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**A MODEL FOR STRESSES IN CIRCULAR MAGNETS
FOR TOROIDAL FUSION DEVICES***

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ABSTRACT

Circular coils in toroidal fusion devices can be subjected to in plane bending moments resulting from the interaction of the current with the inhomogeneous magnetic field. To take into account the support of the coils both externally through the in plane structure (a central cylinder), and internally through the out of plane structure (the coil forms a toroidal assembly which can take up the net inward force by a reaction in the toroidal direction) the coil is modelled as a circular beam on elastic foundation. From the solutions for a single force acting on such a beam, we derive the formulas for an arbitrary $\cos n\theta$ variation of a distributed force. The general system is solved by Fourier expansion of the in plane loads and external reaction forces. The fraction of the net inward force reacted by the central cylinder and by the intercoil structure can be varied continuously, and a minimum for the bending moments is found for a particular value of this fraction (depending on the aspect ratio, the stiffness of the coil, and the intercoil structure). It is suggested that the best way to optimize the design of the toroidal coils is to choose the coil shape such that within other constraints the circumferential length is minimal. To minimize the moments one should optimize the design of the center support cylinder, the design of the out of plane structure and especially the distribution of the reaction force between both. Only a small fraction ($\simeq 20\%$) of the net inward force should be taken up by a central cylinder.

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INTRODUCTION

Except for some notable exceptions, like the spheromak and the torsatron most toroidal systems have, as one of their basic components, magnetic coils that provide the main toroidal field. These magnets make up a rather large fraction of the total system cost, both the conductor and magnet structure being significant cost items, and therefore large efforts have gone into optimizing their design.

There have been basically two approaches.

One approach considers the coils separately, neglecting the out of plane structure, and optimizes the shape of the magnet to reduce bending moments. The first analysis by File, Mills, and Sheffield [1] which used a filament in a simple $1/R$ field, supported by a central cylinder, was later further refined by taking into account the finite thickness of the coils [2] and the ripple in the toroidal field [3,4]. An attempt to consider not only the coil shape as the primary design parameter but also proper design of the center support structure in order to minimize in plane bending stresses, was presented by Ojalvo and Zatz [5].

A second approach has been to model the coils, together with the out of plane structure as a continuous rotationally symmetric shell. Gray, et al. approximated the shell by a membrane [6]. Bobrov and Schultz [7] used Reissner shell theory to analyze orthotropic shells of finite thickness and pointed out that choosing the shape of the coils so as to minimize bending stresses may not be the optimum strategy. The reason is that the additional circumferential length needed to give the coil its bending free shape may more than outweigh the material savings resulting from its bending free properties.

Because of the intrinsic difference in the two approaches, each approach has tended to concentrate on a different way to support the net inward force resulting from the magnetic load. A central column is necessary to take the net force when the coils are considered separately while the shell

approach is easiest when one assumes this force to be completely taken up internally. The importance to distribute appropriately the load between both a shell structure and a central cylinder was evident from the design of the toroidal field coils for TFTR [8]. Montgomery [9] has pointed out that the natural bending free shape depends strongly on the way the coil is supported. It is thus important to have a model that can include both the effect of the out of plane structure as well as the reaction of a central column in taking up the net inward force.

In this report an approach is presented that can achieve this. The coils are considered separately, but the effect of the out of plane structure is included by modeling the coils as a beam on elastic foundation. The model is thus more complete than the first approach, and contrary to the second, allows us to give different properties to the coil and the out of plane structure. With this model we have investigated the continuum between the net centering force being taken up completely internally (through the reaction of the elastic foundation) or being taken up completely by a central column. It is shown that there is an optimum distribution of the reaction force between the reaction provided by the central column and the internal reaction. At this optimum the bending moments are minimum. This, together with the previous approaches, suggests that the best way to optimize toroidal coils would be the following: the coil shape is chosen such that within other constraints (plasma shape, ripple requirements) the circumferential length is minimal; the design of the center support and of the out of plane structure, and the distribution of the force between both are then optimized to minimize the moments.

MODEL

The coil is considered as a circular ring of isotropic properties, with an applied load given by the interaction of the coil current I and the toroidal magnetic field B , for which we take a simple $1/R$ variation

$$p(N/m) = \frac{BI}{2} = \frac{1}{2} \frac{\mu_0 N I^2}{2\pi R}$$

In this formula N is the number of coils. With $R = R_o + a \cos \theta$, where R_o is the major radius of the

geometric center of the ring and a its radius (Fig. 1), we can write this as

$$P(\theta) = \frac{1}{2} \frac{\mu_0 N I^2}{2\pi R_o} \frac{1}{1 + \frac{a}{R_o} \cos \theta}$$

or

$$p(\theta) = p_o \frac{1}{1 + \epsilon \cos \theta} \quad (1)$$

with

$$p_o = \frac{1}{2} \frac{\mu_0 N I^2}{2\pi R_o} \text{ and } \epsilon = \frac{a}{R_o}.$$

The fact that the coil is part of a three dimensional structure is taken into account by assuming a reaction force on the coil proportional to its displacement

$$f(N/m) = ky \quad (2)$$

where f is the reaction force per unit length and y the displacement in the minor radial direction. The reaction constant k can be chosen appropriately to model a very strong interaction (Alcator Bitter plates, where it would not at all be appropriate to consider each coil separately) or a very weak one (if the coils are not wedged and no part of the net inward force is taken up in the toroidal direction we can even take $k = 0$). Note that here k is assumed to be constant and the model can also be used for

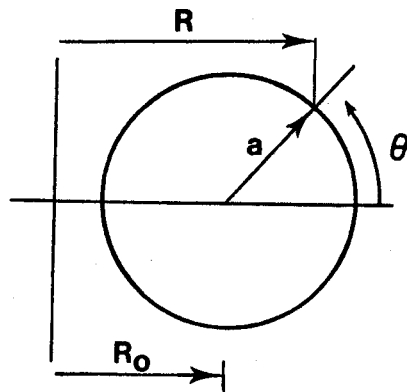


Figure 1 Model for a toroidal coil.

a coil that would be supported at its periphery by springs which would give a reaction proportional to the displacement.

Two different models are used for the central column, either a concentrated force or a distributed force. The distribution of the reaction between the elastic foundation, modeling the out of plane structure, and the central cylinder can be varied continuously.

The solution is derived from the known solutions of a circular ring on an elastic foundation, subjected to a concentrated force P (Fig. 2).

From [10] we have

$$\begin{aligned}
 y &= \frac{Pr^3}{4\alpha\beta EI} \left(\frac{2\alpha\beta}{\pi\eta^2} - A \cosh \alpha\phi \cos \beta\phi + B \sinh \alpha\phi \sin \beta\phi \right) \\
 M &= -\frac{Pr}{2} \left(\frac{1}{\pi\eta^2} + A \sinh \alpha\phi \sin \beta\phi + B \cosh \alpha\phi \cos \beta\phi \right) \\
 Q &= -\frac{P}{2} [(\alpha A - \beta B) \cosh \alpha\phi \sin \beta\phi + (\beta A + \alpha B) \sinh \alpha\phi \cos \beta\phi] \\
 N &= \frac{P}{2} \left(\frac{\eta^2 - 1}{\pi\eta^2} - A \sinh \alpha\phi \sin \beta\phi - B \cosh \alpha\phi \cos \beta\phi \right)
 \end{aligned}$$

where

$$\begin{aligned}
 \eta &= \sqrt{\frac{r^4 k}{EI} + 1} \\
 \alpha &= \sqrt{\frac{\eta - 1}{2}} \\
 \beta &= \sqrt{\frac{\eta + 1}{2}} \\
 A &= \frac{\alpha \cosh \alpha\pi \sin \beta\pi + \beta \sinh \alpha\pi \cos \beta\pi}{\eta (\sinh^2 \alpha\pi + \sin^2 \beta\pi)} \\
 B &= \frac{\alpha \sinh \alpha\pi \cos \beta\pi - \beta \cosh \alpha\pi \sin \beta\pi}{\eta (\sinh^2 \alpha\pi + \sin^2 \beta\pi)} \quad (3)
 \end{aligned}$$

These results, which give $y(\phi)$ for a force applied at $\phi = \pi$, can also be considered as the displacement y at π for a force at ϕ (Fig. 3). If we take the force to be $P = \ell(\phi) r d\phi$ and perform the integration we can find the displacement at $\phi = \pi$ for a distributed force $\ell(\phi)$. It can further

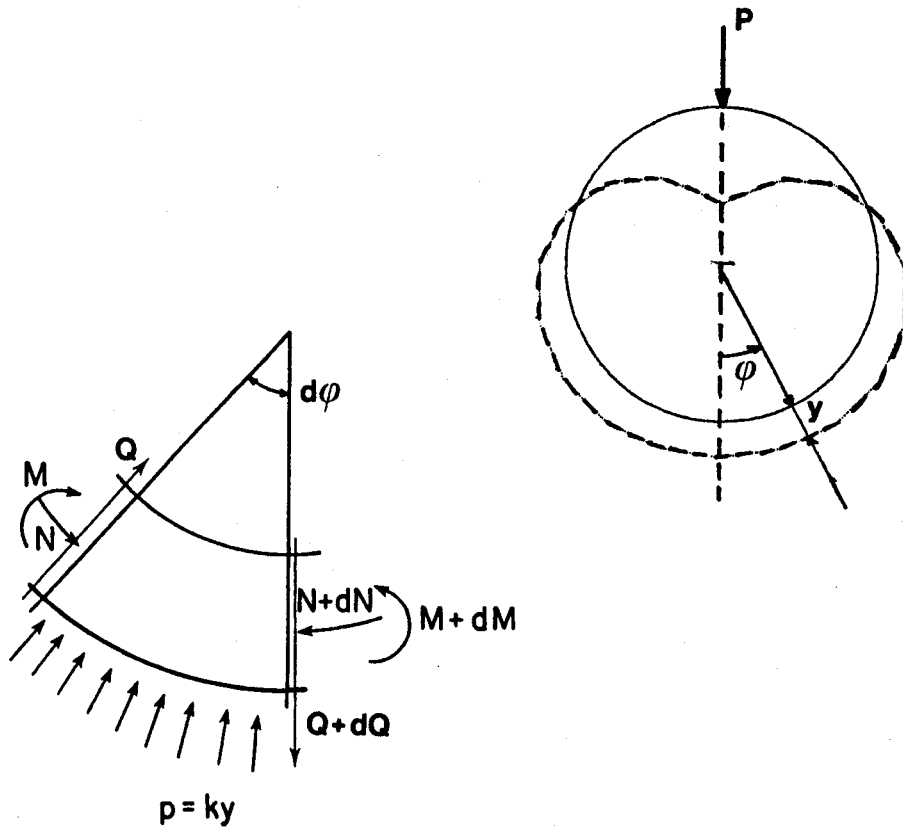


Figure 2 Definition of angles, moments and forces. Also shown is the displacement y for a point force P .

be generalized by making the reference axis not vertical, but at an angle ψ (Fig. 4). Still measuring ϕ from this reference axis we can get the displacement at an arbitrary location, for an arbitrary distribution of the force, and more particularly for

$$P = -p(\theta)rd\phi = -p_o \frac{rd\phi}{1 + \epsilon \cos(\pi + \psi + \phi)}$$

We thus have

$$y(\psi) = \int_{-\pi}^{\pi} \frac{-p_o}{1 + \epsilon \cos(\pi + \psi + \phi)} \times \frac{r^4}{4\alpha\beta EI} \times \left(\frac{2\alpha\beta}{\pi\eta^2} - A \cosh \alpha\phi \cos \beta\phi + B \sinh \alpha\phi \sin \beta\phi \right)$$

In order to perform the integration we expand $\frac{1}{1 + \epsilon \cos \theta}$ in Fourier series as follows.

First use

$$\frac{1}{1 + \epsilon \cos \theta} = 1 - \epsilon \cos \theta + \epsilon^2 \cos^2 \theta - \epsilon^3 \cos^3 \theta + \epsilon^4 \cos^4 \theta \dots$$

then write the powers of $\cos^n \theta$ in terms of $\cos n\theta$... to obtain

$$\frac{1}{1 + \epsilon \cos \theta} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\theta$$

with

$$a_n = \sum_{i=0}^{\infty} (-1)^i \frac{\epsilon^{n+2i}}{2^{n-1+2i}} C \binom{n+2i}{n+i} \quad (4)$$

The series to calculate the coefficients a_n (summation over i) converges rapidly, each term being a factor

$$\frac{\epsilon^2 (n+2i+1)(n+2i+2)}{4 (n+i+1)(i+1)}$$

smaller than the previous one.

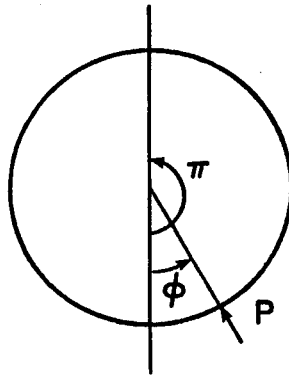


Figure 3 Solution at π for force P at ϕ .

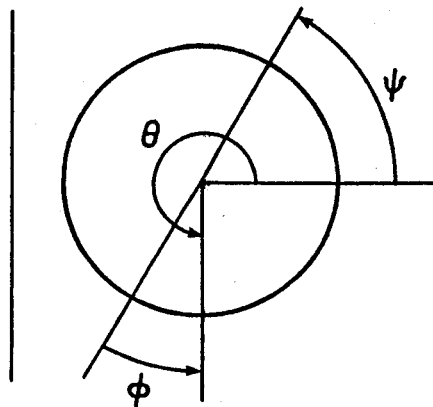


Figure 4 The reference axis now becomes tilted at an arbitrary angle ψ .

For the Fourier series also, only a few terms are necessary as the leading coefficient of a_n is $\frac{\epsilon^n}{2^{n-1}}$.

Performing the integration term by term finally yields

$$\begin{aligned}
 y(\psi) &= -\frac{p_0 r^4}{4\alpha\beta EI} \left(\frac{2\alpha\beta}{\pi\eta^2} \times \frac{a_0}{2} \times 2\pi - A \operatorname{chc}(\psi) + B \operatorname{shs}(\psi) \right) \\
 M(\psi) &= \frac{p_0 r^2}{2} \left(\frac{1}{\pi\eta^2} \times \frac{a_0}{2} \times 2\pi + A \operatorname{shs}(\psi) + B \operatorname{chc}(\psi) \right) \\
 Q(\psi) &= -\frac{p_0 r}{2} \left((\alpha A - \beta B) \operatorname{chs}(\psi) + (\beta A + \alpha B) \operatorname{shc}(\psi) \right) \\
 N(\psi) &= -\frac{p_0 r}{2} \left(\frac{\eta^2 - 1}{\pi\eta^2} \times \frac{a_0}{2} \times 2\pi - A \operatorname{shs}(\psi) - B \operatorname{chc}(\psi) \right) \quad (5)
 \end{aligned}$$

where

$$\begin{aligned}
 \operatorname{chc}(\psi) &= \int_{-\pi}^{\pi} \cosh \alpha\phi \cos \beta\phi \left[\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n(\pi + \psi + \phi) \right] d\phi \\
 &= \sum_{n=0}^{\infty} a'_n \left[\frac{1}{[\alpha^2 + (\beta - n)^2]} \left((\beta - n) \cosh \alpha\pi \sin \beta\pi \cos n\psi + \alpha \sinh \alpha\pi \cos \beta\pi \cos n\psi \right) \right. \\
 &\quad \left. + \frac{1}{[\alpha^2 + (\beta + n)^2]} \left((\beta + n) \cosh \alpha\pi \sin \beta\pi \cos n\psi + \alpha \sinh \alpha\pi \cos \beta\pi \cos n\psi \right) \right] \quad (6)
 \end{aligned}$$

I have defined

$$\begin{aligned}
 a'_n &= a_n \text{ for } n \neq 0 \\
 a'_n &= \frac{a_n}{2} \text{ for } n = 0.
 \end{aligned}$$

Similarly

$$\begin{aligned}
 \operatorname{shs}(\psi) &= \int_{-\pi}^{\pi} \sinh \alpha\phi \sin \beta\phi \left[\sum_{n=0}^{\infty} a'_n \cos n(\pi + \psi + \phi) \right] \\
 &= \sum_{n=0}^{\infty} a'_n \left[\frac{1}{[\alpha^2 + (\beta - n)^2]} \left(\alpha \cosh \alpha\pi \sin \beta\pi \cos n\psi - (\beta - n) \sinh \alpha\pi \cos \beta\pi \cos n\psi \right) \right. \\
 &\quad \left. + \frac{1}{[\alpha^2 + (\beta + n)^2]} \left(\alpha \cosh \alpha\pi \sin \beta\pi \cos n\psi - (\beta + n) \sinh \alpha\pi \cos \beta\pi \cos n\psi \right) \right] \quad (7)
 \end{aligned}$$

$$\begin{aligned}
shc(\psi) &= \int_{-\pi}^{\pi} \sinh \alpha \phi \cos \beta \phi \left[\sum_{n=0}^{\infty} a'_n \cos n(\pi + \psi + \phi) \right] \\
&= \sum_{n=0}^{\infty} a'_n \left[\frac{1}{[\alpha^2 + (\beta - n)^2]} \left(-(\beta - n) \sinh \alpha \pi \cos \beta \pi \sin n\psi + \alpha \cosh \alpha \pi \sin \beta \pi \cos n\psi \right) \right. \\
&\quad \left. + \frac{1}{[\alpha^2 + (\beta + n)^2]} \left(+(\beta + n) \sinh \alpha \pi \cos \beta \pi \sin n\psi - \alpha \cosh \alpha \pi \sin \beta \pi \cos n\psi \right) \right] \quad (8)
\end{aligned}$$

$$\begin{aligned}
chs(\psi) &= \int_{-\pi}^{\pi} \cosh \alpha \phi \sin \beta \phi \left[\sum_{n=0}^{\infty} a'_n \cos n(\pi + \psi + \phi) \right] \\
&= \sum_{n=0}^{\infty} a'_n \left[\frac{1}{[\alpha^2 + (\beta - n)^2]} \left(-\alpha \sinh \alpha \pi \cos \beta \pi \sin n\psi - (\beta - n) \cosh \alpha \pi \sin \beta \pi \sin n\psi \right) \right. \\
&\quad \left. + \frac{1}{[\alpha^2 + (\beta + n)^2]} \left(+\alpha \sinh \alpha \pi \cos \beta \pi \sin n\psi + (\beta + n) \cosh \alpha \pi \sin \beta \pi \sin n\psi \right) \right] \quad (9)
\end{aligned}$$

These formulas completely solve the problem for the loading forces. For the reaction forces we have investigated two cases.

1. A single reaction force applied at $\theta = 0$ or $\theta = \pi$ (Fig. 5). For this the formulas (3) can be used directly.
2. In order to model the reaction of a central column we have chosen a force distributed as $\sin^8 \frac{\theta}{2} \cos \theta$. The 8th power for the sin term is a compromise. A higher power would give too peaked a distribution which would then not be significantly different from a single force. For too low a power we obtain that the effect on the outer section, between $\theta = -\frac{\pi}{2}$ and $\theta = \frac{\pi}{2}$ becomes important so that we would not appropriately model a column in the center.

This distribution is plotted in Fig. 6. In order to get the formulas for this case we have followed the same procedure and expanded $\sin^8 \frac{\theta}{2} \cos \theta$ according to

$$\begin{aligned}
\sin^8 \frac{\theta}{2} \cos \theta &= -1 + 6 \cos^2 \frac{\theta}{2} - 14 \cos^4 \frac{\theta}{2} + 16 \cos^6 \frac{\theta}{2} - 9 \cos^8 \frac{\theta}{2} + 2 \cos^{10} \frac{\theta}{2} \\
&= -0.2187 + 0.3828 \cos \theta - 0.2500 \cos 2\theta + 0.1133 \cos 3\theta \\
&\quad - 0.03125 \cos 4\theta + 0.004 \cos 5\theta \quad (10)
\end{aligned}$$

Again term by term integration is performed and in fact the earlier formulas (5-9) apply, limiting the sum to $n = 5$ with a_n taken from (10).

The magnitude of the reaction force (be it the single force or the distributed force) is varied with a parameter *frac* that measures what fraction of the total net inward force, due to the magnetic loading, is taken up by the reaction, the other fraction being taken up by the overall structure. The net inward force is given by

$$F_i = \int_{-\pi}^{\pi} \frac{-p_o \cos \theta}{1 + \epsilon \cos \theta} r d\theta$$

$$= 2\pi r p_o \frac{1}{\epsilon} \left[\frac{1}{\sqrt{1 - \epsilon^2}} - 1 \right] \quad (11)$$

For the single reaction force we apply

$$F_r = \text{frac} \times 2\pi r p_o \frac{1}{\epsilon} \left[\frac{1}{\sqrt{1 - \epsilon^2}} - 1 \right]$$

to the circular ring at $\theta = 0$ or $\theta = \pi$.

For the distributed force

$$p_r = \text{frac} \times \frac{256}{49} p_o r \frac{1}{\epsilon} \left[\frac{1}{\sqrt{1 - \epsilon^2}} - 1 \right] \sin^8 \frac{\theta}{2} \cos \theta,$$

where the coefficient of $\sin^8 \frac{\theta}{2} \cos \theta$ is such that for *frac* = 1, the net outward force exactly equals the net inward magnetic force.

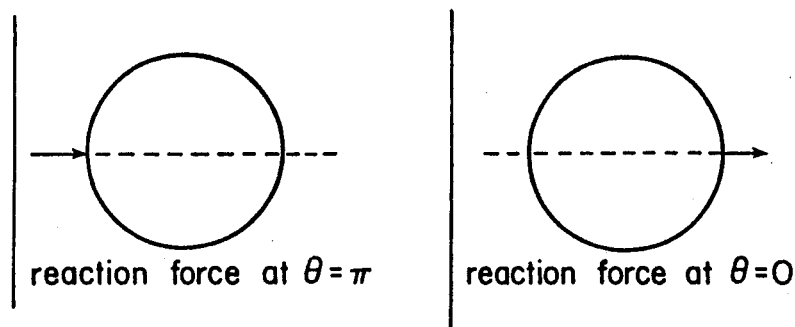


Figure 5 Position of the concentrated reaction.

RESULTS

Displacements, moments, transverse and tangential forces have been calculated for varying values of η (related to the stiffness of the structure k through $\eta = \sqrt{\frac{r^4 k}{EI} + 1}$), and ϵ (inverse aspect ratio $\frac{r}{R}$). Some deformed shapes are shown in Figs. 7 - 9.

Except in such extreme cases as where more than 50% of the net centering force is balanced by a single point force on the inside or on the outside, the transverse force Q is always less than 30% of the tangential force so that in our discussion we concentrate on moment and tangential force only. It is helpful to calculate the sum of the tangential force acting on both legs of the coil in the horizontal midplane, as in Fig. 10. The value of $N_1 + N_2$ is equal to

$$F_z = \frac{\mu_0 N I^2}{4\pi} \log \left(\frac{1 + \epsilon}{1 - \epsilon} \right).$$

If there were no external force, and the coil had to take the total bursting force acting on it internally with hoop stresses then the theoretical minimum for N_1 would be

$$N_1 = \frac{\mu_0 N I^2}{4\pi} \frac{1}{2} \log \left(\frac{1 + \epsilon}{1 - \epsilon} \right)$$

To within less than 10% (for ϵ smaller than 0.5) this is equal to

$$N_1 = \frac{\mu_0 N I^2}{4\pi} \frac{r}{R_0}$$

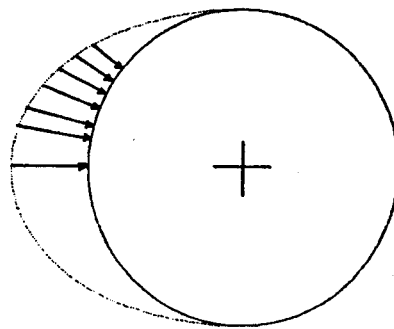


Figure 6 Distributed reaction force.

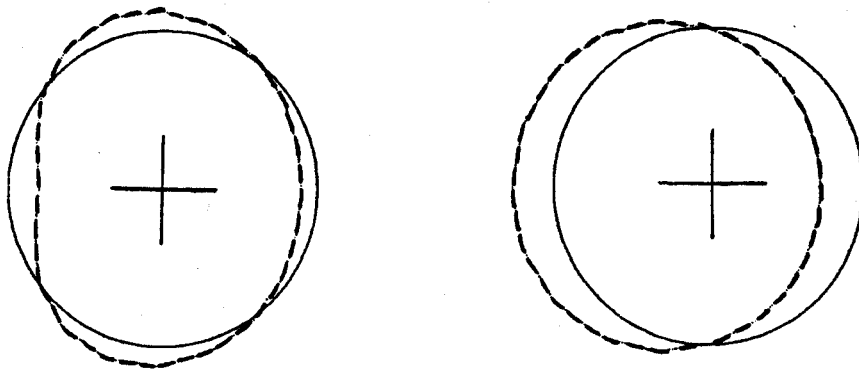


Figure 7 Single force on the inside $\eta = 2$ $\epsilon = 0.45$ $frac = 1$ (left), $frac = 0.25$ (right).

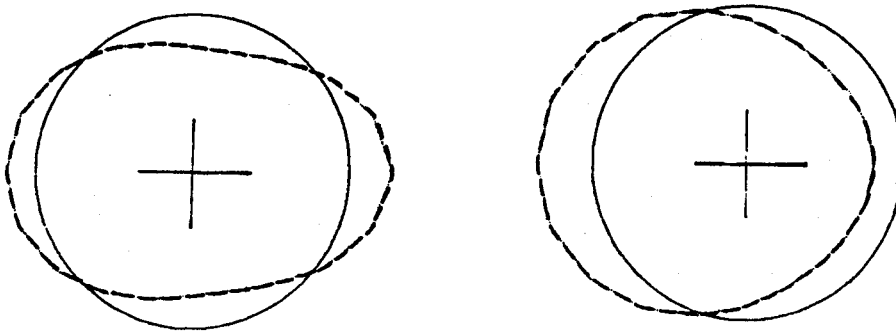


Figure 8 Single force on the outside $\eta = 2$ $\epsilon = 0.45$ $frac = 1$ (left), $frac = 0.25$ (right).

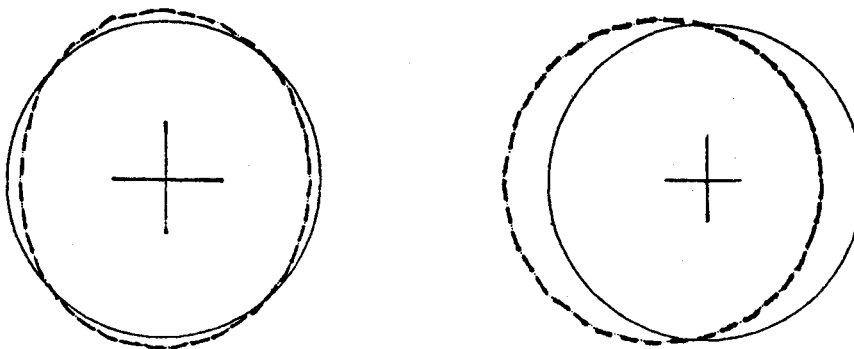


Figure 9 Distributed force on the inside $\eta = 2$ $\epsilon = 0.45$ $frac = 1$ (left), $frac = 0.25$ (right).

which is the value one would obtain assuming a uniform pressure calculated from the current in the coil and half the magnetic field at the center.

The value of N normalized for the case where the total reaction is taken up by a central cylinder (distributed force), is shown in Fig. 11. If the net centering force is taken up internally we obtain the results of Fig. 12. Except for the cases where more than 50% of the net centering force is taken up by a single force on the outer leg, the value of $N_1 / \left(\frac{\mu_0 N I^2}{4\pi} \frac{r}{R_0} \right)$ is always within 25% of 1.

The variation of moments is somewhat more complicated, as it is much more sensitive to ϵ , η and the fraction of the force taken up by an external reaction. Typical values of $M / \frac{p_0 r^2}{2}$ as a function of η and ϵ for the case of a bucking cylinder taking the total net centering force are shown in Fig. 13. Figure 14 assumes no bucking cylinder. The variation between no bucking cylinder and a bucking cylinder taking up all the net centering force is shown in Fig. 15, for fixed ϵ and η . Figure 16 gives the same for a single force. Some conclusions can be drawn for the variation of the moments. First, there is an optimal distribution of the net centering force between the reaction of the cylinder and the part taken up by the internal structure. The smaller the aspect ratio, the larger the optimal fraction is that should be taken up by a bucking cylinder. The moments are usually smaller for a larger aspect ratio, while a stiffer structure always gives smaller moments for any aspect ratio. By properly choosing the fractions taken up and the stiffness of the structure, it should be possible to keep $M / \frac{p_0 r^2}{2}$ well below 0.02.

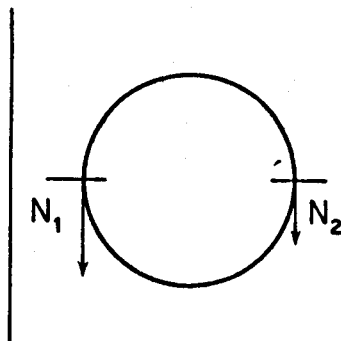


Figure 10 Tangential forces of the horizontal section.

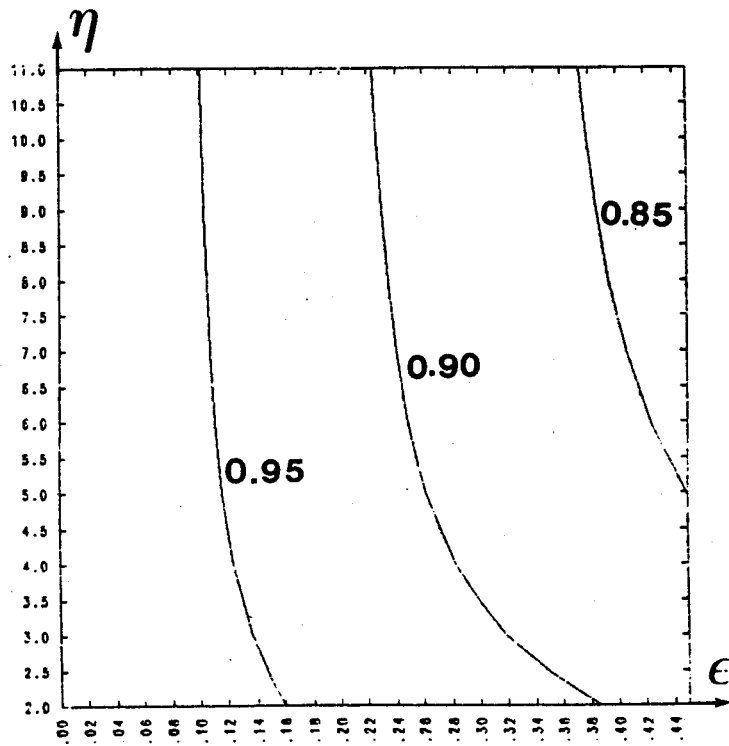


Figure 11 Normalized N for a distributed reaction.

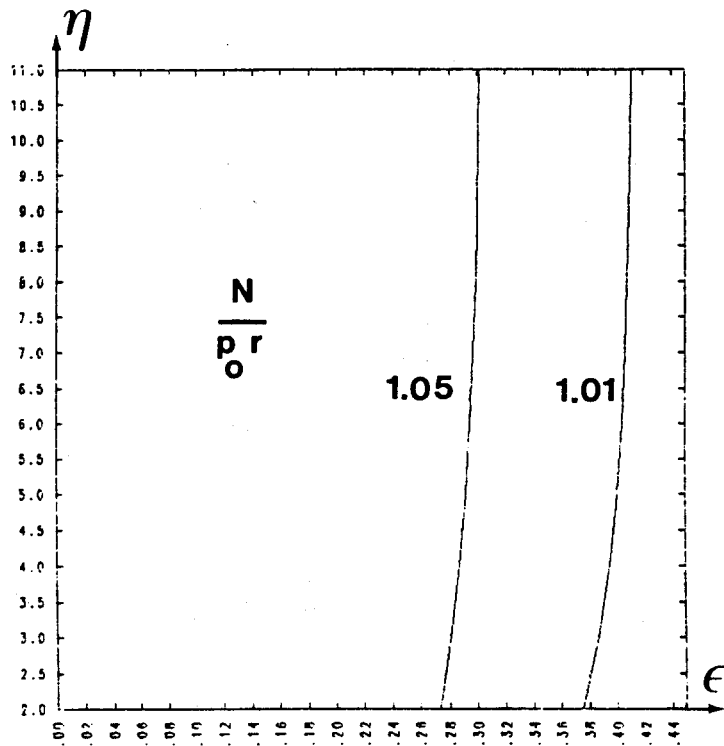


Figure 12 Normalized N for no external reaction.

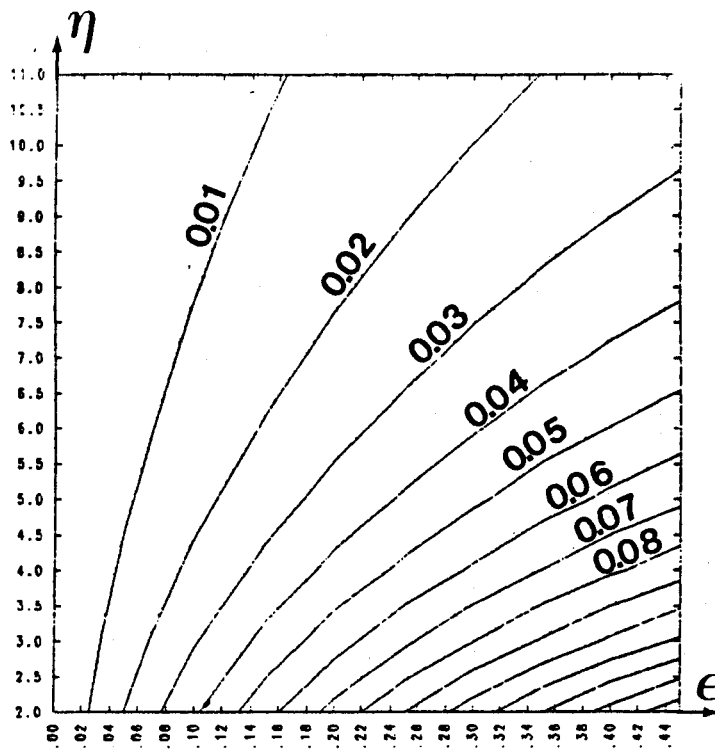


Figure 13 Normalized absolute value of the maximum moment for the case of a distributed reaction.

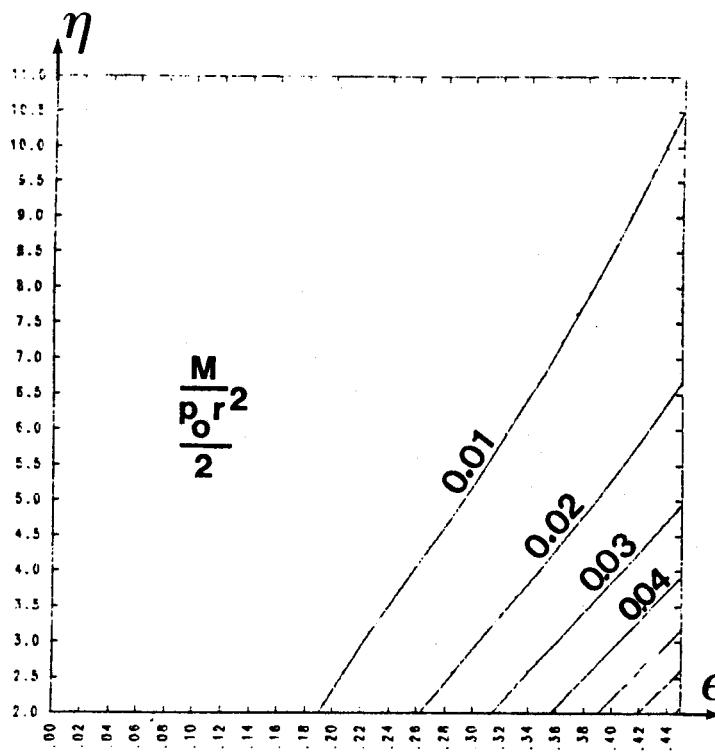


Figure 14 Normalized absolute value of the maximum moment for the case of no external reaction.

As both N and M vary (even though N only varies slightly) we are interested in what the variation of the stresses will be once they are combined. Will stresses due to moments dominate or need the structure be designed essentially for stresses due to N ?

It is important to make the distinction between stresses due to tangential force and stresses due to moments because they scale differently. And so will the total amount of structural material necessary, depending on whether it is dimensioned for bending or tangential stresses.

For a tangential force, if the load quadruples (for a doubling of the magnetic field), the cross section and thus the weight have to quadruple if the maximum allowable stress stays the same. The structural material thus scales as B^2 .

For moments, as $\sigma = \frac{6M}{bh^2}$ (where $h \times b$ are the dimension of the section of the coil, h being measured in the radial direction), if the load quadruples the amount of material does not necessarily have to quadruple. By increasing h , keeping b constant it is possible to keep the same maximum allowable stress with only twice as much material. The structural material in this case scales as B .

One way of distinguishing between dominance of tangential force or moments is by comparing the actual build $\frac{h}{r}$ of the coil with $\frac{6M}{Nr}$.

Indeed let

$$\begin{aligned}\sigma_{max} &= \frac{N}{A} + \frac{6M}{bh^2} \\ &= \frac{N}{A} \left(1 + \frac{6M}{Nh} \right) \\ &= \frac{N}{A} \left(1 + \frac{6M}{Nr} \cdot \frac{r}{h} \right).\end{aligned}$$

If we call $\frac{6M}{Nr} = \left(\frac{h}{r}\right)_o$, then if $\frac{h}{r} > \left(\frac{h}{r}\right)_o$ it means that the normal force dominates, thus giving a structural material weight scaling as B^2 . If $\frac{h}{r} < \left(\frac{h}{r}\right)_o$, the moment dominates and by increasing $\frac{h}{r}$ it is possible to have the structural material weight scale as B . Of course this increase of $\frac{h}{r}$ will be beneficial up to $\frac{h}{r} = \left(\frac{h}{r}\right)_o$ at which point the weight will start to scale as B^2 . It is also possible that for reasons of access, or of power dissipation one would rather not increase h . Then of course weight will scale as B^2 . This can be represented schematically in Fig. 17. Values of $\left(\frac{h}{r}\right)_o$ are given in Figs. 18 and 19. The negative values of $frac$ are for a single reaction force at $\theta = 0$ in both figures. Positive values

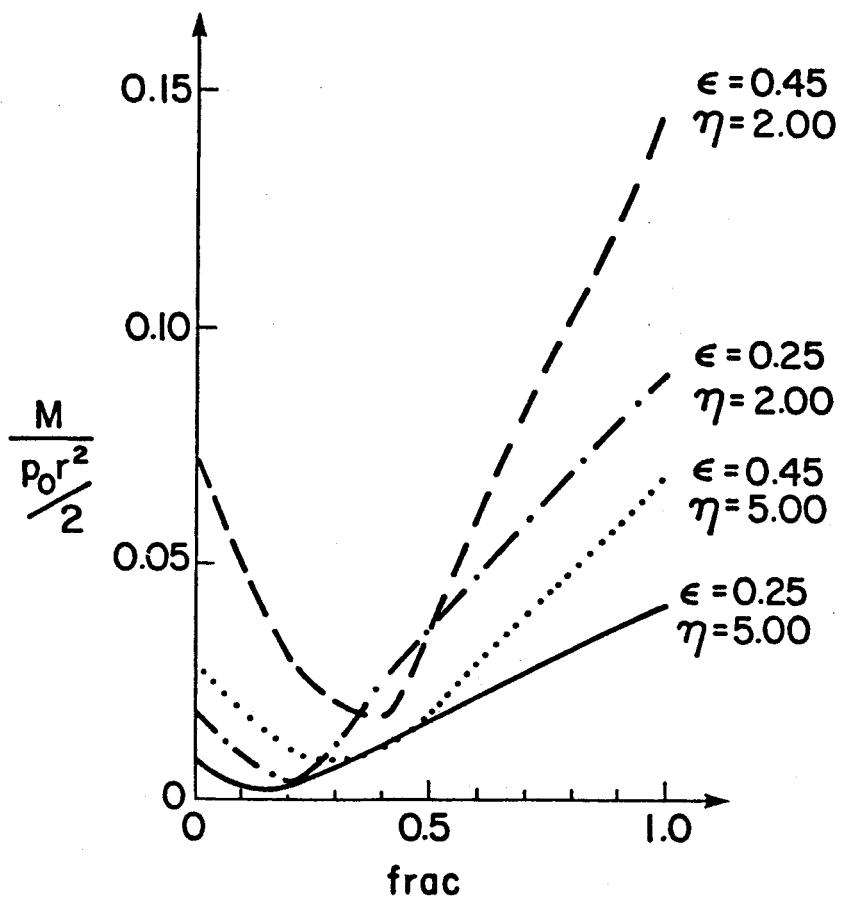


Figure 15 Normalized moment as a function of the fraction taken up by a distributed reaction.

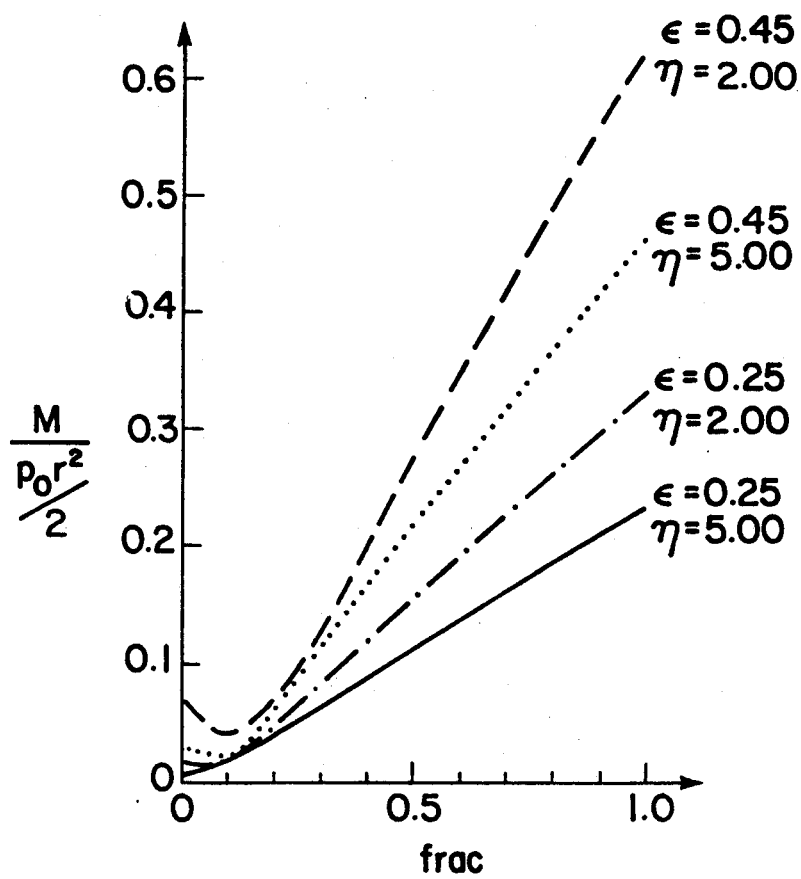


Figure 16 Normalized moment as a function of the fraction taken up by a point reaction.

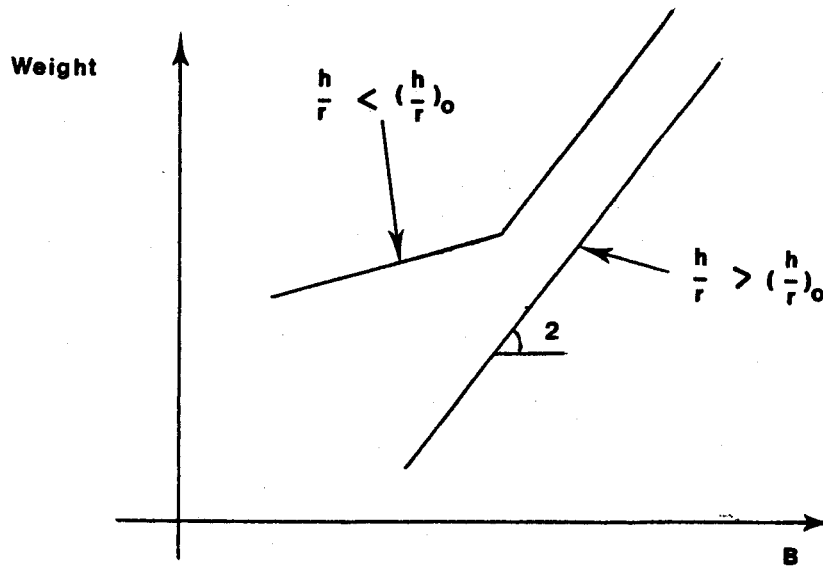


Figure 17 Scaling of the weight with B , shown schematically.

of $frac$ are for a single force at $\theta = \pi$ in Fig. 18, and for a distributed force in Fig. 19. For most present machines, the reaction force is provided by the wedging action of the nose of the coil or by a bucking cylinder. If the wedging action of the nose of the coil is considered as taking up the forces internally in a very local manner on the inside, then our model is not applicable because we have assumed that the reaction constant k is really constant along the periphery of the coil. Alternatively, we can view the vault of the coil casings on the inside as a central cylinder that provides a distributed central reaction on the coils. The model is then applicable and we are thus somewhat in between the case of a single central force ($frac = 0.4 \rightarrow 1.0$ of Fig. 18) and a distributed central reaction ($frac = 0.4 \rightarrow 1.0$ of Fig. 19). The radial build of the coil is usually not larger than $h/r = 0.3$. Thus in general we have $\frac{h}{r} < (\frac{h}{r})_o$ and moments dominate. If however most of the centering force is taken up internally by a well distributed out of plane structure, without central reaction force or bucking cylinder, $(\frac{h}{r})_o$ becomes small so that $\frac{h}{r} > (\frac{h}{r})_o$ is easily satisfied, and the structure has to be dimensioned essentially for the tangential force.

Our analysis thus shows that a more solid distributed intercoil structure would be beneficial. It could take a larger fraction of the inward force thereby reducing the moments in the coil. We can write down for $M/\frac{p_o r^2}{2}$ at the minimum approximately $M/\frac{p_o r^2}{2} < \frac{6}{20}$. Since we had $\frac{N}{p_o r} \sim 1$ we can write $(\frac{h}{r})_o = \frac{6M}{rN} < \frac{3r}{20} = 0.15\epsilon$. Thus for $(\frac{h}{r}) > 0.15\epsilon$, the coil is not dominated by bending stresses. Presently most of the net centering force is taken up by a bucking cylinder or by wedging

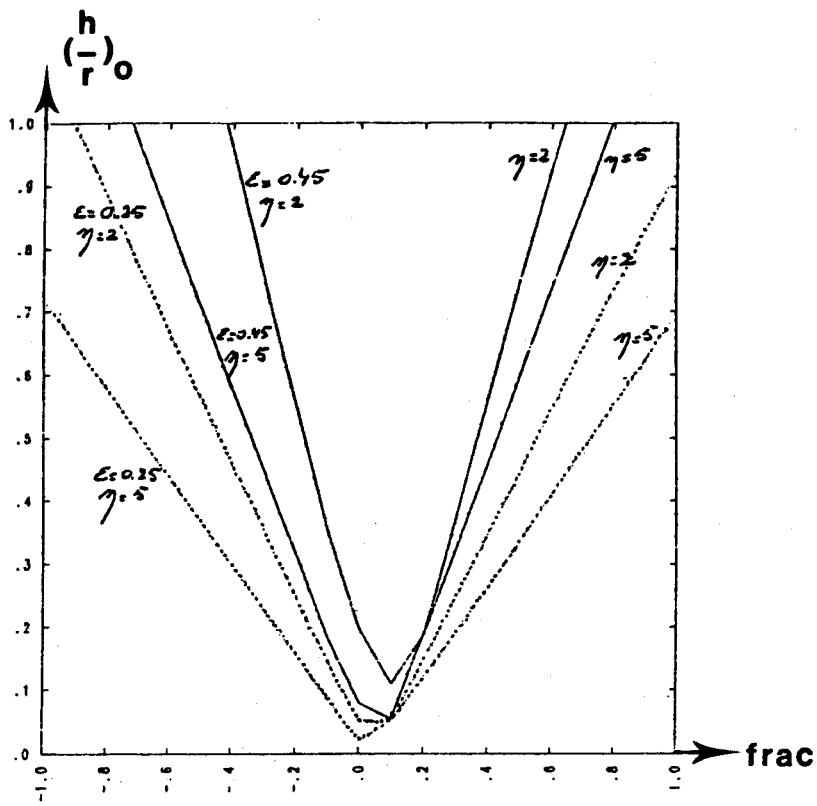


Figure 18 Value of $(h/r)_0$ as a function of $frac$ for a single force.

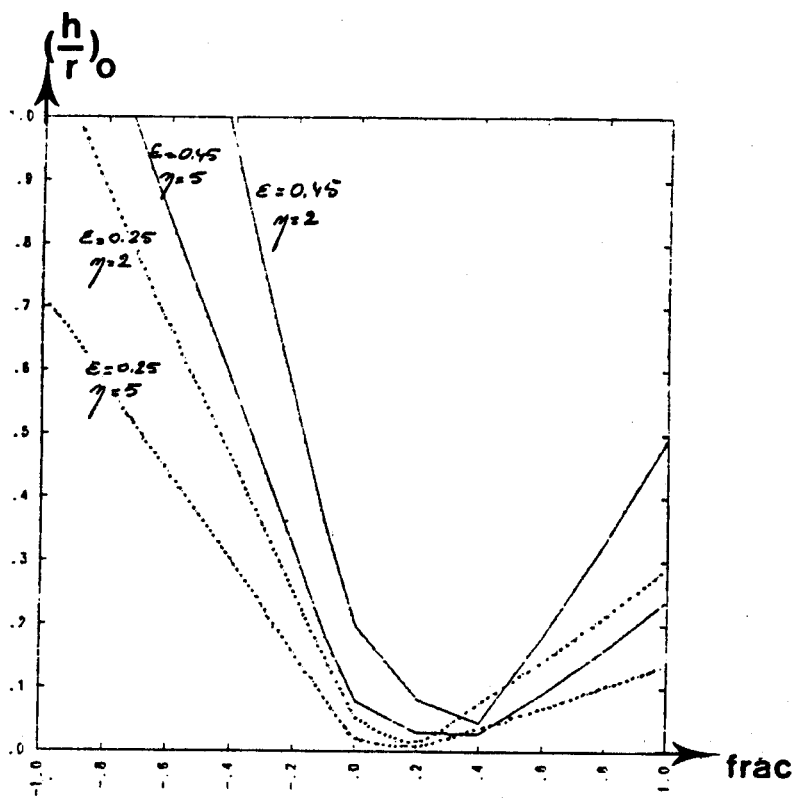


Figure 19 Value of $(h/r)_0$ as a function of $frac$ for a distributed force.

of the nose of the coils only and $M/\frac{E_0 r^2}{2} \simeq \frac{1}{3}$ so that $(\frac{h}{r})_0 \simeq \epsilon$. In order for the coil not to be dominated by bending stresses it is necessary that $\frac{h}{r} > \epsilon$, which is not fulfilled in most present cases.

It is interesting to note that our model confirms the fact pointed out by Bobrov and Schultz [7] that circular coils, when taking into account the structure that supports out of plane stresses, are not necessarily moment dominated. In their model no additional reaction force was included. This model goes further, in that it shows how circular coils may become moment dominated if most of the reaction force is taken up by an external reaction (single force or distributed force). It also shows that there is an optimum in the distribution of the centering force between the out of plane structure and a central bucking cylinder. This optimum, however, contrary to present practice, lies for circular coils in the direction of having a large fraction of the force being taken up by the out of plane structure. As of now the purpose of the out of plane structure is mainly to take up torques and overturning moments resulting from the interaction of poloidal coils with the current in the toroidal coils. Its usefulness in taking up the net centering force should be recognized.

The fact that the addition of a small central reaction reduces the bending moment in the coils can be understood qualitatively in the following way. Assume an infinitely rigid circular coil subjected to the magnetic forces without central support. The net inward force will be taken up through the elastic foundation by a rigid shift of the coil. It is easy to show that the net load on the coil then has a $\cos^2 \theta$ dependence meaning that the coil tries to deform into an oblong shape. The moment at $\theta = \pi$ is such that it tries to reduce the radius of curvature there. Adding a small outward central reaction has the opposite effect, thus decreasing the total moment. Recall further that in our analysis we have assumed k , the reaction due to the out of plane structure, to be constant. A continuous rotationally symmetric toroidal shell would have a larger k (stiffer) near $\theta = \pi$ than near $\theta = 0$. How much larger depends on the aspect ratio. The deformation of the toroidal shell is toward a D shape. The moment at $\theta = \pi$ is then such that it tries to increase the radius of curvature. This is already past the optimum as the addition of a central reaction would increase this moment.

CONCLUSION

A simple method has been devised for the analysis of circular beams on elastic foundations subjected to an arbitrary distribution of in plane loads. By expanding the load distribution in Fourier components, the method can be applied to any particular case. Using this method to analyze the magnetic load on toroidal coils we have shown the importance of the out of plane structure and of the distribution of the net inward force between this structure and a central cylinder. A more solid out of plane structure than normally used would be beneficial, as well as a better distribution of this structure around the coils (wedging of the nose of the coils only is not a good practice). The out of plane structure could take a larger fraction of the inward force thereby reducing the moments in the coil. The best approach to optimize toroidal coils may thus be to choose the coil shape such that within other constraints the circumferential length is minimal; the design of the center support, the out of plane structure and the distribution of the force between both are then optimized to minimize the moments.

REFERENCES

- [1] File, J., Mills, R. G., Sheffield, G. V., *Large Superconducting Magnet Designs for Fusion Reactors*, Matt-848, June 1971, Princeton Plasma Physics Laboratory.
- [2] Weissenburger, D. W., Christensen, U. R., Bialek, J., *Pure Tension Shape of a Thick Torus*, PPPL-1353, July 1977, Princeton Plasma Physics Laboratory.
- [3] File, J., Sheffield, G. V., *A Large Superconducting Magnet for Fusion Research*, in Proceedings of the Fourth International Conference on Magnet Technology (Brookhaven 1972) Ed. Y. Winterbottom Conf-720908 (1972), 240.
- [4] Moses, R. W., Young, W. C., *Analytic Expressions for Magnetic Forces on Sected Toroidal Coils* in Engineering Problems of Fusion Research (Proc. 6th Symp. San Diego 1975) IEEE, New York (1975).

- [5] Ojalvo, I. U., Zatz, I. J., *Structural Support Design Method for Minimizing In Plane Toroidal Field Coil System*, in Engineering Problems of Fusion Research (Proc. 8th Symp. San Francisco 1979) IEEE, New York (1979), Vol. 3, 1463.
- [6] Gray, W. H., Stoddart, W. C. T., Akin, J. E., *A Derivation of a Bending Free Toroidal Shell for Tokamak Fusion Reactors*, Journal of Applied Mechanics, 46 (1979) 120.
- [7] Bobrov, E. S. and Schultz, J. H., *A Model of Toroidal Magnet Systems as Orthotropic Shells of Finite Thickness* in Engineering Problems of Fusion Research (Proc. 9th Symp. Chicago 1981), IEEE, New York (1981), Vol. 1, 115.
- [8] Blumenau, L., Citrolo, J., Bialek, J., *TFTR TF Coil Support Restraint Structure*, in Engineering Problems of Fusion Research (Proc. 8th Symp. San Francisco 1979) IEEE, New York (1979), Vol. 1, 113.
- [9] Montgomery, D. B., *Magnet Development*, Proceedings of the IEEE (1981) 977.
- [10] Hetenyi, M., *Beams on Elastic Foundation*, The University of Michigan Press, Ann Harbor, (1946).

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