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Physics and Technology of the Feasibility of Plasma Sails

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Abstract: Plasma sail technology, in which an artificial magnetosphere is generated around a spacecraft and is coupled to and dragged by the solar wind, has been proposed as a propulsion technique to rapidly travel to the outer regions of the solar system and heliosphere. An examination of the physics and planned implementation of plasma sail propulsion indicates that the proposed technology does not provide a functional, resource-competitive and viable propulsion mechanism using current technologies or technologies expected to be available in the foreseeable future. This conclusion is primarily based on application of the conservation of magnetic flux for the plasma sail system, which follows from the requirement that the divergence of the magnetic field vanish. A functional plasma sail must intercept and couple to a large area of the solar wind, and spacecraft thrust is derived from momentum transfer from the solar wind. To obtain the magnetic flux necessary for a large bubble requires a much larger magnet than those described in the literature. Even for a highly conservative implementation having a minimal 10 km radius of the artificial magnetosphere, along with an optimistic assumption of a 1/r fall-off in the magnetic field and a boundary condition of ~50 nT at the edge of the sail that is needed to balance the solar wind pressure (~2 nPa), one finds that the magnetic flux in the magnetic coil on the spacecraft must be at least ~30 Wb,

with a larger bubble requiring a larger flux. To provide such a large flux for a reasonable mass and power requires a superconducting coil. Utilizing state-of the-art magnet construction parameters, a 30 Wb superconducting coil weighs ~ 3 tons without the necessary refrigeration and power systems. This is orders of magnitude above the previously published 50 kg estimates and renders the proposed technology non-competitive with existing propulsion systems based on a thrust per unit mass metric.

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I. Introduction

Plasma sails have been proposed as an attractive and feasible means of propulsion for exploring the outer solar system and beyond including the Kuiper belt [Winglee, et al., 2000]. The propulsion concept utilizes a magnetic field generated around a spacecraft, forming a bubble that couples to the solar wind and transfers solar wind momentum to the spacecraft. The original concept, called the Mini-Magnetospheric Plasma Propulsion, or M2P2, a compact alternative to the earlier magnetic sail concept [Zubrin and Andrews, 1983; Zubrin, 1993] was interesting and innovative because of its use of the solar wind, which is a ubiquitous energy source in the heliosphere. While the magnetic sail idea is based on the direct deflection of the solar wind by a very large magnet, Winglee et al. [2000] instead suggested that a large magnetic bubble (a mini-magnetosphere) could be formed using a small magnetic coil and plasma source attached to a spacecraft, to efficiently inflate the bubble to a large cross-sectional area. A net force would be exerted on the spacecraft due to the deflection of the solar wind around the bubble. By analogy, such a deflection of the solar wind by the magnetic bubbles (magnetospheres) around the Earth and other magnetized planets has been routinely observed. Inflation of a magnetic field by plasma injection is also observed in natural systems, for example, injection of plasma from the photosphere leads to coronal loops on the Sun, and planetary magnetospheres are inflated by the external injection of plasma from the solar wind.

The net force on the spacecraft is due to the solar wind momentum transferred first to the bubble and then from the bubble to the spacecraft at the electromagnet coil that generates the magnetic field. This momentum transfer is via deflection of solar wind ions from the magnetic bubble, which can only occur if the bubble is larger than a minimum size related roughly to the solar wind ion inertial length ($c/\omega_{pi} \sim 100$km, where ω_{pi} is the ion plasma frequency and c the speed of light). We can represent the net momentum p_{SC} imparted to the spacecraft using the simple equation $p_{SC} = P_{SW} * \pi b^2 * \varepsilon_B * \varepsilon_{SC}$ where P_{SW} is the solar wind momentum per unit area, b is the bubble radius such that πb^2 is the cross-sectional area of the bubble as viewed by the solar wind, ε_B is the momentum coupling efficiency from the solar wind to the bubble, and ε_{SC} is the coupling efficiency from the bubble to the spacecraft (since momentum may be lost by reconnection and transport). Note that this is a simple representation of each component that would need to be investigated in some detail for a complete engineering analysis. For this study, we focus primarily on assessment of b for the best case scenario in which $\varepsilon_B = \varepsilon_{SC} = 1$ and the resources needed to generate a bubble commensurate with the magnitude of b.

In order for the plasma sail propulsion system to be viable, the following elements are necessary: (1) a magnetic bubble of radius of tens of kilometers to effectively deflect the solar wind; (2) transfer of a large fraction of the solar wind momentum to the spacecraft; and (3) a magnetic coil and plasma source utilizing low enough mass and power resources for a practical mission.

The force (thrust) on a plasma sail system is the result of momentum transfer from solar wind ions via a dipole magnetic field generated at the spacecraft, so any push on the dipole may be transferred to the spacecraft. Because the magnetic field of a vacuum dipole field falls off rapidly (r^{-3}) with increasing distance r from the spacecraft and results in a bubble too small to intersect much solar wind, the concept relies on inflating the dipole magnetic field by injecting plasma along magnetic field lines that pass through the coil at the spacecraft. The injected plasma remains confined within the dipole field and generates an internal pressure that stretches the closed magnetic field lines and thereby inflates the flux tube region between neighboring field lines. The intent is that the bubble inflates so that the magnetic field of the dipole falls off more slowly than $1/r^3$, preferably 1/r over a large range of r. It has been noted that, in the absence of the solar wind and inflation, the highest pressure plasma that can be confined by a dipole field results in diamagnetic currents that makes the magnetic field fall off only as $1/r^2$ [Krasheninnikov et al., 1999].

For a plasma sail, the size of the inflated bubble of the closed field lines that link to the spacecraft is one of the most critical issues because it determines the area of the sail intercepting the solar wind, and thus the maximum momentum that could be transferred to the spacecraft. The location of the outer boundary of the bubble, which defines the bubble size, is called the magnetopause and is determined by balancing the local magnetic and plasma pressure with the kinetic pressure of the solar wind (which decreases inversely as the square of the radial distance from the Sun). If the entire inflated field varies as 1/r, then the magnetic field at the bubble's magnetopause decreases at the rate needed so that the bubble area will increase to just balance the decrease in the solar wind pressure. This feature is critical for use of the proposed plasma sail for outer heliospheric missions because it insures that the intercepted solar wind momentum (and thus the thrust) remains constant. However, if the bubble magnetic field varies primarily as $1/r^2$ or $1/r^3$, then the bubble area will increase at a slower rate than the decrease in the solar wind pressure, and the net momentum transferred to the spacecraft will decrease with increasing distance from the Sun.

The plasma sail concept is critically dependent on the magnetic flux produced by the magnet as well as on details of the inflation process. For example, at the equatorial plane of the dipolar field, according to Gauss' law, the magnetic flux inside the magnet coil must be balanced by an equal and opposite magnetic flux outside the coil but within the bubble of closed field lines linked to the spacecraft magnet. Consequently, the size of the magnet becomes the key issue because of the large mass, power, and volume of high magnetic flux magnets and the limited launch and spacecraft resources available, especially for missions traveling to the outer solar system or outer heliosphere.

In Section II, previous studies of the plasma sail concept are examined. Although some laboratory experiments have been conducted on the plasma sail concepts, a discussion of the application of the results requires a somewhat lengthy description of the diagnostics, the interpretation of the measurements and issues of scaling and will not be discussed here. In Section III, the key issue of the required magnetic flux and magnetic field fall-off are discussed. The magnet size required to produce the necessary flux is discussed in Section IV. Conclusions are given in Section V.   Specifics of implementation, including required spacecraft resources and comparison to other propulsion technologies, are presented in the Appendix.

II. Brief review of previous studies

Except for study of existing systems such as the Earth's magnetosphere, no experiments have been performed to validate the plasma sail concept, primarily because due to the large scale size required. Therefore, previous studies of plasma sail concepts have relied on numerical simulations to evaluate the feasibility of the concepts and to derive the physical properties of the bubble structure, the system dynamics, and its scaling relations. Various types of magnetohydrodynamic (MHD) and hybrid (particle ion, fluid electron) codes have been employed in these studies, each of which has both strengths and weaknesses. The primary advantage of the MHD codes is the ability to cover the wide range of scales from the diameter of the magnet coil (< 1m) to the size of the bubble (10-100 km). The validity of MHD results describing the interaction of the plasma sail with the solar wind, however, is somewhat uncertain because the ion inertial length is comparable to the size of the magnetic bubble-solar wind system and the gyro-radii of the confined ions are also comparable to the bubble size throughout much of the system. An important assumption for applying MHD is that both the ion inertial length and gyroradius must be small compared to the bubble radius. When these length scales are not small, the MHD approach will substantially overestimate the momentum transfer. Extending MHD to include the Hall term allows the fluid treatment to remain valid on ion inertial scales, but momentum cannot be effectively coupled to the unmagnetized ions. On the other hand, the principal strength of the hybrid codes is that because they treat the dynamics of the ions kinetically as particles, rather than in a fluid approximation, and include the Hall term directly, they can more accurately model the solar wind interaction with the magnetic bubble. However, they cannot cover the full range of scales of the system and, therefore, usually an initially inflated, large bubble and/or a specific magnetic field profile must be assumed.

Using a series of scaled MHD calculations that include the Hall term, Winglee et al. (2000) concluded that the size of the bubble increases linearly with the magnetic field of the magnet (see their Fig. 2). Results also indicated that B ~ 1/r over a portion of the bubble and the presence of a bow "shock" upstream of the bubble with a standoff distance about 70% of the bubble diameter, which when scaled up to solar wind conditions, corresponded to a bubble radius of 10 km and standoff distance from the magnet of 15 km for a magnet field of 600G and a coil radius of 10 cm. More recent results [Winglee, 2004] concluded that the radial fall-off of B in the equatorial plane is proportional to $r^{-1.2}$ over several orders of magnitude change in bubble radius. This result is also supported by calculations of Khazanov et al. (2003), who ran combined MHD calculations near the magnet and hybrid calculations farther out and obtained a mini-magnetosphere with a well-formed bow shock (~ 80 km) and a magnetopause (~ 40 km) and a much larger bubble (diameter ~ 100 km). To a distance of 30 km from the magnet, they find that the injected plasma and magnetic field radial profiles are essentially identical whether or not the solar wind is present, suggesting that the solar wind is excluded by the large bubble. While these results appear to support the plasma sail concept, there is an issue with magnetic flux conservation in the calculations, as discussed later. In addition, there is some question of how extrapolations are used to arrive at the final full-scale results. More details on these issues are provided in Section III C.

Winske and Omidi (2005) have recently examined the initial phase of the expansion of an injected plasma in a dipole magnetic field embedded in a stationary background plasma, using a two-dimensional hybrid code. They find that for small dipoles the injected ions are unmagnetized and the expansion results in the formation of a diamagnetic cavity with only minor perturbations to the initial dipole field geometry. Large diamagnetic cavities have been produced by material (barium, lithium) releases from satellites (with no on-board dipole magnet) in the solar wind and the magnetotail [Luhr et al, 1986; Bernhardt et al., 1987]. On the other hand, in calculations where the initial dipole field is larger but the injection rate is the same, a similar sized cavity forms but the injected ions remain trapped near the dipole. As these ions then expand outward, they drag the dipole field lines along, resulting in a significant modification of the original dipole field, but nothing as dramatic as what would be required to obtain B ~ 1/r behavior in 3-D.

III. Magnetic flux conservation and implementation of a viable plasma sail

The most crucial issue for assessing the feasibility of plasma sail propulsion is the magnetic flux (and, therefore, magnet size and power) required to maintain a magnetic bubble of sufficient size to yield a specific impulse greater than competing propulsion technologies. Because the magnetic flux issue provides the most compelling evidence that the plasma sail is not competitive with other propulsion technologies, other plasma sail issues related to the transfer of solar wind momentum to the spacecraft and the rate of loss of the inflation plasma (and, therefore, expenditure rate of propellant mass) over time are only briefly discussed. In this section, the required bubble size, associated magnetic flux in the bubble for various magnetic field fall-offs, magnet construction and sizing and related issues are examined.

A. **Opacity of the bubble in the solar wind**

The force on the magnetic bubble of a plasma sail comes from the momentum transferred from the solar wind. As long as the solar wind can be deflected by the bubble, a net force will be exerted on the bubble. For large objects in the solar wind this transfer occurs by a bow shock wave, which converts the momentum into a pressure on the bubble. For smaller bubbles, the thickness of the shock wave remains constant while the distance between the magnetopause and the shock wave decreases, as has been discussed in relation to the type of magnetic disturbance produced by asteroids of various sizes in the solar wind [Omidi et al., 2002]. For a small enough bubble, the shock wave and magnetopause become superimposed. However, as long as the ions are stopped (or significantly deflected) by the magnetic field in the bubble, momentum is still transferred to the bubble.

If the bubble size or magnetopause of a plasma sail becomes too small then the solar wind will no longer be affected by it. The conditions under which a bubble of sufficient cross-section can deflect the solar wind can be defined by the following two arguments. The first argument is that the object must provide a strong enough magnetic field that it deflects solar wind ions at the bow shock or magnetopause (assumed here to be essentially the same distance from the spacecraft), where the directed kinetic energy is converted to thermal energy. This first argument gives a reasonable estimate of the magnetic field at the magnetopause B_{mp} by balancing the local magnetic energy density $B_{mp}^2/8\pi$ with the kinetic energy (or pressure) of the solar wind $m_p n_{sw} V_{sw}^2/2$:

$$m_p n_{sw} V_{sw}^2/2 = B_{mp}^2/8\pi. \tag{1}$$

where m_P is the proton mass, n_{SW} is the solar wind density, and V_{SW} is the solar wind velocity. Using typical solar wind parameters at 1 AU of $n_{sw} = 5$ cm^{-3} and $V_{sw} = 500$ km/sec, Eq. 1 yields $B_{mp} \sim 50$ nT. The interaction cross section πb^2 of the bubble can therefore be approximated using the radial distance b at which the plasma sail magnetic field drops to a value of $B_{mp} \sim 50$ nT. Here, we ignore the plasma pressure due to the injected plasma within the sail.

If we define the magnetopause gyrofrequency $\Omega_{mp} = eB_{mp}/m_p c$ and solar wind ion plasma frequency $\omega_{pi} = (4\pi e^2 n_{SW}/m_p)^{1/2}$, this pressure balance equation becomes $\frac{V_{sw}}{\Omega_{mp}} = \frac{c}{\omega_{pi}}$. Physically, this condition says that the ion gyro-radius in the magnetic field at the magnetopause is comparable to the ion inertial length in the solar wind. This condition implicitly sets a spatial scale of the system by setting the standoff distance of the bow shock/magnetopause from the object - relative to the bubble radius, b.

A second argument follows by noting that if the magnetic field is too weak, the ions will be unmagnetized and the Hall term will dominate over the usual $\vec{V} \times \vec{B}$ Lorentz force in electron momentum balance or Ohm's law, $JB_{mp} > en_{sw} V_{sw} B_{mp}$. Taking the bubble radius b as the representative scale length in Ampere's law and using it to obtain the current density, i.e. $J \sim cB_{mp}/4\pi b$, yields the condition for the bubble to be too small to interact significantly with the solar wind: $1/b > (V_{sw}/\Omega_{mp})(\omega_{pi}/c)^2$. This result, consistent with that obtained by the previous argument, indicates that when the bubble radius is less than the solar wind gyro-radius or inertial length, i.e., $b < V_{sw}/\Omega_{mp} \sim c/\omega_{pi}$, momentum will not beefficiently transferred to the sail from the solar wind, because only the electrons are magnetized. It is found that, for a small sail size ($b << c/\omega_{pi}$) only a weak whistler wave will appear upstream of the bubble [Gurnett, 1995; Omidi et al., 2002], while for larger bubbles ($b \geq c/\omega_{pi}$), a bow shock will exist upstream of the bubble.

Therefore, it is reasonable to assume that the ions will no longer be stopped (or deflected) if the ion inertial length becomes close to the bubble size, although the actual bubble size may be somewhat smaller because inclusion of the electrons can produce electric fields that affect the ions. Thus, we can use the ion inertial length as a rough guide as to when more detailed particle kinetics must be used. For a bubble radius smaller than the ion inertial length, the MHD simulations incorrectly indicate significant momentum transfer and will thereby overestimate the thrust.

B. Flux conservation

Conservation of magnetic flux constrains the bubble size and magnetic field strength of the spacecraft-linked magnetic field lines inside the bubble. Since magnetic monopoles do not exist in nature, flux lines must close on themselves, and the total flux originating at the magnetic coil must either close within the simulation or extend beyond the physical dimensions of the simulation. Therefore, in a simulation the total flux coming from the interior of the coil must be equal to or larger than the total magnetic flux crossing any plane of closed field lines outside the coil. Because dipole field lines must extend out to the edge of the bubble, i.e., to the magnetopause, in order to transfer momentum from the solar wind to the spacecraft, flux conservation, in conjunction with a model for the radial profile of the stretched dipole magnetic field, provides a condition on the size of the bubble given the size of the magnet coil.

The size of the plasma sail bubble is sensitive to the details of how quickly the magnetic field falls off with distance from the spacecraft. The bubble's shape is determined by the complicated interaction of the coil magnetic field and the injected plasma with the solar wind. In vacuum, the decrease of the magnetic field with distance r from the spacecraft for a classical dipole follows $B \propto r^{-3}$. As a result, the total magnetic flux required for a dipole bubble with a cross section large enough to be opaque to the solar wind and to generate enough force for propulsion is quite large. A plasma sail system in which $B \propto r^{-n}$ where $n < 3$ reduces the total magnetic flux required and increases the efficiency of the system relative to a classical dipole configuration. By inflating the bubble using plasma injected at the coil along magnetic field lines, simulations indicate that there are regions in which $B \propto r^{-1}$ can be obtained, resulting in a larger bubble using less magnetic flux than a classical dipole [Winglee et al., 2000; Khazanov, et al., 2003; and Blanco-Cano, et al., 2003], although the spatial extent of these regions are somewhat limited [Winglee, 2004].

To illustrate the sensitivity to fall off of the magnetic field in a qualitative way, we assume the magnetic field B decreases with increasing radial distance r from the spacecraft according to a power law:

$$B = B_{mp}(b/r)^{p}, \quad r \le b$$

where b is the bubble radius, B_{mp} is the magnetic field at the outer edge of the bubble at $r = b$, and $p = 1$, 2, or 3. The case $p = 3$ represents an approximation of the vacuum dipole case (the magnetic sail concept). The case p = 2 gives an estimate of the slowest fall off possible without the solar wind for an axi-symmetric dipole in the absence of plasma losses once the plasma pressure far exceeds the magnetic pressure (large plasma beta >> 1)[Krasheninnikov et al., 1999]. The remaining p = 1 case approximates the desired field for the plasma sail concept and results from a combination of inflation via plasma injection and interaction with the solar wind. If we denote the magnetic flux through the magnet coil onboard the spacecraft by Φ, then for an average magnetic field B_0 inside the coil and a coil radius a, we obtain $\Phi = \pi a^2 B_0$. Since the magnetic flux through the equatorial plane outside the coil must be equal and opposite to this coil flux, this gives

$$\Phi = 2\pi \int_a^b drrB = 2\pi B_{mp} b^2 \times \begin{cases} b/a & p=3 & \text{magsail} \\ \ln(b/a) & p=2 & \beta >> 1 \\ 1 & p=1 & \text{plasma sail} \end{cases} ,$$

where the bubble radius b is defined as the radius at which the magnetic field drops to the magnetopause value of B_{mp}~50 nT and b/a >>1 is assumed. Even for $B \propto 1/r$, which represents the most optimistic case, large magnetic fluxes are required for large bubbles. For example, using b = 10 km, the magnetic flux generated by the onboard magnet must be about 30 webers. For sail sizes of ~100 km, which are likely needed to deflect the solar wind, based on the arguments in the previous subsection, the magnetic flux is much larger, ~3000 Wb.

Thus, for p = 1, we find that $\Phi = \pi a^2 B_o = 2\pi B_{mp} b^2$, or $B_o = B_{mp}(b/a)^2$. This scaling is in contrast to the linear relationship presented by Winglee et al. (2000), $B_{mp} b = B_0 a$, which does not account for flux conservation. When flux conservation is satisfied, the a

= 10 cm, 50 kg coil of Winglee gives a maximum bubble radius of b ~ 100 m instead of 20 km even for the optimistic case of p = 1.

C. Flux conservation and simulations of plasma sails

As we have discussed previously, flux conservation with a r^{-1} fall-off of the magnetic field for a 1 kG magnetic field in the dipole field coil implies that a final bubble radius of only ~ 100 m can be achieved. Yet calculations of bubble expansion obtained in some simulation studies seem to indicate that much larger bubble sizes are possible. We suggest several ways that this discrepancy can be resolved: either magnetic flux is not conserved in the calculations; or if flux is conserved, the scaling of the calculations is not done correctly; or the magnetic field radial profile includes the solar wind field as well as the dipole field.

A possible way that magnetic flux could be continuously added inside or at the inner boundary of the simulation domain in some calculations is if plasma is injected radially outward with velocity $\vec{V}_{in}$ at the inner boundary where the magnetic field lines are dipolar and therefore not aligned with the injected plasma. As a result, a non-vanishing induced electric field $\vec{E} = -c^{-1}\vec{V}_{in} \times \vec{B}_{in}$ is applied, where $\vec{B}_{in}$ is the dipolar magnetic field at the inner boundary. The electric field $\vec{E}$ is closed and in the toroidal direction and so has a non-vanishing curl. The loop integral of the electric field over a closed toroidal contour along with Faraday's law implies that magnetic flux is being injected as time proceeds at the inner boundary. Using Faraday's law, and assuming the magnetic field B_{in} is roughly uniform inside the inner boundary and remains perpendicular to the injected gas, we find $\pi r_{in}^2 dB_{in}/dt = 2\pi r_{in} V_{in} B_{in}$. As a result, B inside the inner boundary increases exponentially with time. In the hybrid simulations, Winske and Omidi [2005] find that magnetic flux is conserved, but they inject the ions in a symmetric manner so that this effect should not occur. In addition, their simulations retain some inherent symmetry because they are only two-dimensional, and they have not been run for very long times ($2V_{in}t/r_{in} \approx 1$) and have not reached steady-state. We recommend that flux conservation should be explicitly checked in the 3-D MHD calculations for times much greater than r_{in}/V_{in}.

Another possible way to reconcile the size of the magnetic bubble obtained from simulations with magnetic flux conservation is related to scaling. The original calculations of Winglee et al. (2000) were actually done for a much smaller system and a much weaker magnet that allows expansion of the bubble only by a factor of several hundred. On the scale of these simulations, the linear relation between system size and coil magnetic field ($B_o a = B_{mp} b$), discussed previously, is roughly valid and conserves flux. The results were then extrapolated by a factor of 100 to estimate the size of a more realistic configuration. However, at this extrapolated scaling, the linear relation is no longer valid.

A third possible source for the differences in the bubble sizes obtained in MHD simulations (e.g., Khazanov et al., [2003]) and from flux conservation estimates may be as follows. The overall size of the cavity that is obtained is determined by the injection rate of the ions, as Winske and Omidi [2005] have shown for weak dipoles, balanced by losses. The expanding plasma excludes the solar wind magnetic field, which is most of the magnetic flux for a large bubble, and leaves the (somewhat stretched) dipole field behind, i.e., the dipolar field lines do not extend to the edge of the bubble. However, it is possible that in the calculations there is sufficient numerical diffusion of the solar wind magnetic field relative to the plasma during the expansion that magnetic flux carried by the solar wind penetrates the bubble, rather than being excluded from it, leading to the measured magnetic field radial profile falling off roughly as r^{-1}, Therefore, it may require a very accurate magnetic field line plotting routine to show the distinction in the radial profiles of the dipolar field lines that remain connected to the magnet and those of the diffused solar wind field.

D. Closed magnetic field loops within the magnetopause

One solution to the problem raised in the preceding discussion is to show that realistic plasma sail configurations can have closed magnetic field loops within the plasma sail ('magnetopause') boundary. This would allow a larger bubble size without requiring that all the flux be linked through the coil. None of the published simulations have provided clear evidence for any region of closed magnetic field loops within the plasma sail. Only the most recent simulation by Winglee (2004) showed an explicit region with reversed magnetic field. This region was primarily confined to the magnetotail region, although it extended also to the flanks. Note that for this simulation, the magnetosphere was located at ~50 coil radii, so the sail size was small (5 m for a 0.1m coil radius).

Indeed, as calculated above, in order to achieve a reasonable bubble size, the required flux in the closed loop outside the coil must be >30 Wb, while the flux through the coil in the simulations is only 2x10 $^{-3}$, a factor of 10^4 difference. If this were to be accomplished by field line loops which do not go through the coil, the loops would have to have a flux of ~5000 times the coil flux, which should be evident in any simulation.

It is also instructive to examine the case of the Earth's magnetosphere, which is often invoked as evidence of similar phenomena occurring in nature. There is no evidence for the occurrence of closed field loops on the dayside, except, perhaps, in the magnetopause boundary, which is narrow and could account for only an extremely small magnetic flux relative to the magnetic flux of the Earth’s dipole. A study of the magnetic field distortion that can be produced by trapped plasma on quasi-dipolar field lines (Hoffmann and Bracken, 1967) showed that even with plasma energy densities that were five times the energy in the initial magnetic field, the perturbation in the field was not large enough to reduce the field to zero. All cases of closed loops have been seen in the region of the stretched magnetotail field lines; e.g., in the magnetotail where the field is highly distorted from a dipole configuration, the field is weak and reconnection occurs, implying that the closed loops are rapidly disconnected from the geomagnetic field.

IV. Implementation of the Plasma Sail concept: An Electromagnet in Space

The preceding discussion has shown that a 10 km sail requires the magnet to generate at least 30 Wb of magnetic flux. In this section, we discuss the implementation of such a magnet and also the scaling relationships between the radial distribution of magnetic flux, mass, and power dissipation for dipole electromagnets. For copper coils, steady state cooling requirements impose a practical limitation on the power dissipation [Bitter, 1961] and thus a limit on the current density. A current density limited design normally leads to a linear scaling of mass to flux, independent of coil radius. In superconducting magnets, allowable material stresses induced by the magnetic field become the limiting factor. The virial theorem imposes an upper limit to the maximum stored energy in a magnet [Moon, 1982 and Tsutsui, 2003], thus also restricting the magnetic flux generated by the magnet. Below several possible magnet types are discussed. Other magnet types have also been studied, but none can provide the required flux for a reasonable weight and power for implementation into a plasma sail.

For a cooled copper coil that is limited by current density, it can be shown that the magnet mass is linearly proportional to flux. A representative steady-state copper coil that produces 30 webers of magnetic flux would have a radius of 1.25 m and mass of 48.5 metric tons. This coil would require 88 MW of power, not including the cooling capacity to maintain a reasonable operating temperature. This electrical power is far beyond any existing capabilities of outer-solar system spacecraft (hundreds of watts for radioisotope thermoelectric generator units) or even the International Space Station (approximately 78 kW).

A superconducting coil, therefore, would be a superior choice when such a high magnetic flux is required, as superconductors have zero steady state power dissipation and allow far greater current densities and thus reduced magnet mass. Superconductors have reduced power and cooling requirements (even considering the inefficiencies of refrigeration required to provide cooling at low temperatures). Higher allowed current densities also permit the addition of non-current bearing, higher stress materials to further strengthen the coil, and thus reduce required mass in a stress limited design. A stress-limited design will have mass scaling roughly as the square of the magnetic flux divided by the magnet radius, suggesting an optimal configuration of a large magnet with a small cross section. The drawback of this shape is that significant flux is wasted on higher order multi-pole components whose magnetic field decreases faster than the dipole magnetic moment.

As an example, consider a stress limited dipole magnet with 2.5 m radius and flux of 30 Wb. Such a superconducting coil would weight 3 tons, have a mean stress of 230 MPa, and carry a current density of 120 A/mm^2 - a value that is obtainable in currently available high temperature superconducting tapes. However, no coil has previously been made with such a high flux/mass ratio as insulation, bonding, and quench protection measures will greatly increase the real coil mass [Moon, 1982]. The required mass for the magnet cryostat and cooling systems will also add to the total system mass.

A large superconducting magnet is currently being constructed for space flight. The Alpha Magnetic Spectrometer (AMS-02) is scheduled to fly within several years on the International Space Station (ISS) [Blau et al., 2002; 2004]. It consists of a 2300 kg coil structure. The magnetic flux, ~0.5 Wb, is low compared to a optimized design of the same mass since several coils in the structure are used solely to remove the fringing field that would interfere with the ISS. It utilizes an open cycle cooling system that is expected to give a mission lifetime of 27-33 months and adds ~30% to the entire mass of the magnet system. Of course, for a longer mission time, it is likely that significantly more power (and therefore mass) would be required to cool the magnet in a closed cycle.

Regardless of which design strategy one chooses, one finds a significantly larger mass than the 10 kg magnet quoted for the M2P2 design to obtain significant magnetic flux, as well as substantially higher power and very large cooling requirements. The impact of this large mass for design of a viable propulsion system is discussed in the Appendix.

V. Conclusions

The plasma sail concept is based on expanding the magnetic field lines of a dipole magnet by injecting plasma. The resulting inflated magnetic bubble deflects the solar wind and imparts momentum that is transferred back to the spacecraft to provide a source of propulsion. Here, we have presented arguments concerning the size of the sail needed to deflect the solar wind and have shown that this size is determined by basic plasma properties of the solar wind plasma, specifically the ion inertial length of the solar wind ions, which is on the order of 100 km. A key feature of the plasma sail concept is that the injection of plasma allows the magnetic field lines of the dipole to be extended so that the field falls off much more slowly than a bare dipole. However, the stretched magnetic field lines must remain attached to the magnet to transfer momentum to the spacecraft. We have applied conservation of magnetic flux and a simple model for the radial profile of the stretched dipole field to conclude that a sail of the desired size requires a magnet of roughly 30 Wb. Even with perfectly efficient transfer of the solar wind momentum to a large sail, only a few Newton of force is produced. The mass of the large magnet required (~ 3 tons) thus implies that the acceleration of the sail will be much less than that currently available from other sources of propulsion.

Appendix: Implications for Propulsion

A functional plasma sail must intercept a large area of the solar wind from which it derives its thrust. Using typical solar wind parameters and taking a sail radius of 10 km, Winglee et al. (2000) computed a force of about 1 N and a resulting acceleration to about 10% of the solar wind speed in about three months, assuming complete transfer of momentum to a 50 kg spacecraft. As discussed above, to inflate the magnetic bubble to an ~10 km radius requires a larger magnet (~ 30 Wb) and that the magnetic field fall off more slowly than a dipole (1/r fall-off over a significant fraction of the sail is optimal). To provide such a large flux for a reasonable mass and power requires a superconducting coil. Utilizing state-of the-art magnet construction parameters, one finds that a 30 Wb superconducting coil weighs ~ 3 tons without the necessary refrigeration and power systems. This value is many orders of magnitude larger than the ~10^{-3} Weber, 10 kg magnet mass baselined in the plasma sail concept described by Winglee et al. (2000). Even assuming the best-case scenario that the total 1 N force incident on the sail is transferred to the spacecraft, the dramatically increased mass reduces the spacecraft acceleration to well below that available from other propulsion systems. Magnets this large have not been built and require high power and a large cooling system, adding to the engineering challenges. It is not clear that the required cooling systems are feasible for a satellite mission. Furthermore, these magnets also waste significant flux on higher order multi-pole components, making it more difficult to obtain the desired magnetic field fall-off with distance. While we have assumed a 10 km radius for the sail, as used by Winglee et al. (2000) this is an order of magnitude smaller than the solar wind ion inertial length and ion gyroradius and is very likely too small to deflect the solar wind, as discussed in Section III. Because the magnetic flux and the force both scale as the square of the bubble radius and the magnet mass scales as the flux squared, the ratio of the force to magnet mass is inversely proportional to the square of the bubble radius. Therefore, if a larger sail is necessary to deflect the solar wind, the disparity between the acceleration base-lined by the proponents and a realistic value is even larger.

There are a number of issues that have not been specifically addressed herein, including the bubble stability and the rate of loss of injected plasma. The required propellant load must be accurately determined. In addition, for any viable spacecraft, the thrust must be controllable (both in magnitude and direction), and there will need to be a secondary propulsion system for thrust in any direction other than anti-sunward.

Table 1 compares plasma sail parameters from Winglee et al. (2000) and those obtained in this study for two different assumptions for the behavior of the magnetic field within the bubble. The final column shows the values that would be obtained for a magnetic sail [Zubrin, 1993]. Comparing the revised plasma sail performance obtained for the more realistic $1/r^2$ fall-off of the magnetic field to that of other propulsion technologies shows that it is not competitive. For example, Noca (2003) and Elliot (2003) present 100kW class nuclear electric propulsion systems capable of providing thrust levels of ~2 N with total spacecraft mass between 10,000 and 20,000 kg, an improvement of more than a factor of ten over the expected plasma sail thrust, even with the assumption that all momentum is transferred to the spacecraft and without including refrigeration, power and propulsion mass. More realistic assumptions would further reduce the competitiveness of the plasma sail. These nuclear electric systems have the added benefit of providing a steerable thrust vector, obviating the need for a large secondary propulsion system, which would be required by a plasma sail system.

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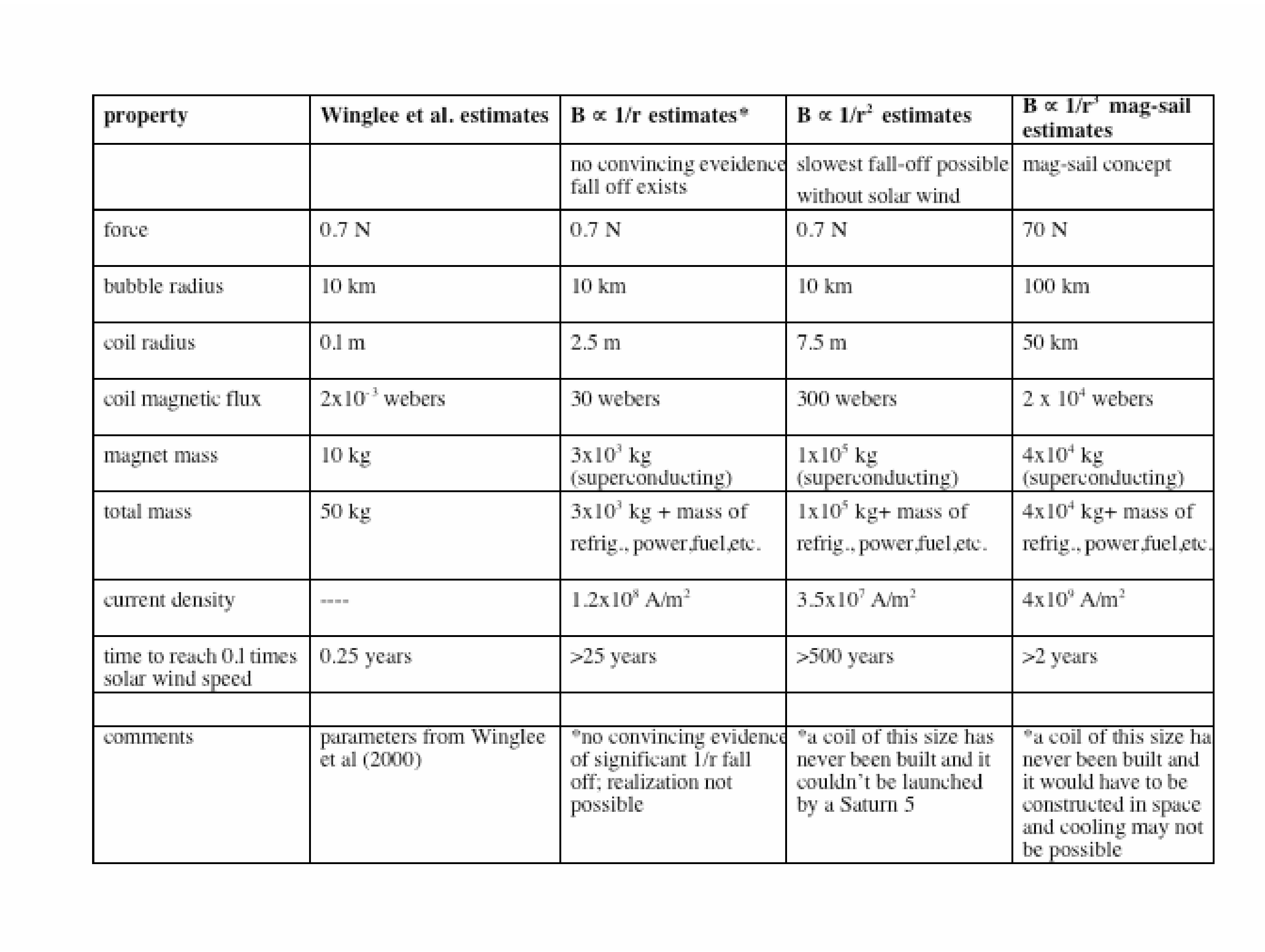

property	Winglee et al. estimates	$B \propto 1/r$ estimates*	$B \propto 1/r^2$ estimates	$B \propto 1/r^3$ mag-sail estimates
		no convincing eveidence fall off exists	slowest fall-off possible without solar wind	mag-sail concept
force	0.7 N	0.7 N	0.7 N	70 N
bubble radius	10 km	10 km	10 km	100 km
coil radius	0.1 m	2.5 m	7.5 m	50 km
coil magnetic flux	2x10^{-3} webers	30 webers	300 webers	2 x 10^{4} webers
magnet mass	10 kg	3x10^{3} kg (superconducting)	1x10^{5} kg (superconducting)	4x10^{4} kg (superconducting)
total mass	50 kg	3x10^{3} kg + mass of refrig., power,fuel,etc.	1x10^{5} kg+ mass of refrig., power,fuel,etc.	4x10^{4} kg+ mass of refrig., power,fuel,etc.
current density	----	1.2x10^{8} A/m^{2}	3.5x10^{7} A/m^{2}	4x10^{9} A/m^{2}
time to reach 0.1 times solar wind speed	0.25 years	>25 years	>500 years	>2 years
comments	parameters from Winglee et al (2000)	*no convincing evidence of significant 1/r fall off; realization not possible	*a coil of this size has never been built and it couldn't be launched by a Saturn 5	*a coil of this size ha never been built and it would have to be constructed in space and cooling may not be possible

Table 1.