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M.I.T. Plasma Fusion Center

ABSTRACT

A formalism is developed for determining the possibility of operation of a gyrotron at both the fundamental, $\omega = \omega_c$, in a TE_{mpq} mode and at the second harmonic, $\omega' = 2\omega_c$, in a $TE_{m'p'q'}$ mode. This formalism is then applied to two physical problems. First, the conditions are derived for conversion of a gyrotron from operation at $\omega = \omega_c$ to operation at $\omega' = 2\omega_c$. Such a conversion is shown to be most practical when the mode at $\omega = \omega_c$ is a higher order mode (m or p large) and/or the beam position is at a higher radial maximum. Several specific examples are considered. Second, the conditions are derived for oscillation of a parasitic mode at $\omega = \omega_c$ in a gyrotron designed for operation at $\omega' = 2\omega_c$. Again, this problem is more severe for higher order modes and beams at larger radii. For a whispering gallery mode $TE_{m'11}$ oscillating at $\omega' = 2\omega_c$, if m' is odd, there is a resonant parasitic whispering gallery mode TE_{m11} at $\omega = \omega_c$, with $m = 0.5\ (m'- 1)$. This would be expected to spoil the $2\omega_c$ operation of all $TE_{m'11}$ modes, with m' odd.

1. INTRODUCTION

In this paper, we consider the general problem of a gyrotron electron gun and resonator system which can be operated at both the fundamental $\omega = \omega_c$, and second harmonic, $\omega' = 2\omega_c$. The present formalism may be applied to several problems of importance in gyrotron research. The first area of application is in the problem of mode competition between modes operating at different harmonics. This problem can be particularly important if a gyrotron designed to operate at $\omega' = 2\omega_c$ can also simultaneously support a parasitic mode operating at $\omega = \omega_c$. Such a parasitic mode might reduce gyrotron efficiency or even prevent $2\omega_c$ operation. A second area of application is in determining whether a gyrotron electron gun, constructed for an experiment at $\omega = \omega_c$, might also be applied to operation at $\omega' = 2\omega_c$ using a different resonator. Such a possibility would allow the application of existing electron guns and magnets to experiments at $2\omega_c$. This would further gyrotron research in general and might allow the achievment of higher frequencies in less time and at reduced cost.

In this paper, a formalism is developed for describing the general features of gyrotron electron beams and resonators. The present approach is similar to that developed in a previous study of gyrotron design [1]. The adiabatic theory of the electron gun [2] is combined with a generalized resonator theory for TE_{mpq} modes. A number of approximations are made to make the analysis reasonable. In particular, the cavity cutoff condition is employed ($k_\perp >> k_{||}$) and, for simplicity, terms of order $k_{||} v_{||} << \omega$ are ignored. Also, higher axial modes, $q > 1$, and cavity tapering effects are ignored. Inclusion of these effects in a more exact treatment might alter the numerical results somewhat, but would not alter the conclusions presented here.

In this paper, the basic theory of gyrotron operation at at ω_c and $2\omega_c$ is first developed. Then the problem of conversion from gyrotron operation at $\omega = \omega_c$ to operation at $\omega' = 2\omega_c$ (or vice versa) is solved. The equations for parasitic modes at ω_c in $2\omega_c$ operation of a gyrotron are developed as a special case of the formalism. All of the general results are then illustrated by various examples, including analysis of specific experiments as well as more general classes of experiments. Solutions are found for parasitic modes in whispering gallery mode gyrotrons. The results and conclusions are summarized in the final section.

2. BASIC THEORY OF ω_c, $2\omega_c$ OPERATION

The possibility of conversion of a gyrotron from ω_c to $2\omega_c$ operation can be analyzed using the adiabatic theory of gyrotron electron guns [2] combined with simple resonator theory. This method of analysis has been reviewed in greater detail elsewhere [1].

We assume that the gyrotron was initially designed for operation at frequency ω in fundamental operation, $\omega \simeq \omega_c$. We then analyze the possibility of device operation at a frequency $\omega' \approx 2\omega_c$. The operation at $2\omega_c$ may be achieved either at the same value of magnetic field, B_o, as in ω_c operation, or at a slightly detuned value of magnetic field, B_o'. This generalized treatment allows us to solve several different problems in gyrotron research. We may analyze the possibility of conversion of a gyrotron from operation at ω_c to operation at $2\omega_c$ (or vice versa), in which case B_o need not be equal to B_o'. Or we may analyze competition between modes oscillating at ω_c and $2\omega_c$ in the same resonator in which case $B_o = B_o'$.

The device at $\omega \approx \omega_c$ is assumed to operate in the TE_{mpq} mode at a magnetic field of B_o. The electron beam, at a voltage U and current I, has a transverse to parallel velocity ratio of g. The ratio of magnetic field at the cavity (B_o) and cathode (B) is α. The beam is situated at the s^{th} radial maximum of the TE_{mpq} mode, at a beam radius R_e in a cavity of radius R_o. The simple equations that describe this gyrotron configuration are:

$$\omega \approx \omega_c = \frac{eB_o}{\gamma mc} \tag{1}$$

$$\gamma = 1 + \frac{U}{511\text{kev}} = (1 - v^2/c^2)^{-1/2} \tag{2}$$

$$g = v_{\perp o} / v_{\parallel o} \tag{3}$$

$$\alpha = B_o/B_k \tag{4}$$

$$\frac{\nu_{mp} R_e}{R_o} = \nu_{m\pm1,s} \tag{5}$$

$$\frac{\omega}{c} = k \simeq k_\perp = \frac{\nu_{mp}}{R_o} \tag{6}$$

These parameters are listed in Table 1. The Doppler shift, $k_\parallel v_\parallel << \omega$, has been omitted from Eq. (1) for simplicity. In Eq.(5), the m±1 signifies a rotating mode when $m \neq 0$. Only one value (the plus or minus sign) will apply, depending on the choice of R_e.

For device operation at $2\omega_c$, we will assume that the operating magnetic field is B_o', which can be different from B_o. For the special case $B_o' = B_o$, we have a treatment of the mode competition problem. In general, we will require that B_o' be close in value to B_o. An electron gun designed for an $\omega \approx \omega_c$ device operating at B_o will only operate properly under the condition $B_o' \simeq B_o$. The possible variation of B_o'/B_o may be quantified as follows. We assume that the magnetic field in the electron gun region is held constant and that the adiabatic theory may be used for the transition region between the gun and the resonator. Then the transverse velocity, $v_{\perp o}'$, at the resonator for a field B_o' is just:

$$\frac{v_{\perp o}'^2}{B_o'} = \frac{v_{\perp o}^2}{B_o} \tag{7}$$

The upper field limit, B_o' (max), is then determined by the condition

$$v^2 > {v'_{\perp o}}^2 \qquad (8)$$

where v^2 is given by Eq. (2). However, in the presence of a velocity spread $\Delta v_{\perp o}$, v^2 must exceed the maximum value of $v_{\perp o}^2$ not the average value. Then,

$$B_o' \text{ (max)} < B_o \frac{v^2}{v_{\perp o}^2} \frac{1}{1 + (2\Delta v_{\perp o}/v_{\perp o})} \qquad (9)$$

In Eq. (9), the maximum value of $\Delta v_{\perp o}/v_{\perp o}$ should be used, not the standard deviation value. Eq. (9) is merely the condition for preventing any part of the electron beam from being reflected by the mirror magnetic field in the resonator region.

The minimum magnetic field, B_o' (min), is only weakly limited by physical conditions, such as the increasing diameter of the electron beam with decreasing B_o'. Perhaps the major limit on decreasing B_o' is the overall efficiency, η. We have

$$\eta = (1 - \frac{Q}{Q_{OM}}) \, \eta_{el} \qquad (10)$$

$$\eta_{el} = \frac{{v'_{\perp o}}^2}{v^2} \eta_\perp = \frac{B_o'}{B_o} \frac{v_{\perp o}^2}{v^2} \eta_\perp \qquad (11)$$

Thus, as B_o' decreases, the transverse velocity $v_{\perp o}'$ decreases and the efficiency decreases, as shown in Eq. (11). The minimum acceptable value of B_o' is somewhat arbitrary and, for the present analysis, will be taken to be 0.85 B_o. The practical range of B_o' values will become clearer in a following section, where examples will be treated.

Since we are using the adiabatic theory for the electron gun and have fixed the parameters at the cathode, then the electron beam parameters at the resonator are determined by B_o'. The transverse velocity, $v_\perp{}'_o$, is given by Eq. (7), while the parallel velocity is given by

$$v_{\parallel}{}'_o = (v^2 - v'_{\perp o}{}^2)^{1/2} \tag{12}$$

where v is given by Eq. (2) since U is fixed. The ratio $g' = v_\perp{}'_o / v_\parallel{}'_o$ is thus determined. The radius of the electron beam at the resonator will be given by:

$$R_e' = R_e \, (B_o/B_o')^{1/2} \tag{13}$$

The resonator equations for $2\omega_c$ operation are given by:

$$\omega' \approx 2\omega_c' = 2 \, \frac{eB_o'}{\gamma mc} \tag{14}$$

$$\frac{\omega'}{c} = k' \simeq k_\perp' = \frac{\nu_{m'p'}}{R_o'} \tag{15}$$

$$\frac{\nu_{m'p'} \, R_e'}{R_o'} = \nu_{m'\pm 2,s'} \tag{16}$$

Eq. (14) is determined by the cyclotron resonance condition, Eq. (15) by the cavity resonance condition for a cavity near cutoff, and Eq. (16) by the necessity for the beam radius to coincide with a maximum of the RF field derivative in $2\omega_c$ operation.

3. OPERATION AT $\omega = 2\omega_c$

A. Solutions with Beam Location Optimized

The analysis of the previous section may be applied to the problem of conversion of a gyrotron from operation at $\omega = \omega_c$ to operation at $\omega' = 2\omega_c$. We assume that a gyrotron was designed for operation at $\omega = \omega_c$ in the TE_{mpq} mode. The problem to be solved is then to determine the appropriate conditions for operation of the gyrotron at $\omega' = 2\omega_c$. For this analysis we allow $2\omega_c$ operation to be achieved at a somewhat different magnetic field, B_o'. Also, we assume that, in general, a different cavity shape will be needed, with the new cavity radius denoted R_o'. Eqs. (13) through (16) represent the basic conditions for $2\omega_c$ operation. These equations may be combined to yield:

$$\nu_{m'\pm2,s'} = \frac{\nu_{m'p'}}{R_o'} R_e' = \frac{\omega'}{c} R_e'$$

$$= 2\,\frac{\omega}{c}\,\frac{B_o'}{B_o}\,R_e\left(\frac{B_o}{B_o'}\right)^{1/2}$$

$$\nu_{m'\pm2,s'} = 2\nu_{m\pm2,s}\,(B_o'/B_o)^{1/2} \tag{17}$$

Eq. (17) is the major result of this analysis. Given operation at $\omega \approx \omega_c$ in the TE_{mpq} mode at the s^{th} radial maximum and a field B_o, Eq. (17) indicates that optimized $2\omega_c$ operation can be achieved at field values B_o' which yield a solution of Eq. (17). Having determined $\nu_{m'\pm2,s'}$, the operating mode $\nu_{m'p'}$ is determined by Eqs. (13)

through (16). In general, the range of B_o' values is restricted by the conditions previously described. Within this range of B_o' values, it is relatively easy to find the solutions, if any, of Eq. (17). This is illustrated later by means of examples. The solution of Eq. (17) determines both $\nu_{m'\pm 2,s'}$ and B_o'. From B_o', we immediately obtain ω' from Eq. (14) and R_e' from Eq. (13). Two possible values of m' are generated by $\nu_{m'\pm 2,s'}$. For each value of m', a series of p' values is possible. The minimum p' value is determined by $\nu_{m'\pm 2,s'} > \nu_{m'p'}$. Finally, for each m'p' value, R_o' is given by Eq. (16). This yields a complete solution.

B. Solutions With Beam Location Not Optimized

In the previous section, we have considered solutions for $2\omega_c$ operation in which the beam location was optimized for the $2\omega_c$ mode. In some cases, it would be allowable for the beam to be somewhat misplaced radially. This might be acceptable if a very large beam power, hundreds of kilowatts to megawatts, is available so that optimum gain is not necessary for reasonable efficiency. If Eq. (17) is not satisfied, the accuracy of beam positioning may be evaluated as follows. Since Eq. (17) is no longer satisfied, we use

$$\frac{\nu_{m'p'} R_e'}{R_o'} = 2\ \nu_{m \pm 1,s} \left(\frac{B_o'}{B_o}\right)^{1/2}$$

Then the reduction in the strength of the maser interaction is given by the ratio:

$$\frac{J^2_{m'\pm 2}\,(\nu_{m'p'},R'_e/R'_o)}{J^2_{m'\pm 2}\,(\nu_{m'\pm 2,s'})}$$

This ratio is unity if Eq. (17) is satisfied.

4. Parasitic Modes at $\omega' = 2\omega_c$

If a gyrotron is designed for operation at $\omega = \omega_c$, it may be possible to simultaneously excite a parasitic mode in the same resonator at $\omega' = 2\omega_c$. Alternatively, a resonator designed for $2\omega_c$ operation may have a parasitic mode at $\omega = \omega_c$. The conditions for either of these situations may be analyzed as a special case of the previous results. The case in which $2\omega_c$ operation is parasitic may not be of wide interest, because the gain at $2\omega_c$ is relatively low and the strength of the parasitic mode should be small. The case in which ω_c operation is parasitic may be of great importance, however, in practice.

We assume that modes at $\omega = \omega_c$ and $\omega' = 2\omega_c$ are simultaneously excited at the same magnetic field value and in the same cavity. Then we have:

$$B_o' = B_o$$

$$R_e' = R_e$$

$$\omega' = 2\omega$$

$$R_o' = R_o$$

From these equations, using Eqs. (15) and (16), we have

$$\nu_{m'p'} = 2\nu_{mp} \tag{18}$$

$$\nu_{m'\pm 2,s'} = 2\nu_{m\pm 1,s} \tag{19}$$

These two equations must both be satisfied in order for modes at ω_c and $2\omega_c$ to both be excited under optimized conditions. In general simultaneous solutions to Eqs. (18) and (19) are rare, especially if

ν_{mp} is not large. This is illustrated in later examples. However, for the whispering gallery modes, Eqs. (18) and (19) are easily satisfied, as illustrated in the next section.

The equations (18) and (19) do not require an exact solution for mode competition to occur. The gain bandwidth of each mode is of order $\beta_{||} \lambda / L$, which amounts to 1-5% in general. Thus, Eq. (18) need only be satisfied to this accuracy. Eq. (19), which determines how closely the beam position is to its optimum, also need not be satisfied exactly. A deviation of order 10% from equality may be acceptable in Eq. (19).

The previous results are easily extended to operation at higher harmonics.

5. Parasitic Modes in Whispering Gallery Mode Gyrotrons Operating at Harmonics

The whispering gallery modes are defined as the modes of the form TE_{m11} (ordinarily with m large). For these modes, we find that the Bessel function roots, ν_{m1}, are given to good accuracy by:

$$\nu_{m1} \approx \frac{m+1}{3}\pi$$

This expression is accurate to within 3% for $m \geq 2$ and to within 1% for $m \geq 20$. An alternate expression

$$\nu_{m1} \cong 1.02\ \frac{m+1}{3}\pi \qquad (20)$$

is accurate to within 0.4% for $15 \geq m \geq 4$.

For a whispering gallery mode gyrotron, $\nu_{mp} = \nu_{m1}$. Also, the condition that $\nu_{m1} > \nu_{m\pm1,s}$ implies for these modes that m-1 must be selected and that s = 1. Then, for each mode ν_{m1} operating at $\omega = \omega_c$ there is a whispering gallery mode that is resonant for $2\omega_c$ operation with

$$m' = 2m + 1$$

$$p' = 1$$

$$s' = 1$$

Using these values of m', p' and s' and Eq. (20), it is easy to show that Eqs. (18) and (19) are satisfied, namely

$$\frac{\nu_{m'1}}{\nu_{m1}} = \frac{m'+1}{m+1} = 2$$

and $$\frac{\nu_{m'\pm 2,s'}}{\nu_{m\pm 1,s}} = \frac{m'-2+1}{m-1+1} = 2$$

These results indicate that parasitic modes operating at a different harmonic are a characteristic of whispering gallery mode gyrotrons. In the case of an experiment designed for $\omega = \omega_c$ operation, the $2\omega_c$ parasitic mode is likely to be weak because of its lower gain. Also, the cavity Q and output coupling might be made unfavorable for this mode. In the case of an $\omega = 2\omega_c$ gyrotron, the parasitic mode at $\omega = \omega_c$ could be quite important. In that case, the parasitic mode TE_{m11} at $\omega = \omega_c$ for a $TE_{m'11}$ operating at $\omega' = 2\omega_c$ is given by:

$$m = \frac{1}{2} (m' - 1)$$

This equation only has solutions for odd values of m'. Therefore, $TE_{m'11}$ mode gyrotrons with m' an odd integer will suffer from a parasitic mode at $\omega = \omega_c$ and may not work well in practice. The mode with m' even, however, should be practical. These results may be extended to operation of a $TE_{m'11}$ mode at $\omega' = n\omega_c$. In such a case, there is a competing mode TE_{m11} at $\omega = \omega_c$ if the value of m given by the following expression is an integer:

$$m = \frac{m' + 1}{n} - 1$$

6. Parasitic Modes in Azimuthally Symmetric Mode Gyrotrons

The specific results for whispering gallery mode gyrotrons are the consequence of the relationship Eq. (20) for those modes. It is easy to show that there is not a similar result for azimuthally symmetric modes. For those modes, with m = 0, there is a different expression for ν_{mp}, namely:

$$\nu_{mp} = \nu_{op} \simeq (p + \frac{1}{4})\,\pi \qquad (21)$$

which is accurate to within 1% for $p \geq 2$. For a mode with ν_{op} at $\omega = \omega_c$ and ν_{op}' at $\omega' = 2\omega_c$, the condition $\nu_{m'p'} \approx 2\nu_{mp}$ would require $4p' = 8p + 1$. This latter equation has no solution for integer values of p and p'. Hence, for azimuthally symmetric modes, there is no possibility of mode competition from other symmetric modes operating at a different harmonic. However, competition from azimuthally asymmetric modes is still possible.

7. Example: A 140 GHz, $\omega = \omega_c$ Gyrotron Operating Between 240 - 320 GHz at $\omega' = 2\omega_c$

As an example, we consider the possible conversion of the M.I.T. 140 GHz gyrotron from ω_c to $2\omega_c$ operation. The operating parameters of this gyrotron for $\omega \simeq \omega_c$ are listed in Table 2.

The range of useful values of magnetic field, B_o', for $2\omega_c$ operation may be estimated as follows using Eq. (9). The value of $v^2/v_{\perp o}^2$ for the electron gun is 1.44. The expected value of $\Delta v_{\perp o} / v_{\perp o}$ is about 0.05. However, a conservative value of about 0.12 must be used to assure that there will be nearly zero electron reflection back into the electron gun. Then B_o' (max) is 6.5T or B_o' (max)/B_o is 1.16. Similarly, B_o' (min) should not be much less than about 4.8T in order to maintain high efficiency. The expected efficiency reduction for that case is at least 15%, exclusive of any reduction in $\eta_\perp$ that results from a reduced $v_\perp$. Hence, we assume an operating range:

$$6.5\text{T} \geq B_o' \geq 4.8\text{T}$$

or

$$1.16 \geq B_o' / B_o \geq 0.85$$

For the TE_{02} mode with the electron beam at the second radial maximum, we have:

$$\nu_{m\pm 1,s} = \nu_{12} = 5.33$$

Then Eq. (17) yields:

$$\nu_{m'\pm 2,s'} = 10.66\ (B_o'/B_o)^{1/2}$$

Using the restriction on the range of B_o'/B_o, we have:

$$11.44 \geq \nu_{m'\pm 2,s'} \geq 9.83$$

Within this range, there are five solutions for $\nu_{m'\pm 2,s'}$, as listed in Table 3. For each solution, the magnetic field, B_o', the emission frequency, $\omega'/2\pi$, and the beam radius, R_e', are uniquely determined. For each solution, there are two possible values of m' and an infinite number of solutions for p', subject only to the condition $\nu_{m'p'} > \nu_{m'\pm 2,s'}$. The minimum p' value for the m' - 2 solution is also shown in Table 3.

Of the solutions shown in Table 3, there is only one solution with azimuthal symmetry, m' = 0. This solution is discussed more fully in the next section.

8. A GYROTRON OPERATING AT 245 GHz, $\omega' = 2\omega_c$

A. Experimental Parameters

In the previous section, solutions for $2\omega_c$ gyrotron operation were obtained for the M.I.T. electron gun designed for 140 GHz operation at $\omega = \omega_c$. Such $2\omega_c$ operation may be of interest in order to test the possibility of achieving high frequency, high efficiency output in second harmonic operation. As was determined in the previous section, one solution for $2\omega_c$ operation is a $TE_{op'q'}$ mode at a frequency near 245 GHz. The parameters for operation in that mode are analyzed in greater detail in the present section.

The mode to be analyzed in this section is the $m' = 0$ solution listed in Table 3. This is the only azimuthally symmetric mode ($TE_{m'p'q'}$, $m'= 0$) that was obtained in the analysis of the previous section. The oscillation frequency, $\omega'/2\pi = 245$ GHz, may be of practical interest because it is within an atmospheric window. For the $m' = 0$ solution, the minimum value of p', which is determined by the condition $R'_e < R'_o$, is found to be $p'=3$. However, the beam will be too close to the wall for that case since $R'_e/R'_o = 0.98$. A more practical case is $m' = 0$ and $p' = 5$, a TE_{051} mode, with $R'_e/R'_o = 0.61$. This case is analyzed more fully in Table 2.

The TE_{051} experiment analyzed in Table 2 appears to have practical significance. The potential efficiency of the 245 GHz gyrotron may be analyzed using the theory of Nusinovich and Erm [3]. The parameters for the calculation are an ohmic Q of 4.2×10^4 and a loaded Q of 8×10^3. All other needed parameters are listed in Table 2. Then the overall efficiency, resonator divided by electron beam power in, is calculated to be 0.24. This efficiency is the final overall efficiency and includes all corrections for finite parallel velocity and ohmic losses in the resonator. This value of efficiency is about double the efficiency previously estimated for $2\omega_c$ operation near 245 GHz using a beam with a voltage of 30kV and a current of 1A [1,4,5]. The reason for the improved efficiency in the present example is the fact that higher voltage and current (in effect, higher beam power) are employed in this gyrotron. In general, the gain of a gyrotron in $2\omega_c$ operation is an increasing function of the product of beam current times beam voltage [3]. This explains the estimated enhanced efficiency of the 65kV,5A gyrotron of Table 2 relative to a 30kV, 1A gyrotron. A $2\omega_c$ experiment at 245 GHz in a TE_{051} mode using the M.I.T. electron gun could thus provide an important test of the predicted increase in efficiency of $2\omega_c$ gyrotrons with electron beam current and voltage.

B. Parasitic Modes

For a gyrotron operating at $\omega = 2\omega_c$, we may also use the present theory to determine which parasitic modes at $\omega = \omega_c$ may be excited. The results for the TE_{051} $2\omega_c$ gyrotron of Table 2 are illustrated in Table 4. There are three possible $\omega = \omega_c$ modes whose frequencies are within a few percent of resonance at the field B'_o (= 4.93T) of this example. The deviation in frequency from resonance is given by the column $2\nu_{mp}/\nu_{m'p'}$. Of The three modes listed, only one mode, the TE_{321} mode, has good overlap with the electron beam position as indicated by the Bessel function ratio in the next to the last column of Table 4. It appears unlikely that the TE_{131}

or TE_{711} modes could be excited. Since the TE_{321} mode is 3% off resonance, it in fact is also unlikely to be excited. However, if the magnetic field is detuned downward by about 3% the TE_{321} mode could have some effect.

9. COMPETING MODES at $\omega' = 2\omega_c$.

Table 3 also indicates that there are solutions for $2\omega_c$ operation at magnetic fields B_o' within 5% of B_o. Such solutions could possibly result in parasitic oscillations at $\omega' = 2\omega_c$, although this is probably unlikely. Parasitic oscillations at $\omega' = 2\omega_c$ in an experiment designed for $\omega = \omega_c$ operation should be weak because the gain at $2\omega_c$ is much smaller that at ω_c. The inverse situation, however, of parasitic oscillations at ω_c in a $2\omega_c$ experiment can be significant and were previously described in Section 8.

For completeness, the parasitic modes at $\omega' = 2\omega_c$ for a 140 GHz gyrotron operating on a TE_{021}, TE_{031} or TE_{511} mode are shown in Table 5. For these experiments, there are in fact a number of parasitic modes operating at $\omega' = 2\omega_c$. The modes which are closest in frequency to the operating mode, as indicated by the ratio $\nu_{m'p'}/2\nu_{mp}$ are shown in the table. In general, these modes are somewhat suppressed because the beam location, which is set for the $\omega = \omega_c$ mode, is not optimized for the $2\omega_c$ mode. The degree of spoiling of the $2\omega_c$ mode by beam location is determined by the ratio of Bessel functions shown in the next to the last column. In only one case in Table 5, where the ratio is zero, is there a complete spoiling of the $2\omega_c$ mode by beam location.

For the TE_{511} experiment, there is a parasitic $2\omega_c$ mode, $TE_{11,1,1}$, which is optimized both in frequency and beam location. This result is an example of the general results for whispering gallery modes previously described in Section 5.

10. EXAMPLE: $2\omega_c$ OPERATION OF A TE_{0p1} , $\omega = \omega_c$ GYROTRON

The case to be considered here is conversion of a TE_{op1} gyrotron from $\omega = \omega_c$ to $\omega = 2\omega_c$ operation. We assume that the electron beam is at the first radial maximum and that p = 1 or 2. Then this case applies to various high power gyrotrons built in the U.S. for plasma heating applications. The purpose of converting the gyrotrons for operation at $\omega = 2\omega_c$ would be to obtain high frequency operation at reduced cost and time, namely by utilizing the same electron gun and magnet system as in $\omega = \omega_c$ operation.

We first seek solutions in which the beam location is optimized, as outlined in Section 3. The condition to be satisfied is Eq. (17). For TE_{op1} modes with the electron beam at the first radial maximum, $\nu_{m\pm1,s}$ is $\nu_{11} = 1.841$. The solution with the nearest B_o' value is a TE_{221} mode operating at $B_o'/B_o = 1.083$. There are no other solutions with values of B_o'/B_o near one. The nearest TE_{op1} mode solution is a TE_{021} mode with $B_o'/B_o = 0.69$. This is so low a value of B_o'/B_o that the projected experiment would be of little interest.

Solutions with unoptimized beam location may be obtained for a variety of modes $TE_{m'p'q'}$ and magnetic fields B_o'. Consider operation at a value of B_o'/B_o equal to 0.5 (110/60) or 0.917, appropriate for conversion of a 60 GHz gyrotron to $2\omega_c$ operation at 110 GHz (or a 28 GHz gyrotron to 51 GHz operation, etc.). In this example, specification of B_o'/B_o yields the quantities ω', R_e' and $\nu_{m'p'}/R_o'$. The mode numbers of the operating mode, (m'p'q'), are not completely determined in this case because the beam location is not optimized. Only the quantity $\nu_{m'p'}/R_o'$ is specified. (The mode numbers are more fully specified in the case where the beam location is optimized.) However, as discussed in Section 3b, we can determine

$$\frac{\nu_{m'p'}\, R'_e}{R'_o} = 2\nu_{m \pm 1,s} \left(\frac{B'_o}{B_o}\right)^{1/2} = 3.526$$

(for the example of Table 6 with $B'_o/B_o = 0.917$). Since we must have $R'_e < R'_o$, we have $\nu_{m'p'} > 3.526$.

We may now seek solutions of the form $m' = 0$. All values of $\nu_{op}{}'$, $p' \geq 1$, exceed 3.526, but we exclude the ν_{o1} solution because the value of R'_e/R'_o (= 0.92) is close to unity and the beam is too close to the wall. Then results for $TE_{op'1}$ modes, $p' = 2,3$ or 4 are given in Table 6. The present information is sufficient to compare the strength of interaction of the listed modes for the actual, unoptimized beam location relative to that for an optimized beam location. This relative strength is determined by the ratio of Bessel functions listed in the last column. The value of 0.87 obtained is not greatly diminished from unity. Consequently, the fact that the beam is not in an optimized location would not be particularly disadvantageous for the case shown.

A complete analysis of this problem would require a number of additional considerations. For the $2\omega_c$ experiments, we could apply the present theory to determine if there are parasitic modes at $\omega = \omega_c$. Also, the efficiency and power dissipation in the walls for the experiments at $\omega' = 2\omega_c$ should also be determined.

11. CONCLUSIONS

In this paper, we have derived a formalism for determining whether a gyrotron oscillating at $\omega = \omega_c$ can be modified (including modification of the resonator) for oscillation at $\omega' = 2\omega_c$. We find that $2\omega_c$ operation must be restricted to a range of magnetic fields B_o' which is close to the field B_o at which ω_c operation occurs. Within the allowed range of B_o' values, a single equation (Eq. 17) yields solutions for optimized operation at $2\omega_c$. Eq. (17) indicates that, if the ω_c operation is in a higher order mode (TE_{mpq} mode, m large) and if the electron beam is at a higher radial maximum (s > 1), there can be many solutions for $2\omega_c$ operation. An example of this is the M.I.T. 140 GHz gyrotron which can operate at $2\omega_c$ on 5 modes in the range 245-317 GHz. On the other hand, if m = 0 and the electron beam is at the first radial maximum (s = 1), there is only one solution for optimized $2\omega_c$ operation, namely a TE_{221} mode at $\omega' = 2.17\ \omega$. However, even for this case (m = 0, s = 1), if the beam is not required to be at the optimum radial location, oscillation can be achieved in a wide range of $\omega' \approx 2\omega_c$ values on TE_{op1} modes $p \geq 2$. The strength of the coupling between the RF field and the electron beam is only diminished by a small amount (10 to 30%) with respect to its value when the beam location is optimized.

The present formalism is also applied to evaluation of the possibility of parasitic oscillation at $\omega = \omega_c$ in a gyrotron designed to operate at $\omega' = 2\omega_c$ (or vice versa). This possibility may be evaluated as a special case of the formalism developed for determining whether a gyrotron can be modified for harmonic operation. The conditions for such parasitic modes are reduced to two simple equations, Eqs. (18) and (19), determining the frequency and beam location of the parasitic mode. In general, for a mode at $\omega' = 2\omega_c$, the higher the mode number (m'p') and the higher the radial maximum (s'), the more likely is the possibility of a parasitic mode at $\omega = \omega_c$. In an example of a TE_{051} mode with s' = 3, there is only one strong parasitic $\omega = \omega_c$ mode, a TE_{321} mode, which is detuned by about 2.7% in frequency. Such a mode might, however, prove troublesome in an actual experiment.

For the whispering gallery modes, TE_{m11}, it is found that parasitic modes are a potential serious problem in harmonic operation. In general, for $TE_{m'11}$ mode operating $\omega' = 2\omega_c$, there is a competing mode TE_{m11} at $\omega = \omega_c$, with $m = 0.5(m' - 1)$, if m' is an odd integer.

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TABLE 1

EXPERIMENTAL PARAMETERS - NOMENCLATURE

	$\omega = \omega_c$	$\omega' = 2\omega_c$
Mode	TE_{mpq}	$TE_{m'p'q'}$
Frequency	$\omega/2\pi$	$\omega'/2\pi$
Cavity Radius	R_o	R'_o
Beam Radius	R_e	R_e'
Voltage	U	U
Current	I	I
Magnetic Field at Resonator	B_o	B_o'
Trans. Velocity	$v_{\perp o}$	$v'_{\perp o}$
Parallel Velocity	$v_{\parallel o}$	$v'_{\parallel o}$
Mirror Ratio	α	α'
Radial Maximum	s	s'

TABLE 2

EXAMPLE - 140 GHz GYROTRON AT $\omega = \omega_c$

	$\omega = \omega_c$	$\omega' = 2\,\omega_c$
Mode	TE_{021}	TE_{051}
Frequency	140 GHz	245 GHz
Cavity Radius	2.39 mm	3.22 mm
Beam Radius	1.82 mm	1.95 mm
Voltage	65 kV	65 kV
Current	5A	5A
B_o	5.64T	4.93T
Trans. Velocity	0.384c	0.359c
Parallel Velocity	0.256c	0.289c
Velocity	0.461c	0.461c
$v_{\perp o}/v_{\parallel o}$	1.5	1.24
Mirror Ratio	25	22
Radial Max	2	3

TABLE 3

SOLUTIONS FOR $2\omega_c$ OPERATION OF THE 140 GHz Gyrotron

$\nu_{m'\pm 2,s'}$	B_o' (T)	$\omega'/2\pi$ (GHz)	R_e' (mm)	m'	B_o'/B_o	p' (Minimum value)
ν_{23} = 9.97	4.93	245	1.95	4,0	0.87	3
ν_{03} = 10.17	5.13	255	1.91	2	0.91	4
ν_{52} = 10.52	5.49	273	1.84	7,3	0.97	2
ν_{91} = 10.71	5.69	283	1.81	11,7	1.01	1
ν_{33} = 11.35	6.39	317	1.71	5,1	1.13	3

TABLE 4

PARASITIC MODES AT $\omega = \omega_c$

FOR A GYROTRON OPERATING AT $\omega' = 2\omega_c$

TE_{051} Experiment

$\nu_{05} = 16.47$ $\nu_{23} = 9.97$ $R_e = 1.95$ mm $R_o = 3.22$ mm

$\omega = \omega_c$ Mode	ν_{mp}	$\frac{\nu_{mp} R_e}{R_o}$	$\frac{2\nu_{mp}}{\nu_{m'p'}}$	$\frac{\nu_{mp} R_e}{\nu_{m\pm1s} R_o}$	$\frac{J_{m\pm1}^2(\nu_{mp} R_e/R_o)}{J_{m\pm1}^2(\nu_{m\pm1s})}$	$\nu_{m\pm1s}$
TE_{321}	8.015	4.85	0.973	0.91	0.90	$\nu_{41} = 5.32$
TE_{131}	8.536	5.17	1.037	1.35	0.08	$\nu_{01} = 3.83$
TE_{711}	8.578	5.19	1.042	0.69	0.18	$\nu_{61} = 7.50$

TABLE 5

PARASITIC MODES AT $\omega' = 2\,\omega_c$

FOR A GYROTRON OPERATING AT $\omega = \omega_c$

1. TE_{021} Experiment

$\nu_{02} = 7.016$ $\nu_{12} = 5.33$ $R_e = 1.82$ mm $R_o = 2.40$ mm

$\omega = 2\omega_c$ MODE	$\nu_{m'p'}$	$\frac{\nu_{m'p'}R_e}{R_o}$	$\frac{\nu_{m'p'}}{2\nu_{mp}}$	$\frac{\nu_{m'p'}R_e}{\nu_{m'\pm2,s}R_o}$	$\frac{J^2_{m'\pm2}(\nu_{m'p'}R_e/R_o)}{J^2_{m\pm2}(\nu_{m'\pm2,s})}$	$\nu_{m'\pm2,s}$
$TE_{12,1,1}$	13.88	10.53	0.989	0.39	0.63	$\nu_{10,1} = 11.77$
TE_{531}	13.99	10.61	0.996	0.93	0.59	$\nu_{33} = 11.35$
TE_{821}	14.12	10.70	1.006	0.91	0.39	$\nu_{62} = 11.73$
TE_{341}	14.59	11.06	1.039	0.94	0.61	$\nu_{14} = 11.71$

2. TE_{031} Experiment

$\nu_{03} = 10.173$ $\nu_{12} = 5.33$ $R_e = 1.82$ mm $R_o = 3.47$ mm

$TE_{18,11}$	20.14	10.56	0.990	0.58	0.00	$\nu_{16,1} = 18.06$
$TE_{10,31}$	20.22	10.61	0.994	1.10	0.71	$\nu_{8,1} = 0.65$
TE_{551}	20.58	10.79	1.012	0.95	0.74	$\nu_{33} = 11.35$

TABLE 5 (cont.)

PARASITIC MODES AT $\omega' = 2\omega_c$

FOR A GYROTRON OPERATIN AT $\omega = \omega_c$

3. TE_{511} Experiment

$\nu_{51} = 6.416$ $\nu_{41} = 5.32$ $R_e = 1.82$ mm $R_o = 2.19$ mm

TE_{431}	12.68	10.54	0.988	0.95	0.76	$\nu_{23} = 9.97$
$TE_{11,11}$	12.83	10.66	1.000	1.00	1.00	$\nu_{91} = 10.71$
TE_{721}	12.93	10.75	1.008	0.98	0.92	$\nu_{52} = 10.52$
TE_{241}	13.17	10.94	1.026	0.93	0.52	$\nu_{03} = 10.17$

TABLE 6

$2\omega_c$ OPERATION OF A TE_{OP1} GYROTRON

$\nu_{m\pm1,s} = \nu_{11} = 1.841$

$2\omega_c$ MODE	$\nu_{m'\pm2,s'}$	B_o'/B_o	ω'/ω	$\nu_{m'p'}$	$\frac{\nu_{m'p'}R_e}{R_o'}$	$\frac{J^2_{m'\pm2,s'}(\nu_{m'p'}R_e'/R_o')}{J^2_{m'\pm2,s'}(\nu_{m'\pm2,s'})}$
OPTIMIZED BEAM LOCATION:						
TE_{221}	$\nu_{01}= 3.832$	1.083	2.17	6.706	3.832	1
TE_{021}	$\nu_{21}= 3.054$	0.688	1.38	7.016	3.054	1
UNOPTIMIZED BEAM LOCATION:						
TE_{021}	$\nu_{21}= 3.054$	0.917	1.83	7.016	3.526	0.87
TE_{031}	$\nu_{21}= 3.054$	0.917	1.83	10.173	3.526	0.87
TE_{041}	$\nu_{21}= 3.054$	0.917	1.83	13.324	3.526	0.87