Source-Indexed Migration Velocity Analysis
with Global Passive Data

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1   Introduction

The reverse-time migration of global seismic data generated by free-surface multiples is regularly used to constrain the crustal structure, but its accuracy is to a large extent determined by the accuracy of the 3-D background velocity model used for wave propagation. To this improve the velocity model and hence the accuracy of the migrated image, we wish to apply the technique of migration velocity analysis (MVA) to global passive data. Applications of MVA in the active setting typically focus on offset- or angle-gather annihilation, a process that takes advantage of data redundancy to form an extended image, and then applies an annihilation operator to determine the success of image formation. Due to the nature of regional-scale passive seismic arrays, it is unlikely that the data in most of these studies will be sufficient to form an extended image volume for use in annihilation-based MVA. In order to make use of the sparse and irregular array design of these arrays, we turn towards a shot-profile moveout scheme for migration velocity analysis introduced by Xie and Yang (2008). In the place of extended image annihilation, we determine the success of the migration velocity model by using a weighted image correlation power norm. We compare pairs of images formed by migrating each teleseismic source by image cross-correlation in the depth direction. We look for a suitable background model by penalizing the amount of correlation power away from zero depth shift. The total weighted correlation power between source-profile images is then used as the error function and optimized via conjugate gradient. We present the method and a proof-of-concept with 2-D synthetic data.

2   Experimental setup

Teleseismic free-surface multiples are global phases that propagate from distant earthquakes, reflect off of the free surface, and are then scattered upward by discontinuities in the crust and mantle. To make the best use of these phases, we design our setup to resemble an active source experiment with the direct phase arrival analogous to the source and the multiples to the reflection data. Modeling the initial teleseismic leg of the wave propagation is difficult to achieve without resorting to either high frequency approximations or great computational expensive, so we limit our area of interest to the region directly below the array which is well illuminated by multiple events. As the direct arrival is complicated and affected by distant structure, we will define the incident field from observation instead of modeling it.

For a first test of our inversion method under realistic teleseismic conditions, we look at a model of a shallow subduction zone. We consider a model region of 150 km surface offset by 100 km depth with numerical grid spacing of 250 m and model data up to 2 Hz. We use the 2-D Helmholtz solver (Wang et al., 2010) to generate a data set which includes 29 different localized plane wave events propagated from the bottom of the domain with incoming incidence angles between 0 and 30°. The source waves propagate upward, are recorded and reflected at the free surface, and are then multiply scattered by discontinuities in the model before being rerecorded at the surface.

The preprocessing chain described in Rondenay (2009) is used to separate the direct arrival from the multiply scattered phases. The traces are aligned via multichannel cross-correlation according to the direct arrival, and a singular value decomposition is performed. The first two eigenvalues are retained as the direct phase, the rest are considered as the scattered field, and both fields are then shifted back to absolute time.

Since data is recorded at the free surface, we must also perform an directional decomposition on each field. Finally, the source function is deconvolved from the incident and scattered wavefields.

As the first step in the inversion procedure, a reverse time migration is performed on the data from each separate teleseismic source. Using the inverse scattering transform, we are able to recover an accurate estimate the singular part of the velocity model, as shown in figure 1(a). In order to apply migration velocity analysis to the image, we eschew stacking the images over all incoming events and instead compare between images formed by different events. If the images from different events are formed in the correct background model, it can be expected that, up to source wavelet and illumination effects, they will be the same. Figure 1(b)-(d) demonstrate the effect of the background model on the source-indexed image gathers. If the model is correct as in figure 1(c), the image gathers should be flat. If the model is incorrect, there can be significant moveout in the image gathers - a slow model leads to images from high incidence angle events forming more shallow than low angle events.
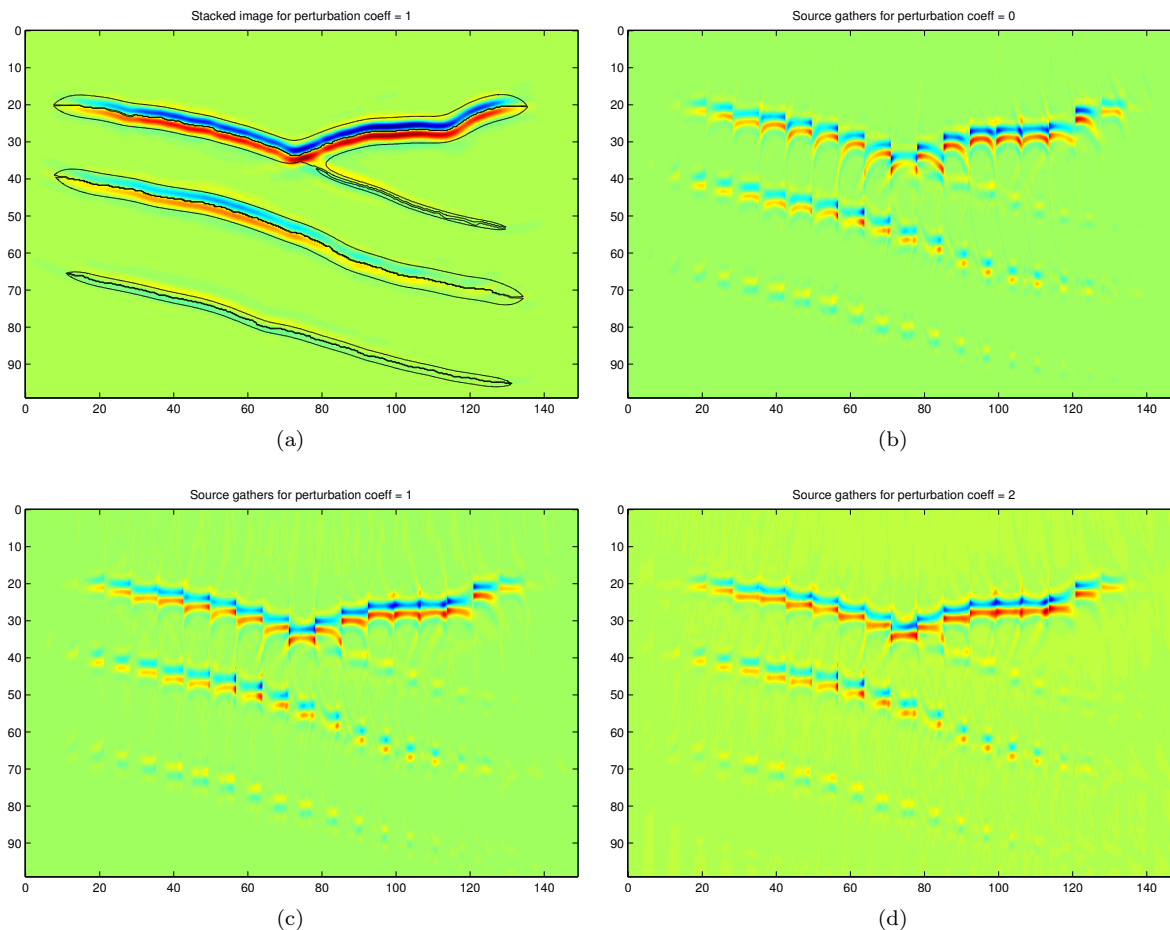

Figure 1: (a) Stacked images for synthetic subduction zone model using inverse scattering transform. Black contour show location of true reflectivity. (b) Source gathers for 1D model. (c) Source gathers for correct smooth model. (d) Source gathers for model with double the difference between the correct model and the 1D model

3   Error functional and inversion

Due to the sparse and irregular distribution of global passive sources and their frequently long and complicated source time functions, previous measures of error are inadequate to deal with with teleseismic studies. The most robust way to characterize the moveout in between images from different sources is to apply the

correlation power error criterium introduced by Van Leeuwen and Mulder (2008). If the background velocity model is correct, then a depth correlation of two images at a given image point should have the majority of its power centered around zero depth shift. The error function is then given by:

$$\mathcal{J} = \sum_{\alpha=1}^{N_\mathcal{I}} \sum_{i,j=1}^{N_s} \frac{1}{S_{ij}^\alpha} \int \int W_B(\Delta z) \left| \mathcal{C}_{ij}^\alpha(\mathbf{x}, \Delta z) \right|^2 \mathrm{d}\Delta z \, \mathrm{d}\mathbf{x}, \tag{1}$$

where correlations $\mathcal{C}_{ij}^\alpha$ are calculated between single-source images indexed i and j at relevant correlation windows α. The weighting function, $W_B(\Delta z) = \exp[-\beta_{B,z}(\Delta z)^2]$, is an negative gaussian designed to penalize correlation power away from zero depth shift. It is dependent on the bandwidth, B of the data, and should have a width of the same order as the wavelength of the imaged reflectivity. S_{ij}^α is the correlation power at zero depth shift, and is used to make sure that the function isn't minimized by shifting the reflectivity outside of the correlation windows. We determine the model gradient via the seismic adjoint method.

For a test of the methodology, we invert for the subduction model starting from a 1-D model. and project the heterogeneity onto a spline basis. The perturbation to the 1D model in the spline basis is displayed in figure 2(a). The inversion result in figure 2(b) demonstrates the ability of the method to recover high-resolution velocity structure. The amplitude of the perturbation is well recovered at shallow depths and near the middle of the model domain. Due to the relative lack of of correlation measurements at the edges of the array and at the deeper parts of the model, the recovery of the model is inaccurate. The bottom left-most region of the reconstruction, in fact, has the wrong sign, demonstrating that our method has no sensitivity in this region of the model.
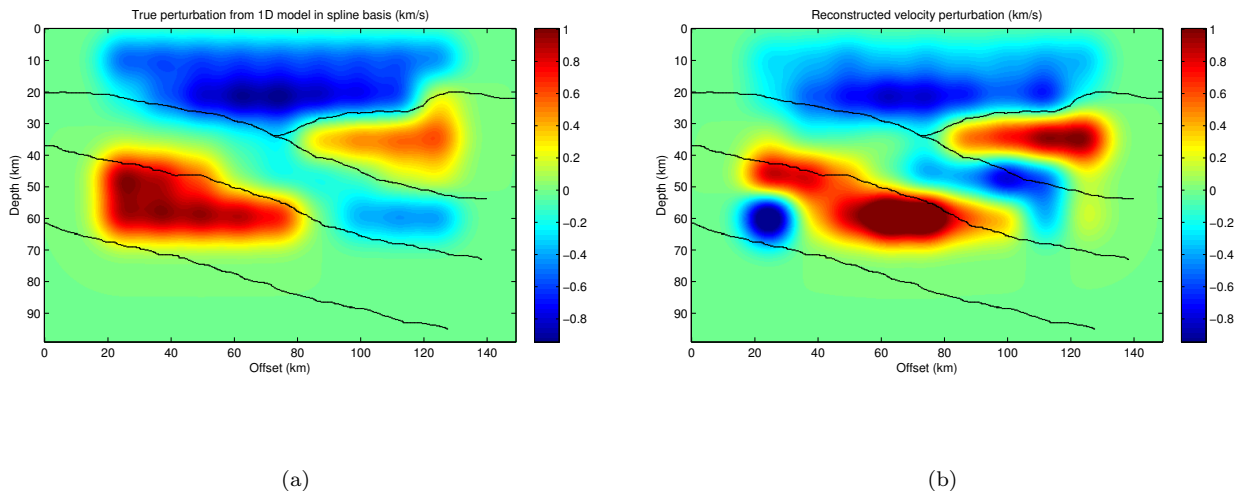

(a) (b)

Figure 2: Synthetic subduction inversion test. (a) Correct perturbation to 1D model projected onto spline basis. Black lines represent location of reflectors. (b) Subduction model recovered by inversion method. Recovery is incorrect at bottom left of model due to poorly illumination of structure by teleseismic data.

The extension of this method to 3D array data is straightforward. The source-indexed power norm criterium is particularly suited to the limitations presented by the global distribution of seismicity due to its ability to handle gaps in incident angle and azimuthal coverage. Further, the method can also be extended to the elastic case with the use of the elastic Helmholtz propagator (Wang et al., 2011).

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