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Abstract. This paper summarizes the theoretical and numerical results we have obtained for the excitation and emission of electron Bernstein waves in spherical tokamak plasmas.

1. INTRODUCTION

The high β operating regime of spherical tokamaks (ST), such as in NSTX and MAST, make them attractive fusion devices. To attain the high β's there is a need to heat and to drive currents in ST plasmas. While ST plasmas are overdense to conventional electron cyclotron (EC) waves, electron Bernstein waves (EBW) offer an attractive possibility both for heating and for driving plasma currents. EBWs, which have no density limits, can propagate into the plasma core for frequencies above the electron cyclotron frequency f_{ce} [1]. Since EBWs are not vacuum modes, they are excited inside the plasma by mode conversion of the traditional X and O modes. From ray tracing analysis we find that EBWs are strongly absorbed by electrons in the region where the wave frequency matches the Doppler broadened electron cyclotron resonance frequency or its harmonics [1]. The strong and localized absorption implies that thermal emission of EBWs can occur for frequencies corresponding to the local Doppler-shifted electron cyclotron frequency. This emission then converts, at the UHR, to the X and O modes which are then observed in the vacuum region.

2. EXCITATION OF ELECTRON BERNSTEIN WAVES

The excitation of EBWs have been studied in a slab geometry model where the x coordinate is in the direction of the inhomogeneity, y is along the poloidal direction, and z along the toroidal direction. Assuming that the magnetic field is along z, the propagation of waves in an inhomogeneous cold plasma is given by

$$K_\perp \frac{d^2 E_y}{d\xi^2} + \left(K_\perp^2 - K_X^2 - \frac{c^2}{\omega^2} k_\parallel^2 K_\perp \right) E_y = \frac{c^2}{\omega^2} k_\parallel K_X F \tag{1}$$

$$\frac{d}{d\xi} \left[\frac{1}{K_\parallel} \frac{dF}{d\xi} \right] + \left(1 - \frac{c^2}{\omega^2} \frac{k_\parallel^2}{K_\perp} \right) F = \frac{K_X}{K_\perp} k_\parallel E_y \tag{2}$$

where $F = i k_\parallel E_x - dE_z/dx$, $\xi = \omega x/c$, c is the speed of light, ω is the wave frequency, $k_\parallel$ is the component of the wave vector $\vec{k}$ along the magnetic field $\vec{B} = B_0(x)\hat{z}$,

$$K_\perp = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}, \quad K_X = -\frac{\omega_{ce}}{\omega} \left(\frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \right), \quad K_\parallel = 1 - \frac{\omega_{pe}^2}{\omega^2} \tag{3}$$

ω_{pe} and ω_{ce} are the electron plasma and cyclotron angular frequencies, respectively.

Let us consider the case when $k_\parallel = 0$. Then, Eqs. (1) and (2) are uncoupled and describe the propagation of the X mode and the O mode, respectively, across the magnetic

field. In this case the X mode equation (1) has a resonance at $K_\perp = 0$ corresponding to the upper hybrid resonance (UHR). Also Eq. (1) has two cutoffs corresponding to the zeros of $K_\perp^2 - K_X^2$. The resonance power absorption of the X mode at the UHR corresponds to the power mode converted to the EBWs [1]. From an analogous analysis carried out in [2], we find that, for the case when the left hand cutoff is close to the UHR, the fraction of the power on the X mode that is converted to EBWs is given by

$$C_{XB} = 4e^{-\pi\eta}\left(1 - e^{-\pi\eta}\right)\cos^2(\phi) \tag{4}$$

where

$$\phi = \frac{\pi}{2} + \frac{1}{2}\cos^{-1}\left(\frac{N_R^2 - N_I^2}{N_R^2 + N_I^2}\right) \tag{5}$$

$N = N_R + iN_I$,

$$
\begin{aligned}
N &= \ln\left(\frac{i\eta}{\eta_1}\right) + \pi\left[\coth\left(\frac{\pi}{2}\eta\right) + \cot\left(\frac{\pi}{2}\eta_1\right)\right] - \frac{1}{\eta_1} - \frac{i}{\eta} \\
&\quad + \psi\left(1 + \frac{\eta_1}{2}\right) - \psi\left(1 + \frac{i\eta}{2}\right)
\end{aligned} \tag{6}
$$

ψ is the Psi function,

$$\eta \approx \left[\frac{\omega_{ce}L_n}{c\alpha}\left(\sqrt{1 + \alpha^2} - 1\right)^2\right]_{UHR} \tag{7}$$

$$\eta_1 \approx \left[\frac{\omega_{ce}L_n}{c\alpha}\left(\sqrt{1 + \alpha^2} + 1\right)^2\right]_{UHR} \tag{8}$$

In Eqs. (7) and (8) the right-hand sides are evaluated at the UHR, L_n is the density scalelength, and $\alpha = \omega_{pe}/\omega_{ce}$. If ϕ is an integer multiple of π, then from (4) we note that $C_{XB} \geq 0.5$ if $0.05 \lesssim \eta \lesssim 0.6$. From (7) we find that the necessary condition for the X-B mode conversion to be better than 50% requires

$$1.3 \times 10^{-2} \text{ T cm} \lesssim |BL_n|_{UHR} \lesssim 0.16 \text{ T cm} \tag{9}$$

This shows that the X-B mode conversion process is efficient for sharp density gradients, i.e., short density scalelengths.

From Eq. (2) it is clear that for $k_\parallel = 0$ the O mode does not couple power to the EBW since there is no resonance absorption of the O mode. For the O mode power to couple to EBW it is necessary to have oblique propagation of the O mode, i.e., $k_\parallel \neq 0$. Then the O mode power can couple to the X mode which in turn can undergo resonance absorption. The necessary condition for optimum coupling of the O mode power to the X mode is [3]

$$n_{\parallel,c} = \frac{ck_\parallel}{\omega} = \left[\left(1 + \frac{\omega}{\omega_{ce}}\right)^{-1/2}\right]_c \tag{10}$$

where the right-hand side is to be evaluated at the spatial location where $\omega = \omega_{pe}$. However, in order for the O mode power not to get reflected back out on the X mode requires that $\eta > 1$ [1]. From (7) we find that this condition is satisfied when the density scalelength is long.

From the above discussion we conclude that the X-B and the O-B mode conversion processes are optimized in different regions of the two-dimensional parameter space

spanned by wave frequency and parallel wavelength. The X-B mode conversion process is optimized for $k_\parallel \approx 0$ and wave frequencies which place the UHR in the short density scalelength region. The O-B mode conversion process is optimized for $k_\parallel$ given in (10) and for wave frequencies which place the UHR in a longer density scalelength region of the plasma. The optimum X-B process occurs closer to the edge of the plasma than the optimum O-B conversion. A more detailed analysis based on an approximate kinetic treatment of the EBW in sheared magnetic fields confirms the basic results of the analysis given above [1].

3. EMISSION OF ELECTRON BERNSTEIN WAVES

From ray tracing analysis we have shown that EBWs are locally and strongly absorbed at the Doppler shifted electron cyclotron resonance or its harmonics [1]. The strong and localized absorption implies that thermal emission of EBWs can occur for frequencies corresponding to the local Doppler-shifted electron cyclotron frequency. This emission then converts, at the UHR, to the X and O modes which are then observed in the vacuum region. The relationships between the scattering coefficients for mode conversion excitation and for mode conversion emission can be established based on global properties of the model description of the mode conversion region [4, 5]. The global properties of our full-wave model equations for the mode conversion region are linearity, energy flow conservation, and (Onsager-like) time reversibility.

The wave fields are assumed to have a WKB form outside the mode conversion region. Referring to Fig. 1, towards the outside, low density region of the plasma (designated as "right") the X and O modes are propagating with energy flow into or out of the mode conversion region, while the EBWs are evanescent with no associated energy flow. Towards the inside, high density region of the plasma (designated as "left") the EBWs are propagating with energy flow into and out of the mode conversion region, while the X and O modes (due to their cutoffs) are evanescent with no energy flow. Let the complex amplitudes of the propagating modes be such that $|a_i|^2$ is the energy flow density into the mode conversion region and $|b_i|^2$ is the energy flow density out of the mode conversion region. ($i = X,\ O,\ B$ designating the X mode, the O mode, or the EBW, respectively.) Then, since the mode conversion equations are linear, the complex field amplitudes a_i
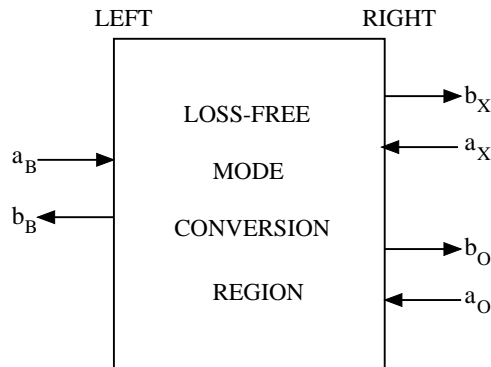

Figure 1: The loss-free mode conversion region

and b_i are related by a scattering matrix

$$
\begin{pmatrix} b_B \\ b_X \\ b_O \end{pmatrix} = \begin{pmatrix} S_B & S_{BX} & S_{BO} \\ S_{XB} & S_X & S_{XO} \\ S_{OB} & S_{OX} & S_O \end{pmatrix} \begin{pmatrix} a_B \\ a_X \\ a_O \end{pmatrix} \tag{11}
$$

or, simply, in matrix notation

$$
\vec{b} = \overset{\leftrightarrow}{S} \cdot \vec{a} \tag{12}
$$

The matrix $\overset{\leftrightarrow}{S}$ is unique since the solution of the linear mode conversion equations with the appropriate boundary conditions is unique.

It can be readily shown [4, 5] that from energy conservation and from the energy flow under time reversibility that

$$
\overset{\leftrightarrow}{S}{}^T = \overset{\leftrightarrow}{S} \tag{13}
$$

where the superscript T indicates the transpose of the matrix. Thus, $\overset{\leftrightarrow}{S}$ is symmetric. This implies that

$$
S_{BX} = S_{XB}, \quad S_{BO} = S_{OB}, \quad S_{XO} = S_{OX} \tag{14}
$$

The symmetry of the scattering matrix $\overset{\leftrightarrow}{S}$ then leads to the following relationships that have important experimental consequences [4, 5]

$$
E_X = C_{XB}, \quad E_O = C_{OB} \tag{15}
$$

where E_X and E_O are the emission coefficients that give the fraction of the emitted EBW power that is mode converted to the X and O modes, respectively, and C_{XB} and C_{OB} are the excitation coefficients that give the fractions of the input power on the X and O modes, respectively, that is converted to the EBWs. These relationships show that the edge plasma characteristics that give maximum power emission from EBWs are also the optimum conditions for obtaining maximum power excitation of EBWs from sources external to the plasma. The monitoring of emission can be readily implemented experimentally. These observations can provide a useful guide for the design and optimization of coupling power to EBWs for heating and current drive in ST plasmas

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