stream-lines, *never* negligible even if $T_i \ll T_e$: $T_{i,\parallel}(\mathbf{x}) = T_i + m_i \mathbf{v}(\mathbf{x})^2$ (The heating depends on the local fluid velocity $\mathbf{v}(\mathbf{x})$ rather than $\mathbf{v_d}$, because the stream-lines are curved in the dust vicinity). The ion temperature perpendicular to the stream-lines is $T_{i,\perp} = T_i$. In the limit $T_i \ll T_e$, integration of the ion pressure at the dust surface yields:

$$F_{im,2}^p = \frac{\pi}{6}(r_p^2 n_\infty T_e)\frac{m_i v_d^2}{T_e}\left[(6 + 8\alpha + 3\alpha^2) - \tilde{a}_0^{-4}\left(6 - 8\tilde{a}_0^{-2}\alpha + 3\tilde{a}_0^{-4}\alpha^2\right)\right], \qquad (6)$$

where $\alpha = -ZeV_p/(m_i v_d \nu_c r_p)$ (we recall that $ZeV_p < 0$); $F_{im,1}^p$ and $F_{im,1}^p$ are *positive*. Eqs. (3,4,5,6) correspond to the mobility model.

Fig. (3) is a plot of forces and floating potential versus collisionality for H^+ ions with $T_i = 0.01ZT_e$, that we now analyze from high to low collisionality. In the continuum limit the mobility model (dashed lines) agrees with SCEPTIC results (symbols), thus validating both works. As the collisionality decreases, the floating potential rises to a maximum at $\nu_c \simeq 0.01\sqrt{T_e/m_i}/r_p$ (a moderate collisionality tends to increase the ion current [11]). The E-field part of the ion-drag follows a similar trend, i.e. peaks when $\lambda_{mfp} \lesssim \lambda_s$ (shielding length); the reason being that collisions favour ion focusing downstream by breaking orbital angular momentum conservation, hence increase the polarization field in the drift direction. This effect, carefully explained in Ref. [3], is not taken into account by Maiorov [14] who therefore arrives at the same erroneous conclusion as Schweigert (i.e. that a negative ion-drag can occur at weak collisionality). The dip in ion-impact force seen at $\nu_c \simeq 0.03\sqrt{T_e/m_i}/r_p$ is

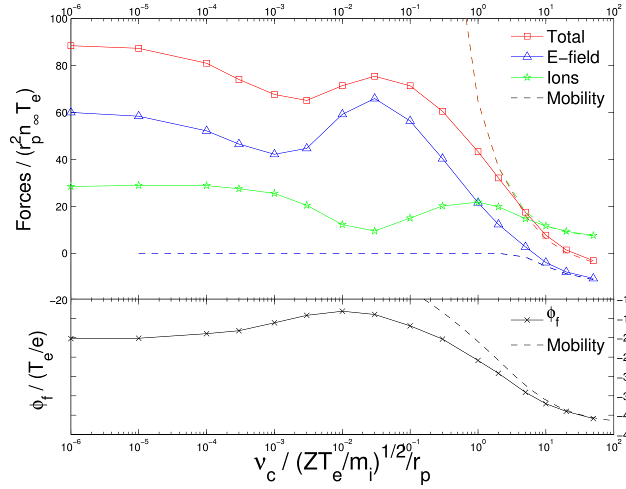

FIG. 3: Ion-drag and dust floating potential against collisionality for H^+ ions; $T_i = 0.01ZT_e$, $v_d = \sqrt{ZT_e/m_i}$ and $\lambda_{De} = 20r_p$. The points correspond to SCEPTIC data at the dust surface, and the dashed lines to the mobility calculations (4,5,6,3).

due to the local floating potential maximum, whose effect is to reduce the energy at which ions are collected. The local minimum in the E-field part of the ion-drag at $\nu_c \sim 10^{-3}\sqrt{ZT_e/m_i}/r_p$ corresponds to $\lambda_{mfp} \gtrsim \lambda_s$. As we keep reducing the collisionality, the floating potential decreases to its "collisionless" ($\nu_{ii} \ll \nu_c \ll \sqrt{ZT_e/m_i}/r_p$) value, causing the E-field part of the ion-drag to increase.

Let us discuss further our results in a regime where the conditions $b_{90} \ll \lambda_s$ and $v_d \ll v_{ti}$ are satisfied. This allows us to directly compare our computations with the result of Ivlev and coauthors, who calculate the electrostatic part of the ion-drag using the linearized plasma equations, not accounting for downstream depletion due to the finite-sized grain (appropriate at weak or moderate collisionality) [3]:

$$F_E^p(v_d) = (r_p^2 n_\infty T_e)\frac{ZT_e}{T_i}\phi_p^2 \frac{4\sqrt{2}}{3} u\left[K\left(\frac{\lambda_D}{l_i}\right) + \sqrt{2\pi}\ln\Lambda\right] \qquad (7)$$

Here $l_i = \sqrt{\pi/8}\lambda_{mfp}$, $\ln\Lambda = \ln(\lambda_D/2b_{90})$, where $u = v_d/v_{ti}$. The function K is given by $K(x) = \mathrm{atan}(x)x + \left(\sqrt{\pi/2}-1\right)x^2/(1+x^2) - \sqrt{\pi/2}\ln(1+x^2)$. We also used the vacuum expression for the dust particle capacitance: $V_p = Q_p/4\pi\epsilon_0 r_p$.

Fig. (4) is a plot of forces and floating potential versus collisionality for equithermal Ar^+ ions and electrons, $\lambda_{De} = 30r_p$ and $v_d = 0.2\sqrt{ZT_e/m_i}$. The ion-drag components and dust floating potential follow a trend similar to what has been observed in Fig. (3), although perhaps with a more monotonic behaviour because at high temperature the ion flow is less sensitive to slight variations in floating potential. Of course the ion-drag peak is located at a higher collisionality ($\lambda_{mfp} \propto \sqrt{T_i}$ is larger here). The agreement between SCEPTIC and Eq. (7) is only qualitative; recall that Eq. (7) is only valid to logarithmic accuracy in λ_D/b_{90}.

Fig. (4) also shows the high collisionality limit of ϕ_f and F_E. While the mobility model does not apply for

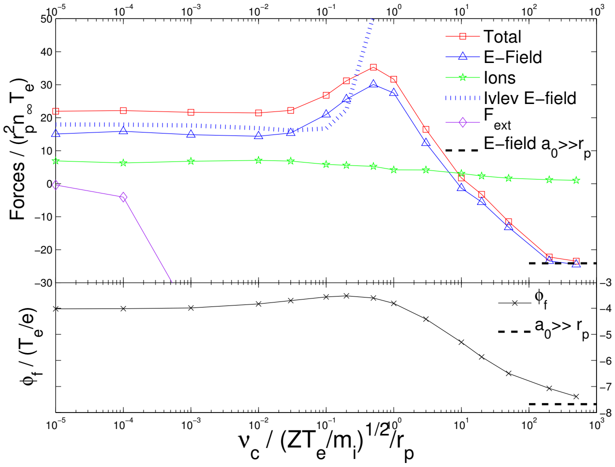

FIG. 4: Ion-drag and floating potential versus collisionality for Ar^+ ions; $T_i = ZT_e$, $v_d = 0.2\sqrt{ZT_e/m_i}$ and $\lambda_{De} = 30r_p$. The points correspond to SCEPTIC data at the dust surface, the dotted line to Eq. (7), and the dashed lines to the limits (8,9). F_{ext} is the background electric force on the dust particle (Eq. (11)), directed upstream.

$T_i = ZT_e$ (It requires $T_i \ll ZT_e$), it is valid in the limit $\tilde{a}_0 \gg 1$, i.e. when E_{ext} strongly dominates the dust Coulombic field. In other words as $\tilde{a}_0 \to \infty$:

$$I_i \to \pi n_\infty r_p^2 v_d, \quad \text{and} \tag{8}$$

$$F_E^p \to \pi r_p^2 n_\infty T_e \phi_p, \tag{9}$$

regardless of T_i/ZT_e (provided of course the formula for vacuum capacitance is applicable, i.e. $\lambda_{De} \gg r_p$). This proves that past a certain collisionality, increasing ν_c does not increase the negativity of F_E, bounded by:

$$F_E^p \geq \pi(n_\infty r_p^2 T_e) \ln\left(\frac{v_d\sqrt{\pi}}{2v_{te}}\right), \tag{10}$$

where the logarithm is the floating potential calculated by equating Eq. (8) with the collisionless electron current.

We have therefore shown that only in the continuum limit ($\nu_c \gg \sqrt{ZT_e/m_i}/r_p$) can the ion-drag reverse. There the ambipolar electric force on the dust particle ($\mathbf{F}_{\text{ext}} = Q_p\mathbf{E}_{\text{ext}}$)

$$\mathbf{F}_{\text{ext}} \sim 4\pi\phi_p(r_p^2 n_\infty T_e)\frac{\lambda_{De}^2}{r_p^2}\frac{\nu_c}{\sqrt{ZT_e/m_i}/r_p}\frac{v_d}{\sqrt{ZT_e/m_i}} \tag{11}$$

is much larger than ion-drag itself (Fig. (4)); in the typical situation where $Q_p \sim 4\pi\epsilon_0 r_p V_p < 0$, $\mathbf{F}_{\text{ext}}$ is directed upstream.

We have also performed extensive simulations, not shown here, where the ion drift is driven by a neutral flow assumed unaffected by the dust particle (kinetic regime); the ion distribution at infinity is then given by a drifting Maxwellian rather than by Eq. (1). The ion-drag is very similar in magnitude to the results presented here, for comparable collisionality and flow velocity. The negative F_{ext} (Eq. (11)) is however replaced by a positive neutral drag.

In conclusion, we have presented fully self-consistent calculations of the ion-drag force on a spherical dust particle over the entire range of charge-exchange collisionality, when the ion flow is driven by an external electric field. We have shown that the ion-drag behaves similarly to the floating potential; it shows a local maximum when $\lambda_{mfp} \lesssim \lambda_s$, and tends to a limit at high ν_c. Although there are physically meaningful parameters where the ion-drag is negative, that occurs only in collisional regimes where the direct electric field force far exceeds the drag.

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