A multi-pass one way method to include turning waves and multiples

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SUMMARY

Conventional one way migration methods exclude turning waves and multiples. We propose an algorithm that uses multiple passes to extend the one way method to efficiently include these wavepaths. A comparison of the images produced by the regular one way algorithm, RTM, and the new method, shows that this new method can significantly improve the image in regions of interest, and in certain situations may even provide more useful information than RTM. The runtime is demonstrated to be in between that of regular one way and RTM, while the physical memory required is considerably lower than that of RTM.

INTRODUCTION

The conventional one way method (Gazdag (1978)) assumes that waves only travel downwards from the source, reflect, and travel upwards to the receivers. In reality, there are many other paths that the waves can take. One class of such paths is called multiples, and refers to waves that have reflected more than once on the path between source and receiver. The extra reflections could take place at the Earth's surface (the free surface), which is particularly common in marine datasets, or could be between reflectors in the subsurface, termed internal multiples. Another class of wavepaths not included in conventional one way migration is turning waves. When wave speed increases with depth, some waves will turn over and begin to propagate upwards. This violates the assumption inherent in the one way method of waves only traveling in one direction.

Arrivals that violate the assumptions of one way migration will not be imaged correctly when that method is used. Such data contain useful information about the subsurface, however. A particular advantage of these wavepaths is their ability to image structures from below, which may, for example, permit better imaging of the flanks of salt bodies. It is therefore desirable to be able to include multiples and turning waves in imaging.

IMAGING WITH ADDITIONAL WAVEPATHS

One Way

Several algorithms for extending the one way algorithm to enable imaging with multiples have been proposed. Attempts can be split into two classes: imaging with free surface multiples, and imaging with internal multiples.

Imaging with free surface multiples has the advantage of being able to record the multiples at one of their bounce points. The problem has received considerable attention, see for example Reiter et al. (1991); Guitton (1999); Berkhout and Verschuur (2006); Muijs et al. (2007). While surface multiples can en-

hance the SNR of the final image and increase the source coverage, they are unlikely to illuminate areas of the subsurface that would be challenging for singly-scattered waves, although they may add information for AVA analysis.

Internal multiples are particularly interesting as they can permit better imaging from below, and of near-vertical structures using doubly-scattered waves. A method for imaging with internal multiples using a one way propagator was proposed recently by Malcolm et al. (2009). In this method the multiples and source wavefields are downward propagated and stored at each depth. Multiplying this data by a standard image 'selects' the wavefield at the depths at which it would undergo reflections. Propagating the stored data upwards in a second upward pass simulates the reflections of the waves. The stored data from each level is added to the upward propagating wave as it is reached so that the upward propagation will include waves reflected from all deeper reflectors. Applying the standard imaging condition will image the underside reflections of first order multiples. Additional applications of alternating upward and downward passes includes higher order multiples. This method has the distinct advantage of not requiring any prior knowledge of the locations of reflectors. This approach uses a series formulation as is done by Weglein et al. (1997) for attenuating multiples. The method also has similarities with that of Jin et al. (2006).

An early proposal for imaging with turning waves using a one way method was made by Claerbout (1985). A regular downward pass is made, followed by an upward pass for regions of the wavefield that correspond to $k_z^2 < 0$ (evanescent waves). This method needs to propagate waves that are travelling almost horizontally, which decreases the accuracy of the one way propagator. A potential solution is to include horizontal passes (Zhang and McMechan (1997); Jia and Wu (2009)), but this can add considerably to the cost of the algorithm.

RTM

Reverse Time Migration is significantly more computationally expensive than one way migration, but its natural ability to include turning waves, and to accurately propagate waves even in the presence of laterally heterogeneous media, mean that it is frequently used in complex areas. It is also possible to image with multiples using RTM, but this requires that the location of multiples-generating reflectors be known before the start of migration. Furthermore, discontinuities included in the velocity model can introduce artifacts into the resulting image as this violates the assumption inherent in the method that only singly-scattered waves are being migrated.

Proposed Method

In this paper a new migration algorithm is proposed. Employing ideas from Claerbout's two-pass turning wave method and the internal multiples algorithm presented by Malcolm, the proposed scheme extends the conventional one way method to efficiently include potentially important additional arrivals.

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The implementation of the new method is described in the following section. The resulting images, runtime, and physical memory required by the enhanced one way method, the conventional one way method, and RTM, are compared in the subsequent Results section.

ALGORITHM

The proposed algorithm consists of two stages. The first is a downward pass. This is similar to the conventional one way method: for each shot, source and data wavefields are downward propagated and an imaging condition is applied. Images from all shots are then stacked to produce an image of the reflectors of singly-scattered downward propagating waves. It differs from the traditional implementation by saving the source and receiver wavefields at each depth.

The second stage is an upward pass. As in the downward pass, each shot is processed independently. Four new upgoing wavefields are created and initially set to zero: waves propagating forwards in time from the source that have turned over, $u_{s,t}$, or reflected from a multiples-generating interface, $u_{s,m}$, and waves propagating backwards in time from the receivers that have turned over, $u_{r,t}$, or reflected from a multiples-generating interface, $u_{r,m}$. At each depth, the source and receiver wavefields saved at that depth during the downward pass are loaded as $u_{s,d}$ and $u_{r,d}$, respectively. The algorithm then proceeds as follows:

1   $u_{s,m} \leftarrow u_{s,m} + u_{s,d} \times image$; (Multiples)
    $u_{r,m} \leftarrow u_{r,m} + u_{r,d} \times image$;
2   $\mathscr{F}_{x,y}(u_{s,d}, u_{r,d}, u_{s,t}, u_{s,m}, u_{r,t}, u_{r,m})$;
3   **forall** k_x, k_y, ω **do if** $\frac{\omega^2}{c^2} < k_x^2 + k_y^2$ **then**
4   $\quad\lfloor\; u_{s,t} \leftarrow u_{s,t} + u_{s,d}; u_{r,t} \leftarrow u_{r,t} + u_{r,d}$ (Turning);
5   `phase shift`$(u_{s,t}, u_{s,m}, u_{r,t}, u_{r,m})$;
6   $\mathscr{F}^{-1}_{x,y}(u_{s,d}, u_{r,d}, u_{s,t}, u_{s,m}, u_{r,t}, u_{r,m})$;
7   `lateral correction`$(u_{s,t}, u_{s,m}, u_{r,t}, u_{r,m})$;
8   `imaging condition`$(u_{s,t}, u_{r,d}; u_{s,d}, u_{r,t}; u_{s,t}, u_{r,t})$;
9   `imaging condition`$(u_{s,m}, u_{r,d}; u_{s,d}, u_{r,m}; u_{s,m}, u_{r,m})$;

In this code, *image* is the reflector image from the first pass, $\mathscr{F}_{x,y}$ is the Fourier transform in x and y, and $\mathscr{F}^{-1}_{x,y}$ is its inverse. The conventional one way phase shift operation is applied to all of the wavefields acted on by `phase shift`, `lateral correction` is the wide-angle correction used in laterally heterogeneous media, and `imaging condition` indicates the application of an imaging condition on pairs of wavefields.

Multiplying a downward propagating wavefield by the reflector image in the frequency-space domain (line 1) has the effect of selecting the wavefield at the locations of reflectors as these will be the only non-zero parts of the image. By adding this to an upgoing wavefield and propagating, reflection is simulated. This allows multiples to be considered.

After transforming to the frequency-wavenumber domain (line 2), the portion of the downgoing source and receiver wavefields that have turned-over ($k_z^2 < 0$) is added to the corresponding upgoing turning wavefields to be propagated upwards

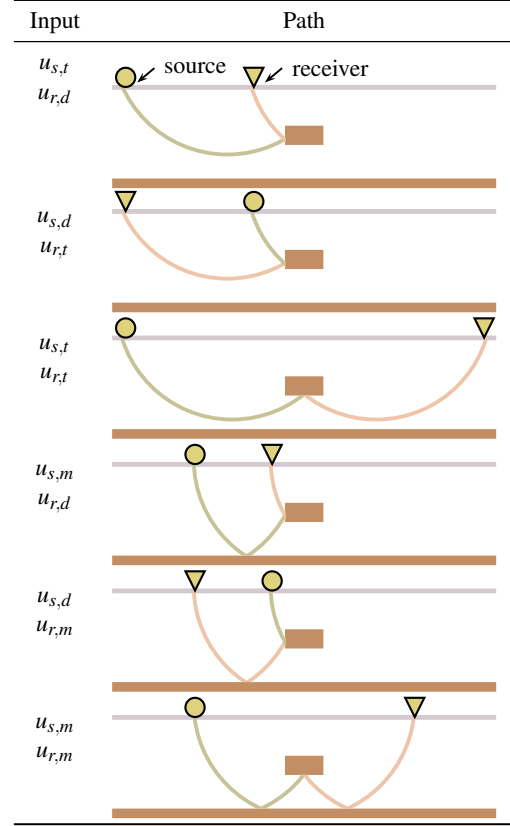| Input	Path
$u_{s,t}$ $u_{r,d}$	
$u_{s,d}$ $u_{r,t}$	
$u_{s,t}$ $u_{r,t}$	
$u_{s,m}$ $u_{r,d}$	
$u_{s,d}$ $u_{r,m}$	
$u_{s,m}$ $u_{r,m}$	

Table 1: Input wavefields in imaging condition to image using different wave paths

(line 4). This includes turning waves in migration.

On line 5, the four upgoing wavefields (the source and receiver waves that have turned, and the source and receiver waves that have reflected) are propagated to the next depth level above.

Returning to the frequency-space domain on line 6 allows the application of corrections to account for lateral heterogeneity in the medium on line 7.

Six additional potential wavepaths are now available for imaging. These are illustrated in Table 1, and correspond to the six pairs on lines 8 and 9.

By loading the saved wavefield from the downward pass at each depth level, the restriction that Claerbout's algorithm can only be used for models with monotonically increasing velocity is relaxed. The problem of propagating through high angles in the presence of lateral velocity variations still exists, but, when a high-accuracy propagator is used, was found empirically to not introduce significant errors for the models tested.

With one additional pass over conventional one way migration, the proposed method exploits more of the recorded data to augment the result with six supplementary wavepaths. Further passes could be performed to include a greater number of wavepaths with successively more turns or reflections, although these are likely to yield wavepaths that make a smaller

contribution to the resulting image due to their small amplitude.

A complication introduced by the inclusion of the additional wave paths can produce severe artifacts. This situation also arises when RTM is used. A solution often used in that case is to postprocess the image with a Laplacian filter. An alternative, proposed by Op 't Root et al. (2010), alters the imaging condition. Either of these methods can be used to attenuate the artifacts in the enhanced one way method.

RESULTS

A section of the synthetic BP model (Billette and Brandsberg-Dahl (2005)), shown in Fig. 1, was used to compare the algorithms. The goal of imaging was primarily to image underneath the salt overhang.
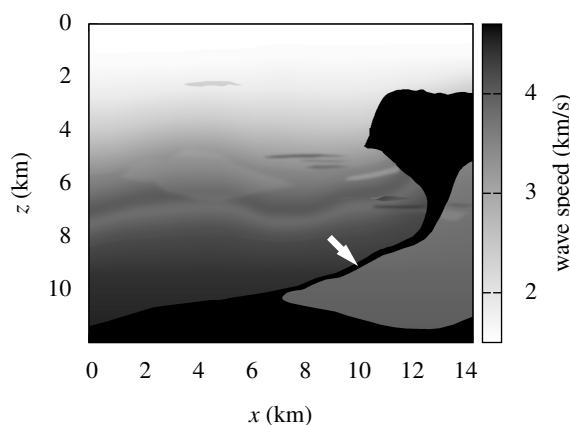

Figure 1: BP velocity model. The white arrow indicates the salt leg that will be used as a multiples-generating interface.

The exact velocity model was used in the first test, with the results shown in Fig. 3. The regular one way method images the top and legs of the salt, but the bottom interface of the overhang is not visible. Even though the exact velocity model was used, even RTM fails to image underneath the overhang, and the resulting image is barely superior to that of the one way method. The one way enhanced method is able to quite accurately image the shape and location of the area of interest. Although the image contains many artifacts, it results in a superior image of the overhang compared to that obtained with RTM.

We now perform a second test to simulate a more realistic situation in which the location of the salt is not known. To this end we recompute the same three images using a velocity model with the salt body replaced by a vertical gradient. The resulting images are shown in Fig. 4. Regular one way and RTM do not image any reflectors in the region of the overhang. The enhanced one way image gives an indication of the location and shape of the structure. The ability of the enhanced one way method to scale the amplitude of images from specific wave paths was the key to this successful result.
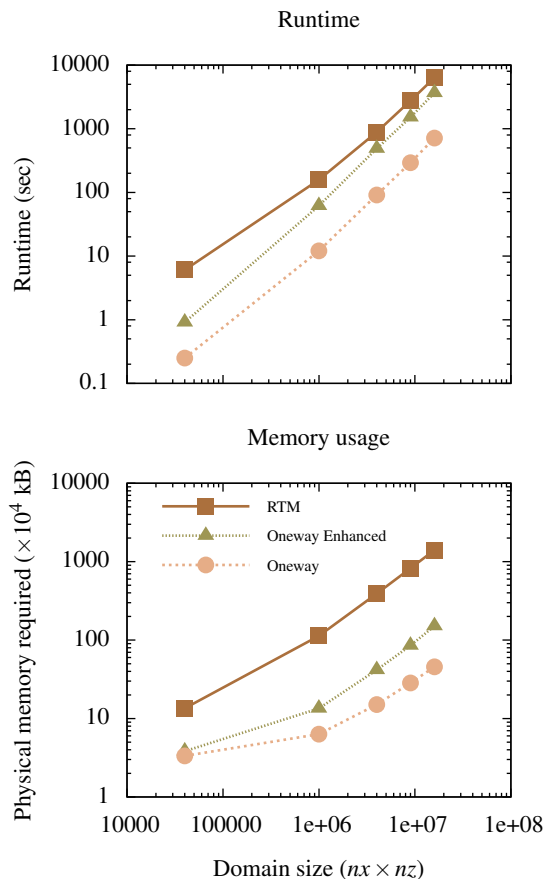

Figure 2: Computational requirements of RTM, the enhanced one way method, and conventional one way migration. The total runtime (above) and maximum physical memory used (below) to migrate a single shot for varying domain size are compared for the three algorithms.

Although introducing additional wavefields necessarily increases the computational requirements of the method, it is demonstrated in Fig. 2 that both the runtime and physical memory needed for the enhanced one way method are significantly lower than that of RTM.

CONCLUSION

An extended one way migration algorithm was proposed. This method uses more of the recorded data for imaging, allowing it to illuminate areas inaccessible with the conventional implementation, such as near-vertical features and underneath salt bodies. The performance was examined by comparing the results with those produced by regular one way migration and RTM. The new algorithm was able to image important areas of the subsurface more clearly than regular one way. It also produced better resolved images of certain features than even RTM. This was achieved while requiring less runtime and physical memory than RTM.

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One way

One way
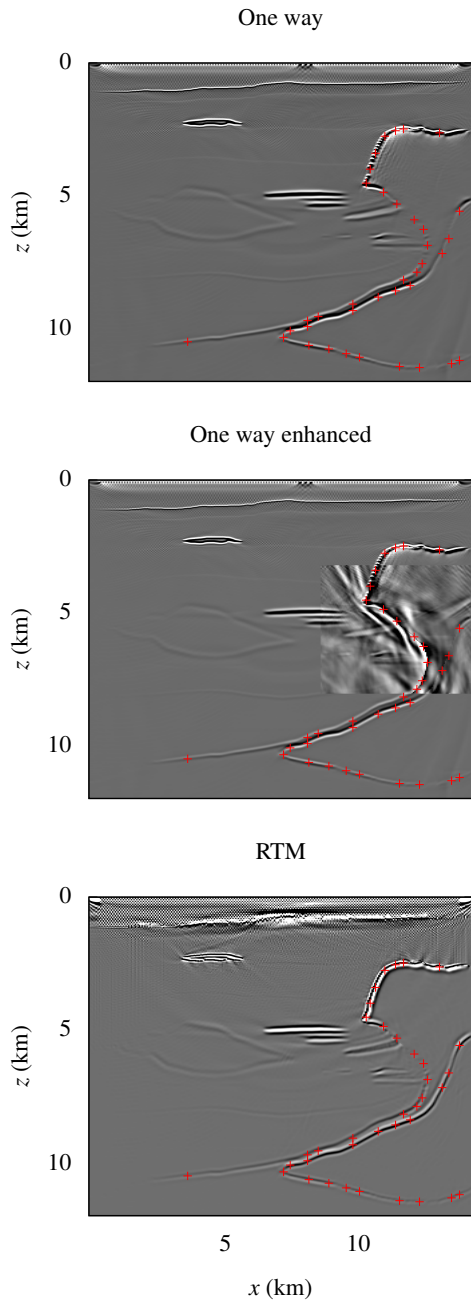

One way enhanced

One way enhanced

RTM

RTM

Figure 3: Image of the BP model when the exact velocity model is used with different migration algorithms. The enhanced one way figure consists of the section around the salt overhang of the image produced by that method, added to the regular one way image.
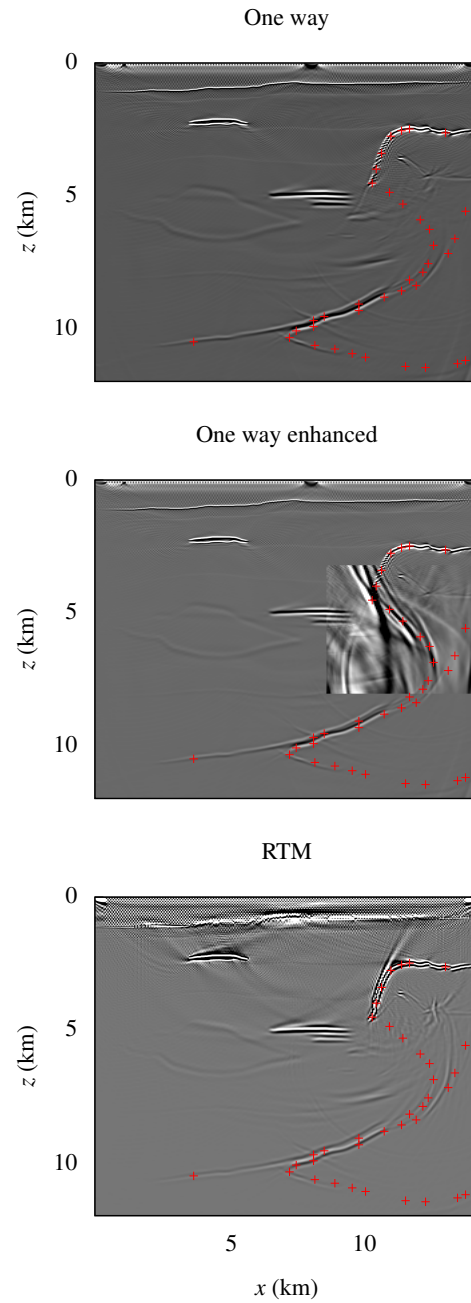
Figure 4: Image of the BP model when only the background velocity model is used during migration. It may be possible to reduce the artifacts present in the images by only using right-going waves in the imaging condition.

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