A Bayesian framework for fracture characterization from surface seismic data

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SUMMARY

We describe a methodology for quantitatively characterizing the fractured nature of a hydrocarbon or geothermal reservoir from surface seismic data under a Bayesian inference framework. Fractures provide pathways for fluid flow in a reservoir, and hence, knowledge about a reservoir's fractured nature can be used to enhance production of the reservoir. The fracture properties of interest in this study (to be inferred) are fracture orientation and excess compliance, where each of these properties are assumed to vary spatially over a 2D lateral grid which is assumed to represent the top of a reservoir. The Bayesian framework in which the inference problem is cast has the key benefits of (1) utilization of a prior model that allows geological information to be incorporated, (2) providing a straightforward means of incorporating all measurements (across the 2D spatial grid) into the estimates at each grid point, (3) allowing different types of measurements to be combined under a single inference procedure, and (4) providing a measure of uncertainty in the estimates. The observed data are taken from a 2D array of surface seismic receivers responding to an array of surface sources. Well understood features from the seismic traces are extracted and treated as the observed data, namely the P-wave reflection amplitude variation with acquisition azimuth (amplitude versus azimuth, or AvAz, data) and fracture transfer function (FTF) data. AvAz data are known to be more sensitive to fracture properties when the fracture spacing is significantly smaller than the seismic wavelength, whereas fracture transfer function data are more sensitive to fracture properties when the fracture spacing is on the order of the seismic wavelength. Combining these two measurements has the benefit of allowing inferences to be made about fracture properties over a larger range of fracture spacing than otherwise attainable. Geophysical forward models for the measurements are used to arrive at likelihood models for the data. The prior distribution for the hidden fracture variables is obtained by defining a Markov random field (MRF) over the lateral 2D grid where we wish to obtain fracture properties. The fracture variables are then inferred by application of loopy belief propagation (LBP) to yield approximations for the posterior marginal distributions of the fracture properties, as well as the *maximum a posteriori* (MAP) and Bayes least squares (BLS) estimates of these properties. Verification of the inference procedure is performed on a synthetic dataset, where the estimates obtained are shown to be at or near ground truth for a large range of fracture spacings.

INTRODUCTION

Fractures are cracks in the earth's crust through which fluid, such as oil, natural gas, or brine, can flow. Knowledge about the presence and properties of fractures in a reservoir can be extremely valuable, as such information can be used to determine preferential pathways for fluid flow and optimize production of the reservoir (Sayers, 2009; Ali and Jakobsen, 2011). Since the presence of fractures in an elastic medium can alter the compliance of the medium and fractures often have a preferred alignment, fractures can cause the medium to exhibit anisotropy (Schoenberg and Sayers, 1995), which refers to changes in seismic wave propagation with direction. This has been exploited to give different techniques for determining fracture properties from seismic data, such as reflection amplitude versus offset and azimuth analysis (Sayers, 2009; Rüger, 1998a; Sayers and Rickett, 1997; Lynn et al., 2010) and shear wave birefringence (Gaiser and Dok, 2001). These methods, however, are only valid when the fracture spacing is small in comparison to the seismic wavelength, so that the seismic wave essentially averages over the fractures (Fang et al., 2012; Willis et al., 2006). This equivalent anisotropic medium assumption, however, breaks down when the spacing between the fractures increases to being on the order of the seismic wavelength. For the case of larger fracture spacings, Willis et al. (2006) proposed a technique, referred to as the scattering index method, to estimate the azimuthal orientation (or strike) of a fracture system based on the scattered seismic energy. Fang et al. (2012) described a series of modifications to this technique to give a more robust methodology for determining fracture orientation, which is referred to as the fracture transfer function method.

In the way of statistical inference methods applied to geophysical problems, Eidsvik et al. (2004) gave a Bayesian framework for determining rock facies and saturating fluid by integrating a forward rock physics model with spatial statistics of rock properties. Specifically in the area of fracture characterization, Ali and Jakobsen (2011) used a Bayesian inference framework to infer fracture orientation and density from seismic velocity and attenuation anisotropy data. Sil and Srinivasan (2009) applied a similar Bayesian inference methodology to determine fracture strikes from seismic and well data. All of the aforementioned statistical studies applied Markov chain Monte Carlo (MCMC) techniques to solve the inference problem. Furthermore, the data models used in all of these studies follow from the equivalent anisotropic medium assumption for small fracture spacings.

DESCRIPTION OF THE PROBLEM

Consider a set of seismic measurements taken from a 2D array of surface receivers over a layered medium responding to a set of surface seismic sources. We assume that fractures may exist in a particular layer of the medium (e.g. the reservoir), and we are interested in inferring from the seismic data whether or not fractures are indeed present and, if so, the properties of the fractures. A simple example of this setup, where the medium consists of flat homogeneous layers, is displayed in Figure 1. In particular, we would like to infer fracture orientation $\varphi =$

$[\varphi_{ij}]$ and the (base 10) log excess fracture compliance $\mathbf{z} = [z_{ij}]$ spatially over a 2D m-by-n lateral grid $\mathbf{L} = \{1,\ldots,m\} \times \{1,\ldots,n\}$. Each grid point corresponds to a square of area ℓ^2, so that the entire grid corresponds to a region of area $mn\ell^2$. Excess fracture compliance is defined as the overall additional medium compliance due to the presence of fractures and is the ratio of the compliance of the individual fractures to the fracture spacing. We make the simplifying assumptions (1) that the normal and tangential excess compliances are equal, hence we need only infer a single log excess compliance value z_{ij} for each grid node, and (2) that the fractures are vertical, so that the fracture orientation φ_{ij} is simply the azimuth (or strike) of the fractures (with respect to North). An excess compliance value of 0 at a particular grid point is taken to mean there are no fractures at that grid point (rendering the value for azimuth arbitrary and meaningless). In order to compare zero and non-zero compliance on a logarithmic scale, we treat an excess compliance of zero as 10^{-13} Pa^{-1}, which is geophysically reasonable as this is an insignificantly small value for excess compliance and results in a negligible effect on seismic wave propagation. We assume that the dataset is rich enough so that for each grid point in $\mathbf{L}$, there are corresponding source-receiver pairs that sample the point at multiple offsets and acquisition azimuths. We further assume that the background velocity structure of the medium is well understood.

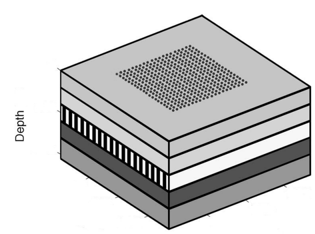

Figure 1: A simple model of the problem setting. The formation consists of five flat homogeneous layers with fractures that may be present in the third layer and measurements obtained from the 2D array of surface seismic receivers. Figure modified from Willis et al. (2006).

In order to relate the fracture properties $\mathbf{x} = (\mathbf{z},\varphi) = [\mathbf{x}_{ij}]$ to the seismic trace dataset, it is necessary to model seismic data as a function of the fracture properties. Unfortunately, simulating the entire seismic trace dataset requires a full elastic 3D forward model of the seismic wavefield, the computational cost of which is prohibitively high, so we instead resort to modeling well understood features of the seismic trace dataset and treat these features as our observed data $\mathbf{y}$. In particular, we choose to model P-wave reflection amplitude as a function of acquisition azimuth (at a fixed angle of incidence) and fracture transfer function (FTF) data, as defined by Fang et al. (2012). We refer to these observed data with variables $\mathbf{y}^{\text{AvAz}}$ and $\mathbf{y}^{\text{FTF}}$, respectively, and let $\mathbf{y} = (\mathbf{y}^{\text{AvAz}},\mathbf{y}^{\text{FTF}})$. Both $\mathbf{y}^{\text{AvAz}}$ and $\mathbf{y}^{\text{FTF}}$ are defined over the grid $\mathbf{L}$, in a manner such that to each grid node of fracture properties $\mathbf{x}_{ij}$ there is an associated data vector $\mathbf{y}_{ij}$.

BAYESIAN INFERENCE FRAMEWORK

In order to arrive at an estimation of the fracture properties from the seismic data, we employ a Bayesian inference framework. The fracture properties and seismic data are treated as random variables, and a stochastic model is used to give the joint distribution of the fracture properties and seismic data $(\mathbf{x},\mathbf{y}) = ((\mathbf{z},\varphi),\mathbf{y})$. In particular, we model the fracture properties as discrete random variables where the alphabet for each of the variables is given by: $10^{\mathsf{z}_{ij}} \in \mathscr{Z} = \left\{10^{-9.0}, 10^{-9.1}, \ldots, 10^{-12.0}, 10^{-13}\right\}$ (in units of Pa^{-1}) and $\varphi_{ij} \in \mathscr{F} = \{0°, 20°, \ldots, 160°\}$, $\forall (i,j) \in \mathbf{L}$. The choice of the alphabets for these variables arises from the level of resolution we can reasonably expect to achieve using seismic measurements. The posterior distribution of the fracture properties given the data $p(\mathbf{x}|\mathbf{y})$ is given by Bayes' rule:

$$p(\mathbf{x}|\mathbf{y}) = \frac{p(\mathbf{x})p(\mathbf{y}|\mathbf{x})}{\sum_{\mathbf{x}'} p(\mathbf{x}')p(\mathbf{y}|\mathbf{x}')} \propto p(\mathbf{x})p(\mathbf{y}|\mathbf{x}) \tag{1}$$

where $p(\mathbf{x})$ and $p(\mathbf{y}|\mathbf{x})$ are the prior distribution of the fracture properties and the distribution of the seismic data given the fracture properties, respectively. To estimate the fracture properties, we desire the MAP estimate $\hat{\mathbf{x}}_{\text{MAP}}$, the posterior marginal distributions $p(z_{ij}|\mathbf{y})$, $p(\varphi_{ij}|\mathbf{y})$ $(\forall (i,j) \in \mathbf{L})$, and the BLS estimate $\hat{\mathbf{x}}_{\text{BLS}}$ of the fracture properties at each grid node given the seismic data. The MAP estimate of the fracture properties is the overall configuration of the fracture properties that maximizes the posterior distribution, that is

$$\hat{\mathbf{x}}_{\text{MAP}} \in \arg\max_{\mathbf{x}} p(\mathbf{x}|\mathbf{y}). \tag{2}$$

The posterior marginal distribution for the fracture properties at a particular node is given by summation of the posterior distribution over all other variables.The BLS estimate of the fracture properties minimizes the expected value of the squared estimation error and is given by the expected value of the fracture properties given the data. So, for the log excess compliance at node (i,j)

$$\hat{z}_{ij,\text{BLS}} = \mathbb{E}\left[\mathsf{z}_{ij}|\mathbf{y} = \mathbf{y}\right] = \sum_{z_{ij}} z_{ij}\, p(z_{ij}|\mathbf{y}). \tag{3}$$

For any reasonably large number of grid nodes mn, the maximization and marginalization operations are intractable, hence we must turn to approximate inference algorithms to perform the estimation.

Prior Model

We arrive at a prior model for the fracture properties x by defining the set of variables x as a Markov random field over an undirected graphical model $\mathscr{G} = (\mathscr{V},\mathscr{E})$ on the 2D grid $\mathbf{L}$ (Koller and Friedman, 2009). We associate with each grid node of $\mathbf{L}$ a vertex in $\mathscr{G}$ so that $\mathscr{V} \equiv \mathbf{L}$. We define the edge set $\mathscr{E}$ over the 2D grid $\mathbf{L}$ so that a particular node shares edges with its four neighbors on the grid $\mathbf{L}$. The graphical model for x prior to observing the data is shown in Figure 2. Intuitively, this means that given the fracture properties of the four nearest neighbors of a particular node, knowledge of the fracture properties of the medium elsewhere on the grid will have no impact

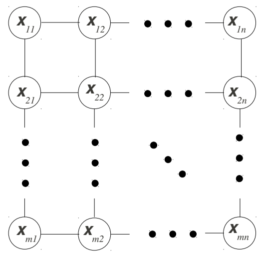

Figure 2: Undirected graphical model $\mathscr{G}$ over which $\mathbf{x}$ is Markov, prior to observing the seismic data

on our belief about the properties at that node. Geologically, we expect the properties of the medium at a particular point in space to be similar to its surrounding properties. This suggests a prior distribution that penalizes differences between a node and its neighbors. In particular we define the prior distribution for the fracture parameters to be

$$p(\mathbf{z}) \propto \exp\left\{-\sum_{(ij,kl)\in\mathscr{E}} \beta_{z_{ij,kl}} (z_{ij} - z_{kl})^2\right\} \tag{4}$$

and

$$p(\varphi) \propto \exp\left\{-\sum_{(ij,kl)\in\mathscr{E}} \beta_{\varphi_{ij,kl}} (\varphi_{ij} - \varphi_{kl})^2\right\}, \tag{5}$$

where $\beta_{z_{ij,kl}}$ and $\beta_{\varphi_{ij,kl}}$ are smoothness parameters which can be manipulated to account for discontinuities such as that those which may arise from a fault. Treating the two different fracture properties as independent *a priori* gives the overall prior distribution for the fracture properties as

$$p(\mathbf{x}) = p(\mathbf{z}, \varphi) = p(\mathbf{z})p(\varphi). \tag{6}$$

Likelihood Model

The seismic data used in this study are extracted from the seismic trace dataset and denoted by $\mathbf{y}^{\text{AvAz}}$ and $\mathbf{y}^{\text{FTF}}$, respectively. We assume that given the fracture parameters $\mathbf{x}$, the two types of seismic data $\mathbf{y}^{\text{AvAz}}$ and $\mathbf{y}^{\text{FTF}}$ are conditionally independent, and hence

$$p(\mathbf{y}|\mathbf{x}) = p(\mathbf{y}^{\text{AvAz}}|\mathbf{x})p(\mathbf{y}^{\text{FTF}}|\mathbf{x}). \tag{7}$$

We now discuss how we model the data to arrive at the likelihood models $p(\mathbf{y}^{\text{AvAz}}|\mathbf{x})$ and $p(\mathbf{y}^{\text{FTF}}|\mathbf{x})$.

Amplitude versus Azimuth Data

In order to arrive at a forward model for the P-wave reflection coefficient of the interface above the fractured layer as a function of acquisition azimuth, we make various simplifying assumptions about the formation and the fractured medium. The layers above the fractured layer are assumed to be isotropic and homogeneous and the background medium of the layer in which the fractures exist is assumed to be homogeneous and isotropic with known medium parameters. We assume that the presence of fractures in the fractured layer causes the layer to behave as an horizontally transverse isotropic (HTI) medium, which is a geophysically valid assumption when the fracture spacing is small compared to the seismic wavelength (Schoenberg and Douma, 1988; Willis et al., 2006). Rüger (1998b) derives the P-wave reflection coefficient is as a function of the incidence and azimuthal phase angles and in terms of the isotropic background and anisotropy parameters. We can relate the fracture properties of the medium to the anisotropy parameters by computing the excess compliance tensor of the fractures and combining it with the background medium compliance tensor (Schoenberg and Sayers, 1995). This gives a deterministic model for the AvAz data. We perturb this model by adding zero-mean, independent, identically distributed (i.i.d.) Gaussian noise to arrive at a stochastic data model $p(\mathbf{y}^{\text{AvAz}}|\mathbf{x})$.

Fracture Transfer Function Data

We compute the fracture transfer function from the seismic trace dataset according to the methodology described by Fang et al. (2012). As FTF captures the intensity of scattering at different acquisition azimuths, we use as our data the azimuth that maximizes the FTF. We then model this as a function of the fracture properties by setting this azimuth to the strike of the fractures. We again perturb the model with i.i.d. Gaussian noise and obtain our stochastic model for the FTF data $p(\mathbf{y}^{\text{FTF}}|\mathbf{x})$.

Inference Algorithms

We return to the graphical model representation of the distribution, as this will play a key role in the inference algorithms used to obtain the posterior marginals and MAP estimate. The updated MRF for the posterior distribution is shown in Figure 3.

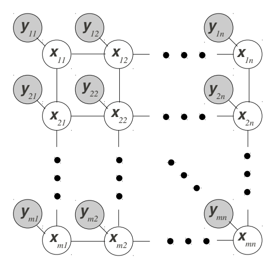

Figure 3: Graphical model showing the Markovianity between the observations $\mathbf{y}$ and the fracture parameters $\mathbf{x}$.

We can write the posterior distribution in terms of factors defined on the nodes and edges of the graph. This allows us to apply belief propagation algorithms to perform approximate inference of the fracture properties $\mathbf{x}$.

Loopy Belief Propagation

Belief propagation (BP) is a technique for performing inference on graphical models which has recently enjoyed much popularity for use amongst a wide-range of applications (Pearl, 1988, 1982; Murphy et al., 1999). Originally formulated for tree graphs (i.e. graphs having no cycles), BP refers to message-passing algorithms for computing either marginal distributions (the sum-product algorithm) or MAP configurations (the max-product algorithm). Applying BP to perform inference on a graph with loops is referred to as loopy belief propagation. While LBP is an approximate algorithm, it has nonetheless been used extensively in various settings and found to often give very good approximations (Murphy et al., 1999). With this in mind, we apply loopy belief propagation on the poste-

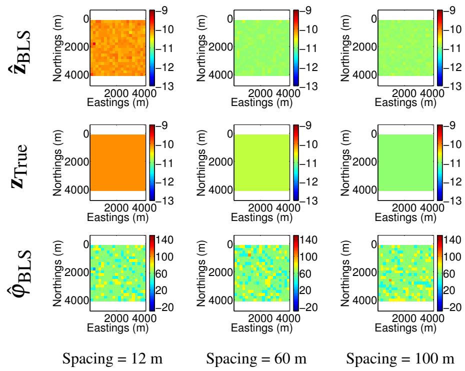

Figure 4: Approximate BLS estimates of the fracture properties computed for models of a single fracture set, with fracture compliance 10^{-9} m/Pa and fracture strike $\varphi_{ij} = 60°$.

rior distribution for the fracture parameters to approximate the MAP configuration and marginal distributions.

RESULTS

We validate our methodology by performing inference on a synthetic data set. The synthetic data are obtained from 3D elastic finite-difference simulations of the seismic wavefield (with a 40 Hz Ricker source) on reservoir models having topology as shown in Figure 1. The results of the inference procedure on a single fracture set are plotted in Figures 4 and 5. Figure 4 shows the approximate BLS estimates of the fracture properties over the 2D grid **L**. The approximate marginal distributions for the fracture properties at a particular node are shown in Figure 5, where peakier posterior marginal distributions represent less uncertainty about the estimates (i.e. due to a lower variance *a posteriori*). The resulting residuals between the estimates and true values are given in Table 1 in terms of the root mean squared (RMS) residuals over all nodes. We see from the results that the estimates for both fracture strikes and excess compliance are very good even up fracture spacings of 100 m, where the dominant seismic wavelength in the fractured layer is 100 m. Figure 6 displays the MAP estimate of two sets of fractures separated by a fault, both with and without *a priori* knowledge of the fault. We observe that the estimates are less correct near the discontinuity when the fault is not known *a priori*.

Spacing	$\varepsilon_{\text{rms},\hat{\mathbf{z}}_{\text{BLS}}}$	$\varepsilon_{\text{rms},\hat{\varphi}_{\text{BLS}}}$	$\varepsilon_{\text{rms},\hat{\mathbf{z}}_{\text{MAP}}}$	$\varepsilon_{\text{rms},\hat{\varphi}_{\text{MAP}}}$
12 m	0.099	10.80°	0.100	10.95°
20 m	0.037	7.49°	0.038	7.62°
40 m	0.154	16.09°	0.118	16.37°
60 m	0.165	11.67°	0.143	11.79°
80 m	0.065	15.14°	0.087	15.39°
100 m	0.091	11.09°	0.101	11.23°

Table 1: Root mean square of residuals between estimates and ground truth. For comparison, note that azimuth φ is discretized into 20° bins and log compliance z has been discretized into bins of size 0.1.

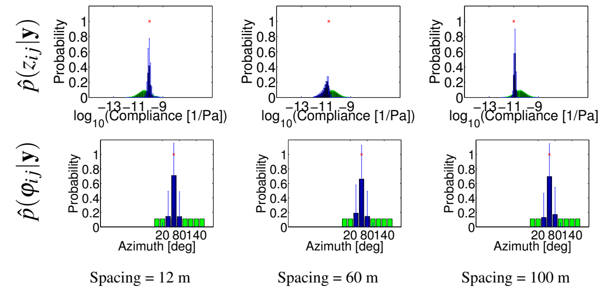

Figure 5: Approximate posterior marginal distributions (blue) of the fracture properties at a single node plotted along with the prior distributions (green). Results are given as mean ± 1 S.D. over all grid nodes. Ground truth shown by red 'x'.

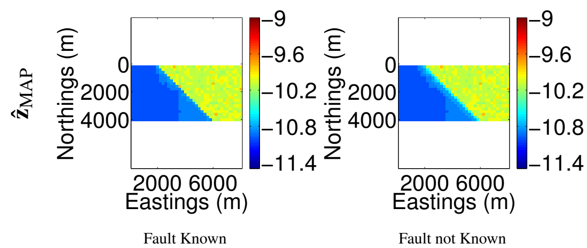

Figure 6: Effect of *a priori* knowledge of the fault on approximate MAP estimates of the fracture properties of a model containing two fracture sets. Notice the incorrect estimates shown by the cyan coloration at the discontinuity when the fault is not known. This vanishes when the fault is known.

CONCLUSIONS AND FUTURE DIRECTION

A methodology for estimation of fracture properties from surface seismic data under a Bayesian inference framework is presented. We have demonstrated that the approximate inference results are very accurate at low fracture spacings and continue to give good estimates up to 100 m spacing, where the dominant seismic wavelength is 100 m. This is significant, as we are able to estimate the fracture properties in a rigorous manner at a greater range of spacings than would otherwise be attainable. We also demonstrated the capability of this framework to handle prior information about geological features, such as a spatial discontinuity arising from a fault. While we presented a very simple case of this, it is not difficult to extend this to more complicated scenarios. A related future direction is to incorporate additional features of the seismic data in the inference procedure. In particular, Zheng et al. (2011) describe a theory for using 3D beam interference to model the scattering properties of a reservoir from reflected seismic P-wave data.

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