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INCREMENTAL COSTS AND OPTIMIZATION OF IN-CORE FUEL MANAGEMENT OF NUCLEAR POWER PLANTS

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ABSTRACT

This thesis is concerned with development of methods for optimizing the energy production and refuelling decision for nuclear power plants in an electric utility system containing both nuclear and fossil-fuelled stations. The objective is to minimize the revenue requirements for refuelling the power plants during the planning horizon; the decision variables are the energy generation, reload enrichment and batch fraction for each reactor cycle; the constraints are that the customer's load demand, as well as various other operational and engineering requirements be satisfied. This problem can be decomposed into two sub-problems. The first sub-problem is concerned with scheduling energy between nuclear reactors which have been fuelled in an optimal fashion. The second sub-problem is concerned with optimizing the fuelling of nuclear reactors given an optimized energy schedule. These two sub-problems when solved iteratively and interactively, would yield an optimal solution to the original problem.

The problem of optimal energy scheduling between nuclear reactors can be formulated as a linear program. The incremental cost of energy is required as input to the linear program. Three methods of calculating incremental cost are considered: the Rigorous Method, based on the definition of partial derivatives, is accurate but time consuming; the Inventory Value Method and the Linearization Method, based respectively on equations of inventory evaluation and linearization, are less accurate, but efficient. The latter two methods are recommended for the early stages of optimization.

The problem of optimizing the fuelling of nuclear reactors has been solved for two cases: the special case of steady state operation, and the general case of non-steady-state operation. The steady-state case has been solved by simple graphic techniques. The results indicate that reactors should be refuelled with as small a batch fraction as allowed by burnup constraints. The non-steady case has been solved by polynomial approximation, in which the objective function as well as the constraints are approximated by a sum of polynomials. The results indicate that the final selection of an optimal solution from a set of sub-optimal solutions is primarily based on engineering considerations, and not on economics considerations.

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CHAPTER 1.0
SUMMARY AND CONCLUSIONS

1.1 Framework for Analyzing the Overall Optimization Problems of Mid-Range Utility Planning

This thesis is concerned with development of methods for optimizing the energy production and refuelling decision for nuclear power plants in an electric utility system containing both nuclear and fossil-fueled stations. The time period under consideration is the so-called mid-range period from five to ten years, within which nuclear fuel management can be varied, for available nuclear plants.

The overall optimization problem of mid-range utility planning can be formulated as follows: given a load forecast for a given electric utility over the span of the planning horizon, given the composition of the electric utility in terms of the capacity, type and location of each generating unit, find the optimal schedule of operation in terms of energy produced by each plant and the reload enrichments and batch fractions for each nuclear plant such that the revenue requirements are minimized and the system constraints and demands are satisfied. The revenue requirement is chosen as the objective function, because it is favored by many electric utilities (CE1, AEP1) and is relatively simple to calculate.

The overall optimization problem is first decomposed into two sub-problems: the first sub-problem consists of finding maintenance and refuelling schedules that satisfy the system constraints; the second sub-problem consists of finding the optimal energy production, reload enrichments and batch fractions for a given time schedule. A computer program for solving the first sub-problem has been developed (CE2). The second sub-problem, formally called system optimization for a given refuelling and maintenance time schedule, is further decomposed into two second level sub-problems.

The first sub-problem at the second level is formally called the optimal energy scheduling problem and consists of finding the optimal energy production of each plant.

The second sub-problem at the second level is formally called the nuclear in-core optimization problem and consists of finding the optimal reload enrichments and batch fractions given an optimal schedule of energy production.

These two sub-problems are to be solved interactively and iteratively until a converged solution of energy production from each plant reload enrichments and batch fractions are obtained. Then the same procedures are repeated for every feasible maintenance and refuelling time schedule. The schedule with the lowest revenue requirement is optimal.

The optimal energy scheduling problem can be formulated mathematically as

Minimize $\overline{TC}^{s} = \overline{TC}^{s\,0} + \sum_{r}^{R}\sum_{j}^{J}\lambda_{rj}\cdot(E_j^r - E_j^{r\,0})$ (1.1)

with respect to E_j^r

Subject to constraints $\sum_{r}^{R} E_j^r = E_j^s$ (1.2)

$E_j^r \leq \Delta t_j \cdot P^r \cdot 8760.$ (1.3)

Where $\overline{TC}^{s}$ = revenue requirement for the system (in \$)

$\overline{TC}^{so}$ = revenue requirement for the system evaluated for an initial feasible solution (in \$)

E_j^r = energy production of unit r in time period j (in MWHe)

E_j^{ro} = energy production for an initial feasible solution (in MWHe)

E_j^s = system demand for time period j (in MWHe)

ΔT_j = duration of time period j (in hours)

P^r = capacity of unit r (in MWe)

λ_{rj} = incremental cost of energy for unit r (in \$/MWHe) and period j.

The crux of the optimal energy scheduling problem is how to calculate the incremental cost.

For fossil fuel generating units, the incremental cost of energy is given simply by the discounted fuel cost for an additional increment of undiscounted energy production. For nuclear generating units, the incremental cost of energy λ_{rj} is given by the change in the revenue requirement for unit r over the entire planning horizon due to an additional increment of energy generated in time period j while holding all the energy production in each of the remaining time periods constant.

$$\lambda_{rj} \simeq \frac{\overline{TC}(\vec{\varepsilon}^{+},\vec{f}^{+})-\overline{TC}(\vec{\varepsilon}^{*}.\vec{f}^{*})}{\Delta E_j^r} \qquad (1.4)$$

Where ε^* and f^* are the optimal reload enrichments and batch fractions for the initial feasible solution E_j^r. ε^+ and f^+ are the optimal reload enrichments and batch fractions for the perturbed solution $E_j^r + \Delta E_j^r$.

For nuclear reactors, the revenue requirement depends mainly on the total energy generated in a cycle, and only weakly on the energy generation pattern within each cycle in which the generation actually takes place. Therefore, under optimal conditions all the incremental costs of energy production within a given cycle have the same value.

$$\lambda_{rj} = \lambda_{rc} \qquad \text{for all } j_{rc} \leq j < j_{rc+1} \tag{1.4a}$$

Various methods of calculating λ_{rc} will be described in Sections 1.2 , 1.4 , 1.5 and 1.8 and in Chapters 3,5,6,9 of the thesis. However, except in Chapter 3 where the optimal energy scheduling problem is solved for a particularly simple case, the application of incremental cost calculation in the optimal energy scheduling problem is not considered in detail in this thesis. Use of incremental costs in optimizing electric generation by nuclear plants is discussed in detail by Deaton (D1).

The nuclear in-core optimization problem can be formulated mathematically as

$$\text{Minimize} \qquad \overline{TC}^{r}\,(E_j^r\,,\ \varepsilon_c^r,\ f_c^r) \tag{1.5}$$

with respect to ε_c^r , and f_c^r

Subject to the constraints

$$\sum_{j_{rc}}^{j_{rc+1}-1} E_j^r = E_c^r \tag{1.6}$$

$$F_c^r(\vec{\varepsilon}^r,\vec{f}^r) = E_c^r \qquad (1.7)$$

$$B_c^r(\vec{\varepsilon}^r,\vec{f}^r) = B^0 \qquad (1.8)$$

where ε_c^r = reload enrichment for reactor r cycle c

$\vec{\varepsilon}^r$ = vector of ε_c^r

f_c^r = batch fraction for reactor r cycle c

$\vec{f}^r$ = vector of f_c^r

j_{rc} = first time period in cycle c

E_c^r = energy for reactor r cycle c

F_c^r = a function of $\vec{\varepsilon}^r$ and $\vec{f}^r$

B_c^r = average discharge burnup for reactor r cycle c

B^0 = maximum allowable discharge burnup.

The general nuclear in-core optimization problem considers variation of both reload enrichments and batch fractions in arriving at the optimum solution. Before solving this general problem, the special problem of varying reload enrichments alone with fixed batch fractions will be considered. This special problem is much easier to solve and has practical applications. Section 1.2 and 1.4 deal with this special problem for steady-state and non-steady state cases respectively. Section 1.5 and 1.9 inclusive deals with the general problem; first with the steady-state case, and later the non-steady state cases.

Two reactors of different sizes are taken as examples: the Zion type 1065 MWe PWR and the San Onofre type 430 MWe PWR. The depletion code CELL-CORE (B1,K1) is chosen to be the standard tool of analysis; the costing code MITCOST1(W1) and

and depletion-costing code COCO(W1) are used interchangeably for the economics calculation.

1.2 Optimal Energy Scheduling Between Two Pressurized Water Reactors of Different Sizes Operating in Steady-State Conditions.

The problem analyzed in that of optimizing energy production from two reactors each refuelled at pre-specified dates with fixed batch fractions after steady-state conditions have been reached. The optimum condition is reached when the incremental cost of energy from a steady-state cycle in one reactor equals the corresponding incremental cost for the second reactor. These incremental costs were obtained by calculating the change in revenue requirement for a steady-state cycle per unit change in cycle energy.

The optimal way of operating this two reactor system as demonstrated in Section 3.4 is to have both reactors generate energy at the same incremental cost. Figure 1.2 shows the

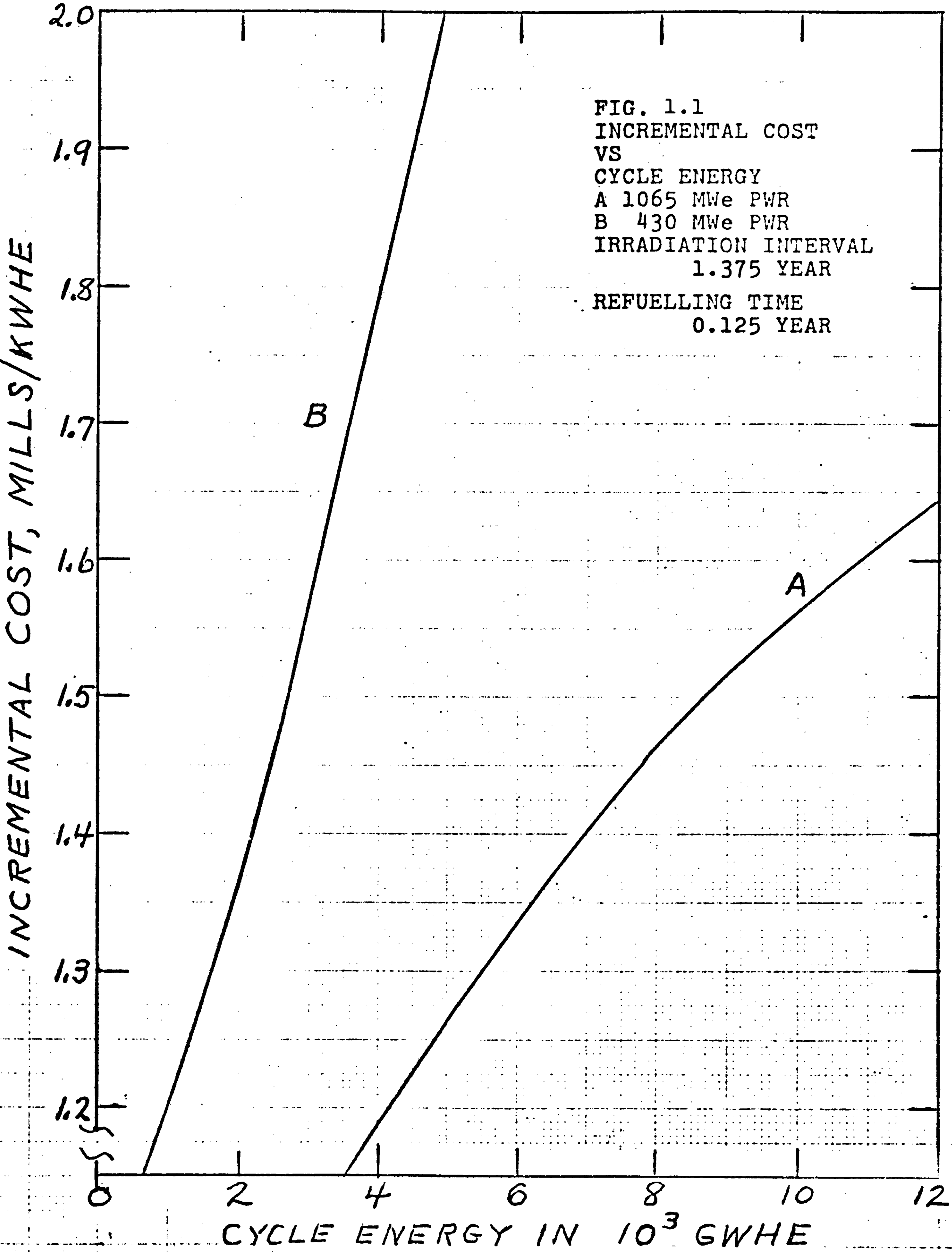


FIG. 1.1
INCREMENTAL COST
VS
CYCLE ENERGY
A 1065 MWe PWR
B 430 MWe PWR
IRRADIATION INTERVAL
1.375 YEAR
REFUELLING TIME
0.125 YEAR
INCREMENTAL COST, MILLS/KWHE
CYCLE ENERGY IN 10^3 GWHE
A
B
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
0
2
4
6
8
10
12

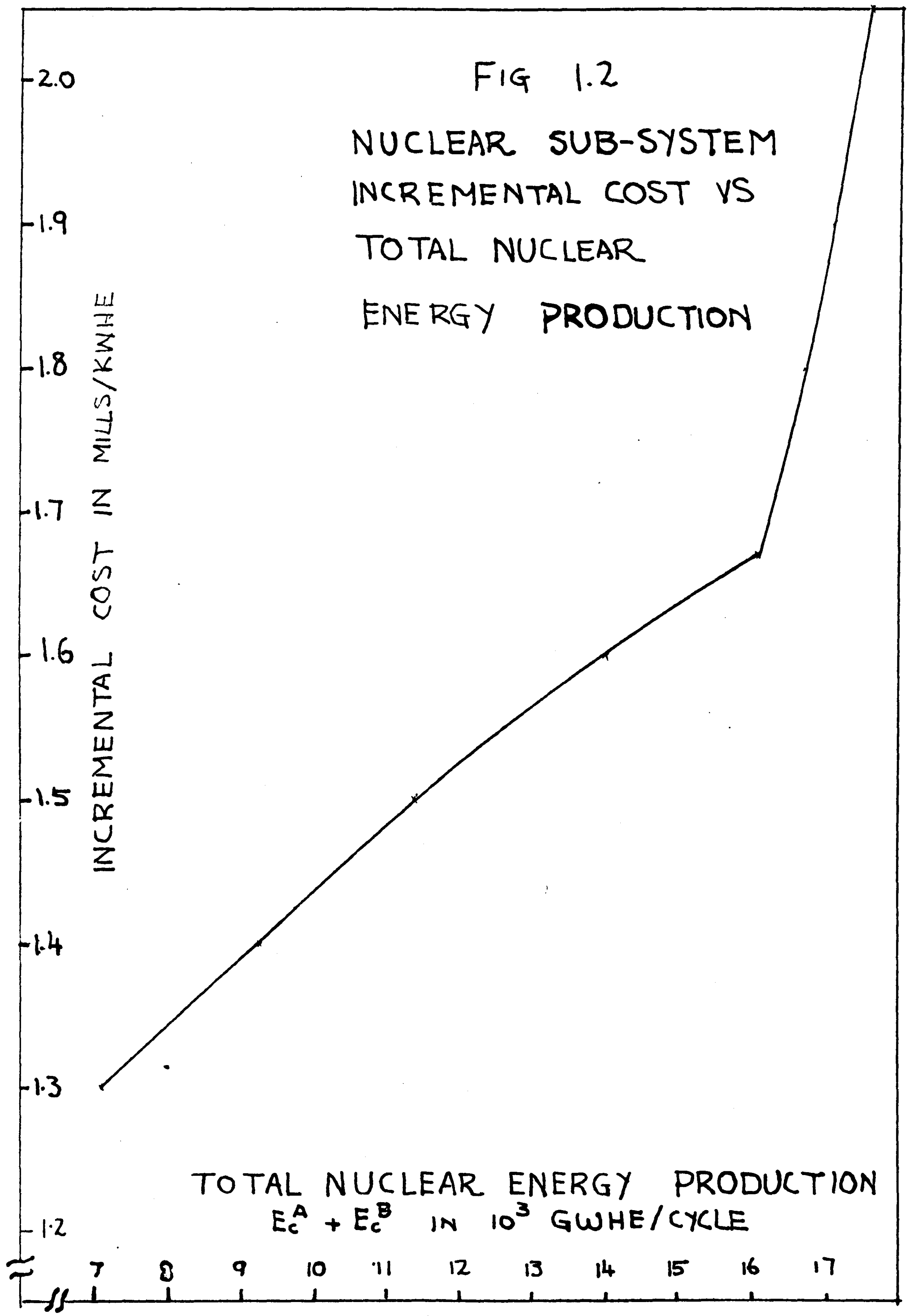


FIG 1.2
NUCLEAR SUB-SYSTEM
INCREMENTAL COST VS
TOTAL NUCLEAR
ENERGY PRODUCTION
INCREMENTAL COST IN MILLS/KWHE
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
TOTAL NUCLEAR ENERGY PRODUCTION
$E_c^A + E_c^B$ IN 10^3 GWHE/CYCLE
7
8
9
10
11
12
13
14
15
16
17

incremental cost versus the sum of energies generated by the two reactors under the equal incremental cost rule. The discontinuity point of the curve indicates that the Zion reactor has reached its capacity limit, and from then on any load increments goes to San Onofre. This curve can be interpreted as the supply curve of the system. If the demand curve is given, the intersection of the two curves give the value of the equilibrium incremental cost, which can be used in turn to calculate the optimal energy production for each of the reactors. A detailed discussion of internal supply and demand curve is presented in Widmers' thesis(W2). Once the optimal energy production of each reactor is know, the corresponding reload enrichment can be found from Figure 1.3.

For this simple problem of steady-state operations, fixed batch fractions and specified time schedule, the problem of optimal energy scheduling and nuclear in-core optimization can be solved easily by a set of graphs. For non-steady state operations, however, the calculation of revenue requirement and incremental cost is much more complex. The following section indicates different ways of calculating the objective function under non-steady state conditions.

1.3 Calculation of Objective Function for Non-Steady State Operations

Under non-steady state operating conditions, the physical state of the reactor does not go through repetitive cycles. Consequently, the end state of the reactor at the end of

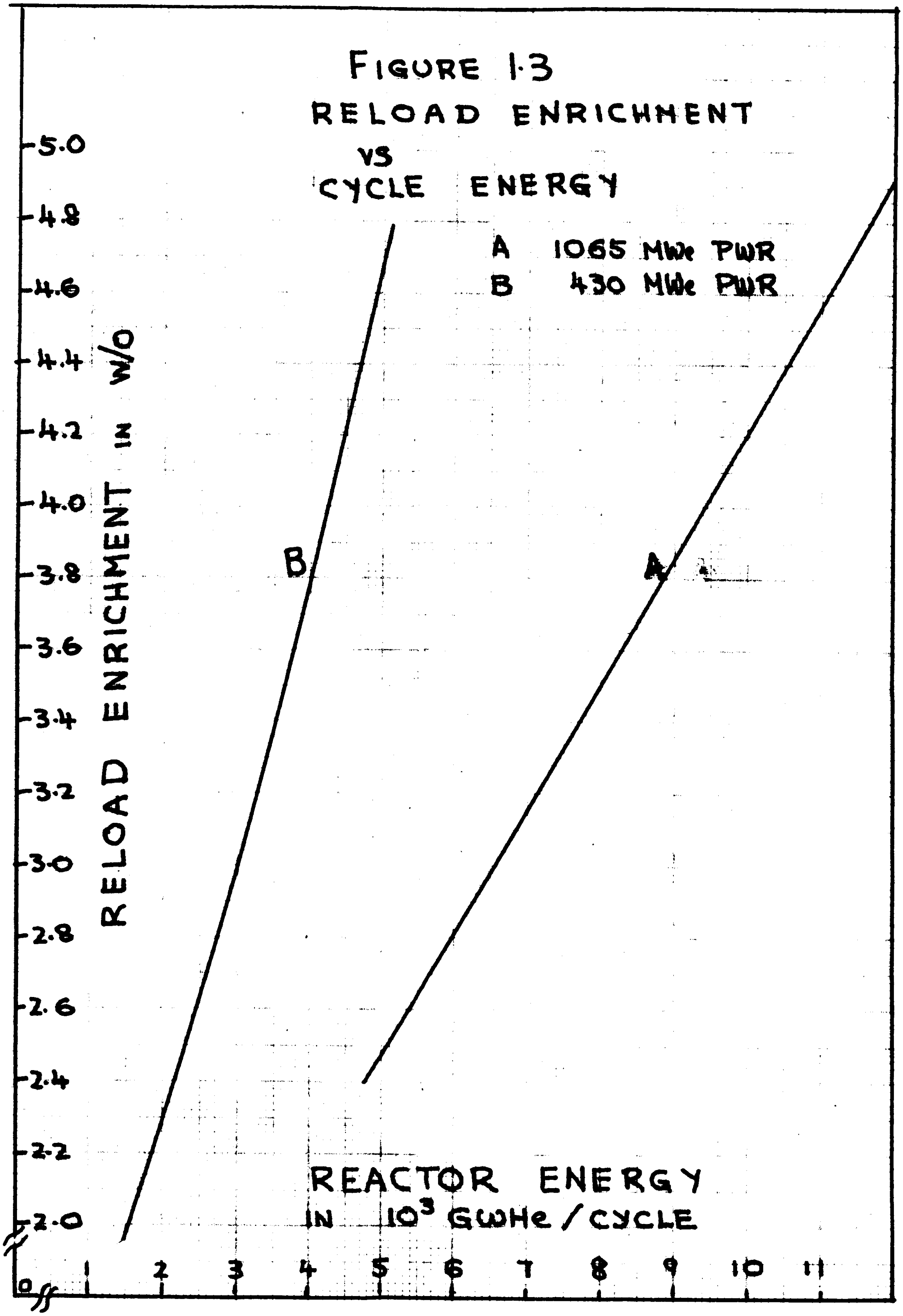


FIGURE 1.3
RELOAD ENRICHMENT
VS
CYCLE ENERGY
A 1065 MWe PWR
B 430 MWe PWR
RELOAD ENRICHMENT IN W/O
REACTOR ENERGY
IN 10³ GWHe / CYCLE
5.0
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
0
1
2
3
4
5
6
7
8
9
10
11
A
B

the planning horizon will not necessarily be the same as the initial state at the beginning of the planning horizon. Consequently, in order for the optimization to be effective, an "end-effect" correction must be incorporated into the calculation of the objective function. The purpose of the end-effect correction is to assign values to core inventories which result in an objective function that varies only with energy production within the planning horizon and not with energy production in the neighboring time periods. If this can be achieved, then optimization can be performed for each individual planning horizon; the collection of such optimal solutions would be the same as the optimal solution for the entire life of the reactor obtained by a one-shot calculation.

The object of the end-effect correction can be stated mathematically as follows:

Let $\overline{TC}_{\infty}$ be the revenue requirement for the entire life of the reactor. Let $\overline{TC}_{I}$ be the revenue requirement for planning horizon I which includes end-effect corrections. The object of the end-effect correction is to equate

$$\frac{\partial \overline{TC}_{\infty}}{\partial E_c^r} = \frac{\partial \overline{TC}_{I}}{\partial E_c^r} \qquad \text{for } E_c^r \text{ within planning horizon I} \tag{1.10}$$

This requirement can be called the condition of "equalized incremental cost."

Two different methods have been investigated for evaluating the end-effect correction. The Inventory Value Method evaluates the worth of the nuclear core as the market value of uranium and plutonium plus a fraction of fuel fabrication, and post irradiation costs. The fraction of fuel fabrication costs assigned to inventory value is $\frac{(N-n)}{N}$, where N is the total number of cycles a batch of fuel remains in the reactor and n is the number of cycles the fuel has been in the reactor at the time the inventory is to be valued. Similiarly, the accrual of post irradiation costs is treated by deducting n/N fraction of their total from the inventory value.

The Unit Production Method evaluates the worth of the nuclear core as the book value of the core based on straight line depreciation according to energy production. In order to obtain the salvage value of the core, the reactor is operated past the end of the planning horizon under some prescribed refuelling strategy until all the batches to be evaluated have been discharged and their salvage value determined.

Table 1.1 compares the incremental costs calculated by the Inventory Value Method and the Unit Production Method with the exact value. The Unit Production Method gives more accurate incremental cost than the Inventory Value Method. However, the Unit Production Method requires more depletion calculations and is very sensitive to the

Table 1.1

Comparison of Exact Incremental Cost with Incremental Cost Calculated by Two Approximate Methods

Method	Incremental Cost for Cycle 1 Mills/KWHe		
	Exact	Approximate	
		Inventory Value	Unit Production
ΔE_1=1029GWHt	1.39	1.43	1.40
=2050GWHt	1.38	1.44	1.40

prescribed refuelling strategy after the planning horizon. Hence, the Inventory Value Method is recommended for use to correct for end effects.

Having a method to correct for end-effects, and consequently an acceptable method for calculating the objective function, efficient ways of calculating approximate incremental costs and reload enrichments for any required set of energies are described in Section 1.4.

1.4 Calculation of Incremental Cost of Nuclear Energy λrc and Reload Enrichments for a Given Set of Required Energies and For Fixed Reload Batch Fraction

Three methods to calculate the incremental cost of nuclear energy λ_{rj} will be described. The first, rigorous, method is based on the definition of λ_{rj}; it is accurate but time consuming. The second method is based on inventory evaluation techniques; it takes less time, but is less accurate. The third method is based on an approximate linear relationship between reload enrichment and cycle energy and again takes less time than the rigorous method but is less accurate.

1.4.1 Rigorous Method

According to Equations (1.4) and (1.4a), the incremental cost of nuclear energy is defined as the partial derivative of the revenue requirement with respect to cycle energy,

$$\lambda_{rc} = \frac{\partial \overline{TC}^r}{\partial E_c^r} \Bigg|_{E_c^r}, \tag{1.10a}$$

which can be replaced by the forward difference

$$\lambda_{rc} \simeq \frac{\overline{TC}^r(E_1^{or}, E_2^{or}, \ldots E_c^{or}+\Delta E, E_{c+1}^{or} \ldots) - \overline{TC}^r(E_1^{or}, E_2^{or}, \ldots E_c^{or}, E_{c+1}^{or} \ldots)}{\Delta E} \quad (1.11)$$

If $\overline{TC}^r$ is known for two values of E_c^r , (e.g. in Equation (1.11) for E_c^{or} and $E_c^{or}+\Delta E$), while all the other E_c^r are constant, λ_{rc} can be evaluated quite easily. However, to obtain the correct enrichments which permit E_c^r to change while all other energies E_c^r, remain unchange is time-consuming and expensive. The correct enrichment for each cycle must be found by trial. To determine all the λ_{rc} in an m-cycle problem requires about $\frac{3m(m+1)}{2}$ trials, using about three trials per cycle.

1.4.2 Inventory Value Method

In Section 1.3, the Inventory Value Method has been shown to evaluate correctly the end effect and gives fairly accurate values of incremental cost. If the Inventory Value Method is applied at the end of the cycle for which incremental cost calculation is desired, then incremental cost of nuclear energy for that cycle can be obtained by analyzing the change in the revenue requirement up to that cycle as energy production changes in that cycle. Thus, all later cycles need not be analyzed.

To calculate all the λ_{rc} in a planning horizon, one can proceed in the forward direction by first changing the energy production of Cycle 1, applying the Inventory Value Method and analyzing the change of revenue requirement up to Cycle 1. This would be repeated for Cycle 2 and so on until all the cycles have been analysed.

For an m-cycle problem, only 2m depletion calculations are required to calculate all the incremental costs.

1.4.3 Linearization Method

This method makes use of the chain rule of partial differentiation

$$\left.\frac{\partial \overline{TC}^r}{\partial \varepsilon_c^r}\right|_{\varepsilon_{c'}^r} = \sum_{c''} \left.\frac{\partial \overline{TC}^r}{\partial E_{c''}^r}\right|_{E_{c'}^r} \cdot \left.\frac{\partial E_{c''}^r}{\partial \varepsilon_c^r}\right|_{\varepsilon_{c'}^r} = \sum_{c''} \lambda_{rc''} \left.\frac{\partial E_{c''}^r}{\partial \varepsilon_c^r}\right|_{\varepsilon_{c'}^r} \tag{1.12}$$

When all $\frac{\partial \overline{TC}^r}{\partial \varepsilon_c^r}$ and $\frac{\partial E_{c''}^r}{\partial \varepsilon_c^r}$ are known, then λ_{rc} can be found by matrix inversion. Evaluation of $\frac{\partial \overline{TC}^r}{\partial \varepsilon_c^r}$ and $\frac{\partial E_{c''}^r}{\partial \varepsilon_c^r}$ is easier than λ_{rc} because reload enrichment E_c^r is an explicit variable that can be controlled. The calculation of each $\frac{\partial \overline{TC}^r}{\partial \varepsilon_c^r}$ requires only (m-c+1) depletion calculations for an m-cycle problem. Hence, to calculate all the λ_{rc}, requires only $\frac{m(m+1)}{2}$ depletion calculations. The relationships between revenue requirement for indefinite planning horizon $\overline{TC}_\alpha$, for finite planning horizon $\overline{TC}$, for the first cycle $\overline{TC}_1$, various batches and cycles are shown schematically on Figure

1.4. Notice that the exact incremental cost given in Table 1.2 is based on the revenue requirement for the indefinite planning horizon, while the Rigorous method is based on the revenue requirement for the finite planning horizon $\overline{TC}^{r}$.

The values of λ_{rc} determined by the three methods for refuelling with fixed batch fraction and variable enrichment are compared in Table 1.2 for the 1065 MWe Zion reactor. The first two cases given below involve perturbations from steady state three-zone operation with 3.2w/o enriched feed, giving E = 7416.5 GWHe/cycle. The magnitude of perturbation ΔE_1 of case 2 is twice as large as that of case 1. The third case involves perturbation from a three-zone transient energy mode of operation of the reactor. The Inventory Value Method is accurate up to ± 4% of the "true" value given by the Rigorous method. The Linearization Method is accurate to ± 4%. For the first few steps of the optimization, when speed is more important than accuracy, the Inventory Value Method or the Linearization Method is recommended. Only at the end of the optimization would one consider using the Rigorous method for its improved accuracy.

Two methods of determining reload enrichments for a given set of required energies and for fixed reload batch fraction will be described. The first method determines

FIGURE 1·4

RELATIONSHIPS BETWEEN THE VARIOUS REVENUE REQUIREMENTS BATCH NUMBER AND CYCLE NUMBER

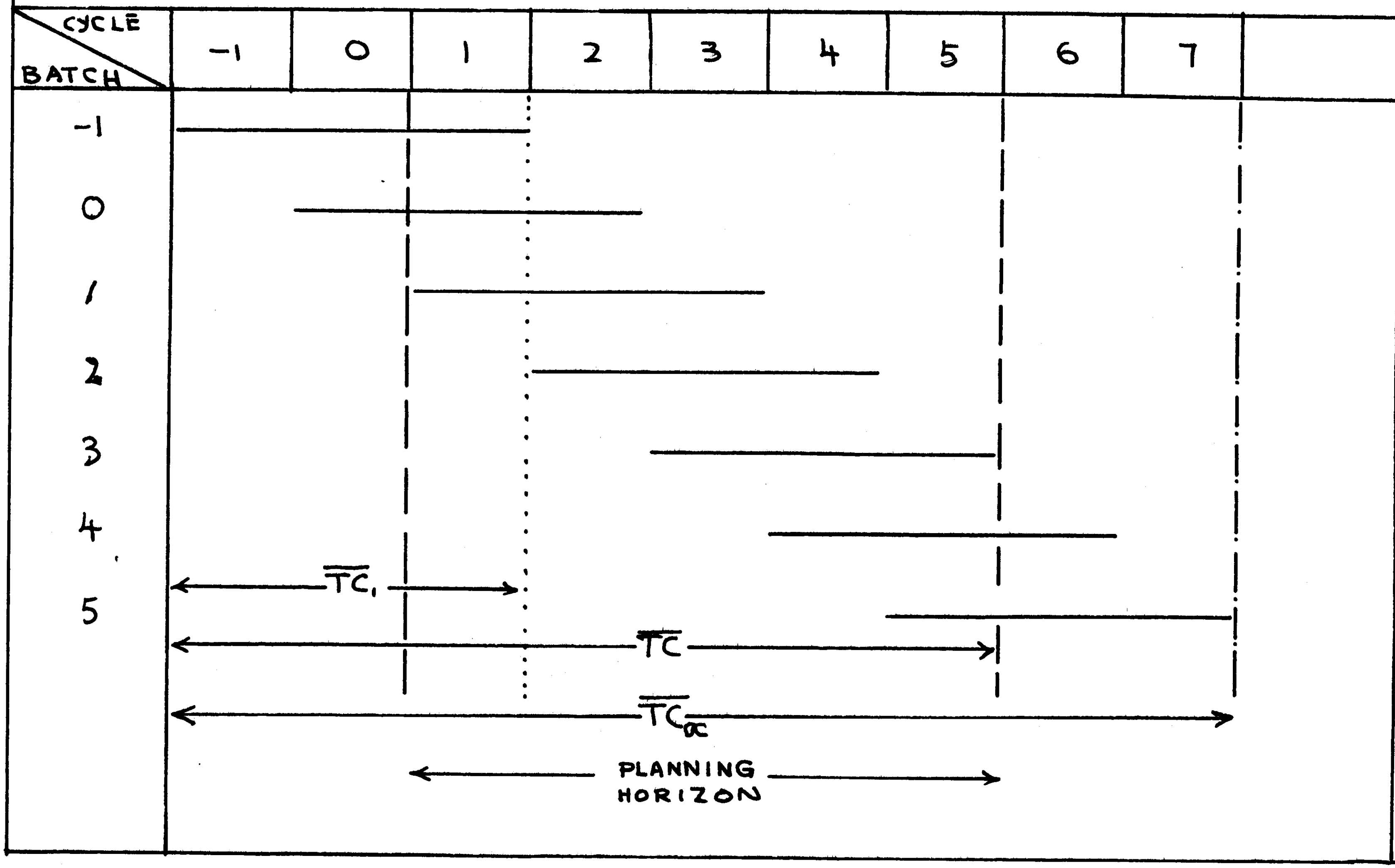


Table 1.2

Incremental Cost of Energy Calculated

by Three Methods

	Incremental Cost by Rigorous Method	Incremental Cost by Linearization Method	Incremental Cost by Inventory Value Method
Case 1	1.42	1.37	1.43
Case 2	1.40	1.37	1.44
Case 3	1.37	1.37	1.43

reload enrichments by trial and error. For a given initial state, two depletion calculations are carried out for one cycle using two values of reload enrichments. The trial enrichment for a given value of cycle energy is then obtained by interpolating between the two values of reload enrichments and the corresponding two values of cycle energies. Three depletion calculations are usually sufficient for any one cycle. Hence, for an m-cycle problem, 3m trials are needed.

The second method determines reload enrichments by an approximate linear relationship between cycle energy and reload enrichment.

$$E_c^r \approx E_c^{0r} + \sum_{c'} \frac{\partial E_c^r}{\partial \varepsilon_{c'}^r} \cdot \{\varepsilon_{c'}^r - \varepsilon_{c'}^{0r}\} \tag{1.13}$$

Since all the coefficients $\frac{\partial E_c^r}{\partial \varepsilon_c^r}$ are made available by the **Linearization** Method in the calculation of incremental cost, the determination of $E_{c'}^r$ is a straightforward operation using matrix inversion. Table 1.3 shows values of reload enrichments calculated by the Trial Method and **Linearization** Method for different sets of cycle energies. Agreement between the two methods is excellent. Hence, either method can be used.

Table 1.3

Reload Enrichments Calculated by (1) Trial Method and (2) Linearization Method

Case 1

		1	2	3	4	5
Cycle		1	2	3	4	5
Energy E_i in 10^3GWHt		22.964	21.935	21.929	21.928	21.933
Enrichment ε_i	(1)	3.359	3.054	3.174	3.196	3.133
(w/o)	(2)	3.360	3.046	3.181	3.191	3.132

Case 2

Cycle		1	2	3	4	5
Energy E_i in 10^3GWHt		23.985	21.919	21.906	21.937	21.970
Enrichment ε_i	(1)	3.557	2.941	3.186	3.235	3.106
(w/o)	(2)	3.557	2.928	3.197	3.225	3.108

Case 3

Cycle		1	2	3	4	5
Energy E_i in 10^3GWHt		23.085	21.535	23.605	20.995	22.164
Enrichment ε_i	(1)	3.359	2.975	3.545	2.833	3.286
(w/o)	(2)	3.360	2.979	3.534	2.836	3.287

1.5 Calculation of Incremental Cost and Nuclear In-Core Optimization for Reactors Operating Under Steady-State Conditions

Starting from this section, batch fractions as well as the reload enrichments are allowed to vary; only refuelling times and energies are fixed. This section deals with reactors operated under steady-state conditions. Hence, there is only one reload enrichment variable and one batch fraction variable for all the cycles. The problem of nuclear in-core optimization under this special circumstance is stated as follows:

$$\begin{aligned} &\text{minimize } \overline{TC}(E^s,\varepsilon,f) \text{ for a given } E^s \\ &\text{with respect to } \varepsilon \text{ and } f \\ &\text{subject to constraints } F(\varepsilon,f) = E^s \\ &\qquad\qquad\qquad\qquad\qquad\quad B(\varepsilon,f) < B^o \end{aligned}$$

the subscripts r, c are omitted because only one reactor is considered and all cycles are the same under steady state conditions. The revenue requirement for the first cycle is chosen to be the objective function.

For any combination of ε and f, the reactor generates a certain energy E^s at a cost $\overline{TC}$. By plotting $\overline{TC}$ vs E^s for all possible combinations of ε and f, the optimal pair can be found directly.

Figure 1.5 shows value of $\overline{TC}$ vs E^s for different combination of ε and f for a Zion type 1065 MWe PWR refuelled in a modified scatter manner. At cycle energies above 7000 Gwhe, a batch fraction f = 0.33 results in lowest revenue requirement. At cycle energies below 7000, a batch fraction of f = 0.25 is preferable.

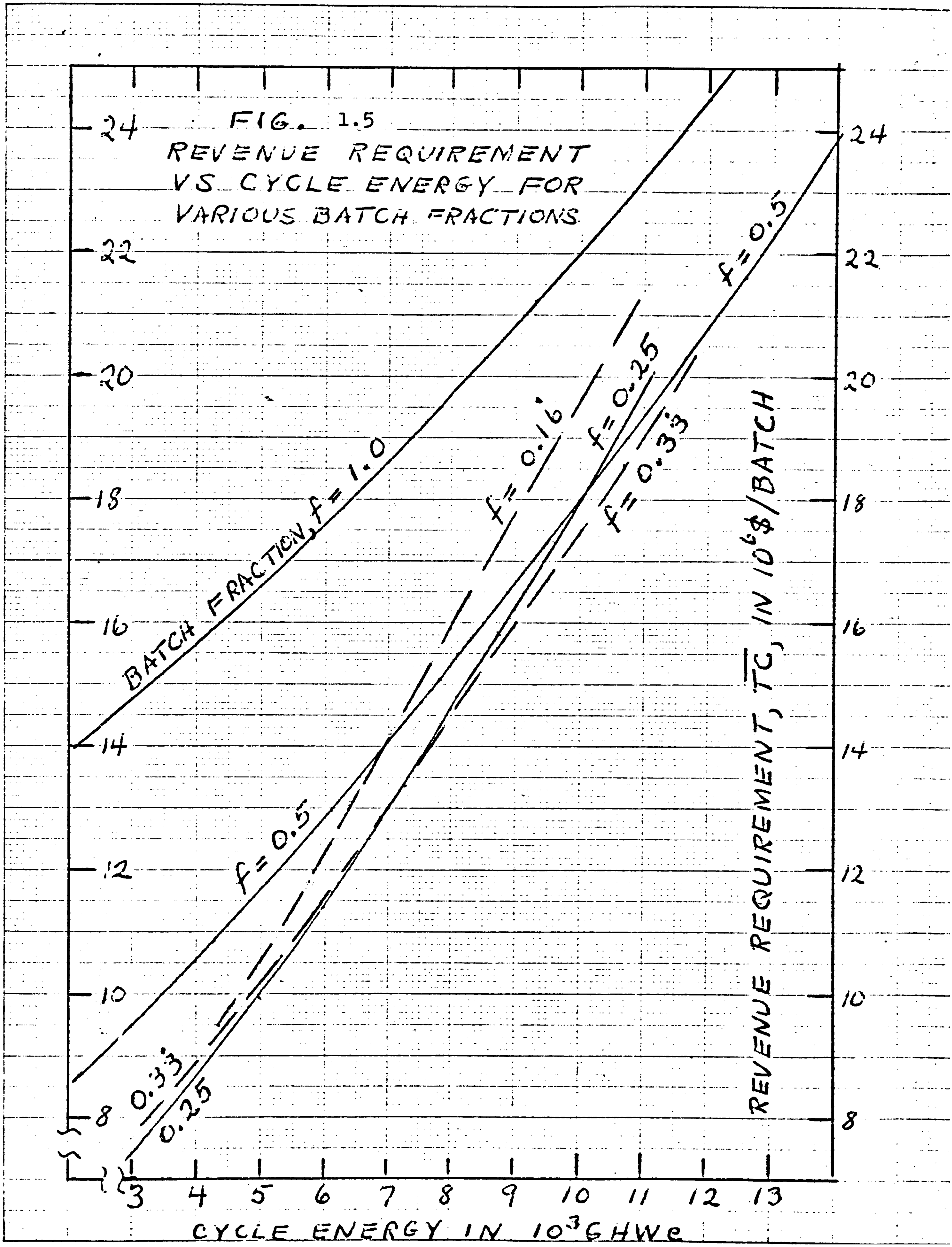


FIG. 1.5
REVENUE REQUIREMENT
VS CYCLE ENERGY FOR
VARIOUS BATCH FRACTIONS
BATCH FRACTION, f = 1.0
f = 0.5
f = 0.16
f = 0.25
f = 0.33
0.33
0.25
REVENUE REQUIREMENT, $\overline{TC}$, IN 10^6\$/BATCH
CYCLE ENERGY IN 10^3GHWe
8
10
12
14
16
18
20
22
24
3
4
5
6
7
8
9
10
11
12
13

In Fig. 1.6, revenue requirement has been replotted against batch fraction at constant cycle energy. In addition, lines of constant average burnup B° are plotted. Only the region to the right of a line of constant burnup is compatible with the burnup constraint (1.8). For example, at the higher cycle energies of 10,650, 9,000 and 7,500 Gwhe, with a burnup constraint of 30 MWD/kg, the optimum batch fraction occurs at the burnup constraint rather than at the lowest value of revenue requirement on the constant energy line, at which

$$\left(\frac{\partial(\overline{TC})}{\partial f}\right)_{E^S} = 0 \qquad (1.14)$$

When the optimum batch fraction is set by the burnup constraint, in steady-state refueling a simple analytic relation obtains between burnup B cycle energy E^S, batch fraction f and entire mass of uranium charged to the core W:

$$B \cdot W \cdot f = E^S \qquad (1.15)$$

Hence, the smallest batch fraction that satisfies the burnup constraint B° is given by $f = E^S/(B°W)$. (1.16)

Figure 1.7 shows the optimal batch fraction as a function of cycle energy for different burnup constraints. For high values of maximum allowable burnup and low cycle energies, the optimal batch fraction is determined by the economic optimization condition Eq.(1.14), whereas at higher cycle energies or lower allowable burnup it is given by Eq.(1.16).

In Figure 1.8 revenue requirement $\overline{TC}$ is plotted against reload enrichment, with lines of constant batch fraction f or cycle energy E or average burnup B^o. The optimal values of reload enrichment and batch fraction to produce specified energy can be read off directly for a specified burnup constraint B^o or minimum revenue requirement.

FIG. 1.6 REVENUE REQUIREMENT VS BATCH FRACTION FOR DIFFERENT LEVELS OF ENERGY

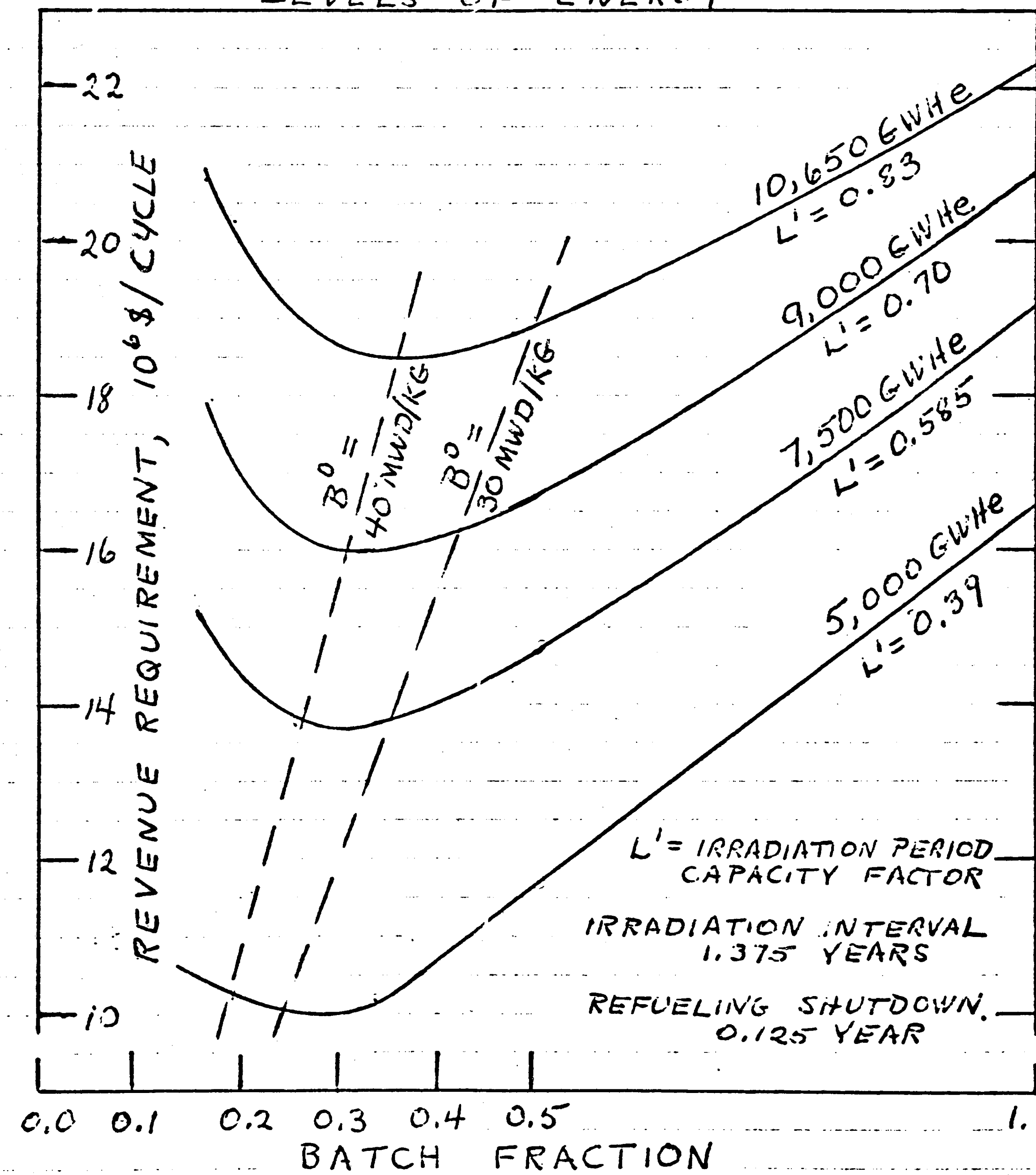


FIG. 1.7 OPTIMAL BATCH FRACTION VS CYCLE ENERGY FOR VARIOUS BURNUP LIMITS B^0

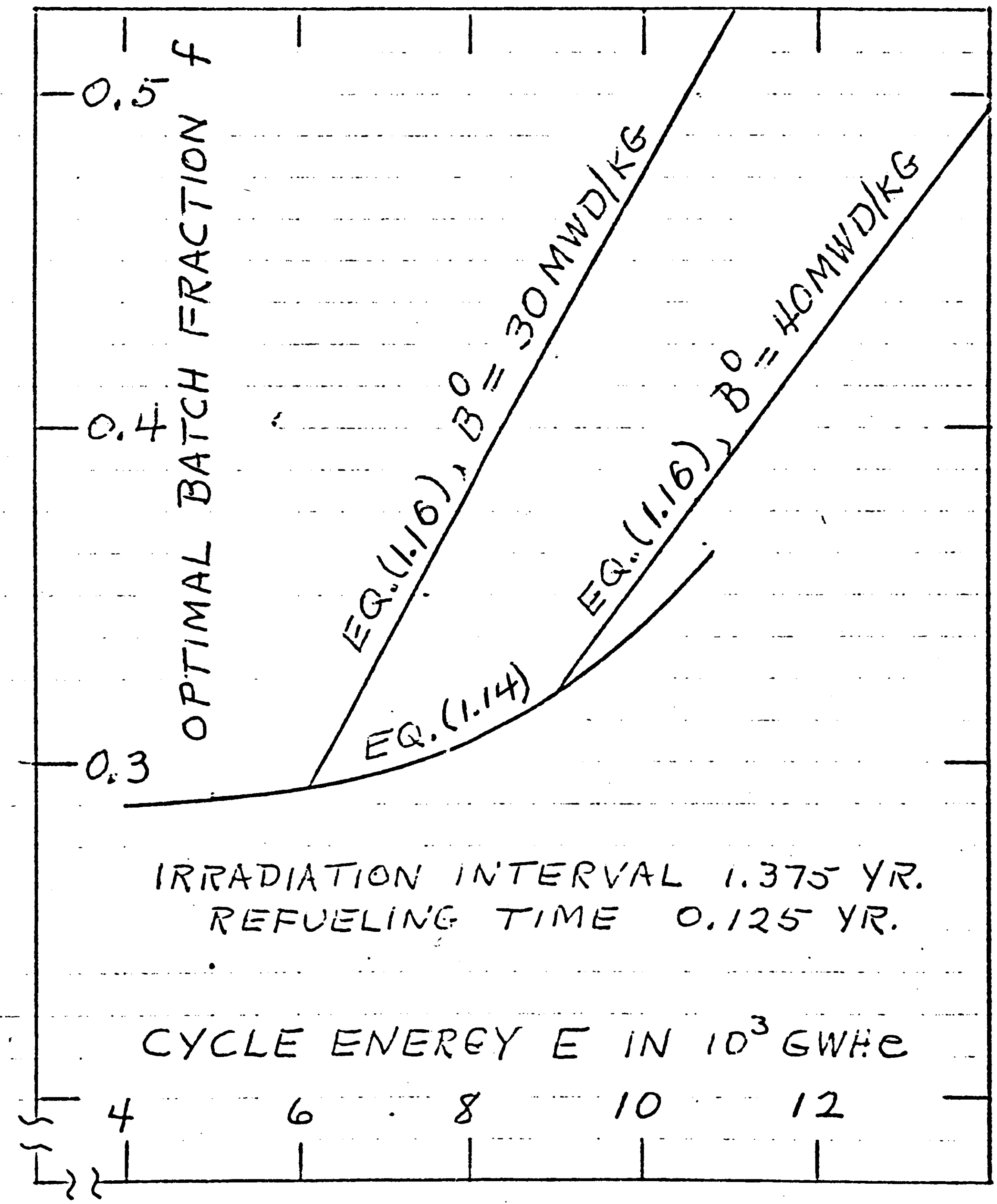
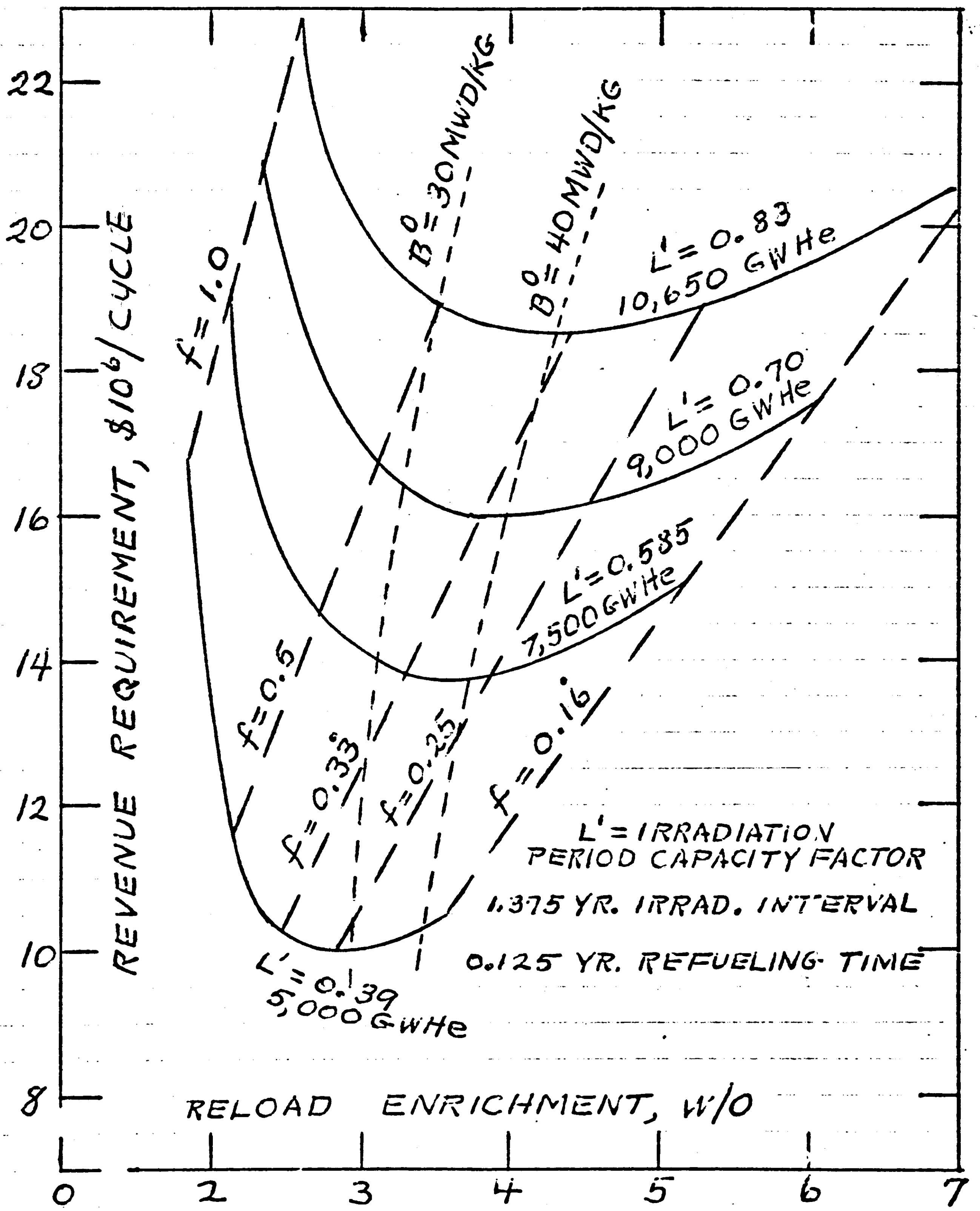


FIGURE 1.8

REVENUE REQUIREMENT VS RELOAD ENRICHMENT FOR VARIOUS LEVELS OF ENERGY



The calculation of incremental cost of energy for the case of variable reload enrichment and batch fraction deserves special attention. According to Equations (1.4) and (1.4a) λ is given as

$$\lambda \equiv \frac{\partial \overline{TC}}{\partial E}(E^S, \varepsilon^*, f^*)$$ where ε^* and f^* are optimal solution for E^S

which can be expanded into the following finite difference relationship

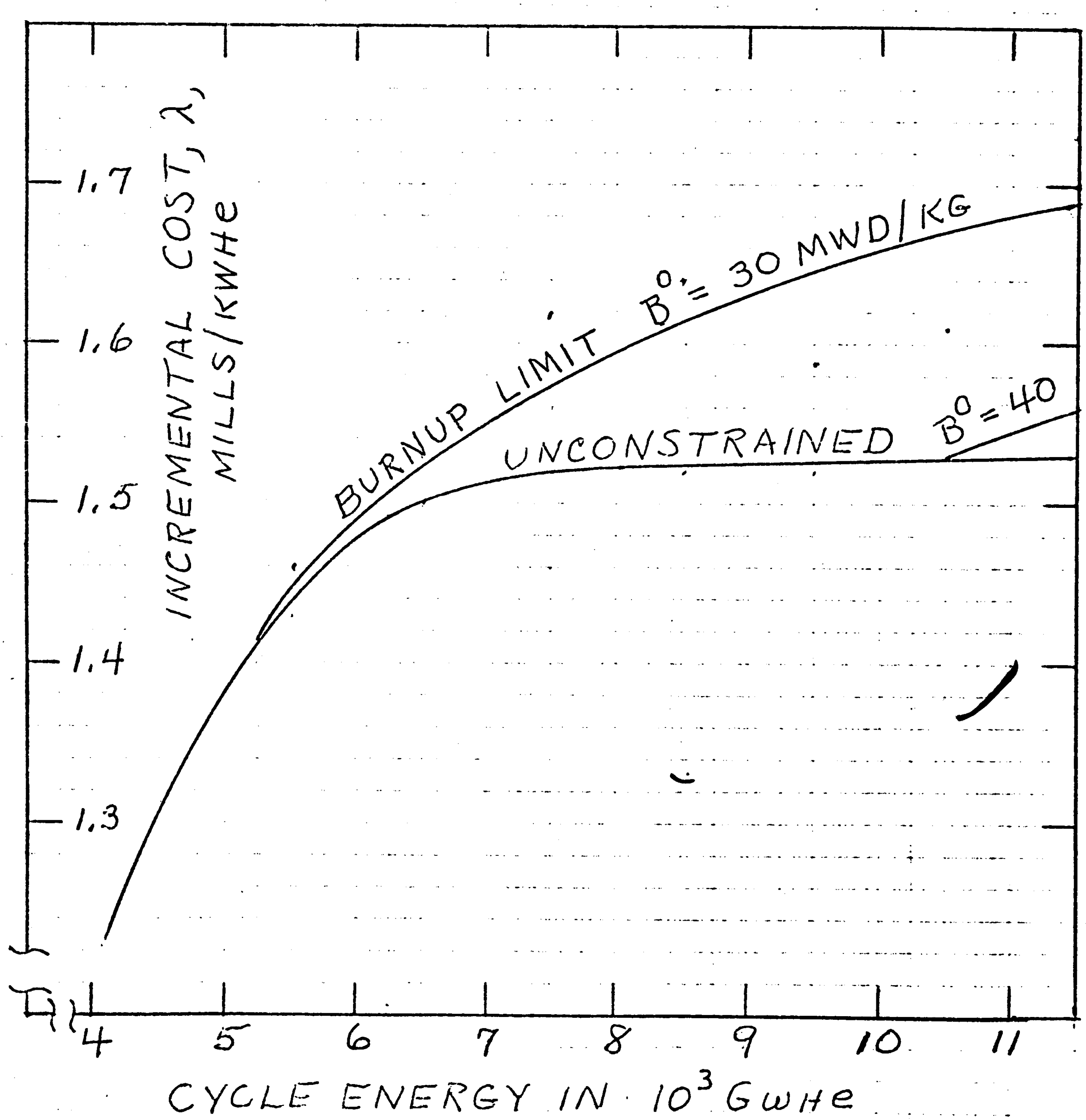
$$\lambda \simeq \frac{TC(E^S + \Delta E,\ \varepsilon\dagger,\ f\dagger) - TC(E^S, \varepsilon^*, f^*)}{\Delta E} \tag{1.17}$$

where $\varepsilon^\dagger$ and $f^\dagger$ are the optimal solution for $E^S + \Delta E$. When there are no constraints on the enrichment and batch fraction, ε and f are those values at which the revenue requirement is a minimum for a particular energy, i.e. the minima of the constant energy lines in Fig. 1.6. When the maximum burnup $B°$ places lower a limit on the batch fraction with which a particular energy may be produced, as in the case at a value of $B°$ of 30 MWD/kg at energies above 5,000 Gwhe, the values of revenue requirement used in Eq. 1.17 are those on the constant burnup line of Fig. 1.6. Fig. 1.9 shows values of incremental

Figure 1.9

INCREMENTAL COST λ VS

CYCLE ENERGY E FOR VARIOUS BURNUP LIMIT B°



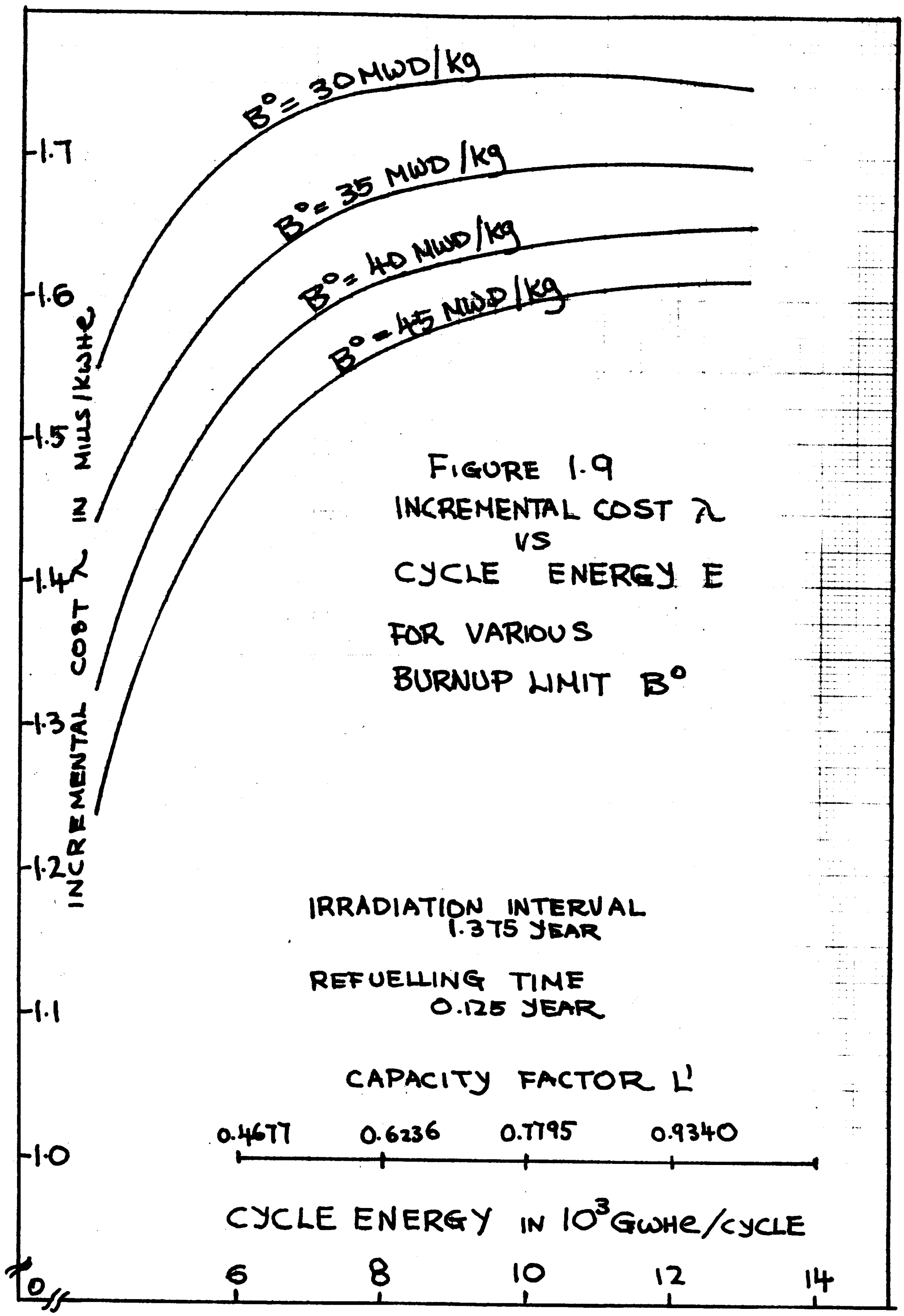


FIGURE 1.9
INCREMENTAL COST λ
VS
CYCLE ENERGY E
FOR VARIOUS
BURNUP LIMIT B°

cost of energy versus cycle energy for different values of burnup limits. Initially, incremental cost increases rapidly with respect to cycle energy but gradually levels off. As the burnup limit decreases, incremental cost increases.

For this special case of steady state operation, the problem of nuclear in-core optimization and the calculation of incremental cost involves a relatively small number of variables and can be handled effectively by graphs. For non-steady state operations, however, there are so many variables that complicated optimization techniques such as piece-wise linear approximation, or polynomial approximation, coupled with total exhaustive search, is required to solve this problem. Sections 1.7 and 1.8 summarize the methods and results of the two approaches. But before that, tests are required to show that the objective function calculated by the Inventory Value Method is suitable for this purpose.

1.6 Test of the Objective Function for the Variable Batch Fraction, Non-Steady State Case

As mentioned earlier in Section 1.3, a method for calculating the objective function for a finite planning horizon is deemed adequate for the purpose of scheduling energy if it gives the same value of incremental cost of energy as an exact calculation in which the entire life span of the reactor is considered.

ie $\frac{\partial \overline{TC}}{\partial E_j}\alpha = \frac{\partial \overline{TC}}{\partial E_j} I$ for all j within planning horizon I

(1.18)

However, for the problem of nuclear in-core optimization, the following additional equations for the partial derivatives are involved:

$$\frac{\partial \overline{TC}}{\partial \varepsilon_c}\alpha = \frac{\partial \overline{TC}}{\partial \varepsilon_c} I \qquad \text{for all c within planning horizon I}$$

$$\frac{\partial \overline{TC}}{\partial f_c}\alpha = \frac{\partial \overline{TC}}{\partial f_c} I \qquad (1.19)$$

If these equalities are maintained throughout the optimization, as demonstrated in Section 7.3, the collection of optimal solutions for each of the finite planning horizons would be the same as the overall optimization performed on the entire life span of the reactor. Table 1.4 shows values of the $\Delta\overline{TC}_\alpha/\Delta\varepsilon_1$ and $\Delta\overline{TC}_1/\Delta\varepsilon_1$ versus enrichment changes $\Delta\varepsilon_1$ and values of $\Delta\overline{TC}_\alpha/\Delta f_1$ and $\Delta\overline{TC}_1/\Delta f_1$ versus batch fraction changes Δf_1 for Cycle 1. It can be seen that the finite planning horizon objective function can be seen to give accurate first order derivatives for Cycle 1. Since nuclear in-core optimization would in all probability be updated on an annual basis, only the first cycle results would actually be utilized. Hence, the main emphasis on accuracy would be placed on the first cycle derivatives.

Having demonstrated that the finite planning horizon

Table 1.4

Effect of Variation of Enrichment and Batch Fraction on Revenue Requirement

$\overline{TC}_{\infty}$ Revenue Requirement for the Indefinite Planning Horizon

$\overline{TC}_I$ Revenue Requirement for the Finite Planning Horizon

Enrichment Changes (w/o)	Revenue Requirement Changes —10^6\$—		$\overline{TC}_I/\Delta\varepsilon_1$ —10^6\$/(w/o)—	$\overline{TC}_{\infty}/\Delta\varepsilon_1$	Error —%—
$\Delta\varepsilon_1$	$\overline{TC}_I$	$\overline{TC}$			
-1.200	-4.5720	-4.5804	3.8100	3.8169	+0.2
-0.434	-1.6648	-1.6746	3.8360	3.8586	+0.6
+0.480	+1.8893	+1.8791	3.9361	3.9148	-0.5
+1.200	+4.6642	+4.6542	3.8868	3.8785	-0.2

Batch Fraction Changes	Revenue Requirement Changes —10^6\$—		$\overline{TC}_I/\Delta f_1$ —10^6\$—	$\overline{TC}_{\infty}/\Delta f_1$	Error —%—
Δf_1	$\overline{TC}_I$	$\overline{TC}$			
-0.8	-2.3494	-2.3623	2.9367	2.9528	+0.5
-0.4	-1.1717	-1.1822	2.9293	2.9554	+0.9
+0.4	+0.7716	+0.7658	1.9290	1.9146	-0.7

objective function is suitable for nuclear in-core optimization, Section 1.7 and 1.8 proceed to describe the piece-wise linear approximation approach and the polynomial approximation approach of solving the optimization.

1.7 The Method of Piece-Wise Linear Approximation for the Problem of Nuclear In-Core Optimization

In the Method of Piece-Wise Linear Approximation, the objective function and the constraints are linearized about an initial feasible solution. For example

$$\overline{TC} \approx \overline{TC}(\vec{\varepsilon}^{\,0}, \vec{f}^{\,0}) + \sum_c \alpha_c \cdot (\varepsilon_c - \varepsilon_c^0) + \sum_c \beta_c \cdot (f_c - f_c^0) \quad (1.20)$$

where

$$\alpha_c \equiv \frac{\partial \overline{TC}(\vec{\varepsilon}^{\,0}, \vec{f}^{\,0})}{\partial \varepsilon_c} \qquad \beta_c \equiv \frac{\partial \overline{TC}(\vec{\varepsilon}^{\,0}, \vec{f}^{\,0})}{\partial f_c}$$

The expansion coefficients α_c and β_c are determined by a number of perturbation cases in which the decision variables are varied one at a time. For example

$$\alpha_c \approx \left\{ \begin{array}{l} \overline{TC}(\varepsilon_1^0, \varepsilon_2^0, \ldots \varepsilon_c^0 + \Delta\varepsilon, \ldots \vec{f}^{\,0}) - \\ \overline{TC}(\varepsilon_1^0, \varepsilon_2^0, \ldots \varepsilon_c^0, \ldots \vec{f}^{\,0}) \end{array} \right\} \Big/ \Delta\varepsilon \quad (1.21)$$

Linear programming can be applied to the set of linearized objective function and constraints. Limiting the changes in $\Delta f/f$ by $\pm$ 1% each time, a new solution can be calculated in the steepest descent direction. The process of linearization and optimization can be repeated on this new solution in an iterative fashion.

Unfortunately, practical mesh spacing setup of the present CELL-CORE depletion code only allows discrete changes of $\Delta f/f$ by 12%. Hence, the linear model must be modified to accommodate changes by large step sizes.

The final form of the equations used is slightly more complicated than the illustrative Equation (1.20). Instead of having a single expansion coefficient for each variable, there are two expansion coefficients, one for positive and one for negative variation of the batch fraction variables. The set of piece-wise linear equations are solved by total exhaustive search. The objective function is calculated for all feasible neighboring points around the initial solution. The neighboring point with the lowest objective function is chosen to be the new solution on which linearization and optimization are to be repeated.

As an example of the application of this method, consider the following sample case A. The reactor under analysis is the Zion type 1065 MWe PWR with initial condition equivalent to the 3.2 w/o three-zone modified scatter refuelled steady-state condition. The planning horizon consists of five cycles. Energy requirement for each of the five cycles is 22750 GWHt, the same value as produced in the steady-state condition. The maximum allowable average discharge burnup is 60 MWD/kg. The Method of Piece-Wise Linear Approximation is applied to solve for the optimal reload enrichments and batch fractions for the five cycles.

Table 1.5 shows the batch fractions, reload enrichments, cycle energies and revenue requirement for the various iterations. The revenue requirement is calculated based on economic parameters similiar to that of TVA, with no income tax obligations. The revenue requirement corrected for target energy decreases in successive iterations. The final solution results in net savings of $1.6 million dollars when compared to the initial solution. However, when the same case is repeated using the economics parameters characteristic of an investor-owned utility which pays income taxes, the Method of Piece-Wise Linear Approximation fails to converge. This is due to the fact that the original initial condition 3.2 w/o three-zone modified scatter refuelling is so close to the optimal solution that piece-wise linear approximation based on step size of 12% is too large for the purpose.

This method of Piece-Wise Linear Approximation is applicable to cases where the objective function has a wide variation over the range of the decision variables, and where the optimal solution is ultimately limited by one or more of the constraints. However, if the objective function is rather flat and the constraints are not active, the Method of Piece-Wise Linear Approximation cannot pinpoint the optimal solution precisely, and a more sophisticated technique like polynomial approximation is needed.

Table 1.5

Reload Enrichments, Batch Fractions, Cycle Energies and Revenue Requirements for Various Number of Iterations Using the Method of Piece-Wise Linear Approximation

		Cycle					Revenue Requirement			
		1	2	3	4	5	For Actual Energy		Corrected for Target Energy	
	ε(w/o) f E(GWHt)						Piece-wise Linear Appro-ximation	CELL-COCO	Piece-wise Linear Appro-ximation	CELL-COCO
	Target Energy	22750.	22750.	22750.	22750.	22750.	10^6 \$			
Iteration Number										
0	ε	3.2	3.2	3.2	3.2	3.2				
	f	0.333	0.333	0.333	0.333	0.333	72.1119	72.1119	72.1119	72.1119
	E	22750.	22750.	22750.	22750.	22750.				
1	ε	3.77	3.37	3.45	3.56	3.42				
	f	0.293	0.293	0.293	0.293	0.293	71.3358	71.1517	71.4971	71.3131
	E	22257.	22725.	22616.	23076.	22769.				
2	ε	5.03	3.03	4.27	2.96	4.57				
	f	0.253	0.253	0.253	0.253	0.253	70.3096	70.5269	70.4969	70.7141
	E	22697.	22534.	22844.	22430.	22646.				
3	ε	3.95	4.25	4.64	4.31	3.61				
	f	0.293	0.253	0.213	0.213	0.213	70.0805	70.4763	70.2485	70.6443
	E	22986.	23133.	22325.	23894.	21253.				

1.8 The Method of Polynomial Approximation for the Problem of Nuclear In-Core Optimization

In the Method of Polynomial Approximation, the objective function and the constraints are approximated by a sum of polynomials in cycle energies and batch fractions. For example

$$\overline{TC} = \sum_{c} \sum_{l=-1} \sum_{m=-2} \sum_{n=-3} \alpha_{clmn} E_c^l \cdot f_c^m \cdot f_{c-1}^n \tag{1.22}$$

$$B_c = \sum_{k=-1} \sum_{l=-1} \sum_{m=-3} \sum_{n=-3} \beta_{cklmn} E_c^k \cdot E_{c-1}^l \cdot f_c^m \cdot f_{c-1}^n \tag{1.23}$$

The expansion coefficients α_{clmn}, β_{cklmn} are multiple regression coefficients based on analysis of a large number of sample cases. For cases considered here, the polynomial can be fitted with an accuracy of $\pm$ 0.1% of $\overline{TC}$ and $\pm$ 5% of B_c using polynomials up to the third order.

The objective function and the constraints in polynomial form can be optimized analytically. Since the energy requirement is implicitly included in Equation (1.22) the only independent variable is the batch fraction f_c.

The objective function $\overline{TC}$ and the discharge burnup B_c are calculated for all possible values of f. The $\overline{TC}$ with the lowest cost satisfying a certain burnup limit B° is chosen as the optimal solution.

The following two sample cases are analyzed by this method. Sample case A is identical to the problem considered in the previous Section 1.7 by the Method of Piece-Wise Linear Approximation, with economic parameters that included income tax. Sample case B differs from sample case A in that the cycle energy requirements are different for different cycles.

Table 1.6 shows values of reload enrichments, batch fractions cycle energies and revenue requirement for sample case A for the seven cases having the lowest costs. AA0 is the base line case, where the batch fractions and reload enrichments are held at the original steady state values. Net savings in the order of 0.3 million dollars are achieved in case AB1 when compared to steady-state operation AA0 through this optimization. Table 1.7 shows values of discharge burnup estimated by the polynomial approximation as compared to the actual values given by CELL-CORE. The results agree within $\pm 5\%$.

Sample case B differs from sample case A in the cycle energy requirement. Cycle energy requirements vary for Sample problem B and are:

E_1=25450. GWHt, E_2=30440. GWHt, E_3=21850. GWHt,
E_4=19340. GWHt, E_5=20880. GWHt

Table 1.8 shows values of reload enrichments, batch fractions, cycle energies and revenue requirements for the five solutions having the lowest costs. BA0 is the base line case, where the batch fractions are held constant at

Table 1.6 B°=50MWD/Kg

Reload Enrichments, Batch Fractions, Cycle Energies and Revenue Requirements for the Various Lowest Cost Cases Using the Method of Polynomial Approximation Sample Case A

Case Number	ε (w/o) f E(GWHt)	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5	Revenue Requirement (10^6\$): For Actual Energy — Polynomial Approximation	For Actual Energy — CELL-COCO	Corrected for Target Energy — Polynomial Approximation (Difference)	Corrected for Target Energy — CELL-COCO
	Target Energy	22750.	22750.	22750.	22750.	22750.				
AA0	ε	3.2	3.2	3.2	3.2	3.2				
	f	0.333	0.333	0.333	0.333	0.333	87.30	87.24	87.30	87.24
	E	22750.	22750.	22750.	22750.	22750.			(+0.06)	
AB1	ε	3.88	4.27	3.42	3.95	2.40				
	f	0.293	0.253	0.253	0.253	0.293	86.43	86.34	86.99	86.90
	E	22690.	23000.	22480.	23100.	20500.			(+0.09)	
AB2	ε	3.88	4.27	2.76	3.77	3.45				
	f	0.293	0.253	0.293	0.293	0.293	87.20	87.33	87.01	87.14
	E	22690.	23000.	22510.	23130.	23070.			(-0.13)	
AB3	ε	3.88	3.33	3.45	3.54	2.94				
	f	0.293	0.293	0.293	0.293	0.333	87.09	87.13	87.02	87.06
	E	22690.	22840.	22560.	22920.	23030.			(-0.04)	
AB4	ε	3.88	4.27	2.77	3.74	2.40				
	f	0.293	0.253	0.293	0.293	0.333	86.26	86.37	87.02	87.13
	E	22690.	23000.	22510.	22980.	19730.			(-0.11)	
AB5	ε	3.88	3.29	3.45	4.50	2.66				
	f	0.293	0.293	0.293	0.253	0.293	86.82	86.89	87.03	87.10
	E	22690.	22700.	22400.	23000.	22300.			(-0.07)	
AB6	ε	3.88	3.29	3.45	3.54	3.45				
	f	0.293	0.293	0.293	0.293	0.293	86.94	87.00	87.04	87.10
	E	22690.	22700.	22460.	22880.	22830.			(-0.06)	
AB7	ε	3.88	4.27	3.42	3.95	3.61				
	f	0.293	0.253	0.253	0.253	0.253	87.23	87.14	87.04	96.95
	E	22690.	23000.	22480.	23090.	23250.			(+0.09)	

Table 1.7 B^{0}=50MWD/Kg

Average Discharge Burnup for the Sublot Experiencing the Highest Exposure for Sample Case A Calculated by (1) Polynomial Approximation Based on Regression Equations (2) CELL-CORE Depletion Calculation

Case Number	Method	Batch Number -2	-1	0	1	2	3	4	5
		MWD/Kg							
AA0	(1)	31.5	31.5	31.5	31.5	31.5	31.5	31.5	31.5
	(2)	31.5	31.5	31.5	31.5	31.5	—	—	—
AB1	(1)	38.6	38.6	38.6	44.2	47.4	40.4	44.4	31.8
	(2)	38.9	38.4	38.1	44.4	46.9	—	—	—
AB2	(1)	38.6	38.6	38.6	44.2	47.4	34.7	43.2	36.4
	(2)	38.9	38.4	38.5	45.2	47.5	—	—	—
AB3	(1)	38.6	38.6	38.6	44.2	39.4	40.9	41.2	36.1
	(2)	38.9	38.6	38.8	44.9	39.4	—	—	—
AB4	(1)	38.6	38.6	38.6	44.2	47.4	34.7	43.2	31.9
	(2)	38.9	38.4	38.5	45.2	47.3	—	—	—
AB5	(1)	38.6	38.6	38.6	44.2	39.4	40.9	49.6	34.3
	(2)	38.9	38.6	38.8	44.5	38.4	—	—	—
AB6	(1)	38.6	38.6	38.6	44.2	39.4	40.9	41.2	40.6
	(2)	38.9	38.6	38.8	44.9	38.7	—	—	—
AB7	(1)	38.6	38.6	38.6	44.2	47.4	40.4	44.4	38.2
	(2)	38.9	38.4	38.1	44.4	47.0	—	—	—

Table 1.8 B^0=50MWD/Kg

Reload Enrichments, Batch Fractions, Cycle Energies and Revenue Requirements for the Various Lowest Cost Cases Using the Method of Polynomial Approximation. Sample Case B

Case Number	ε(w/o) / f / E(GWHt)	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5	Revenue Requirement For Actual Energy: Polynomial Approximation (10^6 \$)	Revenue Requirement For Actual Energy: CELL-COCO (10^6 \$)	Corrected for Target Energy: Polynomial Approximation (Difference)	Corrected for Target Energy: CELL-COCO
	Target Energy	25450.	30440.	21850.	19340.	20880.				
BA0	ε	3.73	4.36	2.40	2.76	3.45				
	f	0.333	0.333	0.333	0.333	0.333	89.36	89.37	89.92	89.93
	E	25510.	30470.	22170.	20280.	17220.			(-0.01)	
BB1	ε	3.74	4.36	2.70	3.88	2.27				
	f	0.333	0.333	0.293	0.253	0.293	88.66	88.71	89.67	89.72
	E	25510.	30470.	21270.	19180.	17930.			(-0.05)	
BB2	ε	4.55	3.79	2.91	3.87	2.61				
	f	0.293	0.333	0.293	0.253	0.293	89.35	89.38	89.67	89.71
	E	25340.	30310.	21790.	19480.	20020.			(-0.04)	
BB3	ε	3.74	4.36	2.70	3.10	2.37				
	f	0.333	0.333	0.293	0.293	0.333	88.61	88.67	89.71	89.76
	E	25510.	30470.	21270.	19260.	17480.			(-0.05)	
BB4	ε	4.55	3.79	2.91	3.09	2.71				
	f	0.293	0.333	0.293	0.293	0.333	89.32	89.38	89.71	89.76
	E	25340.	30310.	21790.	19320.	19930.			(-0.05)	
BB5	ε	4.55	3.79	3.72	2.93	2.93				
	f	0.293	0.333	0.253	0.253	0.293	89.31	89.27	89.72	89.68
	E	25340.	30310.	21790.	19130.	20110.			(+0.04)	

the 0.33 level and serves as a standard for comparing other cases. Net savings of 0.25 million dollars achieved by Case BB5 are realized when compared to base case BAO. Table 1.9 shows values of discharge burnup estimated by the polynomial approximation as compared to the actual values given by CELL-CORE. The same accuracy as in sample case A is achieved.

The results of regression analysis and the optimization procedure indicate that the objective function is rather insensitive to batch fraction changes, if the same cycle energies are produced. In the two sample cases given above, using the base line cases instead of the optimal cases only incurred additional cost of 0.3 million dollars, which is a mere 0.4% of the total revenue requirement. If the base line cases give better engineering margins in terms of discharge burnup, power peaking and shut down reactivity, they should be used instead. The final choice should be based on engineering margins rather than on economics.

Finally, a method of calculating incremental cost of energy under the variable batch fraction, non-steady state operating conditions are given. The method is based on taking finite differences on the regression equation involving $\overline{TC}$. The incremental cost of energy for cycle c is given by

$$\lambda_c \simeq \frac{\overline{TC}(E_1^s, E_2^s, ..E_c^s+\Delta E, ..\vec{f}^{\dagger}) - \overline{TC}(E_1^s, E_2^s, ..E_c^s, ..\vec{f}^{*})}{\Delta E} \qquad (1.24)$$

Table 1.9 B^{o}=50MWD/Kg

Average Discharge Burnup for the Sublot Experiencing the Highest Exposure for Sample Case B Calculated by (1) Polynomial Approximation Based on Regression Equations (2) CELL-CORE Depletion Calculation

Case Number	Method	Batch Number -2	-1	0	1	2	3	4	5
		MWD/Kg							
BA0	(1)	31.5	31.5	31.5	37.2	43.9	31.9	35.0	41.4
	(2)	31.5	31.8	32.8	37.9	42.2	28.5	32.9	41.9
BB1	(1)	31.5	31.5	38.6	43.0	48.2	34.4	44.3	30.9
	(2)	31.5	31.8	39.3	44.9	49.4	—	—	—
BB2	(1)	38.6	38.6	38.6	49.7	43.5	36.2	44.1	33.7
	(2)	39.2	39.8	39.7	52.2	44.0	—	—	—
BB3	(1)	31.5	31.5	38.6	43.0	48.2	34.4	37.8	31.7
	(2)	31.5	31.8	39.3	45.6	50.2	—	—	—
BB4	(1)	38.6	38.6	38.6	49.7	43.5	36.2	37.6	34.6
	(2)	39.2	39.8	39.7	52.7	44.7	—	—	—
BB5	(1)	38.6	38.6	38.6	49.7	43.5	42.9	36.3	36.4
	(2)	39.2	39.8	39.4	51.7	44.1	—	—	—

Notice that the B^{o}=50MWD/Kg limit only applies to the estimated burnup values calculated by the polynomial regression equation. The fact that actual burnup values sometimes exceed 50MWD/Kg indicates that the estimated burnup values are only approximate.

where $f^{\dagger}$ and f^{*} are the optimal batch fractions for the $\vec{E}^{s} + \Delta E_c$ and the $\vec{E}^{s}$ case respectively.

that is : $\overline{TC}(\vec{E}^{s} + \Delta E_c, f^{\dagger}) = \underset{\text{with respect to } f}{\text{minimum}}\ \overline{TC}(\vec{E}^{s} + \Delta E_c, f)$

and $\overline{TC}(\vec{E}^{s}, f^{*}) = \underset{\text{with respect to } f}{\text{minimum}}\ \overline{TC}(\vec{E}^{s}, f)$

Tables 1.10 and 1.11 show values of f^{*}, $f^{\dagger}$, $\overline{TC}$ and λ_c for various values of E_c and for various burnup limits based on the optimal solution of sample case A. Tables 1.12 and 1.13 show the same quantities for sample case B. It can be seen that the incremental cost in a cycle varies irregularly with cycle energy. This is due to the fact that different sets of f are needed to satisfy the burnup constraints for different cycle energy requirements. The variation of $\overline{TC}$ with respect to these different sets of f is not continuous.

1.9 Conclusions

The following conclusions are obtained from this thesis research.

(1) The Inventory Value Method for evaluating worth of nuclear fuel inventories to be used in

Table 1.10

Calculation of Incremental Cost of Energy

Using Regression Equations. Sample Case A

Burnup Limit $B^{\circ} = 45$MWD/Kg

	Batch Fraction for Cycle 1	2	3	4	5	Revenue Requirement — 10^6\$ —	Incremental Cost in Mills/KWHe
Base Case AA1	0.293	0.293	0.293	0.293	0.293	87.01872	
Positive Energy Change ΔE=1000GWHt in Cycle							
1	0.333	0.293	0.293	0.293	0.333	87.5284	1.56
2	0.293	0.293	0.293	0.293	0.333	87.4265	1.22
3	0.293	0.293	0.293	0.293	0.333	87.3890	1.15
4	0.293	0.293	0.293	0.293	0.333	87.3170	0.91
5	0.293	0.293	0.293	0.293	0.333	87.2957	0.845
Negative Energy Change ΔE=-1000GWHt in Cycle							
1	0.293	0.293	0.293	0.293	0.333	86.5642	1.395
2	0.293	0.253	0.253	0.253	0.293	86.5848	1.33
3	0.293	0.293	0.293	0.293	0.333	86.6605	1.095
4	0.293	0.293	0.293	0.293	0.333	86.7226	0.905
5	0.293	0.293	0.293	0.293	0.333	86.7443	0.84

Table 1.11

Calculation of Incremental Cost of Energy

Using Regression Equations. Sample Case A

Burnup Limit B =50MWD/Kg

	Batch Fraction for Cycle 1	2	3	4	5	Revenue Requirement 10^6\$	Incremental Cost in Mills/KWHe
Base Case AB1	0.293	0.253	0.253	0.253	0.293	86.9890	
Positive Energy Change ΔE=1000GWHt in Cycle							
1	0.293	0.253	0.253	0.253	0.293	87.4642	1.46
2	0.293	0.293	0.293	0.293	0.333	87.4265	1.335
3	0.293	0.253	0.293	0.293	0.293	87.3848	1.21
4	0.293	0.253	0.253	0.253	0.293	87.3047	0.965
5	0.293	0.253	0.253	0.253	0.293	87.2748	0.875
Negative Energy Change ΔE=-1000GWHt in Cycle							
1	0.293	0.253	0.253	0.253	0.293	86.5345	1.395
2	0.293	0.253	0.253	0.253	0.293	86.5848	1.24
3	0.293	0.253	0.253	0.253	0.293	86.5860	1.24
4	0.293	0.253	0.253	0.253	0.293	86.6761	0.955
5	0.293	0.253	0.253	0.253	0.293	86.7064	0.865

Table 1.12

Calculation of Incremental Cost of Energy

Using Regression Equations. Sample Case B

Burnup Limit B^{o}=45MWD/Kg

	Batch Fraction for Cycle 1	2	3	4	5	Revenue Requirement —10^6\$—	Incremental Cost —Mills/KWHe—
Base Case BA1	0.333	0.373	0.293	0.253	0.293	89.8251	———
Positive Energy Change ΔE=1000GWHt in Cycle							
1	0.333	0.373	0.293	0.253	0.293	90.2916	1.435
2	0.333	0.373	0.293	0.253	0.293	90.2424	1.28
3	0.333	0.373	0.293	0.253	0.293	90.1845	1.10
4	0.333	0.373	0.293	0.293	0.333	90.1255	0.91
5	0.333	0.373	0.293	0.253	0.293	90.1049	0.915
Negative Energy Change ΔE=-1000GWHt in Cycle							
1	0.333	0.373	0.293	0.253	0.293	89.3766	1.375
2	0.333	0.373	0.293	0.253	0.293	89.4070	1.28
3	0.333	0.373	0.293	0.253	0.293	89.4773	1.07
4	0.333	0.373	0.293	0.253	0.293	89.5224	0.925
5	0.333	0.373	0.293	0.253	0.293	89.5484	0.85

Table 1.13

Calculation of Incremental Cost of Energy

Using Regression Equations. Sample Case B

Burnup Limit B =50MWD/Kg

	Batch Fraction for Cycle 1	2	3	4	5	Revenue Requirement 10^6\$	Incremental Cost Mills/KWHe
Base Case BB1	0.333	0.333	0.293	0.253	0.293	89.6715	
Positive Energy Change ΔE=1000GWHt in Cycle							
1	0.333	0.333	0.293	0.253	0.293	90.1380	1.435
2	0.293	0.333	0.293	0.253	0.293	90.0775	1.25
3	0.333	0.333	0.293	0.253	0.293	90.0309	1.10
4	0.333	0.333	0.293	0.253	0.293	89.9772	0.93
5	0.333	0.333	0.293	0.253	0.293	89.9513	0.86
Negative Energy Change ΔE=-1000GWHt in Cycle							
1	0.293	0.333	0.293	0.253	0.293	89.1628	1.56
2	0.293	0.293	0.253	0.253	0.293	89.1515	1.60
3	0.333	0.333	0.253	0.253	0.293	89.3229	1.07
4	0.333	0.333	0.293	0.253	0.293	89.3687	0.925
5	0.333	0.333	0.293	0.253	0.293	89.3947	0.845

calculating finite planning horizon revenue requirement is adequate for the purpose of scheduling energy and nuclear in-core optimization.

(2) Three methods are proposed for calculating incremental cost of energy for the fixed batch fraction case. The Linearization Method and the Inventory Value method for calculating incremental cost of energy are both suitable for the initial stages of optimal energy scheduling. The Rigorous Method is very time-consuming and expensive and should be used only in the final stages of optimal energy scheduling.

(3) For the problem of nuclear in-core optimization under steady state conditions with variable batch fractions and reload enrichments, the optimal solution is practically always on the boundary of the burnup constraints. Hence, there are strong incentives to increase the burnup limits.

(4) For the problem of nuclear in-core optimization under non-steady state conditions, the Method of Piece-Wise Linear Approximation is applicable for the cases where there are large variations of objective function near the optimal solution. It is not applicable for economic situations where

there is a broad region of optimality.

(5) The Method of Polynomial Approximation gives accurate values of the optimal solutions, even though the objective function is very flat near the optimum.

(6) Since the objective function is insensitive to large variations in batch fractions, selection of the optimal solution can be based primarily on other considerations, such as engineering margins.

1.10 Recommendations

The depletion code CELL-CORE should be modified in order that the batch fraction can be varied continuously. This modification would enable the efficient usage of the Method of Linear Approximation instead of Piece-Wise Linear Approximation or Polynomial Approximation. Once the optimal batch fraction in the continuum is located, the realistic batch fraction to be used in refuelling would be given by the number of integral fuel assemblies which is closest to the continuum optimal solution.

Finally, the algorithm of optimal energy schedule should be modified so that the polynomial equations from regression analysis could be used directly, instead of the present indirect usage which require intermediate calculations of incremental cost. It is recommended that a quadratic programming algorithm, or an even higher order programming algorithm should be used in the optimal energy scheduling procedures, so that the higher order derivatives can be used directly.

CHAPTER 2

INTRODUCTION

2.1 Motivations for Mid-Range Utility Planning

Until recently, procedures for scheduling energy production from different nuclear power plants in an electric utility system have consisted of a relatively simple set of rules. All the nuclear power plants were to be operated base-loaded whenever they were available. They were to be refuelled annually, either in the spring or in the fall when the system demand is at its lowest level. From an economics stand point, the foregoing rules can be justified because nuclear energy, being cheaper than conventional fossil energy, should be used whenever possible to displace the latter. Annual refuelling is desirable from an operational standpoint.

For electric utilities having only a small number of nuclear units, this is a practical and economical way to operate nuclear power units. However, recently the number of nuclear power units in some large utilities, such as Commonwealth Edison and Tennesse Valley Authority, have increased to such a level that the foregoing rules are not sufficient for the following reasons. The combined nuclear generating capacity is so large that all of them cannot be operated base-loaded in periods of low system demand. Another reason is that there are so many nuclear power units that all of them cannot be refuelled annually during the spring and fall without creating some operating and reliability difficulties. For example, refuelling two or more reactors at the same site simultaneously or successively might create excessive strain on the grid in the region to which these reactors belong and might also overload station refuelling and maintenance personel operations. Consequently, the following requirements in refuelling are being considered (CW1)

(i) From the standpoint of area security, no more than one reactor should be down for refuelling for any region at any given time.

(ii) From the standpoint of efficient refuelling operations, reactors should not be refuelled simultaneously or successively at a given site.

(iii) From the stand point of satisfying the system demand, all the nuclear power units should be available in the peak demand periods. Hence, nuclear power units cannot be scheduled for refuelling in the summer if there is a severe summer peak.

Under these requirements annual refuelling can no longer be maintained for all nuclear reactors at all times. In this situation reactors cannot be base-loaded all the time and refuelled annually.

New scheduling methods must be developed that will handle this situation. These methods should provide an optimal operating schedule for energy production for all the generating units(fossil, hydro and nuclear) in a given electric utility spanning a planning horizon of more than five years. Besides specifying energy production for every unit, the schedule should also specify refuelling and maintenance dates for each unit and other refuelling details for nuclear reactors, such as reload enrichments and batch fractions. This overall problem of scheduling is called Mid-Range Utility Planning.

2.2 Formulation of the Overall Optimization Problem for Mid-Range Utility Planning

The overall optimization problem for Mid-Range Utility Planning can be formulated as follows; given a load forecast for a given electric utility over the span of the planning horizon, given the composition of the electric utility in terms of the capacity, type and locations of each generating unit, find the optimal schedule of operation which consists of refuelling and maintenance dates, energy production in each time period for every unit, and (for all nuclear reactors) the reload enrichments and batch fractions for each cycle in the planning horizon.

The objective function for this problem is the revenue requirement directly related to energy production in the planning horizon. This is the capital which if received as revenue by the company at time zero which, invested in the company at the effective rate of return x, would enable the company to pay all fossil and nuclear fuel expenses startup and shutdown costs, other variable operating costs, and all related taxes, pay bond holders and stock holders their required rate of return on outstanding investments on nuclear fuels, and retire all fuel investments at the end of the time horizon. The fuel revenue requirement for the electric utility is the sum of all these revenue requirements for each generating units:

$$\overline{TC}^s = \sum_{r}^{R} \overline{TC}^r \tag{2.1}$$

where $\overline{TC}^s$ is the total revenue requirement for the system

$\overline{TC}^r$ is the revenue requirement for unit r

R: total number of generating units in the system.

The decision variables are

(i) time for maintenance and refuelling for each unit

(ii) energy production of each unit for each period of time in the planning horizon

(iii) for the nuclear generating units, the reload enrichments and batch fractions for each cycle.

In general, there are other parameters specific to the nuclear generating units; such as refuelling pattern, configuration of burnable poison rods, multi-enrichment batches etc. For the sake of simplicity, these parameters are not included in the decision variables.

The constraints for this problem are:

(i) the sum of energy production from all of the generating units must be equal to the total system demand in each period of time.

(ii) Rate of energy production for each unit cannot exceed its rated capacity.

(iii) Each nuclear reactor should operate within its physics and engineering constraints, for example, burnup limits, power peaking factors and reactor shut down margins.

(iv) Other system operating restrictions such as area security, spinning reserve requirements limitations on startup and shutdown frequency etc. must be met.

(v) Refuelling schedules must meet the restrictions as specified in Section 2.1. For a complete listings of the constraints refer to Widmer (W2) or Deaton (D1). For the purpose of this thesis research, only a few of these constraints are explicitly considered, and they will be stated clearly in each chapter. Some of the physics and engineering constraints for nuclear reactors are investigated in greater depth in Kearney's (K1) and Rieck's (R1) thesis research.

2.3 Decomposition of the Overall Problem into Various Sub-Problems

The overall optimization problem of Mid-Range planning can be decomposed into three sub-problems. The first sub-problem deals with the decision variable of maintenance and refuelling times. A computer code has been developed by John Bukovski (CE2) that generates a number of refuelling and maintenance schedules compatible with specified constraints. For each refuelling and maintenance schedule, the second sub-problem involves finding the energy productions, reload enrichments and batch fractions for the generating unit which lead to lowest cost. This is repeated for each time schedule, and the schedule with the lowest cost is chosen to be the optimal solution. The third sub-problem involves separating the problem of optimal energy schedule from nuclear in-core optimization and then the energy variables from the enrichment and batch fraction variables. In essence, this technique of decomposition separates the time dependence from the other decision variables. Hence, the overall optimization problem of mid-range planning reduces to solving for the optimal energy production, reload enrichments and batch fractions based on a given refuelling and maintenance time schedule. This sub-problem is called System Optimization for a given refuelling and maintenance time schedule. This problem can be formulated mathematically as

$$\text{minimize } \overline{TC}^{s} = \sum_{r}^{R} \overline{TC}^{r} \tag{2.2}$$

with respect to E_j^r, ε_c^r, f_c^r

Subject to constraints

$$\sum_{r} E_j^r = E_j^s \tag{2.3}$$

$$E_j^r \leq \Delta t_j \cdot P^r \cdot 8760. \tag{2.4}$$

$$E_c^r = \sum_{j_{rc}}^{(j_{rc+1}-1)} E_j^r \tag{2.5}$$

$$F_c^r(\vec{\varepsilon}^r, \vec{f}^r) = E_c^r \tag{2.6}$$

$$B_c^r(\vec{\varepsilon}^r, \vec{f}^r) \leq B^0 \tag{2.7}$$

where:

E_j^s = system demand in time period j

E_j^r = energy production of unit r in time period j

Δt_j = duration of period j

P^r = capacity of unit r

j_{rc} = period when reactor r cycle c begins

E_c^r = energy production of unit r in cycle c

ε_c^r = reload enrichment for unit r cycle c

$\vec{\varepsilon}^r$ = vector of ε_c^r for all c = $\{\varepsilon_1^r, \varepsilon_2^r \ldots\ldots\}$

$\vec{f}^r$ = vector of f_c^r for all c = $\{f_1^r, f_2^r \ldots\ldots\}$

F_c^r = a function of $\vec{\varepsilon}^r$ and $\vec{f}^r$. This is the energy produced in reactor r in cycle c

B_c^r = a function of $\vec{\varepsilon}^r$ and $\vec{f}^r$. This is the average discharge burnup in reactor r cycle c

B^0 = Maximum allowable average discharge burnup.

Notice that only some of the constraints given in Section (2.2)are considered explicitly in this thesis.

For a system with R units, a planning horizon containing J period and C cycles, RJ + 3RC variables and J + RJ + 2RC constraints are to be considered. A non-linear problem with this number of variables and constraints is difficult to handle. However, this problem can be further decomposed into two sub-problems; one containing only the linear constraints, and the other the linear and the non-linear constraints. The linear sub-problem, which can be called optimal energy scheduling, is concerned with finding the optimal energy production E_j^r for each reactor r in each time period j.

This problem can be stated as follows

Minimize $\overline{TC}^S = \sum_r^R \overline{TC}^r(E_j^r, \vec{\varepsilon}^{r*}, \vec{f}^{r*})$ (2.8)

with respect to E_j^r

Subject to constraints $\sum_r E_j^r = E_j^S$ (2.3)

$E_j^r \leq \Delta t_j \cdot P^r 8760.$ (2.4) where $\vec{\varepsilon}^{r*}$, $\vec{f}^{r*}$ are the optimal reload enrichments and batch fractions for any set of E_j^r.

The non-linear sub-problem which can be called nuclear in-core optimization is concerned with finding the optimum enrichment and batch fraction for reactor r when required to produce energy E_j^r. This problem can be stated as follows.

$$\overline{TC}^r(E_j^r,\vec{\varepsilon}^{r*},\vec{f}^{r*}) = \text{minimum}\,\overline{TC}^r(E_j^r,\vec{\varepsilon}^r,\vec{f}^r) \qquad (2.9)$$

with respect to $\vec{\varepsilon}^r$ $\vec{f}^r$ for a specified set of E_j^r subject to constraints

$$F_c^r(\vec{\varepsilon}^r,\vec{f}^r) = E_c^r \qquad (2.6)$$

$$B_c^r(\vec{\varepsilon}^r,\vec{f}^r) < B^{\circ} \qquad (2.7)$$

$$\sum_{j_{rc}}^{(j_{rc+1}-1)} E_j^r = E_c^r \qquad (2.5)$$

The problem of optimal energy scheduling and the problem of nuclear in-core optimization can be solved sequentially as follows. Based on an initial guess of $\vec{\varepsilon}^{r*}$, $\vec{f}^{r*}$ for all r, the problem of optimal energy scheduling can be solved to yield an initial solution of E_j^r. Then the problem of nuclear in-core optimization is solved for the optimal $\vec{\varepsilon}^{r*}$ $\vec{f}^{r*}$ corresponding to the initial E_j^r. The improved values of $\vec{\varepsilon}^{r*}$ and $\vec{f}^{r*}$ can be used in the problem of optimal energy scheduling to yield better values of E_j^r. This operation continues until the solution of the two-problems remain the same after successive iterations. The converged results are then the optimal solution for the system optimization problem based on one refuelling and maintenance time schedule. The entire procedure would be repeated for all possible time schedules.

The time schedule with the lowest system operating cost is then the global optimum for the overall problem of Mid-range Utility Planning. The various steps of decomposition are summarized in Table 2.1. The problem of optimal energy scheduling is considered by Deaton(D1). A brief description of his solution technique is presented in Section 2.4. The problem of nuclear in-core optimization is discussed in Section 2.5; in Chapter 6,7,8,9, of this thesis, and also by Kearney(K1).

2.4 Brief Description of the Solution Technique for the Problem of Optimal Energy Scheduling

The problem of optimal energy scheduling can be solved by the method of steepest descent. First, the non-linear objective function is linearized about an initial feasible point

$$\overline{TC}^{s} = \sum_{r}^{R} \overline{TC}^{r} = \sum_{r}^{R} \left\{ \overline{TC}^{or} + \sum_{j} \lambda_{rj} (E_j^r - E_j^{or}) \right\}$$

$$\text{where } \lambda_{rj} \approx \frac{\partial \overline{TC}^{r}(E_j^{or}, \vec{\varepsilon}*, \vec{f}*)}{\partial E_j^r} \qquad (2.10)$$

λ_{rj} as defined in Equation (2.10) may be thought of as the incremental cost of energy for unit r in time period j. Notice that in Equation (2.10) the numerator is the revenue requirement, while the denominator is the actual undiscounted energy. If λ_{rj} could be evaluated for a given set of E_j^{or}, $\vec{\varepsilon}*$, $\vec{f}*$. Equation (2.10) is merely a linear equation, which, together with Equations (2.3) and (2.4)

Table 2.1

Various Steps in the Decomposition of the Overall Optimization Problem of Mid-Range Utility Planning

Step Number	Sub-Problem Name	Variables Held Fixed	Variables to be Optimized
(0)	Overall Optimization Problem of Mid-Range Utility Planning	--	1,2,3,4
(1)	System Optimization for a Given Refuelling and Maintenance Time Schedule	1	2,3,4
(2)	Optimal Energy Scheduling	3,4	2
(3)	Nuclear In-Core Optimization	2	3,4

Variables Designation

1 : Refuelling and maintenance time schedule
2 : Energy production for each generating unit
3 : Reload enrichments for each nuclear unit
4 : Batch fractions for each nuclear unit

constitutes a standard linear program. This can be solved easily by Simplex Method(DZ1) or by standard Network(DZ1) programming techniques. Hence, the crux of the problem is to calculate λ_{rj} for a given set of E_j^r , $\vec{\varepsilon}*$, $\vec{f}*$.

For nuclear reactors, the objective function is a unique function of the cycle energy, reload enrichments and batch fractions, $\overline{TC}^r = \overline{TC}^r(E_c^r , \vec{\varepsilon}*, \vec{f}*)$.
Since by Equation (2.5) E_c^r is a linear combination of E_j^r , the derivatives of $\overline{TC}$ with respect to E_j^r is the same as the derivatives of $\overline{TC}$ with respect to E_c^r . In other words

$$\lambda_{rj} = \lambda_{rc} \equiv \frac{\partial \overline{TC}(E_c^r, \vec{\varepsilon}*, \vec{f}*)}{\partial E_c^r} \quad \text{for } j_{rc} \leq j < j_{rc+1} \tag{2.11}$$

Hence the λ_{rj}'s for all reactors belonging to the same cycle are equal. Calculation of λ_{rc} under many different operating conditions is considered in this thesis. Chapter 3 and 6 consider the calculation of λ_{rc} under steady-state operating condition for the fixed batch fraction case and the variable batch fraction case respectively. Chapter 5, and 9 consider the calculation for λ_{rc} under non-steady state operating condition for the fixed batch fraction case and variable batch fraction case respectively. These calculations of incremental cost would serve as inputs into the optimal energy scheduling algorithm. Methods of solving the optimal energy scheduling problem are not considered in this thesis, except in Chapter 3, where an extremely simple problem of optimal energy scheduling for two different size reactors both operating in steady-state is solved by graphical technique.

2.5 The Organization of the General and Special Problem Of Nuclear In-Core Optimization

The general problem of nuclear in-core optimization is presented in Section (2.3) by Equations (2.9), (2.5), (2.6) and (2.1) as a minimization problem in which both reload enrichments and batch fractions are varied to arrive at the lowest cost. However, one can also consider the simpler problem in which the batch fractions are fixed throughout the planning horizon, and only the reload enrichments are varied. For this special problem, there is at most only one set of reload enrichments that would satisfy all the constraints, Equations (2.5), (2.6) and (2.7). This is due to the physics requirement of a reactivity limited nuclear core that, once the reload batch fraction is fixed, selecting the reload enrichment completely determines the energy it can generate in that cycle. Hence, for this special problem in which batch fractions are fixed, nuclear in-core optimization reduces to the problem of finding the correct reload enrichments that satisfy the constraints. Chapter 3 and 5 consider the special problem of fixed batch fractions. Chapter 6,8 and 9 consider the general problem in which both reload enrichments and batch fractions are allowed to vary.

Steady-state and non-steady-state operation of the reactor is also considered in this thesis. For steady state operation, the energy produced, reload enrichments, and batch fractions are the same for every cycle. Since the physical state of the reactor goes through a complete cycle between refuellings, there are no changes in the value of nuclear fuel inventory between the beginning and the ending of the planning horizon. However, for the non-steady-state case, the physical state of the reactor at the end of the planning horizon is not necessarily the same as at the beginning of the planning horizon. Hence, in order to calculate the objective function accurately, changes in monetary value of nuclear fuel inventory between these two points in time must be accounted for. Chapter 4 describes the various methods of evaluating monetary value of nuclear fuels, which can be used in the calculation of the objective function.

Table 2.2 shows the various problems and special cases considered, and the chapters describing them.

2.6 Types of Reactors Analyzed

The general methodology described in this thesis is applicable to different types of light water reactors. However, only the pressurized water reactors are chosen as examples. This is solely a matter of convenience because pressurized water reactors are easier to model and the relevant computer codes are readily available.

Two pressurized water reactors of different sizes are considered: the 430 MWe San Onofre reactor and the 1065 MWe Zion reactor. Detail descriptions of the two reactors can be found in their final safety reports (SO1,Z1). In this thesis research, the overall weight of UO_2 in Zion core is taken to be

Table 2.2

Contents of the Various Chapters in This Thesis

	Steady State Operation	Non-steady State Operation
Special Problem :		
constant batch fractions variable enrichments	Chapter 3	Chapters 4, 5
General Problem :		
variable batch fractions and enrichments	Chapter 6	Chapters 4, 7, 8, 9

90 metric tonnes instead of the normal value of 86 metric tonnes. The San Onofre reactor is normally refuelled in a 4-zone modified scatter manner, in which the fresh fuel is always loaded on to the outer radial zone during its first cycle of irradiation, and scattered throughout the inner zone in a checker board pattern for the remaining cycles of irradiation. The Zion reactor is normally refuelled in a 3-zone modified scatter manner.

2.7 Depletion Code CELL-CORE

CELL (B1) is a point depletion code which generates one group cross-section data as a function of flux-time. These cross-section data are fed into the spatial depletion code CORE (K1) which is a finite-difference, one-group diffusion theory code in R-Z geometry. Refuelling and fuel shuffling are completely automated in CORE. The input consists of some geometrical descriptions of the nuclear core. The output consists of the mass and concentration of each heavy metal isotope in each individual batch of fuel at the end of every cycle. A more detailed description of the various versions of CORE is given in Appendix A.

The twin-code CELL-CORE was chosen to be the depletion tool in this thesis because of simplicity of usage, high speed of calculation and minimal storage space. To do a depletion calculation for a planning horizon consisting of five cycles takes 160 k byte storage and a CPU time of 0.5 minutes on an IBM 370/45. Hence, it is possible to analyse a large number of cases at low cost. Comparison of the results of CORE with other computer codes and experimental data are given by Kearney (K1).

2.8 Economics Code MITCOST1 and COCO

MITCOST (CJ1) is an economics code which calculate the revenue requirement and average fuel cycle cost for an individual batch of fuel. MITCOST1 is a slight modification of MITCOST which is capable of handling batches with residue book value of fabrication, shipping, reprocessing and conversion costs based on methods developed in Chapter 4.

COCO is a modification of the depletion code CORE. The revenue requirement for each batch of fuel is calculated according to the Inventory Value method given in Chapter 4 directly from the physics data provided in the output of the depletion code CORE. Hence, it is no longer necessary to transfer physics data from the CORE code to MITCOST1 to obtain fuel costs data.

Course listings of CELL-CORE, MITCOST1 and COCO are on file with Professor E.A. Mason at M.I.T.

CHAPTER 3.0

OPTIMAL ENERGY SCHEDULING FOR STEADY-STATE OPERATION WITH FIXED RELOAD BATCH FRACTIONS AND SHUFFLING PATTERN

3.1 Defining the Problem

The first of the problems outlined in Section 2.5 to be considered consists of two nuclear reactors with a fixed refuelling schedule and operating at steady-state conditions. This two-unit system is assumed to supply all the steady-state energy demanded by a customer over the entire planning horizon, except at the time of refuelling, when replacement power is purchased. Depending on the incremental cost of electricity, the customer will decide on the steady-state power level he wishes the reactors to supply.

The problem is to find the optimal enrichments for the reload batches for both of the reactors given the customer's demand curve of energy from the system.

Reactor A of the system is the 1065 MWe PWR described in Chapter 2. Reactor B of the system is a 430 MWe PWR similar to San Onofre I. Reactor A is fuelled in a three-zone modified scatter manner. The irradiation interval is fixed to be 1.375 years and refuelling takes 0.125 years. At time 0.0, the reactors start a new cycle.

Reactor B is fuelled in a four-zone modified scatter manner. The irradiation interval and refuelling time are the same as Reactor A.

Hence both reactors are assumed to be operating from time 0.0 to time 1.375 years and, to facilitate this simplified analysis, they are both assumed to be down for refuelling at the same time. This pattern would repeat itself indefinitely into the future.

Both of the reactors can operate at any power level from zero up to their capacity limit. Forced outages are not included in this simple-minded case.

3.2 Defining the Objective Function

The objective function of this problem is the revenue requirement for fuelling these two reactors from their initial loading into the indefinite future in which they are operating under steady state conditions.

The equations of the revenue requirement will be stated without proof.

$$\overline{TC}^{s} = \overline{TCA} + \overline{TCB} \tag{3.1}$$

$$\overline{TCA} = \sum_{b} \frac{R_b^{A}}{(1+x)^{t_b}} \quad \text{sum over all the batches of fuel for reactor A} \tag{3.2}$$

$$\overline{TCB} = \sum_{b} \frac{R_b^{B}}{(1+x)^{t_b}} \quad \text{sum over all the batches of fuel for reactor B} \tag{3.3}$$

$$R_b^{A\text{ or }B} = \sum_{i} \frac{Z_{ib}^{A\text{ or }B}}{(1+x)^{\Delta t_i}} + \frac{\tau}{1-\tau} \sum_{i} \left[\frac{Z_{ib}^{A\text{ or }B}}{(1+x)^{\Delta t_i}} \sum_{i} Z_{ib}^{A\text{ or }B} \frac{\sum_{c} \dfrac{E_c^{A\text{ or }B}}{(1+x)^{\Delta t_c}}}{\sum_{c} E_c^{A\text{ or }B}} \right] \tag{3.4}$$

where $\overline{TC}^{s}$: revenue requirement for the system

$\overline{TCA}$: revenue requirement for reactor A

$\overline{TCB}$: revenue requirement for reactor B

$R_b^{A\text{ or }B}$: revenue requirement for batch b of reactor A or B discounted to the start of irradiation for that batch

x : effective cost of money

t_b : time when batch b is charged to reactor A or B relative to start of planning horizon

$Z_{ib}^{A \text{ or } B}$: various payments associated with a given batch for reactor A or B

Δt_i : time of these various payments relative to the start of irradiation of that batch

$E_c^{A \text{ or } B}$: energy generated from a given batch at cycle c for reactor A or B

Δt_c : time revenue is received for E_c and income tax paid relative to the start of irradiation

3.3 Defining the Decision Variables and the Design Variables

Since the reload batch fractions are fixed for both reactors and there is no time dependence in this problem, the decision variables reduce to E_c^A and E_c^B, energy generated per cycle from reactor A and B respectively. Since there is a one-to-one correspondence between energy per cycle and reload enrichment under these conditions, specifying one determines the other. Reload enrichment is the dependent variable in this case. Since reload enrichment is one of the design parameters in fuel management, it is formally called a design variable for this problem.

3.4 Lagrangian Optimality Condition

The objective function for the system $\overline{TC}^s$ is to be a minimum with respect to the decision variables E_c^A and E_c^B subject to the condition that the energy of each cycle E_c has the specified value E_c^s. That is

$$E_c^A + E_c^B = E_c^s \qquad c = 1, 2, \ldots. \tag{3.5}$$

Under the assumed condition that the batch fraction of each reactor is held constant, $\overline{TC}^A$ is a function only of the energies E_c^A and $\overline{TC}^B$ is a function only of the energies E_c^B. The Lagrangian condition for $\overline{TC}^s$ to be a minimum subject to the constraints (3.5) is

$$\delta[\overline{TC}^s + \sum_c \lambda_c(E_c^A + E_c^B - E_c^s)] = 0 \tag{3.6}$$

or

$$\frac{\partial}{\partial E_c^A} \overline{TC}^s + \lambda_c(E_c^A + E_c^B - E_c^S) = 0 \tag{3.7}$$

$$\frac{\partial}{\partial E_c^B} \overline{TC}^s + \lambda_c(E_c^A + E_c^B - E_c^S) = 0 \tag{3.8}$$

λ_c being the Lagrangian multiplier for cycle c. Carrying out the differentiation:

$$\frac{\partial \overline{TCA}}{\partial E_c^A} = \frac{\partial \overline{TCB}}{\partial E_c^B} = \lambda_c \qquad c = 1, 2, \ldots. \tag{3.9}$$

After steady state conditions are reached, λ_c becomes a constant λ_{ss}, and the terms in $\overline{TCA}$ and $\overline{TCB}$ affected by the steady state energy are of the form $\sum_c \frac{R_{ss}^A}{(1+x)^{t_c}}$ and $\sum_c \frac{R_{ss}^B}{(1+x)^{t_c}}$ respectively, where t_c is the time irradiation starts in cycle C. At steady state the revenue requirements R_{ss}^A and R_{ss}^B are independent of cycle number c. Hence Eq. (3.9) reduces to

$$\frac{dR_{ss}^A}{dE_{ss}^A} = \frac{dR_{ss}^B}{dE_{ss}^B} = \lambda_{ss} \tag{3.10}$$

For the present work, revenue requirements R^A and R^B for steady state batches in reactors A and B respectively were available, calculated from Eq. (3.4). To use Eq. (3.9) directly it is necessary to have the revenue requirements R^A_{ss} and R^B_{ss} for steady state cycles. Fuel in reactor A in a particular batch contributes energy to three cycles, starting when batch of interest is charged, a second starting 1.5 years later and a third starting 3.0 years later. For the present work it was assumed that the revenue requirement for a steady-state batch of reactor A was made up of equal contributions of one-third of the revenue requirements of each of the three cycles to which it contributes energy, each present worthed to the time basis for the batch in question, that is

$$R^A = \frac{R^A_{ss}}{3}\left[1 + \frac{1}{(1+x)^{1.5}} + \frac{1}{(1+x)^3}\right] \tag{3.10a}$$

Similarly, for reactor B, with four-zone fueling, it was assumed that

$$R^B = \frac{R^B_{ss}}{4}\left[1 + \frac{1}{(1+x)^{1.5}} + \frac{1}{(1+x)^3} + \frac{1}{(1+x)^{4.5}}\right] \tag{3.10b}$$

This procedure of bringing the cycle revenue requirements to the time basis of a batch is used instead of bringing the batch revenue requirements to the time basis of a cycle because in a rigorous treatment of this optimization problem the independent variable used to provide the specified energy per cycle is the enrichment of a batch.

3.5 The Optimization Procedures

The optimization procedure was divided into several steps. Through these steps, the following data have been generated:

(1) revenue requirement for each reactor for steady state cycles at different enrichments

(2) incremental revenue requirement, or incremental cost, as a function of cycle energy for each reactor

(3) system incremental cost as a function of system energy

(4) energy per cycle for each reactor as a function of system energy

(5) reload enrichment for each reactor

Step 1

Using the code package CELL-CORE-MITCOST 1, the cycle energy and the revenue requirement per steady state batch for different enrichments were calculated for reactors A and B. The results are shown on Table (3.1), and plotted in the form of revenue requirement per cycle on Figures (3.1, 3.2).

Step 2

By differentiating R_{ss}^{A} with respect to E_{ss}^{A} numerically or graphically, the incremental steady state cycle cost is obtained. The results are given on Figure (3.3) for reactors A and B.

Table 3.1

Cycle Energy and Revenue Requirement for Different Enrichments

Reactor A Zion type 1065 MWe PWR Three-zone Modified Scatter Refuelled Steady State Conditions

Enrichment, (w/o)	Energy per Cycle, GWHe	Revenue Requirement, 10^6\$ Per Batch	Revenue Requirement, 10^6\$ Per Cycle
2.4	4732.6	8.9448	9.9371
2.8	6025.9	10.4375	11.5954
3.2	7251.0	11.9499	13.2756
3.6	8434.1	13.4861	14.9822
4.0	9575.3	15.0320	16.6997
4.4	10687.0	16.5900	18.4305
4.8	11774.7	18.1588	20.1733

Reactor B San Onofre type 430 MWe PWR Four-zone Modified Scatter Refuelled Steady State Condition

Enrichment, (w/o)	Energy per Cycle, GWHe	Revenue Requirement, 10^6\$ Per Batch	Revenue Requirement, 10^6\$ Per Cycle
1.960	1536.7	3.3914	3.9666
2.444	2273.5	4.2371	4.9557
2.913	2940.2	5.0744	5.9350
3.846	4123.6	6.7718	7.9203
4.762	5152.7	8.4588	9.8934

For both reactors, irradiation starts at 0.0 year
irradiation ends at 1.375 years
refuelling time 0.125 years
thermal efficiency 32.6%

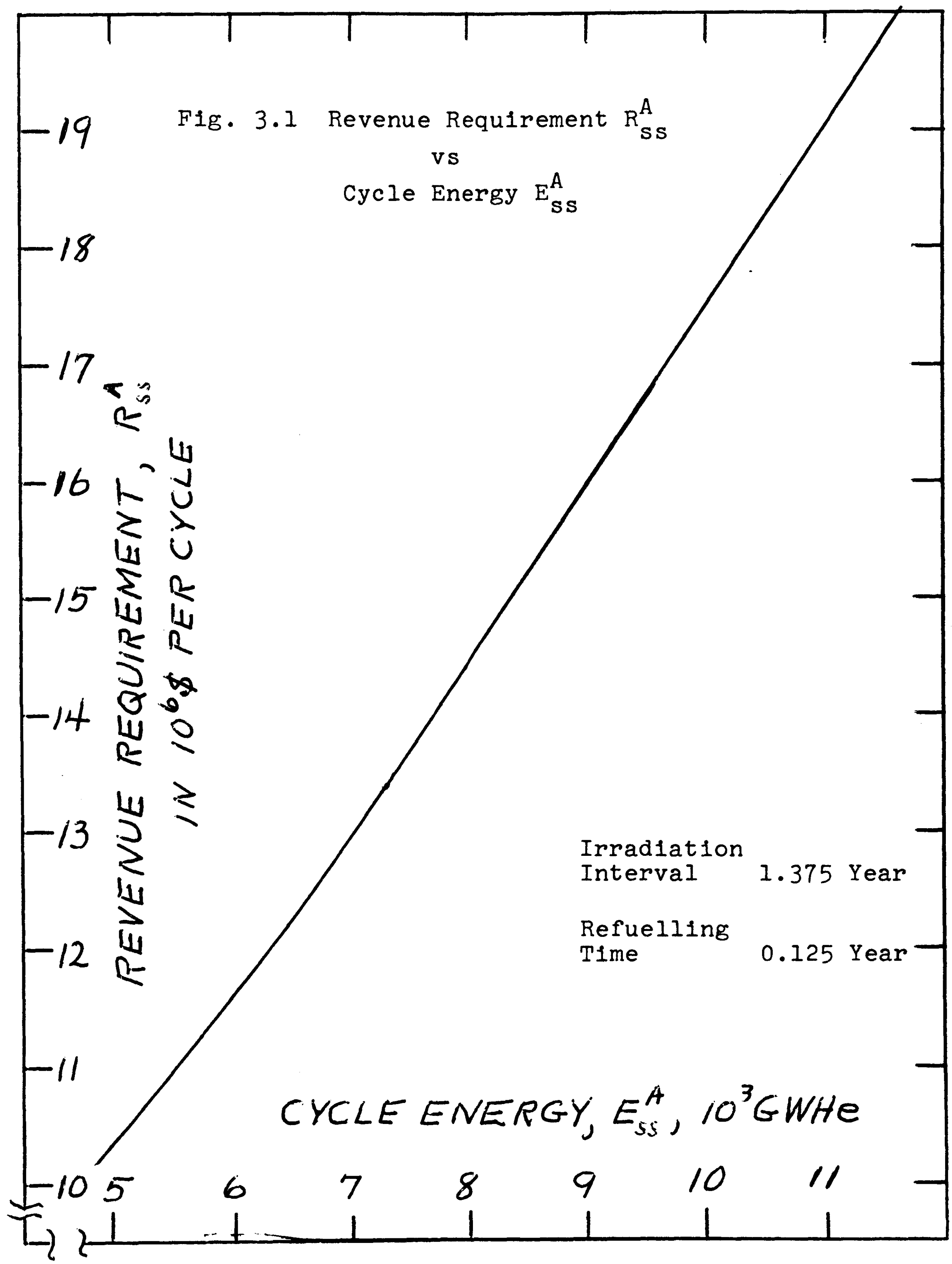


Fig. 3.1 Revenue Requirement R^A_{ss}
vs
Cycle Energy E^A_{ss}
REVENUE REQUIREMENT, R^A_{ss}
IN 10^6\$ PER CYCLE
CYCLE ENERGY, E^A_{ss}, 10^3GWHe
Irradiation Interval 1.375 Year
Refuelling Time 0.125 Year
10
11
12
13
14
15
16
17
18
19
5
6
7
8
9
10
11

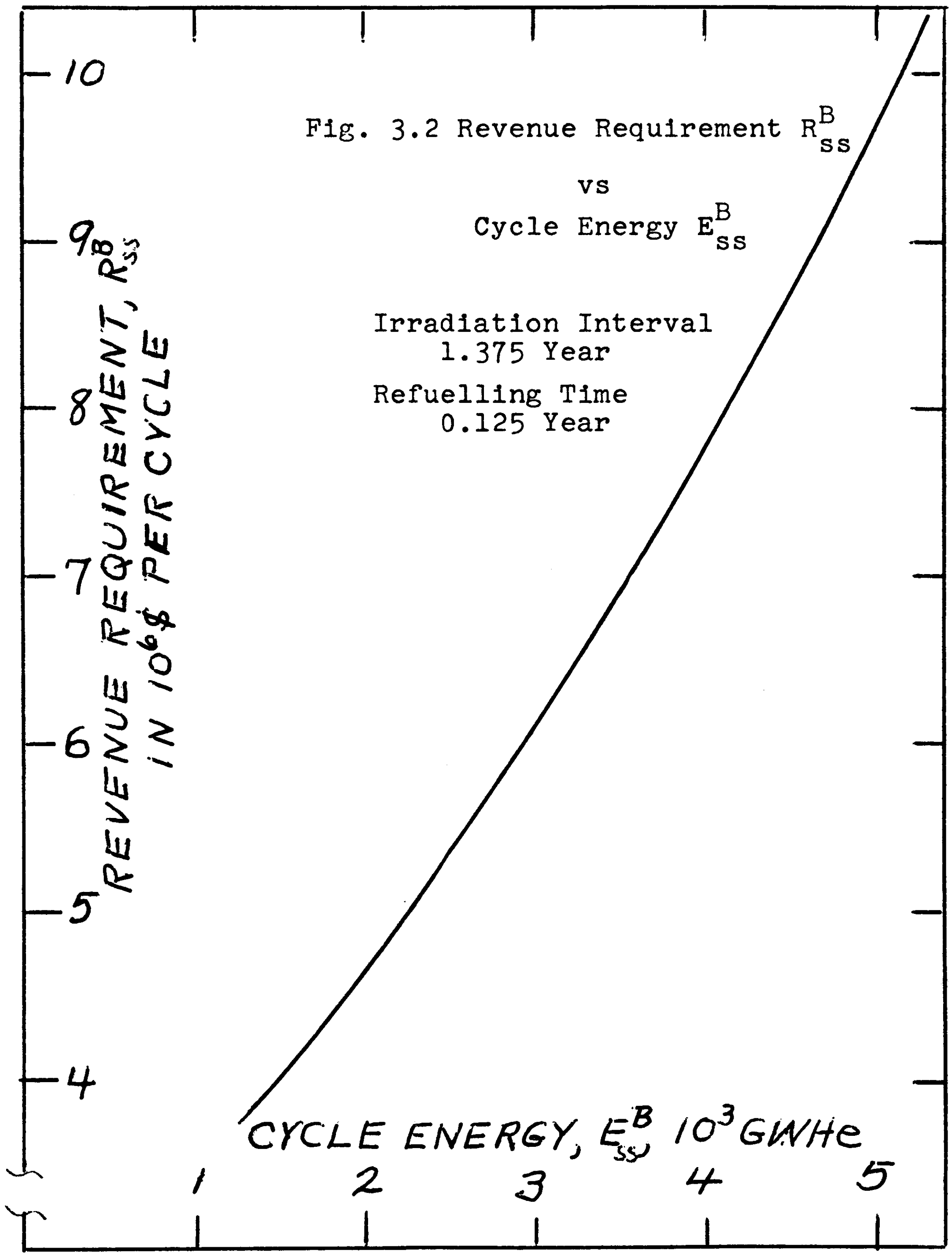


Fig. 3.2 Revenue Requirement R^B_{ss} vs Cycle Energy E^B_{ss}

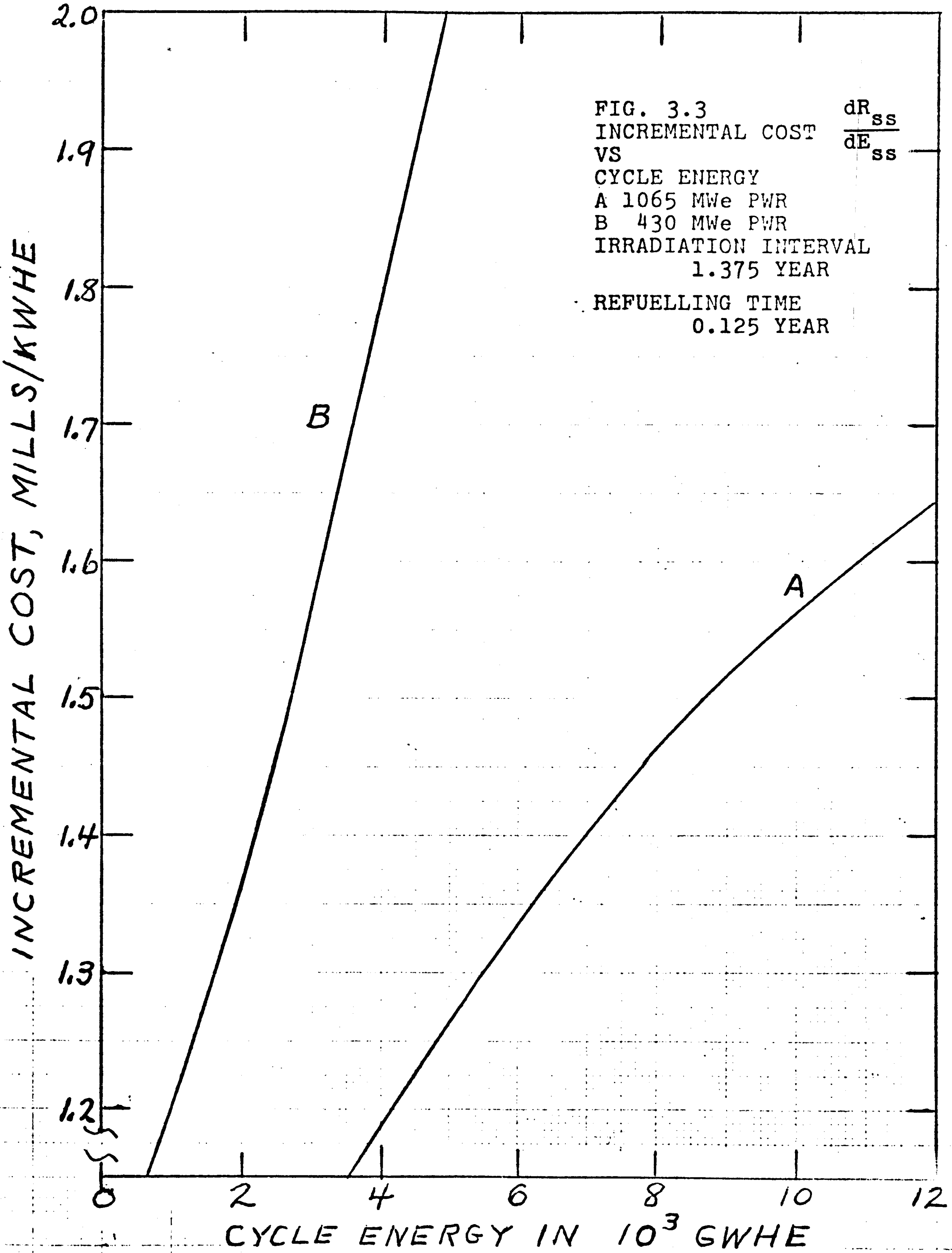


FIG. 3.3
INCREMENTAL COST $\frac{dR_{ss}}{dE_{ss}}$
VS
CYCLE ENERGY
A 1065 MWe PWR
B 430 MWe PWR
IRRADIATION INTERVAL
1.375 YEAR
REFUELLING TIME
0.125 YEAR
A
B
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
0
2
4
6
8
10
12
INCREMENTAL COST, MILLS/KWHE
CYCLE ENERGY IN 10^3 GWHE

Step 3

Since the Lagrangian condition for minimal cost requires that the two reactors have the same incremental cost, the reactors should be operated in the following manner. For any given level of E_c^S (systems demand), the reactors must be loaded such that their incremental costs are the same. Figure 3.4 shows the relationship of E_c^S with respect to the incremental cost of reactor A or B. The ordinate represents the incremental cost for the entire system at that level of E_c^S. Figure 3.4 can be viewed as the supply curve of energy for the system. Notice that for $E_c^S \geqslant 16.7 \cdot 10^3$ GWHe reactor A is base-loaded and any load increment goes to reactor B. Hence the incremental cost for the system is equal to the incremental cost for reactor B from then onwards.

Step 4

Based on the supply curve of energy for the system, the customer can decide on the level of E he wants. Once he decides on a E_c^S, Figure 3.5 would give the energy output from each reactor. Figure 3.5 represents the loading of reactor A or B for a given level of E_c^S under the Lagrangian condition of equal incremental cost.

Figure 3.6 shows the relationship between capacity factor for each reactor versus E_c^S. Notice again that reactor A has unity capacity factor for $E_c^S \geqslant 16.7 \cdot 10^3$ GWHe. This is due to the fact that reactor A has a lower incremental cost than reactor B, and therefore is base-loaded sooner.

Step 5

Finally, the optimum reload enrichment for each reactor

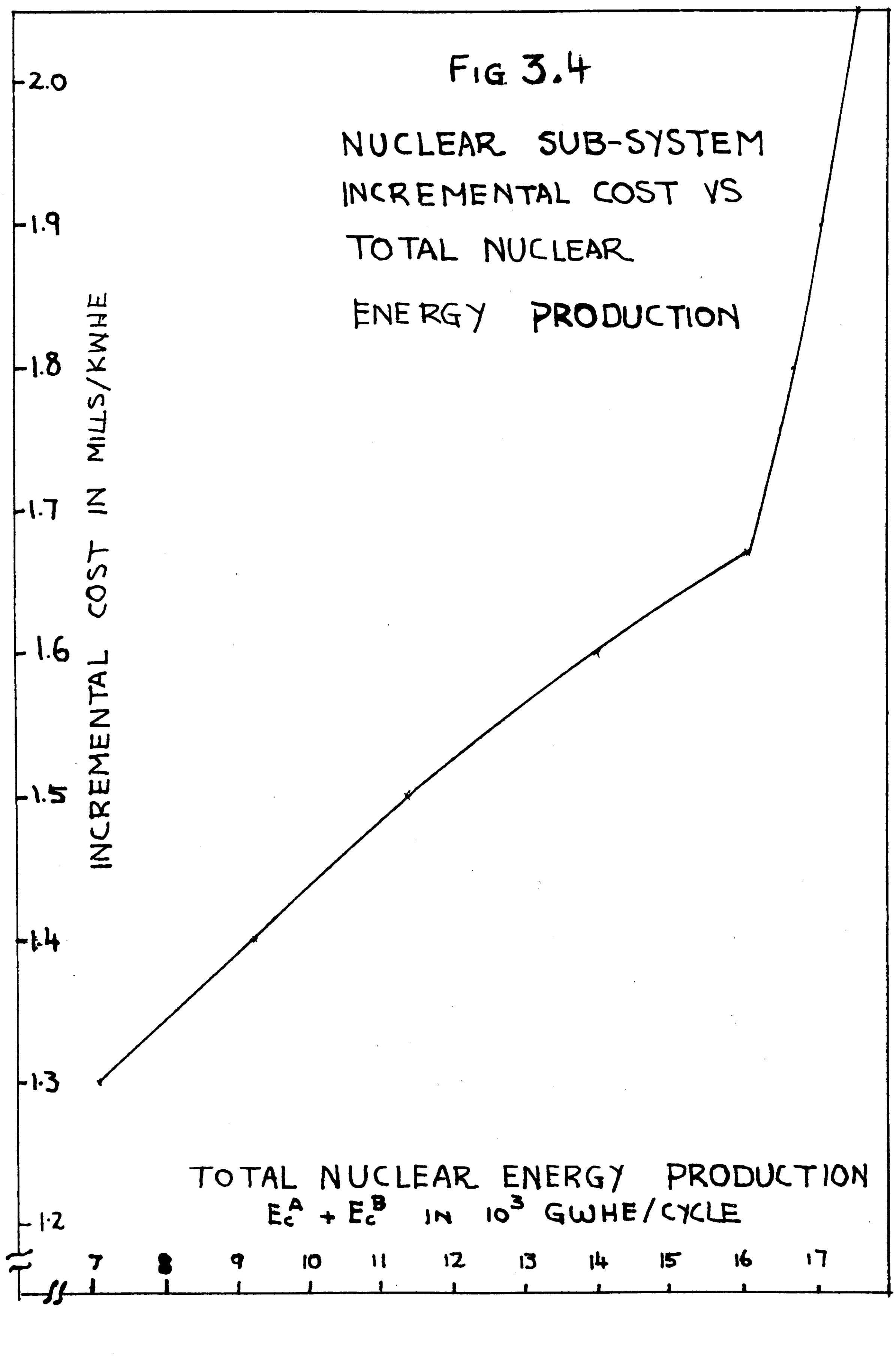
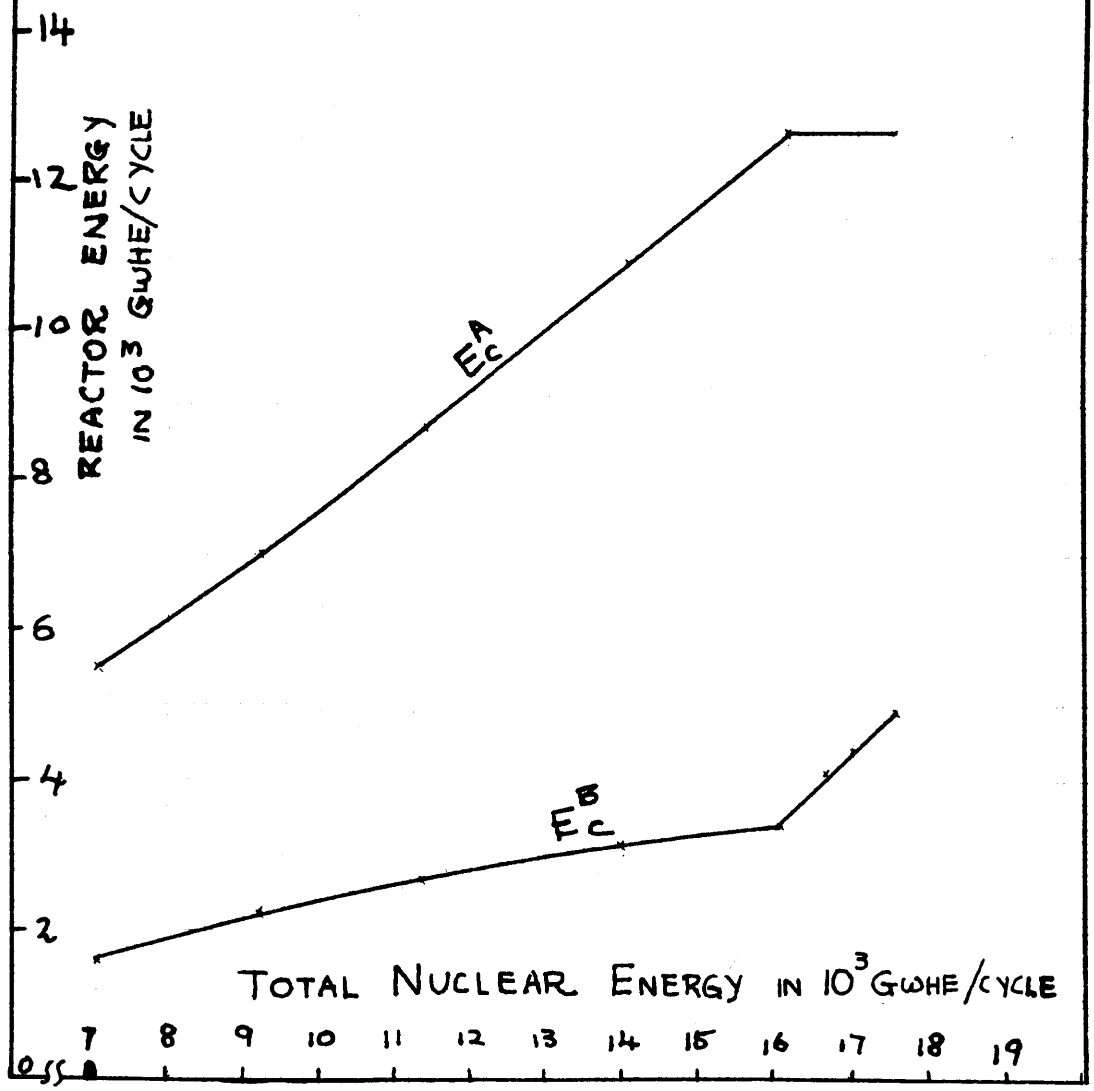


FIG 3.4
NUCLEAR SUB-SYSTEM INCREMENTAL COST VS TOTAL NUCLEAR ENERGY PRODUCTION
INCREMENTAL COST IN MILLS/KWHE
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
TOTAL NUCLEAR ENERGY PRODUCTION
$E_c^A + E_c^B$ IN 10^3 GWHE/CYCLE
7
8
9
10
11
12
13
14
15
16
17

FIG. 3.5

REACTOR ENERGY
VS
TOTAL NUCLEAR ENERGY



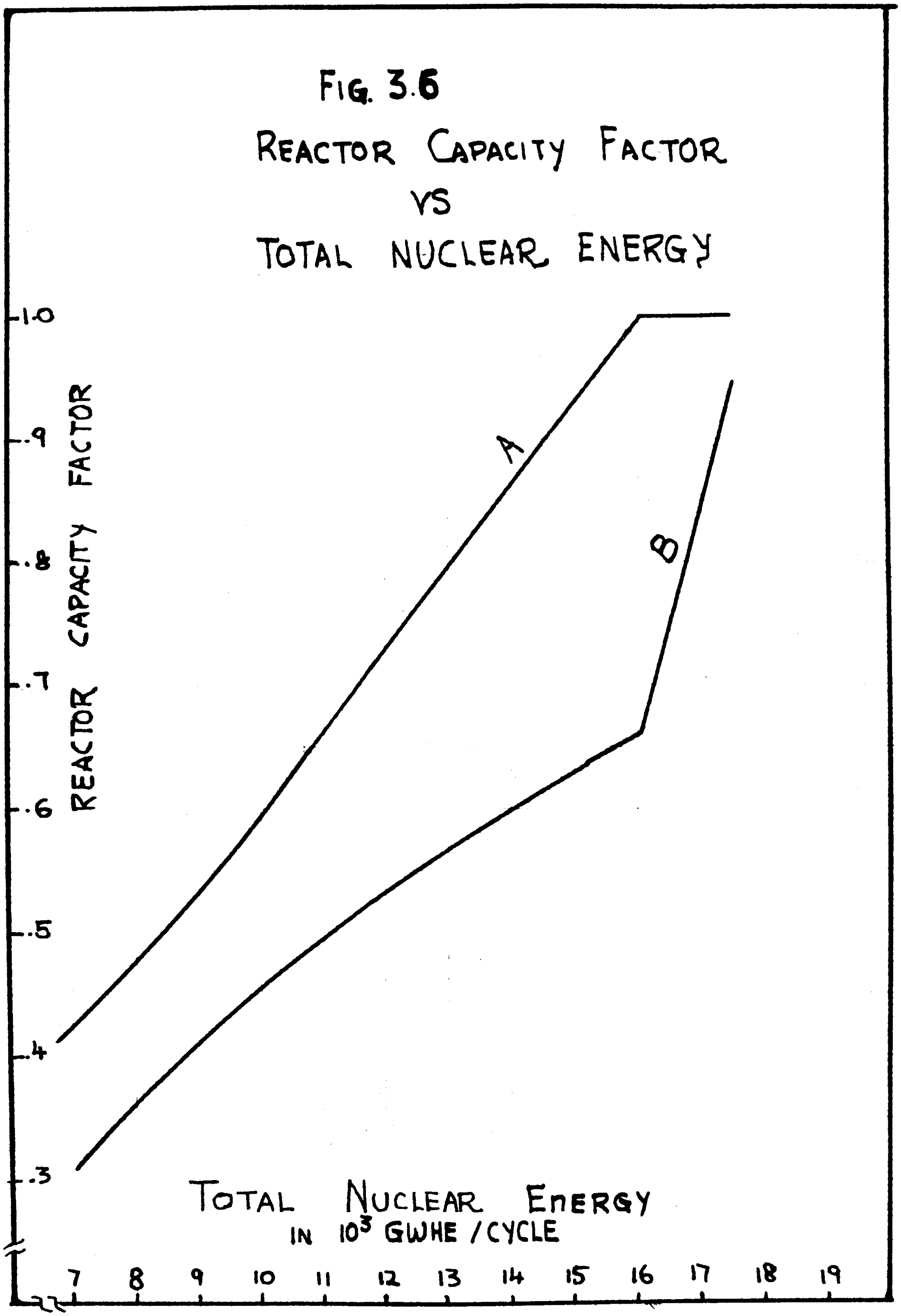


Fig. 3.6
Reactor Capacity Factor
vs
Total Nuclear Energy
1.0
.9
.8
.7
.6
.5
.4
.3
Reactor Capacity Factor
A
B
Total Nuclear Energy
in 10^3 GWHE / Cycle
7
8
9
10
11
12
13
14
15
16
17
18
19

can be inferred directly from cycle energy by Figure 3.7. Specifying the reload enrichments completes the optimization analysis.

3.6 Summary and Conclusions

The problem of optimal energy scheduling for steady-state operation with fixed reload batch fraction and shuffling pattern has been solved in a straight-forward manner using Lagrangian optimality condition and direct calculation of incremental costs. Unfortunately, this problem is too simple to be realistic or of practical interest. Not considered are time behaviour, stochastic events and other refuelling and operation options. However, the important concept of equal-incremental cost operation is illustrated. This sample case shows how incremental cost can be generated from fuel depletion computer codes and applied in the energy scheduling for the whole system.

The problem of optimal energy scheduling between generating units will not be considered further in this thesis. Development of simulation method to make similar optimizations from beginning to end involving many reactors and fossil plants in a time varying framework is the subject of two other thesis projects (Deaton (D1) and Kearney (K1)). This simple example serves as a bridge linking the calculation of incremental costs to the problem of overall system simulation and optimization.

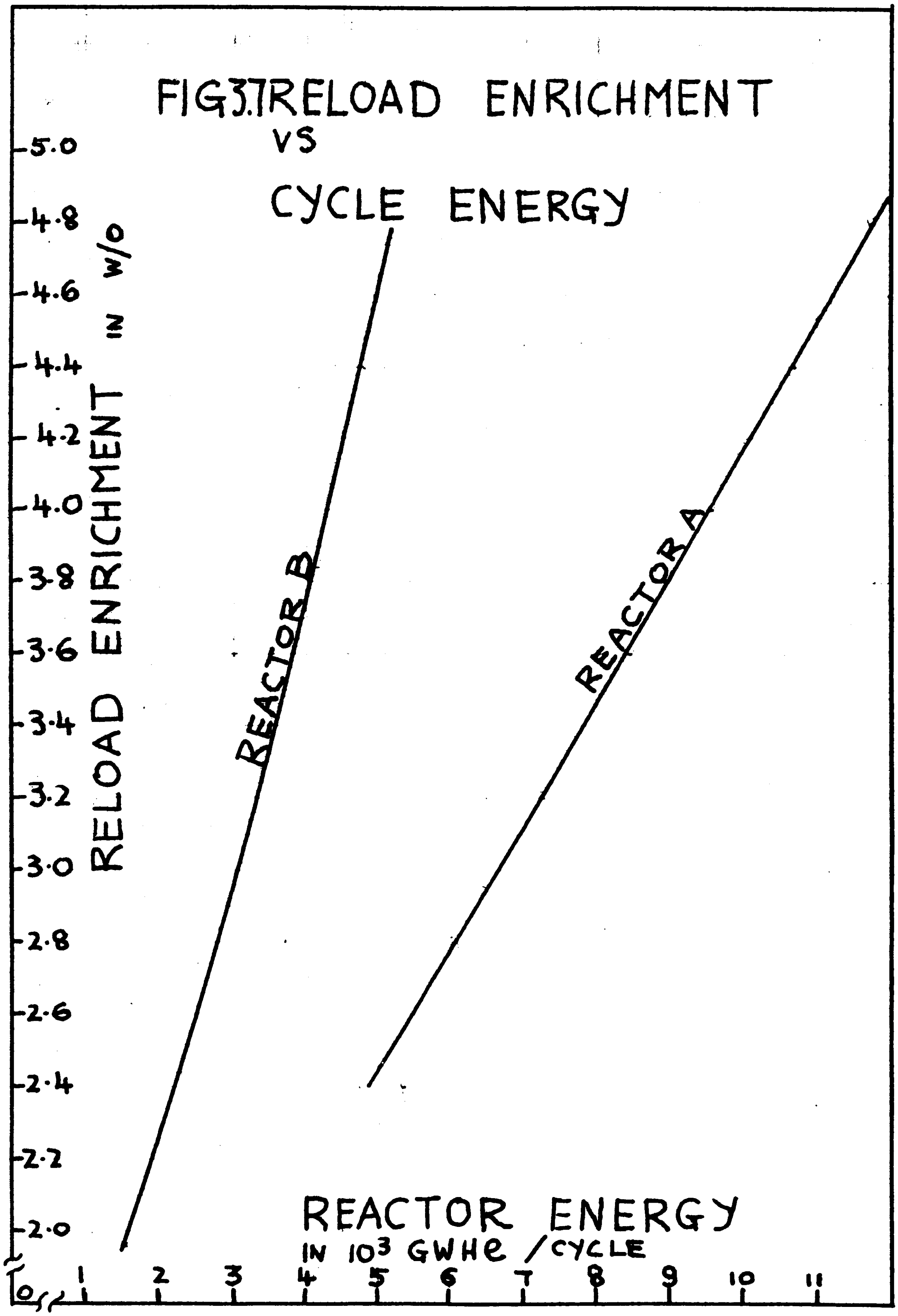


FIG3.7RELOAD ENRICHMENT
VS
CYCLE ENERGY
RELOAD ENRICHMENT IN w/o
5.0
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
REACTOR B
REACTOR A
REACTOR ENERGY
IN 10³ GWHe /CYCLE
0
1
2
3
4
5
6
7
8
9
10
11

CHAPTER 4.0
OBJECTIVE FUNCTION FOR NON-STEADY STATE CASES

4.1 Introduction

The second of the problems outlined in Section 2.5 is concerned with the calculation of the objective function for a finite time horizon. In principle, the complete optimization problem would provide a solution for the indefinite time horizon provided that pertinent information about the system is available. However, the future is always uncertain, and the farther away it is, the greater the uncertainty there is regarding its characteristics. Hence, after some time in the future, information about the system is so uncertain that optimization based on this information becomes irrelevant.

For practical purposes, optimization is usually performed for a finite time horizon for which information is available with some degree of certainty. In this circumstance, one would like to have an optimization procedure such that when it is applied successively to a sequence of finite time periods, the collection of optimal solutions would be the same as the optimal solution for the entire duration of the time periods based on the same input data. In other words, one would like to optimize for the individual pieces and at the same time arrive at a global optimal. Any optimization procedures having such a characteristic possess the property of separability.

The development of an optimization procedure possessing the property of separability begins with the definition of the objective function. The objective function is defined as the total fuel cycle cost in a given time period. However, due to the physical nature of multi-batch refuelling, the physics, and hence the economics of fuel cost for different batches are not separable from each other. To make the optimization procedure possess the property of separability, a mechanism must be developed to decouple the fuel cycle cost calculations in one time period from the other. The proposed mechanism involves the treatment of fuel inventories at the end points of the time period.

For the case in which the corporate income tax rate is taken as zero (e.g., government-owned utilities) but there are carrying charges, a rigorous and consistent treatment of the fuel inventories at the end points is developed. For the case where income taxes apply (e.g., investor-owned utilities) the treatment is not completely rigorous. This is mainly due to the fact that income tax laws are difficult to apply to fuel batches which are in the reactor at the end of a time period and are subject to undecided future operations.

Hence, two definitions of objective function are used, one for the case of no income tax and the other for the case of finite income tax.

4.2 Objective Function Defined For The Case With No Income Tax

4.2.1 Formulating the Problem

First consider the optimization problem for the indefinite time horizon (unspecified but not infinite in length). The output variables are the cycle energies E_c^r for Reactor r in Cycle c. The objective function for Reactor r is the present value of all the fuel cycle expenditures in the future.

$$\overline{TC}_{\infty} = \overline{TC}_{\infty}^{N} + \overline{TC}_{\infty}^{S}$$

$$\overline{TC}_{\infty}^{N} = \sum_{i} \frac{Z_i^N}{(1+x)^{t_i}} \qquad (4.1)$$

$$\overline{TC}_{\infty}^{S} = \sum_{i} \frac{Z_i^S}{(1+x)^{t_i}}$$

where the summation includes all the fuel cycle expenditures.

Z_i^N expenditures and credits for uranium and plutonium

Z_i^S expenditures for service, or processing, components which include fabrication, shipping, reprocessing and conversion.

This formulation separates the variable and fixed components of the fuel cycle cost. Uranium and plutonium costs are directly related to energy production. Service components costs are necessary to maintain the operation of the reactor, but they are not related directly to the level of energy production.

The objective function for the finite horizon case is defined as the present value of all the fuel cycle expenditures associated with that finite time period. For the nuclear component of the cost, an inventory adjustment term is included.

$$\overline{TC}_{I} = \overline{TC}_{I}^{N} + \overline{TC}_{I}^{S}$$

$$\overline{TC}_{I}^{N} = \sum_{jI} \frac{Z_{jI}^N}{(1+x)^{t_{jI}}} + \frac{V_{\text{initial}}^{I}}{(1+x)^{t_{I'}}} - \frac{V_{\text{final}}^{I}}{(1+x)^{t_{I''}}} \qquad (4.2)$$

$$\overline{TC}_{I}^{S} = \sum_{jI} \frac{Z_{jI}^S}{(1+x)^{t_{jI}}}$$

where $\sum_{jI}$ sums over all the fuel cycle expenditures in time period I.

V^I is the inventory adjustment term.

t_{jI} : time for the various fuel cycle expenses

$t_{i'}$: time when time period I begins

$t_{i''}$: time when time period I ends

4.2.2. The Condition of Consistency

The sum of the objective functions for all the time periods must be equal to the objective function for the indefinite time horizon.

$$\sum_{I}^{n} \overline{TC}_I = \overline{TC}_\alpha \tag{4.3}$$

n: number of time periods in the indefinite time horizon.

Substituting Equation (4.1) for $\overline{TC}_\alpha$, and Equation (4.2) for $\overline{TC}_I$, Equation (4.3) reduces to

$$\sum_i \frac{z_i^S}{(1+x)^{t_i}} + \sum_i \frac{z_i^N}{(1+x)^{t_i}} + \sum_I \left\{ \frac{V^I_{initial}}{(1+x)^{t_{I'}}} - \frac{V^I_{final}}{(1+x)^{t_{I''}}} \right\} = \sum_i \frac{z_i^N}{(1+x)^{t_i}} + \sum_i \frac{z_i^S}{(1+x)^{t_i}} \tag{4.4}$$

since the sum of partial sum is equal to the total sum.

$$\sum_I \sum_{jI} = \sum_i$$

From Equation (4.4) the consistency condition results:

$$\sum_I \frac{V^I_{initial}}{(1+x)^{t_{I'}}} = \sum_I \frac{V^I_{final}}{(1+x)^{t_{I''}}} \tag{4.5}$$

4.2.3 The Condition of Equalized Incremental Cost

Equalized incremental cost: Since reactors are energy producing devices, and fuel cycle cost is a measure of the cost associated with energy production, the relationship between cost and energy output must be preserved in the finite horizon case. In other words, the variation of objective function with respect to energy in the finite time horizon must be the same as that of the indefinite time horizon. If this equality is maintained, optimal energy scheduling based on the finite planning horizon objective function is the same as that based on the indefinite planning horizon objective function. Hence the requirement is that the incremental cost of energy be the same in both cases.

$$\frac{\partial \overline{TC}_I}{\partial E_c^r} = \frac{\partial \overline{TC}_\alpha}{\partial E_c^r} \quad \text{for those cycles } c \text{ which are in time period } I \tag{4.6}$$

Since service component costs in period I depend on what happens in period I, and do not depend on what happens in the other time periods,

$$\frac{\partial \overline{TC}_I^S}{\partial E_c^r} = \frac{\partial}{\partial E_c^r}\sum_{jI} \frac{Z_{jI}^S}{(1+x)^{t_{jI}}} = \frac{\partial}{\partial E_c^r}\sum_{i} \frac{Z_i^S}{(1+x)^{t_i}} = \frac{\partial \overline{TC}_\alpha^S}{\partial E_c^r} \tag{4.7}$$

Hence, (4.6) reduces to

$$\frac{\partial \overline{TC}_I^N}{\partial E_c^r} = \frac{\partial \overline{TC}_\alpha^N}{\partial E_c^r} \tag{4.8}$$

Hence, the problem of developing separable optimization procedures reduces to the problem of finding $V_{initial}^I$ and V_{final}^I such that Equation (4.5) and Equation (4.8) are satisfied.

Equation (4.5) can be satisfied quite easily by equating the present worth of $V_{initial}^I$ and V_{final}^{I-1}.

that is $$\frac{V_{initial}^I}{(1+x)^{t_{I'}}} = \frac{V_{final}^{I-1}}{(1+x)^{t_{(I-1)''}}} \tag{4.9a}$$

and by taking $V_{initial}^1 = 0$ and $V_{final}^n = 0$ (4.9b,c)

where n is the last time period

Equation (4.9a) is equivalent to the requirement that the value of ending inventory in one time period must be equal to the value of beginning inventory in the following time period. To simplify the notation, V^I will represent $V^I_{initial}$ and V^{I-1}_{final} .

$$V^I = V^I_{initial} = V^{I-1}_{final} \tag{4.10}$$

4.3 Three Methods of Evaluating Fuel Inventories

Three different methods of evaluating V^I have been developed. Each one of them satisfies the consistency condition (4.5). By performing some sample calculations, one can determine whether **any** of them satisfies the equal incremental cost condition Equation (4.8). The methods are described below and the sample calculations are given in the next Section 4.4.

4.3.1. Nuclide Value Method

V^I is equated to the market value of nuclear material, i.e., value of uranium and plutonium inside the reactor at the beginning of time period I.

$$V^I = \$\text{value} (U,Pu) \tag{4.11}$$

The value of separative work is calculated for each individual batch, and it is summed up with the value of uranium and plutonium.

4.3.2 Unit Production Method

V^I is equated to the book value of nuclear material in the fuel batches in the reactor at the beginning of time period I. Book value is determined by linear depreciation as a function of energy production.

$$V^I = \sum_b \left[\left\{ \frac{\text{Initial value - salvage value}}{\text{total energy generation}} \right\} \cdot \left\{ \begin{array}{l} \text{Energy generation} \\ \text{in time period I} \end{array} \right\} + \text{salvage value} \right]$$

the summation over b runs over all the batches of fuel in the reactor at the beginning of time period I.

Since $\overline{TC}_1$ involves the beginning inventory V^1 as well as the ending inventory V^2, calculation of $\overline{TC}_1$ requires projecting into time period 2 to obtain total energy generation and nuclide salvage value for some batches.

Hence, this method is subject to forecast error. Moreover, projecting the salvage value for all the fuel batches remaining in the reactor at the end of the time period requires many more cycles of depletion calculation. For a planning horizon of five cycles concerning a reactor refuelled in a three-zone modified scatter manner, this method may require 2 or more cycles of depletion calculations, equivalent to a 40% increase in computational effort.

4.3.3 Constant Value Method

$V^I/(1+x)^{t_{I'}}$ is equated to a constant. Physically this implies that the relative changes of the present value of fuel inventories value from one time period to the other are ignored.

$$\frac{V^I}{(1+x)^{t_{I'}}} = \text{constant} \tag{4.12}$$

4.4 Results of Two Sample Cases

Two sample cases are presented below.

The first case consists of a perturbation in energy in the first cycle of a steady-state operating condition. The reactor is the 1065 MWe PWR described in Chapter 2.

The reactor is considered to have been operating on a 3.16 w/o three-zone modified scatter refuelling steady-state condition for a long time. At time zero, the reload enrichment for batch 1 is changed so that energy production in that cycle is increased. For the succeeding cycles, energy production is brought back to the former steady-state level by adjusting the reload enrichments. This operation continues until the reactor is back to its original steady-state condition again.

The second case is similar to the first case except that the perturbation magnitude is doubled. Again, the reload enrichments are adjusted in the succeeding cycles to bring back the energy production to its former steady-state level until the reactor is again in steady-state condition.

Table 4.1 shows the reload enrichments and cycle energies for the steady-state case and the two perturbed cases. For the two perturbed cases, the results of the first five cycles are shown. Note that the reactor has nearly settled back to its initial condition by the fifth cycle.

From the data from the depletion codes, the economics calculations can be carried out. Hence the objective function for the indefinite future $\overline{TC_{\infty}}$ can be calculated, using Equation (4.1).

Table 4.1

Feed Enrichment and Energy per Cycle for Steady State Case and the Two Perturbed Cases

Steady State Case

Cycle	1	2	3	4	5
Enrichment (w/o)	3.16	3.16	3.16	3.16	3.16
Cycle Energy GWHt	21935.	21935.	21935.	21935.	21935.

First Perturbed Case (ΔE=1029GWHt in Cycle 1)

Cycle	1	2	3	4	5
Enrichment (w/o)	3.359	3.054	3.174	3.196	3.133
Cycle Energy GWHt	22964.	21935.	21929.	21928.	21933.

Second Perturbed Case (ΔE=2050GWHt in Cycle 1)

Cycle	1	2	3	4	5
Enrichment (w/o)	3.557	2.941	3.186	3.235	3.106
Cycle Energy GWHt	23985.	21919.	21906.	21939.	21970.

Note: The cycle energies in the two perturbed cases for Cycles 2 through 5 were not converged to exactly the same energies as occurred in the basic steady state case. The differences in total energy for the four cycles are:

1st Case $\sum_{2}^{5} E_c(\text{Perturbed}) - \sum_{2}^{5} E_c(\text{Base}) = -15$ GWHt (0.02%)

2nd Case $= -6$ GWHt (0.007%)

This each of the complete convergence introduces an insignificant error in the calculated incremental costs.

$$
\begin{aligned}
\overline{TC}_{\alpha} &= \overline{TC}_{\alpha}^{N} + \overline{TC}_{\alpha}^{S} \\
\overline{TC}_{\alpha}^{N} &= \sum_{i}^{N} \frac{Z_i}{(1+x)^{t_i}} \\
\overline{TC}_{\alpha}^{S} &= \sum_{i}^{S} \frac{Z_i}{(1+x)^{t_i}}
\end{aligned}
\qquad (4.1)
$$

For three-zone fueling, the perturbation affects the salvage value of the two fuel batches that come before the fuel batch loaded into the perturbed cycle, and the initial and final value of the four fuel batches that come after it. Hence a total of seven fuel batches are affected by the perturbation. The other fuel batches in the indefinite time horizon are not affected by the perturbation.

The number of batches included in $\overline{TC}_{\alpha}$ and $\overline{TC}_1$ and $\overline{TC}_2$ is shown schematically in Figure 4.1. Only the batches that are affected by the perturbation are included. $\overline{TC}_{\alpha}$ includes all seven batches (-1 to 5 inclusive) for a total of eight cycles.

$\overline{TC}_1$ includes only the first three batches (-1, 0, 1) for the first three cycles. $\overline{TC}_1$ is credited with the value of fuel inventories of batch 0 and -1 at the end of the first cycle. $\overline{TC}_2$ includes the last six batches for the last six cycles. $\overline{TC}_2$ is charged with initial value of fuel inventories of batch 0 and -1 at the beginning of the second cycle.

Part A of Table 4.2 gives the objective function for the batches whose values are affected by changes in energy in Cycle 1. The first column gives the result of exact calculation

FIGURE 4.1

RELATIONSHIPS BETWEEN THE VARIOUS REVENUE REQUIREMENTS BATCH NUMBERS AND CYCLE NUMBER

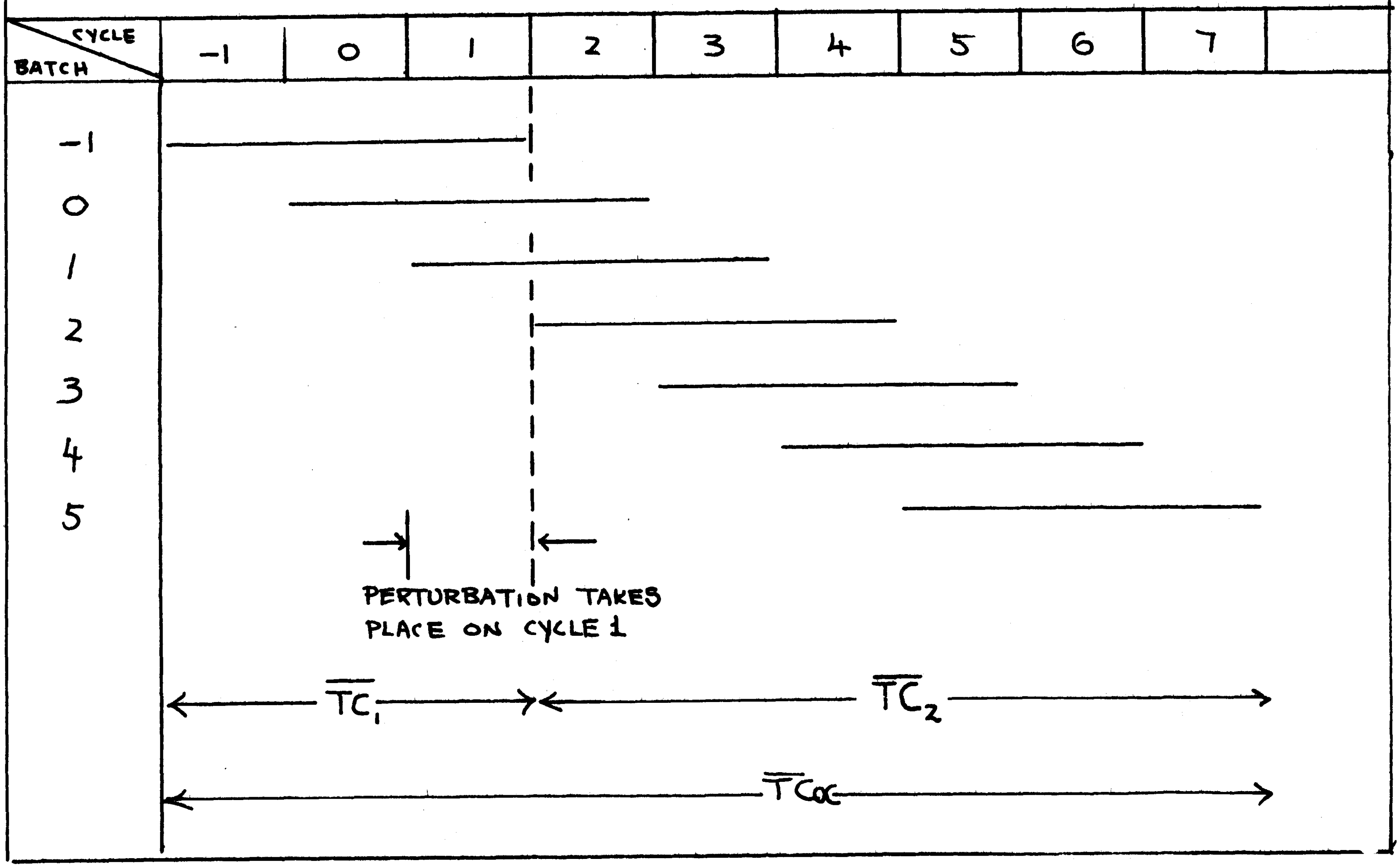


Table 4.2

Comparison of Exact Incremental Cost with Incremental Cost Calculated by Three Approximate Methods. (No Income Tax)

Method	Exact	Nuclide Value	Unit Production	Constant Value
Quantity Calculated	$\overline{TC}_{\infty}$	$\overline{TC}_1$	$\overline{TC}_1$	$\overline{TC}_1$
Batches Included	7 (-1,0,1,2,3,4,5)	3 (-1,0,1)	3 (-1,0,1)	3 (-1,0,1)

Part A	Revenue Requirement 10^6 \$			
Steady State	62.3515	25.8651	25.0157	35.2680
Additional Energy in Cycle 1				
ΔE_1=1029GWHt	62.7428	26.2693	25.3782	35.9983
=2050GWHt	63.1245	26.6740	25.7430	36.7316

Part B	Incremental Cost for Cycle 1 Mills/KWHe[+]			
$\Delta E_1 \equiv$1029GWHt	1.17	1.20	1.08	2.18
2050GWHt	1.16	1.21	1.09	2.19

+ $\text{Mills/kwhe} = 10^3 \cdot \Delta TC / 10^6 \cdot \Delta E_1 \cdot \eta$

† η = thermal efficiency=0.326

Irradiation time =1.375 year
Refuelling time =0.125 year

of the objective function for batches -1, 0, 1, 2, 3, 4, and 5. The second, third and fourth columns give the results of calculation of the objective function by three different approximate methods. For these columns, results are given for only batches -1, 0, 1, since these are the only batches whose contribution to the objective function are changed by change of energy in cycle 1, under the assumptions of these approximate methods.

The first row of Part A gives the objective function for the stated number of batches for the unperturbed case. The second row gives the objective function for an increase in energy production ΔE in cycle 1 of 1000 GWHt, with unchanged energy production in all following periods. The third row gives corresponding information for an energy increase of 2000 GWHt in cycle 1.

Part B gives incremental costs as defined in Equation (4.13), for the two values of ΔE. The first column gives exact incremental costs over the entire five cycles. The last three columns give approximate incremental costs calculated by each of the three methods for evaluating the initial and final inventories for the first cycle. These incremental costs are calculated from Equation (4.13).

$$\frac{\Delta \overline{TC}_{\alpha}}{\Delta E_1} = \frac{\overline{TC}_{\alpha}(E_1 + \Delta E_1) - \overline{TC}_{\alpha}(E_1)}{\Delta E_1} \tag{4.13a}$$

$$\frac{\Delta \overline{TC}_1}{\Delta E_1} = \frac{\overline{TC}_1(E_1 + \Delta E_1) - \overline{TC}_1(E_1)}{\Delta E_1} \tag{4.13b}$$

From the results of Table 4.2, the Constant Value Method clearly gives poor agreement with the exact values for the incremental cost. Accounting for the changes in inventory is necessary for calculation of the objective function in periods of finite duration.

Both the Nuclide Value Method and the Unit Production Method give incremental cost close to the exact value. Hence both of them satisfies the equalized incremental cost condition of Equation (4.6). Since both of the methods are consistent they can be accepted as a valid way to evaluate changes in inventory value.

As mentioned under Section 4.3, the Unit Production Method requires forecast of performance of future cycles. However, for these sample cases, the future operation of the reactor after Cycle 1 has been explicitly specified. Hence Table 4.2 a, b, show values of the objective function with no forecast error.

In practical application of this method, when the future is uncertain, the **Unit Production Method** may give less accurate results for incremental costs due to uncertainty in future discharge burnup and salvage values. Moreover, predicting these values may increase computational effort to a large extent. Hence, the Nuclide Value Method, which is consistent, accurate in calculating incremental cost, and free from forecast error, is recommended for calculating the objective function for the case of no income tax.

4.5 Objective Function Defined for the Case with Income Tax

4.5.1 Objective Function for the Indefinite Time Horizon

The objective function for the indefinite time horizon is defined to be the "revenue requirement", which is given by Equation (4.14).

$$\overline{TC}_{\alpha} = \sum_{b} \frac{1}{1-\tau} \cdot \left(P_{wc}^{b} - \tau P_{wd}^{b}\right) \tag{4.14}$$

where

$P_{wc}^{b} = \sum_{b} \frac{Z_{ib}}{(1+x)^{t_{ib}}}$ present value of fuel cycle expenses

$P_{wd}^{b} = \sum_{ib} Z_{ib} \times \frac{P_{we}^{b}}{E^{b}}$ discounted depreciation credit

$P_{we}^{b} = \sum_{jb} \frac{E_{j}^{b}}{(1+x)^{t_{jb}}}$ discounted electricity generated

$E^{b} = \sum_{jb} E_{j}^{b}$ total energy generated by batch b

τ = income tax rate

For the derivation of Equation (4.14) refer to Benedict (B2) and Grant (G1). This definition of objective function is consistent with the cost code MITCOST.

4.5.2 Objective Function for the Finite Time Horizon

Objective function for the finite time horizon can be derived in a manner analogous to the derivation in Section 4.2. Again, it is necessary to introduce an inventory value for those fuel batches that are in the reactor at the end of a time period. Since depreciation credit is calculated for each batch individually, an inventory value must be assigned on the per batch basis. Defining $V^b(t)$ as the residue value of fuel batch b at time t, the objective function for the finite time horizon is given by

$$\overline{TC}_I = \sum_b \frac{1}{1-\tau} \cdot (P^b_{wc} - \tau \cdot P^b_{wd}) \tag{4.15}$$

where the summation runs over all the fuel batches that have ever been in the reactor during that time period.

For those fuel batches that are charged and discharged from the reactor in the time period, P^b_{wc}, P^b_{wd} are defined earlier.

For those fuel batches that are in the reactor at the beginning of the time period at time t_I, but are not in the reactor at the end of the time period

$$P^b_{wc} = \frac{V^b(t_{I'})}{(1+x)^{t_{I'}}} + \sum_{i'} \frac{Z_{i'}}{(1+x)^{t_{i'}}} \tag{4.16}$$

$$P^b_{wd} = \left\{ V^b(t_{I'}) + \sum_{i'} Z_{i'} \right\} \frac{P^b_{we}}{E^b} \tag{4.17}$$

where

$\sum_{i'}$ sum over expenses in this time period only

P^b_{we}: Present worth of electricity generated by this fuel batch in this time period

E^b : Electricity generated by this fuel batch in this time period

For those fuel batches that are in the reactor at the end of the time period at time $t_{I''}$ but are not in the reactor at the beginning of the time period

$$P^b_{wc} = \sum_{i''} \frac{Z_{i''}}{(1+x)^{t_{i''}}} - \frac{V^b(t_{I''})}{(1+x)^{t_{I''}}} \qquad (4.18)$$

$$P^b_{wd} = \sum_{i''} \left\{ Z_{i''} - V^b(t_{I''}) \right\} \cdot \frac{P^b_{we}}{E^b} \qquad (4.19)$$

where $\sum_{i''}$ sums over expenses in this time period only

P^b_{we} : present worth of electricity generated by this fuel batch in this time period

E^b : electricity generated by this fuel batch in this time period

If the reactor operator purchases the fuel batches at value $V^b(t_{I'})$ at the beginning of the time period, and sells them at $V^b(t_{I''})$ at the end of the time period, the objective function defined in Equation (4.15) is the revenue requirement for this time period.

4.5.3 Conditions of Consistency and Equalized Incremental Cost

Again, the property of separability is required. Hence the objective function defined in Equation (4.15) should satisfy the consistency and equalized incremental cost conditions.

$$\sum_{I}^{n} \overline{TC}_I = \overline{TC}_\infty \tag{4.3}$$

n: number of time periods in the indefinite time horizon

$$\frac{\partial}{\partial E_c^r}\left(\overline{TC}_I\right) = \frac{\partial}{\partial E_c^r}\left(\overline{TC}_\infty\right) \tag{4.6}$$

for those cycles c that are in time period I

Unfortunately, due to the effect of tax credits, it is no longer possible to satisfy the consistency condition exactly by imposing the equality of Equation (4.9).

$$\frac{V^b(t_{I''})}{(1+x)^{t_{I''}}} = \frac{V^b(t_{(I+1)'})}{(1+x)^{t_{(I+1)'}}} \tag{4.9}$$

Inconsistency comes from the fact that the depreciation base for the finite time horizon case is different from that of the indefinite horizon case.

Hence, the problem of separability reduces once again to the problem of finding values of $V^b(t)$ that come closest to satisfying the consistency and equalized incremental cost conditions.

Two different methods of evaluating $V^b(t)$ have been examined. They are the Inventory Value Method and the Unit Production Method. The Constant Value Method is not applicable in this case because neglecting the relative changes of the present value of fuel inventories is not consistent with tax regulations.

4.6 Two Methods of Evaluating Fuel Inventories V^b

4.6.1 Inventory Value Method

$V^b(t)$ is equated to the market value of nuclear material of fuel batch b at time t , plus the book value of fabrication and appreciated value of shipping, reprocessing, and conversion. The value of the service cost is determined by linear depreciation based on the Unit Production Method.

$$V^b(t_{I'}) = \$\text{value } (U,Pu) + \$\text{value FSRC}$$

where $value FSRC = book value of fabrication, shipping, reprocessing and conversion

$$= \text{initial value} - \frac{(\text{initial value} - \text{final value})}{\text{total energy generation}} \cdot \begin{Bmatrix} \text{energy} \\ \text{generated} \\ \text{up to } I' \end{Bmatrix}$$

initial value = Z_F: fabrication cost

final value $= -(Z_S + Z_R + Z_C)$: post-irradiation costs

Thus, $ value FSRC varies linearly with respect to energy production from an initial value of the fabrication cost to a final value equal to the sum of post-irradiation costs. Since $V^b(t_{I'})$ depends on the total amount of energy generated by fuel in the reactor, projected into future operations, this method is subject to forecast uncertainty. A forecasting rule is given below in Equation (4.26) to project total energy generation. No depletion calculations are involved.

$$E^b = (N/n) \cdot E^b_I \qquad (4.26)$$

E^b : total energy generation for batch b

E^b_I : total energy generation up to time $t_{I'}$

n : number of cycles the fuel batch has been in the reactor up to time

N : total number of cycles the fuel batch is expected to go through before discharge

Since E^b_I and n are already known at time $t_{I'}$, the only parameter to predict is N. Predicting N is much easier than predicting E^b directly. This rule of thumb is useful when very little or no information is available for predicting the future. Even though this rule is crude, incremental cost calculations based on the Inventory Value Method using this rule of thumb give fairly accurate results (See Table 4.4). If enough information is available to predict E^b reliably, E^b should be used instead of this approximate value.

4.6.2 Unit Production Method

$V^b(t)$ is equated to the book value of nuclear material and service cost (FSRC) for batch b in time t . Book value is determined by linear depreciation using the Unit Production Method.

$$V^b(t_{I'}) = \text{initial value of nuclides and FSRC} - \left\{ \begin{array}{l} \text{initial value of nuclides and FSRC} \\ - \text{salvage value of nuclides and FSRC} \end{array} \right\} \Big/ \left\{ \begin{array}{l} \text{total} \\ \text{energy} \\ \text{generation} \end{array} \right\} \times \left\{ \text{energy generation up to } t_{I'} \right\}$$

where Initial value of nuclides, FSRC = $Z_U + Z_F$

Salvage value of nuclides, FSRC = $Z_{U'} + Z_{Pu} - Z_S - Z_R - Z_C$

In this method $V^b(t_{I'})$ depends on both the total amount of energy to be generated by the fuel in the reactor, projected into future operations, and on the composition of the fuel when discharged after these future operations. This requires running depletion calculations. Hence, the depletion calculations must be carried out until all the fuel batches in time period I have been discharged from the reactor. This would provide enough data for calculating salvage value as well as total energy. In order to complete the calculation for time period I, it is necessary to predict system behaviour for time period 2. This is much more difficult than predicting E^b and requires more computation effort.

4.7 Results of Two Sample Cases

The sample cases of Section 4.4 are used again to test the degree of consistency and equality of incremental cost for the two methods.

Similar to the treatment in Section 4.4, the objective function $\overline{TC}_\alpha$ includes all seven batches (-1, 0, 1, 2, 3, 4, and 5) affected by the perturbation. $\overline{TC}_1$ includes the first three batches, credited with the inventory value of batch 0 and 1 at the end of cycle 1. $\overline{TC}_2$ includes the last six batches, charged with the inventory value of batch 0 and 1 at the beginning of cycle 2.

If the methods of evaluating inventory worth possess the property of consistency, then $\overline{TC}_\alpha = \overline{TC}_1 + \overline{TC}_2$. Hence, any difference between $\overline{TC}_\alpha$ and $\overline{TC}_1 + \overline{TC}_2$ is a measure of inconsistency for the two methods.

Part A of Table 4.3 gives the objective function for the batches whose values are affected by changes in energy in Cycle 1. The first column gives the result of exact calculation of the objective function for the indefinite time horizon $\overline{TC}_\alpha$. The second column gives the result of using the Inventory Value Method for calculating the objective function for time period 1, $\overline{TC}_1$. The third column gives values of $\overline{TC}_2$. The fourth column gives the sum of $\overline{TC}_1$ and $\overline{TC}_2$; it should be compared with column 1. Part B is a similar table for the Unit Production Method.

Table 4.3

Test of Inconsistency Between the Exact Value and the Approximate Methods

Part A

Revenue Requirement

Method	Exact	Inventory Value Method		
Quantity Calculated	$\overline{TC}_\alpha$	$\overline{TC}_1$	$\overline{TC}_2$	$\overline{TC}_1+\overline{TC}_2$
	$10^6\$$			
Steady State case	75.8458	30.7900	44.9734	75.7634
Additional Energy in Cycle 1				
ΔE_i=1029 GWHt	76.3106	31.2713	44.9588	76.2301
=2050 GWHt	76.7661	31.7532	44.9339	76.6872

Part B

Revenue Requirement

Method	Exact	Unit Production Method		
Quantity Calculated	$\overline{TC}_\alpha$	$\overline{TC}_1$	$\overline{TC}_2$	$\overline{TC}_1+\overline{TC}_2$
	$10^6\$$			
Steady State case	75.8458	30.1342	45.7538	75.8879
Additional Energy in Cycle 1				
ΔE_i=1029 GWHt	76.3106	30.6041	45.7333	76.3375
=2050 GWHt	76.7661	31.0729	45.7073	76.7802

From the results in Table 4.3, the magnitude of inconsistency can be seen to be quite small for both methods in all three cases, but the Unit Production Method in comparison has the smaller measure of inconsistency.

Table 4.4 shows the incremental cost for the two methods. Incremental costs calculated from the Unit Production Method give better agreement in general.

4.8 Conclusions

The Unit Production Method provides the most consistent and accurate evaluation of $V^b(t)$. However, to use this method in a practical case, the information required as input is difficult to obtain. Moreover, more depletion calculations are required.

On the other hand, the Inventory Value Method requires the minimal amount of projections and computations, at some loss of consistency and accuracy. For this kind of scoping optimization which requires evaluation of many different alternatives, computational speed is the major concern. Using a fast optimization algorithm, a large number of cases can be evaluated in order to eliminate those that are far from optimal and locate those that may be optimal. Then a more accurate algorithm can be used to evaluate those limited number of near optimal cases.

Hence, the Inventory Value Method for evaluating $V^b(t)$ is recommended for scoping calculation of the objective function for the finite horizon case.

Table 4.4

Comparison of Exact Incremental Cost with Incremental Cost Calculated by Two Approximate Methods

Method	Exact	Approximate: Inventory Value	Approximate: Unit Production
	Incremental Cost for Cycle 1 Mills/KWHe		
ΔE_1=1029GWHt	1.39	1.43	1.40
=2050GWHt	1.38	1.44	1.40

CHAPTER 5.0

CALCULATION OF RELOAD ENRICHMENT AND INCREMENTAL COST OF ENERGY FOR GIVEN SCHEDULE OF ENERGY PRODUCTION WITH FIXED RELOAD BATCH FRACTION AND SHUFFLING PATTERN

5.1 Defining the Problem

The problem here is to calculate the reload enrichments and incremental cost of energy for successive cycles of a particular reactor given the energy requirements for each cycle and the refuelling schedule. The initial state of the reactor is specified. Reload batch fraction and shuffling pattern for each cycle are fixed. Under these restrictive conditions, there is only one unique solution for this problem. This can be understood quite easily by analyzing the relationships between the variables.

If the initial state of the reactor is specified and if the reload batch fraction and shuffling pattern for the first cycle are fixed, the only refuelling option is the reload enrichment. If the energy for the first cycle is given, the reload enrichment for the first cycle is fixed. This in turn specifies the end condition of the first cycle. The above argument can be repeated for the second, third and subsequent cycles. Hence, if the energy requirements for successive cycles are specified there is only one sequence of reload enrichments for this case.

The economics of the fuel cycle is a unique function of the physical state of the fuel cycle. Since the physical state of the fuel cycle is uniquely specified, the economics of the system is also uniquely defined. Hence, incremental costs for the various cycles can be explicitly evaluated.

5.2 One-Zone Batch refuelling case

For a batch refuelled one-zone reactor, the calculation of reload enrichment and incremental cost of energy is straight forward. Energy output depends entirely on the reload enrichment for that cycle. There is no inter-coupling between cycles.

Figure 5.1 shows the relationship between cycle energy and reload enrichment for this one-zone case. For a sequence of cycle energies, the sequence of reload enrichments for successive cycles can be read off directly.

Since there is no inter-coupling between cycles, the fuel costs for different cycles are also decoupled.

The objective function is given by

$$\overline{TC} = \sum_{b}^{5} \frac{1}{1-\tau} \cdot (P_{wc}^{b} - \tau \cdot P_{wd}^{b}) \tag{5.1}$$

$$= \sum \frac{R_b}{(1+x)^{t_b}}$$

where R_b = revenue requirement for batch b

t_b = irradiation starts for cycle b

The specific refuelling schedule is given in Table 5.1

Table 5.1

Refuelling Schedule (in years)

Cycle	Irradiation Starts	Ends
1	0.0	1.463
2	1.588	3.151
3	3.176	4.639
4	4.764	6.227
5	6.352	7.815

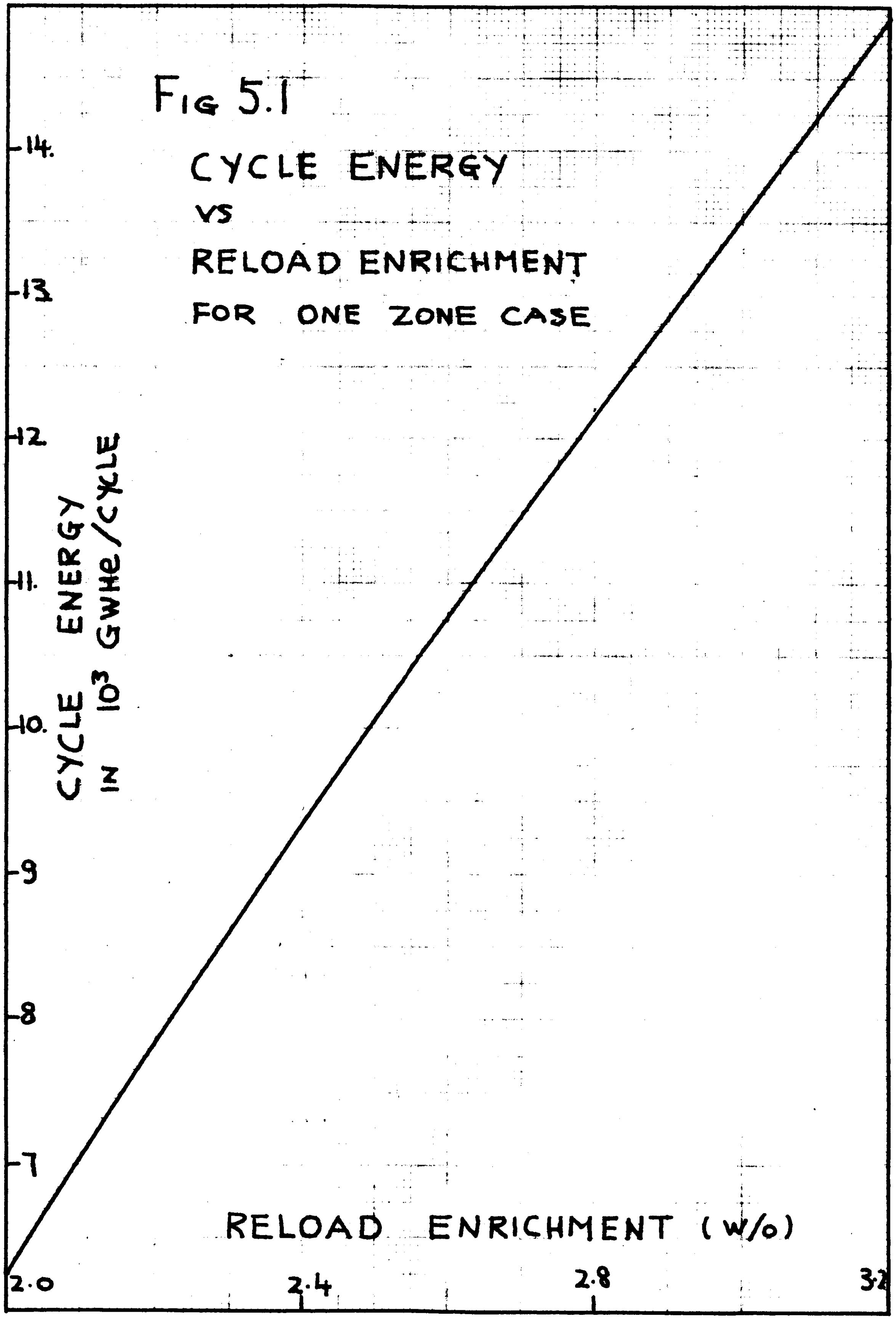


Fig 5.1
CYCLE ENERGY
VS
RELOAD ENRICHMENT
FOR ONE ZONE CASE
14.
13
12
11.
10.
9
8
7
CYCLE ENERGY
IN 10³ GWHe/CYCLE
RELOAD ENRICHMENT (w/o)
2.0
2.4
2.8
3.2

Figure 5.2 shows the relationship between R_b and cycle energy. For a given sequence of cycle energies, the sequence of R_b's can be read off directly.

The incremental cost of energy for Cycle c is equal to the slope of the curve of R_b vs E curve. Notice that for the same cycle energy, the incremental cost are different for different cycles due to the present worth factor. Figure 5.3 shows the relationship between incremental cost and energy per cycle.

Hence for the batch refuelling case, the reload enrichment and incremental cost of energy for each cycle can be calculated directly once the cycle energy and the refuelling schedule are specified.

5.3 Multi-Zone Refuelling

In the more general case, only a part of the reactor core is replaced during each refuelling. Energy generated in any cycle originates from the fissioning of the fresh reload fuel and the partially burnt fuel remaining in the reactor. As a result, energy generated in one cycle depends not only on the reload fuel for that cycle, but also in the reload fuel for the preceding cycles. In this way, all the fuel cycles are coupled together. Hence, the calculation of reload enrichments and incremental cost is no longer straightforward.

Three methods are developed for the calculation. The first method is the Rigorous Method based on the definition of the incremental cost. The second method, called

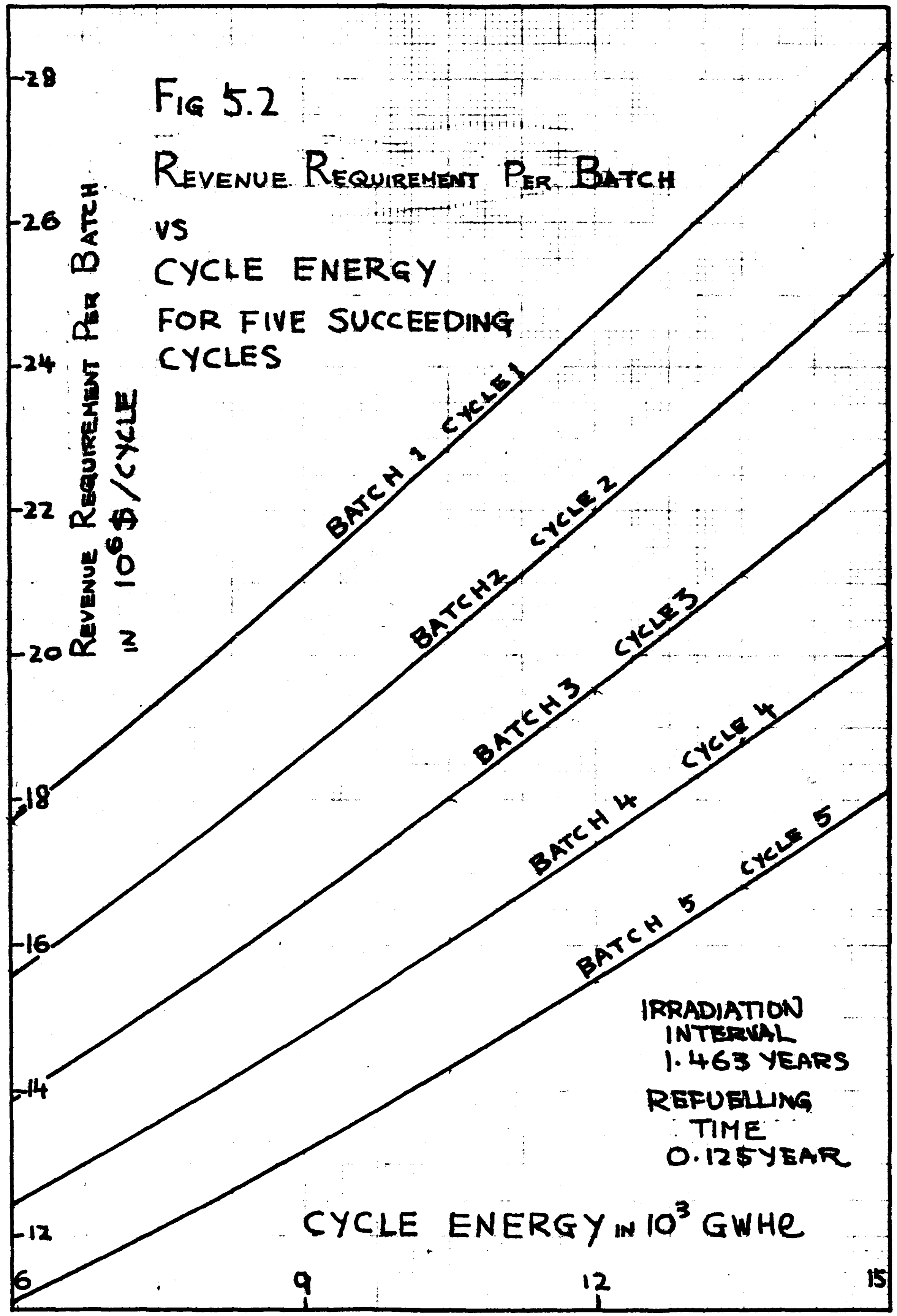


Fig 5.2
Revenue Requirement Per Batch
vs
Cycle Energy
for five succeeding
cycles
Revenue Requirement Per Batch in 10^6 $/cycle
28
26
24
22
20
18
16
14
12
Batch 1 Cycle 1
Batch2 Cycle 2
Batch 3 Cycle 3
Batch 4 Cycle 4
Batch 5 Cycle 5
Irradiation interval 1.463 years
Refuelling time 0.125 year
Cycle Energy in 10^3 GWHe
6
9
12
15

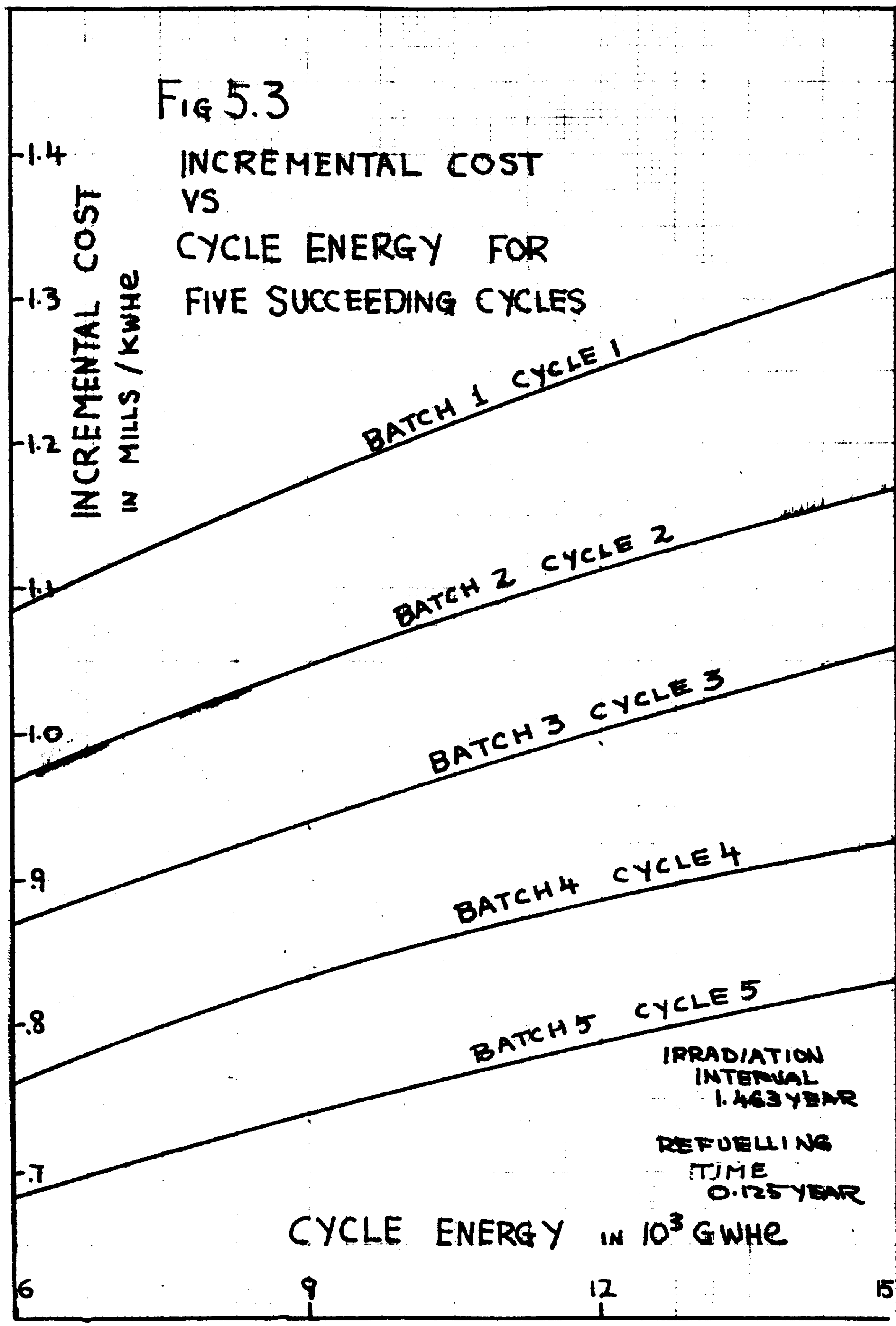


Fig 5.3
INCREMENTAL COST
VS
CYCLE ENERGY FOR
FIVE SUCCEEDING CYCLES
INCREMENTAL COST
IN MILLS / KWHe
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
BATCH 1 CYCLE 1
BATCH 2 CYCLE 2
BATCH 3 CYCLE 3
BATCH 4 CYCLE 4
BATCH 5 CYCLE 5
IRRADIATION
INTERVAL
1.463 YEAR
REFUELLING
TIME
0.125 YEAR
CYCLE ENERGY IN 10³ GWHe
6
9
12
15

Linearization Method is based on approximate linear relationship between objective function and reload enrichments. The third method, called the Inventory Value is based on an analysis of the variation of the revenue requirement calculated for the perturbed cycle alone.

5.3.1 The Rigorous Method

The incremental cost of energy λ_c is defined as the partial derivative of the revenue requirement with respect to cycle energy

$$\lambda_c \equiv \frac{\partial \overline{TC}}{\partial E_c} \bigg|_{E_{c'}} \tag{5.2}$$

which can be replaced by the forward difference

$$\lambda_c \simeq \frac{\overline{TC}(E_1^0, E_2^0, \ldots E_c^0 + \Delta E, E_{c+1}^0 \ldots) - \overline{TC}(E_1^0, E_2^0, \ldots E_c^0, E_{c+1}^0 \ldots)}{\Delta E} \tag{5.3}$$

If $\overline{TC}$ is known for two values of E_c (eg. in Equation (5.3) for E_c^o and $E_c^o + \Delta E$) while all other $E_{c'}$ are constant, λ_c can be evaluated quite easily. However, to obtain the correct enrichments which permit E_c to change while all other energies $E_{c'}$ remain unchanged is time consuming and computationally expensive. The correct enrichment for each cycle must be found by trial. To determine all the λ_c in an m-cycle problem requires about $\frac{3m\ (m+1)}{2}$ trials, using about three trials per cycle.

5.3.2 Linearization Method

Due to the complicated inter-coupling effects between various batches and cycles, energy production in any one cycle depends on the reload enrichments of all the preceding cycles.

$$E_c = E_c(\varepsilon_1, \varepsilon_2 \ldots\ldots \psi^0) \tag{5.4}$$

where ψ^0 is the initial state of the reactor prior to Cycle 1.

For small changes of enrichments from a given base case, the energy production per cycle can be approximated by the linear relation

$$E_c - E_c^0 \approx \sum_{c'} \frac{\partial E_c}{\partial \varepsilon_{c'}} \cdot (\varepsilon_{c'} - \varepsilon_{c'}^0) \tag{5.5}$$

where $\varepsilon_{c'}^0$: reload enrichment for cycle c' for the base case

E_c^0 : energy production for cycle c for the base case.

Equation (5.5) can be put in matrix form

$$\vec{E} = \mathbb{A}\, \Delta\vec{\varepsilon} + \vec{E}^0 \tag{5.6}$$

where $\vec{E} = \text{col}\,\{E_1 \ldots\ldots E_C\}$

$\mathbb{A}$ = lower diagonal matrix

$$a_{cc'} = \begin{cases} \Delta E_c / \Delta\varepsilon_{c'} & \text{for } c' \leqslant c \\ 0 & \text{for } c' > c \end{cases}$$

$\Delta\vec{\varepsilon} = \text{col}\{\Delta\varepsilon_1 \ldots\ldots \Delta\varepsilon_C\}$

$\vec{E}^0 = \text{col}\{E_1^0 \ldots\ldots E_C\}$

Solving for $\Delta\vec{\varepsilon}$, Equation (5.6) becomes

$$\Delta\vec{\varepsilon} = \mathbb{A}^{-1}(\vec{E} - \vec{E}^0) \tag{5.7}$$

Adding $\vec{\varepsilon}^0$ on both sides

$$\vec{\varepsilon} = \vec{\varepsilon}^0 + \Delta\vec{\varepsilon} = \mathbb{A}^{-1}(\vec{E} - \vec{E}^0) + \vec{\varepsilon}^0 \tag{5.8}$$

where $\vec{\varepsilon} = \text{col}\,\{\varepsilon_1 \ldots \varepsilon_C\}$

$\vec{\varepsilon}^0 = \text{col}\,\{\varepsilon^0 \ldots \varepsilon_C^0\}$

If the elements in the matrix $\mathbb{A}$ are known, the reload enrichments can be calculated for any specified set of cycle energy.

$\mathbb{A}$ is lower diagonal. Equation (5.8) can be solved by forward elimination.

The success of this method depends on the accuracy of the elements of matrix $\mathbb{A}$. If $\vec{E}$ is close to $\vec{E}^{\circ}$ or one of the $\vec{E}$'s from which the coefficients are calculated, the method can be very accurate.

The objective function $\overline{TC}$ for a finite time period can be treated in a similiar manner. The objective function depends on the physical state of the system, and consequently it has the same set of independent variables.

$$\overline{TC} = \overline{TC}\,(\varepsilon_1 \ldots\ \varepsilon_C,\ \psi^{0}). \tag{5.9}$$

However, by the chain rule of differentiation,

$$\frac{\partial \overline{TC}}{\partial \varepsilon_c} = \sum_{c'}^{C} \frac{\partial \overline{TC}}{\partial E_{c'}} \cdot \frac{\partial E_{c'}}{\partial \varepsilon_c} = \sum_{c'}^{C} \lambda_{c'} \cdot \frac{\partial E_{c'}}{\partial \varepsilon_c} \tag{5.10}$$

Equation (5.10) can be inverted to solve for $\lambda_{c'}$.

Rewriting Equation (5.10) in matrix notation

$$\overrightarrow{\left\{\frac{\partial \overline{TC}}{\partial \varepsilon}\right\}} = \mathbb{A}^{T} \cdot \vec{\lambda} \tag{5.11}$$

where

$$\overrightarrow{\left\{\frac{\partial \overline{TC}}{\partial \varepsilon}\right\}} = \text{col.}\left\{\frac{\partial \overline{TC}}{\partial \varepsilon_1}\quad \frac{\partial \overline{TC}}{\partial \varepsilon_2}\ \cdots\cdots\ \frac{\partial \overline{TC}}{\partial \varepsilon_C}\right\}$$

$$\vec{\lambda} = \text{col.}\left\{\frac{\partial \overline{TC}}{\partial E_1}\quad \frac{\partial \overline{TC}}{\partial E_2}\ \cdots\cdots\ \frac{\partial \overline{TC}}{\partial E_C}\right\}$$

Inverting Equation (5.11) to solve for $\vec{\lambda}$,

$$\vec{\lambda} = (\mathbb{A}^T)^{-1} \overrightarrow{\left\{\frac{\partial \overline{TC}}{\partial \varepsilon}\right\}} \tag{5.12}$$

If matrix $\mathbb{A}$ and the vector $\overrightarrow{\left\{\frac{\partial \overline{TC}}{\partial \varepsilon}\right\}}$ are known, $\vec{\lambda}$ can be calculated directly.

The matrix $\mathbb{A}$ and the vector $\overrightarrow{\left\{\frac{\partial \overline{TC}}{\partial \varepsilon}\right\}}$ are determined by a series of perturbation calculations. Using the steady state case as the base line, the perturbed case consists of a positive change in enrichment in the first cycle alone. Reload enrichments for the succeeding cycles are kept to the original steady state value. Cycle energy for the first few cycles would be increased. This effect would slowly damp out. By analysing the dampening effect in cycle energy, the elements in matrix $\mathbb{A}$ can be determined.

For example

$$a_{11} = \frac{\partial E_1}{\partial \varepsilon_1} = \frac{E_1(\varepsilon_1^0+\Delta\varepsilon_1) - E_1(\varepsilon_1^0)}{\Delta\varepsilon_1} \tag{5.13}$$

$$a_{21} = \frac{\partial E_2}{\partial \varepsilon_1} = \frac{E_2(\varepsilon_1^0+\Delta\varepsilon_1, \varepsilon_2^0) - E_2(\varepsilon_1^0, \varepsilon_2^0)}{\Delta\varepsilon_1} \tag{5.14}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$a_{51} = \frac{\partial E_5}{\partial \varepsilon_1} = \frac{E_5(\varepsilon_1^0+\Delta\varepsilon_1, \varepsilon_2^0 \ldots) - E_5(\varepsilon_1^0, \varepsilon_2^0 \ldots)}{\Delta\varepsilon_1} \tag{5.15}$$

Similiarly, $\frac{\partial \overline{TC}}{\partial \varepsilon_c}$ can be calculated.

$$\frac{\partial \overline{TC}}{\partial \varepsilon_1} = \frac{\overline{TC}(\varepsilon_1^0+\Delta\varepsilon_1, \varepsilon_2^0 \ldots) - \overline{TC}(\varepsilon_1^0, \varepsilon_2^0 \ldots)}{\Delta\varepsilon_1} \tag{5.16}$$

Table 5.3 and Table 5.6 show the application of this method in sample problem 1, 2 and 3.

5.3.3 Inventory Value Method

The Inventory Value Method consists of two parts. Part 1 deals with the calculation of reload enrichments by trial and error. Part 2 calculates incremental cost of energy by making use of the data generated in Part 1.

Part 1 Given an initial state of the reactor, the reload enrichments for succeeding cycles for a specified sequence of cycle energies can be determined by trial and error. This method is primitive and costly, but it can be made as accurate as one likes.

For a given initial state, a given requirement of cycle energy, a guess is made for the reload enrichment for the first cycle. A depletion run is made using the guessed value for the reload enrichment. If the resulting cycle energy is too high (low), the reload enrichment is decreased (increased). The depletion run for this cycle is repeated. The cycle energy for the adjusted reload enrichment is obtained. A third trial on the reload enrichment can be made using interpolation, or extrapolation based on previous results.

$$\varepsilon^{(i+1)} = \frac{\varepsilon^{(i)} - \varepsilon^{(i-1)}}{E^{(i)} - E^{(i-1)}} \cdot (E^{(i)} - E^{\circ}) + \varepsilon^{(i)} \qquad (5.17)$$

Where E° = target value
$E^{(i)}$ = cycle energy for the i-th trial
$\varepsilon^{(i)}$ = reload enrichment for the i-th trial

This method converges very rapidly. Usually three trials of the enrichment are required for an accuracy of ±0.1%. With experience, the number of trials can be reduced to two.

After the reload enrichment for the first cycle has converged, the whole procedure can be repeated for the second cycle.

For an m - cycle problem, at most 3m depletion runs are required to determine the reload enrichments.

Part 2 Incremental costs can be calculated using data generated in the trial and error procedures.

In Chapter 4, it has been shown that the Inventory Value Method correctly evaluates the end effect and gives fairly accurate values of incremental cost. If the Inventory Value Method is applied at the end of the cycle for which incremental cost calculation is desired, then incremental cost of nuclear energy for that cycle can be obtained by analyzing the change in the revenue requirement up to that cycle as energy production changes in that cycle.

Consider the first cycle in the planning horizon in which the initial state is well specified. After using the trial and error procedures to calculate the correct reload enrichment for the target energy, there would be at least three depletion runs available for that cycle with different enrichments and cycle energies.

From the output of the depletion runs, the revenue requirement up to the end of Cycle 1 can be calculated for each enrichment or cycle energy. Incremental cost of energy for the first cycle $\frac{\Delta\overline{TC}}{\Delta E_1}$ can be approximated by

$$\lambda_1 \approx \frac{\Delta\overline{TC}_1}{\Delta E_1} \approx \frac{\overline{TC}_1(E_1') - \overline{TC}_1(E_1'')}{(E_1' - E_1'')} \tag{5.18}$$

Where E_1' and E_1'' correspond to different trial energies for the first cycle.

The same method can be applied for Cycle 2, 3... etc. Hence, the incremental cost of energy for all the cycles can be approximated.

From Equation (5.18) it may be noted that only two data points are required for each calculation of incremental cost. If more than two depletion runs are available for each cycle, higher order coefficients can be calculated.

Figure (5.4) shows the relationships between $\overline{TC}$, $\overline{TC}_1$ (revenue requirements up to cycle 1) batches and cycle for the example in which the incremental cost of energy for Cycle 1 is required.

5.4 Results For Three Sample cases

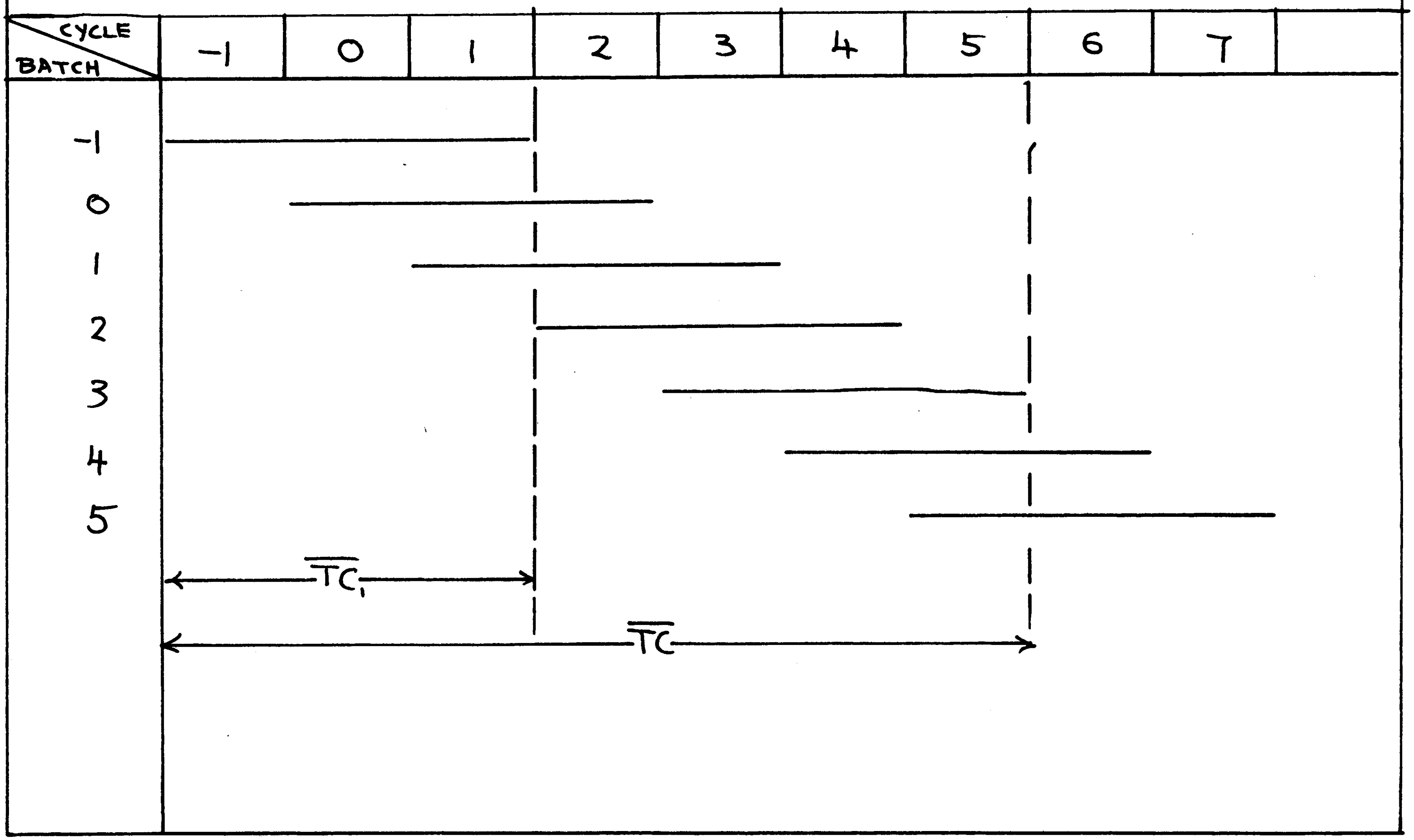
Three sample cases are considered in this section. The first two sample cases deal with perturbation in a steady-state operating condition. The third sample case deals with non-steady state operating condition. The third case supposedly is more realistic.

5.4.1 Sample Case 1 & 2

Sample Cases 1 & 2 are the same cases considered in Section 4.4. The initial state of the 1065 MWe Zion type

FIGURE 5.4

RELATIONSHIPS BETWEEN THE VARIOUS REVENUE REQUIRMENTS BATCH NUMBERS AND CYCLE NUMBER



reactor is given by the steady-state operating condition of three-zone, 3.16 w/o refuelling, with energy generation of 21935 GWHt per cycle. The energy production in Cycle 1 of the planning horizon is increased to 22964 GWHt per cycle for case 1, and 23985 GWHt per cycle for case 2 by increasing the reload enrichment. The energy productions in the remaining cycles of the planning horizon are kept constant at the 21935 GWHt level by adjusting the reload enrichments.

Table 5.2 shows the reload enrichments, cycle energies and revenue requirements for the base line case and the two perturbed cases. Incremental cost of energy calculated by the three methods are presented in the last three columns. The Inventory Value Method gives better results than the Linearization Method when compared to the exact values given by the Rigorous Method.

Table 5.3 shows the calculations required by the Linearization Method. From a set of five enrichment perturbation cases, the coefficients $\frac{\partial \overline{TC}}{\partial \varepsilon_c}$ and $\frac{\partial E_c}{\partial \varepsilon_c}$, were calculated. Solving the set of linear equations, the incremental cost of energy $\frac{\Delta \overline{TC}}{\Delta E_c}$ were determined, and are given in the last row of the table.

Finally Table 5.4 shows values of reload enrichment calculated by trial and error and by the Linearization Method. They agree within 0.3%.

5.4.2 <u>Sample Case 3</u>

This is a case with non-steady state initial condition and varying cycle length and cycle energy. Refuelling intervals

Table 5.2

Incremental Cost of Energy for Sample Cases 1 and 2 Calculated By Three Different Methods

		Enrichment and Cycle Energy					Revenue Requirement		Incremental Cost Method of Calculation:		
	ϵ(w/o) E(GWHt)								Rigorous	Inventory Value	Linearization
	Cycle	1	2	3	4	5	$\overline{TC}$	$\overline{TC}_1$			
							10^6\$		Mills/KWHe		
Base Case	ϵ E	3.162 21935	3.162 21935	3.162 21935	3.162 21935	3.162 21935	69.983	30.790	—	—	—
Case 1	ϵ E	3.359 22964	3.054 21935	3.174 21929	3.196 21928	3.133 21933	70.461	31.271	1.42	1.43	1.37
Case 2	ϵ E	3.557 23985	2.941 21919	3.186 21906	3.235 21937	3.106 21970	70.929	31.753	1.40	1.44	1.37

Refuelling Time Schedule For These Two Cases

Cycle	Irradiation starts	Irradiation ends
	Years	
1	0.0	1.375
2	1.5	2.875
3	3.0	4.375
4	4.5	5.875
5	6.0	7.375

Table 5.3

Calculation of Incremental Cost Using the Method of Linearization for Sample Case 1and 2

		Enrichment and Cycle Energy					Revenue Requirement	$\Delta\overline{TC}/\Delta\epsilon$	Incremental Cost
	ϵ(w/o) E(GWHt)						10^6\$	10^6\$/(w/o)	Mills/KWHe (Mills/KWHt)
Base Case	ϵ	3.162	3.162	3.162	3.162	3.162			
	E	21935	21935	21935	21935	21935	69.9837	—	—
Perturbation									
Cycle 1	ϵ	3.557	3.162	3.162	3.162	3.162			
	E	23985	23126	22424	21791	21929	71.5094	3.8526	1.3646 (0.4448)
	$\Delta E/\Delta\epsilon_1$	5181.	3010.	1236.	-364.	-15.			
Cycle 2	ϵ	3.162	3.557	3.162	3.162	3.162			
	E	21935	23985	23126	22424	21791	71.3511	3.4531	1.2408 (0.4045)
	$\Delta E/\Delta\epsilon_2$	—	5181.	3010.	1236.	-364.			
Cycle 3	Δ	3.162	3.162	3.557	3.162	3.162			
	E	21935	21935	23985	23126	22424	71.2338	3.1569	1.1176 (0.3643)
	$\Delta E/\Delta\epsilon_3$	—	—	5181.	3010.	1236.			
Cycle 4	ϵ	3.162	3.162	3.162	3.557	3.162			
	E	21935	21935	21935	23985	23126	70.9535	2.4490	0.9178 (0.2992)
	$\Delta E/\Delta\epsilon_4$	—	—	—	5181.	3010.			
Cycle 5	ϵ	3.162	3.162	3.162	3.162	3.557			
	E	21935	21935	21935	21935	23985	70.5965	1.5473	0.9163 (0.2987)
	$\Delta E/\Delta\epsilon_5$	—	—	—	—	5181.			

Table 5.4

Reload Enrichment Calculated By Trial Method and By Linearization Method

Sample Case 1

Cycle		1	2	3	4	5
Energy/Cycle	GWHt	22964.	21935.	21929.	21928.	21933.
Enrichment						
Trial Method	ϵ(w/o)	3.359	3.054	3.174	3.196	3.133
Linearization Method	ϵ(w/o)	3.360	3.046	3.181	3.191	3.132

Sample Case 2

Cycle		1	2	3	4	5
Energy/cycle	GWHt	23985.	21919.	21906.	21937.	21970.
Enrichment						
Trial Method	ϵ(w/o)	3.557	2.941	3.186	3.235	3.106
Linearization Method	ϵ(w/o)	3.557	2.928	3.197	3.225	3.108

alternate between twelve and eighteen months. Cycle energies follow a similiar pattern to the refuelling intervals. The incremental cost of energy for Cycle 1 is obtained by decreasing energy production in that cycle by 1000 GWH+ while keeping energy production in other cycles the same as the base case. Table 5.5 gives values of reload enrichments, cycle energies, revenue requirements and incremental costs. The accuracy of the Inventory Value Method is comparable to the previous results. The apparent accuracy of the Linearization Method is just coincidental.

Table 5.6 shows the calculations required by the Linearization Method. The perturbation cases are the same as given in Table 5.3, except that refuelling times are different.

Finally Table 5.7 shows the values of reload enrichment calculated by the trial and error method and the Linearization Method. The same order of accuracy is obtained in this case as in the previous two cases.

5.5 Conclusions

The Linearization Method is least accurate among the three methods. However, once the coefficients are calculated, incremental costs and reload enrichments for any cycles can be obtained very easily. The Inventory Value Method is more accurate in terms of incremental costs. However, the trial method of calculating reload enrichments is awkward. Either the Linearization Method or the Inventory Value Method can be used to estimate incremental cost to be used in the beginning.

Table 5.5

Incremental Cost of Energy for Sample Case 3 Calculated by Three Different Methods

		Enrichment and Cycle Energy					Revenue Requirement		Incremental Cost Method of Calculation:		
	ϵ(w/o) E(GWHt)								Rigor-ous	Inventory Value	Linear-ization
Cycle		1	2	3	4	5	$\overline{TC}$	$\overline{TC}_1$			
							10^6\$		Mills/KWHe		
Base Case	ϵ	3.557	2.864	3.557	2.864	3.260	70.837	31.580	—	—	—
	E	24105.	21532.	23621.	20999.	22172.					
Changed Case	ϵ	3.359	2.975	3.545	2.833	3.286	70.383	31.107	1.37	1.43	1.37
	E	23085.	21535.	23605.	20995.	22164.					
									Error	4%	0.05%

Refuelling Time Schedule For This Case

Cycle	Irradiation starts	Irradiation ends
	Years	
1	0.0	1.375
2	1.5	2.375
3	2.5	3.875
4	4.0	4.875
5	5.0	6.375

Table 5.6

Calculation of Incremental Cost Using the Method of Linearization for Sample Case 3

		Enrichment and Cycle Energy					Revenue Requirement	$\Delta TC/\Delta\epsilon$	Incremental Cost
	ϵ (w/o) E(GWHt)						10^6\$	10^6\$/(w/o)	Mills/KWHe (Mills/KWHt)
Base Case	ϵ	3.162	3.162	3.162	3.162	3.162	69.5936	—	—
	E	21935.	21935.	21935.	21935.	21935.			
Perturbation									
Cycle 1	ϵ	3.557	3.162	3.162	3.162	3.162	71.0973	3.8007	1.3687 (0.4462)
	E	23985.	23126.	22424.	21791.	21929.			
	$\Delta E/\Delta\epsilon_1$	5181.	3010.	1236.	-363.	-15.			
Cycle 2	ϵ	3.162	3.557	3.162	3.162	3.162	70.9090	3.3248	1.1509 (0.3752)
	E	21935.	23985.	23126.	22424.	21791.			
	$\Delta E/\Delta\epsilon_2$	—	5181.	3010.	1236.	-363.			
Cycle 3	ϵ	3.162	3.162	3.557	3.162	3.162	70.8707	3.2280	1.1647 (0.3797)
	E	21935.	21935.	23985.	23126.	22424.			
	$\Delta E/\Delta\epsilon_3$	—	—	5181.	3010.	1236.			
Cycle 4	ϵ	3.162	3.162	3.162	3.557	3.162	70.5645	2.4542	0.8810 (0.2872)
	E	21935.	21935.	21935.	23985.	23126.			
	$\Delta E/\Delta\epsilon_4$	—	—	—	5181.	3010.			
Cycle 5	ϵ	3.162	3.162	3.162	3.162	3.557	70.2515	1.6631	0.9847 (0.3210)
	E	21935.	21935.	21935.	21935.	23985.			
	$\Delta E/\Delta\epsilon_5$	—	—	—	—	5181.			

Table 5.7

Reload Enrichment Calculated By the Trial Method and by the Linearization Method

Sample Case 3

Cycle		1	2	3	4	5
Energy/cycle	(GWHt)	23085.	21532.	23605.	20995.	22164.
Enrichment						
Trial Method	ϵ(w/o)	3.359	2.975	3.545	2.833	3.286
Linearization Method	ϵ(w/o)	3.360	2.979	3.534	2.836	3.287

CHAPTER 6.0

CALCULATION OF OPTIMAL RELOAD ENRICHMENT AND RELOAD BATCH FRACTION FOR REACTORS OPERATING IN STEADY STATE CONDITION AND MODIFIED SCATTER REFUELLING

6.1 Introduction

The problem of nuclear in-core optimization can be formulated as follows: given a refuelling schedule and a fixed energy demand, find the optimal combination of reload enrichment and reload batch fraction such that the fuel cycle cost is minimized. In this chapter, the special case of steady-state operation is considered in which the size of the irradiation interval and the energy demand are the same cycle after cycle. Refuelling is done in a modified-scatter manner. Fresh fuel elements are always put on the outside annulus and once-irradiated fuel elements are scattered throughout the inner core. Under these restrictive conditions, the state of the reactor is uniquely defined, as the reload enrichment and reload batch fraction are specified. For a given combination of reload enrichment and batch fraction, there is a unique fuel cost and a unique cycle energy.

6.2 Mathematical Formulation of the Problem and Optimality Conditions

The problem of nuclear in-core optimization in the steady-state case can be stated mathematically as

$$\text{Minimize } \overline{TC}\ (\varepsilon, f) \tag{6.1}$$

$$\text{subject to constraints } E(\varepsilon, f) = E^S \tag{6.2}$$

$$B(\varepsilon, f) < B^\circ \tag{6.3}$$

Where $\overline{TC}$: revenue requirement for a single cycle
ε : reload enrichment
f : batch fraction
E^S : energy demanded by the system on this reactor
B° : burnup limitations.

The revenue requirement for a single cycle is chosen to be the objective function because in steady state conditions, the revenue requirement for a single cycle is equal to that of the succeeding cycles. Equation (6.2) is the constraint that the energy demand must be satisfied. Equation (6.3) is the limitation on discharge burnup.

Notice that for reactivity limited burnup, specifying the cycle energy and reload batch fraction completely determines the reload enrichment. Hence cycle energy and reload batch fraction can be taken as the independent variables, and reload enrichment as the dependent variable. Equations (6.1) (6.2) and (6.3) can be rewritten as

$$\text{Minimize } \overline{TC}\ (E^S, f) \tag{6.4}$$

$$\text{Subject to constraints } B(E^S, f) < B^\circ \tag{6.5}$$

The non-linear Kuhn-Tucker optimality conditions for Equations (6.1) and (6.3) are

$$B^\circ - B(E^S, f^*) \geqslant 0 \tag{6.6}$$

$$f^* \geqslant 0 \tag{6.7}$$

$$-\pi \cdot \frac{\partial B}{\partial f}(E^S, f^*) \leqslant \frac{\partial \overline{TC}}{\partial f}(E^S, f^*) \tag{6.8}$$

$$\pi \geqslant 0 \tag{6.9}$$

$$\pi \cdot (B^\circ - B(E^S, f^*)) = 0 \tag{6.10}$$

$$\frac{\partial \overline{TC}}{\partial f} \cdot f^* + \pi \cdot \frac{\partial B}{\partial f} \cdot f^* = 0 \tag{6.11}$$

Equations (6.6) and (6.7) state that at (E^s, f^*) the burnup constraint is satisfied. Equations (6.8) and (6.9) state that at (E^s, f^*) the objective function cannot be further minimized. Equations (6.10) and (6.11) state that either (B^o - $B(E^s, f^*)$)is zero or $\frac{\partial \overline{TC}}{\partial f}$ is zero. Physically that means the optimal solution(E^s, f^*)either lies on the boundary of the constraints, or it is at a local minimum. Combining Equation (6.8) and Equation (6.1) reduces to

$$\frac{\partial \overline{TC}(E^s, f^*)}{\partial f} = -\pi \frac{\partial B(E^s, f^*)}{\partial f} \tag{6.12}$$

For steady-state refuelling, the average discharge burnup $B(E^s, f)$ can be expressed in analytic form, in terms of the cycle energy E^s and reload batch fraction f

$$B(E^s, f) \cdot W \cdot f = E^s \tag{6.13}$$

or $$B(E^s, f) = \frac{E^s}{W \cdot f} \tag{6.14}$$

where W is the mass of uranium for the entire core before irradiation.

Substituting Equation (6.14) into Equation (6.10) results in

$$\pi \cdot (B^o - \frac{E^s}{W \cdot f^*}) = 0 \tag{6.15}$$

If the maximum allowable burnup is high eg. $B^o > 60$ MWD/kg, Equation (6.6) would never be zero in the practical range E^s.

Hence, according to Equation (6.10) π would be zero.

In this case, the condition at optimum would be

$$\frac{\partial \overline{TC}}{\partial f} (E^s, f^*) = 0 \tag{6.16}$$

However, if the maximum allowable burnup is low, eg. $B° < 30$ MWD/kg, π is not equal to zero, and hence

$$B° - E^S/(W \cdot f^*) = 0 \qquad (6.17)$$

$$f^* = E^S/(W \cdot B°) \qquad (6.18)$$

At these lower maximum allowable burnups, the optimal batch fraction can be expressed as a linear function of $(E^S/B°)$.

6.3 Graphic Solution for Optimal Batch Fraction

A direct way of solving this problem is to calculate $\overline{TC}$ for all possible choices of E^S and f. Since $\overline{TC}$ is a smooth varying function of these variables, calculating $\overline{TC}$ on a coarse mesh of E^S and f would give an adequate representation of the function. Results shown on Table 6.1 are based on the Zion type 1065 MWe Pressurized water reactor. Figure 6.1 shows $\overline{TC}$ versus E^S for various values of f.

In Fig. 6.2, revenue requirement has been replotted against batch fraction at constant cycle energy. In addition, lines of constant average burnup B° are plotted. Only the region to the right of a line of constant burnup is compatible with the burnup constraint (6.3).

At the higher cycle energies of 10,650, 9,000 and 7,500 Gwhe, with a burnup constraint of 30 MWD/kg the optimal batch fraction occurs at the intersection of the constant burnup line and the constant energy line. At the lowest cycle energy of 5,000 Gwhe, the optimal batch fraction occurs at the lowest point on the constant energy line, at which condition (6.16) is met.

Table 6.1

Table of Revenue Requirement Per Cycle, Energy Per Cycle and Average Discharge Burnup versus Batch Fraction and Reload Enrichment

Enrichment (w/o)	Batch Fraction	1/1	1/2	1/3	1/4	1/6
2.0	$\overline{TC}$	17.837	10.798			
	E	6278.	4287.			
	B	8.879	12.129			
2.4	$\overline{TC}$	21.224	12.879	10.057		
	E	9259.	6092.	5311.		
	B	13.097	17.235	22.539		
2.8	$\overline{TC}$	24.712	15.015	11.595	9.799	
	E	12127.	7801.	6026.	4938.	
	B	17.155	22.068	25.571	27.938	
3.2	$\overline{TC}$	28.272	17.192	13.278	11.236	9.065
	E	14906.	9441.	7251.	5959.	4348.
	B	21.085	26.708	30.771	33.718	36.907
3.6	$\overline{TC}$		19.399	14.982	12.668	10.232
	E		11032.	8434.	6899.	5053.
	B		31.209	35.791	39.035	42.889
4.0	$\overline{TC}$		21.629	16.700	14.122	11.404
	E		12577.	9575.	7827.	5730.
	B		35.582	40.634	44.285	48.635
4.4	$\overline{TC}$		23.880	18.430	15.583	12.585
	E		14089.	10687.	8720.	6385.
	B		39.861	45.352	49.339	54.195
4.8	$\overline{TC}$			20.174	17.052	13.769
	E			11775.	9593.	7019.
	B			49.968	54.277	59.564
5.3	$\overline{TC}$				18.901	
	E				10660.	N.A.
	B				60.316	
7.0	$\overline{TC}$					20.339
	E					10253.
	B					87.021

$\overline{TC}$ (10^6\$)
E (GWHe)
B (MWD/Kg)

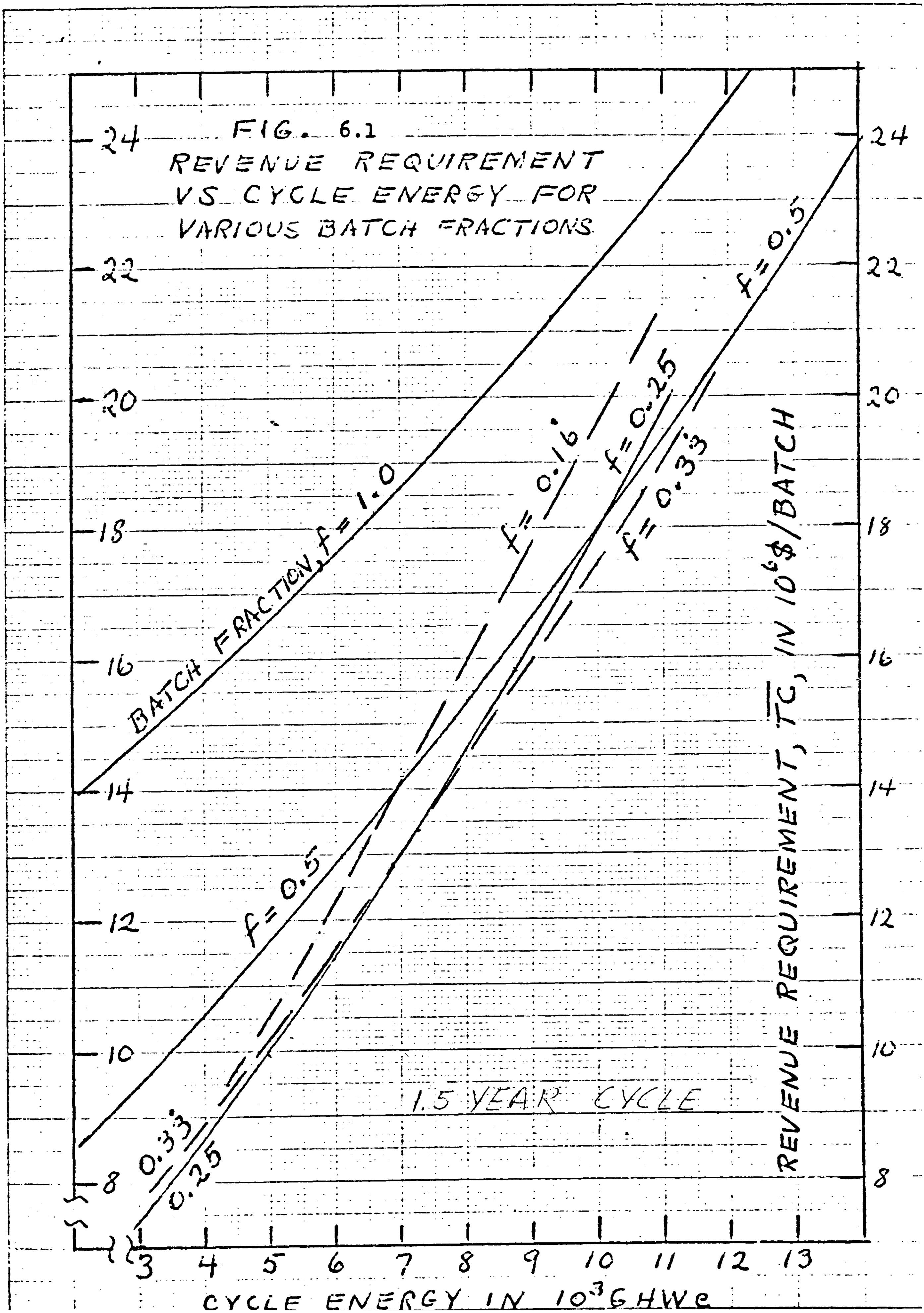


FIG. 6.1
REVENUE REQUIREMENT VS CYCLE ENERGY FOR VARIOUS BATCH FRACTIONS
BATCH FRACTION, f = 1.0
f = 0.5
f = 0.16̇
f = 0.25
f = 0.33̇
f = 0.5
0.33̇
0.25
1.5 YEAR CYCLE
REVENUE REQUIREMENT, $\overline{TC}$, IN 10^6 \$/BATCH
CYCLE ENERGY IN 10^3 GHWe
8
10
12
14
16
18
20
22
24
3
4
5
6
7
8
9
10
11
12
13

Figure 6.2 shows the variation of revenue requirement with respect to batch fraction for various cycle energies. The curves are rather flat near the minimum. Hence, in the vicinity of the minimum, economics plays a less important role than engineering and physical considerations.

Figure 6.3 shows the variation of revenue requirement with respect to reload enrichment for various cycle energies and batch fractions. Here the two independent variables E and f and the two dependent variables are shown on the same graph. The values of E* and f* can be read off directly for any minimal points.

6.4 Interpretation of the Lagrangian Multiplier π

When the maximum allowable burnup is high,

$$B^{\circ} > B(E^{s},f^{*})$$

according to Equation (6.10), π is zero. In this case π is a passive parameter which has no physical meaning. When the maximum allowable burnup is low,

$$B^{\circ} = B(E^{s},f^{*})$$

π would not be zero in general. In this case the optimal solution is on the boundary of the burnup constraint. For such cases the objective function can be further minimized by raising the burnup limitation. However, there are certain penalties that can be expressed in monetary terms resulting from raising the burnup limitation. Let the penalty be ρ dollars per unit increment of burnup limitation.

FIG. 6.2 REVENUE REQUIREMENT VS BATCH FRACTION FOR DIFFERENT LEVELS OF ENERGY

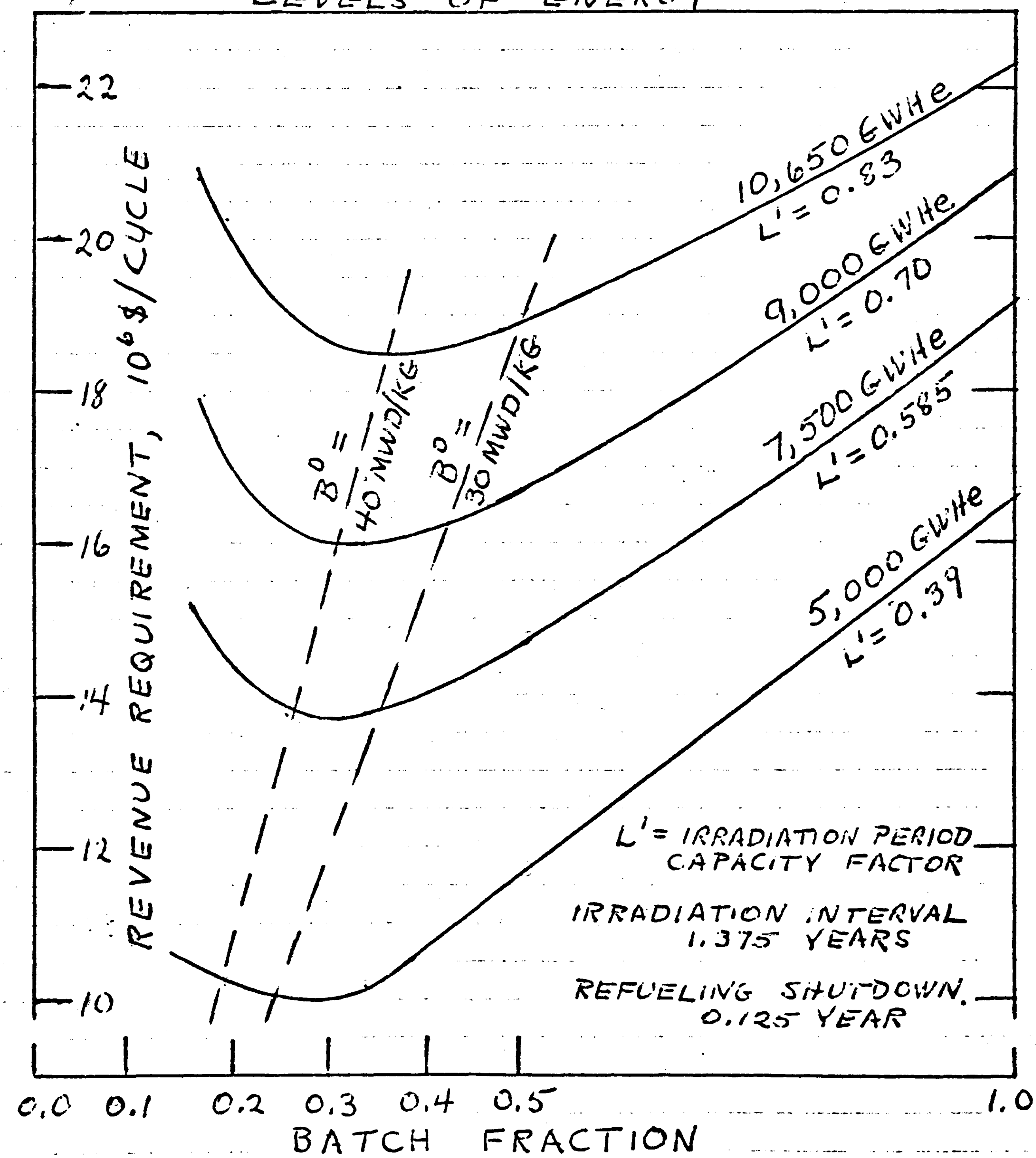
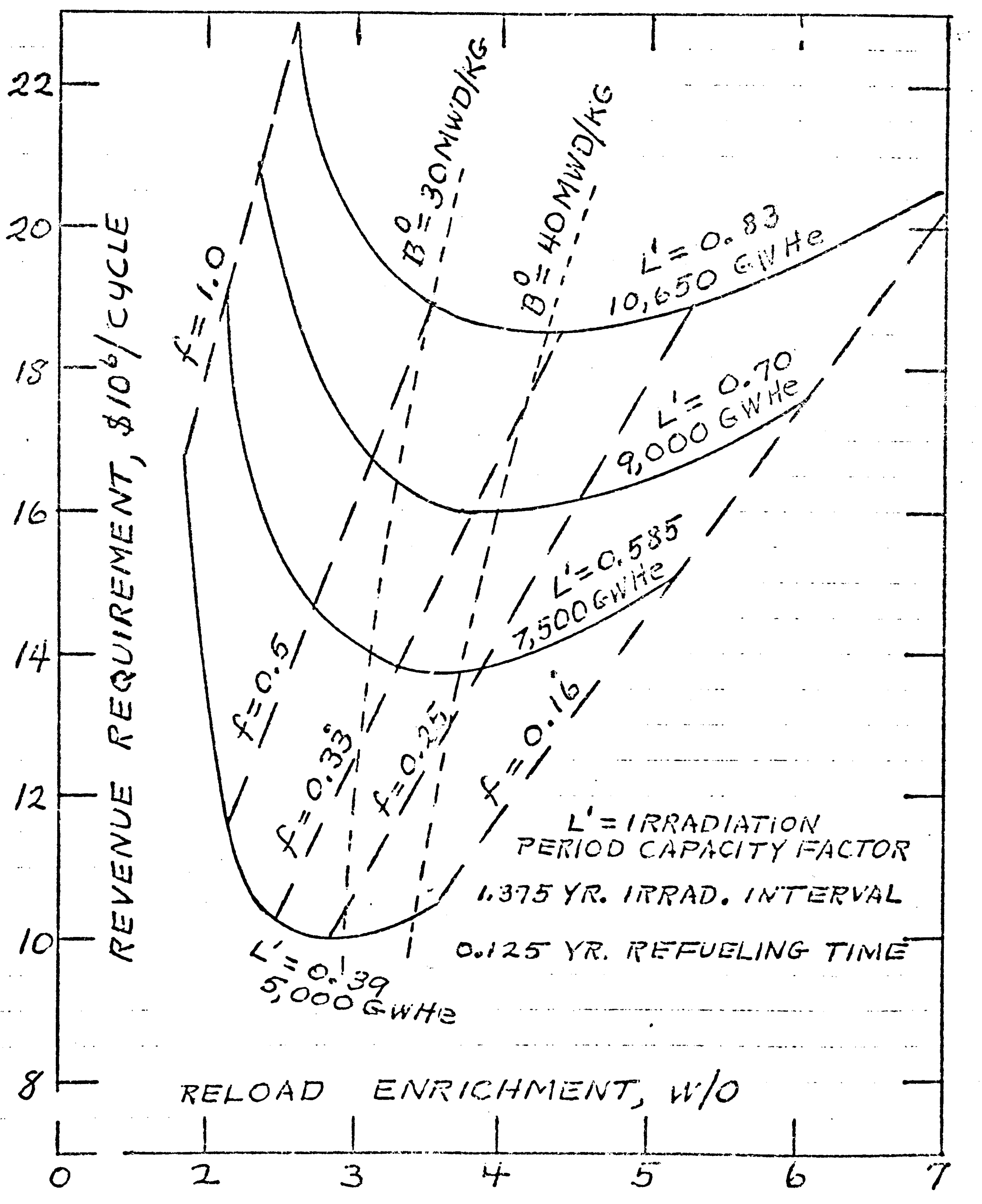


FIGURE 6.3

REVENUE REQUIREMENT VS RELOAD

ENRICHMENT FOR VARIOUS LEVELS OF ENERGY



Decreasing batch fraction by ∂f would result in a saving of $\frac{\partial \overline{TC}}{\partial f}(E^s, f^*)$. ∂f dollars

If this saving is more than the penalty $\rho \cdot \frac{\partial B}{\partial f}(E^s, f^*)$. ∂f there would be an incentive for decreasing the batch fraction further. The penalty ρ^* for which one is indifferent to decrease or not to decrease ∂f is

$$\rho^* = \frac{\partial \overline{TC}}{\partial f}(E^s, f^*) \ . \ \frac{\partial B}{\partial f}(E^s, f^*) \tag{6.19}$$

Since $\frac{\partial \overline{TC}}{\partial f} > 0$ according to Equation (6.8) and $\frac{\partial B}{\partial f} < 0$, ρ^* would be negative, meaning that it is a penalty. Comparing Equation (6.19) with Equation (6.12) reveals that

$$\pi = -\rho^* \tag{6.19}$$

Therefore, one can interpret π as the maximum price one would be willing to pay to increase the maximum allowable burnup.

6.5 <u>Calculation of Incremental Cost of Energy λ</u>

Since the objective function $\overline{TC}$ and the constraints $B^o \geq B$ are functions of two variables, cycle energy E^s and batch fraction f^*, defining incremental cost deserves special attention.

> Let f^* be the optimal batch fraction for the problem
> minimize $\overline{TC}(E^s, f)$ with respect to f
> subject to constraints $B^o \geq B(E^s, f)$
> Let $f^\dagger$ be the optimal batch fraction for the problem
> minimize $\overline{TC}(E^s + E, f^\dagger)$ with respect to f
> subject to constraints $B^o \geq B(E^s + \Delta E, f)$

Incremental cost of energy is defined formally as λ where

$$\lambda \equiv \lim_{\Delta E \to 0} \frac{\overline{TC}(E^S + \Delta E, f^\dagger) - \overline{TC}(E^S, f^*)}{\Delta E} \tag{6.21}$$

This equation can be simplified for the following two special cases.

Case (a): The maximum allowable burnup B^o is very high, such that

$$B^o > B(E^S, f^*)$$

$$B^o > B(E^S + \Delta E, f^\dagger)$$

In this case $\pi = 0$ according to Equation (6.10). Therefore according to Equation (6.11)

$$\frac{\partial TC(E^S, f^*)}{\partial f} = 0 \qquad (6.22)$$

$$\frac{\partial TC(E^S + \Delta E, f^\dagger)}{\partial f} = 0 \qquad (6.23)$$

Equation (6.22) and Equation (6.23) could be solved individually for f^* and f^+. Substituting f^* and f^+ into Equation (6.21) would yield the incremental cost of energy λ.

Case (b): The maximum allowable burnup $B°$ is low, such that

$$B° = B(E^S, f^*) = \frac{E^S}{W \cdot f^*}$$

$$B° = B(E^S + \Delta E, f^+) = \frac{E^S + \Delta E}{W \cdot f^+}$$

or
$$f^* = \frac{E^S}{W \cdot B°} \qquad (6.24)$$

$$f^+ = \frac{E^S + \Delta E}{W \cdot B°} \qquad (6.25)$$

Substituting f^* and f^+ into Equation (6.21) would again yield the incremental cost of energy λ. Note that in any case, incremental cost of energy λ is not given by the partial derivative of total cost $\overline{TC}$ with respect to cycle energy E holding batch fraction f constant, but is given by Equation (6.21) with the f^* and f^+ determined by either Equations (6.22) and (6.25) or Equations (6.24) and (6.22)

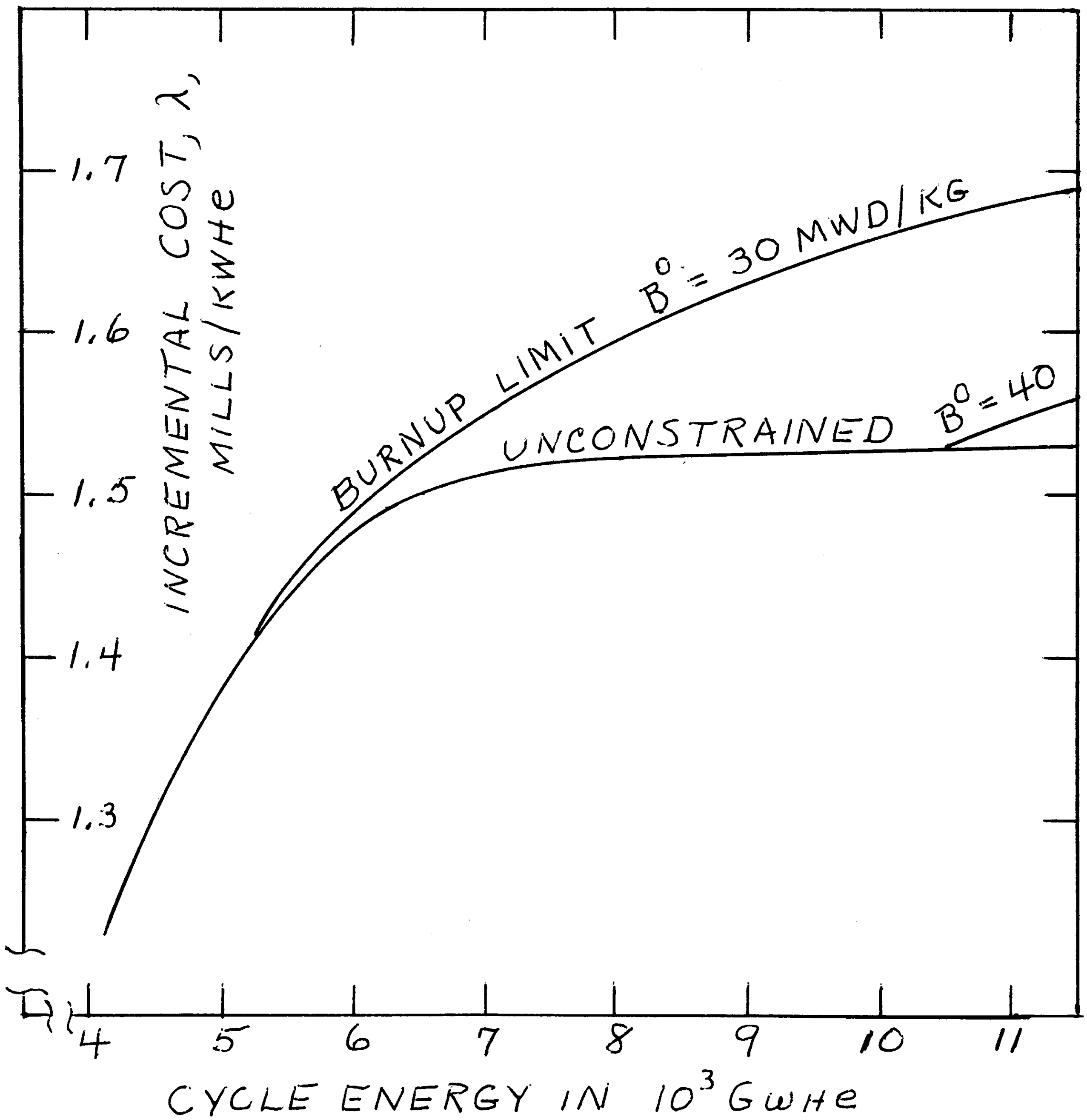
$$\lambda \neq \left.\frac{\partial \overline{TC}}{\partial E}\right|_f$$

Figure 6.4 shows incremental cost of energy λ versus cycle energy E^S for various burnup limitations. For the same cycle energy E^S, incremental cost of energy increases with

Figure 6.4

INCREMENTAL COST λ VS

CYCLE ENERGY FOR VARIOUS BURNUP LIMITS B°



decreasing allowable burnup levels. When the burnup constraint is not controlling, incremental cost first increases, then levels off with increasing cycle energy.

6.6 Effects of Shortening the Irradiation Interval

Fuel cycle calculations are repeated for refuelling interval of one year based on the same depletion calculations given in this chapter. The results are shown on Figures 6.5 and 6.6.

The following differences can be seen between the cases of 1.5 year and 1 year refuelling interval. The revenue requirements for all cycles are lower for the 1 year case. This is due to a shorter time period in which carrying charges are based. The optimum batch fraction for a given cycle energy is somewhat lower. But the overall trends of the two cases are very similar. Hence, for small variations of refuelling interval, the behavior of the incremental cost and optimal solutions would not be greatly changed.

6.7 Conclusions

For steady-state refuelling, the problem of nuclear in-core optimization can be solved directly by graphic techniques. For a specified cycle energy, the optimal batch fraction is given by the smallest value compatible with burnup limitation for nearly all practical cases. The explanation is that the savings in service components

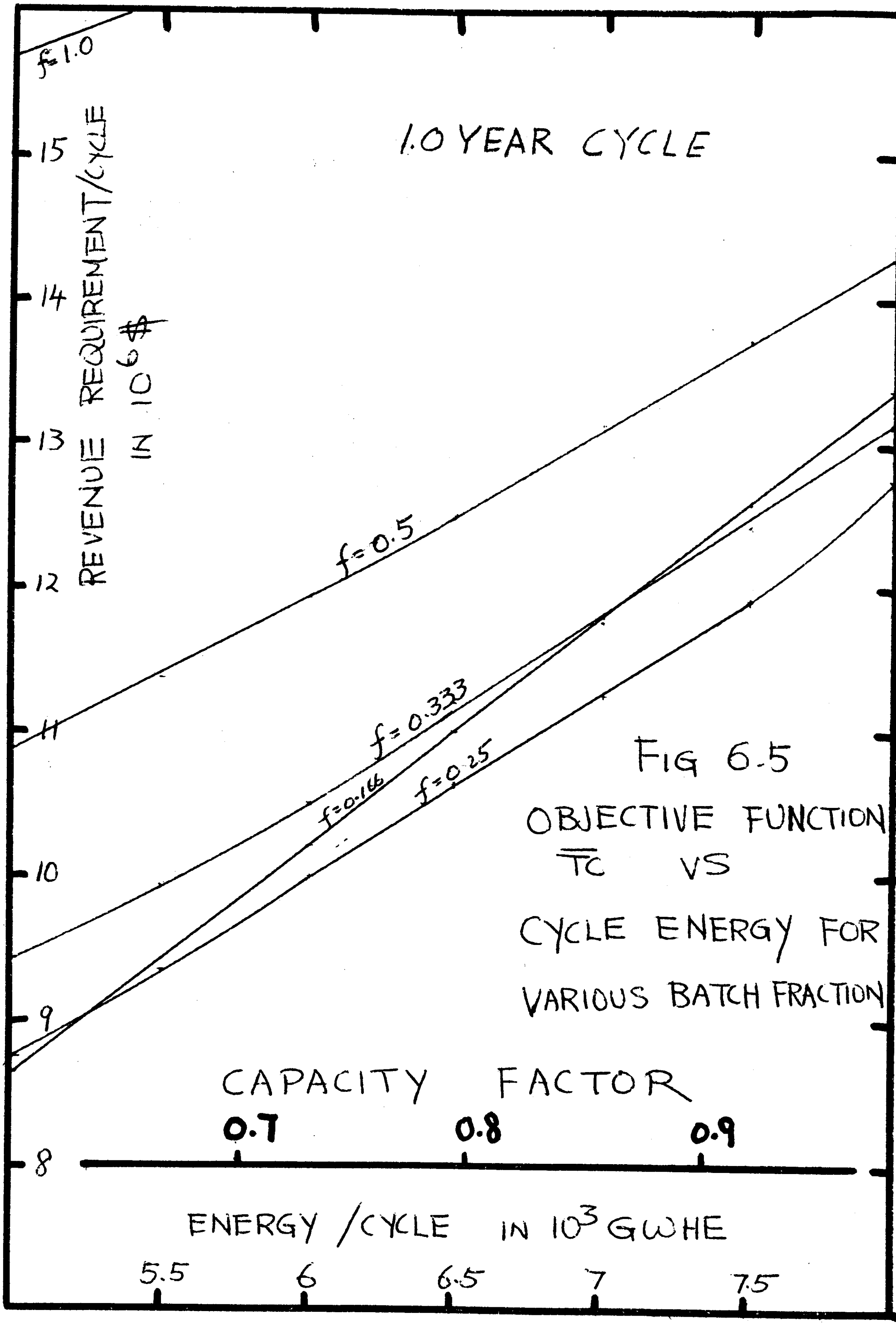


FIG 6.5
OBJECTIVE FUNCTION $\overline{TC}$ VS CYCLE ENERGY FOR VARIOUS BATCH FRACTION

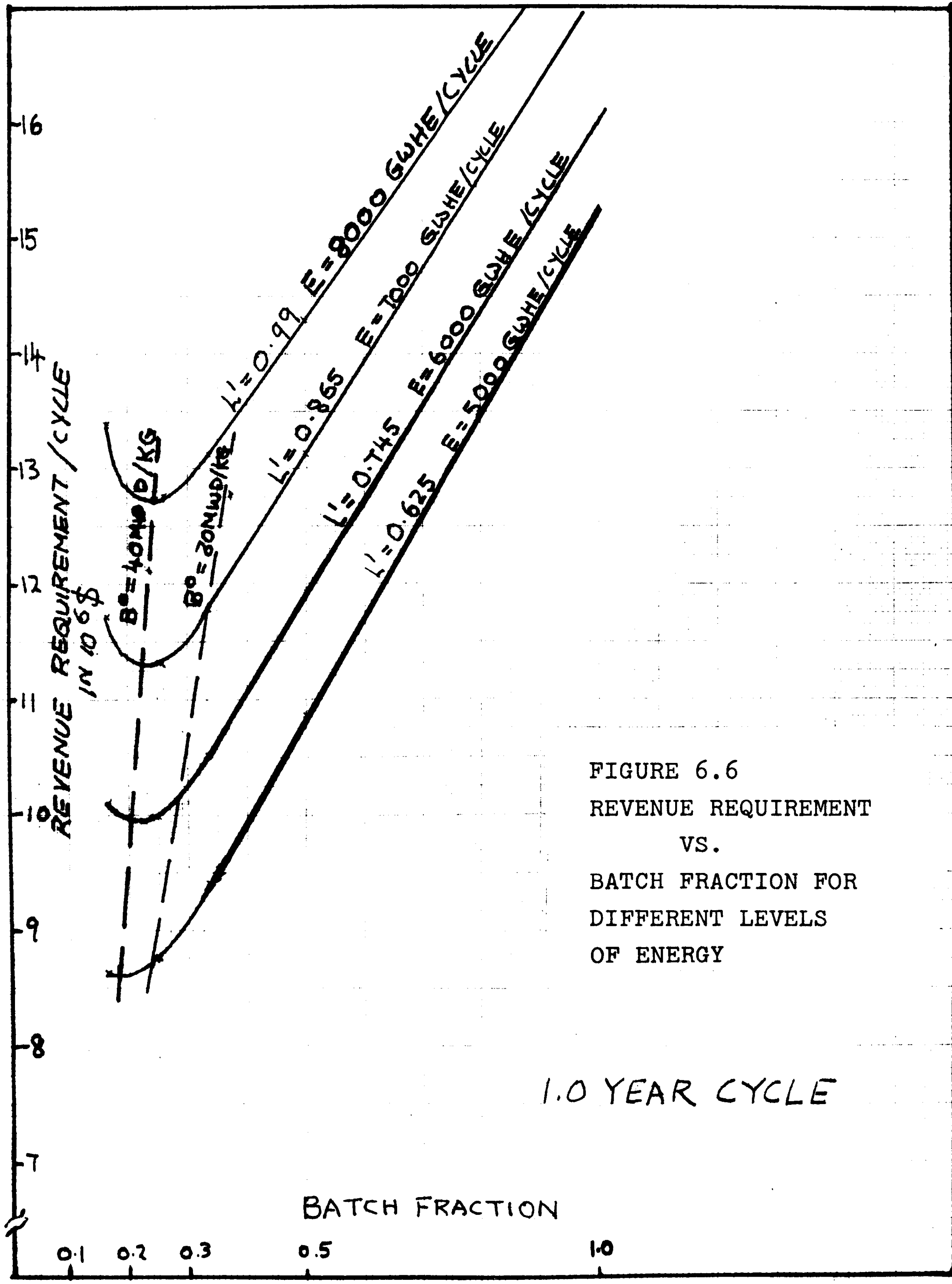


FIGURE 6.6
REVENUE REQUIREMENT
VS.
BATCH FRACTION FOR
DIFFERENT LEVELS
OF ENERGY

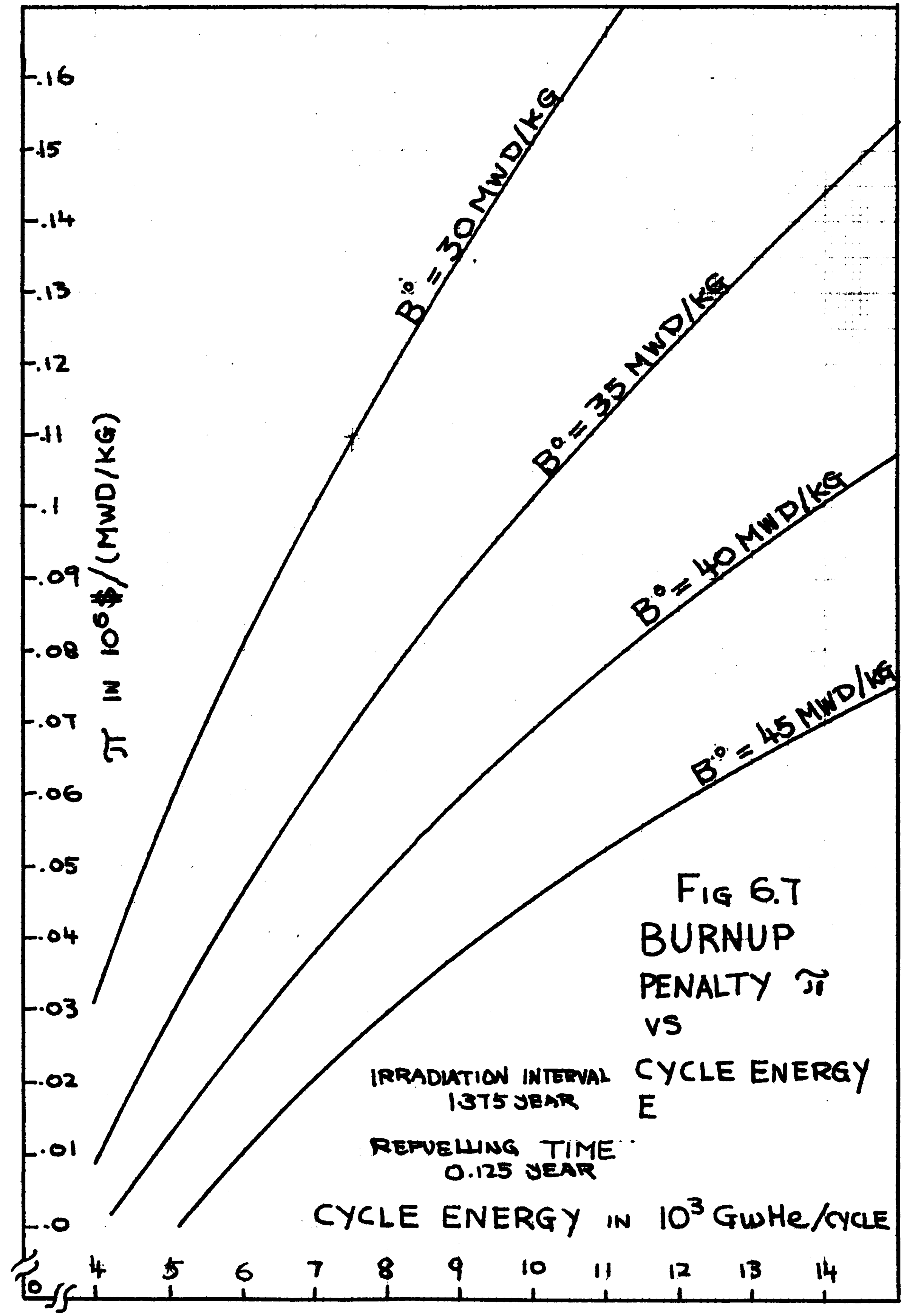


Fig 6.7
BURNUP PENALTY π
VS
CYCLE ENERGY
E

In this case $\pi = 0$ according to Equation (6.10). Therefore according to Equation (6.11)

$$\frac{\partial TC(E^S, f^*)}{\partial f} = 0 \tag{6.22}$$

$$\frac{\partial TC(E^S + \Delta E, f^+)}{\partial f} = 0 \tag{6.23}$$

Equation (6.22) and Equation (6.23) could be solved individually for f^* and f^+. Substituting f^* and f^+ into Equation (6.21) would yield the incremental cost of energy λ.

Case (b): The maximum allowable burnup B^o is low, such that

$$B^o = B(E^S, f^*) = \frac{E^S}{W \cdot f^*}$$

$$B^o = B(E^S + \Delta E, f^+) = \frac{E^S + \Delta E}{W \cdot f^+}$$

or
$$f^* = \frac{E^S}{W \cdot B^o} \tag{6.24}$$

$$f^+ = \frac{E^S + \Delta E}{W \cdot B^o} \tag{6.25}$$

Substituting f^* and f^+ into Equation (6.21) would again yield the incremental cost of energy λ. Note that in any case, incremental cost of energy λ is not given by the partial derivative of total cost $\overline{TC}$ with respect to cycle energy E holding batch fraction f constant, but is given by Equation (6.21) with the f^* and f^+ determined by either Equations (6.22) and (6.25) or Equations (6.24) and (6.22)

$$\lambda \neq \left.\frac{\partial \overline{TC}}{\partial E}\right|_f$$

Figure 6.8 shows incremental cost of energy λ versus cycle energy E^S for various burnup limitations. For the same cycle energy E^S, incremental cost of energy increases with

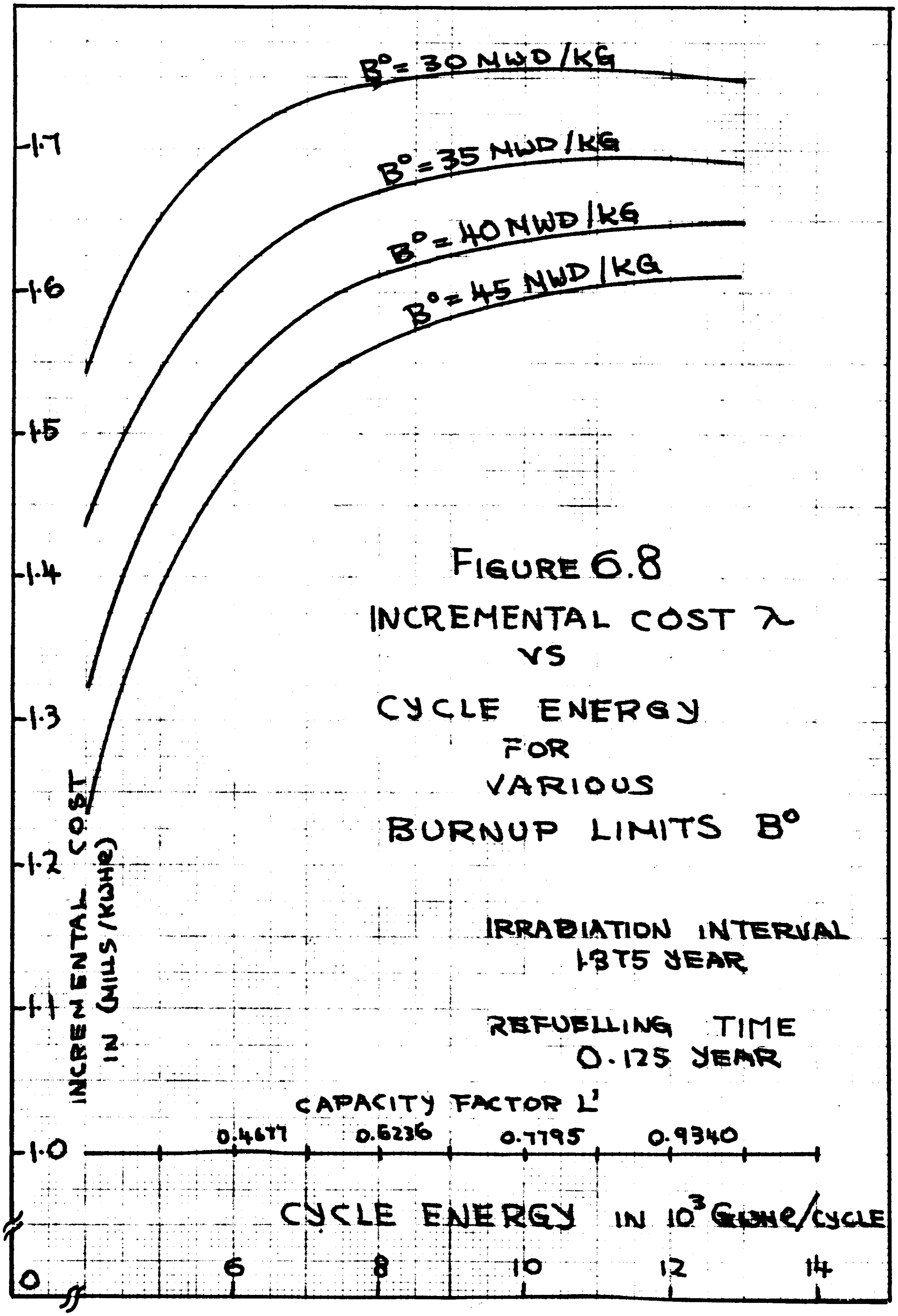


FIGURE 6.8
INCREMENTAL COST λ
VS
CYCLE ENERGY
FOR
VARIOUS
BURNUP LIMITS B°

IRRADIATION INTERVAL
1.375 YEAR

REFUELLING TIME
0.125 YEAR

decreasing allowable burnup levels. For the same burnup, incremental cost first increases, then levels off and finally decreases for increasing cycle energy.

6.6 Effects of Shortening the Irradiation Interval

Fuel cycle calculations are repeated for refuelling interval of one year based on the same depletion calculations given in this chapter. The results are shown on Figures 6.9, 6.10, 6.11, 6.12.

The following differences can be seen between the cases of 1.5 year and 1 year refuelling interval. The revenue requirement for all cycles are lower for the 1 year case. This is due to a shorter time period in which carrying charges are based. Incremental cost of energy shows a wider spread for the range of burnup limits considered. But the overall trends of the two cases are very similar. Hence, for small variations of refuelling interval, the behavior of the incremental cost and optimal solutions would not be greatly changed.

6.7 Conclusions

For steady-state refuelling, the problem of nuclear in-core optimization can be solved directly by graphic techniques. For a specified cycle energy, the optimal batch fraction is given by the smallest value compatible with burnup limitation for nearly all practical cases. The explanation is that the savings in service components

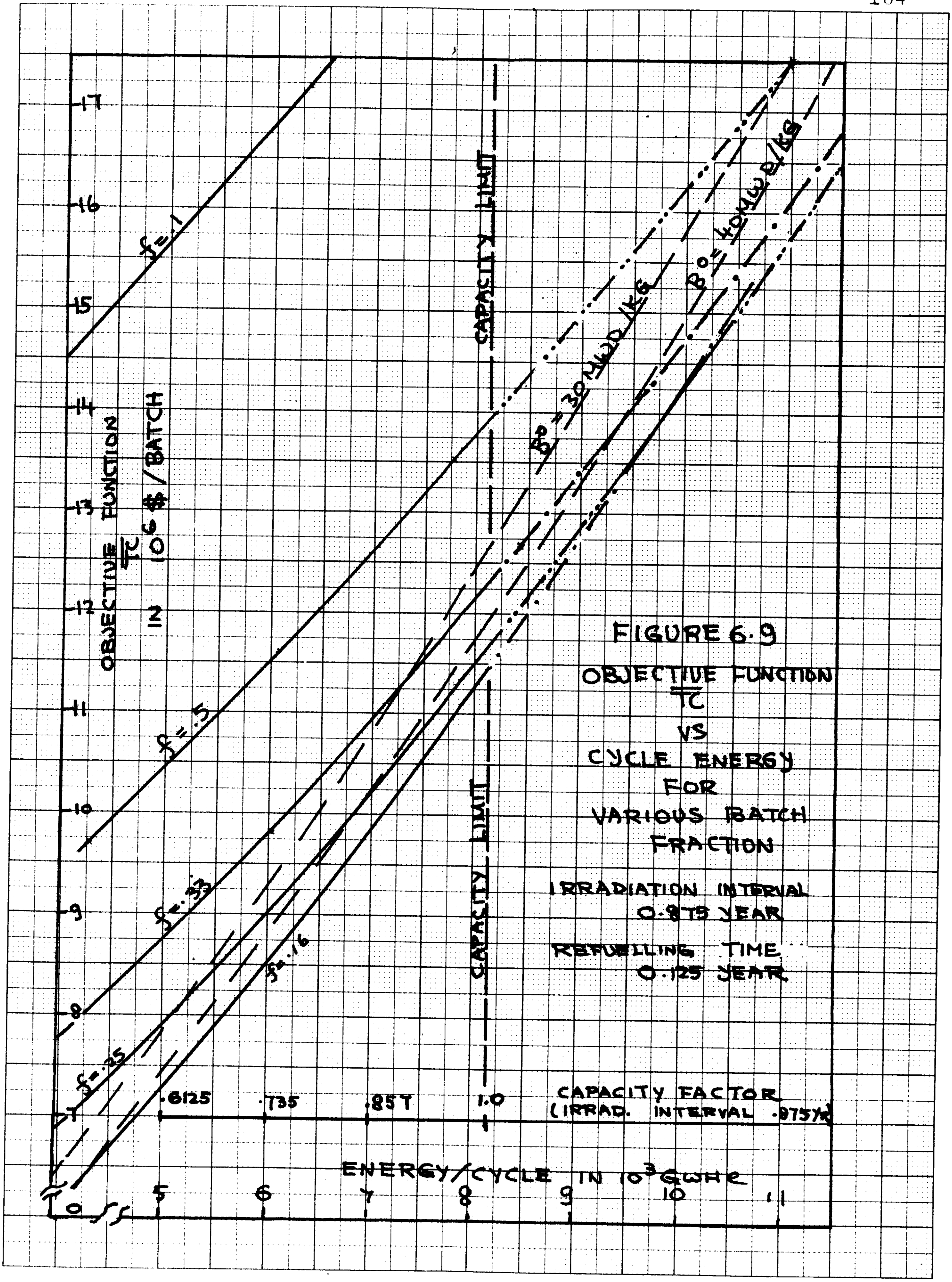


FIGURE 6.9

OBJECTIVE FUNCTION $\overline{TC}$ VS CYCLE ENERGY FOR VARIOUS BATCH FRACTION

IRRADIATION INTERVAL 0.875 YEAR

REFUELLING TIME 0.125 YEAR

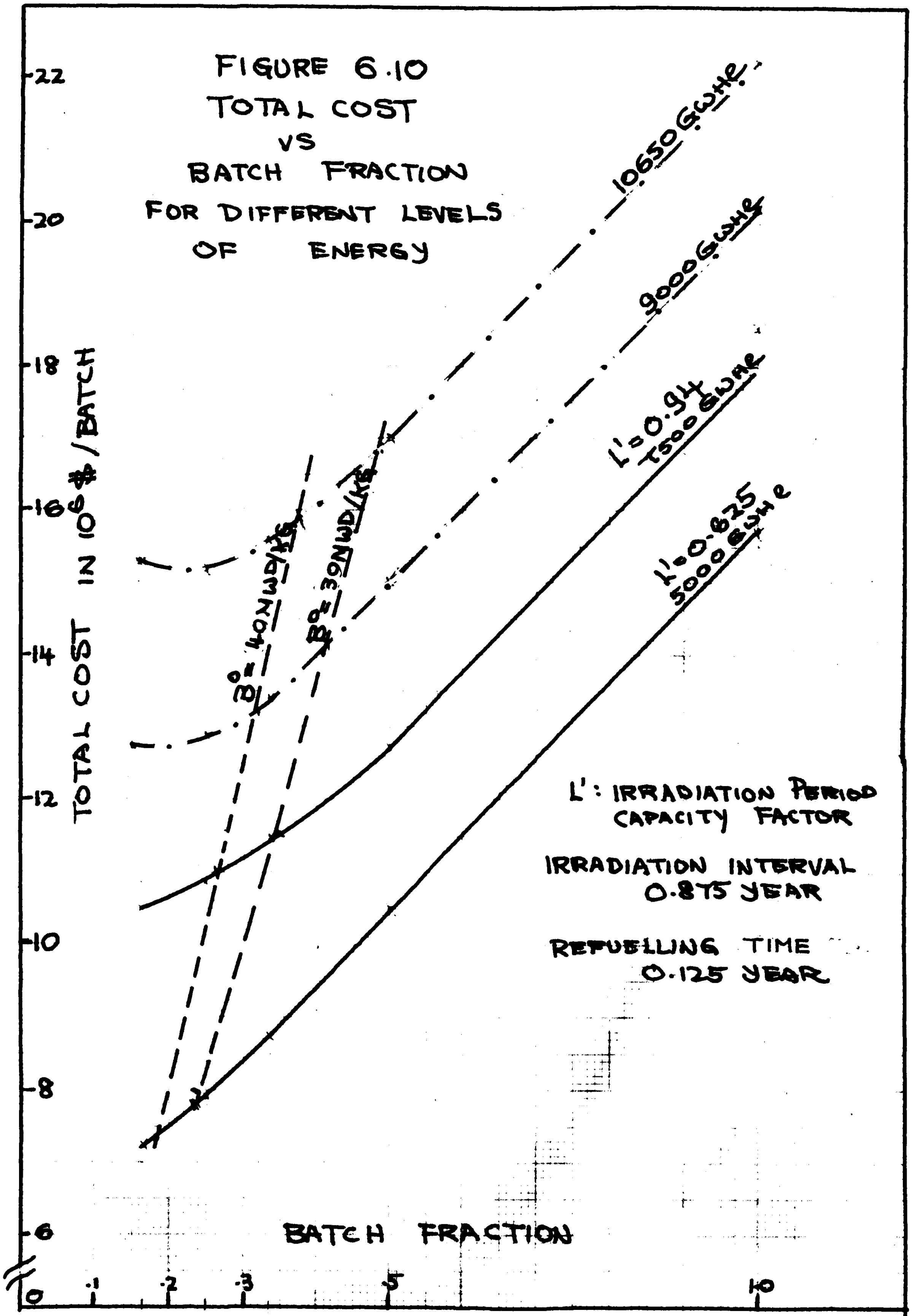


FIGURE 6.10
TOTAL COST
VS
BATCH FRACTION
FOR DIFFERENT LEVELS
OF ENERGY
TOTAL COST IN 10^6 $/BATCH
22
20
18
16
14
12
10
8
6
0
.1
.2
.3
.5
1.0
BATCH FRACTION
10650 GWHe
9000 GWHe
L' = 0.94
7500 GWHe
L' = 0.825
5000 GWHe
B° = 40 MWD/KG
B° = 30 MWD/KG
L': IRRADIATION PERIOD CAPACITY FACTOR
IRRADIATION INTERVAL 0.875 YEAR
REFUELLING TIME 0.125 YEAR

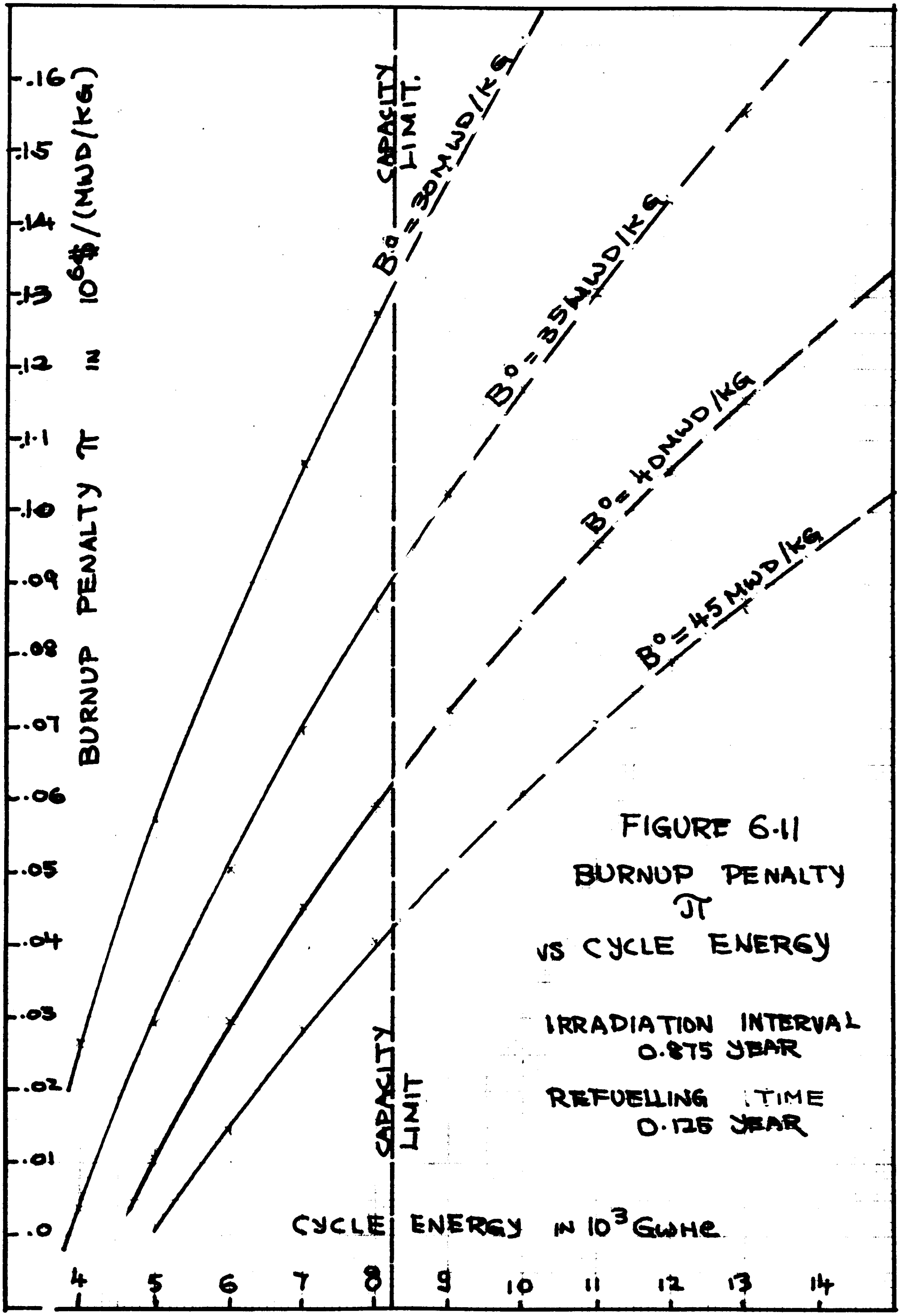
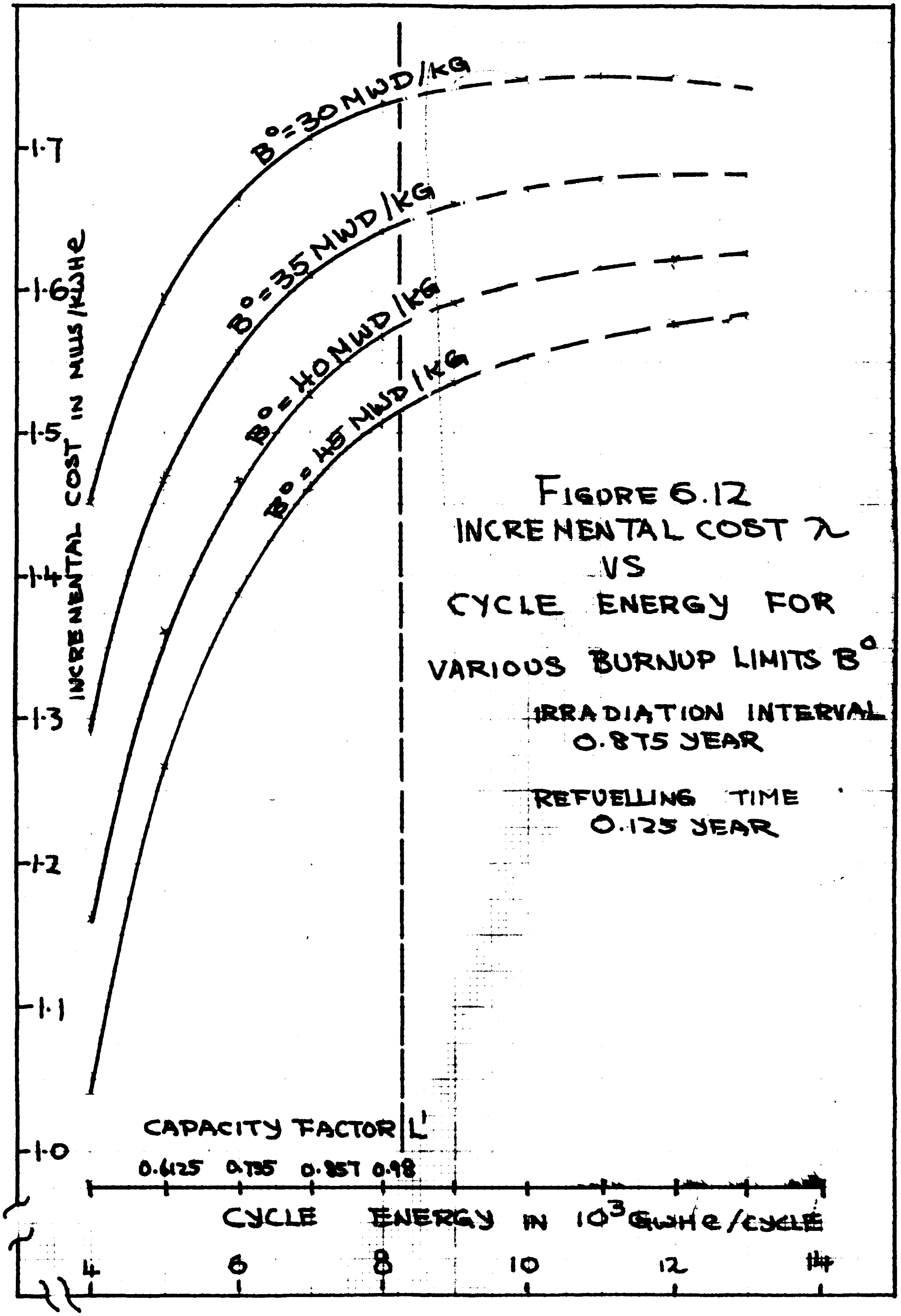


FIGURE 6.11
BURNUP PENALTY π
VS CYCLE ENERGY



B° = 30 MWD/KG
B° = 35 MWD/KG
B° = 40 MWD/KG
B° = 45 MWD/KG
INCREMENTAL COST IN MILLS/KWHe
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
CAPACITY FACTOR L'
0.6125 0.735 0.857 0.98
CYCLE ENERGY IN 10³ GWHe/CYCLE
4
6
8
10
12
14
FIGURE 6.12
INCREMENTAL COST λ
VS
CYCLE ENERGY FOR
VARIOUS BURNUP LIMITS B°
IRRADIATION INTERVAL
0.875 YEAR
REFUELLING TIME
0.125 YEAR

costs resulting from a smaller batch fraction outweights the additional enrichment cost, carrying charges and income taxes. Finally, the incremental cost of energy increases with cycle energy, but levels off at $E^S \simeq 10{,}000$ GWHe/cycle. The incremental cost of energy also increases with decreasing allowable burnup levels.

CHAPTER 7.0

NUCLEAR IN-CORE OPTIMIZATION FOR NON-STEADY STATE FORMULATION OF THE PROBLEM

7.1 Introduction

Having solved the steady state nuclear in-core optimization problem in Chapter 6, this chapter considers the general non-steady state nuclear in-core optimization problem outlined in Chapter 2 Section 2.5. The general problem of nuclear in-core optimization can be stated as follows: given a refuelling and maintenance schedule, and a specified sequence of cycle energy demand for a given reactor in the planning horizon, find the optimal combination of reload enrichments and batch fractions such that the fuel cycle cost is minimized and the engineering constraints are satisfied. A typical planning horizon consists of five cycles with a total duration of about seven years. In general the cycle energy demand for each of the five cycles would be different from each other. Consequently, the reload enrichment and batch fraction for each cycle would be different and hence the reactor supplying this energy is said to be operating in a non-steady state manner. At the beginning of the planning horizon, the reactor is in a certain well specified initial state. This initial state would play an important role in the overall optimization. In addition to satisfying the cycle energy demand, the optimal combination of reload enrichments and batch fractions should also satisfy engineering constraints, such as burnup limitations, power peaking, control poison margins and other safety considerations. Only when all these constraints have been satisfied does the economics optimization has any practical significance.

7.2 Mathematical Formulation of the Problem

For full-power reactivity-limited burnup, cycle energy and discharge burnup are unique functions of the reload enrichments and batch fractions of all the preceding cycles. Hence, reload enrichments and batch fractions can be considered as independent variables, while cycle energy and discharge burnup can be considered as dependent variables. The objective function: revenue requirement for the planning horizon, is also a variable dependent on reload enrichments and batch fractions.

Thus, the problem of non-steady state in-core optimization can be mathematically stated as

minimize $\overline{TC}(\vec{\varepsilon},\vec{f},\psi)$ (7.1)

with respect to $\vec{\varepsilon}$ and $\vec{f}$

subject to constraints

$$E_c(\vec{\varepsilon},\vec{f})=E_c^s \tag{7.2}$$

$$B_c(\vec{\varepsilon},\vec{f})<B^0 \tag{7.3}$$

where $\overline{TC}$: is the revenue requirement for this reactor for the planning horizon

E_c: energy generated in cycle c

E_c^S: energy demanded by the system on cycle c of the reactor

B_c: average discharge burnup of Cycle c

B^o: maximum allowable burnup

$\vec{\varepsilon}$: a vector consisting of all the reload enrichments

$\vec{f}$: a vector consisting of all the batch fractions

Ψ : initial condition of the reactor

Equation (7.2) is the requirement that the cycle energy demand be satisfied. Equation (7.3) is the requirement that the average discharge burnup be within technical limits. In general, other engineering constraints, such as power peaking and control poison margin, etc. should be imposed on the system. However, for simplicity, only the burnup constraint is considered. Other constraints can be incorperated with no major difficulties.

The Kuhn -Tucker optimality conditions for the optimal solution $\vec{\varepsilon}$, $\vec{f}$ are

$$E_c(\vec{\varepsilon}^*,\vec{f}^*)=E_c^S \qquad \text{for all } c \tag{7.4}$$

$$B_c(\vec{\varepsilon}^*,\vec{f}^*)<B^0 \tag{7.5}$$

$$\vec{\varepsilon}^*>0 \tag{7.6}$$

$$\vec{f}^*>0 \tag{7.7}$$

$$\frac{\partial}{\partial \varepsilon_{c'}}\sum_c^C\{\pi_c\cdot(B^0-B_c)+\lambda_c\cdot(E_c^S-E_c)\}\leqslant\frac{\partial\overline{TC}}{\partial\varepsilon_{c'}} \qquad \text{for all } c' \tag{7.8}$$

$$\frac{\partial}{\partial f_{c'}}\sum_c^C\{\pi_c\cdot(B^0-B_c)+\lambda_c\cdot(E_c^S-E_c)\}\leqslant\frac{\partial\overline{TC}}{\partial f_{c'}} \qquad \text{for all } c' \tag{7.9}$$

$$\pi_c\geqslant 0$$

$$\lambda_c\geqslant 0$$

$$\sum_c^C\{\pi_c\cdot(B^*-B_c)+\lambda_c\cdot(E_c^S-E_c)\}=0 \tag{7.10}$$

$$\sum_c^C\left\{\frac{\partial\overline{TC}}{\partial\varepsilon_c}\times\varepsilon_c^*+\frac{\partial\overline{TC}}{\partial f_c}\times f_c^*\right\}=\sum_{c'}^C\varepsilon_{c'}^*\times\frac{\partial}{\partial\varepsilon_{c'}}\sum_c^C\{\pi_c\cdot(B^0-B_c)+\lambda_c\cdot(E_c^S-E_c)\}$$
$$+\sum_{c'}^C f_{c'}^*\times\frac{\partial}{\partial f_{c'}}\sum_c^C\{\pi_c\cdot(B^0-B_c)+\lambda_c\cdot(E_c^S-E_c)\} \tag{7.11}$$

where λ_c: is defined as the incremental cost of nuclear energy for the c-cycle

π_c: is defined as the burnup penalty for the c-cycle

Since the dependent variables are not analytically differentiable, the optimality conditions are not useful in a practical sense. Calculation of the incremental cost and burnup penalty directly from these equations is not feasible.

Methods of solving the nuclear in-core optimization problem are given in Chapters 8 and 9. Calculation of incremental cost is given in Chapter 9 .

7.3 Exact and Approximate Calculation of the Objective Function

The objective function $\overline{TC}_I$ is defined as the revenue requirement for the reactor in the planning horizon I. The method of calculating $\overline{TC}_I$ is given in Section 4.3.2, with end state correction based on the Inventory Value Method. In principle, it includes the revenue requirement for all the batches discharged from the reactor in the planning horizon. The treatment for these batches is exact.

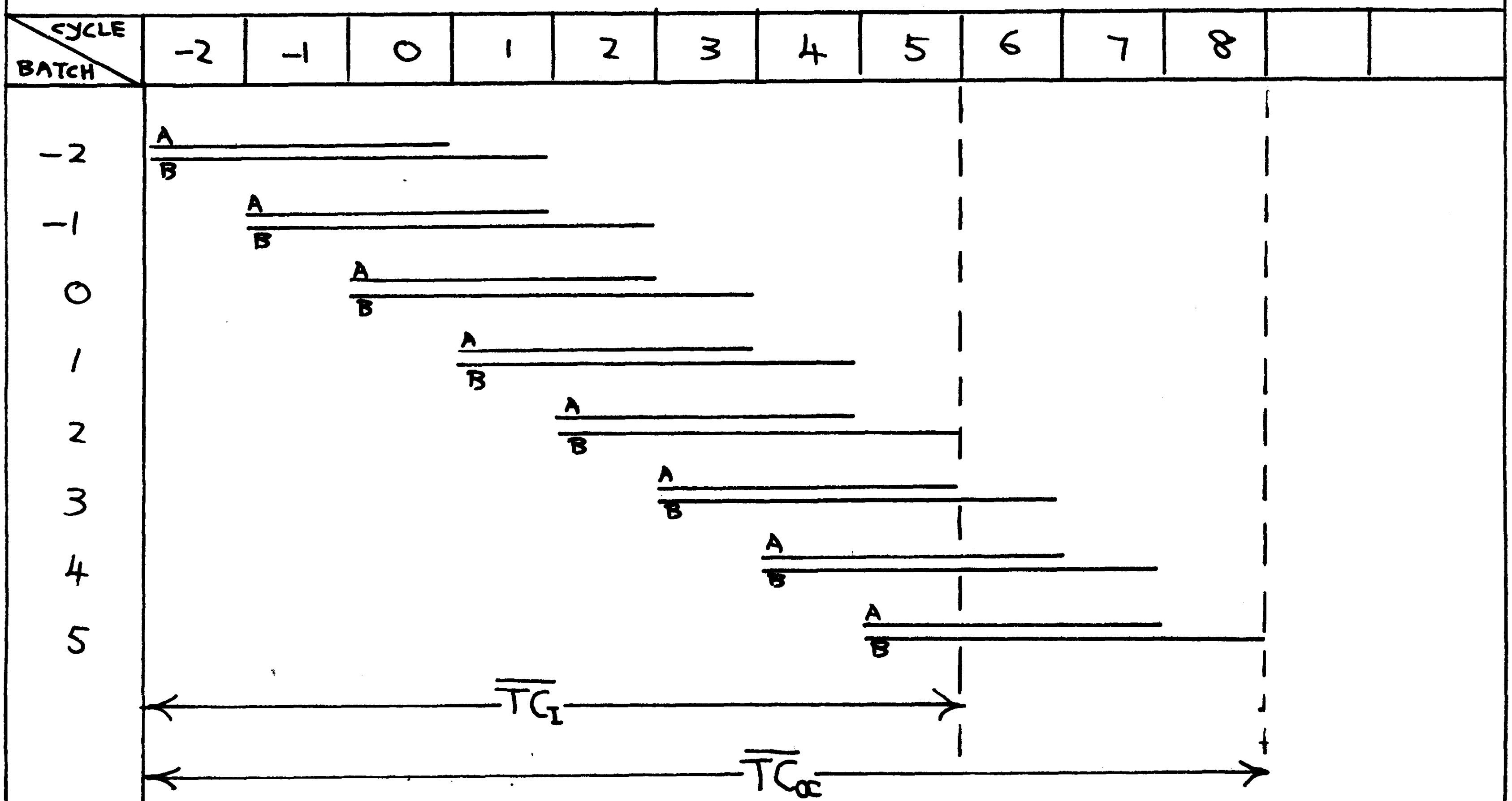
Those batches that remain in the reactor core at the end of the planning horizon are assigned a value $V^b(\theta_{I'})$ that reflects the nuclide value and residual book value of fabrication, shipping, reprocessing and conversion.

For these batches, the calculation of revenue requirement is only approximate because of these service costs.

Hence, the accuracy of the approximate $\overline{TC}_I$ is compared to an exact revenue requirement $\overline{TC}_\infty$ based on a pre-specified fuel strategy. The number of batches included in $\overline{TC}_I$ and $\overline{TC}_\infty$ are shown schematically in Figure 7.1. The result of the test would hopefully demonstrate that optimization based on the approximate $\overline{TC}_I$ is equivalent

FIGURE 7.1

RELATIONSHIPS BETWEEN THE VARIOUS REVENUE REQUIREMENTS BATCH NUMBER AND CYCLE NUMBER



A SUBLOT OF FUEL DISCHARGED IN EARLIER CYCLE

B SUBLOT OF FUEL DISCHARGED IN LATER CYCLE to optimization based on the exact $\overline{TC}_\alpha$.

If $\vec{\varepsilon}^*$ and $\vec{f}^*$ is the optimal solution based on an exact calculation of $\overline{TC}_\alpha$, according to Kuhn-Tucker optimality conditions Equations (7.8), (7.9) and (7.11) would be

$$\frac{\partial \overline{TC}_\alpha(\vec{\varepsilon}^*,\vec{f}^*)}{\partial \varepsilon_{c'}} > \frac{\partial}{\partial \varepsilon_{c'}} \sum_c^C \{\pi_c(B^0-B_c(\vec{\varepsilon}^*,\vec{f}^*)) + \lambda_c(E_c^S - E_c(\vec{\varepsilon}^*,\vec{f}^*))\} \tag{7.12}$$

$$\frac{\partial \overline{TC}_\alpha(\vec{\varepsilon}^*,\vec{f}^*)}{\partial f_{c'}} > \frac{\partial}{\partial f_{c'}} \sum_c^C \{\pi_c(B^0-B_c(\vec{\varepsilon}^*,\vec{f}^*)) + \lambda_c(E_c^S - E_c(\vec{\varepsilon}^*,\vec{f}^*))\} \tag{7.13}$$

$$\begin{aligned} \sum_c^C \left\{ \frac{\partial \overline{TC}_\alpha(\vec{\varepsilon}^*,\vec{f}^*)}{\partial \varepsilon_c} \times \varepsilon_c^* + \frac{\partial \overline{TC}_\alpha(\vec{\varepsilon}^*,\vec{f}^*)}{\partial f_c} \times f_c^* \right\} = \\ \sum_{c'}^C \varepsilon_{c'}^* \cdot \frac{\partial}{\partial \varepsilon_{c'}} \sum_c^C \{\pi_c \cdot (B^0-B_c) + \lambda_c \cdot (E_c^S - E_c) \quad \} \\ + \sum_{c'}^C f_{c'}^* \cdot \frac{\partial}{\partial f_{c'}} \sum_c^C \{\pi_c \cdot (B^0-B_c) + \lambda_c \cdot (E_c^S - E_c) \quad \} \end{aligned} \tag{7.14}$$

However, if one requires $\vec{\varepsilon}^*$, $\vec{f}^*$ to be the optimal solution based on the approximate objective function $\overline{TC}_I$, $\vec{\varepsilon}^*$, $\vec{f}^*$ should also satisfy the Kuhn-Tucker optimality condition for $\overline{TC}_I$. Hence, the Kuhn-Tucker conditions for $\vec{\varepsilon}^*$, $\vec{f}^*$ and $\overline{TC}_I$ are exactly similar to that of Equations (7.12), (7.13) and (7.14) with $\overline{TC}_I$ replacing $\overline{TC}_\alpha$. Since the right sides of the equations are not affected by the replacement, the value of the left hand side of the equations should be the same for $\overline{TC}_I$ and $\overline{TC}_\alpha$. In other words, we should show that

$$\left.\frac{\partial \overline{TC}_I(\vec{\varepsilon}^*,\vec{f}^*)}{\partial \varepsilon_c}\right|_{\vec{f}} = \left.\frac{\partial \overline{TC}_\alpha(\vec{\varepsilon}^*,\ \vec{f}^*)}{\partial \varepsilon_c}\right|_{\vec{f}} \tag{7.15}$$

$$\left.\frac{\partial \overline{TC}_I(\vec{\varepsilon}^*,\vec{f}^*)}{\partial f_c}\right|_{\vec{\varepsilon}} = \left.\frac{\partial \overline{TC}_\alpha(\vec{\varepsilon}^*,\ \vec{f}^*)}{\partial f_c}\right|_{\vec{\varepsilon}} \tag{7.16}$$

Therefore, if each partial derivatives for $\overline{TC}_\alpha$ is equal to the corresponding derivative of $\overline{TC}_I$, then optimization based on either of them is equivalent.

7.4 Comparison of the Exact and Approximate Methods

The partial derivatives of $\overline{TC}_I$ are compared to those of $\overline{TC}_\alpha$ in a series of sample cases.

Planning horizons for each of the sample cases consist of five cycles. However, to calculate $\overline{TC}_\alpha$ it is necessary to know the reload enrichment and batch fraction up to the eighth cycle. The reload enrichments and batch fractions for the sixth, seventh and eighth cycle are taken to be 3.2 w/o and 0.33 respectively. Calculations are based on the Zion type 1065 MWe PWR. At time zero, the reactor is down for refuelling after it has been refuelled in a three-zone modified scatter manner with 3.2 w/o reload enrichment until steady state has been reached. The energy requirement for each of the next five cycles is 22750 GWHt. The maximum allowable average discharge burnup is 32 MWD/kg. Under these restrictive conditions, the optimal reload enrichments and batch fractions are ε=3.2w/o and f=0.33 for the next five cycles. In other words, the reactor is already optimized before the planning horizon.

The reload enrichment or the batch fraction for the first cycle is varied in order to evaluate the partial derivative of the objective function with respect to enrichment or batch fraction. The partial derivatives for $\overline{TC}_\alpha$ are

$$\left.\frac{\partial \overline{TC}_\alpha}{\partial \varepsilon_1}\right|_{\vec{f}^\dagger} \simeq \frac{\overline{TC}(\varepsilon_1^\dagger+\Delta\varepsilon_1, \varepsilon_2^\dagger \ldots \vec{f}^\dagger) - \overline{TC}(\varepsilon_1^\dagger, \varepsilon_2^\dagger \ldots \vec{f}^\dagger)}{\Delta \varepsilon_1} \tag{7.17}$$

$$\left.\frac{\partial \overline{TC}_\alpha}{\partial f_1}\right|_{\vec{\varepsilon}^\dagger} \simeq \frac{\overline{TC}(\vec{\varepsilon}^\dagger, f_1^\dagger+\Delta f_1, f_2^\dagger \ldots) - \overline{TC}(\vec{\varepsilon}^\dagger, f_1^\dagger, f_2^\dagger \ldots)}{\Delta f_1} \tag{7.18}$$

Partial derivatives for $\overline{TC}_I$ are similiar to Equations (7.17) and (7.18) by replacing $\overline{TC}_\alpha$ with $\overline{TC}_I$.

Figure 7.1 shows schematically the number of batches included in $\overline{TC}_I$ and $\overline{TC}_\alpha$, the last three of which bring the reactor sufficiently close back to steady state condition. $\overline{TC}_\alpha$ consists of eight batches irradiated from Cycle -2 to Cycle 8 for a total of eleven cycles. $\overline{TC}_I$ consists of the same eight batches irradiated from Cycle -2 to Cycle 5 for a total of eight cycles, with the last three batches given approximate ending inventory value based on their discharge composition and burnup. Table 7.1 shows the values of $\overline{TC}_I$ and $\overline{TC}_\alpha$ for the optimal case and the cases in which reload enrichment or batch fraction is varied. Figure 7.2 and Figure 7.3 show the value of $\overline{TC}_I$ and $\overline{TC}_\alpha$ plotted against ε_1 and f_1 respectively.

The accuracy of the partial derivatives on ε_1 is within $\pm$ 0.6%. The accuracy of partial derivatives on f_1 is within $\pm$ 0.9%. The accuracy of partial derivatives on ε_2, ε_3 ..and

Table 7.1

Exact and Approximate Revenue Requirements for Various Enrichments and Batch Fractions

Enrichment (w/o) ε_1	ε_2	ε_3	ε_4	ε_5	Batch Fraction f_1	f_2	f_3	f_4	f_5	Revenue Requirement Approximate (10^6\$)	Exact (10^6\$)
3.2	3.2	3.2	3.2	3.2	0.333	0.333	0.333	0.333	0.333	87.6426	93.5606
2.0	3.2	3.2	3.2	3.2	0.333	0.333	0.333	0.333	0.333	83.0706	88.9801
2.8	3.2	3.2	3.2	3.2	0.333	0.333	0.333	0.333	0.333	85.9778	91.8859
3.7	3.2	3.2	3.2	3.2	0.333	0.333	0.333	0.333	0.333	89.5319	95.4396
4.4	3.2	3.2	3.2	3.2	0.333	0.333	0.333	0.333	0.333	92.3067	98.2147
3.2	3.2	3.2	3.2	3.2	0.253	0.333	0.333	0.333	0.333	85.2932	91.1982
3.2	3.2	3.2	3.2	3.2	0.293	0.333	0.333	0.333	0.333	86.4709	92.3783
3.2	3.2	3.2	3.2	3.2	0.373	0.333	0.333	0.333	0.333	88.4142	94.3263

Enrichment Changes (w/o) $\Delta\varepsilon_1$	Revenue Requirement Changes (10^6\$) $\Delta\overline{TC}_I$	$\Delta\overline{TC}_\alpha$	$\Delta\overline{TC}_I/\Delta\varepsilon_1$ (10^6\$)	$\Delta\overline{TC}_\alpha/\Delta\varepsilon_1$ (10^6\$)	Error (%)
-1.200	-4.5720	-4.5804	3.8100	3.8169	+0.2
-0.434	-1.6648	-1.6746	3.8360	3.8586	+0.6
+0.480	+1.8893	+1.8791	3.9361	3.9148	-0.5
+1.200	+4.6642	+4.6542	3.8868	3.8785	-0.2

Batch Fraction Changes Δf_1	Revenue Requirement Changes (10^6\$) $\Delta\overline{TC}_I$	$\Delta\overline{TC}_\alpha$	$\Delta\overline{TC}_I/\Delta f_1$ (10^6\$)	$\Delta\overline{TC}_\alpha/\Delta f_1$ (10^6\$)	Error (%)
-0.8	-2.3494	-2.3623	2.9367	2.9528	+0.5
-0.4	-1.1717	-1.1822	2.9293	2.9554	+0.9
+0.4	+0.7716	+0.7658	1.9290	1.9146	-0.7

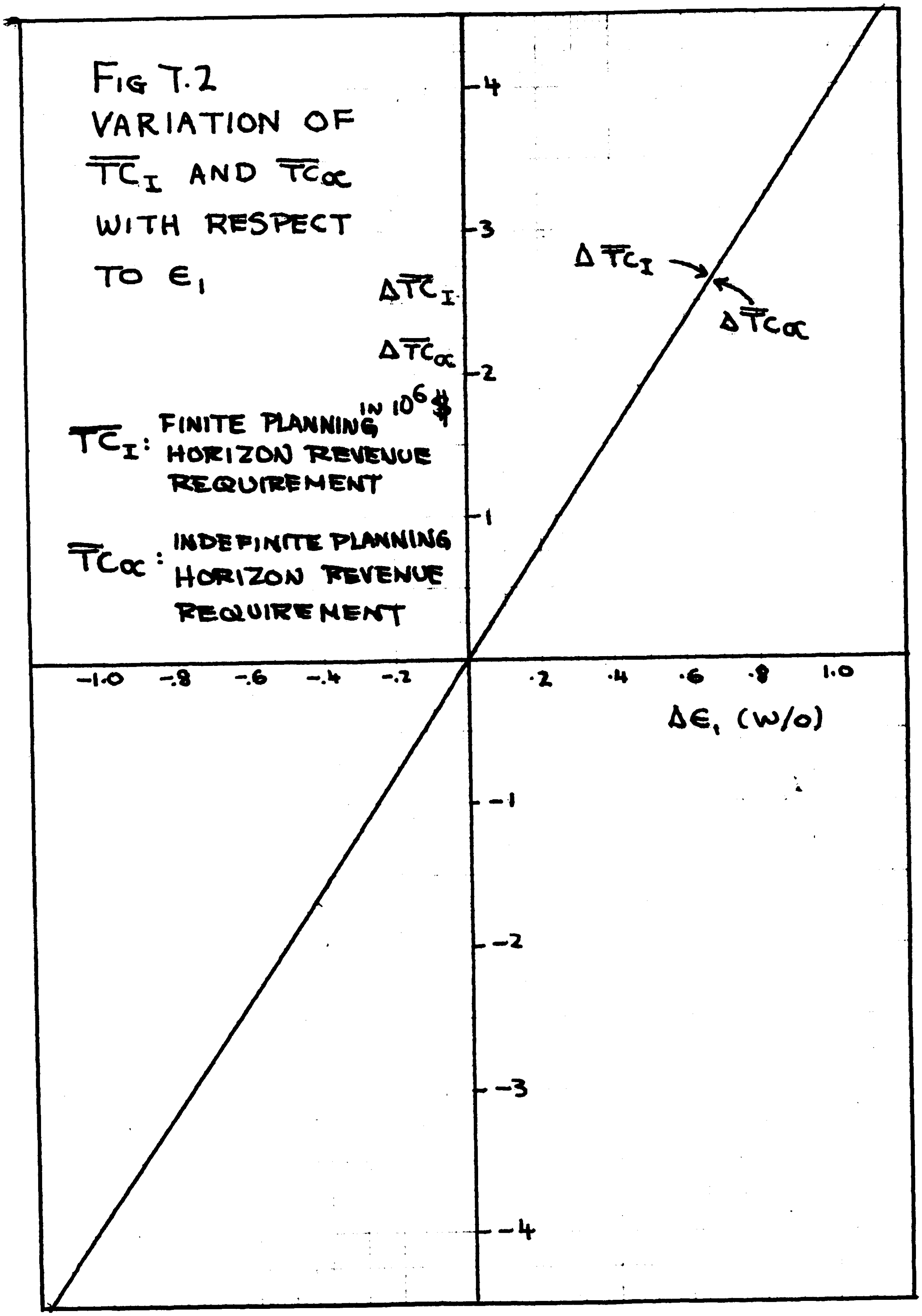


FIG T.2
VARIATION OF $\overline{TC}_I$ AND $\overline{TC}_\infty$ WITH RESPECT TO ϵ_1
$\Delta\overline{TC}_I$
$\Delta\overline{TC}_\infty$
IN 10^6 \$
$\overline{TC}_I$: FINITE PLANNING HORIZON REVENUE REQUIREMENT
$\overline{TC}_\infty$: INDEFINITE PLANNING HORIZON REVENUE REQUIREMENT
4
3
2
1
-1
-2
-3
-4
-1.0
-.8
-.6
-.4
-.2
.2
.4
.6
.8
1.0
$\Delta\epsilon_1$ (w/o)
$\Delta\overline{TC}_I$
$\Delta\overline{TC}_\infty$

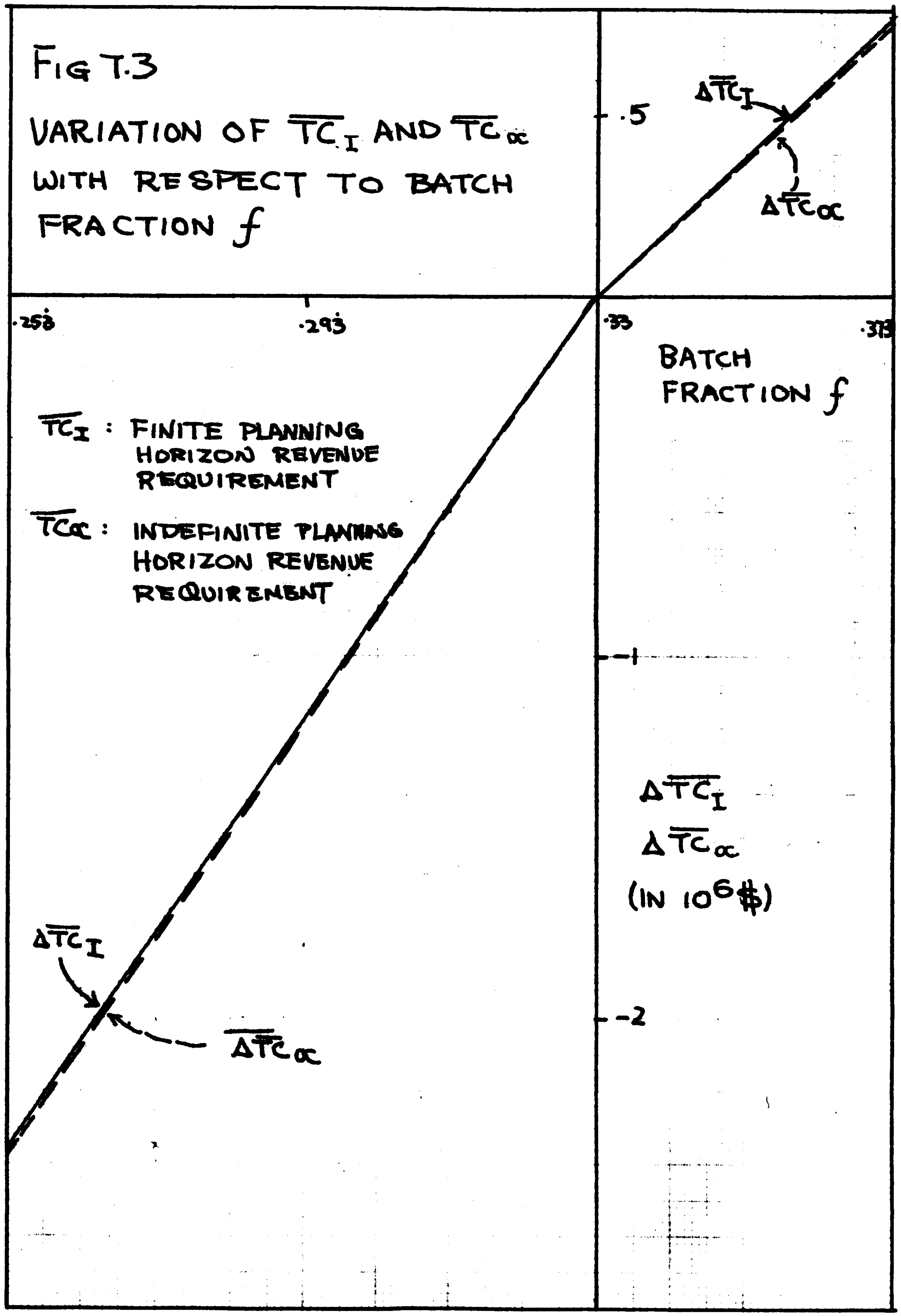


Fig T.3
VARIATION OF $\overline{TC}_I$ AND $\overline{TC}_\alpha$ WITH RESPECT TO BATCH FRACTION f
.5
$\Delta\overline{TC}_I$
$\Delta\overline{TC}_\alpha$
.253
.293
.33
.373
BATCH FRACTION f
$\overline{TC}_I$: FINITE PLANNING HORIZON REVENUE REQUIREMENT
$\overline{TC}_\alpha$: INDEFINITE PLANNING HORIZON REVENUE REQUIREMENT
-1
-2
$\Delta\overline{TC}_I$
$\Delta\overline{TC}_\alpha$
(IN 10^6\$)

$f_2 \ldots f_5$ would be progressively worse. This is due to the fact that the end state correction would have a larger effect on the subsequent cycles. However, this optimization would be repeated on an annual basis. Hence, it is only the first cycle results that would actually be utilized. Therefore, the main emphasis on accuracy would be placed on the first cycle derivatives.

This degree of accuracy is adequate for a survey type of calculation in which a large number of cases are analysed. After eliminating most of the sub-optimal cases, the exact objective function would then be used for the final optimization.

As a final test, the values of $\overline{TC}_I$ and $\overline{TC}_\alpha$ are calculated for a complicated case in which the reload enrichments and batch fractions are different for all the cycles. The difference of $\overline{TC}_I$ between this complicated case and the optimal base case in the preceding sections is compared to the same difference for $\overline{TC}_\alpha$. Table 7.2 shows the value of $\overline{TC}_I$ and $\overline{TC}_\alpha$ for the two cases. The discrepancy in this case is substantially larger. This is due to the fact that as enrichment and batch fraction changes take place near the end of the planning horizon, the end-effect correction would not be able to handle these changes accurately. Nevertheless, this serves the purpose of comparing $\overline{TC}_I$ and $\overline{TC}_\alpha$ under the worse possible situation.

Table 7.2

Exact and Approximate Revenue Requirement Calculated for the Base Case and the Case in which the Reload Enrichments and Batch Fractions for All the Cycles are Changed

	Cycles 1	2	3	4	5	Revenue Requirement Approximate (10^6 \$)	Exact (10^6 \$)
ϵ(w/o) / f / E(GWHt)							
Base Case							
ϵ	3.20	3.20	3.20	3.20	3.20		
f	0.333	0.333	0.333	0.333	0.333	87.6426	93.5605
E	22750.	22750.	22750.	22750.	22750.		
Changed Case							
ϵ	4.57	3.26	4.31	2.83	3.26		
f	0.293	0.373	0.253	0.253	0.293	90.2296	96.2674
E	25450.	30440.	21850.	19340.	20880.		
Absolute Change						2.5870	2.7069
Percentage Error						4.6%	

7.5 Conclusions

The nuclear in-core optimization problem is formulated as a non-linear optimization problem. Kuhn-Tucker conditions for optimium ε^* and f^* are derived. The accuracy of the approximate objective function $\overline{TC}_I$ is compared with an exact objective function $\overline{TC}_\infty$ under pre-specified end conditions.

The approximate objective function $\overline{TC}_I$ has been demonstrated to be adequate for a survey type of optimization aiming at eliminating a large number of sub-optimal cases.

CHAPTER 8.0

NUCLEAR IN-CORE OPTIMIZATION FOR NON-STEADY STATE BY METHOD OF PIECE-WISE LINEAR APPROXIMATION

8.1 Introduction

In principle, the Method of Linear Approximation consists of the following three steps;

(i) Linearization of the objective function and the constraints about a given feasible point.

(ii)Finding the steepest direction in which the objective function decreases most rapidly.

(iii)Choosing an increment step size and proceeding in this steepest direction.

The entire process is repeated at this new point until either all the derivatives of the objective function are zero or succeeding steps show no significant improvement over the previous steps. This method is useful when the objective function and the constraints are linear or quasi-linear. This method also assumes that an initial feasible solution is available.

8.2 The Optimization Algorithm

Starting from an initial feasible solution $\vec{\varepsilon}^{0}$and $\vec{f}^{0}$;

$$\overline{TC}^{0}= \overline{TC}(\vec{\varepsilon}^{0},\vec{f}^{0})$$

$$\left.\begin{aligned} E_c(\vec{\varepsilon}^{0},\vec{f}^{0})&=E_c^S \\ B_c(\vec{\varepsilon}^{0},\vec{f}^{0})&=B^{0} \end{aligned}\right\} \quad \text{for all c}$$

the objective function and the constraints are linearized about the neighborhood of $\vec{\varepsilon}^{0},\vec{f}^{0}$

$$\overline{TC}(\vec{\varepsilon},\vec{f}\ ) \simeq \overline{TC}^0+\sum_c\{\alpha_c(\varepsilon_c-\varepsilon_c^0)+\beta_c(f_c-f_c^0)\} \tag{8.1}$$

$$E_k(\vec{\varepsilon},\vec{f}) \simeq E_k(\vec{\varepsilon}^0,\vec{f}^0)+\sum_c\{\gamma_{kc}(\varepsilon_c-\varepsilon_c^0)+\delta_{kc}(f_c-f_c^0)\} \tag{8.2}$$

$$B_k(\vec{\varepsilon},\vec{f}) \simeq B_k(\vec{\varepsilon}^0,\vec{f}^0)+\sum_c\{\xi_{kc}(\varepsilon_c-\varepsilon_c^0)+\zeta_{kc}(f_c-f_c^0)\} \tag{8.3}$$

where the coefficients are represented by:

$$\alpha_c\equiv\frac{\partial\overline{TC}(\vec{\varepsilon}^0,\vec{f}^0)}{\partial\varepsilon_c}$$

$$\beta_c\equiv\frac{\partial\overline{TC}(\vec{\varepsilon}^0,\vec{f}^0)}{\partial f_c}$$

$$\gamma_{kc}\equiv\frac{\partial E_k(\vec{\varepsilon}^0,\vec{f}^0)}{\partial\varepsilon_c}$$

$$\delta_{kc}\equiv\frac{\partial E_k(\vec{\varepsilon}^0,\vec{f}^0)}{\partial f_c}$$

$$\xi_{kc}\equiv\frac{\partial B_k(\vec{\varepsilon}^0,\vec{f}^0)}{\partial\varepsilon_c}$$

$$\zeta_{kc}\equiv\frac{\partial B_k(\vec{\varepsilon}^0,\vec{f}^0)}{\partial f_c}$$

The expansion coefficients α_c,β_c etc. are determined by a number of variational cases, in which the variables ε_c, f_c are varied one at a time. For example,

$$\alpha_c\simeq\frac{\overline{TC}(\varepsilon_1^0\ldots\varepsilon_c^0+\Delta\varepsilon_c\ldots\vec{f}^0)-\overline{TC}(\varepsilon_1^0\ldots\varepsilon_c^0\ldots\vec{f}^0)}{\Delta\varepsilon_c} \tag{8.4}$$

$$\delta_{kc}\simeq\frac{E_k(\vec{\varepsilon}^0,f_1^0\ldots f_c^0+\Delta f_c\ldots)-E_k(\vec{\varepsilon}^0,f_1^0\ldots f_c^0\ldots\ldots)}{\Delta f_c} \tag{8.5}$$

Since Equations (8.1) (8.2) and (8.3) are valid only in the vicinity of $\vec{\varepsilon}^0$ and $\vec{f}^0$, $\Delta\varepsilon_c$ and Δf_c should be limited to small values, for example $\Delta f_c/f_c \leqslant 0.01$.

Linear programming can be applied to Equations (8.1) (8.2) and (8.3) to determine the next optimal point. By imposing the additional requirement that $|f_c-f_c^0|/f_c<0.01$, the next optimal point would be forced to lie inside the region in which the equations are valid.The objective function for this new optimal point is calculated, and compared with the previous objective function $\overline{TC}(\vec{\varepsilon}^{\,0},\vec{f}^{\,0})$. If the new solution is an improvement over the previous one, the entire procedure of linearization and optimization is repeated for this new point. Figure 8.1 is the flow chart of the Method of Linear Approximation. The iteration terminates when the new solution shows no improvement over the previous one.

Unfortunately, this method is not applicable to the situation in which batch fraction changes are restricted to large discrete values due to the special requirements in the depletion code CELL-CORE. The smallest batch fraction changes allowed by the code is $\Delta f/f$=12%. Over this large range of batch fraction changes, the linear approximation does not hold. Hence, the Method of Piece-Wise Linear Approximation is introduced, and this requires a seperate coefficient for positive or negative changes in the batch fraction. For example, if $(f_c-f_c^0)$ is positive, the expansion coefficients multiplying $(f_c-f_c^0)$ and $(\varepsilon_c-\varepsilon_c^0)$ in Equations (8.1) and (8.2) are $\alpha_c^+, \beta_c^+, \gamma_{kc}^+, \delta_{kc}^+$respectively. On the other hand, if $(f_c-f_c^0)$ is negative, the expansion coefficients are $\alpha_c^-, \beta_c^-, \gamma_{kc}^-, \delta_{kc}^-$ respectively. Definitions of the positive and negative coefficients are given in Table 8.1.

Figure 8.1

Flow Chart for Method of Linear Approximation

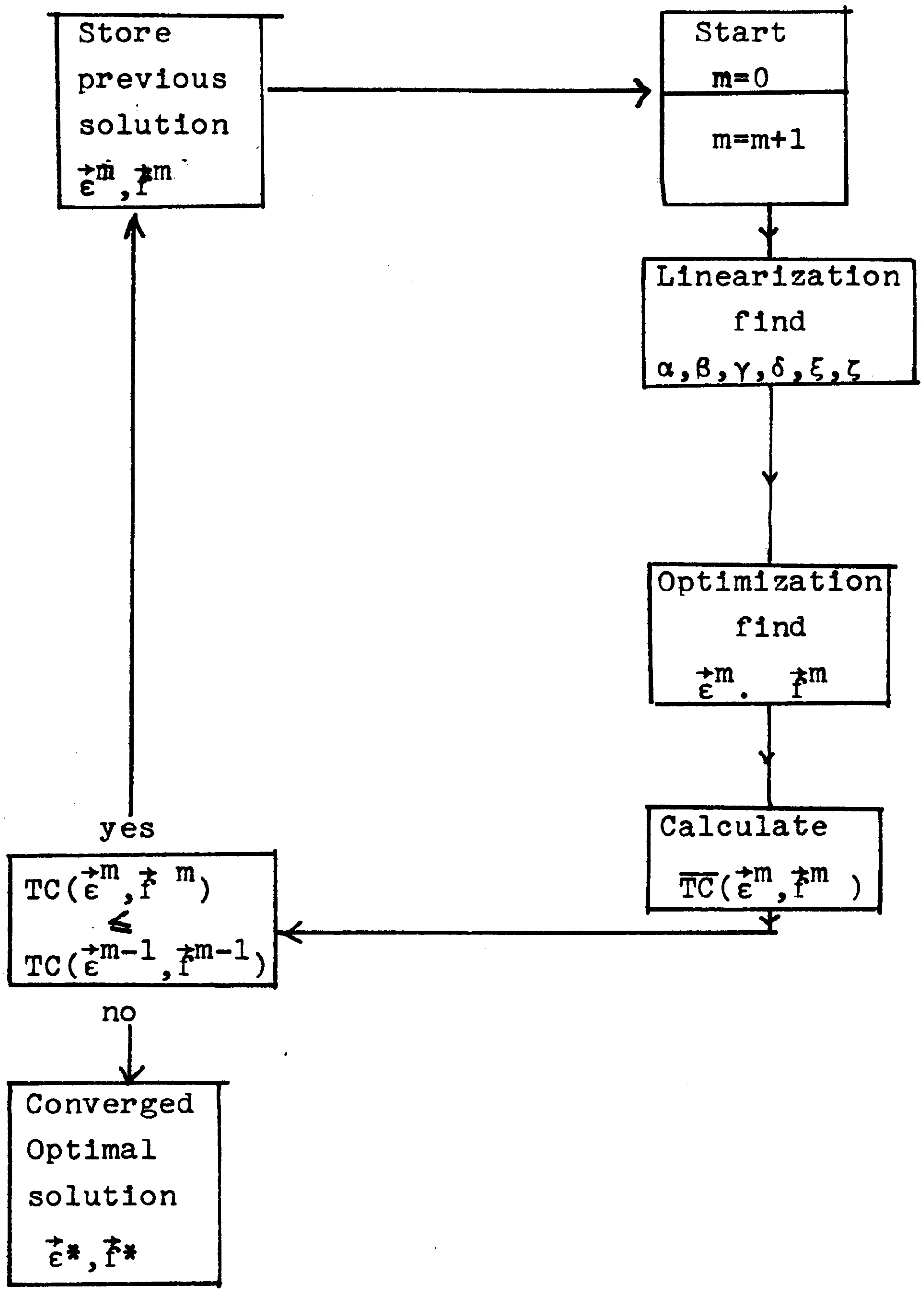


Table 8.1

Definitions of the Various Linear Expansion Coefficients

$$\alpha_c^+ \equiv \left.\frac{\Delta\overline{TC}}{\Delta\varepsilon_c}\right|_{\Delta f_c>0} = \frac{\overline{TC}(\varepsilon_1^0 \ldots\ldots \varepsilon_c^0+\Delta\varepsilon_c \ldots f_1^0 \ldots f_c^0+\Delta f_c \ldots) - \overline{TC}(\varepsilon_1^0 \ldots \varepsilon_c^0 \ldots f_1^0 \ldots f_c^0+\Delta f_c \ldots)}{\Delta\varepsilon_c}$$

$$\alpha_c^- \equiv \left.\frac{\Delta\overline{TC}}{\Delta\varepsilon_c}\right|_{\Delta f_c<0} = \frac{\overline{TC}(\varepsilon_1^0 \ldots\ldots \varepsilon_c^0+\Delta\varepsilon_c \ldots f_1^0 \ldots f_c^0+\Delta f_c \ldots) - \overline{TC}(\varepsilon_1^0 \ldots \varepsilon_c^0 \ldots f_1^0 \ldots f_c^0+\Delta f_c \ldots)}{\Delta\varepsilon_c}$$

$$\beta_c^+ \equiv \left.\frac{\Delta\overline{TC}}{\Delta f_c}\right|_{\Delta f_c>0} = \frac{\overline{TC}(\vec{\varepsilon}^{\,0}, f_1^0 \ldots f_c^0+\Delta f_c \ldots) - \overline{TC}(\vec{\varepsilon}^{\,0}, f_1^0 \ldots f_c^0 \ldots)}{\Delta f_c}$$

$$\beta_c^- \equiv \left.\frac{\Delta\overline{TC}}{\Delta f_c}\right|_{\Delta f_c<0} = \frac{\overline{TC}(\vec{\varepsilon}^{\,0}, f_1^0 \ldots f_c^0+\Delta f_c \ldots) - \overline{TC}(\vec{\varepsilon}^{\,0}, f_1^0 \ldots f_c^0 \ldots)}{\Delta f_c}$$

$$\gamma_{kc}^+ \equiv \left.\frac{\Delta E_k}{\Delta\varepsilon_c}\right|_{\Delta f_c>0} = \frac{E_k(\varepsilon_1^0 .. \varepsilon_c^0+\Delta\varepsilon_c .. f_1^0 \ldots f_c^0+\Delta f_c \ldots) - E_k(\varepsilon_1^0 \ldots \varepsilon_c^0 \ldots f_1^0 .. f_c^0+\Delta f_c \ldots)}{\Delta\varepsilon_c}$$

$$\gamma_{kc}^- \equiv \left.\frac{\Delta E_k}{\Delta\varepsilon_c}\right|_{\Delta f_c<0} = \frac{E_k(\varepsilon_1^0 .. \varepsilon_c+\Delta\varepsilon_c .. f_1^0 \ldots f_c^0+\Delta f_c \ldots) - E_k(\varepsilon_1^0 \ldots \varepsilon_c^0 \ldots f_1^0 .. f_c^0+\Delta f_c \ldots)}{\Delta\varepsilon_c}$$

$$\delta_{kc}^+ \equiv \left.\frac{\Delta E_k}{\Delta f_c}\right|_{\Delta f_c>0} = \frac{E_k(\vec{\varepsilon}^{\,0}, f_1^0 \ldots f_c^0+\Delta f_c \ldots) - E_k(\vec{\varepsilon}^{\,0}, f_1^0 \ldots f_c^0 \ldots)}{\Delta f_c}$$

$$\delta_{kc}^- \equiv \left.\frac{\Delta E_k}{\Delta f_c}\right|_{\Delta f_c<0} = \frac{E_k(\vec{\varepsilon}^{\,0}, f_1^0 \ldots f_c^0+\Delta f_c \ldots) - E_k(\vec{\varepsilon}^{\,0}, f_1^0 \ldots f_c^0 \ldots)}{\Delta f_c}$$

Using this more complicated form of the equations, step size $\Delta f_c/f_c$ up to 12% can be handled at the expense of doubling the number of coefficients to be calculated.

The equations involving average discharge burnup Equation (8.3) however, do not require additional elaboration. The following approximation for burnup is accurate within ±5% of the actual value.

$$B_b(\vec{\varepsilon},\vec{f}) \simeq B(n_b)\times(1+\xi(n_b)\times(\varepsilon_b - c^0)) \qquad (8.6)$$

where n_b: is the number of cycles of irradiation before discharge for batch b

ε_b: reload enrichment for batch b

$B(n_b)$: average discharge burnup for a 3.2w/o enriched batch which remains in the reactor for n_b cycles before discharge. For the Zion type 1065 MWE PWR, typical values of $B(n_b)$ are

$B(3)=31.5$ MWD/Kg

$B(4)=38.6$ MWD/Kg

$B(5)=44.3$ MWD/Kg

$\xi(n_b)$ a constant multiplying the enrichment of batch b For the Zion type 1065 MWE PWR, typical values of $\xi(n_b)$ are

$\xi(3)=0.34$

$\xi(4)=0.21$

$\xi(5)=0.23$

c^0: a dummy constant equal to steady state refuelling enrichment (w/o). For the Zion type 1065 MWe PWR, the value of c^0 is 3.2.

The various values for $B(n_b)$, $\xi(n_b)$, $c^{\bullet}$ are determined by multiple regression ananlysis based on a large number of burnup data points. Equation (8.6) together with the modified form of Equations (8.1) and (8.2) cannot be solved by Linear Programming. They are solved by exhaustive search, in which all possible combinations of f_c are calculated. Since the equations are valid over a small region, and the depletion code CELL-CORE only allows discrete charges in batch fraction, there is a finite number of combinations of f_c. If the batch fractions are restricted to vary by one mesh size at a time, there are 3^m combinations for an m-cycle problem. A five-cycle problem consists of 243 possible combinations of f_c. The corresponding ε_i can be calculated by Equation (8.2). Finally ε_i and f_i can be substituted into Equations (8.1) and (8.6) to solve for the objective function and the discharge burnup. Only those cases which satisfy the burnup constraint are retained.

Finally, the objective function of all the feasible cases are compared, and the new solution for this step is found. The entire process of linearization and exhaustive search is repeated for this new solution.

8.3 Results for Sample Case A with No Income Tax

The reactor under analysis is the Zion type 1065 MWe PWR, with initial condition equivalent to the 3.2 w/o three-zone modified scatter refuelled steady state condition.

The planning horizon consists of five cycles. Energy requirement for each of the five cycles is 22750 GWHt. The maximum allowable average burnup is 60 MWD/Kg. The Method of Steepest Descent is applied to solve for the optimal reload enrichments and batch fractions for the five cycles.

The objective function for this problem consisted of revenue requirements for eight batches in the five cycle planning horizon. Income tax rate is zero in this case. For the more general case where there are income taxes, refer to Section 8.4 or Chapter 9. Figure 8.2 shows the relationships between the objective function, batches and cycle. The objective function calculation is based on economics environment similar to that of a government operated utility which does not have to pay income tax.(refer to Appendix B) The Nuclide Value method given in Section 4.3.1 is used to evaluate end effects. Since there is no depreciation tax credit for this case, future disposition of each sublot of fuel remaining in the reactor core does not affect the objective function. However, according to Equation (8.6) the future disposition of each sublot of fuel must be known before one can estimate the discharge burnup. For this case alone, the assumption is made that the reactor would be refuelled with f = 0.253 for all subsequent cycles after the planning horizon.

Table 8.1 shows the result of the optimization. Table 8.2 shows the average discharge burnup for the various

FIG 8.2 RELATIONSHIPS BETWEEN $\overline{TC}$, BATCH AND CYCLE

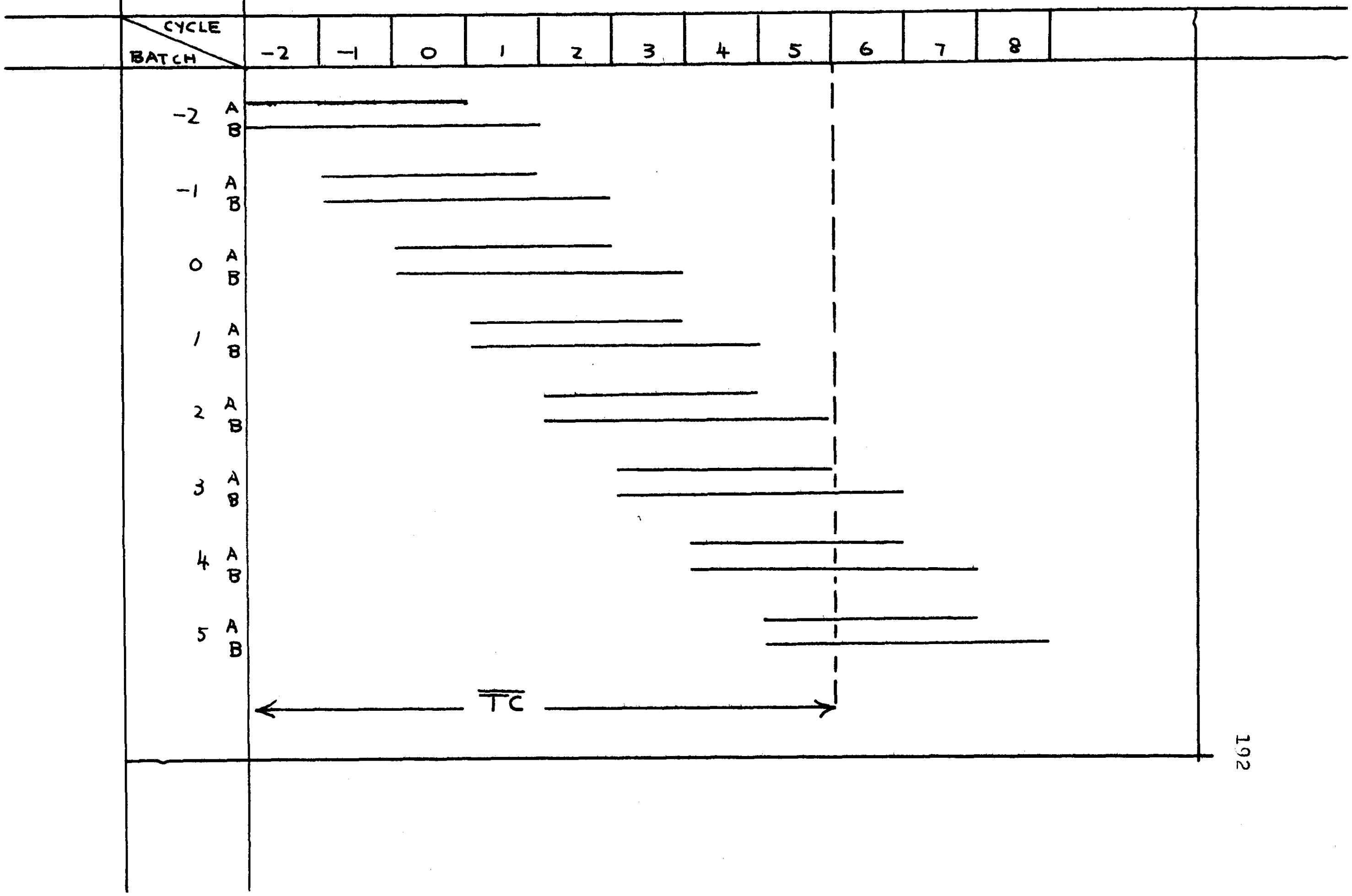


Table 8.1

Reload Enrichments, Batch Fractions, Cycle Energies and Revenue Requirements for the Various Number of Iterations Using the Method of Piece-wise Linear Approximation

		Cycle					Revenue Requirement (10^6 \$)			
		1	2	3	4	5	For Actual Energy		Corrected for Target Energy	
	ε(w/o) f E(GWHt)						Piece-wise Linear Approximation	CELL COCO	Piece-wise Linear Approximation	CELL COCO
Target Energy		22750.	22750.	22750.	22750.	22750.				
Iteration Number										
0	ε	3.2	3.2	3.2	3.2	3.2				
	f	0.333	0.333	0.333	0.333	0.333	72.1119	72.1119	72.1119	72.1119
	E	22750.	22750.	22750.	22750.	22750.				
1	ε	3.77	3.37	3.45	3.56	3.42				
	f	0.293	0.293	0.293	0.293	0.293	71.3358	71.1517	71.4971	71.3131
	E	22257.	22725.	22616.	23076.	22769.				
2	ε	5.03	3.03	4.27	2.96	4.57				
	f	0.253	0.253	0.253	0.253	0.253	70.3096	70.5269	70.4969	70.7141
	E	22697.	22534.	22844.	22430.	22646.				
3	ε	3.95	4.25	4.64	4.31	3.61				
	f	0.293	0.253	0.213	0.213	0.213	70.0805	70.4763	70.2485	70.6443
	E	22986.	23133.	22325.	23894.	21253.				

Table 8.2

Average Discharge Burnup for the Sublot Experiencing the Highest Exposure for Sample Case A

Calculated by (1) Piece-wise Linear Approximation

(2) CELL-CORE Depletion Calculation

Burnup in MWD/kg

Iteration Number	Method	Batch Number -2	-1	0	1	2	3	4	5
		MWD/Kg							
0	(1)	31.5	31.5	31.5	31.5	31.5	31.5	31.5	31.5
	(2)	31.5	31.5	31.5	31.5	31.5	31.5	31.5	31.5
1	(1)	38.6	38.6	38.6	43.2	40.0	40.7	41.5	40.4
	(2)	38.9	38.6	38.7	43.8	39.6	—	—	—
2	(1)	38.6	38.6	38.6	52.8	37.3	46.9	36.7	49.2
	(2)	38.6	38.1	38.3	54.1	34.4	—	—	—
3	(1)	38.6	38.6	44.3	51.5	54.4	58.1	54.9	48.3
	(2)	39.0	38.5	45.0	52.2	—	—	—	—

batches. The optimal solution consists of the smallest possible batch fraction permitted by the discharge burnup constraint. Further savings in excess of $1.6 million could be realized if a higher discharge burnup were allowed.

8.4 Results for Sample Case A with Income Tax

If income tax of 50% is included in the calculation of the objective function, the Method of Piece-Wise Linear Approximation fails to produce an optimal solution. Table 8.3 shows the results for two iterations. The actual revenue requirement corrected for target energy increased in the second iteration. Hence, the method fails to produce a better solution.

This failure is due to the fact that the objective function for this particular case is very flat when income taxes is included. Moreover, the base case is so close to the optimal solution that the Method of Piece-Wise Linear Approximation based on first order derivatives cannot give good estimate of the trends. Hence, higher order approximation is required for optimization in this situation.

8.5 Conclusions

The Method of Piece-Wise Linear Approximation is simple and straight forward. However, the energy constraints are only approximately satisfied. This is particularly true when optimal solution is far away from the cases in which the expansion coefficients are determined.

Table 8.3

Reload Enrichments, Batch Fractions, Cycle Energies and Revenue Requirements with Income Taxes for the Various Number of Iteration Using the Method of Piece-wise Linear Approximation

Iteration Number		Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5	Revenue Requirement (10^6 \$) For Actual Energy: Piece-wise Linear Approximation	For Actual Energy: CELL COCO	Corrected for Target Energy: Piece-wise Linear Approximation	Corrected for Target Energy: CELL COCO
	ϵ(w/o) f E(GWHt)									
	Target Energy	22750.	22750.	22750.	22750.	22750.				
0	ϵ	3.2	3.2	3.2	3.2	3.2				
	f	0.33	0.33	0.33	0.33	0.33	87.2407	87.2407	87.2407	87.2407
	E	22750.	22750.	22750.	22750.	22750.				
1	ϵ	3.77	2.77	3.29	3.88	2.67				
	f	0.29	0.33	0.33	0.29	0.33	86.4105	86.6273	87.1015	87.3183
	E	22257.	22384.	22618.	22259.	22422.				

This method is useful for cases where the objective function has a wide variation over the range of the decision variables (as in this no income tax case) and where the optimal solution is ultimately limited by one or more of the constraints. However, if the objective function is rather flat, as in this case with income tax, and the constraints are not active, the Method of Piece-Wise Linear Approximation cannot pin-point the optimal solution precisely.

CHAPTER 9.0

NUCLEAR IN-CORE OPTIMIZATION FOR NON-STEADY STATE BY METHOD OF POLYNOMIAL APPROXIMATION

9.1 Introduction

In Chapter 7, the problem of nuclear in-core optimization was formulated in terms of finding the combination of reload enrichments and batch fractions such that the energy and burnup constraints are satisfied and the objective function minimized. However, experience has shown that the energy constraints are difficult to satisfy accurately (within $\pm 1\%$). As a result, the objective function calculated for a particular combination of reload enrichments and batch fractions has to be corrected for this error in meeting the energy constraints. However, the objective function has been found to vary smoothly with energy and batch fraction. This well-behaved property of the objective function can be exploited to solve the foregoing problem by incorporating the dependent variable, cycle energy, directly in the objective function and eliminating the the decision variable, reload enrichment, altogether. The corresponding mathematical transformations are given below. Repeating Equations (7.1), (7.2), and (7.3)

$$\text{Minimize } \overline{TC}(\vec{\varepsilon},\vec{f},\psi) \text{ with respect to } \vec{\varepsilon},\ \vec{f}\ . \tag{7.1}$$

Subject to constraints

$$E_c(\vec{\varepsilon},\vec{f}) = E_c^s \tag{7.2}$$

$$B_c(\vec{\varepsilon},\vec{f}) < B^0 \tag{7.3}$$

Equation (7.2) can be inverted to yield

$$\vec{\varepsilon} = \vec{g}(\vec{E}^s,\vec{f}) \tag{9.1}$$

which can be substituted into (7.1) and (7.3) to give

$$\text{minimize } \overline{TC}(\vec{E}^s,\vec{f}) \text{ with respect to } \vec{f} \tag{9.2}$$

subject to constraints

$$B_c(\vec{E}^s, \vec{f}) < B^0 \qquad (9.3)$$

Since $\vec{E}^s$ are specified energy requirements, the decision variables in this problem are only the batch fractions. Since the initial state ψ is the same in all cases considered, it has been omitted from Equation (9.2).

The functional form of Equations (9.2) and (9.3) cannot be derived from theory. However, it can be approximated by fitting polynomials in $\vec{E}^s$ and $\vec{f}$ to a large set of data with the same initial condition ψ. The analytic expressions that result from multiple regression analysis can then be optimized by conventional techniques. Section 9.3 describes how the polynomials are chosen. Sections 9.4 and 9.5 present the results of the regression analysis. Before that, there are some brief comments about the objective function and the end conditions.

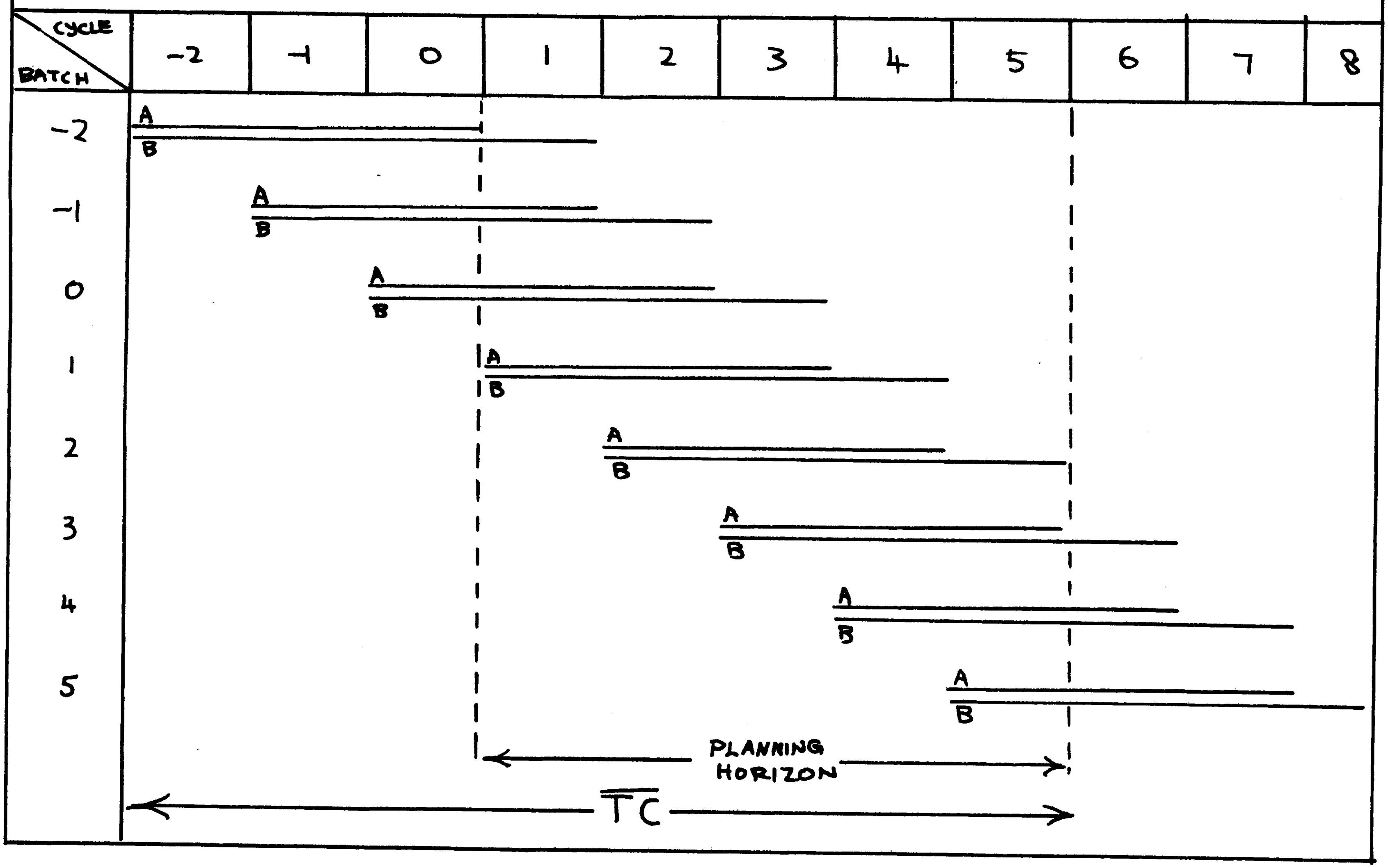
9.2 Brief Comments About the Objective Function and the End Conditions

The objective function $\overline{TC}$ is defined in Chapter 7 (Equation 7.1) and represents the revenue requirement for all the batches involved in the operation of the reactor in the planning horizon. For those batches that remain in the reactor after the end of the time horizon, the end values are evaluated by the Inventory Value Method as outlined in Chapter 4. For a typical five-cycle problem, the relationships between objective function, batches andcycles are given on Figure 9.1.

However, in order to arrive at a value for the tax depreciation credit and discharge burnups for all the fuel batches,

FIGURE 9.1

RELATIONSHIPS BETWEEN REVENUE REQUIRE. BATCH NUMER AND CYCLE NUMBER



end conditions have to be specified in the last three cycles after the planning horizon. The end conditions are specified in terms of the reload batch fractions for the sixth, seventh and eighth cycles. The choice for these batch fractions could be arbitrary, but choice based on realistic assumptions on the future operation mode of the reactor in these cycles could minimize this error due to truncation of the time horizon. However, if the wrong choice is made the optimization would be affected, at most for the last three cycles of the planning horizon.

The optimum batch fractions for the first two cycles in the planning horizon would not be affected. Since this optimization problem would be updated annually, this error would not cause any great difficulty. For the sample cases analyzed in this chapter, the reload batch fractions for Cycles 6, 7, and 8 would be 0.253 throughout. This choice is based on the fact that f = 1/4 is the optimal batch fraction for the steady state case if the burnup limitation is 45 MWD/kg and cycle energy requirement ranges from 7000 GWHe to 9000 GWHe.

9.3 Choice of the Polynomials

The following behaviors are observed when the objective function varies over energy and batch fraction .

(1) Objective function increases as more energy is produced.

(2) Objective function increases as batch fraction increases.

(3) Objective function increases as enrichment increases even as energy production is kept the same.

(4) Objective function increases when batch fractions vary greatly from cycle to cycle.

When the batch fraction changes, inefficiencies are introduced, such as discharging fuel lots which are not yet fully depleted, and retaining fuel lots which are over depleted. Inefficiencies like these would not take place in a constant batch fraction process in which fuel batches are discharged at nearly the same burnup.

Based on the foregoing observations, the following functional form of the objective function is constructed.

$$\overline{TC} = \sum_c \alpha_c E_c + \sum_c \beta_c f_c + \sum_c \gamma_c \frac{E_c}{f_c} + \sum_c \delta_c (f_c - f_{c-1})^2 \quad (9.4)$$

The first term represents the linear change of objective function due to energy changes. The second term represents the linear change of objective function due to batch fraction changes. The third term represents the linear change of objective function due to enrichment changes. Energy production is found to be directly related to fissile content of the core. At the same time, fissile content is directly related to reload enrichment times the batch size. Hence, reload enrichment can be approximated as proportional to energy divided by batch fraction. The last term of Equation (9.4) represents the linear change of the objective function due to the absolute variation of batch fraction from cycle to cycle. While Equation (9.4) was a fairly accurate representation of the objective function, a more accurate, more complex equation involving 18 terms was used, which resulted in a multiple correlation coefficient of 0.99891 and a standard error of estimate of 0.0774 million dollars. The equation for this more complex objective function is given in Table 9.1.

The burnup constraint Equation (9.3) could be represented adequately by Equation (8.6) for an error band of ±5%. This band width is considered adequate for an inequality constraint. Lacking information on peak discharge burnup, refinement in accuracy in predicting average discharge burnup is not warranted. However, Equation (8.6) involves reload enrichment as one of its independent variables. Reload enrichment evaluated as a function of cycle energy and batch fraction has to be obtained in order to use Equation (8.6). Following the argument that reload enrichments are related to cycle energies divided by batch fractions, a set of polynomials was constructed around this argument. The regression equations for all the reload enrichments are given in Tables 9.2 to 9.6. These equations are used exclusively for the calculation of average discharge burnup. In no way does the accuracy of these equations affect the objective function.

Figure 9.2 is a plot of the standard estimate of error versus cycle number. The curve represents the results of regression analysis of cases having batch fraction ranges from $0.2\dot{5}3$ to $0.3\dot{7}3$.

On the same figure, the actual observed error in enrichment is plotted. Most of the data points lie within 10% of the actual enrichment. Since in (8.6), the estimated burnup is represented as a linear function of enrichment, the effect of 10% error in enrichment is equivalent to a 10% error in

Table 9.1

Regression Equation for Revenue Requirement

$$
\begin{aligned}
\overline{TC} = 87.240720 + 0.06551 &+ 2.28342\times(E_1-E_{01}) \\
&+ 4.40931\times(E_2-E_{02}) \\
&+ 2.26829\times(E_3-E_{03}) \\
&+ 2.47006\times(E_4-E_{04}) \\
&+ 2.46467\times(E_5-E_{05}) \\
&+ 1.13642\times(f_1-f_{01}) \\
&+ 0.81828\times(f_3-f_{03}) \\
&+ 0.64499\times(f_4-f_{04}) \\
&+ 1.30984\times(E_1^3/f_1^2-E_{01}^3/f_{01}^2) \\
&+ 0.94908\times(E_4^2/f_4^2-E_{04}^2/f_{04}^2) \\
&+ 0.76090\times(E_3^3/f_3^2-E_{03}^3/f_{03}^2) \\
&+ 0.20903\times(E_5^3/f_5^2-E_{05}^3/f_{05}^2) \\
&- 5.27670 \\
&+ 0.48486\times(f_1-3.333)^2 \\
&+ 0.15590\times(f_3-f_2)^2 \\
&+ 0.13438\times(f_4-f_3)^2 \\
&+ 0.22128\times(f_5-f_4)^2 \\
&+ 0.07579\times(f_5-f_2)^2
\end{aligned}
$$

Constants in Regression Equation

i	E_{0i}	f_{0i}
1	2.275	3.333
2	2.275	3.333
3	2.275	3.333
4	2.275	3.333
5	2.275	3.333

UNITS

$\overline{TC}$	Revenue Requirement in 10^6\$
E_i	Energy for Cycle i in 10^4GWHt
f_i	10×Batch Fraction for Cycle i

Statistical Properties of Regression Equation

Correlation Coefficient $\rho=0.99891$

F Value $F=3191.$

Standard Estimate of Error$=0.07740\cdot 10^6$\$

Table 9.2

Regression Equation for Enrichment for Cycle 1

$$
\begin{aligned}
\varepsilon_1 = -1.53588 &+ 2.84647 \times E_1 \\
&+ 16.85454 / f_1 \\
&- 18.28799 \times E_1 / f_1 \\
&+ 46.35364 \times E_1 / f_1^2 \\
&- 44.67946 \times / f_1^2 \\
&+ 0.22981 \times E_1^3 / f_1^2
\end{aligned}
$$

UNITS

ε_i Enrichment for Cycle i in (w/o)

E_i Energy for Cycle i in 10^4 GWHt

f_i 10×Batch Fraction for Cycle i

Statistical Properties of Regression Equation

Correlation Coefficient $\rho=0.99969$

F Value $F=29637.$

Standard Estimate of Error $=0.02115$ w/o

Table 9.3

Regression Equation for Enrichment for Cycle 2

$$
\begin{aligned}
\varepsilon_2 = 0.97686 &+ 1.28069\times E_2 \\
&- 10.71479\times E_2/f_2 \\
&+ 34.56815\times E_2/f_2^2 \\
&- 14.59538\ /f_2^2 \\
&+ 0.14029\times E_2^3\ /f_2^2 \\
&- 62.52820\times E_1^2\ /(f_1^3\times f_2^2) \\
&+ 1.31300\ /f_1 \\
&+ 1.12580\times E_2\times f_1/(E_1\times f_2)
\end{aligned}
$$

UNITS

ε_i Enrichment for Cycle i in (w/o)

E_i Energy for Cycle i in 10^4 GWHt

f_i 10×Batch Fraction for Cycle i

Statistical Properties of Regression Equation

Correlation Coefficient $\rho=0.99805$

F Value $F=7741.$

Standard Estimate of Error=0.05428 w/o

Table 9.4

Regression Equation for Enrichment for Cycle 3

$$
\begin{aligned}
\varepsilon_3 = 2.00669 & + 1.23508 \times E_3 \\
& - 11.87128 \times E_3/f_3 \\
& + 34.89198 \times E_3/f_3^2 \\
& - 18.24043/f_3^2 \\
& + 0.17167 \times E_3^3/f_3^2 \\
& - 61.54253 \times E_2^2/(f_2^3 \times f_3^2) \\
& - 2.25149/f_2 \\
& + 1.83274 \times E_3 \times f_2/(E_2 \times f_3) \\
& + 20.63379 \times E_1^2/(f_1^3 \times f_3^2) \\
& + 37.85361 \times E_1^2/(f_1^3 \times f_2^2) \\
& - 3.60271/f_1 \\
& + 0.77057 \times E_2 \times f_1/(E_1 \times f_2)
\end{aligned}
$$

UNITS

ε_i Enrichment for Cycle i in (w/o)

E_i Energy for Cycle i in 10^4 GWHt

f_i 10×Batch Fraction for Cycle i

Statistical Properties of Regression Equation

Correlation Coefficient $\rho = 0.99651$

F Value $F = 4106.$

Standard Estimate of Error $= 0.06838$ w/o

Table 9.5

Regression Equation for Enrichment for Cycle 4

$$
\begin{aligned}
\varepsilon_4 = 2.86942 &+ 1.42475\times E_4 \\
&- 6.98729\times E_4/f_4 \\
&+29.96323\times E_4/f_4^2 \\
&-11.06240/f_4^2 \\
&-52.88219\times E_3^2/(f_3^3\times f_4^2) \\
&- 0.37538\times E_4\times f_3/(E_3\times f_4) \\
&+10.15228\times E_2^2/(f_2^3\times f_4^2) \\
&+29.75157\times E_2^2/(f_2^3\times f_3^2) \\
&-1.53779\times E_3\times f_2/(E_2\times f_3) \\
&-22.62199\times E_1^2/(f_1^3\times f_3^2) \\
&-28.70589\times E_1^2/(f_1^3\times f_2^2) \\
&+ 4.00576/f_1 \\
&- 1.72619\times E_1\times f \quad (E_2\times f_1)
\end{aligned}
$$

UNITS

ε_i Enrichment for Cycle i in(w/o)

E_i Energy for Cycle i in 10^4 GWHt

f_i 10×Batch Fraction for Cycle i

Statistical Properties of Regression Equation

Correlation Coefficient $\rho=0.99235$

F Value $F=1828.8$

Standard Estimate of Error=0.07854 w/o

Table 9.6

Regression Equation for Enrichment for Cycle 5

$$
\begin{aligned}
\varepsilon_5 = 0.43445 &+ 1.35606\times E_5 \\
&-11.21701\times E_5/f_5 \\
&+35.92697\times E_5/f_5^2 \\
&-19.04411/f_5^2 \\
&-48.65585\times E_4^2/(f_4^3\times f_5^2) \\
&+ 1.41909\times E_5\times f_4/(E_4\times f_5) \\
&+ 6.50443\times E_3^2/(f_3^3\times f_5^2) \\
&+24.18600\times E_3^2/(f_3^3\times f_4^2) \\
&-14.66613\times E_2^2/(f_2^3\times f_4^2) \\
&-25.58575\times E_2^2/(f_2^3\times f_3^2) \\
&+ 9.66152\times E_1^2/(f_1^3\times f_3^2) \\
&+29.41844\times E_1^2/(f_1^3\times f_2^2) \\
&- 3.47285/f_1 \\
&+ 1.56183\times E_2\times f_1/(E_1\times f_2)
\end{aligned}
$$

UNITS

ε_i Enrichment for Cycle i in (w/o)

E_i Energy for Cycle i in 10^4 GWHt

f_i 10×Batch Fraction for Cycle i

Statistical Properties of Regression Equation

Correlation Coefficient ρ=0.98721

F Value F=1268.

Standard Estimate of Error=0.09459 w/o

average discharge burnup. Comparisons of actual and predicted average discharge burnup will be presented later in Tables 9.8 and 9.10.

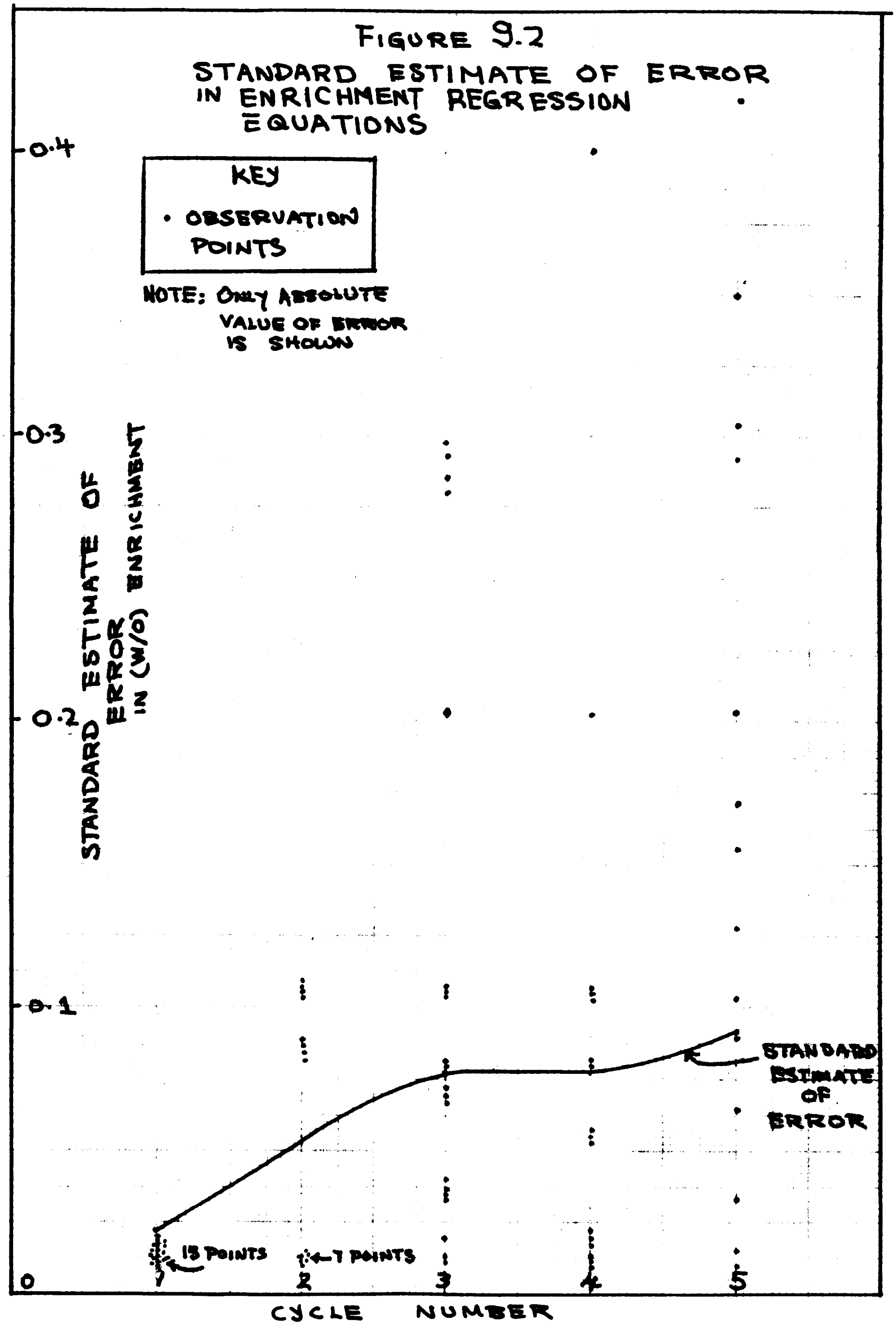


FIGURE 9.2
STANDARD ESTIMATE OF ERROR IN ENRICHMENT REGRESSION EQUATIONS
KEY
• OBSERVATION POINTS
NOTE: ONLY ABSOLUTE VALUE OF ERROR IS SHOWN
STANDARD ESTIMATE OF ERROR IN (W/O) ENRICHMENT
0.4
0.3
0.2
0.1
0
1
2
3
4
5
13 POINTS
7 POINTS
STANDARD ESTIMATE OF ERROR
CYCLE NUMBER

9.4 Regression Analysis on Objective Function

The equation given on Table 9.1 is the result of analyzing 135 separate cases. This equation predicts the objective function for any selection of cycle energy E_c and batch fraction f_c to an accuracy of $\pm 0.1\%$ of the true value. Other independent tests besides the regression analysis have been performed to confirm this result. Using this representation of the objective function, an analysis of its sensitivity to changes in cycle energy E_1 or batch fraction f_1 can be made.

Figure 9.3 shows the variation of $\overline{TC}$ due to changes in E_1 for different values of f_1 holding $f_2=f_3=f_4=f_5=0.33$ and $E_2=E_3=E_4=E_5=22750$GWHt. The behavior of the objective function in the non-steady state is very similar to that of the steady state (ref. to Figure 6.1). The objective function for a smaller batch fraction increases more rapidly with energy than that for a larger batch fraction. This is due mainly to the disproportionate increase of uranium and plutonium depletion cost.

The many cross-overs between lines of different batch fractions imply that the optimal batch fraction for any given level of cycle energy increases as cycle energy increases. This trend is again similar to that in the steady-state results.

Figure 9.4, which shows the variation of the objective function with respect to batch fraction for cycle 1 for different levels of cycle energy E_1 holding all the other f's and E's at the steady-state 3.2 w/o, 1/3 batch fraction level, is another way of plotting the data shown in Figure 9.3. The

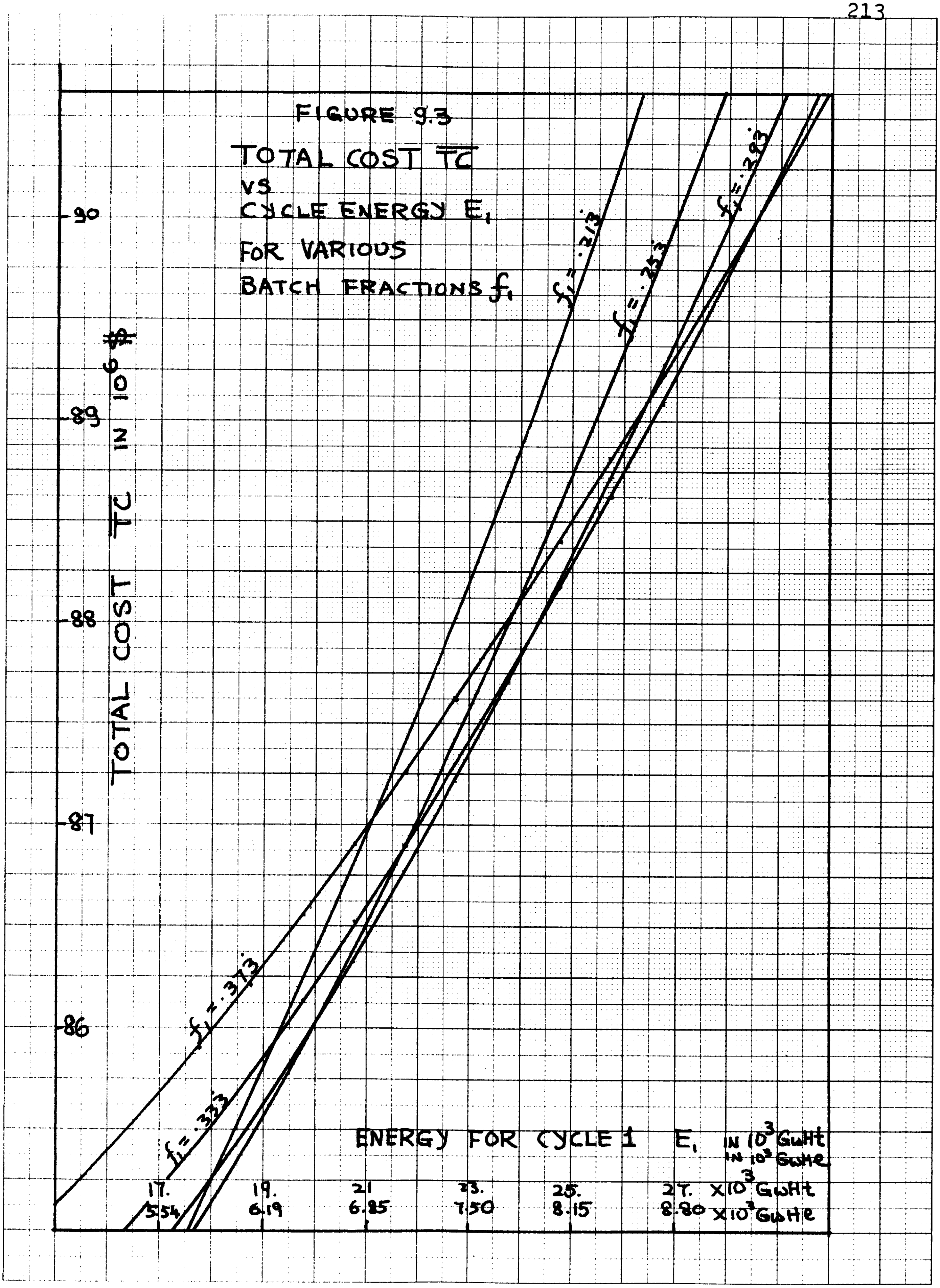


FIGURE 9.3
TOTAL COST $\overline{TC}$
VS
CYCLE ENERGY E_1
FOR VARIOUS
BATCH FRACTIONS f_1
TOTAL COST $\overline{TC}$ IN 10^6 \$
90
89
88
87
86
$f_1 = .313$
$f_1 = .353$
$f_1 = .293$
$f_1 = .373$
$f_1 = .333$
ENERGY FOR CYCLE 1 E_1 IN 10^3 GWHt
IN 10^3 GWHe
17. 19. 21 23. 25. 27. $\times 10^3$ GWHt
5.54 6.19 6.85 7.50 8.15 8.80 $\times 10^3$ GWHe

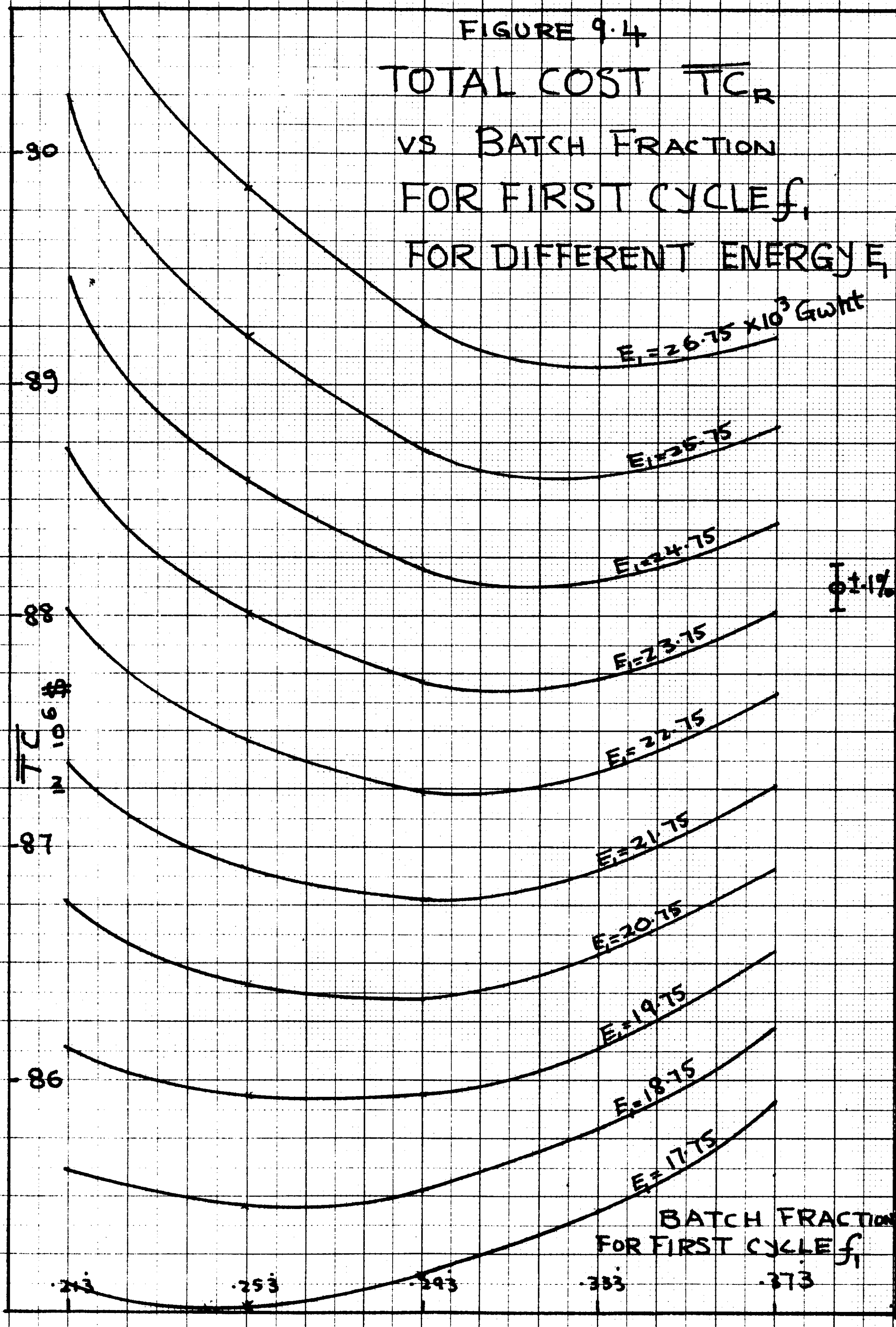


FIGURE 9.4
TOTAL COST $\overline{TC}_R$
VS BATCH FRACTION
FOR FIRST CYCLE f_1
FOR DIFFERENT ENERGY E_1
$E_1 = 26.75 \times 10^3$ GWHt
$E_1 = 25.75$
$E_1 = 24.75$
$E_1 = 23.75$
$E_1 = 22.75$
$E_1 = 21.75$
$E_1 = 20.75$
$E_1 = 19.75$
$E_1 = 18.75$
$E_1 = 17.75$
±.1%
90
89
88
87
86
$\overline{TC}_2$ 10^6 \$
BATCH FRACTION
FOR FIRST CYCLE f_1
.213
.253
.293
.333
.373

trend of higher optimal batch fraction at higher cycle energy is illustrated more clearly. Refer to Figure 6.5 for comparisons between steady-state and non-steady state results.

Figure 9.5 shows the variation of the objective function with respect to batch fraction for cycle 1 for different values of f_2 while holding the remaining f's and all the E's at the 3.2 w/o 1/3 batch fraction steady-state level. There is a small cross-coupling effect between f_1 and f_2 in determining the value of $\overline{TC}$. If an error of $\pm 0.1\%$ in the objective function can be tolerated, it is possible to optimize each cycle independently and neglect the cross-coupling effects altogether.

In all these figures, the objective function varies by less than $\pm 0.25\%$ over the practical range of f_1. In other words, the objective function is very flat around the region of 0.33 reload batch fraction. Thus near the optimal solution, there are many sub-optimal solutions with roughly the same total cost. For a saving of $\pm 0.25\%$, there is very little incentive to find "the optimal solution." Instead, one should concentrate on optimizing other considerations such as engineering safety and reliability within this range of batch fractions.

9.5 Optimization Algorithm

Based on the equations given on Tables 9.1-9.6, the objective function is calculated for all possible combinations of f's which produce the specified cycle energy demand and satisfy the burnup constraints. These combinations are then ranked in ascending order in terms of their cost. The lowest

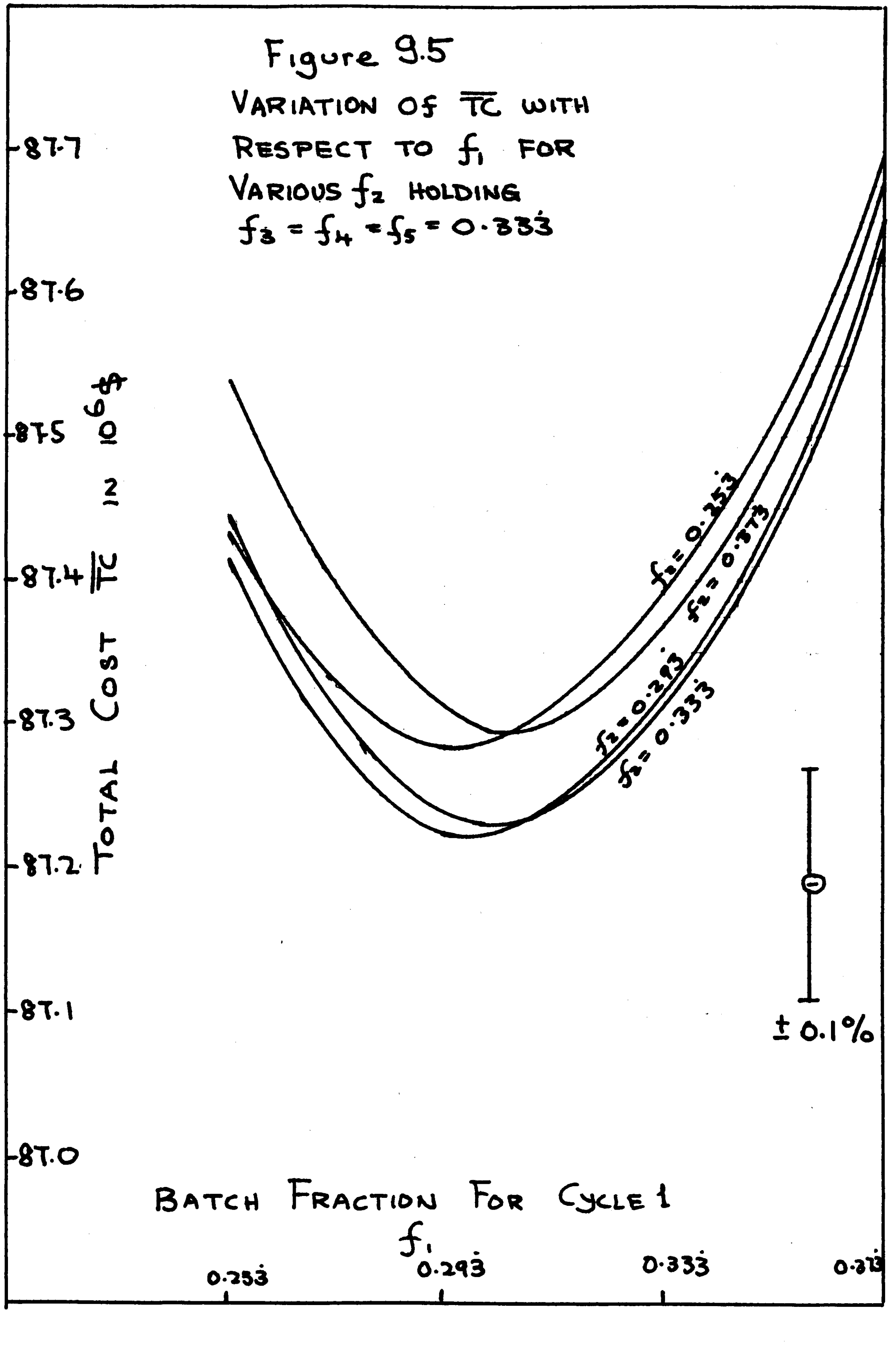


Figure 9.5
VARIATION OF $\overline{TC}$ WITH RESPECT TO f_1 FOR VARIOUS f_2 HOLDING $f_3 = f_4 = f_5 = 0.33\dot{3}$

five combinations are subjected to further tests.

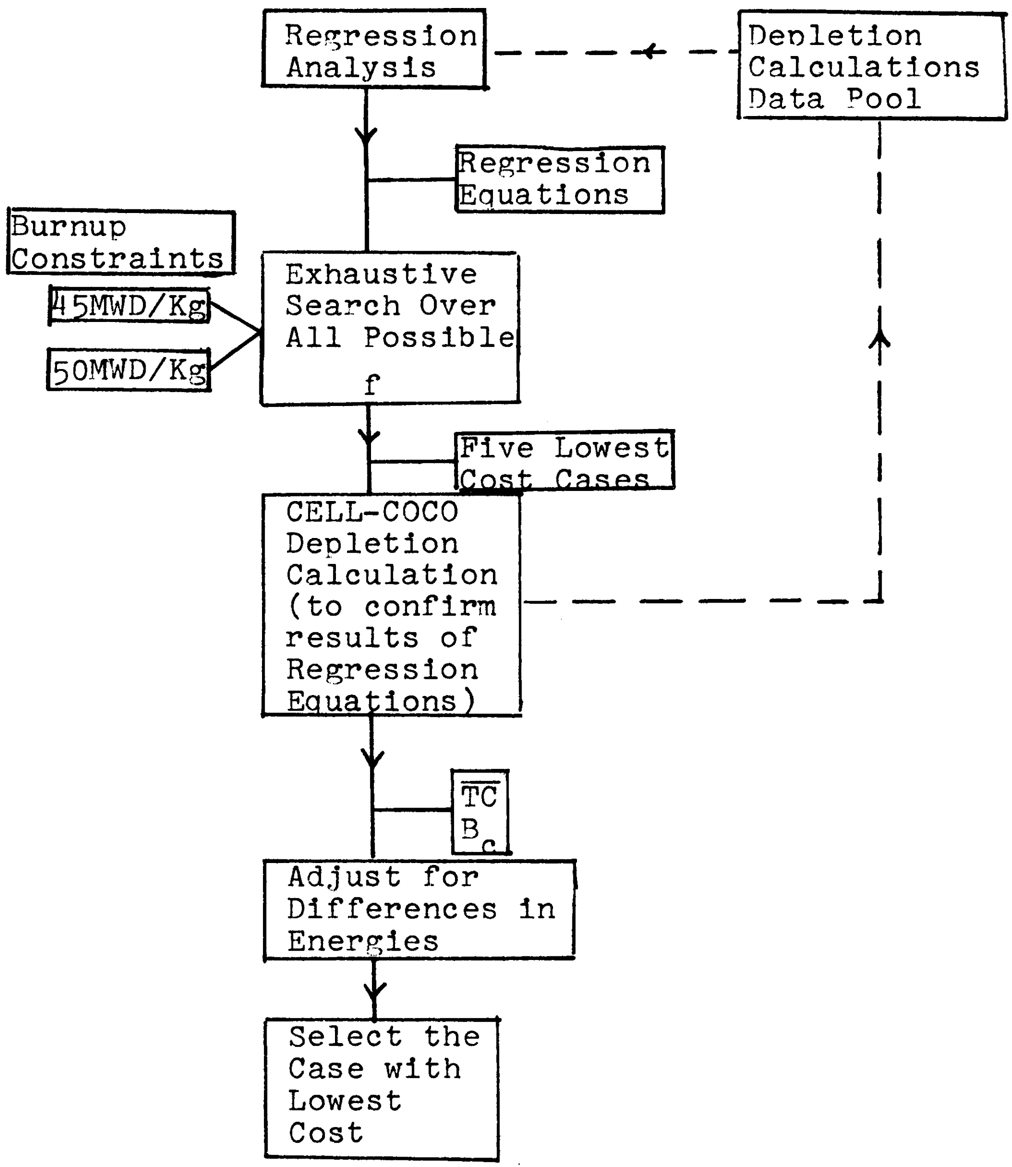
Further tests consist of carrying out the depletion calculations based on the estimated reload enrichments and batch fractions. The actual values of the objective function and average discharge burnups are obtained. These are compared with the values predicted by the regression equations. If the estimations for reload enrichments are so far off that the resulted cycle energies are significantly different from the the specified cycle energy demand, the objective function should be adjusted to reflect this difference. The case that satisfies the constraints with the lowest adjusted objective function is the optimal case for a particular optimization problem.

Hence, for any set of cycle energies, a maximum of five depletion calculations are required. Moreover, as more problems are solved, the additional depletion data can be incorporated into the regression equations. In this manner, the regression equations are made valid over a larger and larger range.

The above procedures can be summarized in the flow chart given on Figure 9.6. The computer code CELL-CORE is used for the depletion calculations in this thesis research. In practice, one would like to use more elaborate physics models for the depletion calculations; such as PDQ-5 or Citation, those that would give more accurate values of discharge burnups, power peaking and shut-down margins, etc.

Figure 9.6

Optimization Algorithm



9.6 Results of Sample Cases A and B

The 1065 MWe Zion Type PWR is chosen for analysis. For both cases the reactor starts with steady state condition for 3.2 w/o, three-zone modified scatter refuelling, which produces 22750 GWHt per cycle. Economics parameters used in evaluating the objective function are given in Appendix B.

Sample case A consists in finding the optimal combination of batch fraction f's that produces the same amount of energy, 22750 GWHt, in each cycle for five succeeding cycles and satisfies the 45 or 50 MWD/kg maximum allowable discharge burnup. Table 9.7 shows the optimal set of batch fractions, for the 45 MWD/kg case. $\overline{TC}_R$ is the objective function based on actual energy production predicted by the regression equations, while $\overline{TC}_{CC}$ is the objective function calculated by CELL-COCO. The last two columns on the right shows the values of $\overline{TC}_R$ and $\overline{TC}_{CC}$ after correcting for differences in cycle energy between the actual values and the target values.

Case AA0 is the base line case in which the reactor continues to refuel with 3.2 w/o reload enrichment and three-zone modified scatter refuelling. It serves as a standard with which other cases are to be compared.

Case AA1 with an adjusted cost of \$87.06 million is the optimal solution for this problem with burnup constrained to be less than 45 MWD/kg. The net savings is \$0.18 million (or 0.3%) over the base line case.

Table 9.8 shows the values of the predicted discharge burnup based on Equation (8.6) and the actual discharge burnup from CELL-CORE. The values of the predicted burnup

Table 9.7

B^{o} =45MWD/Kg

Reload Enrichments, Batch Fractions, Cycle Energies and Revenue Requirements for the Various Lowest Cost Cases Using the Method of Polynomial Approximation Sample Case A

Case Number	∈(w/o) / f / E(GWHt)	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5	Revenue Requirement, For Actual Energy: Polynomial Approximation (10^6 $)	Revenue Requirement, For Actual Energy: CELL COCO (10^6 $)	Revenue Requirement, Corrected for Target Energy: Polynomial Approximation (Difference) (10^6 $)	Revenue Requirement, Corrected for Target Energy: CELL COCO (10^6 $)
	Target Energy	22750.	22750.	22750.	22750.	22750.				
AA0	∈	3.200	3.200	3.200	3.200	3.200				
	f	0.333	0.333	0.333	0.333	0.333	87.30	87.24	87.30	87.24
	E	22750.	22750.	22750.	22750.	22750.			(+.06)	
AA1	∈	3.88	3.33	3.45	3.54	2.94				
	f	0.293	0.293	0.293	0.293	0.333	87.09	87.13	87.02	87.06
	E	22690.	22840.	22560.	22920.	23030.			(-.04)	
AA2	∈	3.88	3.29	3.45	3.54	3.45				
	f	0.293	0.293	0.293	0.293	0.293	86.94	87.00	87.04	87.10
	E	22690.	22700.	22460.	22880.	22830.			(-.06)	
AA3	∈	3.23	3.88	3.33	3.54	2.57				
	f	0.33	0.293	0.293	0.293	0.33	86.76	86.81	87.11	87.16
	E	22850.	22870.	22860.	22960.	20800.			(-.05)	
AA4	∈	3.88	3.33	3.45	3.54	2.67				
	f	0.293	0.293	0.293	0.293	0.373	87.34	87.33	87.11	87.10
	E	22690.	22840.	22560.	22920.	23620.			(+.01)	
AA5	∈	3.23	3.88	3.33	3.54	3.08				
	f	0.33	0.293	0.293	0.293	0.293	86.88	86.91	87.13	87.16
	E	22850.	22870.	22860.	22960.	21190.			(-.03)	

Table 9.8 B°=45MWD/Kg

Average Discharge Burnup for the Sublot Experiencing the Highest Exposure for Sample Case A Calculated by (1) Polynomial Approximation Based on Regression Equations (2) CELL-CORE Depletion Calculation

Case Number	Method	Batch Number -2	-1	0	1	2	3	4	5
		MWD/Kg							
AA0	(1)	31.5	31.5	31.5	31.5	31.5	31.5	31.5	31.5
	(2)	31.5	31.5	31.5	31.5	31.5			
AA1	(1)	38.6	38.6	38.6	44.2	39.4	40.9	41.2	36.1
	(2)	38.9	38.6	38.8	44.9	39.4	—	—	—
AA2	(1)	38.6	38.6	38.6	44.2	39.4	40.9	41.2	40.6
	(2)	38.9	38.6	38.8	44.9	38.7	—	—	—
AA3	(1)	31.5	38.6	38.6	38.8	44.2	39.3	44.2	33.3
	(2)	31.6	38.9	38.6	39.2	45.0	—	—	—
AA4	(1)	38.6	38.6	38.6	44.2	39.4	40.9	41.2	33.6
	(2)	38.9	38.6	38.8	44.9	39.9	—	—	—
AA5	(1)	31.5	38.6	38.6	38.8	44.2	39.3	41.2	37.8
	(2)	31.6	38.9	38.6	39.2	44.7	—	—	—

for batches discharged after the planning horizon are estimated based on end conditions pre-specified in Section 9.2. No corresponding values are given from CELL-CORE. It can be seen that for most cases, the error between estimated and actual burnups are less than 5%.

Table 9.9 shows the results for the case of 50 MWD/kg maximum allowable burnup. Case AB1 with an adjusted cost of $86.90 million is the optimal solution, with a net savings of 0.34 million, or 0.39% over the base line case. Table 9.10 shows the burnup values. For 50 MWD/kg maximum allowable burnup, it is possible to refuel with batch fraction $=0.25\dot{3}$ for all cycles. But due to the high initial enrichment required for Cycle 1, it is not economical to do so. Hence, in this case of 50 MWD/kg burnup limit, the optimal solution is not given by the strategy with the smallest feasible batch fraction, whereas the previous case of 45 MWD/kg burnup limit, the optimal solution is dictated by burnup constraints.

Sample case B consists of finding the optimal combination of batch fraction f's that produces the following energy requirements and satisfies the 45 or 50 MWD/kg maximum allowable discharge burnup.

Cycle energy requirements for sample case B are

E_1=25450 GWHt, E_2=30440GWHt, E_3=21850GWHt, E_4=19340GWHt
E_5=20880 GWHt

B° =50MWD/Kg

Table 9.9

Reload Enrichments, Batch Fractions, Cycle Energies and Revenue Requirements for the Various Lowest Cost Cases Using the Method of Polynomial Approximation Sample Case A

Case Number	ε(w/o) / f / E(GWHt)	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5	Revenue Requirement, For Actual Energy: Polynomial Approximation (10^6 $)	Revenue Requirement, For Actual Energy: CELL-COCO (10^6 $)	Revenue Requirement, Corrected for Target Energy: Polynomial Approximation (Difference) (10^6 $)	Revenue Requirement, Corrected for Target Energy: CELL-COCO (10^6 $)
	Target Energy	22750.	22750.	22750.	22750.	22750.				
AA0	ε	3.2	3.2	3.2	3.2	3.2				
	f	0.333	0.333	0.333	0.333	0.333	87.30	87.24	87.30	87.24
	E	22750.	22750.	22750.	22750.	22750.			(+0.06)	
AB1	ε	3.88	4.27	3.42	3.95	2.40				
	f	0.293	0.253	0.253	0.253	0.293	86.43	86.34	86.99	86.90
	E	22690.	23000.	22480.	23100.	20500.			(+0.09)	
AB2	ε	3.88	4.27	2.76	3.77	3.45				
	f	0.293	0.253	0.293	0.293	0.293	87.20	87.33	87.01	87.14
	E	22690.	23000.	22510.	23130.	23070.			(-0.13)	
AB3	ε	3.88	3.33	3.45	3.54	2.94				
	f	0.293	0.293	0.293	0.293	0.333	87.09	87.13	87.02	87.06
	E	22690.	22840.	22560.	22920.	23030.			(-0.04)	
AB4	ε	3.88	4.27	2.77	3.74	2.40				
	f	0.293	0.253	0.293	0.293	0.333	86.26	86.37	87.02	87.13
	E	22690.	23000.	22510.	22980.	19730.			(-0.11)	
AB5	ε	3.88	3.29	3.45	4.50	2.66				
	f	0.293	0.293	0.293	0.253	0.293	86.82	86.89	87.03	87.10
	E	22690.	22700.	22400.	23000.	22300.			(-0.07)	
AB6	ε	3.88	3.29	3.45	3.54	3.45				
	f	0.293	0.293	0.293	0.293	0.293	86.94	87.00	87.04	87.10
	E	22690.	22700.	22460.	22880.	22830.			(-0.06)	
AB7	ε	3.88	4.27	3.42	3.95	3.61				
	f	0.293	0.253	0.253	0.253	0.253	87.23	87.14	87.04	86.95
	E	22690.	23000.	22480.	23090.	23250.			(+0.09)	

Table 9.10 B° =50MWD/Kg

Average Discharge Burnup for the Sublot Experiencing the Highest Exposure for Sample Case A Calculated by(1) Polynomial Approximation Based on Regression Equations (2) CELL-CORE Depletion Calculation

Case Number	Method	Batch Number -2	-1	0	1	2	3	4	5
		MWD/Kg							
AA0	(1)	31.5	31.5	31.5	31.5	31.5	31.5	31.5	31.5
	(2)	31.5	31.5	31.5	31.5	31.5	—	—	—
AB1	(1)	38.6	38.6	38.6	44.2	47.4	40.4	44.4	31.8
	(2)	38.9	38.4	38.1	44.4	46.9	—	—	—
AB2	(1)	38.6	38.6	38.6	44.2	47.4	34.7	43.2	36.4
	(2)	38.9	38.4	38.5	45.2	47.5	—	—	—
AB3	(1)	38.6	38.6	38.6	44.2	39.4	40.9	41.2	36.1
	(2)	38.9	38.6	38.8	44.9	39.4	—	—	—
AB4	(1)	38.6	38.6	38.6	44.2	47.4	34.7	43.2	31.9
	(2)	38.9	38.4	38.5	45.2	47.3	—	—	—
AB5	(1)	38.6	38.6	38.6	44.2	39.4	40.9	49.6	34.3
	(2)	38.9	38.6	38.8	44.5	38.4	—	—	—
AB6	(1)	38.6	38.6	38.6	44.2	39.4	40.9	41.2	40.6
	(2)	38.9	38.6	38.8	44.9	38.7	—	—	—
AB7	(1)	38.6	38.6	38.6	44.2	47.4	40.4	44.4	38.2
	(2)	38.9	38.4	38.1	44.4	47.0	—	—	—

Table 9.11 shows the three lowest cost combinations of $\overline{TC}_R$ for the case of 45 MWD/kg burnup limit. Case BAO is the reference case in which the batch fractions are held constant at the 0.33 level and is used as a standard for comparing other cases.

Case BA1 with an adjusted cost of \$89.65 million is the optimal solution for case B with burnup constraint equal to 45 MWD/kg. The net savings is 0.28 million compared to BAO. Table 9.12 shows estimated and actual burnup values for cases BA1-BA3. Table 9.13 shows the set of optimal solutions for the case of 50 MWD/kg burnup limit. Case BB5 with an adjusted total cost of \$89.68 million is the optimal solution. However, BB5 is not cheaper than BA1 despite the more relaxed burnup constraints. Due to the fact that the objective function is so flat near the optimal, the regression equations with a $\pm 0.1\%$ error cannot always succeed in identifying "the optimal solution" among the neighboring sub-optimals. Table 9.14 shows estimated and actual burnup values for cases BB1-BB5.

From case BA1 or BB5, one can identify some interesting relations between optimal batch fractions and cycle energy requirements. Where the cycle energy level is high, the optimal batch fraction is relatively large, and conversely. This phenomenon has already been observed in Figure 9.4 and in the steady state results in Figure 6.5. Since this case is similar to the first example given in J. Kearney's thesis (K1), it is possible to make a comparison between the Method of Dynamic Programming and the Method of Polynomial Approximation.

Table 9.11 $B^{o} = 45$ MWD/Kg

Reload Enrichments, Batch Fractions, Cycle Energies and Revenue Requirements for the Various Lowest Cost Cases Using the Method of Polynomial Approximation Sample Case B

Case Number	ϵ(w/o) f E(GWHt)	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5	Revenue Requirement: For Actual Energy, Polynomial Approximation	For Actual Energy, CELL-COCO	Corrected for Target Energy, Polynomial Approximation (Difference)	Corrected for Target Energy, CELL-COCO
	Target Energy	25450.	30440.	21850.	19340.	20880.	10^6 \$			
BA0	ϵ	3.73	4.36	2.40	2.76	3.45				
	f	0.333	0.333	0.333	0.333	0.333	89.36	89.37	89.92	89.93
	E	25510.	30470.	22170.	20280.	17220.			(-0.01)	
BA1	ϵ	3.74	3.73	3.25	3.68	2.71				
	f	0.333	0.373	0.293	0.253	0.293	89.53	89.36	89.83	89.65
	E	25520.	30100.	22030.	19200.	20150.			(+0.18)	
BA2	ϵ	3.74	3.73	3.24	2.93	2.77				
	f	0.333	0.373	0.293	0.293	0.333	89.50	89.36	89.83	89.70
	E	25520.	30100.	22030.	19250.	19890.			(+0.13)	
BA3	ϵ	3.74	4.36	2.70	2.66	2.37				
	f	0.333	0.333	0.293	0.333	0.373	88.88	88.91	89.87	89.89
	E	25510.	30470.	21270.	19740.	17310.			(-0.02)	

Table 9.12

$B^c = 45 MWD/Kg$

Average Discharge Burnup for the Sublot Experiencing the Highest Exposure for Sample Case B Calculated by (1) Polynomial Approximation Based on Regression Equations

(2) CELL-CORE Depletion Calculation

Case Number	Method	Batch Number -2	-1	0	1	2	3	4	5
		MWD/Kg							
BA0	(1)	31.5	31.5	31.5	37.2	43.9	31.9	35.0	41.4
	(2)	31.5	31.8	32.8	37.9	42.2	28.5	32.9	41.9
BA1	(1)	31.5	31.5	31.5	43.0	43.0	39.0	42.6	34.5
	(2)	31.5	31.8	32.8	44.9	44.5	—	—	—
BA2	(1)	31.5	31.5	31.5	43.0	43.0	39.0	36.3	35.0
	(2)	31.5	31.8	32.8	45.3	45.3	—	—	—
BA3	(1)	31.5	31.5	38.6	43.0	43.9	34.4	34.1	31.7
	(2)	31.5	31.8	39.3	46.2	41.5	—	—	—

B° = 50MWD/Kg

Table 9.13

Reload Enrichments, Batch Fractions, Cycle Energies and Revenue Requirements for the Various Lowest Cost Cases Using the Method of Polynomial Approximation Sample Case B

Case Number	ε(w/o) / f / E(GWHt)	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5	Revenue Requirement (10^6 \$): For Actual Energy: Polynomial Approximation	For Actual Energy: CELL-COCO	Corrected for Target Energy: Polynomial Approximation (Difference)	Corrected for Target Energy: CELL-COCO
	Target Energy	25450.	30440.	21850.	19340.	20880.				
BA0	ε	3.73	4.36	2.40	2.76	3.45				
	f	0.333	0.333	0.333	0.333	0.333	89.36	89.37	89.92	89.93
	E	25510.	30470.	22170.	20280.	17220.			(-0.01)	
BB1	ε	3.74	4.36	2.70	3.88	2.27				
	f	0.333	0.333	0.293	0.253	0.293	88.66	88.71	89.67	89.72
	E	25510.	30470.	21270.	19180.	17930.			(-0.05)	
BB2	ε	4.55	3.79	2.91	3.87	2.61				
	f	0.293	0.333	0.293	0.253	0.293	89.35	89.38	89.67	89.71
	E	25340.	30310.	21790.	19480.	20020.			(-0.04)	
BB3	ε	3.74	4.36	2.70	3.10	2.37				
	f	0.333	0.333	0.293	0.293	0.333	88.61	88.67	89.71	89.76
	E	25510.	30470.	21270.	19260.	17480.			(-0.05)	
BB4	ε	4.55	3.79	2.91	3.09	2.71				
	f	0.293	0.333	0.293	0.293	0.333	89.32	89.38	89.71	89.76
	E	25340.	30310.	21790.	19320.	19930.			(-0.05)	
BB5	ε	4.55	3.79	3.72	2.93	2.93				
	f	0.293	0.333	0.253	0.253	0.293	89.31	89.27	89.72	89.68
	E	25340.	30310.	21790.	19130.	20110.			(+0.04)	
B_*^*	ε	4.57	3.26	4.31	2.83	3.26				
	f	0.293	0.373	0.253	0.253	0.293	89.94	89.82	89.94	89.82
	E	25450.	30440.	21850.	19340.	20880.			(+0.12)	

Table 9.14

B^{o}=50MWD/Kg

Average Discharge Burnup for the Sublot Experiencing the Highest Exposure for Sample Case B Calculated by (1) Polynomial Approxiamtion Based on Regression Equations

(2) CELL-CORE Depletion Calculation

Case Number	Method	Batch Number -2	-1	0	1	2	3	4	5
		MWD/Kg							
BA0	(1)	31.5	31.5	31.5	37.2	43.9	31.9	35.0	41.4
	(2)	31.5	31.8	32.8	37.9	42.2	28.5	32.9	41.9
BB1	(1)	31.5	31.5	38.6	43.0	48.2	34.4	44.3	30.9
	(2)	31.5	31.8	39.3	44.9	49.4	—	—	—
BB2	(1)	38.6	38.6	38.6	49.7	43.5	36.2	44.1	33.7
	(2)	39.2	39.8	39.7	52.2	44.0	—	—	—
BB3	(1)	31.5	31.5	38.6	43.0	48.2	34.4	37.8	31.7
	(2)	31.5	31.8	39.3	45.6	50.2	—	—	—
BB4	(1)	38.6	38.6	38.6	49.7	43.5	36.2	37.6	34.6
	(2)	39.2	39.8	39.7	52.7	44.7	—	—	—
BB5	(1)	38.6	38.6	38.6	49.7	43.5	42.9	36.3	36.4
	(2)	39.2	39.8	39.4	51.7	44.1	—	—	—
B*	(1)	38.6	31.5	38.6	50.7	39.1	47.8	35.5	39.1
	(2)	39.2	31.3	39.9	52.3	39.5	46.3	32.5	38.4

Notice that the B^{o}=50MWD/Kg limit only applies to the estimated burnup values calculated by the polynomial regression equation. The fact that actual burnup values sometimes exceed 50 MWD/Kg indicates that the estimated burnup values are only approximate.

Case B$^*_*$ is the optimal solution arrived at by Dynamic Programming. B$^*_*$ is more expensive than BA1 by $0.17 million dollars. However, the total savings of B$^*_*$ from the base line BA0 is only $0.11 million dollars. This is in great contrast with the $2.5 million dollars saving reported by Kearney. The difference is probably due to the different methods of calculating $\overline{TC}$.

Finally, it is important to notice that in the vicinity of "the optimal solution", there are many sub-optimal solutions with roughly the same total cost. Some of these solutions may have higher engineering margins in terms of discharge burnup, power peaking and shut-down reactivity. Hence, the final choice should be based on these considerations as well.

9.7 <u>Estimates of Burnup Penalty π</u>

The concept of burnup penalty π was introduced in Chapter 6, and it is defined for the non-steady state case by Equation (7.10) in Chapter 7. For each cycle, there would be a separate value for burnup penalty π_c, which can be interpreted as the additional cost that would be incurred if the burnup limitation on Cycle c were decreased by one unit.

Since the actual optimization algorithm solves by exhaustive search instead of by Equations (7.10) and (7.11), burnup penalty is not calculated explicitly. However, the order of magnitude of π_c can be infered by inspecting Tables 9.7, 9.8, 9.9, and 9.10.

Tables 9.8 and 9.10 show that the discharge burnup is well within the limit for almost all of the batches except one in all cases. In other words, a single batch in each case controls the values of the batch fractions. Hence, by definition, the burnup penalties for those cycles not on the border of the burnup constraints have the value zero.

The burnup penalty for the controlling batch can be estimated by

$$\pi_c \simeq \frac{\overline{TC}(\vec{E}^s,\vec{f}^*) - \overline{TC}(\vec{E}^s,\vec{f}^{**})}{\Delta B_c^0}$$

where $\overline{TC}(\vec{E}^s,\vec{f}^*)$ is the optimal solution for $\vec{E}^s$ and $\vec{B}^0$ and $\overline{TC}(\vec{E}^s,\vec{f}^{**})$ is the optimal solution for $\vec{E}^s$ and $\vec{B}^0+\Delta B_c^0$.

For sample case A, π_2 for the second fuel batch is given by the difference in $\overline{TC}_R$ between case AA1 and AB1 divided by the increment in B.

$$\pi_2 \simeq \frac{(87.02-86.98)\cdot 10^6\$}{5(\text{MWD/Kg})} = \frac{0.04}{5} = 0.008\cdot 10^6\$/(\text{MWD/Kg})$$

This value of π is much smaller than that given in Figure 6.7 for the steady state case. Similar results are obtained for sample case B. Hence there is very little incentive to increase the maximum allowable burnup limit above the 45 MWD/Kg level.

9.8 Incremental Cost

Incremental cost of energy is defined as the additional cost that would be incurred if an additional unit of energy is produced in an optional fashion. In other words, if the reactor is optimized for one set of cycle energy E^s .

$$\overline{TC}(\vec{E}^s,\vec{f}*) = \text{minimum of } \overline{TC}(\vec{E}^s,\vec{f}) \text{ with respect to } \vec{f}$$

$$\text{and } B^0 < B_c(\vec{E}^s,\vec{f}*)$$

and for the second set of cycle energies, $\vec{E}^s + \Delta E_c$ the reactor is reoptimized.

$$\overline{TC}(\vec{E}^s + \Delta E_c, \vec{f}\dagger) = \text{mininum of } \overline{TC}(\vec{E}^s + \Delta E_c, \vec{f}) \text{ with respect to } \vec{f}$$

$$\text{and } B^0 > B_c(\vec{E}^s + \Delta E_c\ , \vec{f}\dagger)$$

then the incremental cost of energy from the c-cycle is given by

$$\lambda_c \simeq \frac{\overline{TC}(\vec{E}^s + \Delta E_c, \vec{f}\dagger) - \overline{TC}(\vec{E}^s\ ,\vec{f}*\ )}{\Delta E_c} \qquad (9.5)$$

The values of $\overline{TC}$ are obtained from the regression equations. In principle, one can use the actual $\overline{TC}$ calculated from CELL-COCO. However, for the purpose of this calculation, the additional efforts involved in doing all the depletion analysis are not justified. Tables 9.15, 16 show the values of $\overline{TC}(\vec{E}^s,\vec{f}*)$ and $\overline{TC}(\vec{E}*+\Delta E_c,\vec{f}\dagger)$ for various ΔE_c for sample case A. Also shown are the various $\vec{f}*$ and $\vec{f}\dagger$. For many cases, $\vec{f}*$ are seen to be the same as $\vec{f}\dagger$. For these cases, more or less energy can be generated using the same combination of $\vec{f}*$. However, for those cases that $\vec{f}\dagger$ are not equal to $\vec{f}*$, either the $\vec{f}*$ are not the least costly combination at the new set of $\vec{E}^s + \Delta E_c$, or the $\vec{f}*$ are not feasible in terms of discharge burnup. For $\Delta E_c > 0$, feasibility considerations change the $\vec{f}*$ to $\vec{f}\dagger$; on the other hand, for $\Delta E_c < 0$, economics considerations cause the change. Tables 9.15, 16 also show the incremental cost for various cycles as a function of energy. In general, the incremental cost

Table 9.15

Calculation of Incremental Cost of Energy Using Regression Equations. Sample Case A

Burnup Limit B^{0} = 45MWD/Kg

	Batch Fraction for Cycle 1	2	3	4	5	Revenue Requirement ——10^6\$——	Incremental Cost Mills/KWHe
Base Case AA1	0.293	0.293	0.293	0.293	0.333	87.01872	————
Positive Energy Change ΔE=1000GWHt in Cycle							
1	0.333	0.293	0.293	0.293	0.333	87.5284	1.56
2	0.293	0.293	0.293	0.293	0.333	87.4265	1.22
3	0.293	0.293	0.293	0.293	0.333	87.3890	1.15
4	0.293	0.293	0.293	0.293	0.333	87.3170	0.91
5	0.293	0.293	0.293	0.293	0.333	87.2957	0.845
Negative Energy Change ΔE=-1000GWHt in Cycle							
1	0.293	0.293	0.293	0.293	0.333	86.5642	1.395
2	0.293	0.253	0.253	0.253	0.293	86.5848	1.33
3	0.293	0.293	0.293	0.293	0.333	86.6605	1.095
4	0.293	0.293	0.293	0.293	0.333	86.7226	0.905
5	0.293	0.293	0.293	0.293	0.333	86.7443	0.84

Table 9.16

Calculation of Incremental Cost of Energy

Using Regression Equations. Sample Case A

Burnup Limit B^0= 50MWD/Kg

	Batch Fraction for Cycle					Revenue Requirement	Incremental Cost
	1	2	3	4	5	10^6 $	Mills/KWHe
Base Case AB1	0.293	0.253	0.253	0.253	0.293	86.9890	———
Positive Energy Change ΔE=1000GWHt in Cycle							
1	0.293	0.253	0.253	0.253	0.293	87.4642	1.46
2	0.293	0.293	0.293	0.293	0.333	87.4265	1.335
3	0.293	0.253	0.293	0.293	0.293	87.3848	1.21
4	0.293	0.253	0.253	0.253	0.293	87.3047	0.965
5	0.293	0.253	0.253	0.253	0.293	87.2748	0.875
Negative Energy Change ΔE=-1000GWHt in Cycle							
1	0.293	0.253	0.253	0.253	0.293	86.5345	1.395
2	0.293	0.253	0.253	0.253	0.293	86.5848	1.24
3	0.293	0.253	0.253	0.253	0.293	86.5860	1.24
4	0.293	0.253	0.253	0.253	0.293	86.6761	0.955
5	0.293	0.253	0.253	0.253	0.293	86.7064	0.865

increases as more energy is produced. However, for those cases in which $\vec{f}^* \neq \vec{f}$, incremental cost would have a negative slope. In the limit that $\Delta E_c \to 0$, λ_c would approach infinity for those cases that $\vec{f}^* \neq \vec{f}$. This is mainly due to the fact that in the present model one can only change batch fraction by a discrete amount, and the objective functions of the two discrete combinations of $\vec{f}^*$ and $\vec{f}^\dagger$ have a finite difference. If batch fractions could be varied in a continuous fashion, these singularities would not be present and the incremental cost would vary continuously in a pattern similar to Figure 6.8.

Table 9.17 and Table 9.18 show values of the objective function and the incremental costs for various ΔE_c for sample case B. The same phenomenon of negative sloping incremental cost is observed.

9.9 Summary and Conclusions

Using cycle energies $\vec{E}$ and batch fractions $\vec{f}$ as independent variables, a set of regression equations based on polynomials in these independent variables has been obtained. These predict the objective function to an accuracy of within $\pm$ 0.1% and average discharge burnup to an accuracy of within $\pm$10%. An optimization algorithm based on the principle of exhaustive search is developed. For every specified set of cycle energies, this algorithm results in five or more sub-optimal cases that bracket the optimal solution. These cases can be analysed further by more elaborate depletion codes.

The results of the regression analysis and the optimization procedures indicate that the objective function for

Table 9.17

Calculation of Incremental Cost of Energy

Using Regression Equations. Sample Case B

Burnup Limit B = 45MWD/Kg

	Batch Fraction for Cycle 1	2	3	4	5	Revenue Requirement —10^6\$—	Incremental Cost Mills/KWHe
Base Case BA1	0.333	0.373	0.293	0.253	0.293	89.8251	————
Positive Energy Change ΔE=1000GWHt in Cycle							
1	0.333	0.373	0.293	0.253	0.293	90.2916	1.435
2	0.333	0.373	0.293	0.253	0.293	90.2424	1.28
3	0.333	0.373	0.293	0.253	0.293	90.1845	1.10
4	0.333	0.373	0.293	0.293	0.333	90.1255	0.91
5	0.333	0.373	0.293	0.263	0.293	90.1049	0.915
Negative Energy Change ΔE=-1000GWHt in Cycle							
1	0.333	0.373	0.293	0.253	0.293	89.3766	1.375
2	0.333	0.373	0.293	0.253	0.293	89.4070	1.28
3	0.333	0.373	0.293	0.253	0.293	89.4773	1.07
4	0.333	0.373	0.293	0.253	0.293	89.5224	0.925
5	0.333	0.373	0.293	0.253	0.293	89.5484	0.85

Table 9.18

Calculation of Incremental Cost of Energy Using Regression Equations. Sample Case B

Burnup Limit B = 50MWD/Kg

	Batch Fraction for Cycle					Revenue Requirement	Incremental Cost
	1	2	3	4	5	10^6\$	in Mills/KWHe
Base Case BB1	0.333	0.333	0.293	0.253	0.293	89.6715	———
Positive Energy Change ΔE=1000GWHt in Cycle							
1	0.333	0.333	0.293	0.253	0.293	90.1380	1.435
2	0.293	0.333	0.293	0.253	0.293	90.0775	1.25
3	0.333	0.333	0.293	0.253	0.293	90.0309	1.10
4	0.333	0.333	0.293	0.253	0.293	89.9772	0.93
5	0.333	0.333	0.293	0.253	0.293	89.9513	0.86
Negative Energy Change ΔE=-1000GWHt in Cycle							
1	0.293	0.333	0.293	0.253	0.293	89.1628	1.56
2	0.293	0.293	0.253	0.253	0.293	89.1515	1.60
3	0.333	0.333	0.253	0.253	0.293	89.3229	1.07
4	0.333	0.333	0.293	0.253	0.293	89.3687	0.925
5	0.333	0.333	0.293	0.253	0.293	89.3947	0.845

cases A and B is insensitive to batch fraction changes, if the same cycle energies are produced. Hence, engineering considerations should be the principal criteria in the selection process for those problems. Since batch fraction cannot be varied continuously, incremental cost of energy varies in a series of discrete jumps. This problem would have been less severe if the increments in batch fraction had been made smaller.

CHAPTER 10.0

CONCLUSIONS AND RECOMMENDATIONS

10.1 Conclusions

The following conclusions are obtained from this thesis research.

(1) The Inventory Value Method for evaluating worth of nuclear fuel inventories to be used in calculating finite planning horizon revenue requirement is adequate for the purpose of scheduling energy and nuclear in-core optimization.

(2) Three methods are proposed for calculating incremental cost of energy for the fixed batch fraction case. The Linearization Method and the Inventory Value method for calculating incremental cost of energy are both suitable for the initial stages of optimal energy scheduling. The Rigorous Method is very time consuming and expensive and should be used only in the final stages of optimal energy scheduling.

(3) For the problem of nuclear in-core optimization under steady state conditions with variable batch fractions and reload enrichments, the optimal solution is practically always on the boundary of the burnup constraints. Hence, there are strong incentives to increase the burnup limits.

(4) For the problem of nuclear in-core optimization under non-steady state conditions, the Method of Piece-Wise Linear Approximation is applicable for the cases where there are large variations of objective function near the optimal solution. It is not applicable for economic situations where there is a broad region of optimality.

(5) The Method of Polynomial Approximation gives accurate values of the optimal solutions, even though the objective function is very flat near the optimum.

(6) Since the objective function is insensitive to large variations in batch fractions, selection of the optimal solution can be based primarily on other considerations, such as engineering margins.

10.2 Recommendations

The depletion code CELL-CORE should be modified in order that the batch fraction can be varied continuously. This modification would enable the efficient usage of the Method of Linear Approximation instead of Piece-Wise Linear Approximation or Polynomial Approximation. Once the optimal batch fraction in the continuum is located, the realistic batch fraction to be used in refuelling would be given by the number of integral fuel assemblies which is closest to the continuum optimal solution.

Finally, the algorithm of optimal energy schedule should be modified so that the polynomial equations from regression analysis could be used directly, instead of the present indirect usage which require intermediate calculations of incremental cost. It is recommended that a quadratic programming algorithm, or an even higher order programming algorithm should be used in the optimal energy scheduling procedures, so that the higher order derivatives can be used directly.

Biographical Note

Hing Yan Watt was born in Kowloon, Hong Kong on March 4, 1948. He received his elementary and secondary education on this island city, and was graduated from St. Paul's College in June 1966.

In September 1966, he enrolled at Massachusetts Institute of Technology where he studied in the Department of Civil Engineering. He was elected to membership in Chi Epsilon engineering honorary society in 1968. He received his Bachelor of Science degree in Civil Engineering in June 1969.

In September 1969, he entered the Department of Nuclear Engineering at M.I.T. and was granted the degree of Master of Science in August 1970.

Mr. Watt is married to the former An-Wen Cheng of Shanghai, China.

Appendix A

Brief Description of the Several Versions of CORE

Code Name	Time of Development	Description of Refuelling Options	Homogenization of Fuel Batches	Economics Calculations
MOVESCIV	Early 1971	N-zone modified scatter refuelling (Batch fraction cannot be changed in adjacent cycles)	Fuel properties homogenized only once when they are scattered from the outer annulus into the inner region.	--
CORE	January 1972	(same as MOVESCIV except it is much faster)		--
CORE	April 1972	Non-integral batch fraction. Variable batch fraction in adjacent cycles	Fuel properties in the inner region are homogenized at the beginning of every cycle	--
COCO	November 1972	(same as CORE(April 1972))		Fuel cycle calculations on per batch basis. Ending inventory calculated by Inventory Value Method

Appendix B

Economics and Fuel Cycle Cost Parameters

Fuel Cycle Financing

Investor-owned utility	
Fraction of bond financing	0.55
Fraction of preferred stock	0.10
Fraction of common stock	0.35
Rate of return on bonds, fraction per year	0.08
Rate of return on preferred stock, fraction per year	0.08
Rate of return on common stock, fraction per year	0.13
Tax rate	0.50
Government-owned utility	
Fraction of bond financing	1.00
Rate of return on bonds, fraction per year	0.0755

Lead Times: Time of transaction prior to the beginning of irradiation , in days

Purchase of uranium concentrate	127
Conversion of U_3O_8 to UF_6	127
Enrichment	97
Plutonium purchase	97
Fabrication	40

<u>Lag Times</u>: Time of transactions after the end of the irradiation, in days

Shipping	182
Reprocessing	212
Conversion of UNH to UF_6	212
Credit for reprocessed fuel	212

<u>Lag time for receipt of revenue</u>:

60 days after the mid-point of the generation period; one single payment

<u>Charges for materials and services</u>

Price of U_3O_8, \$/ib	8.00 #
Conversion of U_3O_8 to UF_6, \$/kg U	2.20 #
Enrichment \$/kg SWU	32.00
Enrichment plant tails composition, w/o U-235	0.25
Fabrication, \$/kg U	70.00
Shipping, \$/kg initial fuel metal	4.00
Reprocessing, \$/kg initial fuel metal	30.57
Conversion of UNH to UF_6, \$/kg U	5.60

<u>Process Yields</u>

Fabrication	0.99
Reprocessing	0.99
Conversion of U_3O_8 to UF_6	0.995
Conversion of UNH to UF_6	0.995

#Consistent with a natural UF_6 price of \$23.46/kg U

Appendix C
NOMENCLATURE

$\overline{TC}$ Revenue requirement

$\overline{TCA}$ Revenue requirement for Reactor A

$\overline{TCB}$ Revenue requirement for Reactor B

$\overline{TC}^S$ Revenue requirement for nuclear sub-system

$\overline{TC}_\infty$ Revenue requirement for the indefinite horizon

$\overline{TC}_\infty^N$ Revenue requirement for the indefinite horizon nuclide component

$\overline{TC}_\infty^S$ Revenue requirement for the indefinite horizon service component

$\overline{TC}_I$ Revenue requirement for planning horizon I

$\overline{TC}_I^N$ Revenue requirement for planning horizon I nuclide component

$\overline{TC}_I^S$ Revenue requirement for planning horizon I service component

$\overline{TC}_1$ Revenue requirement up to and including Cycle 1

$\overline{TC}^r$ Revenue requirement for reactor r in the planning horizon

R Revenue requirement for a batch

R_b^A Revenue requirement for Reactor A Batch b

R_b^B Revenue requirement for Reactor B Batch b

Z_i Component i of the various fuel cycle expenses,$

Z_i^N Nuclide component of the fuel cycle expense

Z_i^S Service component of the fuel cycle expense

Z_U Cost of U feed as UF_6

$Z_{U'}$ Credit for U discharge as UF_6

Z_F Fuel fabrication cost

Z_S	Shipping cost
Z_R	Reprocessing cost
Z_C	Conversion cost
Z_{Pu}	Plutonium credit
V	Value of nuclear fuel
$V^I_{initial}$	Value of nuclear fuel at the beginning of planning horizon I
V^I_{final}	Value of nuclear fuel at the end of planning horizon I
V^I	Value of nuclear fuel at hte end of planning horizon I
$V^b(t_{I'})$	Value of nuclear fuel batch b at time $t_{I'}$
λ	Incremental cost of energy
λ_{rc}	Incremental cost of energy for reactor r cycle c
λ_c	Incremental cost of energy for cycle c
π	Burnup penalty
π_c	Burnup penalty for cycle c
π_b	Burnup penalty for batch b
ρ	Negative of burnup penalty $(-\pi)$
ε_c	Enrichment for cycle c w/o U-235
f_c	Batch fraction for cycle c
B	Average discharge burnup
B_c	Average discharge burnup for cycle c
B_b	Average discharge burnup for batch b
B^0	Maximium allowable burnup limit
Ψ	Initial state of the reactor at the beginning of time horizon
τ	Corporate income tax rate
x	Effective cost of money, per year

t Time

t_b Time when batch b is charged to the reactor

t_i Time when fuel cycle expense i is paid

t_c Time when cycle c begins

t_{j_I} Time when fuel cycle expense j in time horizon I is paid

$t_{I'}$ Time when planning horizon I begins

$t_{I''}$ Time when planning horizon I ends

$\mathbb{A}$ Coefficient matrix of derivative of energy with respect to enrichment

α_c Derivative of revenue requirement with respect to enrichment of Cycle c

β_c Derivative of revenue requirement with respect to batch fraction of Cycle c

γ_{kc} Derivative of energy for Cycle k with respect to enrichment of Cycle c

δ_{kc} Derivative of energy for Cycle k with respect to batch fraction of Cycle c

ξ_{kc} Derivative of discharge burnup of Cycle k with respect to enrichment of Cycle c

ζ_{kc} Derivative of discharge burnup of Cycle k with respect to batch fraction of Cycle c

$\xi(n_b)$ Burnup coefficient for a batch of fuel that has been irradiated for n_b cycles

$\bar{\alpha}_c$ Multiple regression coefficient

$\bar{\beta}_c$ Multiple regression coefficient

$\bar{\gamma}_c$ Multiple regression coefficient

$\bar{\delta}_c$ Multiple regression coefficient

Superscripts

$*\dagger$ Denote optimal values

Superscripts

+ Coefficients evaluated at positive values

- Coefficients evaluated at negative values

Appendix D
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