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ABSTRACT

High frequency ($\geq$ 200 GHz) gyrotrons are potentially useful for several important applications, including plasma heating and radar. For electron cyclotron resonance heating of a moderate-size, high power density tokamak power reactor to ignition temperatures, a gyrotron frequency around 200 GHz appears to be necessary. The design of high frequency gyrotrons is discussed. Analysis of overall gyrotron efficiency indicates that high efficiency may be obtained in fundamental electron cyclotron frequency (ω_c) emission at high frequencies. The linear theory of a gyrotron operating at the fundamental frequency is derived for the TE_{mpq} modes of a right circular cylinder cavity. An analytic expression is given for the oscillator threshold or starting current vs. magnetic field.

I. INTRODUCTION

Gyrotrons are a specific version of the electron cyclotron resonance maser (CRM) developed in the Soviet Union by A.V. Gaponov and coworkers [1-3] in the 1960's. A CRM is a fast wave interaction electron beam tube which operates in a strong magnetic field and emits radiation at the electron cyclotron frequency, ω_c, or its harmonics. Special features of the gyrotron include a cylindrically symmetric, temperature limited, magnetron injection type electron gun and a "quasi-optical" cavity which is relatively large compared to a wavelength (of order 2λ - 5λ in diameter by 5λ - 20λ long). Because of the relatively large cavity, gyrotrons are useful in generating high power at wavelengths in the millimeter and submillimeter range. Powers of 22 kW, CW at 150 GHz, 1.5 kW, CW at 326 GHz and 1.1 MW, pulsed at 100 GHz have been achieved [3,4]. Gyrotrons should prove useful in several applications, including heating of magnetically confined plasmas at electron cyclotron resonance and high frequency radar.

II. PLASMA HEATING WITH GYROTRONS

The high frequency, high power radiation generated by gyrotrons is useful for electron cyclotron resonance heating (ECRH) of tokamak plasmas. Successful ECRH experiments using gyrotrons have been carried out in the Soviet Union on the TM-3 tokamak [5]. Those experiments were performed at magnetic fields of up to 25 kG, at power levels of up to 60 kW in 1 msec. pulses and at both fundamental (ω_c) and second harmonic ($2\omega_c$) frequencies. Bulk heating of the plasma, with a central temperature rise of up to 200 eV, was observed. Similar experiments are now planned for tokamak plasmas in several other countries, including the U.S.

We have investigated theoretically the characteristics of tokamak power reactors which would be bulk heated by ECRH [6]. A set of reactor operating characteristics was derived based on wave propagation and reactor engineering requirements. Of the heating modes considered, ordinary wave heating at $\omega \approx \omega_c$ appears to be the most promising. For this mode, $\omega \approx \omega_c$ should exceed the plasma frequency, ω_p, for good penetration into the center of the plasma. Operation of a moderate-size tokamak reactor at a fusion power density of 5 - 10 MW/m^3 leads to a requirement of a relatively high plasma density, n_o, of about $4 - 5 \times 10^{14}$cm^{-3}. Since n_o determines ω_p, setting $\omega \approx \omega_c \approx \omega_p$ results in frequencies ($\omega/2\pi$) in the 200 GHz range for ECRH of reactor plasmas. For this case, the average plasma β, which is defined as the thermal/magnetic pressure ratio, is limited to less than 3.9% for a central temperature below 15 keV. For extraordinary wave heating at $2\omega_c$, higher β operation (up to 7.8%) is allowed relative to ordinary wave heating at ω_c, and the tokamak toroidal magnetic field requirement is lowered (by about $\sqrt{2}$) but higher gyrotron frequencies (by about $\sqrt{2}$) are then needed.

III. DESIGN OF HIGH FREQUENCY GYROTRONS

We have found that a complete design of a high frequency gyrotron oscillator can be carried out using a combination of analytic expressions and numerical results primarily derived from the Soviet literature [7]. The design utilizes an adiabatic electron gun theory [8], numerical resonator mode analysis [9] and computer calculations of cavity efficiency [10, 11]. The present design approach is useful for preliminary design and for evaluation of technological feasibility. A completely optimized final design, however, would obviously require more extensive computational work and modeling.

In the present discussion, we concentrate on one aspect of gyrotron design, namely the evaluation of the overall gyrotron efficiency, η. Using our model [7], we have calculated the efficiency vs. wavelength for moderate output power gyrotron oscillators operating at the fundamental (ω_c) or second harmonic ($2\omega_c$) in the wavelength range 0.5 to 5.0 mm. The calculation assumes an electron beam with V = 30 kV, I = 1 A and a perpendicular to parallel velocity ratio of 1.5 in the cavity region. The cavity is assumed to oscillate in a TE_{031} mode with the electron beam coinciding with the second radial maximum of the E field.

The overall efficiency is given by [10]

$$\eta = \left[1 - \frac{Q}{Q_{OM}}\right] \eta_{e\ell} = \left[\frac{Q_{OM}}{Q_D + Q_{OM}}\right] \eta_{e\ell} \tag{1}$$

where Q_{OM} is the ohmic Q, Q_D is the diffractive Q and Q is the total resonator Q, i.e.,

$$Q^{-1} = Q_{OM}^{-1} + Q_D^{-1} \tag{2}$$

$\eta_{e\ell}$ is given by [10]:

$$\eta_{e\ell} = \frac{1}{1 + (v_{\parallel}/v_{\perp})^2}\, \eta_{\perp} \tag{3}$$

where $\eta_{\perp}$ is the efficiency for converting transverse energy of the electron beam into electromagnetic energy inside the gyrotron cavity.

The theory used for calculating $\eta_{\perp}$ is that of Nusinovich and Erm [11] as modified in Gaponov et al. [10]. This theory assumes a Gaussian RF field distribution along the gyrotron axis. The theory is presented as a set of plots of efficiency as a function of normalized beam current, I_o, and normalized cavity length, μ [10]. In order to evaluate I_o and μ, we use the assumed values of V, I, $v_{\perp}/v_{\parallel}$ and TE_{031} mode. The ohmic Q, Q_{OM}, is evaluated for a TE_{031} mode copper cavity. The calculations are performed for both an ideal cavity with Q_{OM} and for a cavity with $0.75\ Q_{OM}$, which corresponds to more realistic losses through fabrication errors and wall heating.

The cavity diffractive Q, Q_D, is a function of cavity length and shape. For the present analysis, we assume a cavity with untapered walls, such as that shown in Fig. 3b of Ref. [9]. For such a cavity, a Gaussian RF field distribution, $\exp -(2z/L_c)^2$, is a reasonable approximation, where L_c is roughly equal to the cavity length L and therefore $L_c = L$ is assumed here. The diffractive Q, Q_D, is given by

$$Q_D = \kappa\ 4\pi \left(\frac{L}{\lambda}\right)^2 \tag{4}$$

where, for the cavity shape assumed, κ is between 2 and 3 [9].

With the above assumptions and procedure, the overall efficiency, η, may be calculated as a function of wavelength, λ, and of cavity length, L.

For each λ, L was varied to optimize η. These optimized values are plotted in Fig. 1. Both fundamental and second harmonic emission were treated, and both the ideal Q_{OM} and the value 0.75 Q_{OM} were used, marked "minimum" and "realistic" losses, respectively.

The existence of an optimum length L may be explained qualitatively as follows. For very small lengths, I_o and μ are small and the interaction between the electron beam and cavity field is weak. A small value of $\eta_\perp$ is obtained and hence a small value of η. As L increases, $\eta_\perp$ increases. However, Q_D increases as L^2 and eventually $Q_D > Q_{OM}$, so that Eq. (1) provides a decreasing value of η even if $\eta_\perp$ is still increasing with L. Figure 1 also shows experimental results of Zaytsev et al. [4] for fundamental (upper point) and second harmonic emission. The experimental results were obtained for beam parameters and cavity modes that, although not identical to those used here, are roughly equivalent. However, the cavity profile is not specified in Ref. [4].

Figure 1 indicates that, for the present analysis which assumes moderate electron beam power and a TE_{031} cavity mode, fundamental operation is significantly more efficient than second harmonic operation. This difference in efficiency is explained as follows. The transverse efficiency, $\eta_\perp$, can achieve a large value, almost 0.80, in either ω_c or $2\omega_c$ operation, according to the theory of Nusinovich and Erm. However, in second harmonic operation the gain is lower and relatively large values of cavity Q are required to achieve a large $\eta_\perp$. At short wavelengths, Q_{OM}, which goes as $\lambda^{1/2}$, is small so that a large cavity Q may exceed Q_{OM} and lead to a low overall efficiency (cf. Eq. (1)). The present results for the relative efficiency of ω_c and $2\omega_c$ are to some extent dependent on the beam parameters and model assumed. Somewhat different results m ght be obtained if a different cavity shape and RF field profile were assumed. The relative efficiency in $2\omega_c$ operation will also increase if the beam voltage and/or current are raised.

IV. LINEAR CRM THEORY

We have derived an expression for the starting current, I_{st} of a gyrotron operating at the fundamental frequency ($\omega \approx \omega_c$) for the case of a right circular cylinder cavity. The method used is similar to that used in a calculation by Hirshfield et al. [13] for the TE_{011} mode. The present calculation is an extension to the general TE_{mpq} mode. Basically, starting with the Vlasov and Maxwell's equations, the perturbation of the distribution function of the electron beam due to the RF E and B fields in the cavity can be calculated using the method of characteristics. This in turn is used to calculate the power transfer that occurs between the beam and the resonant field. Under certain conditions the field gains energy and emission is possible. Relativistic effects are included by retaining the velocity dependence of the electron mass and cyclotron frequency.

Our expression for I_{st} was derived for a weakly relativistic electron beam. For simplicity, we present here only the result for an electron beam with no spread in parallel or perpendicular velocity, ie., $f(v) \sim \delta(v_\perp - w)\delta(v_{||} - u)$. It can be written as:

$$I_{st} = -\frac{\pi}{2}\left(\frac{m_e\varepsilon_o}{e}\right)\left(\frac{1}{Q}\right)\left(\frac{q\pi}{k_\perp^2}\right)\left(\frac{u^3\omega^2}{w^2}\right) F_r G^{-1}(x)\left[\left(\frac{\omega u}{k_{||}w^2}\right) + H_\omega(x)\right]^{-1}$$

where

$$F_r = \frac{(\nu_{mp}^2 - m^2)J_m^2(\nu_{mp})}{[(J'^2_m(k_\perp r) + (m/k_\perp r)^2 J_m^2(k_\perp r)]}$$

$$G(x) = (1-x^2)^{-2}\sin^2\left((x+1)\pi q/2\right)$$

$$H_\omega(x) = x - \frac{1}{2G(x)}\frac{dG(x)}{dx}\left(\frac{\omega\omega_c}{k_{||}^2c^2} - x^2\right)$$

$$x = \frac{(\omega_c - \omega)}{k_{||} u}$$

Here m_e is the relativistic electron mass, ω is the frequency of the standing wave, ω_c is the relativistic gyrofrequency of the electrons, ν_{mp} is the p^{th} zero of $J'_m(x) = 0$, $k_{||} = q\pi/L$, $k_\perp = \nu_{mp}/R$ where R is the cavity radius, and r is the radial distance of the beam from the cavity axis. This equation is valid in the limit of small electron gyroradius (ie, $k_\perp r_L \approx \beta_\perp << 1$). Note that the starting current is positive and emission is possible only when $H_\omega(x) < -(\omega u/k_{||} w^2)$.

The above equation was compared to Hirshfield's result [2] and found to reduce to his expression for the TE_{011} mode, except for a factor of two[14]. Agreement is found in the weakly relativistic limit with results presented by Symons and Jory [15] for the TE_{01q} mode. Chu [16] has recently solved this problem for the TE_{0pq} modes (i.e. m = 0) using a fully relativistic approach. By comparing his expression with our equation, it is found that using the fully relativistic approach introduces rather small changes as long as the beam voltage is low and the device operates at the fundamental frequency. For example, if we assume w = 1.5 u, the present weakly relativistic expression agrees with the fully relativistic expression within 9% for a beam energy of 30 keV or less and to within 18% for a beam energy of 60 keV or less. This level of accuracy should be satisfactory for most purposes since the present calculations are idealized in two major respects. First, the sinusoidal distribution of RF field along the magnetic axis, which is assumed in the present calculation and those in refs. 13, 15 and 16, is only an approximation to the distribution in a real gyrotron cavity. Gaponov et al [10] have calculated I_{st} for an RF electric field with a Gaussian rather than a sinusoidal dependence along the cavity axis. Comparing his equation with our result shows that a Gaussian distribution can reduce the starting current by as much as a factor of two. Secondly, the velocity spread in the beam, which, for simplicity, was omitted from the expression for I_{st} presented here, can also significantly alter the value of I_{st}. The present calculation should, however, prove useful in predicting the possibility of mode competition in a gyrotron since I_{st} is given for all TE_{mpq} modes of the resonator.

V. CONCLUSIONS

High frequency gyrotrons would be useful for several applications, including bulk ECRH of tokamak plasmas to ignition temperatures. For a tokamak reactor, about 75 MW of heating in a pulse of order 5 - 10 sec. is required, which could be achieved by grouping together a number of 100 - 200 kW sources. For good wave penetration in a moderate size, high power density tokamak reactor, a frequency of about 200 GHz is required [6]. Theoretical considerations as well as recent Soviet results [3] indicate that the required efficient, high-power, high-frequency gyrotrons needed for tokamak reactor heating should be feasible.

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FIGURE CAPTION

Fig. 1. Theoretical overall gyrotron efficiency vs. wavelength is plotted for fundamental (ω_c) and second harmonic ($2\omega_c$) operation. The model and assumed parameters are described in the text.

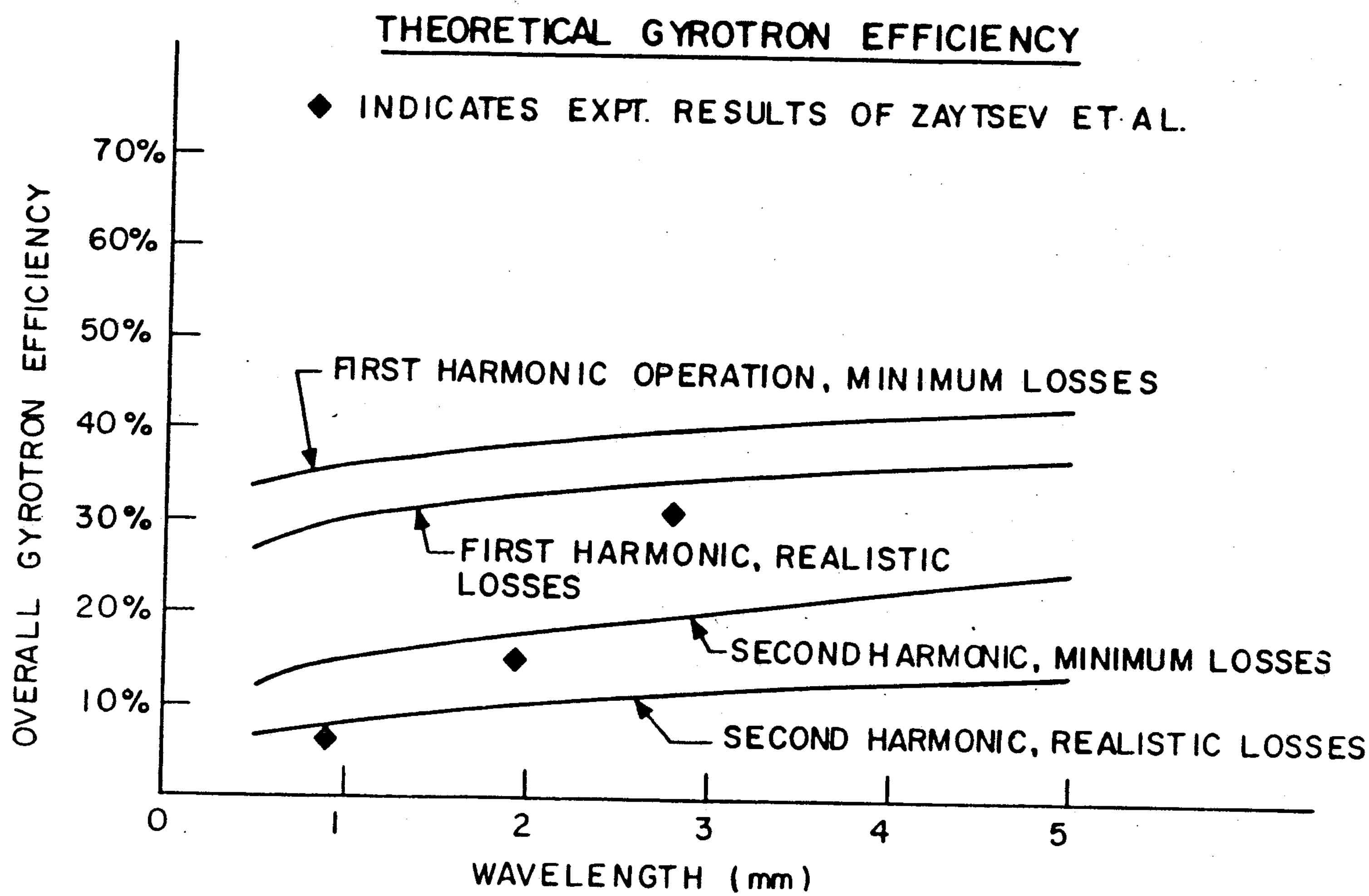
THEORETICAL GYROTRON EFFICIENCY
◆ INDICATES EXPT. RESULTS OF ZAYTSEV ET AL.
OVERALL GYROTRON EFFICIENCY
70%
60%
50%
40%
30%
20%
10%
FIRST HARMONIC OPERATION, MINIMUM LOSSES
FIRST HARMONIC, REALISTIC LOSSES
SECOND HARMONIC, MINIMUM LOSSES
SECOND HARMONIC, REALISTIC LOSSES
0
1
2
3
4
5
WAVELENGTH (mm)

Fig. 1