

# Laser Radar Point-Target Localization at High Photon Efficiency

Jeffrey H. Shapiro,<sup>1</sup> James Schneeloch,<sup>2</sup>  
Gregory A. Howland,<sup>2</sup> and John C. Howell<sup>2</sup>

<sup>1</sup>*Research Laboratory of Electronics, Massachusetts Institute of Technology,  
Cambridge, Massachusetts 02139*

<sup>2</sup>*Department of Physics and Astronomy, University of Rochester,  
Rochester, NY 14627  
e-mail: jhs@mit.edu*

**Abstract:** Minimum error-probability laser radar point-target localization is analyzed, including the effects of dark counts, background counts, and target speckle. Results from preliminary table-top experiments are reported.

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Laser radars offer superior resolution in comparison with their microwave counterparts. Laser radar, however, suffers from the ill-effects of target-induced speckle and atmospheric propagation. Moreover, laser radar's superior resolution capability can dictate a longer time to interrogate a particular target region. Thus a useful scenario to consider is for a high-resolution laser radar to be cued by a lower-resolution microwave system. An interesting question that arises is determining the ultimate sensitivity when quantum-mechanical resources—nonclassical transmitter states and nonstandard reception techniques—are provided. This paper addresses the photon efficiency for the cued-sensor scenario in which a point target is to be localized.

Suppose that a pulsed-laser transmitter floodlights a volume known to contain a point target, and an  $M_s$ -pixel photon-number resolving array detects the returned light. The localization task is to determine the target's transverse location within these  $M_s$  pixels and its range within  $M_r$  range bins. In the absence of background light and detector dark counts, this scenario can yield an extraordinary number of bits per detected photon (bdp), viz., a  $32 \times 32$  pixel array combined with 15 cm range resolution and a 1 km uncertainty in target range will yield  $\log_2(M_s M_r) = 22.7$  bits from the detection of one photon. There is still, however, the possibility of no detections, even though the target is present. Forcing the radar to randomly choose among the  $M = M_s M_r$  possible target locations when no photons are detected then reduces bdp to 3.3 when the error probability— $\Pr(e) = (M - 1) \exp(-\eta n_T) / M$ , where  $n_T$  is the average number of transmitted photons and  $\eta$  is the radar-to-target-to-radar transmissivity—is  $10^{-3}$ . This performance is realized with  $n_S \equiv \eta n_T = 6.91$ . With a number-state transmitter and the optimum quantum receiver, we have [1]

$$\Pr(e) = \frac{M - 1}{M^2} \{ [1 + (M - 1)p]^{1/2} - (1 - p)^{1/2} \}^2, \quad \text{with } p = (1 - \eta)^{n_T}, \quad (1)$$

yielding  $\Pr(e) = 10^{-3}$  at  $\eta = 10^{-4}$  and  $n_T = 6.88/\eta$ , showing that little is to be gained from the nonclassical transmitter and nonstandard receiver in this case. Thus we focused on the effects of dark counts, background light, and speckle on point-target localization with a floodlight laser-transmitter and a detector array.

Our calculations are for table-top experiments aimed at verifying the theory. We assume a 50 ps transmitter pulse duration, a  $32 \times 32$  detector array with 100 dark-counts/sec on each array element, and a 50 cm uncertainty in target range. Figure 1(a) shows the erasure (no detections) and error (incorrect localization) probabilities versus  $n_S$  when dark counts are the only nonideality, and Fig. 1(c) shows these probabilities when there is also  $9.4 \times 10^3$  background-counts/sec on each array element, arising from a  $1 \text{ W/m}^2$ -Sr- $\mu\text{m}$  background spectral radiance, 1 nm optical bandwidth, and 50% detector quantum efficiency. Background counts drive up the error probability, but have little effect on the erasure probability because that is dominated by the probability that no target-return photons are detected. Figures 1(b) and (d) show the mutual information and bdp for the dark-counts only and dark-counts plus background-counts cases, respectively. Both situations can provide more than 2 bdp with  $\Pr(\text{erasure}) \leq 10^{-3}$  and  $\Pr(\text{error}) \leq 10^{-3}$ .

Figure 2 shows  $\Pr(\text{erase})$  and  $\Pr(\text{error})$  [in (a)] and the mutual information and bdp [in (b)] when, in addition to dark and background counts, the target produces fully-developed speckle. Here we see a substantial performance degradation has been incurred, but 2 bdp can still be obtained.

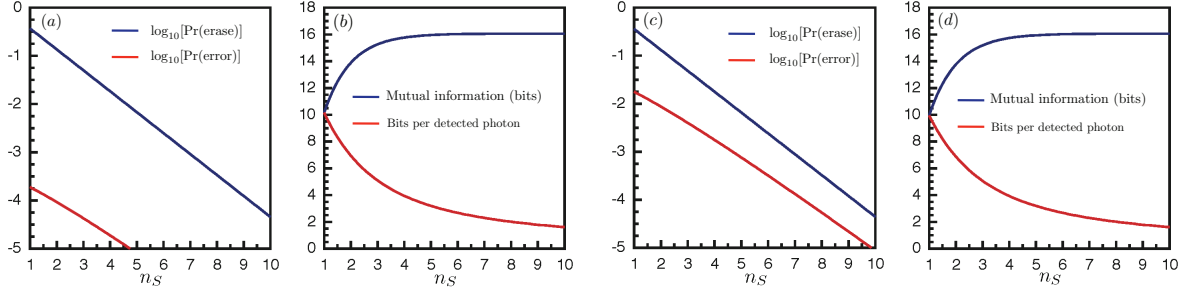


Fig. 1.  $\Pr(\text{erase})$  and  $\Pr(\text{error})$  [(a) and (c)] and mutual information (MI) and bdpd [(b) and (d)] for dark-count limited operation [(a) and (b)] and background-limited operation [(c) and (d)].

In a preliminary experiment, we have used the Fig. 3 arrangement to emulate point-target localization. Light from a low power HeNe laser is attenuated to the single-photon level with neutral-density (ND) filters and coupled into a single-mode fiber for spatial filtering. The fiber's output illuminates a one-to-one imaging system with digital micro-mirror devices (DMDs) in the object and image planes. The first DMD introduces a point target into the system, and the second emulates a detector array by scanning the light from its elements onto a geiger-mode avalanche photodiode.

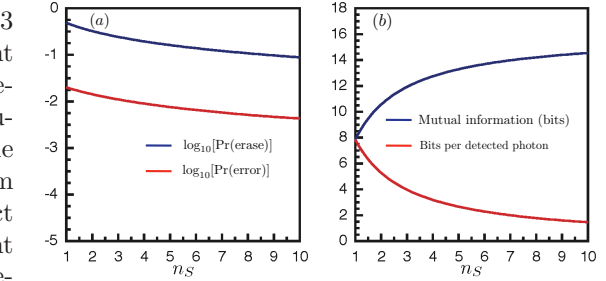


Fig. 2.  $\Pr(\text{erase})$  and  $\Pr(\text{error})$  [in (a)] and MI and bdpd [in (b)] for dark-limited operation with speckle.

Figure 4 compares results from an experiment in which the target is to be localized within a  $16 \times 16$  pixel ( $M = 256$ ) array in dark-count limited operation (600 dark-counts/sec on each pixel): (a) and (b) show the performance when a single pulse interrogates each pixel; (c) and (d) show the performance when each pixel is interrogated until at least one count occurs. This pulse-until-detect (PUD) protocol completely suppresses erasures, but at the expense of more errors. Figure 4 shows that this error-probability penalty is not severe, and that our experiments are in excellent agreement with theory.

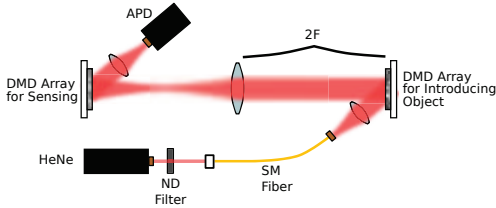


Fig. 3. Setup for point-target localization experiment.

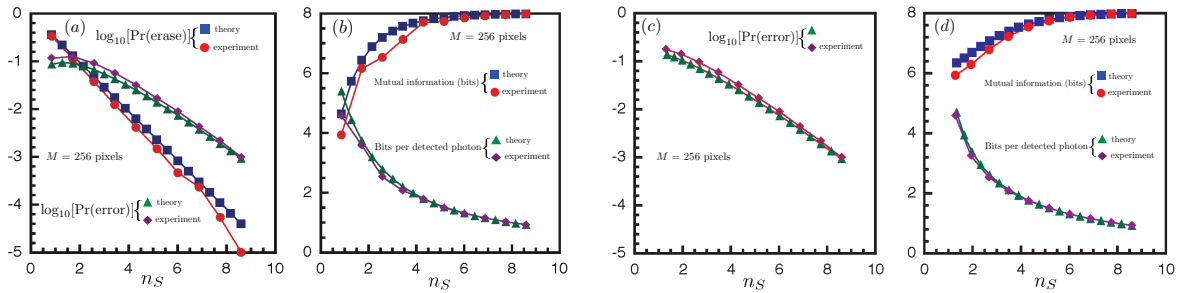


Fig. 4.  $\Pr(\text{erase})$  and  $\Pr(\text{error})$  for dark-count limited operation [(a) and (c)] and MI and bdpd [(b) and (d)] without [(a) and (b)] and with [(c) and (d)] the PUD protocol.

## References

1. C. W. Helstrom, *Quantum Detection and Estimation Theory* (Academic, New York, 1976).