Double-beam stacking to infer seismic properties of fractured reservoirs

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Summary

We develop a theory for using 3D beam interference to infer scattering properties of a fractured reservoir using reflected seismic P data. For the sake of simplicity, we use Gaussian beams. The scattering properties are important to infer fracture spacing, orientation and compliance. The method involves the interference of two beams, one from the source region and the other from the receiver region. Each beam is formed by first windowing the data in space and time and then performing f-k filtering. The interference pattern depends on frequency, the incident angle, the reflection angle, and the azimuth. We try to interpret the interference pattern using local Born scattering in the target region. This interpretation is motivated by the observation that full-wave finite difference simulation of waves propagating through a set of vertical fractures using Schoenberg's linear-slip boundary condition and fracture compliances consistent with those inferred from field and laboratory data shows that single scattering dominates in the reflection data. The methodology is versatile in that by adjusting the window sizes we can obtain plane wave interference as well as interference for a single shot or receiver gather. By suitable choice of pairs of source and receiver beams, the spatially varying fracture properties as well as the fracture orientation can be inferred.

Introduction

Reliable assessment of properties of fractures is critical for enhanced oil recovery. The type of information we are interested in includes fracture orientation, fracture density or spacing and fracture compliance. Widely used seismic methods to characterize fractured reservoirs include shear wave splitting (Hudson, 1981; Vetri *et al.*, 2003) and the amplitude-versus-angle-and-azimuths (AVAz) for P waves (Ruger and Tsvankin, 1997). These methods regard the vertically fractured medium as an equivalent anisotropic medium (HTI) with a horizontal symmetry axis. It is essentially a long-wavelength approximation, which requires there are many fractures per wavelength. If the fracture spacing is close to the wavelength, one needs scattering theory to characterize the fractures (Tura *et al.*, 1992; Gibson *et al.*, 1993; Willis *et al.*, 2006; Burns *et al.*, 2007). Recognizing that the seismic response along and perpendicular to the fracture strike is different, Zhang (2005) used *f-k* analysis to study scattered signals by fractures. The moveout of the scattered waves seen in Figure 1 follows that for the singly backscattered waves when the fracture compliance is similar to that found in laboratory and field studies; e.g. about 10^{-9} m/Pa (see e.g., Bakku *et al.*, 2011). However, complex overlaying geological structures will make the CDP based method less accurate and the uneven illumination can also cause bias in the P wave AVAz analysis. So we need a method, which can account for complex wave phenomenon in the overlying structures. The method should also be able to extract spatially varying fracture information.

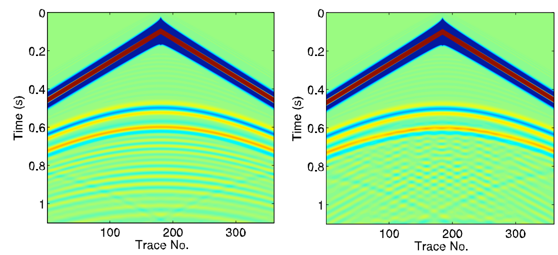

Figure 1. Seismic gathers for a line of receivers parallel (left) and perpendicular (right) to the fracture strike. The background velocity is 2500m/s; finite-difference scheme plus the Schoenberg (1980) linear slip boundary condition is used to simulate wave propagation. The fracture spacing is 60m. The source wavelet is a Ricker with f_0=40 Hz. The fracture compliance is 10^{-9}m/Pa. The vertical extent of the fractures is 40m. The top of the fracture zone is at 1000m depth.

Here we aim at developing such a new scheme, which we call the *double-beam stacking method*. The method is a phase-space method and it can provide spatially varying fracture properties for a wide range of scales. Therefore, it is localized in the spatial as well as in the angular domains (Figure 2), necessary for balancing the uneven illumination. Fracture information within the interference zone (pink area in Figure 2) is extracted.

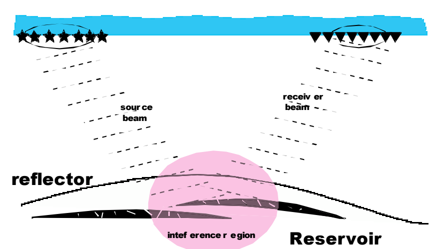

Figure 2. Interference geometry for double beams. Stars are sources and triangles are receivers. The pink ellipse indicates the interference zone within which the fracture properties can be inferred.

Theory and Method: beam interference

The 5-dimensional seismic data can be represented as $p\left(\mathbf{x}_s, \mathbf{x}_g, t\right)$ where symbols $\mathbf{x_s}$, $\mathbf{x_g}$ and t are source location, receiver location and time, respectively. The double-beam stacking is an *f-k* analysis for the localized data, resulting in a 10-dimensional dataset (Figure 2):

$$B\left(\mathbf{x}_s^0,\mathbf{x}_g^0,t_0;\mathbf{k}_s,\mathbf{k}_g,!\ \right)=\#p_w\left(\mathbf{x}_s,\mathbf{x}_g,t\right)e^{i!\,t\quad i\mathbf{k}_s\cdot\mathbf{x}_s\quad i\mathbf{k}_g\cdot\mathbf{x}_g}d^2\mathbf{x}_s d^2\mathbf{x}_g dt$$

where $p_w\left(\mathbf{x}_s,\mathbf{x}_g,t\right)$ is the windowed data

$$p_w\left(\mathbf{x}_s,\mathbf{x}_g,t\right)=p\left(\mathbf{x}_s,\mathbf{x}_g,t\right)w_s\left(\mathbf{x}_s\ !\ \mathbf{x}_s^0\right)w_g\left(\mathbf{x}_g\ !\ \mathbf{x}_g^0\right)w_t\left(t\ !\ t_0\right)$$

where W_s, W_g and W_t are windowing functions for sources, receivers and time, respectively. If the source window width W_s is zero, then we have the case of common source gather. Likewise, if $W_g=0$, we have the common receiver gather. If $W_s\ !\ 0$ and $W_g\ !\ 0$, we get beams. t_0 is the center of the time window and it is determined as the traveltime for waves from the source beam center $\mathbf{x}_s^0$ to the target then reflected back to the receiver beam center $\mathbf{x}_g^0$. The form of the beams can be taken as Gaussian beams (Cerveny, 1982; Hill, 1990; 2001). Moreover, converging beams, which converge to the target from the surface, can also be constructed. Imaging using beams is a very active area of research and many objects such as Wigner optics, X-waves, Bessel beams, Gaussian packets, coherent states, curvelets are intimately related. If the beam widths are infinite, then we have plane wave extrapolation such as the double-square-root operator, plane wave migration, offset plane waves etc. The local angle information for waves is essential to perform the illumination correction (Wu *et al.*, 2004). For the reverse time migration, there is no angle information. To obtain this information, one has to do local angle decomposition of the wavefield. The double-beam stacking is a phase-space method which by its design possesses both space and wavenumber properties of the wavefield. Before we go into the inverse problem of finding fracture orientation and spacing, let us take a look on how fractures scatter seismic waves. We first consider plane wave scattering by periodic structures and then consider scattering of Gaussian beams by periodic structures. Scattering by non-periodic structures is a straightforward extension.

Anaytical Results: plane waves

It has been recognized that the Born approximation is good for understanding the interference patterns shown in Figure 1. Assume we have a set of vertical fractures that are equally spaced along the x direction and let a plane wave be incident upon the fractures from above (Figure 3). The incident field upon the fractures is $\exp\left[i\mathbf{k}_s\cdot\left(\mathbf{r}-\mathbf{r}_s\right)\right]$ and the scattered field at wavenumber $\mathbf{k}_g$ is

$$\begin{aligned}u_{scatt}\left(\mathbf{k}_g,\mathbf{k}_s\right)&\propto\iiint_V\epsilon\left(\mathbf{r}\right)e^{i\mathbf{k}_s\cdot\left(\mathbf{r}-\mathbf{r}_s\right)+i\mathbf{k}_g\cdot\left(\mathbf{r}_g-\mathbf{r}\right)}d^3\mathbf{r}\\&=\tilde{\epsilon}\left(\mathbf{k}_g-\mathbf{k}_s\right)\exp\left[-i\mathbf{k}_s\cdot\mathbf{r}_s+i\mathbf{k}_g\cdot\mathbf{r}_g\right]\end{aligned}$$

where ϵ can be thought as the scattering function caused by fractures. Assuming the fracture system is periodic along x and the spacing between two adjacent fractures is a, its Fourier transform (Poisson summation formula) is

$$\tilde{\epsilon}\left(k_x\right)=\frac{1}{a}\sum_{n=-\infty}^{\infty}\delta\left(k_x^e-n\frac{2\pi}{a}\right)$$

where $\mathbf{k}^e=\mathbf{k}_g-\mathbf{k}_s$ is the *exchange horizontal wavenumber*. If the fracture system is localized at $\mathbf{r}=\mathbf{r}_0$ or non-periodic, i.e.,

$$\hat{\epsilon}\left(\mathbf{r}\right)=W\left(\mathbf{r}-\mathbf{r}_0\right)\epsilon\left(\mathbf{r}\right)$$

where W is some spatial window, the Fourier transform of $\hat{\epsilon}$ is

$$\tilde{\hat{\epsilon}}\left(\mathbf{k}\right)=\tilde{\epsilon}\left(\mathbf{k}\right)*\tilde{W}\left(\mathbf{k}\right)$$

where * denotes convolution. For example, if W is Gaussian, then

$$W\left(\mathbf{r}-\mathbf{r}_0\right)\rightarrow\tilde{W}\left(\mathbf{k}\right)=e^{-\frac{1}{4}\left|\mathbf{k}\right|^2L^2}e^{-i\mathbf{k}\cdot\mathbf{r}_0}.$$

So we have

$$\tilde{\hat{\epsilon}}\left(\mathbf{k}\right)=\int\epsilon\left(\mathbf{k}'\right)W\left(\mathbf{k}-\mathbf{k}'\right)d\mathbf{k}'\cdot$$

Because the spectral spacing of $\epsilon\left(\mathbf{k}\right)$ is $2\pi/a$, the width of spectral leakage of the window should be less than that, i.e., $\frac{2}{L}\ll\frac{1}{m}\frac{2\pi}{a}\Rightarrow\frac{L}{a}\gg\frac{m}{\pi}$. So the aperture L of the window W should be at least a couple of times bigger than the fracture spacing a lest we cannot resolve the spectrum after convolution.

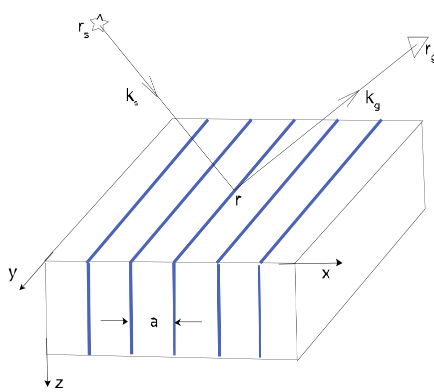

Figure 3. Schematic showing scattering by a set of parallel fracture. Fracture planes are vertical and parallel to the *y* direction.

Let φ be the angle measured clockwise from the -*y* direction and θ_1 and θ_2 are the incident angles for $\mathbf{k}_s$ and

$\mathbf{k}_g$ with respect to the vertical direction, the exchange wavenumber along the direction normal to the fractures is

$$k_x^e = k\left(\sin\theta_2 - \sin\theta_1\right)\sin\varphi$$

where we assume the azimuths for $\mathbf{k}_s$ and $\mathbf{k}_g$ are the same φ. We have

$$\sum_{n=-\infty}^{\infty}\delta\left[k\left(\sin\theta_2 - \sin\theta_1\right)\sin\varphi - n\frac{2\pi}{a}\right]$$
$$= \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\delta\left[\frac{1}{\lambda}\left(\sin\theta_2 - \sin\theta_1\right)\sin\varphi - n\frac{1}{a}\right]$$

and in this case, $\left|k_x^e\right| \le 1$, therefore, λ should be smaller than $2a$, i.e., $\lambda \le 2a$. Along the normal direction $\sin\varphi = 1$, we have for $n = 1$:

$$\left|\sin\theta_2 - \sin\theta_1\right| = \lambda / a$$

and Figure 4 shows the non-zero values of $\tilde{\epsilon}\left(k_x\right)$ for an interference pattern for the double beams.

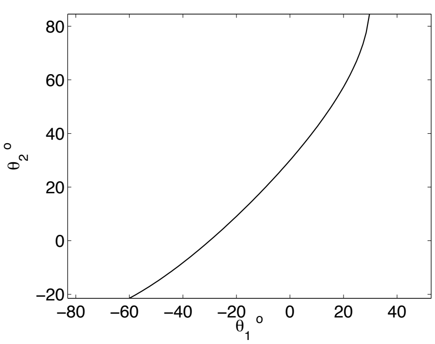

Figure 4 Expected stacking pattern for two plane waves at $\lambda / a = 0.5$.

Anaytical Results: Gaussian beams

If we use Gaussian windowing for the space and no time windowing, we obtain the interference pattern for Gaussian beams. In addition, if we use a Gaussian windowing for the time, we obtain the fracture interference pattern for the Gaussian packets. In this section, we present the case for the Gaussian beams in a locally homogeneous medium. If the medium is inhomogeneous, one can use the generalized formulation for Gaussian beams/packets using the dynamical ray tracing (Cerveny, 1982; Hill, 2001). However, in a local homogeneous medium, the double-beam stacking reads:

$$B\left(\mathbf{p}_s,\mathbf{p}_g,\omega\right) = \frac{1}{\Delta}\sum_{n=-\infty}^{\infty}\int_{\mathbf{k}_g}\int_{\mathbf{k}_s}\frac{\eta_{sz}\eta_{gz}}{k_{sz}k_{gz}}\delta\left(\xi - n\frac{2\pi}{a}\right)$$
$$\times e^{-\frac{1}{4}\left|\mathbf{k}_s+\mathbf{p}_s\right|^2 L_s^2} e^{-\frac{1}{4}\left|\mathbf{k}_g-\mathbf{p}_g\right|^2 L_g^2} e^{i\left(k_{sz}+k_{gz}\right)z} e^{i\left(\mathbf{k}_g+\mathbf{k}_s\right)\cdot\mathbf{h}} d^2\mathbf{k}_g d^2\mathbf{k}_s$$

where $\xi = k_{gx} - k_{sx}$ is the exchange horizontal wavenumber in the x direction and vertical wavenumbers $k_{sz} = \sqrt{k^2 - k_{sx}^2 - k_{sy}^2}$ and $k_{gz} = \sqrt{k^2 - k_{gx}^2 - k_{gy}^2}$. $\mathbf{h}$ is the vector from the source beam center to the receiver beam center and $h = \left|\mathbf{h}\right|$. $\mathbf{p}_s$ and $\mathbf{p}_g$ are the horizontal wavenumbers for the incident and reflected Gaussian beams, respectively; and η_{sz} and η_{gz} are their corresponding vertical wavenumbers. We can use the paraxial approximation to expand the vertical wavenumbers around $\mathbf{k}_s = \mathbf{p}_s$ and $\mathbf{k}_g = \mathbf{p}_g$, respectively, and then integrate analytically. The integral is composed of three Gaussian integrals that are easy to be calculated. If the fracture spacing is on the order of the wavelength, only the terms with $n = 0, \pm 1$ are significant and represent propagating waves. Other terms are evanescent waves, which cannot be recorded in the reflection geometry. $n = 0$ corresponds to specular reflection and $n = \pm 1$ corresponds to the traditional "grating" diffractions in optics. Next, we investigate how the beam interference pattern *B* changes with (1) different *h;* (2) different fracture spacing *a*; and (3) different azimuths.

It is evident that if the beam widths are infinite, the double-beam interference pattern along the symmetry axis is the same as that of the plane waves (Figure 4). When the beam widths are finite and we vary the distance *h*, we obtain an energy packet from the double-beam stacking, moving along the interference curve for the plane waves as shown in Figure 5.

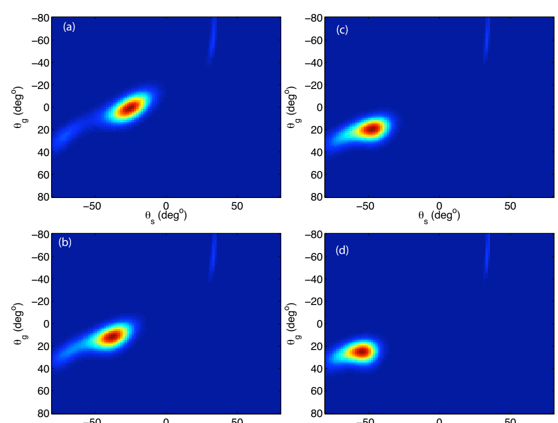

Figure 5. Gaussian beam interference pattern for different source-receiver distances (a) $h = 100$ m; (b) $h = 200$ m; (c) $h = 300$ m and (d) $h = 400$ m. $L_s = L_g = 100$ m and z=400m; $f = 60$ Hz and $a = 100$ m. The line connecting the source and receiver beam centers is along the x axis. $n = -1$. The background velocity is 2500m/s.

The effect of the fracture spacing on the beam interference pattern is dramatic. To demonstrate this numerically, we use a set of parallel fractures whose symmetry axis is the x axis. We assume the depth to the top of the fracture is z =1000m. The distance between the beam centers is 200m. $L_s = L_g = 100$ m. The orientation of the source and receiver beam centers is the same as the fracture symmetry axis. The main energy of the beam interference shifts to different locations for different fracture spacing (Figure 6).

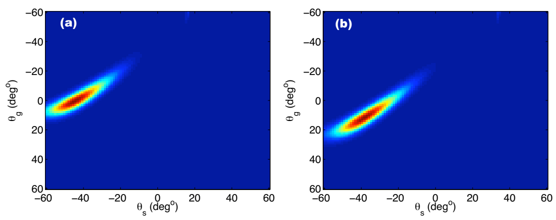

Figure 6. Gaussian beam interference pattern for (a) $a = 60m$ and (b) $a = 100m$ at $n = -1$.

Although the spacing of the fractures is important, the orientation of the fracture is another important property, which is related to the preferential direction of permeable flow. Azimuthally, the beam interference pattern also varies (**Figure 7**).

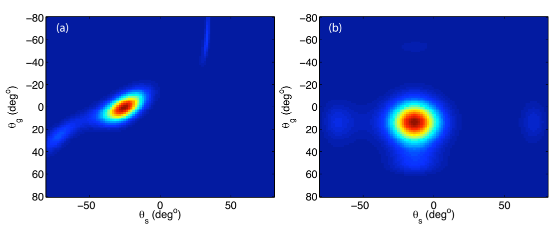

Figure 7. Gaussian beam interference pattern for source-receiver azimuth (a) $\varphi = 0$ degrees (perpendicular to fractures) and (b) $\varphi = 90$ degrees (parallel to fractures). $L_s = L_g = 100$ m and z=400m; $f = 60$ Hz and $a = 100$ m. $n = -1$.

Conclusions

We have intoduced a *double-beam stacking* method, in which the interference of two beams produce a characteristic pattern that depends on fracture spacing and orientation. Analytical results for the cases of interfering plane waves and Gaussian beams are derived. When the beam width is infinite, we obain interferometric plane waves. Our theory is based on a single scattering model. The use of single-scattering is consistent with our observations made using the 3D finite-difference algorithm with the Schoenberg's linear slip boundary condition and fracture compliances that are of similar in magnitude to those measured in the field and the laboratory. It shows that in the reflection data the interference pattern (or the moveout) resembles that of the singly scattered waves. The double-beam stacking pattern changes with the beam center distance, source-receiver azimuth with respect to the fracture orientation and the fracture spacing. However, it is a scattering method, working most effectively when the wavelength and the fracture spacing are comparable (on the order of 10s of meters in typical seismic surveys). For closely spaced fractures (e.g., on the order of meters), effective-medium methods should be used.

Acknowledgments

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