MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF NUCLEAR ENGINEERING
Cambridge 39, Massachusetts

MEASUREMENTS OF REACTOR PARAMETERS IN SUBCRITICAL AND CRITICAL ASSEMBLIES: A REVIEW

by

Irving Kaplan

August 15, 1962

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AEC Research and Development Report
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(TID-4500, 17th Edition)

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I. INTRODUCTION

In the present state of reactor design, theorists are resigned to the need for supplementing their calculations with experimental information. In view of the complexity of design problems and the high cost of uncertainties in design it seems likely that the need for experimental data will continue and even increase. The use of exponential and zero-power critical experiments has grown in recent years and more attention is being paid to the measurement of microscopic nuclear quantities in these assemblies. It is no longer enough to determine the critical buckling of a multiplying system, and increasing effort is being put into the development of methods for measuring the various ratios that are related to the thermal utilization, the resonance escape probability and the fast fission effect. There is also increasing emphasis on the need to understand effects related to the energy spectrum of the thermal neutrons, and the determination of these effects is receiving increasing attention, both theoretical and experimental.

The experimental work mentioned involves a serious problem. The measurements are made in assemblies which are very different from the actual reactor for which the information they yield is needed. The exponential assembly and the zero-power critical assembly are usually much smaller than the reactor will be at its design operating conditions. Most experimental work is done at room temperature rather than at the desired operating temperatures, and poisons of one sort or another may make the flux distributions in the actual reactor quite different from those in the experiments. For these and other reasons, the experiments usually supply information about the nuclear parameters of a small subcritical or barely critical assembly at room temperature, without fission product poisons, and for a reactor core in its early infancy rather than at a more advanced age. Because of these difficulties, experimental research has been pushed to higher temperatures, as in the pressurized exponential facility at the Savannah River Laboratory and the pressurized critical facility at the Knolls

Atomic Power Laboratory, and it will probably be extended in other directions.

In spite of the difficulties mentioned above, experimental research has been extremely useful. For example, it has made possible at least two kinds of comparison between theory and experiment. First, the theorist can often make predictions about the initial, low power, nuclear properties of a clean reactor and these can be compared with the actual properties, without too much extrapolation, by means of measurements in exponential (1) and zero-power critical experiments (2). In such a comparison, the experiment represents a low-order approximation to the actual reactor. Second, the experiments provide information with which theoretical methods can be tested (3-10); these methods may then be used for design calculations on reactors quite different from those corresponding to the experiments. Agreement between the calculations and the experimental results then increases the theorists' confidence in the extrapolation of the theoretical methods. It seems certain that both of these ways of using experimental information will continue to be helpful. In addition, experimental information can be incorporated directly into "recipes" for reactor design – perhaps the most important practical use of such information (11-25).

Two types of experimental assemblies have been mentioned so far: exponential and zero-power critical. In practice, the use of these assemblies may be expensive and time consuming. Thus, an exponential pile used for the study of the properties of a typical natural uranium, graphite assembly may contain as much as 25 tons of graphite and six tons of uranium. These material requirements impose severe limitations on the number and kinds of experiments that can be made. Critical experiments are much more demanding with respect to material requirements, as well as with respect to control equipment. A need has, therefore, developed for simpler, more flexible assemblies for the measurement of nuclear parameters. Several devices or methods for meeting this need will be discussed briefly. It will then be possible to state the general problem to be treated in this paper.

A method that reduces the material requirements of the usual exponential or critical experiment is the "substitution," or two-region critical experiment (26), used mainly for the measurement of the

critical buckling. The critical buckling of a reference lattice is measured in a critical facility. This lattice is then used for the outer zone of a critical assembly. A number of cells at the center of the assembly are replaced by cells of the lattice under investigation, and the difference between the bucklings of the reference lattice and the two-region assembly is measured. This method has been used extensively for lattices moderated by D_2O, where the measurement of change in buckling can be made conveniently by measuring the change in height of the D_2O required for criticality. Only small amounts of the material to be tested are needed, and the ease of making the measurements is such that over 250 such measurements could be made at the Savannah River Laboratory over a period of about two years.

A second kind of facility which permits the use of small amounts of test material is represented by the Physical Constants Test Reactor (PCTR) (27, 28) at the Hanford Laboratory of the General Electric Company. The PCTR is a cube of graphite, seven feet on a side, with a cavity, two feet by two feet by seven feet, located at its center. It is made critical by enriched uranium distributed on the boundary of the central cavity. Several cells of the lattice to be tested are placed in the cavity and are poisoned with a thermal neutron absorber in such a way that the reactivity of the resulting assembly is the same as that of the reactor with the cavity. Since the buckling of the cavity is zero, that of the poisoned test region is also zero, and a value of $k_\infty - 1$ can be obtained from a two-group analysis of the experiment. An attempt is made to ensure that the energy spectrum of the neutrons incident on the test sample, as defined by some suitable integral spectral index, is the same as the spectrum which would exist in an infinitely large sample of the poisoned medium. This is accomplished in the PCTR by using a test sample considerably smaller than the central void region; the test sample can be surrounded by a buffer region of the same composition, which should help make the spectra in the test and buffer regions match. The main purpose of the PCTR is, then, to supply a null-reactivity method for measuring the infinite-medium multiplication factor, k_∞. Such a method, if successful, would give the value of k_∞, with the use of only a very small volume of the lattice array to be tested. The PCTR

can also be used for measurements of intracell flux distributions and of ratios related to f, p, and ϵ.

A third method for making simplified lattice measurements involves the irradiation of a "miniature" lattice with neutrons from a reactor (5, 8, 29). The miniature lattice may consist of a small number of lattice cells, that is, small compared to the number of cells in an exponential assembly, or it may contain much shorter fuel elements than are needed for an exponential assembly. Lattices with fuel elements only 20 inches long in H_2O moderator have been irradiated at the Brookhaven National Laboratory in a tunnel under the graphite-moderated research pile. Some of these rods, in lattices moderated by D_2O, and by mixtures of D_2O and H_2O, have also been irradiated at the Medical Facility of the M.I.T. Reactor. Miniature lattices cannot be used for the measurement of the critical buckling, but can be used for the measurement of intracell flux traverses and for ratios associated with p, f, and ϵ.

The use of the various assemblies mentioned raises some serious questions. First: does a measurement of a particular nuclear parameter in one of the different types of assemblies mentioned give the information desired for the design of an actual reactor? Second: how do the results of measurements made in different assemblies compare with each other and with measurements made in actual reactors? Third: if a measurement made in one type of assembly, e.g., a miniature lattice, does not give the same result as is obtained in another type of assembly, e.g., an exponential or critical assembly, can it be corrected so as to give agreement? The purpose of this paper is to review literature bearing on these questions with the object of trying to find some answers. A literature search has been made with the object of finding measurements that can be compared, but it is doubtless incomplete. More questions may be raised than can be answered, but the general problem seems to be one that will grow, rather than diminish, in importance, and even a start on the problem should be helpful.

This paper has its origin in several problems and observations. The first of these was the application of primitive pile theory, "adjusted" with the aid of the results of exponential experiments, to the design of

the Brookhaven uranium-graphite research pile, and the subsequent comparison between the design calculations and the actual behavior of the pile during its initial operation. Questions arose which have still not been completely answered. The broad use of this general method has added to the number of such questions. The second motivating factor was the paper by Wingfield and Hennelly (30) reporting work at the Savannah River Laboratory on the comparison of buckling measurements in exponential and critical assemblies moderated with D_2O. This work indicates that systematic differences may, indeed, exist between the buckling values obtained in exponential and critical assemblies. A third factor was the statement by R. Avery of the Argonne National Laboratory that analogous discrepancies have been observed in fast exponential and critical assemblies. Another general problem that raised questions was the investigation, at M.I.T. (29), of the possibilities of using a miniature lattice in the work of the M.I.T. Heavy Water Lattice Research Project.

The emphasis in this paper will be on the experimental results obtained in thermal systems and on some of the problems raised by the existing data rather than on the correlation of theory and experiment. The latter problem is a very large one which should be treated in separate studies.

II. GRAPHITE-MODERATED REACTORS

A. THE CRITICAL BUCKLING

Exponential and critical experiments were first used for natural uranium-graphite piles, and it is both useful and interesting to start by considering this type of system. Fermi and his colleagues at the Metallurgical Laboratory applied the results of exponential experiments to the first chain reacting pile, CP-1, which was the first zero-power critical experiment (31). The critical pile was a rotational ellipsoid with polar radius of 309 cm and equatorial radius of 388 cm; the effective critical radius of the ellipsoid was 355 cm and the experimental critical buckling was 78.3×10^{-6} cm^{-2}. The derived value of $(k_\infty)_{av}$ was 1.054. The lattice cell was a cube 8.25 inches on a side, and the central region of the pile contained about 6 tons of uranium metal. Exponential experiments on a lattice of this central type had yielded a material buckling of 101.7×10^{-6} cm^{-2}, which, with the calculated value of the migration area, gave $k_\infty = 1.067$. The greatest part of the volume of the pile contained lumps of UO_2; exponential experiments had given, for a lattice of this material, $B^2 = 59 \times 10^{-6}$ cm^2, and, with the calculated value of the migration area, $k_\infty = 1.039$. There were three small additional regions of small importance, and the use of one-group statistical weight theory led to a predicted value of 73.4×10^{-6} for the critical buckling. According to Fermi, "This (result) is an indication that the values of B^2 and of the reproduction factors calculated from exponential experiments have been slightly underestimated, the correct values (of k_∞) being about 0.003 or 0.004 higher than the predicted values." In view of the mixture of lattices in CP-1 and the use of one-group statistical weight theory, the discrepancy of about 6 per cent between the value of the actual buckling of the zero-power critical assembly, and the value derived from the results of exponential experiments may be taken to represent satisfactory agreement between the two types of experiments.

The next uranium-graphite pile for which information is available is the X-10 pile at the Oak Ridge National Laboratory (32). The pile, loaded to criticality for the first time, represents a critical assembly whose buckling can be compared with that of an early exponential experiment. The lattice cell of the pile contains a rod 1.40 cm in radius, clad in aluminum, with a volume ratio of aluminum to uranium of 0.10. The rod is surrounded by an air channel 20.25 cm^2 in area; the lattice spacing is 8.0 inches and the ratio of the number of carbon atoms to that of uranium atoms is close to 100 to 1. The experimental value of the critical buckling, obtained from the critical loading and the use of two-group reflector theory was 92×10^{-6} cm^{-2}. Early exponential experiments at the Metallurgy Laboratory (13) indicated a value of about 100×10^{-6} cm^{-2} at a carbon-to-uranium atom ratio of 100. When this value is corrected for the presence of the aluminum and the air channels, the result is in the neighborhood of 90 to 95×10^{-6} cm^{-2}, in good agreement with the critical experiment value.

The construction of the Brookhaven graphite research reactor permitted a more precise comparison between the exponential and critical values of the buckling (33, 12). It was possible to make detailed studies of the subcritical reactor, including an exponential experiment at a loading of 100 channels as compared with 387 channels required for criticality. The problem was complicated by the presence of air channels, the central gap (for the air coolant), and an anisotropy in the migration area of about 6 per cent. Both the exponential and critical loadings were surrounded by an effectively infinite radial reflector and a finite axial reflector. Nevertheless, the axial component of the critical buckling could be determined in the two cases with the results:

$$B_z^2(\text{exponential}) = (95.7 \pm 3.5) \times 10^{-6}\ cm^{-2},$$

$$B_z^2(\text{critical}) = (92.4 \pm 0.7) \times 10^{-6}\ cm^{-2},$$

which agree within the experimental uncertainties.

Another set of exponential and critical experiments was made during the initial operation of the French natural uranium-graphite reactor, G1 (34-37). This pile, like the Brookhaven graphite pile, is divided into two parts by a central, transverse air gap. Two zero-power critical experiments were made. In one, uranium rods were

loaded only on one side of the gap. The resulting "flat" pile required 508 loaded half-channels for criticality; the critical radius was 254.4 cm, and the height 372 cm. In the second, or "long," pile fuel was loaded on both sides of the gap in 329 channels; the critical radius was 204.8 cm and the height 752 cm. The values of the critical buckling, corrected for the anisotropy of the migration area, were $(103.8\pm1.4) \times 10^{-6}$ cm^2 for the "flat" pile, and $(104.1\pm1.4)\times 10^{-6}$ cm^2 for the "long" pile. In a third experiment, a "bottle" pile was constructed. A cylinder 1.54 meters in radius was loaded on one side of the gap, and was surrounded by a cage of tubes of cadmium loaded in channels around the cylinder. The cylinder formed an exponential pile; neutrons were supplied by loading the other side of the gap in such a way that the entire "bottle" pile was just critical. The radial and axial bucklings of the exponential pile were found to be $(201.5\pm1.2)\times 10^{-6}$ cm^{-2} and $(-91.5\pm1.3)\times 10^{-6}$ cm^{-2}, respectively, or a total buckling of $(110.0\pm1.8) \times 10^{-6}$ cm^{-2}. Correction for the anisotropy of the migration area gave an effective buckling of $(103.9\pm2.6)\times 10^{-6}$ cm^{-2} for the exponential experiment, in good agreement with the two critical experiment values.

In the four cases discussed, the agreement between the values of the critical buckling obtained in exponential and critical experiments is good. But, in each case, the comparison is complicated by other conditions for which corrections must be made, and the range of lattices covered is very small. Some additional comparisons have been made by Heineman (38) who points out that only a few critical experiments have been made for natural uranium-graphite reactors, and that some of these critical systems were designed for high power operation and could not be compared directly with the relatively clean exponential pile assemblies. When corrections were made for the differences in design, the residual differences in buckling were reasonably small (14). Similar results were obtained with a Calder Hall Reactor. Finally, two direct comparisons of buckling measurements in exponential and critical assemblies with slightly enriched uranium have been reported (16). The results are given in Table 1.1. The results indicate that the same values of the buckling were obtained in the exponential and critical assemblies within the uncertainties of the measurements. In both the exponential and critical experiments, however, the curvature of the flux along the

TABLE 1.1

Bucklings Measured in Exponential and Critical Assemblies of Slightly Enriched Uranium and Graphite

	Lattice	Buckling (Corrected for Anisotropy) Exponential Measurement (cm^{-2})	Critical Measurement (cm^{-2})
(1)	1.2 inch diameter rod 1.5 inch diameter air channel 7.875 inch lattice spacing U^{235} concentration: 1.3 × normal	$(318 \pm 3) \times 10^{-6}$	$(322 \pm 2) \times 10^{-6}$
(2)	1.2 inch diameter rod 1.5 inch diameter air channel 7.875 inch lattice spacing U^{235} concentration: 1.6 × normal	$(467 \pm 3) \times 10^{-6}$	$(468 + 2) \times 10^{-6}$

axis of the lattice was small, that is, the exponential pile was not far from critical, and the bucklings were not expected to be very different.

The experimental evidence discussed so far indicates that in the limited range of uranium-graphite lattices for which results have been obtained in both exponential and critical experiments, no significant differences have been observed between the values of the critical buckling obtained in the two types of assemblies.

In the subcritical and critical experiments done with the Brookhaven pile, the radial reflector saving was studied in some detail. It was determined from radial flux traverses at subcritical fuel loadings between 100 and 300 channels (33); the result obtained was 3.00 ± 0.07 lattice units, where 1 lattice unit is 8 inches. This value was very close to 61 cm, in good agreement with the value calculated with two-group theory for the particular graphite used in the pile. Radial flux traverses were also measured during the period of initial critical operation of the pile, at fuel loadings from 277 to 461 channels, with the critical loading at 389 channels. The result obtained from these measurements was

3.05 ± 0.09 lattice units (120). Under the conditions of the experiments, therefore, the reflector saving was constant, within the experimental uncertainties, over a range of loadings from 100 channels to 461 channels, and agreed very well with the value predicted by two-group theory.

Measurements of the material buckling have been made in uranium-graphite exponential piles of different sizes at Hanford, where a comprehensive program of exponential experiments has been under way for some years (13, 39-42). To permit a less expensive survey of fuel arrangements than would be possible in piles eight feet on a side, the usual size of exponential experiments at Hanford, measurements have been made in small piles, four feet long and five or six feet wide as well as in larger piles (43, 44). Some of the small piles were only 5 or 6 lattice units wide and 8 or 9 units high. The small piles have the inherent difficulty that the inferred value of the material buckling is quite sensitive to the value of the transverse extrapolation length. The relative values of the material buckling obtained for different lattices from measurements in a small pile were found to be consistent so long as measured values of the extrapolation length were used but not in agreement with the values obtained in large piles. The latter values are less sensitive to uncertainties in extrapolation length, and were used to normalize the small pile results. The buckling was calculated from the usual formula:

$$B_m^2 = \frac{\pi^2}{(a+2\lambda_T)^2} + \frac{\pi^2}{(b+2\lambda_L)^2} - \frac{1}{B_{11}^2}, \qquad (2.1)$$

where a and b are the physical width and length of the pile, respectively; λ_T and λ_L are the transverse and longitudinal extrapolation lengths, respectively, with the longitudinal direction taken to be along the fuel rods; B_{11} is the vertical relaxation length of the flux in the fundamental mode, and was determined by making a least squares fit to vertical traverse activations after correcting for harmonics and finite height; λ_T and λ_L were determined by fitting cosine curves to horizontal traverse data. The extrapolation lengths obtained in this way were found to depend on the arrangement of the neutron sources used in the experiments, e.g., on whether they are clustered near the vertical axis of the pile or split (spread apart) to reduce harmonic effects; they also seem to depend on

the position in the cell at which the activation measurements are made. Finally, the extrapolation lengths were found to vary with the energy of the neutrons: the λ's for epicadmium neutrons were consistently larger than those for subcadmium neutrons. Since the values of λ_T and λ_L seem to depend in complicated ways on the lattice as well as on the details of how the measurements are made, the transverse and longitudinal bucklings, defined by

$$B_T^2 = \frac{\pi^2}{(a+2\lambda_T)^2}, \tag{2.1a}$$

$$B_L^2 = \frac{\pi^2}{(b+2\lambda_L)^2} \tag{2.1b}$$

also varied for fixed values of a and b. The Hanford workers found (43) that the uncertainties (variations) in λ_T could be as large as an inch, and the corresponding variation in transverse buckling could be as large as $55\mu B$ ($1\mu B = 10^{-6}$ cm^{-2}) in a pile 4 feet wide, $16.4\mu B$ in a pile 6 feet wide, $6.9\mu B$ in a pile 8 feet wide, and $3.5\mu B$ in a pile 10 feet wide. These uncertainties, combined with analogous uncertainties or variations in the longitudinal buckling, may introduce significant variations into the final values of the material buckling obtained for different lattices and conditions of measurement even though the physical size of the exponential pile is fixed.

These results emphasize the difficulty of determining accurate values of the buckling from measurements in small piles only a few cells wide and the need for better understanding of the theory of small assemblies.

The extrapolation distance for thermal neutrons has been studied carefully at the Brookhaven National Laboratory in stacks of graphite $4 \times 4 \times 4$ feet (130). The neutrons came from the thermal column of a reactor, and had a spatial distribution matched to the fundamental mode of diffusion in the stack. Thermal neutron flux traverses with bare indium foils were used to obtain the thermal neutron diffusion length; extrapolation distances were obtained by fitting the observed thermal neutron fluxes in the transverse directions to a cosine function.

The distance was taken to be 0.7104 λ_{tr} and was used to determine the value of λ_{tr}. The result obtained was

$$\lambda_{tr} = 2.77 \pm 0.06 \text{ cm},$$

for graphite of density of 1.60 g/cm^3. This value is in good agreement with the value calculated from the average scattering cross section of graphite, $\bar{\sigma}_s = 4.8 \pm 0.2$ barns, namely $\lambda_{tr} = 2.74 \pm 0.11$ cm. The experimental determination of the extrapolation distance, even in small graphite stacks, may be accurate enough to yield a satisfactory value of the transport mean free path in graphite.

There is still a problem, however, because the value of the transport mean free path consistent with the results of pulsed neutron measurements is 2.60 ± 0.05 cm for graphite of density 1.60 g/cm^3, somewhat lower than the value obtained from the extrapolation distance. In an attempt to get further information bearing on this problem, the transport mean free path of thermal neutrons was also determined, by the Brookhaven workers, by means of a "poison" technique. This method (131) is to poison the moderator with a material of known thermal neutron absorption cross section and to measure the diffusion length of the mixture as a function of the amount of poison. If the poison has a negligible moderating effect, a graph of the reciprocal diffusion area against the poison concentration gives a straight line whose slope is proportional to the transport cross section of the moderator, and whose intercept is proportional to the absorption cross section of the moderator. In the Brookhaven experiments, the graphite was poisoned with copper. The value obtained for the transport mean free path was $\lambda_{tr} = 2.74 \pm 0.03$ cm, for a graphite density of 1.60 g/cm^3, in agreement with the value obtained from the extrapolation distance, but somewhat higher than that obtained from pulsed neutron experiments.

The diffusion of thermal neutrons in a graphite stack was studied at Harwell (131) with somewhat different results from those obtained at Brookhaven. Four cylindrical antimony-beryllium photo neutron sources in a graphite pedestal supplied neutrons to a square stock of graphite 4 feet on a side. Thermal neutron flux traverses gave a value of the transport mean free path of graphite of 4.38 ± 0.28 cm, considerably greater than that obtained from the scattering cross section, in the

Brookhaven experiments, and from pulsed neutron measurements.

It is evident that even experiments in graphite moderator present difficulties in the measurement of transverse bucklings.

B INTRACELL LATTICE PARAMETERS

1. Intracell Thermal Neutron Density (or Flux) Distribution and the Thermal Utilization

Programs of measurements in subcritical and critical assemblies now often include measurements of intracell lattice parameters in addition to the determination of the buckling. Only a small amount of work has been done in critical assemblies of uranium and graphite, and only a few comparisons can be made between results obtained in subcritical and critical assemblies. In the case of the intracell thermal neutron density (or flux) distribution, from which the thermal utilization can be derived, the main purpose of the work on uranium-graphite lattices has been correlation between theory and experiment, with emphasis on the development of semiempirical methods of improving diffusion theory calculations (45-49). Nearly all of the measurements have been made in exponential experiments or in the PCTR.

The early experimental work on the Brookhaven natural uranium-graphite pile permitted a comparison between values of the thermal utilization in the subcritical pile and in the actual critical pile. The thermal utilization of the moderator (f_{mod}) is related to the cadmium ratio; if the latter is measured, e. g., for indium foils, and the small amount of absorption in the aluminum rod jackets is taken into account, the thermal utilization of the fuel can be obtained. The subcritical Brookhaven pile was found to have a constant cadmium ratio of 6. 10 (33) at loadings between 100 and 300 channels (387 loaded chains were needed for zero-power criticality), leading to a value of 0. 899 for f, with an uncertainty of about 1. 5 per cent. A measurement was made (50) of the thermal neutron distribution in a lattice cell of the fully loaded pile operating at a power level of 40 kilowatts, and at a temperature close to room temperature. The thermal utilization was then calculated from the formula:

$$f = \frac{N_u \sigma_{au} \bar{\phi}_u V_u}{N_u \sigma_{au} \bar{\phi}_u V_u + N_M \sigma_{au} \bar{\phi}_M V_M + N_{al} \sigma_{al} \bar{\phi}_{al} V_{al}}, \qquad (2.2)$$

where the N's represent atomic densities in number of atoms per cm^3, the σ's are absorption cross sections, the $\bar{\phi}$'s represent average fluxes, the V's represent volumes, and u, M and al stand for uranium, graphite (moderator) and aluminum, respectively. The values of the cross sections were those at 2200 meters per sec, and the uranium cross section was corrected for deviation from 1/v-dependence. The resulting value of the thermal utilization was 0.890 with an uncertainty of about one-half per cent. The experimental values of the thermal utilization in the critical pile and in the subcritical pile at loadings as low as one-fourth that at criticality, therefore, agreed within the uncertainties of the measurements.

The thermal utilization of the French pile, G1, was also obtained from a measurement of the flux distribution in a cell of the cold critical pile (36, 51). The lattice was very similar to the lattice of the Brookhaven pile, and the measured value 0.895 ± 0.005 agreed well with that obtained for the latter pile.

Measurements of the thermal utilization have been made in the PCTR (28, 52). The results agree, on the average, to within one per cent with values obtained from measurements in exponential assemblies (39). The PCTR seems, therefore, to provide a satisfactory assembly for the measurement of the thermal utilization with a small lattice sample.

2. The U^{238} Cadmium Ratio, Conversion Factor, and Resonance Escape Probability

Measurements of these quantities are even fewer in number in uranium-graphite lattices than measurements of the thermal utilization. It is only in recent years that reasonably satisfactory experimental methods have been developed (5, 9, 55*), and they have not yet been applied extensively to lattice studies. The measurements that have been made have been used mainly for correlation with theoretical methods, and little comparison is possible of results obtained in different types of assemblies.

A measurement was made, in the critical Brookhaven natural uranium-graphite pile, of the ratio of resonance capture in U^{238} in a

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*Reference (55) contains an extensive bibliography.

fuel rod to thermal capture in U^{238} (56). This ratio denoted by S, can be related to the resonance escape probability by means of the formula (5):

$$p = \frac{1 + [(\epsilon-1)a_{28}/\epsilon(\nu_{28}-1-a_{28})\mathscr{L}_1]}{1 + [f\Sigma\mathscr{L}_2\mathscr{L}_3 S(1-\delta/S)(1+\delta)]} , \qquad (2.3)$$

where f, ϵ, are obtained from measurements, $\mathscr{L}_1$, $\mathscr{L}_2$, $\mathscr{L}_3$ are non-leakage probabilities in the fast, resonance, and thermal energy regions, respectively, $\Sigma = \frac{\Sigma_{abs}(U^{238})}{\Sigma_{abs}(\text{fuel})}$, and δ is defined by the fact that δ/S is the fraction of the epicadmium capture that is 1/v. The values of the non-leakage probabilities were obtained from the known buckling of the pile and the calculated values of the thermal diffusion area and partial ages. The resulting value of p may be regarded as an " experimental" value: the result for the Brookhaven pile lattice was $p = 0.886 \pm 0.006$. Similar measurements were made in the French pile G1 on two lattices, one with natural uranium rods 2.6 cm in diameter, the second with rods 3.2 cm in diameter (36, 57). With the use of a formula analogous to equation (2.3), p was found to be 0.898 ± 0.005 for the 2.6 cm rods, and 0.863 ± 0.009 for the 3.2 cm rods. The lattice with the 2.6 cm rods was very close in constitution to the Brookhaven lattice and the two results agree reasonably well. The second G1 lattice contained more uranium than the first, and the value of p was correspondingly smaller. The Brookhaven and French experiments yielded values of p which agreed with the values calculated from the formula for p used in the early "recipe" for the design calculations of natural uranium-graphite piles (11, 12).

Analogous measurements made in a Hanford pile (58) on somewhat larger uranium rods gave results which were about three per cent smaller than the values calculated from the formula of the old recipe. This disagreement is not surprising and is consistent with the fact that much less work has been done on the problem of resonance absorption in uranium-graphite assemblies than in uranium-water systems.

Foil techniques have been used in the United Kingdom to measure the ratio of plutonium production to U^{235} consumption, and values of the conversion factor have been obtained in exponential piles (16, 59).

The results have been used in a two-group correlation of theoretical and experimental information, but cannot be compared with values for critical assemblies because of the absence of data for the latter.

Measurements from which values of the conversion ratio and resonance escape probability can be obtained have also been made in the PCTR (54). They have been used with experimental values of k_∞, f and ϵ to arrive at values of η for lattices, and this type of correlation will be considered in section II, 8, 4.

3. The Fast Fission Factor

Measurements of R, the ratio of the fission rate in U^{238} to that in U^{235}, were made in the Brookhaven natural uranium-graphite pile (5, 12, 60). This ratio is also called δ, or δ_{28} in the literature. It can be related to the fast fission factor by the formula

$$\epsilon = \frac{R}{25}(\nu_{28} - 1 - \alpha_{28}) = \text{constant} \cdot R, \qquad (2.4)$$

where ν_{25} is the average number of neutrons emitted per fission of U^{235}, ν_{28} is the average number of neutrons emitted per fission of U^{238} induced by fission neutrons, and α_{28} is the average value, over the fission neutron energy spectrum, of the ratio of the capture cross section of U^{238} to the fission cross section. The results of measurements in graphite lattices have usually been reported in terms of ϵ, and different workers have used different values for the nuclear constants in equation (2.4). It is probably best, therefore, to convert the reported values of ϵ to the measured values of R, and compare the latter, as has been done with some of the results by Fleishman and Soodak (61). In the Brookhaven experiments, rods 0.750, 1.10 and 1.345 inches in diameter, respectively, were irradiated in a fuel channel and also in the graphite reflector. In the latter case, the rod was far enough from the lattice so that the experiment represented a measurement of the fast fission effect in an isolated rod. The values of R obtained were:

0.750 inch diameter rod in fuel channel:	0.0395
0.750 inch diameter rod in reflector:	0.0485
1.10 inch diameter rod in fuel channel:	0.0610
1.10 inch diameter rod in reflector:	0.0628
1.345 inch diameter rod in fuel channel:	0.0826
1.345 inch diameter rod in reflector:	0.0772

Within the precision of the experiments, the fast fission effect was independent of the position of the rod, indicating that the interaction effect is negligible. The Brookhaven lattice spacing was 8 inches and this effect would be expected to be small. Measurements were also made at Hanford (62), in the 7.5-inch uranium-graphite lattice of the KW pile, in two rods 1.33 inches in diameter. Values of 0.0662 and 0.0605 were obtained for R, somewhat lower than those obtained at Brookhaven.

Additional measurements of the U^{238}:U^{235} fission ratio have been made at Harwell (63, 64). A single rod of natural uranium, surrounded by an air channel whose diameter was varied from 1.2 inches to 3.75 inches, gave values with an average of 0.0691, with uncertainties of about 3.5 per cent in the individual measurements; natural uranium rods of the same diameter, in an exponential lattice with a spacing of 8 inches, give a value of 0.0769, again with uncertainties of about 3.5 per cent. These results agree reasonably well with the Brookhaven results and may indicate a small interaction effect. A lattice of 1.2 inch diameter rods with a U^{235} concentration 1.6 times natural, and with a spacing of 4.95 inches, gave R = 0.0964, showing a definite interaction effect. Finally, single rods 2.25 inches in diameter gave values of R averaging 0.120. The Brookhaven and Harwell results indicate that when there is no interaction effect, the same values of R are obtained in a critical or exponential pile as for a single rod, as would be expected. Insufficient work has been done on the interaction effect in graphite-moderated lattices to permit any conclusions to be drawn with respect to possible differences between results in exponential and critical assemblies.

The values of the constants to be used in equation (2.4) have varied over the years: the earliest value of the coefficient of R used at Brookhaven was 0.557 (3); the most recent value is 0.708 (65). The

latter value is based on the following values of the constants ν_{25}, ν_{28} and α_{28}: $\nu_{25} = 2.43 \pm 0.02$ (66-68); $\nu_{28}/\nu_{25} = 1.160 \pm 0.020$, (69), so that $\nu_{28} = 2.82 \pm 0.05$. The value $\alpha_{28} = 0.107$ was obtained at Brookhaven (3) from the expression:

$$\alpha_{28} = \frac{\int_{E_o}^{\infty} \sigma_\gamma^{28}(E) N(E) dE}{\int_{E_o}^{\infty} \sigma_f^{28}(E) N(E) dE} . \qquad (2.5)$$

Here, N(E) represents the energy distribution of the fission neutrons from U^{235}, σ_γ^{28} and σ_f^{28} are the radiative capture and fission cross sections of U^{238}, respectively, and E_o is the threshold energy for U^{238} fission. Numerical integration of equation (2.5) gave $\alpha_{28} = 0.107$.

Measurements have also been made at Hanford in the PCTR (54), in a test cell surrounded by eight buffer cells with the same lattice spacing (7-1/2 inches). Natural uranium rods 1.330, 1.679, and 1.923 inches in diameter gave R = 0.064, 0.081, and 0.100, respectively, in reasonably good agreement with the Brookhaven and Harwell results, as well as with the Hanford results cited earlier in this section. The cases studied in the PCTR would be expected to show little, if any, interaction effect, except possibly for the lattice with 1.923 inch diameter rods. If the lattice spacing were decreased to such an extent that the interaction effect were to become important, measurements in the PCTR might become difficult to interpret, and experiments of this kind may be of interest from the standpoint of the use of small assemblies for the measurement of reactor parameters.

Finally, the correlation of integral experiments, including measurements of U^{238}:U^{235} fission ratios, and high energy cross sections has been reviewed by Chernick and coworkers (70) from a somewhat different viewpoint from that of this paper.

4. The Multiplication Factor, k_∞, for an Infinite Lattice

Values of k_∞ (actually of $k_\infty - 1$) have been determined from measurements made in the PCTR (27, 28, 38) with much smaller lattice samples than those used in exponential piles. Values of $k_\infty - 1$ can also be obtained

from critical experiments; they depend on measured values of the buckling and on delayed neutron fractions. Both PCTR and critical experiments were made on a uranium-graphite lattice with a ratio of carbon to uranium atoms of 70.62; the uranium rods were surrounded by a coolant channel in which air or water could be used. The values of $k_\infty - 1$ obtained in the two types of experiment were in agreement, within the limits of the uncertainties cited, i..e., to 0.002 or 0.003 in k_∞. Another comparison of this kind was made for a lattice containing rods 1.36 inches in diameter at a spacing of 8-3/8 inches and with a value of 90.77 for the ratio of carbon to uranium atoms; the agreement was again good. Values of $k_\infty - 1$ obtained in the PCTR have also been compared with the results of exponential experiments, by using calculated values of the migration area. The agreement was reasonably good, although not quite as good as that between PCTR and critical measurements. As a result, the Hanford workers consider that the value of the migration area of a lattice is best derived from a PCTR measurement of k_∞ and an exponential pile measurement of the material buckling. This kind of combined experimental study should be very useful.

It has been mentioned that values of f, p, and ϵ can also be obtained from PCTR measurements (54). These can be combined with the value of k_∞, and a value of η can be derived. The precision of the value of η obtained in this way is not great, of course, since the various experimental uncertainties are compounded in η. A set of experiments was made with rods 1.679 inches in diameter at a lattice spacing of 7-1/2 inches. In one case, the rods were cooled with air. In a second case, the rods were cored, with a 0.75 inch diameter central channel and the control coolant was air or water. The three experiments yielded values of 1.335, 1.328, and 1.329, respectively, for η. Although these results are reasonably consistent, they disagree with the result obtained for η when a series of lattice experiments was analyzed at Hanford (52) by means of the method of Mummery (15); the latter procedure gave $\eta = 1.313 \pm 0.013$, in agreement with the value obtained from the early exponential experiments at the Metallurgical Laboratory.

The results of the PCTR experiments have been generally

encouraging, with the result that the method is now used extensively at Hanford and has also been applied to lattices moderated with D_2O, H_2O, and organic moderators. Because of its great flexibility, the PCTR deserves more detailed studies of its theory and behavior. One such study is under way at M. I. T. with the cooperation of the Hanford group. More extensive studies of the ratios related to ϵ, p, and f should also be undertaken, together with detailed comparisons with results measurements in critical and exponential assemblies. These studies would do much to help overcome the difficulties of doing critical and exponential experiments in large, expensive, and clumsy lattices.

III. URANIUM-HEAVY WATER LATTICES

A. CRITICAL BUCKLING

More extensive comparisons of buckling measurements in critical and exponential experiments have been made for lattices of natural uranium metal rods in D_2O than for uranium-graphite lattices. The following discussion is based to a considerable extent on the paper of Wingfield and Hennelly (30) and on subsequent work at the Savannah River Laboratory (30, 71, 72).

The early studies indicated the possibility of differences between values of the buckling obtained for uranium-heavy water lattices in critical assemblies and in exponential assemblies. In both studies, the fuel rods were exchanged between cooperating laboratories. In one set of experiments, buckling measurements were made on lattices of single columns of natural uranium metal slugs, 3.80 cm in diameter, clad with 1.0 mm of aluminum, first in the exponential facility at North American Aviation, Inc. (NAA) (73), and later in the critical facility, ZEEP (21), at Chalk River. In this comparison, the bucklings obtained from the critical experiments were approximately 30μB ($1\mu B = 10^{-6} cm^{-2}$) smaller than those obtained from the exponential experiments. In the second study, the buckling measurements were made on clusters of 13 and 19 solid uranium oxide rods, 1.56 cm in diameter, clad in 0.8 mm aluminum, and with a distance of 1 mm between jackets, in the Swedish exponential facility ZEBRA (74, 75) and in the French critical facility, AQUILON (18). The results obtained from the critical experiments were within 30μB of those obtained from the exponential experiments but seemed to show systematic differences. The possibility that there may be real differences between critical and exponential buckling measurements led to the experiments at the Savannah River Laboratory.

In the SRL experiments, buckling measurements were made (30) on 42 lattices of 1.00 inch natural uranium rods about 6.5 feet long, in heavy water, in an exponential assembly, the SE. The fuel assemblies

studied were single rods and clusters of 3, 7, and 19 rods, respectively; the lattice spacings were varied from 3.00 to 21.59 inches. The bucklings of fourteen of the lattices were also obtained from measurements (76) in the Process Development Pile (PDP), a critical facility. In the first set of exponential measurements, the radial buckling was taken as constant at a value of 925μB, determined experimentally from measurements on two lattices with moderator-to-fuel volume ratios of 65.5 and 16.4, respectively. The PDP and SE results were found to be approximately in agreement, but detailed analysis of the results showed systematic trends, as shown in Fig. 3.1. This figure shows a graph of $[B_m^2(SE)-B_m^2(PDP)]$ versus moderator-to-uranium volume ratio. The straight lines in the figure were all drawn through the point $V_m/V_f = 1.25$ which corresponds to a lattice of single rods with a spacing of 1.5 inches, and is the limiting case for each of the clustered arrangements. The graph indicates that the differences between the values of the buckling measured in the PDP and SE depend on the type of fuel assembly and on the moderator-to-fuel volume ratio.

Wingfield and Hennelly have also shown that agreement can be obtained between the earlier NAA and ZEEP (exponential-critical) results with rods 1.5 inches in diameter if the appropriate values of $[B_m^2(SE)-B_m^2(PDP)]$ from the single rod curve of Fig. 3.1 are applied to the data. They have suggested the possibility that the observed SE-PDP differences may be generally applicable to exponential-critical measurements of buckling in heavy water. They have also suggested several possible causes for the differences between the exponential (SE) and critical (PDP) bucklings. First, a theoretical correction could be applied to the exponential data for the change in the radial buckling B_r^2 due to the change in extrapolation distance for lattices with different transport mean free paths. Calculations based on simple two-group theory indicated a correction of about 15μB over the entire range of lattices studied, but this correction would account only partially for the observed differences between the SE and PDP experiments. Second, there may be systematic variations in the radial buckling in the exponential experiments that cannot be calculated from simple two-group theory. Third, there may be differences in some of the nuclear parameters, e.g., the resonance escape probability, owing to differences in the

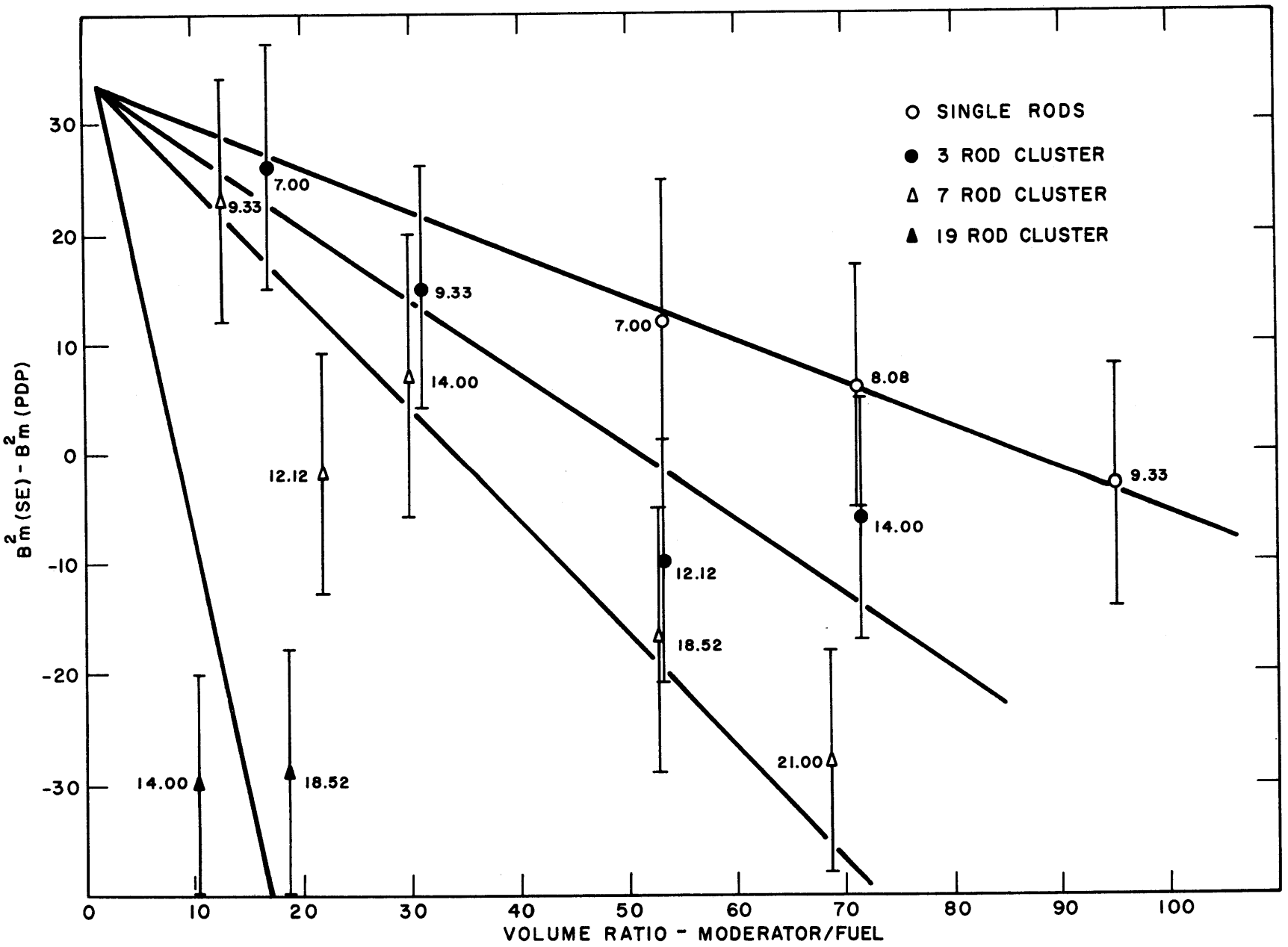

FIG. 3.1 - $B^2m(SE) - B^2m(PDP)$ vs Vm/Vf FOR NATURAL U RODS (0.998" DIA.) - 99.75% D_2O (From Ref. 30).

sizes of the lattices in the two facilities. Fourth, there may be changes in the neutron diffusion properties of the lattices with larger fuel assemblies.

The possibility that variations in the radial buckling in the exponential assembly (SE) might contribute to the observed differences has been investigated experimentally (71). The radial buckling was measured by activating gold pins at radial positions throughout the SE tank and fitting the activations by least squares to the appropriate J_o Bessel function distributions. Measurements were made for at least one lattice of each of the fuel assembly types investigated in the original SE-PDP comparison as well as for the case of D_2O only in the SE. The results are shown in Fig. 3.2. This figure is a composite graph which shows the differences between values of the buckling measured in the SE and PDP as well as the measured values of the radial buckling. The differences between the measured radial bucklings and the previously assumed constant value of 925μB correspond closely to the differences between exponential and critical material bucklings. The value of the radial buckling when only D_2O was in the exponential tank was 955μB, greater than that obtained for any of the lattices. A result similar to the last has also been obtained in exponential experiments at M.I.T. (77). Lattices of uranium rods 1.01 inch in diameter with spacings of 4-1/2, 5 and 5-3/4 inches, had measured radial bucklings of 1411, 1412, and 1420μB, respectively, with a statistical uncertainty of about 5μB. With D_2O only in the tank the measured radial buckling was 1470μB; although the uncertainty in this measurement was greater than in the lattice measurements, the difference between the lattice bucklings and the value for D_2O was significantly greater than the experimental uncertainties. Similar effects have also been mentioned in connection with uranium-graphite subcritical assemblies (42b). Experiments are now in progress in the same exponential tank on lattices of 0.25 inch diameter rods of uranium metal slightly enriched in U^{235}, at different lattice spacings, and it will be interesting to see what values are obtained for the radial buckling.

In the M.I.T. experiments, the buckling has also been measured for epicadmium neutrons, but so far in only a small number of lattices. The results obtained for the radial buckling of a lattice of one inch

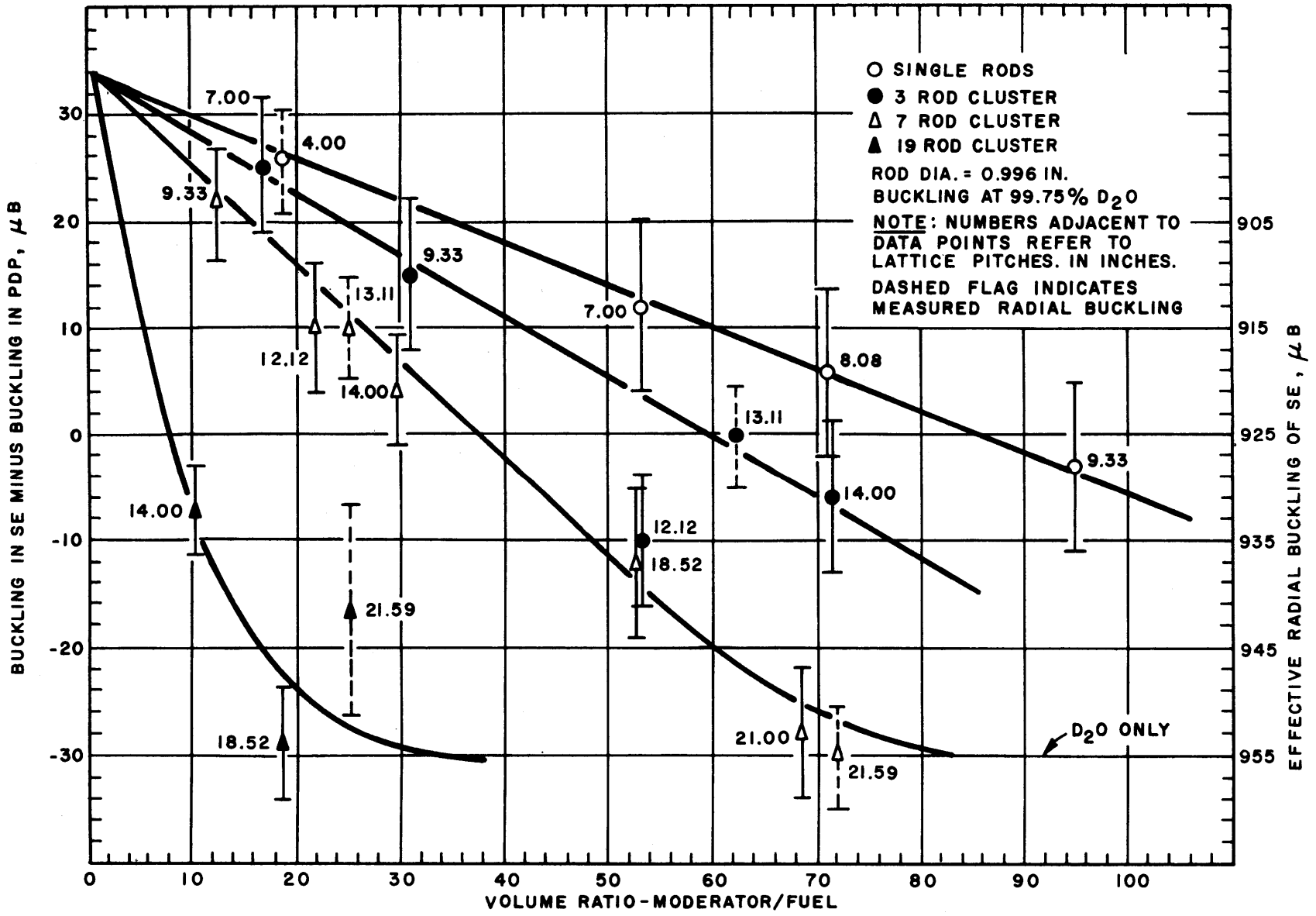

FIG. 3.2- EFFECT OF MODERATOR TO FUEL RATIO ON DIFFERENCES BETWEEN EXPONENTIAL AND CRITICAL BUCKLINGS FOR NATURAL U RODS IN D_2O (From Ref. 71).

diameter uranium metal rods with a spacing of 4-1/2 inches were:

Experiment Number	Radial Buckling B_r^2 (μB)	Foils Used in the Measurements
20	1414	1/4-inch bare Av foils
26	1416	1/4-inch bare Av foils
23	1404	1/2-inch bare Av foils
25	1423	Cd-covered 1/2-inch Av foils
50	1410	Cd-covered Av foils

The average value of the radial buckling in experiments 20, 26, and 23 is 1411μB with an assigned uncertainty of $\pm$ 6μB, which is thought to include possible systematic errors as well as the statistical uncertainty. The experiments with cadmium-covered foils had somewhat larger uncertainties because of the smaller number of activation points. Within the limits of the uncertainties the same values were found for the radial buckling, for both the total and epicadmium neutron distributions. The cadmium ratio in the moderator was constant radially across the tank, with a value of 10.5; only the outermost value, 10.8, differed from the value obtained for the other points, corresponding to a change in the epicadmium neutron flux of about 0.3 per cent. It seems reasonable to conclude, therefore, that in this lattice the radial buckling was the same for both thermal (subcadmium) and epicadmium neutrons. When the same one-inch diameter rods were at a spacing of 5-3/4 inches, the radial buckling obtained with bare gold foils was 1420 $\pm$ 5μB, and with cadmium-covered foils 1392 $\pm$ 9μB. Although the difference between the two measurements is somewhat greater than the quoted uncertainties, it is too small to be considered significant.

The M.I.T. experiments have also shown small deviations, of the order of a few per cent, from exact separability of macroscopic and microscopic thermal neutron density distributions. These deviations appear only near the outer boundary of the exponential tank and seem to differ from the oscillations observed at Hanford (78).

The effects of reflectors and voids on exponential measurements in uranium-D_2O lattices have recently been studied at the Savannah

River Laboratory (118). The experimental method used was to measure the vertical relaxation length, $1/\kappa$, for a lattice that completely filled an exponential tank, then to remove fuel assemblies symmetrically from the periphery and redetermine the vertical relaxation length for each core size. The radial buckling for each reduced core, $B_r^2(RC)$ was determined from the relation:

$$B_r^2(RC) = B_r^2(FC) - \frac{M_z^2}{M_r^2}[\kappa^2(FC)-\kappa^2(RC)] , \qquad (3.1)$$

where (FC) and (RC) refer to full and reduced core loadings, respectively, and M_z^2 and M_r^2 are the vertical and radial components of the migration area, respectively. Some of the fuel assemblies contained voids, some did not. The results showed that even thin reflectors must be taken into account when ascribing a value of the radial buckling to a particular lattice loading, and that large changes in the radial buckling may occur when voids are created in a lattice.

Lattices of uranium rods in D_2O have been studied in Sweden in an exponential facility (ZEBRA) of small diameter (74, 75). This tank is 3 meters high and 1.0 meters in diameter, giving a radial buckling of about 2100μB. The maximum value of the material buckling in natural uranium-heavy water lattices is in the neighborhood of 800μB, so that the axial buckling is about 1300μB; an uncertainty of 0.2 per cent in the relaxation length leads to an uncertainty of 5μB. The axial neutron distribution is measured with a special detector arrangement which suppresses the higher harmonics in the radial distribution. Although the radial distribution could be measured, it was calculated from the tank size and shape with an accuracy that was thought to be 20μB. This facility has also been used for substitution experiments. Early experiments (74) yielded values of the material buckling somewhat higher than the NAA (73) values, which were obtained in a tank 5 feet in diameter. Both Cohen (73) and Persson et al. (74, 75) have mentioned the possibility that there may be systematic deviations depending on the radius of the tank.

The small Swedish exponential facility was also used for the buckling measurements on lattices of oxide rods, mentioned at the

beginning of this section, which gave results differing somewhat from results obtained in substitution experiments in the French critical facility AQUILON. Two possible explanations have been offered for the discrepancy: uncertainty in the calculated radial bucklings in the exponential experiments; and anisotropy in neutron diffusion. In the latter case, an exponential experiment characterized by a radial buckling B_r^2 and a vertical buckling κ^2 must be compared with the same lattice in a critical experiment characterized by B'^2_r and B'^2_z. The leakages are then related by the equation

$$M_r^2 B'^2_r + M_z^2 B'^2_z = M_r^2 B_r^2 - M_z^2 \kappa^2 \quad . \tag{3.2}$$

For the Scandinavian lattices, $B_r^2 - \kappa^2$ was about one-fifth of B_r^2, and a 2 per cent difference between M_r^2 and M_z^2 (if such a difference exists) would cause about a 10 per cent difference in the buckling. In the Saclay assembly, B'^2_z and B'^2_r were nearly the same, and the anisotropy might account for the discrepancy between the two sets of experiments. Anisotropies of this order have been observed in uranium-graphite lattices when the fuel rods were surrounded by air channels (12). They have also been observed in lattices of slightly enriched uranium rods in ordinary water, in which the ratio of water to uranium volumes was between 2 and 5 (7). It is not clear, however, that the fuel clusters in the Scandinavian-French experiments could cause enough anisotropy to account for the observed discrepancies in buckling.

Later measurements of the radial buckling in ZEBRA (99) indicate that calculated values may have been too high by about 10μB, but this change alone is not enough to account for the apparent differences between the critical and exponential results. There is still some question as to whether the differences are real or just represent experimental uncertainties (24). Careful analysis and a method such as that used by Wingfield and Hennelly must be used before it can be established that the differences are real and systematic. Thus, the experimental results in the case of the 19-rod clusters do not seem to show the discrepancies observed with the 13-rod clusters although the discussion of the results, e. g., by Persson (75), does not distinguish between the results obtained with the two different fuel assemblies.

Measurements have been made at the Argonne National Laboratory on UO_2-Tho_2-D_2O lattices in 3 foot diameter and 5 foot diameter exponential tanks, and in critical assemblies (82). Unfortunately, the number of comparable experiments is small and the experimental uncertainties are considerably larger than in the uranium-heavy water lattices. Measurements were made on three lattices in both the 3 foot and 5 foot exponential tanks. In each case, the values obtained were greater in the 5 foot tank. The pairs of values are, with the value for the 5 foot tank listed first: 65μB as compared to 49μB, 665μB and 607μB, 1060μB and 887μB; the differences are 16, 58, and 173μB, respectively. The uncertainties in the measurements may have been 50μB or more. Preliminary critical experiments seemed to give results in somewhat better agreement with the 3 foot exponentials than with the 5 foot exponentials. But these lattices have been studied only to a relatively small extent and it would probably be rash to attempt to make serious inferences from the data.

The use of a very small subcritical assembly has been studied at M.I.T. (29) where a tank 21 inches high and 20 inches in diameter was used. One of the purposes of the study was to investigate what lattice experiments could profitably be done in such a tank. The lattices studied contained uranium rods 0.25 inch in diameter, 16 inches long, and with a U^{235} concentration of 1.43 per cent; the moderator was water with D_2O concentrations of 99.80, 90.27 and 80.23 mole per cent, respectively. Axial and radial flux traverses and various cell parameters were measured at lattice spacings of 0.880, 1.128, and 1.340 inches, or moderator-to-uranium volume ratios of 12.0, 20.8, and 30.0, respectively. A set of 290 rods was used for the experiments so that the number of cells was not small, although the lattices could properly be characterized as "miniature" both because of their height and diameters. The source neutrons were supplied by the medical facility of the M.I.T. Reactor which gave a reasonably flat (plane) flux of thermal neutrons. Axial and radial flux traverses were calculated on the basis of age-diffusion theory, which was used because of the exploratory nature of the work. The slowing-down density at the gold resonance, along the central axis, was also calculated. It was found that the simple theory used predicted the shape of the axial and radial fluxes and the slowing-down

density quite well for the nine lattices studied. The axial traverses provide the most informative comparison between theory and experiment; the radial traverses were nearly identical in the nine lattices. Satisfactory quantitative agreement between the theoretical and experimental axial neutron distributions could be obtained by adjusting the extrapolation distance d_a at the axial boundary and the value of the ratio, ERI/σ_o, where ERI represents the effective resonance integral of the gold foils used and σ_o is the microscopic absorption cross section of gold at 2200 m/sec. The values needed for d_a were, in every case, greater than 0.71 λ_{tr}; the extrapolation distance 1.0 λ_{tr} was used for the radius in the calculation of the theoretical curves. In the lattices moderated by 99.8 per cent D_2O, the value of d_a that led to satisfactory agreement between theory and experiment was 2.0 λ_{tr}. At D_2O concentrations of 90.3 and 80.2 per cent, the values used were 1.8 λ_{tr} and 1.4 λ_{tr}, respectively. These results, like those reported from the Hanford and Savannah River Laboratories, point to the existence of serious problems in the interpretation of flux traverses measured in subcritical assemblies; the problems seem to become more severe as the assemblies become smaller.

The material buckling could not be measured in the M.I.T. Small Subcritical Assembly because the leakage from the assembly determines the axial relaxation length almost independently of the material buckling. The material buckling for lattices of interest is so much smaller than the radial buckling that the inverse relaxation length is hardly affected by changes in the material buckling, and it would be practically impossible to measure the latter accurately, even in the absence of source and end effects. The assembly is, however, quite useful for the measurement of other lattice parameters, and some of those measurements will be discussed later.

B. INTRACELL LATTICE PARAMETERS

1. Intracell Flux Distribution, Disadvantage Factors, Thermal Utilization

Measurements of the intracell distribution of the thermal neutron density are now a usual part of the experimental program at nearly all laboratories working on the physics of heavy water moderated reactors. A detailed comparison of results obtained at the different laboratories presents some difficulties, however, because of the use of fuel rods of different sizes, different lattice spacings, and fuel assemblies of different design. Sometimes the results are given as graphs of the neutron distribution, sometimes as values of disadvantage factors, and sometimes as values of the thermal utilization. Disadvantage factors can be obtained directly from activation measurements, while the thermal utilization depends on the values of effective absorption cross sections which depend, in turn, on the neutron energy spectrum and the energy dependence of absorption cross sections. The measurements have been used in two ways: to provide values of the thermal utilization for use in a "recipe" for reactor design calculations; and to provide a basis with which to compare some theoretical treatment, either to test the validity of a formula or to supply experimental values from which an empirical "constant" can be derived. The need for the latter procedure has arisen from the use of oversimplified formulas based on one-group models, or from ignorance of the spectrum of neutron energies, which may have serious consequences when absorption cross sections have significant non-1/v dependence. The development of theoretical methods (83-89) for determining the distribution in both space and energy of the thermal neutrons in a lattice cell has increased the need for careful, detailed measurements of intracell neutron distributions and has encouraged work in this field.

In view of the difficulties facing a detailed comparison of experimental results, the analysis to be used in this section will differ somewhat from that used in other sections of this report. We discuss first

an empirical correlation of data concerning the disadvantage factor in lattices in which the fuel is a single uranium metal fuel rod. Sato (90, 60) has shown that the experimental intracellular thermal flux distribution for heavy water lattices obtained at North American Aviation, Inc. (73), as well as data on uranium rods in graphite-moderated and water-moderated lattices can be described by the formulas:

$$\phi(r, r_o, \Sigma) = \frac{I_o(\kappa r) + e^{-4\Sigma r_o} I_o(4\Sigma r)}{1 + e^{-4\Sigma r_o}}, \quad r_o < \lambda; \tag{3.3}$$

$$\phi(r, r_o, \Sigma) = \frac{I_o(\kappa r) + \gamma(\Sigma)\, r_o\, e^{-4\Sigma(r_o - r)}}{1 + \gamma(\Sigma)\, r_o\, e^{-4\Sigma r_o}}, \quad r_o \geq \lambda. \tag{3.4}$$

In these equations,

r_o is the radius of the uranium rod,

Σ is the total macroscopic cross section of uranium,

κ is obtained from the equation

$$\frac{\kappa}{\Sigma} = \tanh \frac{\kappa}{\Sigma_s}, \tag{3.5}$$

$\lambda = 1/\Sigma$ is the total mean free path in uranium,

$\gamma(\Sigma)$ is a constant for a given U^{235} concentration, with the values

$\gamma(\Sigma) = 0.15$ for uranium containing 0.49% U^{235},

$= 0.22$ for uranium containing 0.71% U^{235},

$= 0.24$ for uranium containing 0.91% U^{235}.

The values of $\gamma(\Sigma)$ were determined by fitting the formulas to the experimental data and could probably be improved by using data accumulated since 1958. The average value of the flux in the rod can be obtained analytically, and the disadvantage factor for the uranium rod can be calculated from the appropriate semiempirical formula.

Some direct comparisons of experimental values of disadvantage factors are possible without too much interpolation and show good agreement among values obtained in different experimental facilities. Thus, Canadian measurements in ZEEP give values of F (the fuel disadvantage

factor) and FF_m (the cell disadvantage factor) in good agreement with values obtained at NAA; French values obtained in AQUILON also agree with NAA values, as do the results of measurements at M.I.T. on lattices of one-inch diameter rods at different lattice spacings (92). The latter measurements, made in an exponential facility also yield values of the thermal utilization, f, which are consistent with the results of measurements made in critical lattices at the Savannah River Laboratory (93). Disadvantage factors have been measured for single rod lattices and for clusters of uranium rods, and for clusters of uranium oxide rods, in heavy water, in a miniature lattice irradiated in the thermal column of the Swedish reactor R1 (91). The results were in reasonably good agreement with theory and, where comparable, with results obtained in exponential assemblies.

Measurements of quantities related to the thermal absorption in uranium-heavy water lattices can also be made in the PCTR, and in another similar facility, the Pawling Lattice Test Rig (PLATR) of the United Nuclear Corporation (94). The latter facility is used primarily for measurements of k_∞ for lattices of uranium and of uranium oxide rod clusters for which detailed comparisons are made between calculated and measured values of k_∞ (95, 96). A thermal flux traverse through the cell and a determination of the thermal utilization are made as part of every determination of k_∞. Work with this facility should, therefore, provide a fruitful source of data for comparison with results obtained for cluster lattices in other facilities, in particular, in the PDP and SE at the Savannah River Laboratory.

Measurements have also been made in the Small Subcritical Assembly at M.I.T. (29). Leakage and source effects have been studied theoretically, and measurements of intracell flux traverses should be possible which would agree with traverses from exponential and critical assemblies. Further experimental work is in progress along these lines.

The work on measurements of quantities related to the thermal absorption of lattices of natural uranium rods in heavy water may be summarized by the statement that, for single rod lattices, the results obtained in different facilities (critical, subcritical, etc.) are in reasonably good agreement. The measurements on lattices of fuel rod clusters require more detailed analysis and comparison.

2. The U^{238} Cadmium Ratio, Conversion Factor, and Resonance Escape Probability

Relatively few measurements of the U^{238} cadmium ratio or conversion factor are available and, as in the case of uranium-graphite lattices, few comparisons can be made of results obtained in different types of assemblies. Values of the resonance escape probability have been derived from U^{238} cadmium ratios in critical assemblies in the PDP at the Savannah River Laboratory (93); the lattices contained one-inch diameter uranium rods at spacings of 7.00, 8.08, 9.33, and 12.12 inches, respectively. The results are consistent with values of p obtained at M.I.T. (55) in exponential experiments on lattices of one-inch diameter uranium rods at spacings of 4.5, 5.0, and 5.75 inches, respectively. In those two sets of experiments, the resonance escape probability was calculated from the measured values of the U^{238} cadmium ratio either by using equation (2.3) or an equation similar to it. Few, if any, other decently comparable experimental results have been reported, and there seems to be a real gap here in the research being done on uranium-heavy water lattices.

The measurement of U^{238} and U^{235} cadmium ratios in a small subcritical assembly has been investigated at M.I.T. (80). Such measurements may depend strongly on leakage and source effects. A relationship has been derived between a cadmium ratio measurement made in an exponential assembly and one made in a critical assembly of the same materials, provided that a relatively simple theory, e.g., age-diffusion theory is used. This relationship depends on the position at which the measurement is made in the subcritical assembly and takes into account the effect of the resonances in U^{238}. It can be used to correct the experimental value obtained in the subcritical assembly to give the value that should be obtained in a critical facility. In the practical exponential assemblies that have been used, the correction factor is very close to unity, and the value of the cadmium ratio obtained in the exponential assembly should agree with the value obtained in a critical assembly. The results cited above, obtained at the Savannah River Laboratory and at M.I.T., are in agreement with this conclusion. For the M.I.T. Small Subcritical Assembly, however, the correction factor for a single measurement of the U^{238} cadmium ratio may be as large as 20 or 30% for lattices

moderated with D_2O. But when measurements were made at two different positions in the assembly, at which the corrections were quite different, the corrected values agreed well, to within 1 or 2 per cent and, in some cases, better. The agreement, or rather, consistency in the resulting values of p was considerably better. These results show that it is possible to correct the values of the U^{238} cadmium ratio obtained in the Small Subcritical Assembly so as to give consistent "critical assembly" or "large" exponential assembly values. The experiments were made in lattices of 0.25-inch diameter, slightly enriched rods at small lattice spacings and need to be compared with the results of measurements in the M.I.T. exponential assembly; such measurements are under way.

3. The U^{238}:U^{235} Fission Ratio and the Fast Fission Factor

A detailed summary of methods and results for these quantities in uranium-heavy water lattices is given in a recent report (97). Detailed comparisons are again difficult, but values of R (called δ_{28} in reference 97) and ϵ obtained for lattices of single rods at the Savannah River Laboratory in the PDP critical assembly and in M.I.T. exponential assemblies are consistent. Comparison of results for clusters of uranium or uranium oxide rods require compilation and analysis.

4. The Multiplication Factor, k_∞, for an Infinite Lattice

Measurements of k_∞ have been made in the PLATR (94), and have been compared (98) with values obtained in the PCTR; values obtained for similar lattices in the two facilities are in good agreement. As in the case of the PCTR, comparison with results of exponential and critical experiments would involve the use of slowing-down and diffusion areas. Such comparisons show reasonably good agreement among the different kinds of experiments (119). So far, however, the emphasis has been mainly on the comparison of calculated and measured values (95, 96).

5. The Effective Value of Eta

The results of buckling measurements have been combined with calculations to derive effective values of η (eta), as has been done for uranium-graphite lattices. The most extensive comparison of this kind seems to be that of Pershagen, Anderson and Carlvik (20) who applied

their method of analysis to several sets of experimental results, including results obtained in critical and exponential assemblies. Their results are listed in Table 3.1. They have calculated the value of the quantity

$$"\eta" = \frac{\epsilon B^2 (1+L^2B^2)}{pf}, \qquad (3.6)$$

as a function of the volume ratio V_{mod}/V_{fuel}, using measured values of B^2 and calculated values of ϵ, p, f, L^2, and Σ, and have drawn the following conclusions with respect to matters of interest to this paper:

(a) The "η" values are fairly independent of the size and shape of the fuel elements and of the lattice spacings in each group of measurements.

(b) There is a significant difference between the values of "η" obtained from critical and exponential experiments. Pershagen, Anderson, and Carlvik have suggested that the discrepancy may be due to anisotropic neutron leakage effects which tend to overestimate the buckling obtained from exponential experiments or to systematic errors in the evaluation of the radial buckling in exponential experiments.

Another analysis similar to that of Pershagen et al. has been made by Bolton (104), who considered the results of Swedish exponential experiments (75), French critical and substitution experiments (18) and Canadian critical experiments (21). Bolton's theoretical treatment differs from that of Pershagen et al., and leads to somewhat different conclusions. His analysis yields a resonance integral independent of the lattice spacing and an effective value of η (U^{235}) for 2200 m/sec neutrons. He obtains values of 1.971 ± 0.022 and 1.995 ± 0.010 for uranium metal and uranium oxide, respectively. These values are in reasonable agreement but are lower than that (2.077 ± 0.010) given by direct measurements (105). He also says that there is an indication that the results of Swedish exponential experiments give effective values of η (U^{235}) greater than those of French and Canadian critical and substitution experiments, by about 0.5 to 1 per cent. But Bolton considers that this variation is comparable with differences in values of $\eta(U^{235})$ deduced from experiments in one facility, and could possibly be due to errors in the theoretical model rather than to the measured bucklings.

TABLE 3.1

Average "η" Values Obtained from the Analysis of Critical and Exponential Buckling Measurements in U- and UO_2-D_2O Rod and Rod Cluster Lattices

Type of Fuel Element	Type of Measurements	Laboratory	Reference	Number of Measurements Analyzed	Average Value	Δ Average Absolute Deviation
1. U rods	Exponential	AI	73	22	1.316	0.004
2. U rods and Clusters	Exponential	AE	74, 99	32	1.320	0.004
3. U rods and Rod Clusters	Critical	Chalk River, Saclay	100, 101 18	27	1.303	0.003
4. UO_2 Rod Clusters	Exponential	AE	102, 75	20	1.302	0.006
5. UO_2 Rod Clusters	Critical	Chalk River	103	4	1.290	0.003

The moral seems to be that this type of analysis is useful to the reactor designer in that it yields values of η which help in the correlation of the measurements and for predicting, by the use of specified calculational methods, the buckling of unknown lattices. It does not, however, lead to a deeper understanding of what is happening in neutron-multiplying systems; for this, more detailed and subtle theoretical treatments are needed. Such a treatment has been developed for lattices of slightly enriched uranium rods in ordinary water (10) and is to be extended to lattices with heavy water.

No entirely satisfactory method is available for measuring η in a lattice, but some attempts have been made at the Savannah River Laboratory (91). These measurements determine the value of η for neutrons with energies below the cadmium resonance. They are based on the comparison, in the fuel and in a well-thermalized flux, of the ratio of U^{235} fission to 1/v-activation (Cu, Mn), taken to represent the absorption in U^{238}. Measurements, in the PDP, in critical lattices of single uranium rods and clusters of uranium rods, yielded a value of $\eta = 1.313 \pm 0.004$. Although this value cannot be compared with the values listed in Table 3.1, it does not differ strongly from the values deduced from buckling measurements and theory, and listed in that table.

The general status of the physics of D_2O-moderated reactors has recently been summarized in an excellent review by J. L. Crandall (133), which will be highly useful to readers of the present report.

IV. URANIUM-WATER LATTICES

The work that has been done, at the Brookhaven National Laboratory and at the Bettis Atomic Power Division of Westinghouse on lattices of slightly enriched uranium in ordinary water (3-10), occupies a unique position in reactor physics. It has been the most systematic and detailed experimental and theoretical research study attempted on any neutron-multiplying system, and has set a high standard for work in basic reactor physics. The general aim has been to measure as many lattice parameters as possible and to parallel the experimental work with theoretical research. This program has resulted in the development of new experimental and theoretical methods. Thus, experimental work on the measurement of the U^{238}:U^{235} fission ratio has led to advances in the theory of the fast fission effect (106, 107, 70). The development of reasonably precise methods of measuring neutron absorption in U^{238} has inspired advances in the theory of resonance absorption in lattices (108-115). Finally, the existence of data on individual lattice parameters (intracell flux distributions, U^{238} and U^{235} cadmium ratios, U^{238}:U^{235} fission ratios, etc.) has made possible the development of a more fundamental theoretical treatment of these lattices (7, 10) as well as a more precise, though still approximate, theory (6, 8).

A. CRITICAL BUCKLING

The critical buckling has been measured for many lattices, and in both exponential and critical assemblies. The measurement of the radial buckling presents some problems that are different from those met in bare uranium-graphite or bare uranium-heavy water assemblies. The difficulty in defining the outer boundary of a water-moderated lattice and the small thermal diffusion length of water made it simpler to use an effectively infinite water reflector rather than a bare assembly. The determination of the radial buckling therefore involved the determination of the reflector saving for each lattice. In other words, the radial buckling and

the reflector saving were treated as unknowns and were determined simultaneously. This could be done in two ways: in one, both radial and axial flux traverses were made in the usual way, and the extrapolated radial core flux was fitted to a J_o Bessel function; in the second, B^2 and λ, the reflector saving, were found from axial measurements alone. The material bucklings obtained with the two methods were found to agree very well.

During the first part of the program, exponential experiments were done at Brookhaven, and critical experiments were done at Bettis with the same fuel rods. Later, both exponential and critical experiments were done at Brookhaven; on some lattices, critical experiments were done at both laboratories. The fuel rods used had diameters of 0.600, 0.387, and 0.250 inch, respectively; the water-to-uranium volume ratios used were 1, 1.5, 2, 3, and 4: the U^{235} concentrations of the fuel were 1.027, 1.143 and 1.299 weight per cent.

Values obtained for the buckling in exponential and critical experiments have been compared by Price (116); they are in good agreement, within the limits of the experimental uncertainties. Results which are interesting, although not understood, have also been obtained for the reflector saving. For a given rod diameter and U^{235} concentration, the reflector saving decreases with increasing lattice spacing, although the reflector thickness is effectively infinite. For a given rod diameter, U^{235} concentration, and lattice spacing (water-to-uranium volume ratio), the reflector saving is independent of the loading radius, and is the same for a subcritical and a critical assembly. Although there is no obvious way of comparing these results for the reflector saving with those for the extrapolation distance in uranium-graphite lattices and the radial buckling in uranium-heavy water lattices, the three types of lattices seem to have some common aspects. Peculiar and not-understood effects occur at the radial boundaries, but consistent and presumably correct values of the radial buckling can be obtained, provided that the measured values of the not-understood parameters are used.

B. INTRACELL LATTICE PARAMETERS

1. Thermal Utilization, Resonance Escape Probability, and U^{238}:U^{235} Fission Ratio

These parameters were measured at Brookhaven in exponential assemblies and in miniature lattices, and at Bettis in critical assemblies. The miniature lattices used at Brookhaven had neutron multiplications of about 3, and the neutron spectrum in the central part of the lattice, where the measurements were made, was characteristic of a critical assembly. The way in which a miniature lattice can be used and its properties described have been studied at M.I.T. (29). It is convenient to discuss a subcritical assembly in terms of the dimensionless ratios, R/L, H/L (where R and H are the extrapolated radius and height, respectively, and L is the thermal diffusion length), and k_∞, and τ_o/L^2, where τ_o is the Fermi age to thermal energies. For a given set of values of the parameters listed, it is possible to predict the distance from the source end of the assembly at which the cadmium ratio for a given detector becomes constant in the assembly. A procedure has been developed for a cylindrical assembly whose height and diameter are equal, for which age-diffusion theory is used, and for which the cadmium ratio is that for a nuclide with a resonance energy at which the neutron age is $0.7\tau_o$ (about 5 to 25 ev). The first two limitations were made to simplify the mathematical treatment; the nuclide described corresponds approximately to U^{238}. The "asymptotic region" of the assembly may be defined as the region in which the value of the cadmium ratio for the nuclide described has a value within one per cent of the value in a critical assembly. The value of the uranium cadmium ratio is used as an integral spectral index. In terms of this criterion, the Brookhaven miniature water lattices had relatively large asymptotic regions, and measurements made in them should yield results in agreement with those obtained in "large" exponential and critical experiments without the need for corrections. In contrast, the M.I.T. heavy-water moderated, Small Subcritical Assembly

lattices had no asymptotic region, and the results obtained in these lattices had to be corrected.

In reference (116), values of f, p, and the U^{238}:U^{235} fission ratio obtained in the Brookhaven miniature lattices are found to be in good agreement with those obtained in critical assemblies, and in good agreement with the theory just outlined.

2. Migration Area

In the Brookhaven and Bettis experiments, it was also possible to determine the migration area, M^2, in both exponential and critical assemblies. The migration area is not measured directly, but is a derived quantity. The interpretation of the experiments from which values of M^2 are derived has been discussed in detail (5, 8, 117). The Brookhaven determinations of M^2 were based on two different experimental procedures. In the first, the moderator was poisoned by the addition of boric acid solution, which changed f and B^2; by varying the amount of poison in a given lattice and using one of the simpler descriptions of the neutron cycle, a value of M^2 could be obtained for the lattice. The second procedure involved combining information obtained with lattices differing only in the U^{235} concentration of the rods used. The Bettis determinations of M^2 were based on reactivity measurements (in terms of water height) in a critical assembly. The three different types of experiments again yielded results in good agreement.

3. The Multiplication Factor, k_∞, for an Infinite Lattice, and Eta

In the Brookhaven and Bettis experiments, B^2 was measured, and ϵ, p, f and M^2 were determined from measured quantities. Hence, by using some of the simpler theoretical models, values of k_∞ were determined for the various lattices studied, and values of η for the uranium at the three U^{235} concentrations were derived; from these values, a value of η for natural uranium was obtained: η (nat U) = 1.309 ± 0.006. This result also contains the result for η obtained from measurements made on the Brookhaven natural uranium-graphite pile. A set of measurements made at Hanford (52) yielded the value 1.313 ± 0.013 in excellent agreement

with the value obtained from the Brookhaven measurements. Since the Bettis critical experiments gave results for all the lattice parameters measurements which were in good agreement with the Brookhaven results, they should also yield a value of η very close to that obtained at Brookhaven.

V. SUMMARY AND RECOMMENDATIONS

A. THE CRITICAL BUCKLING

The comparisons that have been made in the earlier sections of this report indicate that the results of measurements of the buckling made in subcritical and critical assemblies give the same results, provided that proper precautions are taken. Thus, the subcritical assemblies should not be too small. Although it is not yet clear what is meant by "too small", the problem of the size of a subcritical assembly is related to the problem of determining the radial buckling with adequate precision. The radial buckling in a cylindrical assembly depends on the radial extrapolation distance or on the reflector saving. The latter distances may depend on the radius of the assembly, as indicated by the Hanford experiments on small exponential piles, by the Savannah River Laboratory and M.I.T. measurements on uranium-heavy water lattices, and by the Brookhaven and Bettis work on uranium-water lattices. Consistent results are obtained for the radial buckling, provided that the latter is always derived from measurements. It does not suffice to add $0.71\lambda_{tr}$ to the radius to obtain the extrapolation distance for a bare assembly because the measured extrapolation distance varies in ways that are not yet understood. The reasons for these variations are not known and further work, both theoretical and experimental, is needed, especially on small assemblies.

The problem of the radial buckling seems to be related to the heterogeneity of the lattices, which raises two problems. One problem is that of the separability of the macroscopic and microscopic (intracell) neutron distributions; the second is that of the anisotropy of the slowing down and diffusion of neutrons. Although there is some evidence bearing on those problems, experiments at different laboratories have yielded different results. The problem of separability in a bare assembly is related to that of the extrapolation distance; the determination of the extrapolation distance, however, is clouded by experimental difficulties

such as the indefiniteness of the outer boundary of a lattice and the back-scattering of neutrons from the walls of the room in which the assembly is located. The anisotropy, as measured by the ratio M_z^2/M_r^2 may be related to the size of the assembly; unpublished experiments at Brookhaven on uranium oxide-water lattices point to a dependence of the anisotropy on the radial buckling. It is clear that more detailed and precise experiments are needed as well as more detailed theoretical studies of heterogeneous assemblies, both small and large. Theoretical methods have been developed which may be useful, for example, those of references (121-123).

B. INTRACELL LATTICE PARAMETERS

1. The Intracell Thermal Neutron Distribution and Related Quantities

The values of the disadvantage factors and the thermal utilization obtained in subcritical assemblies (exponential, miniature, PCTR type) seem to agree, and to give values appropriate to a critical assembly. Where comparisons are possible, e.g., between PCTR values and values obtained in exponential piles, or between values obtained in exponential and critical assemblies, the agreement seems to be satisfactory. The disadvantage factors, however, depend on averages of flux distributions and cross sections, and are less sensitive to differences in neutron energy distribution than the spatial distribution of the neutron density; the thermal utilization is still less sensitive. Attention is now being focused on the details of the neutron distribution in a lattice cell in both energy and position. It is not yet possible to examine the energy spectrum in detail experimentally, but some information can be obtained about "effective" neutron cross sections and temperature by using detectors whose absorption cross sections depend on energy in different ways. The development of the THERMOS code (83-85) has provided a powerful tool for the study of thermal neutron distribution and absorption in a lattice cell and has made possible the calculation of the thermal neutron distribution with high accuracy for some types of lattices (84, 124). Some discrepancies between theory and experiment have been observed (92, 124, 125) and have encouraged further theoretical and experimental work.

2. The U^{238} Cadmium Ratio, Conversion Factor, and Resonance Escape Probability

Less can be said about these quantities than about any of the other reactor parameters discussed in this report. Fewer measurements have been made, and there are still experimental problems to be overcome in the methods available (55, 126, 127). The only detailed comparisons between results obtained in subcritical and critical assemblies have been for uranium-water lattices. Theoretical work on small assemblies

(29) indicates that results obtained in the usual kind of exponential assemblies should be satisfactory, but that measurements in small assemblies may give erroneous results. In general, this branch of reactor physics requires much further work, and it is urged that programs of reactor physics measurements include measurements of the U^{238} (and U^{235}) cadmium ratios and of the conversion ratio.

3. The U^{238}:U^{235} Fission Ratio and the Fast Fission Effect

These quantities have been studied in single rods and in lattices of various types. Where comparisons are possible, the results of measurements in subcritical and critical assemblies are in reasonably good agreement. There is some possibility of errors in small assemblies in which the interaction effect may be important. This effect has been investigated thoroughly in uranium-water lattices, but much less has been done in uranium-graphite and uranium-heavy water lattices. More detailed studies of the interaction effect in clusters also seem advisable.

4. The Multiplication Factor, k_∞, for an Infinite Lattice

Measurements from which values of this quantity can be obtained are being made in the PCTR and the PLATR, where the main emphasis is on the comparison with values obtained from design recipes. Although this work is important in connection with practical reactor design, its value would be greatly enhanced by measuring parameters related to p and ϵ, as well as k_∞ and f. Thus, measurements of the U^{238} (and U^{235}) cadmium ratios, the ratio of neutron capture in U^{238} to fission of U^{235}, and the U^{238}:U^{235} fission ratio would make possible the determination of the effective value of η. Alternatively, if η were computed, e.g., by means of the THERMOS code, two determinations of k_∞ would be possible, one based on the poison technique used for PCTR and PLATR, the other based almost entirely on measurements of individual parameters. This kind of experimentation would yield additional information about the lattices, would extend the usefulness of the PCTR type of assembly and might yield information about the general method on which the PCTR is based. Although some work along these lines has been started (54), much more is possible by way of exploiting the possibilities of the PCTR and PLATR.

C. EXPERIMENTS ON SINGLE FUEL ELEMENTS

An attempt has been made to determine lattice parameters by means of measurements on a single fuel element (128). The results obtained, together with the method of analysis used, have been reported to give results that agree with results obtained in exponential assemblies (128, 129). Further theoretical and experimental work on this type of measurement seems worthwhile.

Notes Added in Proof

A. To Section III, B, 1 (p. 34)

A comparison has recently been made (134) of results obtained with three types of final assemblies of natural uranium oxide clusters in heavy water. Measured values of the material buckling were combined, in the two-group criticality equation, with calculated values of the age and diffusion area to obtain values of k_∞. These were compared with experimental values measured in the PLATR, and the agreement was good.

B. To Section II, B (pp. 15-17)

The determination of extrapolation distances also presents a serious problem in pulsed neutron measurements on small assemblies. Little experimental information is available on the extrapolation distance appropriate to pulsed systems. The influence of the geometrical configuration and the buckling on the extrapolation distance when neutrons are injected into water has recently been investigated (135). Under diffusion theory, the extrapolation distance depends on both configuration and buckling, but a linear extrapolation distance can be defined which is a configuration-independent function of the buckling alone. Transport effects and multigroups may be separated, and transport corrections derived from one-group calculations may be superimposed on multigroup diffusion results.

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