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Electromagnetic Wall Torques from Magnetically Confined Plasmas

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Abstract

A direct method to measure the localized magnetic shear stress and hence electromagnetic torque or force exerted on a wall by a fluctuating magnetic field is presented. A model of the wall is needed when the magnetic measurements do not include the normal component. Explicit formulas for thin and thick walls are given. Estimates of the force in illustrative Alcator C-Mod tokamak plasmas provide evidence against the theory that resistive magnetic drag determines mode locking.

1   Introduction

Propagating magnetic fluctuations interact with fixed conducting structures in such a way as to transfer momentum to them. As a result, for example, magnetic perturbations generated by MHD instabilities experience a drag in the wall, which they generally pass on to the plasma in which they are embedded. These processes are extremely important in understanding the possible stabilization of MHD instabilities by the effects of the walls [1], and most previous discussions of this topic focus on the way in which the magnetic perturbations are affected by the presence of the wall and transfer their influence back to the plasma, see for example [2, 3, 4]. In these calculations an extremely simplified model of the conducting wall has almost universally been adopted. The present paper focusses instead on the interaction with the wall to show first, that it is possible, and indeed in many cases essential, to adopt more realistic models for the wall effect; second, that regardless of the actual wall structure it is in principle possible to *measure* the momentum flux into the wall; and third to illustrate actual cases in which magnetic measurements bear on the importance of magnetic fluctuation momentum transfer to the plasma.

2   Magnetic Stress Calculation

Consider the situation shown schematically in figure 1. The plasma is considered to be enclosed by a surface **S** which contains inside it no fixed conducting components capable of affecting the momentum. If necessary, the volume enclosed by this surface will be considered to be periodic in a particular direction so that the contributions from the ends of the volume cancel and can be ignored. We are thus considering a toroidal plasma to be represented approximately by a periodic linear approximation. In fact, the analysis can be carried out for
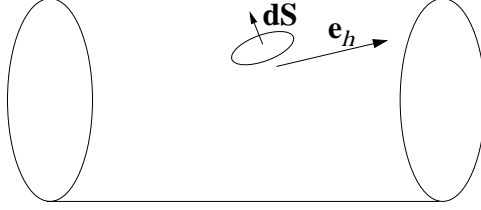
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Figure 1: Surface over which the integration of the momentum flux is to occur.

a torus, in terms of torque and angular momentum rather than force and linear momentum, *mutatis mutandis.* No significant new physics is introduced thereby, so we will analyse the linear case for simplicity. We consider the force/momentum in a particular direction specified by the unit vector $\mathbf{e}_h$. This direction need not be the direction of periodicity.

Changes of the total momentum of the plasma within this surface can occur only if momentum is transferred across the surface $\mathbf{S}$. That transfer can be mediated either by particles, as represented by the pressure tensor (including viscous stress) in a fluid description, or by electric or magnetic fields. The rate at which electromagnetic fields transfer momentum across the surface is given, perfectly generally, by the integral over the surface,

$$\frac{dM_h}{dt} = \int \mathbf{e}_h.\mathbf{T}.\mathbf{dS} \tag{1}$$

of the Maxwell stress tensor,

$$\mathbf{T} \equiv \epsilon_0 \left[\mathbf{EE} - \frac{1}{2}E^2\mathbf{I} \right] + \frac{1}{\mu_0} \left[\mathbf{BB} - \frac{1}{2}B^2\mathbf{I} \right] \tag{2}$$

We focus on the magnetic components and write, ignoring $\mathbf{E}$,

$$\frac{dM_h}{dt} = \int \mathbf{e}_h.\frac{1}{\mu_0} \left[\mathbf{BB} - \frac{1}{2}B^2\mathbf{I} \right].\mathbf{dS} \tag{3}$$

This equation is true for the steady-state components as well as any fluctuations. Thus, if there were a steady momentum transfer from the plasma to the magnetic field inside this volume, which might arise if the volume enclosed one end of a magnetic mirror, for example, this momentum transfer is, in principle, measurable by performing this integral. In practice, however, such an approach would face severe difficulties for a low-β plasma because there would be large values of the stress tensor components whose sign changed from place to place in the integral cancelling each other out. That cancellation is exact, of course, when there is no net magnetic force on the plasma. In practice taking the sensitive difference of large quantities in order to constuct the integral would require accuracy far beyond what is normally possible in routine magnetic measurements.

If however, we are concerned only with the magnetic *fluctuations* we shall see that it is plausible that we might be able to measure the fields accurately enough to construct this integral because the momentum transfer associated with those fluctuations does not necessarily involve large cancellations over the extent of the integral.

So, let us suppose that the magnetic field is composed of a steady part and a fluctuating part

$$\mathbf{B} = \langle \mathbf{B} \rangle + \tilde{\mathbf{B}} \qquad (4)$$

where angular brackets indicate an average over time much longer than the fluctuation period. When this decomposition is substituted into equation 3, and then averaged, the cross terms between $\langle \mathbf{B} \rangle$ and $\tilde{\mathbf{B}}$ will average to zero, and we are left with a term arising purely from the fluctuations

$$\frac{d\langle M_h \rangle}{dt} = \int \mathbf{e}_h . \frac{1}{\mu_0} \left\langle \tilde{\mathbf{B}} \tilde{\mathbf{B}} - \frac{1}{2} \tilde{B}^2 \mathbf{I} \right\rangle . \mathbf{dS} \quad . \qquad (5)$$

The extent to which this quantity can meaningfully be measured experimentally will depend largely on whether or not there are major cancellations. The $\tilde{B}^2$ term has a sign that is that of $\mathbf{e}_h . \mathbf{dS}$ and will generally therefore experience significant cancellations if it is non-zero, because its sign changes from place to place on the surface. In the circumstance that the surface normal is perpendicular to the direction of momentum $\mathbf{e}_h . \mathbf{dS} = 0$ this term vanishes. This will be the case if we take an axisymmetric surface in a tokamak, for example. (Equivalent to a translationally symmetric surface, independent of h, here.) In a stellarator, or other non-symmetric system, it appears likely that this fluctuating magnetic pressure term will render impractical the direct measurement of momentum transfer through this integral, unless it can be constructed over a surface that is perpendicular to $\mathbf{h}$, utilizing the periodicity of the system.

Let us therefore consider only the case $\mathbf{e}_h . \mathbf{dS} = 0$ (and omit the end surfaces by our periodic assumption). The only remaining magnetic stress tensor component is $\mathbf{e}_h . \langle \tilde{\mathbf{B}} \tilde{\mathbf{B}} \rangle . \hat{\mathbf{n}} = \langle \tilde{B}_h \tilde{B}_n \rangle$, where $\hat{\mathbf{n}}$ is the surface normal direction. This component is measureable in practice by simultaneous measurements of the tangential ($\mathbf{e}_h$) and normal components of the fluctuating field and forming the average.

Generally, of course, we do not have such measurements all over the surface in question. Therefore it will normally be necessary either to perform this measurement locally in a part of the surface that is representative of the whole, or else to construct a model that enables us to derive an estimate of the full integral from partial measurements.

The important point is that this measurement is completely independent of the complicated details of the mode behavior inside the plasma. The momentum transfer to the plasma region can be obtained at the measurement surface directly from measured fields. To the best of the present author's knowledge, a direct reconstruction of the fluctuating magnetic stress tensor average has not been measured on tokamaks, although that would be an interesting experiment. However there are models that can be used to derive an estimate of the term from the routine field measurements for simplified approximations of experimental situations.

3 Wall Penetration Models

As a simple model, widely used since the earliest fusion research [5], we approximate the configuration as having circular cylindrical symmetry but suffering from a magnetic perturbation that is a function only of the helical coordinate $h \equiv m\theta + kz$, and the radius r.

Magnetic measurements are generally made in close proximity to the wall, and it is usually only the poloidal component $\tilde{B}_\theta$ that is measured. However, in vacuum, the field can be represented as the gradient of a scalar potential, ϕ. Then the component of $\tilde{\mathbf{B}}$ that is perpendicular to r points in the gradient direction

$$\mathbf{e}_h \equiv \frac{\nabla h}{|\nabla h|} = \frac{1}{g}\left(\mathbf{e}_\theta + (kr/m)\mathbf{e}_z\right) \tag{6}$$

where $g \equiv \sqrt{1+(kr/m)^2}$. This fact allows us immediately to derive the required h-component of $\tilde{B}$ from $\tilde{B}_\theta$ measurements alone, knowing that $\tilde{B}_z = (kr/m)\tilde{B}_\theta$.

If the perturbation is harmonic in the helical coordinate, proportional to $\exp ih$, and we adopt a complex representation, dropping the tilde on the complex terms for brevity, then the vacuum field equation for a scalar magnetic potential, satisfying Laplace's equation $\nabla^2 \phi = 0$, consist of constants p and q times modified Bessel functions,

$$\phi(r) = pI_m(kr) + qK_m(kr). \tag{7}$$

The application of boundary conditions is more convenient, however, if we use a helical flux-function, ψ, to represent the field as

$$\mathbf{B} = \frac{1}{g}(\mathbf{e}_r \wedge \mathbf{e}_h) \wedge \nabla\psi = \frac{1}{g}\frac{\partial\psi}{\partial r}\mathbf{e}_h - \frac{m}{r}\frac{\partial\psi}{\partial h}\mathbf{e}_r \tag{8}$$

In terms of ψ, the vacuum solutions are then

$$\psi(r) = i\frac{kr}{m}\left[pI'_m(kr) + qK'_m(kr)\right]. \tag{9}$$

The model that has most widely been used for the magnetic transfer of momentum to the wall is to approximate the wall as a concentric thin resistive sheet, and either ignore any helical current outside the sheet [2, 3] or sometimes impose a perfectly conducting boundary further outside it [4]. Extremely rare examples of thick-wall calculations include references [6, 7]. The burden of our present argument is that no matter what model is adopted, the entire effect of the wall is contained in the boundary condition at the inner surface of the wall relating ψ and $(\partial\psi/\partial r)$, or in other words, the relationship (phase and amplitude) between B_r and B_h. This statement holds true even for "active" walls (see e.g. [8]) involving powered feedback, provided the feedback algorithm is linear.

Two (passive wall) cases represent the idealized extremes of wall behavior, the thin wall referred to above, and the thick wall, in which the perturbed field does not escape through the outer face of the wall. In either case, the solution of the problem follows from elementary matching of solutions at the inner radius, b, of the wall.

3.1 Thin Wall

For the thin wall, the matching condition is

$$\left[b\frac{\partial\psi}{\partial r}\right]_{b-}^{b+} = \tau_\delta \frac{\partial\psi}{\partial t}, \tag{10}$$

where $\tau_\delta \equiv \mu_o \sigma b \delta$ is the time-constant of the wall whose conductivity is σ and thickness $\delta \ll b$ and the notation $b-$ and $b+$ denote radii just inside or just outside the wall. Taking $p = 0$ in the outer region ($r = b+$), to make the solution analytic at $r \to \infty$ the outer solution has logarithmic derivative

$$\frac{1}{L} \equiv \left. \frac{\partial \psi}{\partial r} \middle/ \psi \right|_{b+} = 1/b + kK_m''(kb)/K_m'(kb) . \tag{11}$$

If small argument approximations of the Bessel function are warranted, i.e. when $(kr)^2 \ll 4m$, this becomes $1/L = -m/b$.

Considering harmonic time variation, $\partial/\partial t \to -i\omega$, we can then substitute into the previous equation to find

$$\left. \frac{\partial \psi}{\partial r} \right|_{b-} = \left(\frac{1}{L} + \frac{i\omega\tau_\delta}{b} \right) \psi_b. \tag{12}$$

This is an equation relating the radial field to the tangential field at $r = b-$:

$$B_r = -\frac{m}{r} \frac{\partial \psi}{\partial h} = -\frac{m}{r} i\psi = -i\frac{m}{r} \frac{1}{\left(\frac{1}{L} + \frac{i\omega\tau_\delta}{b}\right)} \left. \frac{\partial \psi}{\partial r} \right|_{b-} = -i\frac{m}{b} \frac{g}{\left(\frac{1}{L} + \frac{i\omega\tau_\delta}{b}\right)} B_h, \tag{13}$$

which naturally has the property that $B_r \to 0$ as $\omega\tau_\delta \to \infty$ in which limit the wall becomes perfectly conducting and no field can penetrate normal to it.

The time-averaged component of the magnetic shear stress is

$$\frac{1}{\mu_0} \langle \tilde{B}_h \tilde{B}_r \rangle = \frac{1}{2\mu_0} \Re(B_h B_r^*) = \frac{1}{2\mu_0} \Re\left(i\frac{m}{b} \frac{g}{\left(\frac{1}{L} - \frac{i\omega\tau_\delta}{b}\right)} \right) |B_h|^2, \tag{14}$$

where $\Re(x)$ denotes real part of x and B_h is the *amplitude* of the oscillating component; so $|B_h|^2$ is twice the mean-square value of B_h. This component gives the magnetic momentum flux rate across the surface $r = b$ at any point, i.e. the force per unit area exerted by the wall on the plasma. Within the helically symmetric model it is uniform over the wall surface. However it is noticeable that $\tau_\delta/b = \mu_0\sigma\delta$ is dependent only on the wall properties and not the mode; so in the limit $kb \ll 2\sqrt{m}$ when $1/L = -m/b$, the only dependence of the complex factor on the mode is through the ratio m/b which is the local value of the *total* wave-number of the perturbation, which we will denote k_h. Thus, in the thin wall limit we can regard the magnetic stress at the wall as given by

$$\langle T_{hr} \rangle = f(k_h, \omega) \frac{1}{2\mu_0} |B_h|^2, \tag{15}$$

where the function f is given by

$$f = \Re\left(\frac{1}{i - \omega\tau_h} \right) = \frac{-\omega\tau_h}{1 + (\omega\tau_h)^2}, \tag{16}$$

and the time-constant is $\tau_h = \mu_0\sigma\delta/k_h$. It is clear that this form is not dependent on geometry. We could equally well have done the calculation in a slab. Moreover, in so far as k_h can be regarded as a well-defined quantity, that is, provided the variation of the magnetic perturbation along the wall can be represented as $\propto \exp ik_h\ell$ where ℓ is distance parallel to $\mathbf{e}_h$, the expression for f can be used for any geometry; even one without a convenient symmetry.

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3.2 Thick Wall

The opposite limit, rarely considered in the tokamak literature but probably often more important, is when the wall is so thick that we can suppose that no field fluctuation penetrates to its outer surface. In other words, the solution domain may be divided into two, the vacuum region (for $r < b$), and a conducting wall region for all $r > b$. For this to be a reasonable approximation in practice, the solution must fall off very rapidly within the wall. Since the equation governing the flux in the wall is effectively

$$\nabla^2 \psi = -i\omega\mu_0\sigma\psi \tag{17}$$

we can ignore the value of $k_h^2 \approx m^2/r^2$ provided it is much less than $\omega\mu_0\sigma$ and then the solution in the wall is

$$\psi \propto \exp\left[\pm r\sqrt{-i\omega\mu_0\sigma}\right] = \exp\left[\pm r(i-1)\sqrt{\omega\mu_0\sigma/2}\right] \tag{18}$$

which shows that we are consistent in ignoring the outer wall interface provided the wall is much thicker than the standard skin-depth, $\sqrt{2/\omega\mu_0\sigma}$ and, by the way, that the thin-wall approximation is invalid if the wall is this thick, that is, if $\omega\mu_0\sigma\delta^2 > 1$. Only the decaying solution (plus sign) is allowed for the thick wall case, and then the boundary condition required for the vacuum solution is

$$\left.\frac{\partial\psi}{\partial r}\right|_b = (i-1)\sqrt{\omega\mu_0\sigma/2}\ \psi_b. \tag{19}$$

No discontinuity occurs at $r = b$ in this case.

The calculation proceeds just as before, arriving at

$$\langle T_{hr}\rangle = f(k_h, \omega)\frac{1}{2\mu_0}|B_h|^2, \tag{20}$$

but now the function f is given by

$$f = \Re\left(\frac{-i}{(i+1)\sqrt{\omega\tau_t}}\right) = \frac{-1}{2\sqrt{\omega\tau_t}}\ , \tag{21}$$

in which the thick wall time-constant is $\tau_t = \mu_0\sigma/2k_h^2$.

Again we have what is really a local expression for the shear stress, to a large degree independent of the geometry.

4 Example from Alcator C-Mod.

We illustrate the models discussed in the previous section by application to the Alcator C-Mod tokamak. A typical equilibrium is shown in figure 2. Notable features of this tokamak are the thick conducting stainless-steel wall, whose shape is not at all conformal to the plasma surface, conducting (molybdenum) tiles lining the wall, and a large gap on the outboard side
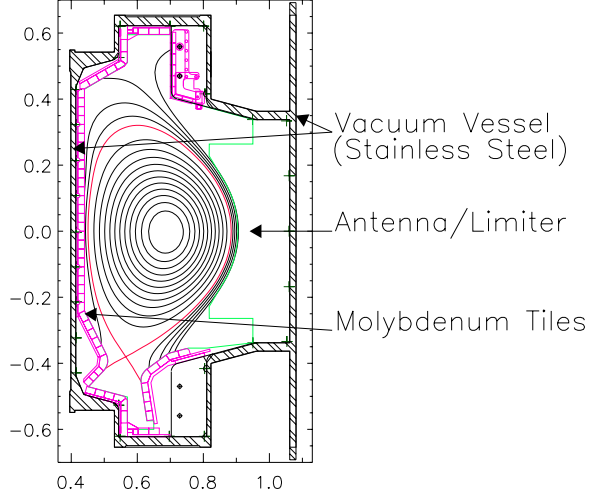
Figure 2: Plasma equilibrium and adjacent conducting surfaces in Alcator C-Mod

in which there are toroidally localized antennas and limiters. Clearly such a wall is far from the idealized cylindrical case, nevertheless the *localized* formulas, equations 16 and 21 can be used with reasonable confidence to estimate the contribution of particular parts of the wall to toroidal torques.

First we point out that the fluctuations observed are practically never in the thin-wall limit. Even if we take account only of the stainless steel wall, 1.3 cm thick, ignoring the molybdenum tiles, which are small and have gaps between them, one obtains $\mu_0\sigma\delta^2 = 4.2 \times 10^{-4}$ s. So any fluctuations faster than this timescale, i.e. with frequency greater than $1/(2\pi \times 4.2 \times 10^{-4}) = 375$ Hz experience "thick" walls. Practically all observed magnetic fluctuations associated with instabilities are faster than this unless they are already "locked" (i.e. stationary).

Moreover, the presence of the tiles, which have electrical conductivity ($\sigma \approx 2.\times 10^7 \Omega^{-1}\mathrm{m}^{-1}$) ten times higher than the stainless-steel wall, would make the time constants even longer, if the tile currents are effective in shielding the fluctuating fields. (Although they usually appear not to be, presumably because of the gaps between them.)

In figure 3 we show examples of poloidal (vertical) field time-derivative ($\dot{B}_v$) measurements, made at the inboard midplane between the wall and the tiles, of large MHD instabilities. One that locks, and one that does not. The former is a tearing mode at or near the $q = 2$ surface caused by excessive gas influx and leading to a disruption, the latter arises from a large "snake" at the $q = 1$ surface, caused by pellet injection. Table 1 shows a summary of their characteristics. The effective vertical wave-number of these modes at the inner wall (k_v) is determined from the relative phase of the mode at coils ranging up the wall. They are both $n = 1$ modes with toroidal wave-number therefore $k_\phi = 2\pi/R = 15$ m^{-1}. The frequencies and amplitudes of the modes are quoted at their slowest (prior to locking). Field perturbations at the outboard limiter (not wall) position are similar in magnitude, which confirms that the inboard measurements are not substantially shielded by the tiles.
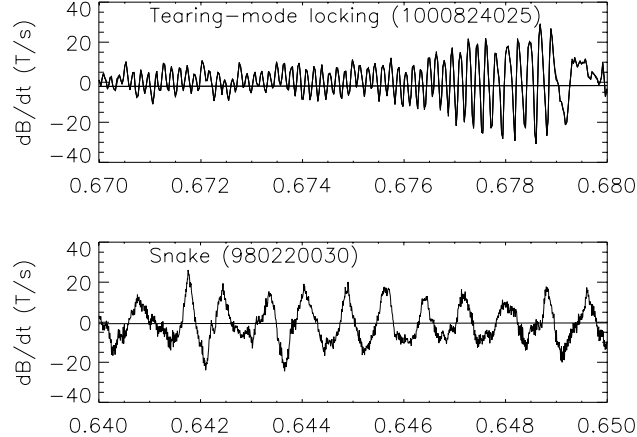
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Figure 3: Examples of MHD $\dot{B}$ oscillations at the inner wall.

Table 1: Comparison of observed modes.

Shot reference	1000824025	980220030
Mode Character	Tearing Mode	Snake
Frequency $\omega/2\pi$ (kHz)	4.5	0.8
$\dot{B}_v$ Amplitude (T/s)	19	10
Field Amplitude (mT)	0.67	1.99
k_v (m^{-1})	16	19
Wall time constant τ_t (ms)	2.6	2.2
Stress $\langle T \rangle$ (N/m^2)	0.017	0.41
Mode slowing time-scale (ms)	40	.4

The shear drag stress at the inner wall is calculated using the thick-wall formula as

$$\langle T \rangle = \frac{1}{2\sqrt{\omega \tau_t}} \frac{|\dot{B}_v|^2}{2\mu_0 \omega^2} \left(1 + \frac{k_\phi^2}{k_v^2} \right) \tag{22}$$

The total force on the plasma is then obtained by integrating over the inner wall. The field fluctuation profile provides an effective area of about 1.5 m^2 over which the quoted stress acts, giving the force as $1.5\langle T \rangle$. Here we ignore the contribution of conducting structures such as antennas and limiters on the outboard side. The fluctuating fields are similar in magnitude at them, but their toroidal extent is substantially smaller.

These plasmas have quite similar shapes and densities ($\bar{n}_e = 3.6 \times 10^{20}$ m^{-3}) leading to total deuterium plasma mass approximately 1.1×10^{-6} kg. The velocity of the mode is ω/k_h and then the ratio of the mass times velocity to the total magnetic drag force gives a time-scale for slowing down, indicated in the last row of the table.

The remarkable conclusion of these estimates is that the tearing mode, which locks in about 2 ms has too little magnetic drag by a factor of 20 to slow the whole plasma down in

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this short a time, whereas the snake, which does not lock and in fact speeds up from time to time , apparently has more than enough drag to slow the whole plasma down.

The resolution of this apparent contradiction must surely be that the rotation of the plasma in the case of the snake is in fact *not* determined by the magnetic drag of the mode on the wall, but rather by some balance of forces intrinsically larger than this drag force. However, if this is true for the snake, it must presumably also be true *a fortiori* for the tearing mode in which the force as estimated from magnetic measurements is far smaller, mostly because the field perturbation is smaller. We conclude that the experimental estimate of the magnetic drag forces from direct measurements does not support the common view that in mode locking a predominant effect is wall drag. Other factors appear to be responsible for the mode velocity and the locking itself may depend on more direct interaction with non-axisymmetric perturbations.

5   Summary

The Maxwell stress tensor provides a natural direct approach to measuring the local momentum flux due to magnetic fluctations. This approach can give direct measurements of wall torques and forces in tokamaks. Linear wall models are completely specified when the complex ratio of the helical flux and its derivative is specified at the wall. These boundary conditions, when known, are sufficient, even with existing measurements, to provide quantitative drag stress estimates that are independent of the behavior of the perturbation in the plasma. However, the commonly adopted thin-wall approximation is not appropriate for Alcator C-Mod. A thick-wall approximation is more appropriate. Applying the estimates to illustrative cases shows that though the drag forces may be large enough to be significant for large-amplitude MHD modes, modes that lock actually have much smaller drag force than other modes that do not lock. Therefore it would appear that magnetic fluctuation wall drag cannot be the predominant factor in determining mode frequency.

Acknowledgements

References

[1] Strait E J *et al* 1999 Nucl. Fusion **39** 1977.

[2] Nave M F F and Wesson J A 1990 Nucl. Fusion **30** 2575.

[3] Fitzpatrick R 1993 Nucl. Fusion **33** 1049.

[4] Finn J M and Sovinec C R 1998 Phys. Plasmas **5** 461 1998.

[5] Kruskal M D, Johnson J L, Gottlieb M B, and Goldman L M 1958 Phys. Fluids **1** 421.

[6] Nalesso G F and Costa S (1980) Nucl. Fusion **20** 443.

[7] Gimblett C G 1986 Nucl. Fusion **26** 617

[8] Fitzpatrick R and Jensen T H 1996 Phys. Plasmas **3** 2641.