MITNE-NUREG-205

A METHOD FOR RISK ANALYSIS OF NUCLEAR REACTOR ACCIDENTS

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by

Mitsuru Maekawa

Submitted to the Department of Nuclear Engineering on July 13, 1976, in partial fulfillment of the requirements of the degree of Doctor of Philosophy.

ABSTRACT

A method is developed for deriving a set of equations relating the public risk in potential nuclear reactor accidents to the basic variables, such as population distributions and radioactive releases, which determine the consequences of the accidents. The equations can be used to determine the risk for different values of the basic variables without the need of complex computer programs and can be used to determine the variable values which are needed to satisfy various risk criteria. The equations will provide considerable savings of time and effort in determining the consequences of the nuclear reactor accidents.

The methodology developed in this study consists of two steps. The first step involves fitting the risk distributions of frequency versus consequence to parametric distributions which contain a small number of parameters. The second step involves deriving the equations which relate the distribution parameters to the basic variables of interest. Regression techniques are used for this second step.

The methodology is demonstrated for examples based on the results of the Reactor Safety Study. The calculated distributions of early fatalities in nuclear reactor accidents and the historical records of fatalities in hurricanes, tornadoes, earthquakes and dam failures are examined to determine an appropriate family of parametric distributions. From these examinations, the Weibull distribution is found to be appropriate for all of the examined events.

A set of equations is then derived which relate the population distribution and the parameters of the Weibull distributions for early fatalities from PWR accidents. The derived equations are straightforward and useful in analyses of population effects on risk. Regression equations relating the parameters to the characteristics of radioactive releases are also derived. The derived equations again are straightforward and useful for evaluating release effects on risk.

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CHAPTER I

INTRODUCTION

I.1 Objective of Study

In October, 1975 the final report of the Reactor Safety Study was published by the U. S. Nuclear Regulatory Commission (Ref. 1). The principal purpose of the Reactor Safety Study was to make a realistic estimate of the public risks that could be involved in potential accidents in commercial nuclear power plants and to provide a perspective to compare them with non-nuclear risks to which our society are already exposed. Though the Reactor Safety Study was focused on an estimate of the total risk of the nuclear power plants existing or being planned, the risk estimation methods developed in the Study can provide help with regard to decision making involving regulations, site planning, plant design and other areas relating to the safety of nuclear power plants.

To apply risk results in decision making, it is of use to prepare a set of equations that give the relationship between the risks and the basic variables that determine and control the consequences of nuclear reactor accidents. With the risk expressed in terms of the basic variables, decision can be made on the basic variables which give acceptable risk. For example, in selection of a site for a nuclear power plant, the population distribution may be one of the basic variables of interest. Relating the risk to the population distribution would then allow investigation and decision on acceptable population distribution. If this can be done, it may result in considerable savings in time and effort in the decision making process.

The objective of this thesis is to develop a method for obtaining a set of equations that describe the relationship of the public risk in potential nuclear reactor accidents to the basic variables that drive and control the consequences of the accidents. The method will be demonstrated in a limited number of examples based on the results of the Reactor Safety Study.

I.2 Basic Concepts of Risk

Since risk is a commonly used word that can convey a variety of meanings to different people, certain concepts of risk will be discussed here. A dictionary definition of risk is "the possibility of loss or injury to people and property". The major elements for defining risk will be consequence and likelihood. The following four types of consequences were considered in the Reactor Safety Study.

a. Early fatalities (i.e., fatalities that occur within one year of the accident).

b. Early injuries (i.e., people needing medical care).

c. Late health effects attributable to the accident.

d. Property damage

In this thesis, early fatalities will be studied specifically as an example in developing the method to relate the risk to the basic variables. The developed method may be applicable to other types of consequences.

The likelihood is expressed by the frequency of occurrence of accidents. For frequent events, the frequency can be estimated from the historical records in the past. However, many potential accidents, such as nuclear accidents, occur at such a low frequency that they have not been observed. In these cases the frequency is obtained by calculational models using basic components and system failure data.

Combining the two major elements of likelihood and consequence, risk is then described by the distribution of frequency vs. magnitude of consequence, which will be called "risk distribution" in this thesis. Two expressions of the risk distribution will be used in the following chapters. One is a "frequency distribution" (denoted by f(x), which is defined by:

$$F[x_a \leq x \leq x_b] = \int_{x_a}^{x_b} f(x)dx \tag{1.1}$$

where $F[x_a \leq x \leq x_b]$ is the number of events per unit time that the magnitude of consequence is between x_a and x_b. Another expression is a "complementary cumulative distribution" (denoted by $F^C(x)$), which presents the frequency of consequences being greater than the magnitude x. The relation of the two expressions is given by:

$$F^C(x) = \int_{x}^{\infty} f(x)dx \tag{1.2}$$

For example, Figs. 1.1 and 1.2 show the complementary cumulative distributions of early fatalities in nuclear reactor accidents as well as other man-made and naturally occurring risks. Fig. 1.3 shows the frequency distribution of early fatalities in nuclear accidents in a form of a histogram.

The risk distributions can be summarized by certain characteristics of the distributions, (called "risk characteristics" in this study) such as:

1. Frequency at a specific magnitude of consequence :

   For example, from Fig. 1.1 the frequency of fatalities being greater than 1,000 is about 10^{-6} per year for 100 nuclear plants, whereas it is 10^{-3} per year for chlorine release.

2. Magnitude of consequence at a specific frequency:

   For example, from Fig. 1.1 the number of fatalities at a chance of one in 10,000 years is less than 10 for 100 nuclear plants, whereas it is greater than 5,000 for chlorine release.

3. Risk moments, which is defined by:

$$M_t(\xi) = \int_0^{\infty} f(x) \cdot (x - \xi)^t \cdot dx \qquad (1.3)$$

where

$M_t(\xi)$ is the t-th risk moment about ξ. The first risk

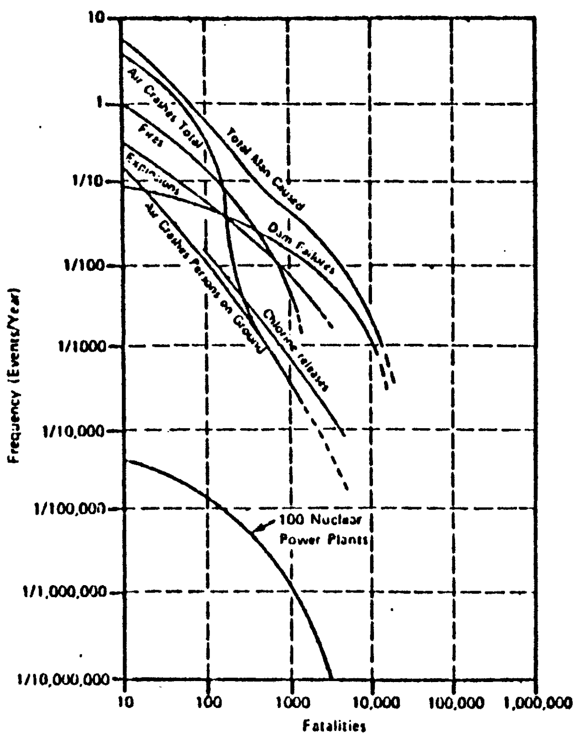

Fig. 1.1 Complementary Cumulative Distribution of Fatalities due to Man-Caused Events

Note: From Fig.1-1 in the Main Report of WASH-1400(Ref-1)

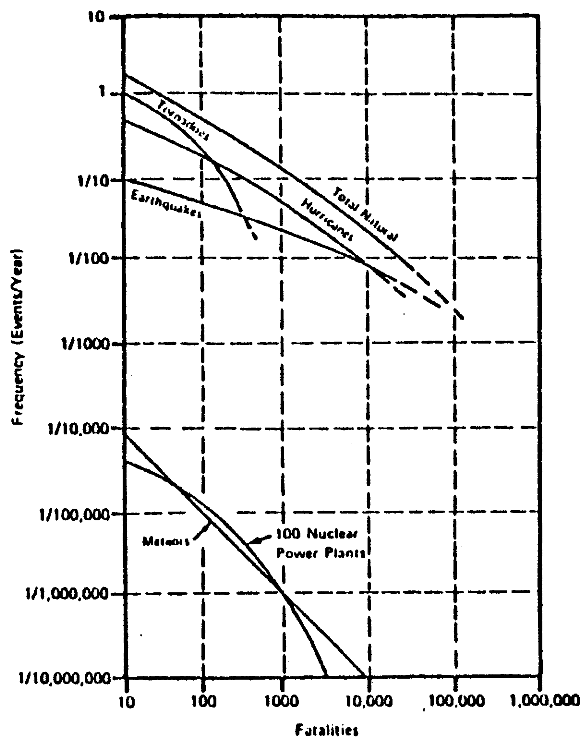

Fig.1.2 Complementary Cumulative Distribution of Fatalities due to Natural Evevts

Note: From Fig.1-2 in the Main Report of WASH-1400(Ref-1)

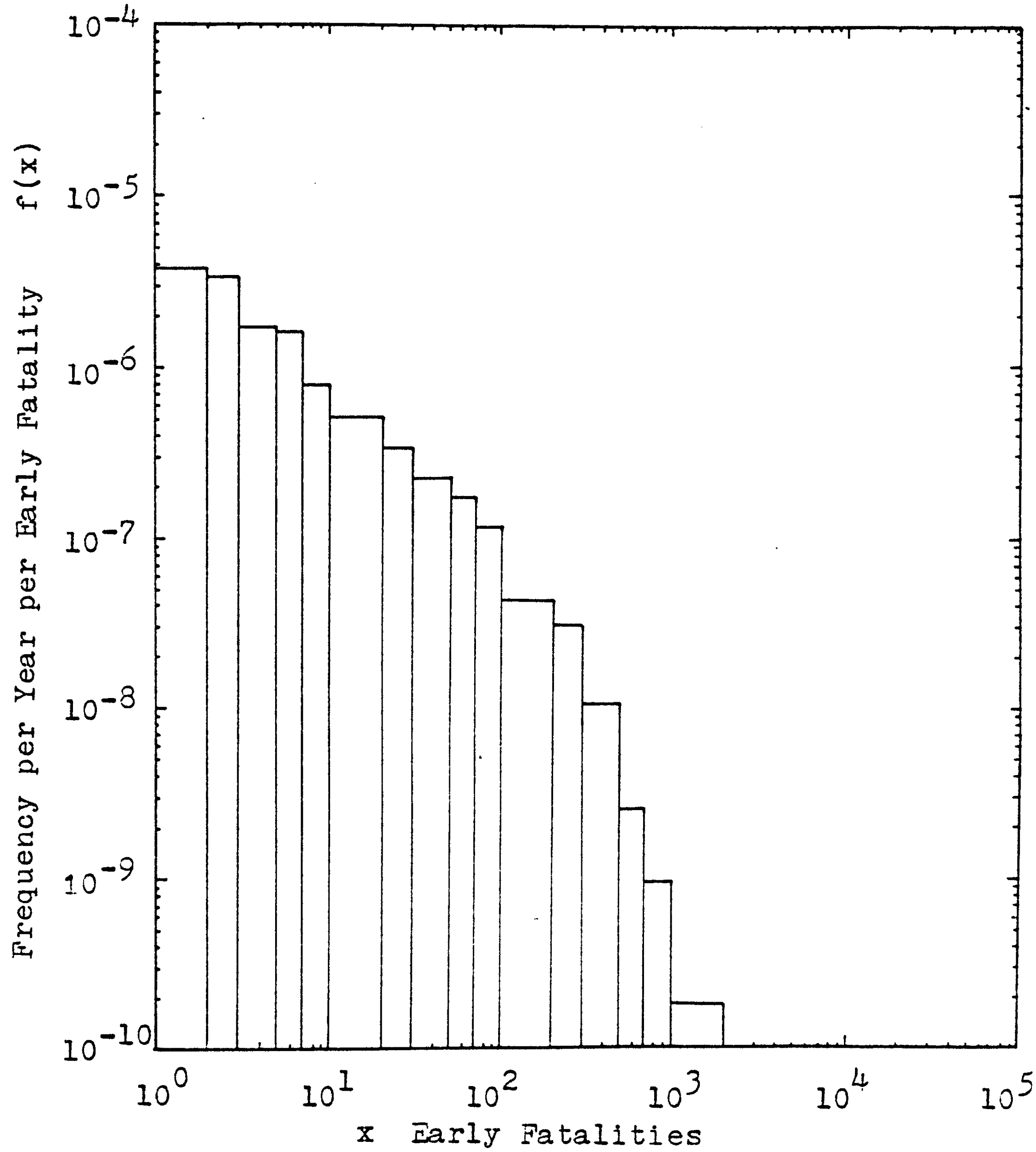

Fig.1.3 Frequency Distribution of Early Fatalities for U.S. 100 Commercial Nuclear Power Plants

Note: Calculated from the results of WASH-1400(Ref-1) by $f(x) = (F^C(x+\Delta x) - F^C(x))/\Delta x$.

moment about the origin can be interpreted as an expected magnitude of consequence per unit time. For example, the expected early fatality per year is 4.6×10^{-3} for 100 nuclear power plants and 55,000 for automobile accidents in U. S. (Ref. 1).

I.3 Outline of the Approach

The approach developed in this thesis is presented by two major steps. They are:

(1) The risk distributions are fitted to parametric distributions involving only a small number of parameters. To determine an appropriate parametric distribution, the fatalities distributions of nuclear and non-nuclear risks are examined. Once an appropriate parametric distribution is selected, the entire curve and any risk characteristic can be estimated from the distribution parameters.

(2) A set of equations are derived to relate the distribution parameters to the basic variables of interest. In this study, regression techniques are used to derive the equations.

The fitting of the risk distribution will be studied in Chapters II and III. A general approach of selection of candidate parametric distributions, fitting techniques and criteria of adequate fits will be discussed in Chapter II. In Chapter III an application is given of the fitting techniques and the criteria to the examination of the fatalities distributions of nuclear and non-nuclear risks.

The regression analysis to relate the distribution parameters to the basic variables will be studied in Chapters IV, V and VI. In Chapter IV a discussion will be given of general approaches of the regression techniques. In Chapter V an application will be given of regression analysis relating the distribution parameters to population distribution variables. In Chapter VI another application will be given relating the parameters to radioactive release variables.

In Chapter VII, the methodologies developed in this study are summarized and a discussion is given of further possible extensions.

I.4 Method of Risk Estimation

A brief discussion will be made about the methods of risk estimation developed in the Reactor Safety Study, particularly about the consequence model, because the numerical values of the risk estimates in this thesis are based on the results of the consequence calculation. More detailed information about the Reactor Safety Study can be found in WASH 1400 (Ref. 1).

I.4.1 Outline of Reactor Safety Study

The Reactor Safety Study was divided into three major tasks shown in Figure 1.4. Task 1 included the identification of potential accidents and quantification of both the probability and magnitude of the associated radioactive releases to the environment. Task II used the radioactive source term defined in Task I and calculated how the radioactive materials are distributed in the environment and what effects they have on public health and property. Task III compared the risk of nuclear reactor accidents estimated in Task II with a variety of non-nuclear risks to provide a perspective of the magnitude of the nuclear risks.

1.4.2 Outline of Consequence Model

The consequence model was developed in Task II in the Reactor Safety Study to predict the consequences from the radioactive releases defined by Task I. The consequence predictions served as the primary input to Task III. The consequences of a given radioactive release depend upon how the radioactive materials are dispersed in the environment, upon

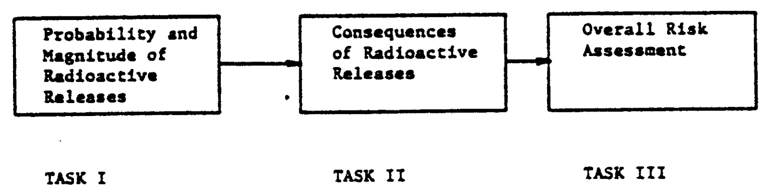

Fig.1.4 Major Tasks of the Reactor Safety Study

Note: Reproduced from Fig.4.1 in the Main Report of WASH-1400 (Ref-1).

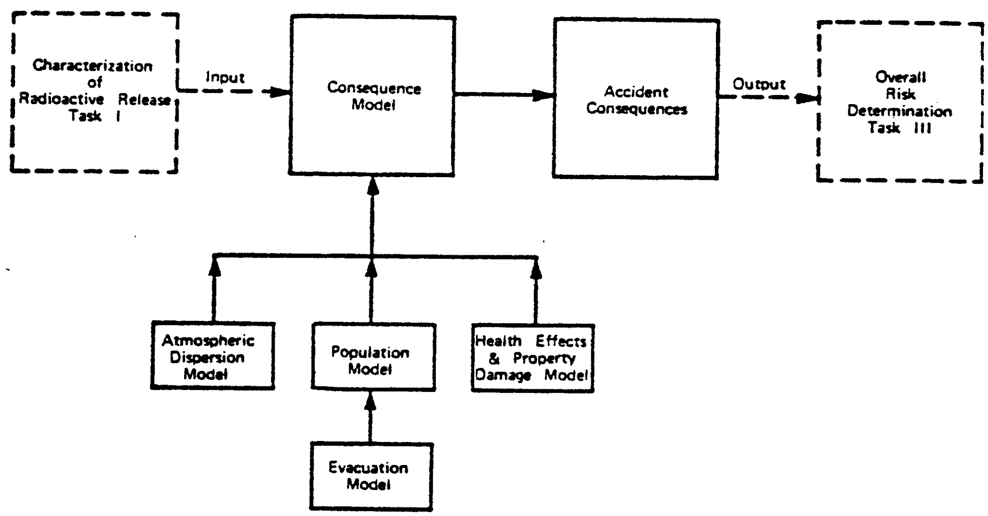

Fig. 1.5 Schematic Outline of the Consequence Model

Note: Reproduced from Fig.4.6 in the Main Report of WASH-1400 (ref-1) the number of people and amount of property exposed, and upon the effects of radiation exposure on people and contamination of property. These major elements of the consequence predictions are indicated in Fig. 1.5, which shows the principal subtasks involved in Task II.

The dispersion of the radioactivity is determined principally by the release conditions and the weather conditions at the time of release. The release conditions are described by the release categories. Each one of the release categories identifies the amount of radioactivity released, the amount of heat released with radioactivity, and the elevation of the release. (See Table 6.1 in Chapter 6.)

The standard Gaussian plume model is used to predict the way the radioactivity is dispersed in the atmosphere. The weather data used in the model is obtained from hour by hour meteorological records covering a one year period. Ninety weather samples are taken and each sample is thus assigned a probability of 1/90. The starting times are determined by systematic selection from the meteorological data. One quarter of the data points are chosen from each season of the year and half from each group are taken in the daytime. This procedure is used to reduce sampling errors to acceptable levels. The weather stability and wind velocity is assumed to change according to the weather recordings, but the wind direction is assumed not to change.

To determine the population that could be exposed to potential radioactive releases, census bureau data was used to determine the number of people as a function of distance from a reactor in each of the sixteen 22-1/2 degree sectors around each of the 68 sites at which the first 100 reactors now in use or planned are located.

Each reactor was assigned to one of six typical meteorological data sets and a sixteen sector composite population was developed for each set. The grouping of population sectors was performed in such a way that the sectors of high population form seperate sectors and the sectors of low population are grouped into composite sectors with average population of the grouped sectors.

The consequence model calculates the dose from five potential exposure modes; the external dose from the passing cloud, the dose from internally deposited radionuclides which are inhaled from the passing cloud, external dose from the radioactive materials deposited on the ground, the dose from internally deposited radionuclides which are inhaled after resuspension and the internal dose from ingestion of contaminated food.

The potential health effects considered are early fatalities, early illnesses and late health effects. The probabilities of early fatalities are computed by using a dose-effect relationship. For bone marrow dose, the probability of early fatalities varies from 0.01 to 99.99% for doses of 320 and 750 rads respectively with a median value of 510 rads. The number of fatalities are estimated by the number of people exposed to radiation multiplied by the probabilities of early fatalities estimated from dose. Early illnesses and late health effects are estimated in a similar way to early fatalities.

The consequence model also provides for prediction of economical damage due to radioactive contamination. It includes evacuation cost, loss of agricultural crops, decontamination cost, relocation cost and property damage.

I.4.3 Calculation Conditions for Individual Site

The consequence model outlined in the previous subsection was developed in the Reactor Safety Study to estimate the total risk of the first 100 nuclear power plants now in use or planned. The composite population model was generated for these 100 reactors. In this thesis, however, the population distribution of individual sites are used to estimate the risks of nuclear power plants, site by site. The population distributions of the individual sites which this study uses are obtained from the census bureau data (Ref. 2). The following assumptions are made in the individual site calculations.

1. Meteorological data sets typical of the eastern valleys area are used for all of the individual site calculations. The characteristics of the eastern valley meteorological conditions are given in Appendix C.
2. The frequency distribution of the wind direction is assumed to be uniform over 16 directions.
3. The radioactive inventory of 3200 MWt reactor is assumed.
4. The probabilities and magnitudes of radioactive releases are assumed to be the same as used in the Reactor Safety Study (Ref. 1). The estimates in the Reactor Safety Study were based on the analyses of Surry Power Station for PWR's and Peach Bottom Atomic Power Station for BWR's. (See Table C.2 in Appendix C).

Because of the assumptions listed above, the estimated risks will be different from the "real" risks of the individual reactors. In order to estimate the "real" risk of a specific reactor, the following data will be required.

1. Meteorological data based on the records observed at the specific site.
2. Radioactive inventory based on the capacity of the specific plant.
3. Estimates of the radioactive releases and their probabilities based on the analysis of the system of the specific plant.

In addition to limitations of the data, the refinement of the consequence model is now under way in U. S. Nuclear Regulatory Commission. Therefore the numerical values in this thesis need further refinement before applying to actual decision making. In this sense, the purpose of this thesis may be interpreted as being one of developing approaches and techniques which are applicable to risk decision, which may be used regardless of the specific data and application.

CHAPTER II

BASIS FOR FITTING OF RISK DISTRIBUTIONS

II.1 Introduction

Risk is described by a distribution of the frequency of occurrence versus the magnitude of consequence. A risk distribution can be summarized by certain risk characteristics. However, any single risk characteristic alone, such as a risk moment, does not provide a complete information about the risk distribution. For example, the first risk moment about the origin of a fatalities distribution does not give any information whether the fatalities are caused by low frequency large consequence events (such as hurricanes) or high frequency small consequence events (such as auto accidents). Theoretically an infinite number of risk characteristics is required to describe the risk distribution, which results in an infinite number of equations to relate the risk distribution to other basic variables. As a compromise, the risk distribution will be fitted to a parametric distribution which only involves a small number of unknown parameters. Once the parameters have been determined, various risk characteristics can then be derived from the fitted parametric distribution. Also a limited number of equations are sufficient to identify the relationship of the risk distribution to the basic variables. In this chapter, the general approach of fitting will be discussed. The approach will be applied to the fatalities distributions of nuclear and non-nuclear risks in Chapter III.

The fitting approach can be divided into three fundamental steps, i.e., selection of candidate distributions to be examined, estimation of unknown constants by fitting and determination of adequate fits based on certain criteria. The fundamental steps will be discussed in the following sections.

One of the special characteristics of the risk analysis is that the extreme consequences as well as lesser consequences are of interest. For example, people sometimes view a single large consequence event more unfavorably than numerous small events having the same total number of fatalities. Therefore the extreme consequence, i.e., the tail behavior of the distribution, will be emphasized in selection of the candidate distributions, the fitting techniques and the criteria of adequate fits. The lesser consequence, i.e., main body behavior of the distribution will also be considered to obtain average risk values with small fitting errors.

II.2 Basis for Selection of Candidate Distributions

A number of candidate parametric distributions will be considered in Chapter III to fit the calculated risk distributions in Figs. 1.1 and 1.2. These calculated distributions to be fitted are called "data distribution" in this thesis. They were obtained by the historical records or by the calculational models using basic component and system failure data. The selection of the candidate parametric distributions will be based on the following considerations:

(1) Domain where the independent variable of the distribution is defined: The domain of the candidate distributions will be determined by the range of the available data. For certain non-nuclear risks, the available historical records are limited to major incidents having consequences greater than

a certain value. The lower end of the domain will be determined by the incident of the smallest consequence recorded or calculated.

(2) Number of modes of the distribution: The mode is a number of peaks in the frequency distribution. When the data distribution is bi-modal and neglecting one of the modes significantly harms the analysis, bi-modal candidate distributions will be considered.

(3) Symmetric or skewed: The skewness is an asymmetric behavior of the frequency distribution. When the distribution peak is to the right of the mean, the distribution is negatively skewed. When the peak is to the left of the mean, it is positively skewed.

(4) Tail behavior: As the tail behavior is of interest in the analysis, a number of candidate distributions with different tail behaviors will be considered for extrapolation sensitivities.

(5) Number of parameters: The distributions with the smaller number of parameters are preferred to keep the model simple.

II.3 Fitting Technique

Having selected candidate distributions, the values of unknown parameters of the candidate distributions will be estimated from the historical data or calculation results. Various techniques have been developed for obtaining estimates of these unknown parameters. Though the best technique may be different for each of the candidate distributions, two general techniques will be discussed here briefly in context of fitting to the risk distributions. General discussion about fitting techniques can be found in standard statistics text books (Ref-3, Ref-4, Ref-5 and Ref-6).

II.3.1 Method of Moments

Let a random variable Y have a frequency distribution given by $f_Y(y:\tau_1,\ldots,\tau_k)$ where τ's represent its k parameters. Let M_m be the m-th moment of $f_Y(y:\tau_1,\ldots,\tau_k)$ about a given magnitude ξ, that is:

$$M_m = \int (y-\xi)^m \cdot f_Y(y:\tau_1,\ldots,\tau_k)\,dy \tag{2.1}$$

Clearly, M_m is a function of the k parameters and hence M_m can be written as:

$$M_m = M_m(\tau_1,\ldots,\tau_k) \tag{2.2}$$

Let $Y_1,Y_2,\ldots,Y_n$ be a random sample of size n from $f_Y(y:\tau_1,\ldots,\tau_k)$. The m-th sample moment $\tilde{M}_m$ are:

$$\tilde{M}_m = \frac{1}{n}\sum_{i=1}^{n}(Y_i-\xi)^m \tag{2.3}$$

The moment estimators $\hat{\tau}_j$, j=1,...,k of the k-parameters are obtained by solving the following k equations:

$$\tilde{M}_m = M_m(\tau_1,\tau_2,\ldots,\tau_k) \qquad m=1,2,\ldots,k \tag{2.4}$$

The advantages of this method are that the calculational procedure is simple for many distributions and also the estimate of the first risk moment (average risk value if $\xi=0$) is not affected by fitting because it is used to estimate the parameters. The disadvantage is that the residual mean square which will be defined later is usually larger than that of the method of least squares.

II.3.2 Method of Least Squares

Suppose that there exist n observable variates $Y_1, Y_2, \ldots, Y_n$ with variance σ_Y^2, which are expressed by:

$$Y_1 = G(x_1 : \tau_1, \tau_2, \ldots, \tau_k) + e_1 \sigma_Y$$

$$Y_2 = G(x_2 : \tau_1, \tau_2, \ldots, \tau_k) + e_2 \sigma_Y$$

$$\vdots$$

$$Y_n = G(x_n : \tau_1, \tau_2, \ldots, \tau_k) + e_n \sigma_Y \tag{2.5}$$

where $G(x : \tau_1, \tau_2, \ldots, \tau_k)$ is a candidate function with k parameters $\tau_1, \tau_2, \ldots, \tau_k$. $\{e_i\}$ are assumed to be errors observing Y_i with $E\{e_i\} = 0$, where E refers to the expectation.

Let $y_1, y_2, \ldots, y_n$ be the observed value of the variates. The estimates $\hat{\tau}_1, \ldots, \hat{\tau}_k$ of the k parameters are obtained by minimizing:

$$\Delta^2 = \frac{1}{n-k} \sum_i [y_i - G(x_i : \tau_1, \ldots, \tau_k)]^2 \tag{2.6}$$

The advantage of this method is that it gives small value of the residual mean square. One of the disadvantages is that it sometimes requires a large computation time. Also the risk moments derived from the estimated parameters are associated with fitting errors.

In applying this method to the fitting of the risk distributions, the following options exist:

(1) The parametric function $G(x : \tau_1, \ldots, \tau_k)$ can be fitted to either the complementary cumulative distribution or the frequency distribution.

(2) The function can be fitted to y, ln y, or any other transformation of y.

This method will be applied to the fitting of the risk distributions in Appendix E. The logarithmic transformation of the complementary cumulative frequency will be used because the fractional errors of the complementary cumulative frequency have comparable magnitude than the absolute errors.

II.4 Criteria of Adequate Fits

After the fitting of the data distributions to the candidate parametric distributions is completed, one family of the distributions will be selected for the study of the relationship to the basic variables. The following criteria are proposed for the selection:

(1) The fitted parametric distribution should be within any error spreads associated with the data distribution (for example, within 90% confidence bounds). The data distributions of non-nuclear risks have estimation errors due to the limited number of available historical records. The data distributions of nuclear risks have errors due to the sampling used in the computer program and the uncertainties of the parameters used in the consequence model. The largest discrepancy in the fitted distribution should be within any estimated error bounds of the data distribution.

(2) The fitted distribution should have a small residual mean square, which is defined by:

$$s^2 = \frac{1}{n-k} \sum_{i=1}^{n} [y_i - G(x_i : \hat{\tau}_1, \ldots, \hat{\tau}_k)]^2 \qquad (2.7)$$

where y_i, x_i are the observed values, $G(x:\tau_1, \ldots, \tau_k)$ is a candidate function and $\hat{\tau}_1, \ldots, \hat{\tau}_k$ are the estimated values of the parameters. This criterion of the residual mean square can be taken as a relative measure to be used in comparing different possible fits. Specifically in this study, the residual mean square is evaluated for a natural logarithm of the complementary cumulative distribution as:

$$s^2 = \frac{1}{n-k} \sum_i [\ln \tilde{F}_i^C - \ln F^C(x_i : \hat{\tau}_1, \ldots, \hat{\tau}_k)]^2 \tag{2.8}$$

where x_i is the magnitude of consequence of sample data i and $\tilde{F}_i^C$ is its complementary cumulative frequency. $F^C(x:\tau_1,\ldots,\tau_k)$ is the candidate distribution. $\hat{\tau}_1,\ldots,\hat{\tau}_k$ are the estimated values of the parameters. The natural logarithmic transformation is used here because the fractional errors of the frequencies are of more interest than the absolute errors.

(3) Systematic errors should be small. When the tendencies to overpredict or underpredict over the ranges of the data are observed, the fitted distributions cannot be extrapolated to the range where the historical records or the calculation results are not available.

II.5 Summary

In this chapter, a general approach was presented for selection of a parametric distribution to fit the risk distributions. These risk distributions are obtained by the historical records or by the calculational models. The approach consists of three fundamental steps, i.e., selection of candidate parametric distributions, estimation of the unknown parameters and selection of adequate fitting distributions based on the criteria. The selection of candidate parametric distributions is based on the number of parameters and the properties of the data distribution, involving the domain of the independent variables, number of modes, skewness and tail behaviors. Two fitting techniques are specifically discussed: method of moments and method of least squares. The method of moments is simple and does not have fitting error of the risk moments, but it usually causes larger residual mean squares than the method of least squares. The method of least squares has small residual mean squares, but requires more computational work and causes fitting errors in the estimates of the risk moments. The criteria of adequate fits are based on the largest deviation, the residual mean squares and the systematic errors.

CHAPTER III

FITTING OF FATALITIES DISTRIBUTIONS OF NUCLEAR AND NON-NUCLEAR RISKS

III.1 Introduction

The general approach of the distribution fitting is applied to the fatalities distributions of nuclear and non-nuclear events in this chapter. Though nuclear risks are of major interest in this thesis, non-nuclear risks are studied here to find whether both types of risks can be described by the same family of distributions.

In Section III.2, candidate distributions are selected using the general criteria discussed in Section II.2. In Section III.3 the fitting technique is applied to the selected candidate distributions. The candidate distributions are evaluated by the historical records of non-nuclear risks in Section III.4 and by the risk estimates of nuclear risks in Section III.5.

III.2 Candidate Distributions

III.2.1 Selection of Candidate Distributions

The distribution of early fatalities of the average reactor as computed in WASH-1400 (Ref. 1) is shown on different scales as histograms in Figs. 3.1 and 3.2. The following behaviors are observed.

(1) The domain of the independent variable is positive.

(2) The histogram does not appear to have a mode.

(3) The histogram distribution is positively skewed.

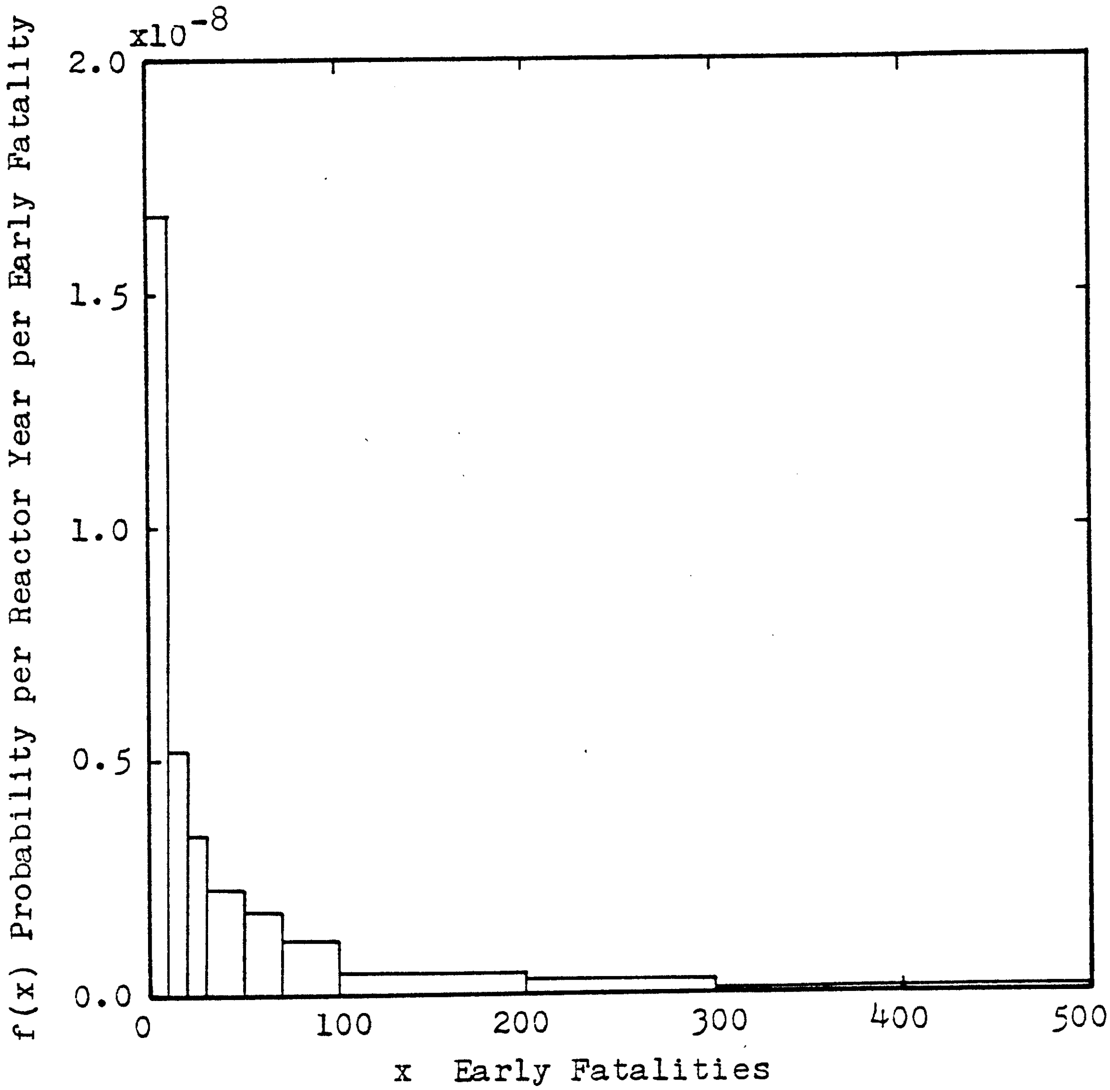

Fig.3.1 Histogram of the Early Fatalities Distribution of the Average of the U.S.100 Reactors (Linear Scale)

Note: Calculated from the results in WASH-1400(Ref-1)

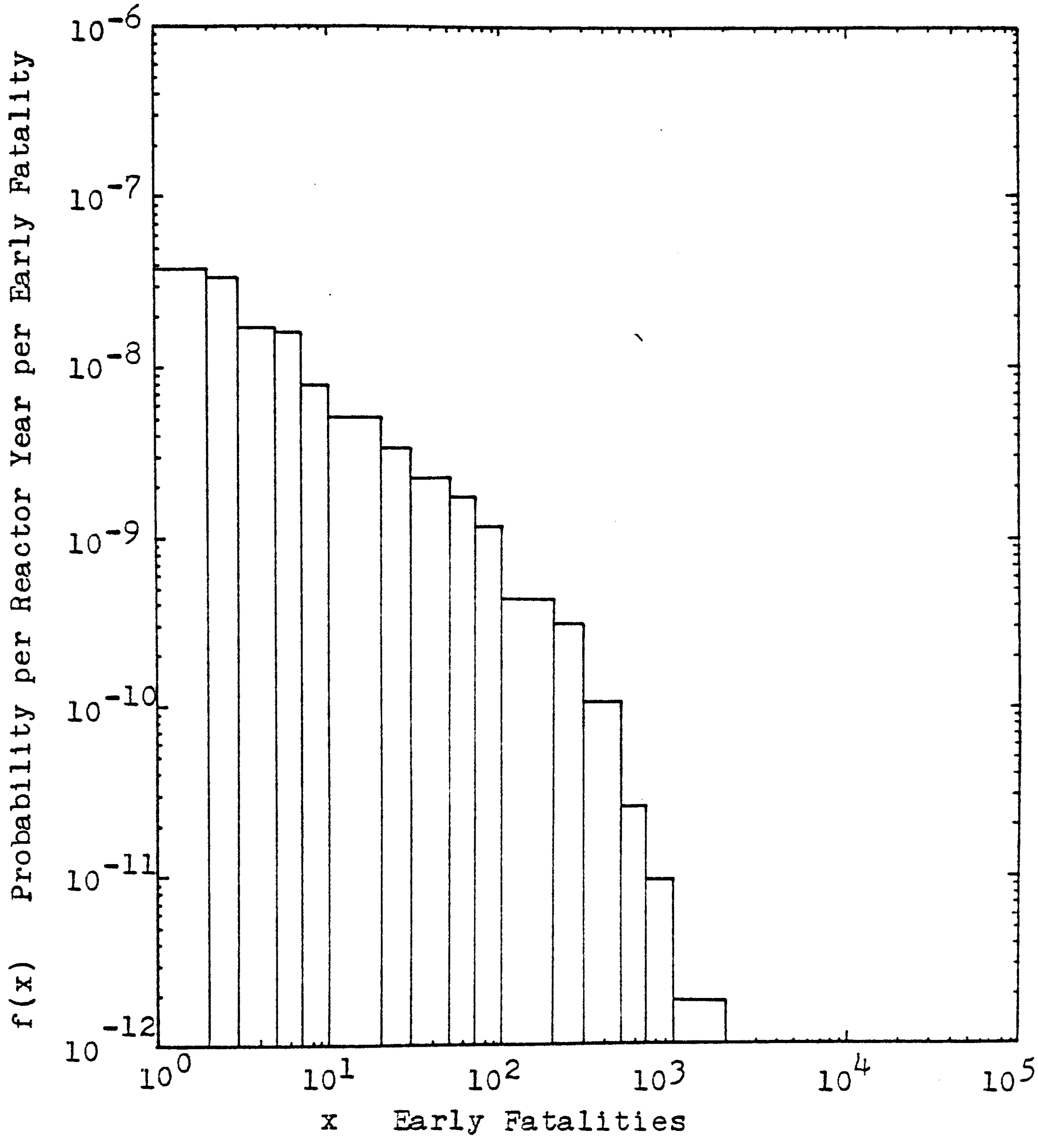

Fig. 3.2 Histogram of the Early Fatalities Distribution of the Average of U.S. 100 Nuclear Reactors (Logarithmic Scale)

Note: Calculated from the results in WASH-1400(Ref-1)

(4) The histogram distribution has a long tail. The tail behavior appears to be similar to an exponential.

The frequency distributions of other nuclear and non-nuclear risks have similar behaviors to that of the average reactor, as shown in Figs. 3.8, 3.10, 3.12, 3.14, 3.18 and 3.20 later in this chapter. Based on the behaviors of these data, the following four candidate distributions are selected in this study.

(1) Exponential

(2) Gamma

(3) Weibull

(4) Lognormal

The distributions above have the following common properties:

(a) They have no mode or at most one mode.

(b) They are positively skewed.

(c) The above distributions cover different tail behaviors, such as decreasing slower than the exponential, exponentially decreasing and decreasing more rapidly than the exponential.

In fitting these distributions, the following additional considerations are made.

The selection of the domain of the independent variables depends on the availability of data. For certain non-nuclear risks, the available historical records are limited to major incidents that have consequences greater than some value. For example, the records of tornadoes used in this study cover the incidents having greater than 20 fatalities. For the sake of fitting, the lower end of the domain is therefore defined by x_o which is the lower limit of the available data. The upper end of the domain is taken to be infinity. Though the fatalities can not exceed some physical limit (such as the population on the earth), the probability beyond that limit will be so small in the candidate distributions that the upper end should not effect the estimate of the parameters and moments.

The integrals of the frequency distributions, such as Fig. 3.1, over the defined domain are not always unity. The dimension of the data are also number of events per unit time. In fitting the distributions, a normalization constant α is therefore introduced, which is defined as the frequency per unit time that the consequences are larger than the lower end of the domain x_o. The candidate distribution f(x) will then be defined by the following form:

$$f(x) = \alpha \cdot \overline{f}(x) \qquad (3.1)$$

where $\overline{f}(x)$ is a probability density function, the integral of which over the defined domain is unity. For example, for the exponential, the density $\overline{f}(x)$ is given by:

$$\bar{f}(x) = \frac{1}{\theta} \cdot \exp\left[- \frac{(x - x_o)}{\theta} \right] \tag{3.2}$$

where θ is a scale factor of an exponential distribution. Then the frequency distribution of the exponential is given by:

$$f(x) = \alpha \cdot \bar{f}(x) = \frac{\alpha}{\theta} \cdot \exp\left[- \frac{(x - x_o)}{\theta} \right] \tag{3.3}$$

Other candidate distributions also have corresponding probability distributions which have been studied in various fields of statistical analysis. The discussion in this thesis is based on the unnormalized frequency distributions $f(x)$ rather than the normalized density distribution $\bar{f}(x)$. Similarly, the term "risk moments" are used in this study because they are the integrals of the unnormalized frequency distribution $f(x)$. From Eq. (3.1), the properties and the risk moments of the unnormalized distribution are simply obtained from those of the densities $\bar{f}(x)$.

III.2.2 Exponential Distribution

The exponential is defined by:

$$f(x) = \frac{\alpha}{\theta} \cdot \exp\left[-(x-x_o)/\theta\right] \tag{3.4}$$

where $x \geq x_o$, $x_o \geq 0$, $\alpha > 0$, $\theta > 0$.

If α and x_o are treated as known constants determined from the area and domain of the data distribution, respectively, then the exponential is a one-parameter distribution with a scale factor θ. The complementary cumulative distribution is given by:

$$F^C(x) = \int_x^\infty f(x)dx = \alpha \cdot \exp\left[-(x-x_o)/\theta\right] \tag{3.5}$$

The risk moments about x_o are given by:

$$M_1 = \alpha \cdot \theta \tag{3.6}$$

$$M_2 = 2 \cdot \alpha \cdot \theta^2 \tag{3.7}$$

$$M_m = \alpha \cdot \theta^m \cdot (m+1)! \tag{3.8}$$

The exponential with $\theta = 1$, $\alpha = 1$ and $x_o = 0$ is illustrated in Fig. 3.3, 3.4, 3.5 and 3.6 on different scales.

III.2.3 Gamma Distribution

The distribution is defined by:

$$f(x) = \alpha \cdot \frac{(x-x_o)^{\beta-1}}{\theta^\beta \cdot \Gamma(\beta)} \cdot \exp\left[-\frac{(x-x_o)}{\theta}\right] \tag{3.9}$$

where $x \geq x_o$, $x_o \geq 0$, $\alpha > 0$, $\theta > 0$, $\beta > 0$ and $\Gamma(\cdot)$ is the Gamma function. For given α and x_o, the distribution is a two-parameter distribution with a scale factor θ and a shape factor β. When β is integer, the complementary cumulative distribution is given by:

$$F^C(x) = \alpha \cdot \exp\left[-\frac{(x-x_o)}{\theta}\right] \cdot \sum_{j=0}^{\beta} \left(\frac{x-x_o}{\theta}\right)^j \frac{1}{\Gamma(j+1)} \quad (3.10)$$

When β is not integer, $F^C(x)$ is not expressed by a closed form. The risk moments about x_o are given by:

$$M_1 = \alpha \cdot \theta \cdot \beta \quad (3.11)$$

$$M_2 = \alpha \cdot \theta^2 \cdot \beta \cdot (\beta+1) \quad (3.12)$$

$$M_m = \alpha \cdot \theta^m \cdot \beta \cdot (\beta+1)\dots(\beta+m-1) \quad (2.13)$$

If $\beta > 1$, the frequency distribution has a mode at $x = x_o + \theta \cdot (\beta-1)$. If $\beta=1$, the gamma reduces to the exponential. If $\beta < 1$, the frequency distribution does not have a mode and is continuously decreasing. If $\beta < 1$ and x approaches x_o, the frequency distribution goes to infinity, but the integral over any finite range about x_o is always finite. The gamma has an exponential tail, regardless of the values of β and θ. Its behavior with $\theta = 1$, $\alpha = 1$ and $x_o = 0$ is also illustrated in Figs. 3.3, 3.4, 3.5, and 3.6 for different values of β.

III.2.4 Weibull Distribution

The distribution is defined by:

$$f(x) = \alpha \cdot \left(\frac{\beta}{\eta}\right) \cdot \left(\frac{x-x_o}{\eta}\right)^{\beta-1} \cdot \exp\left[-\left(\frac{x-x_o}{\eta}\right)^{\beta}\right] \quad (3.14)$$

where $x \geq x_o$, $x_o \geq 0$, $\alpha > 0$, $\beta > 0$ and $\eta > 0$.

For given α and x_o, the Weibull is a two-parameter distribution with a scale factor η and a shape factor β. The complementary cumulative distribution is given by:

$$F^C(x) = \alpha \cdot \exp\left[-\left(\frac{x-x_o}{\beta}\right)^{\beta}\right] \quad (3.15)$$

The risk moments about x_o are given by:

$$M_1 = \alpha \cdot \eta \cdot \Gamma\left(1 + \frac{1}{\beta}\right) \quad (3.16)$$

$$M_2 = \alpha \cdot \eta^2 \cdot \Gamma\left(1 + \frac{2}{\beta}\right) \quad (3.17)$$

$$M_m = \alpha \cdot \eta^m \cdot \Gamma\left(1 + \frac{m}{\beta}\right) \quad (3.18)$$

If $\beta > 1$, the frequency distribution has a mode at $x = x_o + \eta \cdot (1-1/\beta)^{1/\beta}$. If $\beta = 1$, the Weibull reduces to an exponential. If $\beta < 1$, the frequency distribution does not have a mode and is continuously decreasing. If $\beta < 1$ and x approaches x_o, f(x) goes to infinity, but the integral over any finite range about x_o is always finite. The rate of decrease in the tail depends on the value of β. If $\beta < 1$, the Weibull decreases more slowly than the exponential. If $\beta > 1$, the Weibull decreases more rapidly than the exponential. The Weibull behavior with $\eta = 1$, $\alpha = 1$ and $x_o = 0$ is also illustrated for different values of β in Figs. 3.3, 3.4, 3.5 and 3.6.

III.2.5 Lognormal Distribution

The distribution is defined by :

$$f(x) = \alpha \cdot \frac{1}{(x-x_o) \cdot \sigma \cdot \sqrt{2\pi}} \exp\left[-(\ln(x-x_o) - \mu)^2/2\sigma^2\right] \qquad (3.19)$$

where $x \geq x_o$, $x_o \geq 0$, $\alpha > 0$, and $\sigma > 0$. For given α and x_o, the lognormal is a two-parameter distribution with a mean μ and standard deviation σ for the normal variable $\ln(x-x_o)$. The complementary cumulative distribution is given by:

$$F^C(x) = \alpha \cdot \frac{1}{\sigma\sqrt{2\pi}} \int_{\ln(x-x_o)}^{\infty} \exp\left[-(\xi - \mu)^2/2\sigma^2\right] d\xi \qquad (3.20)$$

The risk moments about x_o are given by:

$$M_1 = \alpha \cdot \exp\left[\mu + \tfrac{1}{2}\sigma^2\right] \qquad (3.21)$$

$$M_2 = \alpha \cdot \exp\left[2\mu + 2\sigma^2\right] \qquad (3.22)$$

$$M_m = \alpha \cdot \exp\left[m\mu + \tfrac{1}{2}m^2\sigma^2\right] \qquad (3.23)$$

The frequency distribution has a mode at $x = x_o + \exp[\mu - \sigma^2]$. The tail of the lognormal decreases more slowly than the exponential. The lognormal behavior with $\alpha = 1$, $x_o = 0$, $\mu = 0$ is illustrated in Figs. 3.3, 3.4, 3.5 and 3.6 for different values of σ.

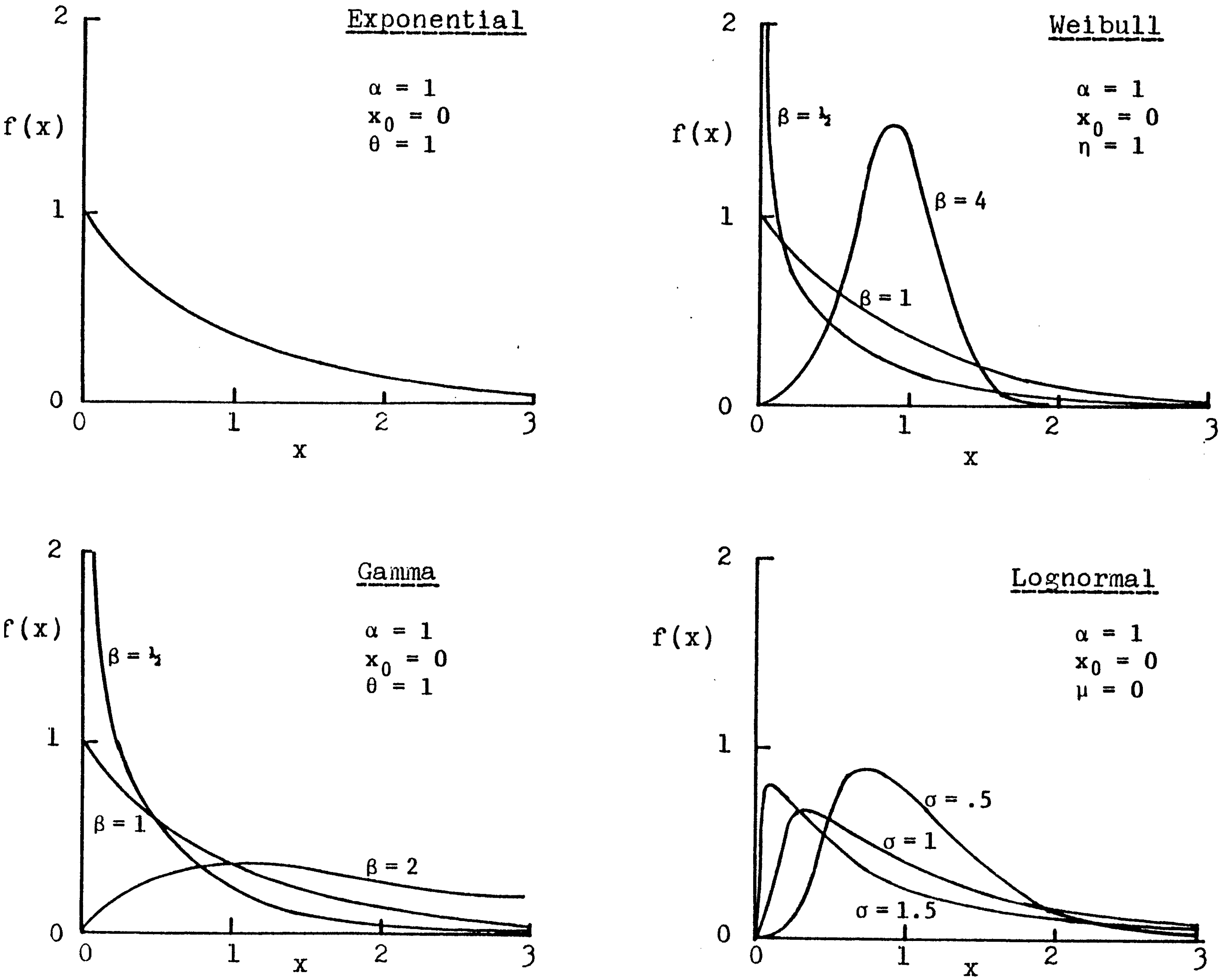

Fig. 3.3 Frequency Distributions of Candidate Families (Linear Scale)

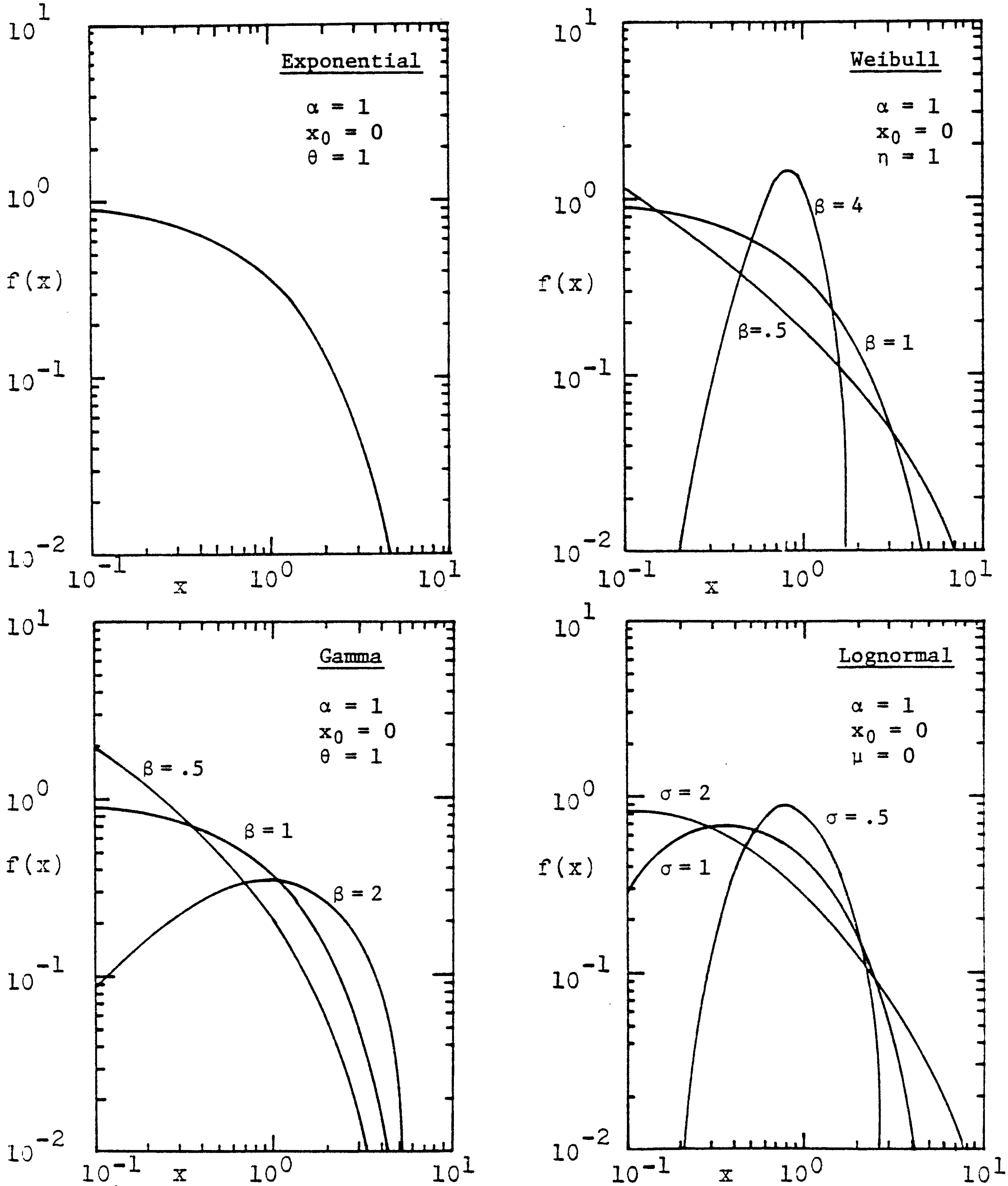

Fig. 3.4 Frequency Distributions of the Candidate Families (Logarithmic Scale)

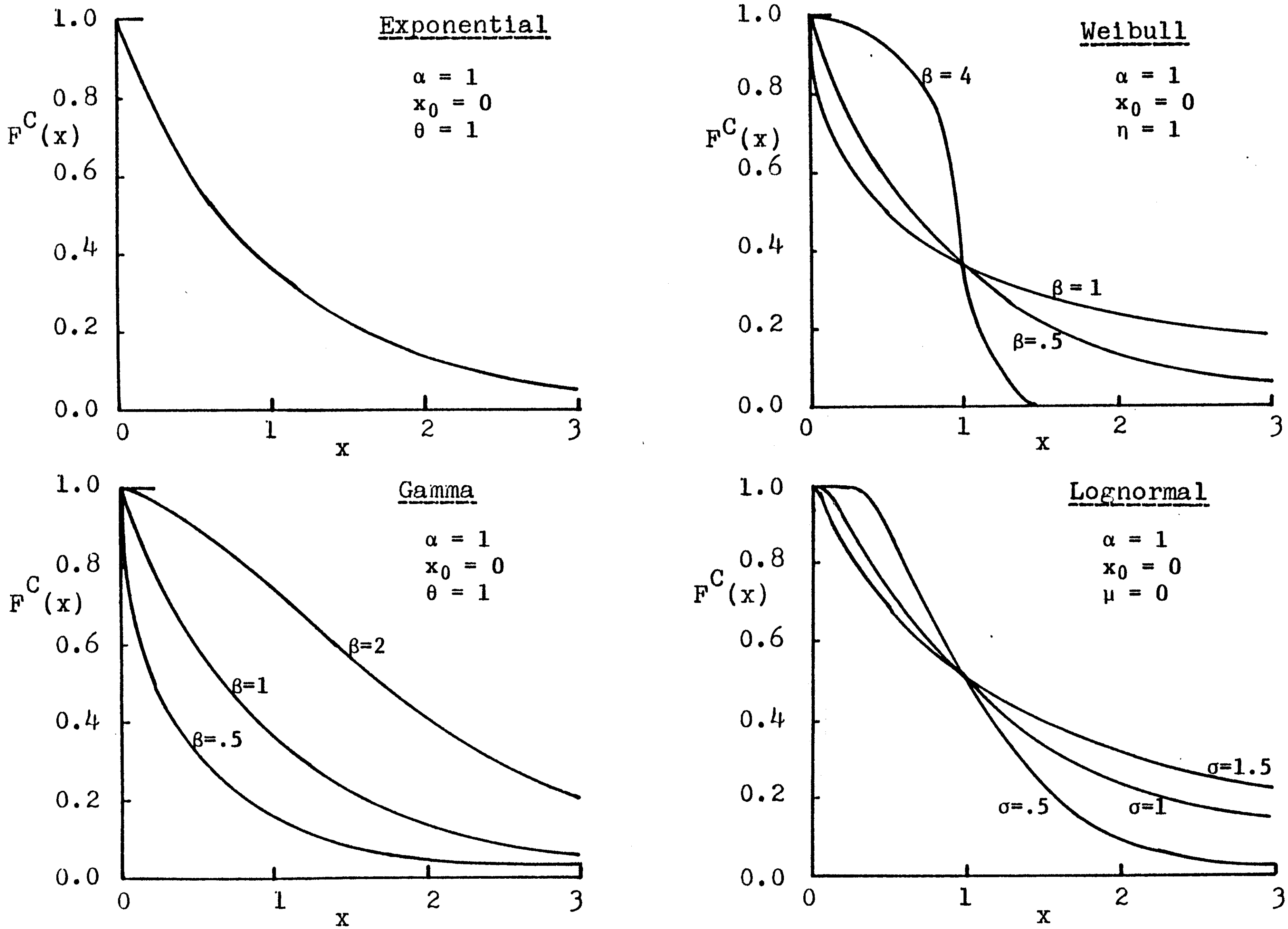

Fig.3.5 Complementary Cumulative Distributions of Candidate Families(Linear Scale)

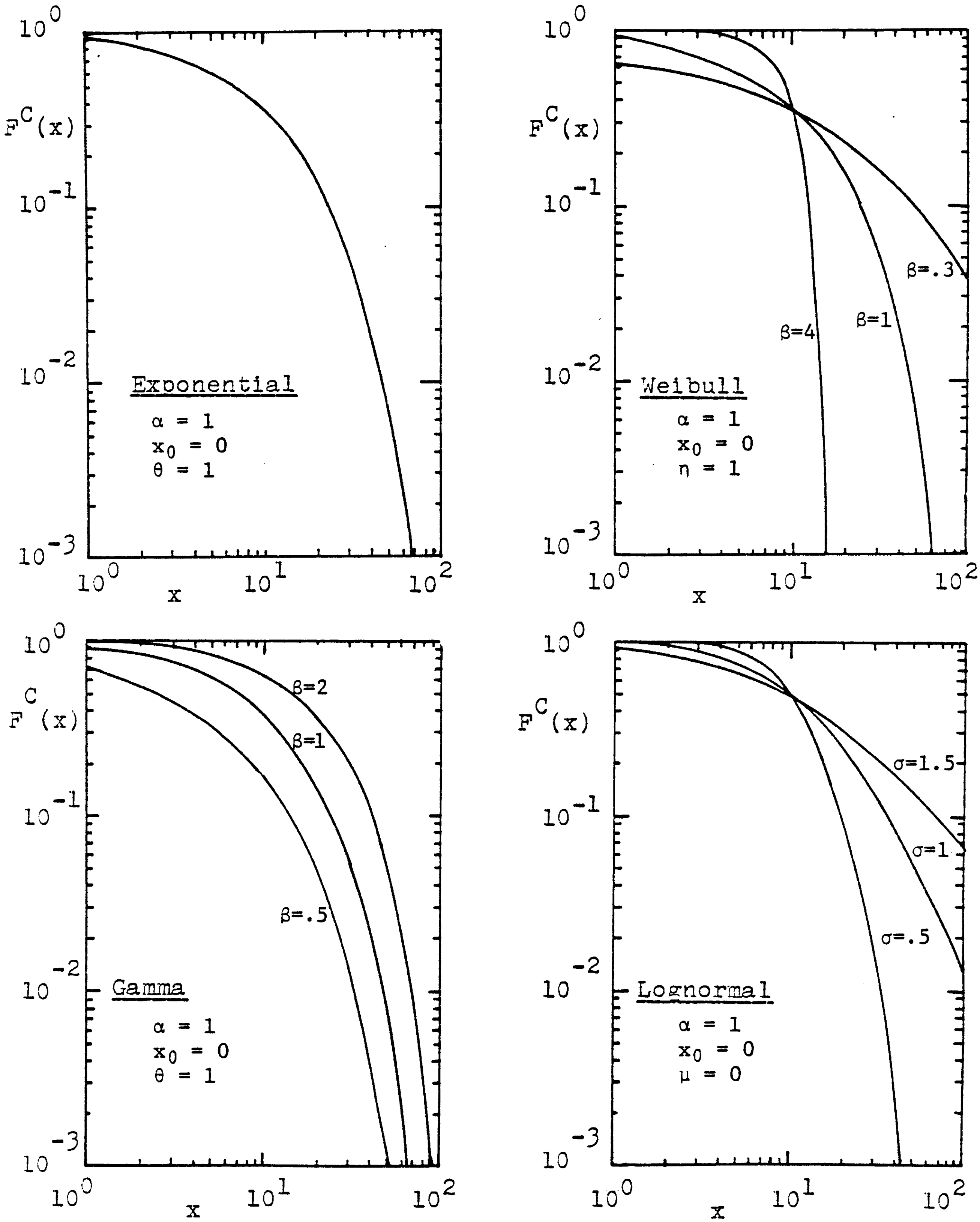

Fig. 3.6 Complementary Cumulative Distributions of Candidate Families (Logarithmic Scale)

III.3 Fitting Techniques

Two candidate fitting techniques were discussed in Section III.3. They are the method of moments and method of least squares. The method of moments is selected in this study because its computation procedure is simple and also because the risk moments will be used to investigate the relation with more basic variables. The moments estimation will be compared with the method of least squares in Appendix E. The method of moments is applied to the candidate distributions in the following way.

(1) Exponential

Since this is a one-parameter distribution the first risk moment about x_o is used to estimate the scale factor θ.

$$\theta = \frac{M_1}{\alpha} \tag{3.24}$$

(2) Gamma

The scale factor θ and the shape factor β are estimated from the first two risk moments about x_o by solving Eqs. (3.11) and (3.12), which give:

$$\beta = \frac{M_1^2}{M_2\alpha - M_1^2} \tag{3.25}$$

$$\theta = \frac{M_2\alpha - M_1^2}{\alpha M_1} \tag{3.26}$$

(3) Weibull

The scale factor η and the shape factor β are estimated from the first two risk moments about x_o by solving Eq. (3.16) and (3.17). The quantity β is given by:

$$\frac{[\Gamma(1 + \frac{1}{\beta})]^2}{[\Gamma(1 + \frac{2}{\beta})]} = \frac{M_1{}^2}{M_2\alpha} \qquad (3.27)$$

A table which evaluates the left hand side of Eq. (3.27) versus values of β is given in Appendix D for a range of $0.1 \leq \beta < 1.1$. Also $\Gamma(1 + \frac{1}{\beta})$ and $\Gamma(1 + \frac{2}{\beta})$ are given in Appendix D. Using these tables to derive β, η is then estimated by:

$$\eta = \frac{M_1}{\alpha\ \Gamma(1 + \frac{1}{\beta})} \qquad (3.28)$$

(4) Lognormal

The mean μ and the standard deviation σ of the normal distribution for $\ell n\ x$ are estimated from the first two risk moments by:

$$\mu = 2\ \ell n\ (\frac{M_1}{\alpha}) - 1/2\ \ell n\ (\frac{M_2}{\alpha}) \qquad (3.29)$$

$$\sigma^2 = \ell n\ (\frac{M_2}{\alpha}) - 2\ \ell n\ (\frac{M_1}{\alpha}) \qquad (3.30)$$

III.4 Fitting of Non-Nuclear Risk Distributions

III.4.1 Source of the Data

The candidate distribution families are fitted here to the historical records of the non-nuclear risks. The purpose of this analysis is to investigate whether non-nuclear and nuclear risks can be described by one distribution family. The non-nuclear risks investigated here are those from hurricanes, earthquakes, tornadoes and dam failures. Except for tornadoes, the historical records are summarized in WASH-1400 (Ref. 1). The historical record of the major tornadoes is listed in the 1976 World Almanac (Ref. 7).

The frequency versus consequence distributions of non-nuclear risks are calculated by ranking the historical observations in a descending order based on the magnitudes of the consequences. The estimates of the complementary cumulative frequency at a specific value x is calculated from the number of observations having consequences greater than the specified value.

$$F^C(x) = \frac{\kappa}{T} \tag{3.31}$$

where $F^C(x)$ is the calculated complementary cumulative frequency at x, κ is the number of the observations having consequences greater than x and T is the time period in which the observations are recorded, The frequency distribution is calculated by grouping the observations into certain number of the classes based on the magnitude of consequence.

The calculated frequency f(x) is given by:

$$f(x) = \frac{\Delta\kappa}{T \cdot \Delta x} \tag{3.32}$$

where Δx is the width of the class and $\Delta\kappa$ is the number of the observations in the class.

The first two risk moments about x_o are estimated from the historical records as:

$$M_1 = \frac{1}{T} \sum_i (x_i - x_o) \tag{3.33}$$

$$M_2 = \frac{1}{T} \sum_i (x_i - x_o)^2 \tag{3.34}$$

The confidence bounds of the calculated complementary cumulative frequencies were estimated in WASH-1400 (Ref. 1). Table 3.1 gives the confidence factors versus the number of the observations having consequences greater than the value of interest. These confidence factors are reproduced from WASH-1400 (Ref. 1). The 95% upper bound is computed by multiplying the estimated complementary cumulative value by the corresponding confidence factor in Table 3.1 and the 5% lower bound is computed by dividing it by the corresponding confidence factor. One of the criteria of the adequate fits discussed in Section II.3 is interpreted as follows. The largest deviation of the fitted curve should be within the 90% confidence bounds calculated from Table 3.1.

TABLE 3.1

Confidence Factors

No. of observations greater than a particular value	95% Upper bound (a)	5% Lower Bound (b)
1000	1.05	1.05
100	1.2	1.2
50	1.3	1.3
20	1.4	1.5
10	1.7	1.8
5	2.1	2.5
1	4.7	10.4

(a) Estimated frequency should be multiplied by this value to obtain upper confidence bound

(b) Estimated frequency should be divided by this value to obtain lower confidence bound

III.4.2 Hurricanes

The historical records of the fatalities in hurricanes are summarized in Ref. 1[1]. 46 fatal incidents were recorded in 73 years. The estimate of the normalization constant is then,

$$\alpha = \frac{46}{73 \text{ years}} = .63/\text{year}$$

Though the fatality of less than 1 is not physically real, the domain of the consequence is taken to be greater than 0, because it does not cause major errors in the fitting procedure[2]. The risk moments estimated from the data are:

$$M_1 = \frac{1}{73} \sum_i x_i = 172.3$$

$$M_2 = \frac{1}{73} \sum_i x_i^2 = 5.64 \times 10^5$$

---

[1] See Table 6.8 in Main Report of WASH-1400 (Ref. 1)

[2] The risk moments about $x_o = 1$ are,

$$M_1 = 171.6$$

$$M_2 = 5.64 \times 10^5$$

The differences from the risk moments about $x_o = 0$ are not significant.

From the risk moments, the parameters of the exponential, gamma, Weibull and lognormal distributions are estimated. The parameter of the exponential distribution is estimated from the first risk moment by Eq. (3.22). The parameters of the other distributions are estimated by Eqs. (3.23) through (3.28). The residual mean squares are calculated by Eq. (2.10). The results are summarized in Table 3.2. The fitted complementary cumulative distributions using the parameter estimates are given in Fig. 3.7 along with the data. The band attached to the data points are the 90% confidence bounds discussed in Section III.4.1. The fitted frequency distributions are given in Fig. 3.8 with the histogram of the data calculated by Eq. (3.32).

The fitted candidate distributions are now evaluated by the criteria discussed in Sections II.5 and III.4.2.

The exponential distribution in Fig. 3.7 is out of the confidence bounds, overestimating the complementary cumulative frequency (denoted by c.c.f. in the following discussion) by a factor of more than 2 in the range of 10 to 500 fatalities and underestimating the c.c.f. by a factor of more than 100 at the largest consequence of the observed data. The gamma distribution is also out of the confidence bounds, underestimating the c.c.f. by a factor of 2 for less than 10 fatalities. The lognormal distribution overestimates the c.c.f. for low consequence range and underestimates it for the largest consequence, but the distribution is within the confidence bounds of the data. The Weibull distribution does not show any apparent systematic error in the range of less than 1000 fatalities, but underestimates the c.c.f. for the largest consequence.

TABLE 3.2

Estimates of the Parameters of the Fatalities

Distribution in Hurricanes

$x = 0$, $\alpha = .630$, $M_1 = 1.72 \times 10^2$, $M_2 = 5.64 \times 10^5$

Candidate Distribution	Estimates of Parameters		Residual Mean Square
Exponential	$\theta = 2.73 \times 10^2$		10.9
Gamma	$\beta = .091$	$\theta = 3.01 \times 10^3$	.31
Weibull	$\beta = .387$	$\eta = 7.48 \times 10^1$	.11
Lognormal	$\mu = 4.37$	$\sigma = 2.49$	.21

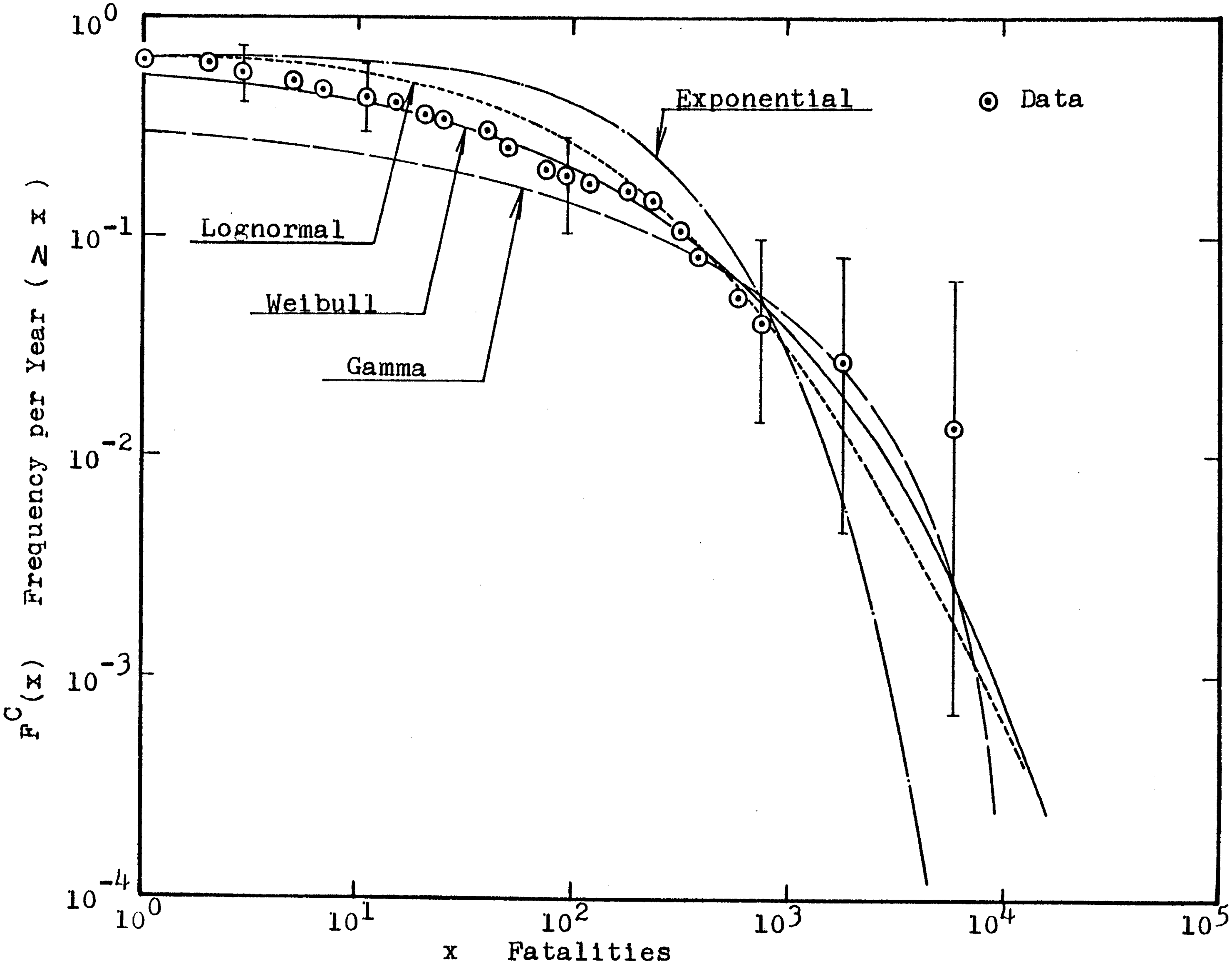

Fig.3.7 Complementary Cumulative Distribution of Fatalities due to Hurricanes

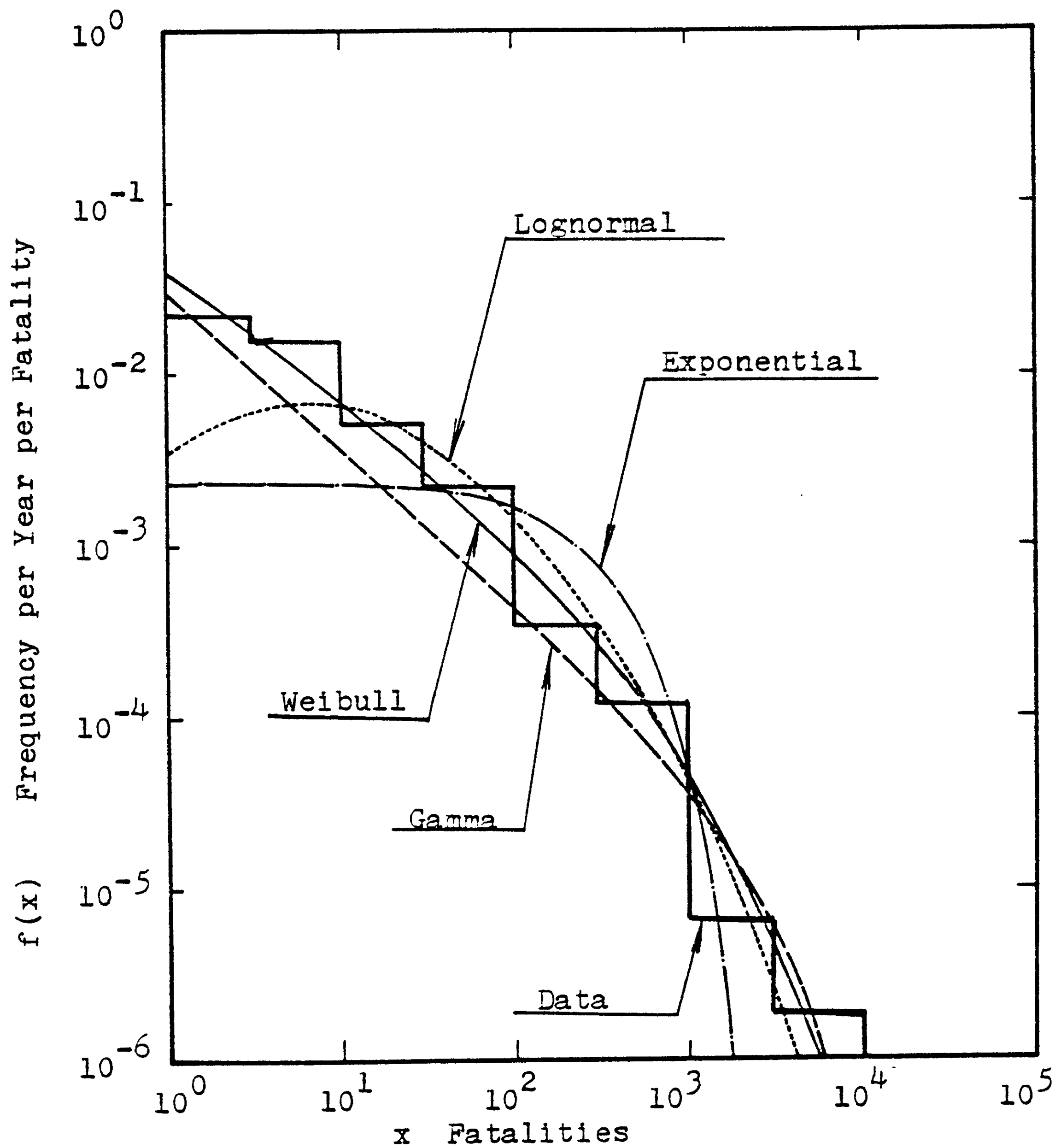

Fig.3.8 Frequency Distribution of Fatalities due to Hurricanes

The Weibull distribution is within the confidence bounds of the data.

Table 3.2 shows that the Weibull has the smallest residual mean square. The lognormal and gamma are the next. The exponential has the largest residual mean square.

III.4.3 Earthquakes

The historical records of the fatalities were given in Ref. 1[1]. 12 fatal incidents were recorded in 73 years. The domain is taken to be greater than zero as was done in the hurricane distributions. The estimates of the normalization constant and the first two risk moments are given in Table 3.3. As before, the parameters of the candidate distributions are estimated from the first two risk moments. The results of fitting are given in Table 3.3, Figs. 3.9 and 3.10.

The exponential distribution in Fig. 3.9 is out of the confidence bounds, underestimating the c.c.f. by a factor of more than 100 for the largest consequence. The other three distributions are within the confidence bounds. The gamma distribution in Fig. 3.9 slightly underestimates the c.c.f. for the low consequence region and also for the largest consequence. The lognormal and the Weibull underestimate the c.c.f. for the largest consequence.

The residual mean square of the Weibull is the smallest. The gamma and lognormal are the next. The exponential has the largest residual mean square.

[1]See Table 6.9 in the Main Report of WASH-1400 (Ref. 1)

Table 3.3

Estimates of the Parameters of the Fatalities

Distribution in Earthquakes

$x_o = 0$, $\alpha = .164$, $M_1 = 1.53 \times 10^1$, $M_2 = 8.13 \times 10^3$

Candidate Distribution	Estimates of Parameters		Residual Mean Square
Exponential	$\theta = 9.31 \times 10$		2.96
Gamma	$\beta = .212$	$\theta = 4.38 \times 10^2$	.27
Weibull	$\beta = .511$	$\eta = 4.84 \times 10^1$	.26
Lognormal	$\mu = 3.66$	$\sigma = 1.74$	.42

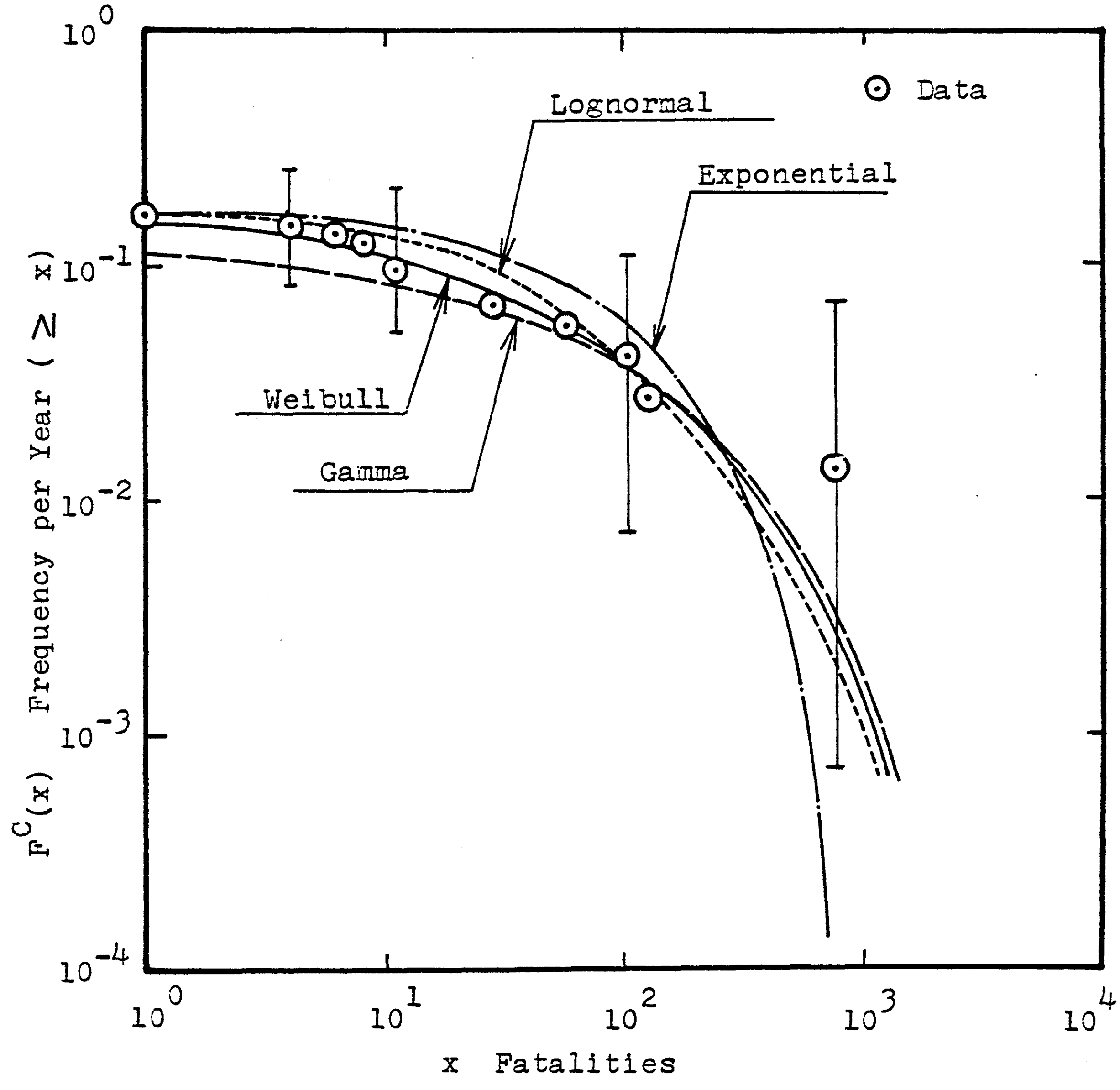

Fig 3.9 Complementary Cumulative Distribution of Fatalities due to Earthquakes

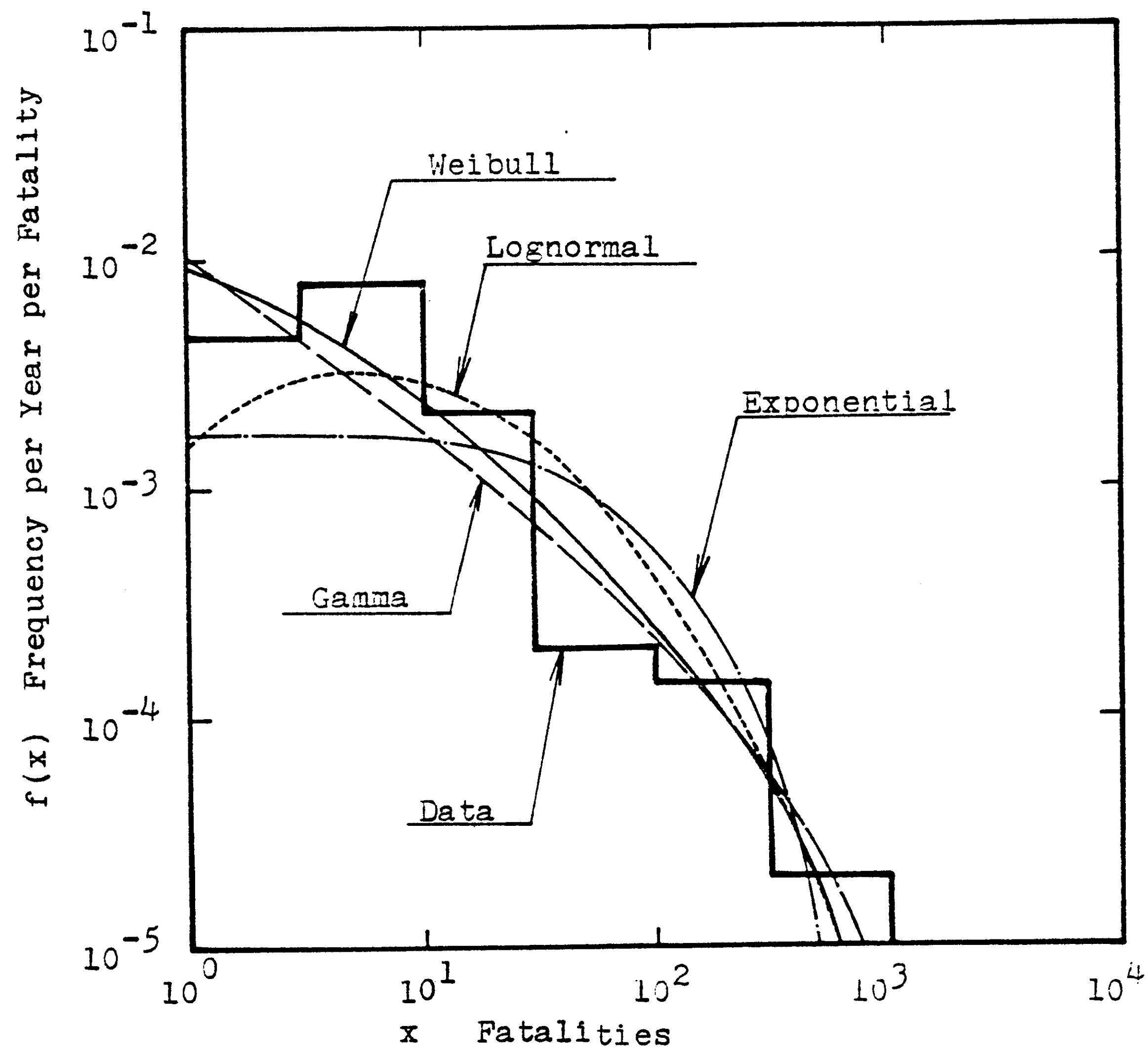

Fig.3.10 Frequency Distribution of Fatalities due to Earthquakes

III.4.4 Tornadoes

The historical records of the major tornadoes in Ref. 7 are summarized in Table 3.4. 38 incidents were recorded in 47 years that caused more than 20 fatalities. As the records below 20 fatalities are not found in Ref. 7, the domain of the fatalities is taken to be greater than 20. The normalization constant and the first two risk moments about $x_o = 20$ which are estimated from Table 3.4 are given in Table 3.5. The results of fitting are given in Table 3.5, Figs. 3.11 and 3.12.

The exponential distribution in Fig. 3.11 is out of the confidence bounds of the data, underestimating the c.c.f. by a factor of more than 100 for the largest consequence of the data. The other three distributions underestimate the c.c.f. for the largest consequence, but they are within the confidence bounds of the data. The residual mean square of the Weibull distribution in Table 3.5 is the smallest. The lognormal and the gamma are the next. The exponential has the largest residual mean square.

Table 3.4

Fatalities of U.S. Major Tornadoes
(1925 - 1971) (a)

Number	Date (month/year)	Lives Lost
1	3/25	689
2	4/65	271
3	3/32	268
4	4/36	216
5	3/52	208
6	4/36	203
7	4/47	169
8	6/44	150
9	6/53	116
10	5/53	114
11	2/71	110
12	4/45	102
13	5/27	92
14	6/53	90
15	5/55	80
16	3/42	75
17	4/27	74
18	9/27	72
19	3/66	61
20	1/49	58
21	3/66	57
22	11/26	53
23	4/42	52
24	5/57	48
25	5/30	41
26	4/29	40
27	12/53	38
28	5/68	34
29	3/48	33
30	4/67	33
31	1/69	32
32	9/38	32
33	1/46	30
34	6/58	30
35	5/60	30
36	5/70	26
37	4/70	25
38	2/59	21

(a) From "The World Almanac and Book of Facts 1976", Newspaper Enterprise Association, Inc.

Table 3.5 Estimates of the Parameters of the Fatalities Distribution in Tornadoes

$x_0 = 20$, $\alpha = .810$, $M_1 = 6.62 \times 10^1$, $M_2 = 1.67 \times 10^4$

Candidate Distributions	Estimates of Parameters		Residual Mean Square
Exponential	$\theta = 8.17 \times 10^1$		.66
Gamma	$\beta = .479$	$\theta = 1.71 \times 10^2$	.11
Weibull	$\beta = .708$	$\eta = 6.53 \times 10^1$	.086
Lognormal	$\mu = 3.84$	$\sigma = 1.12$	.093

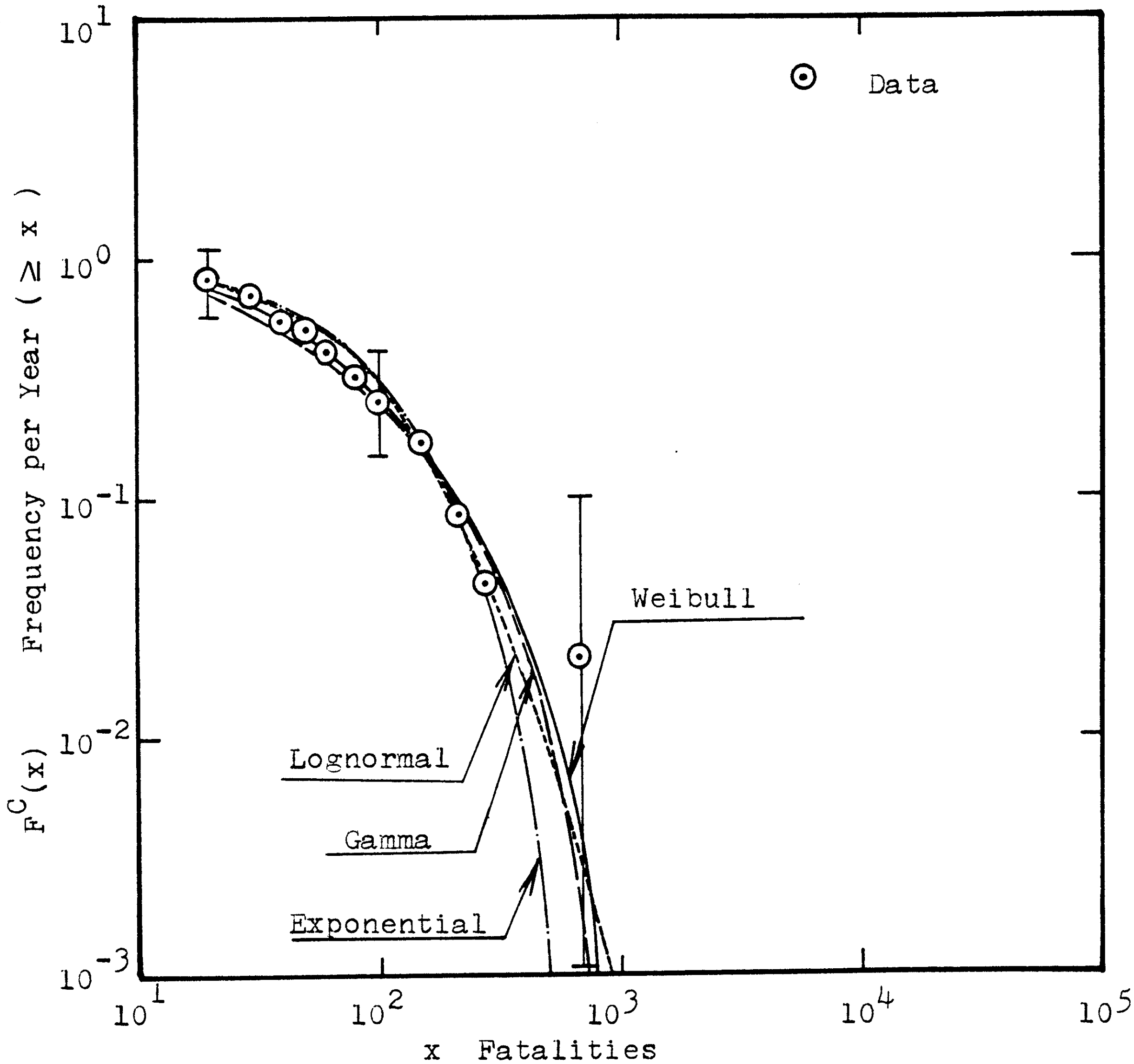

Fig. 3.11 Complementary Cumulative Distribution of Fatalities due to Tornadoes

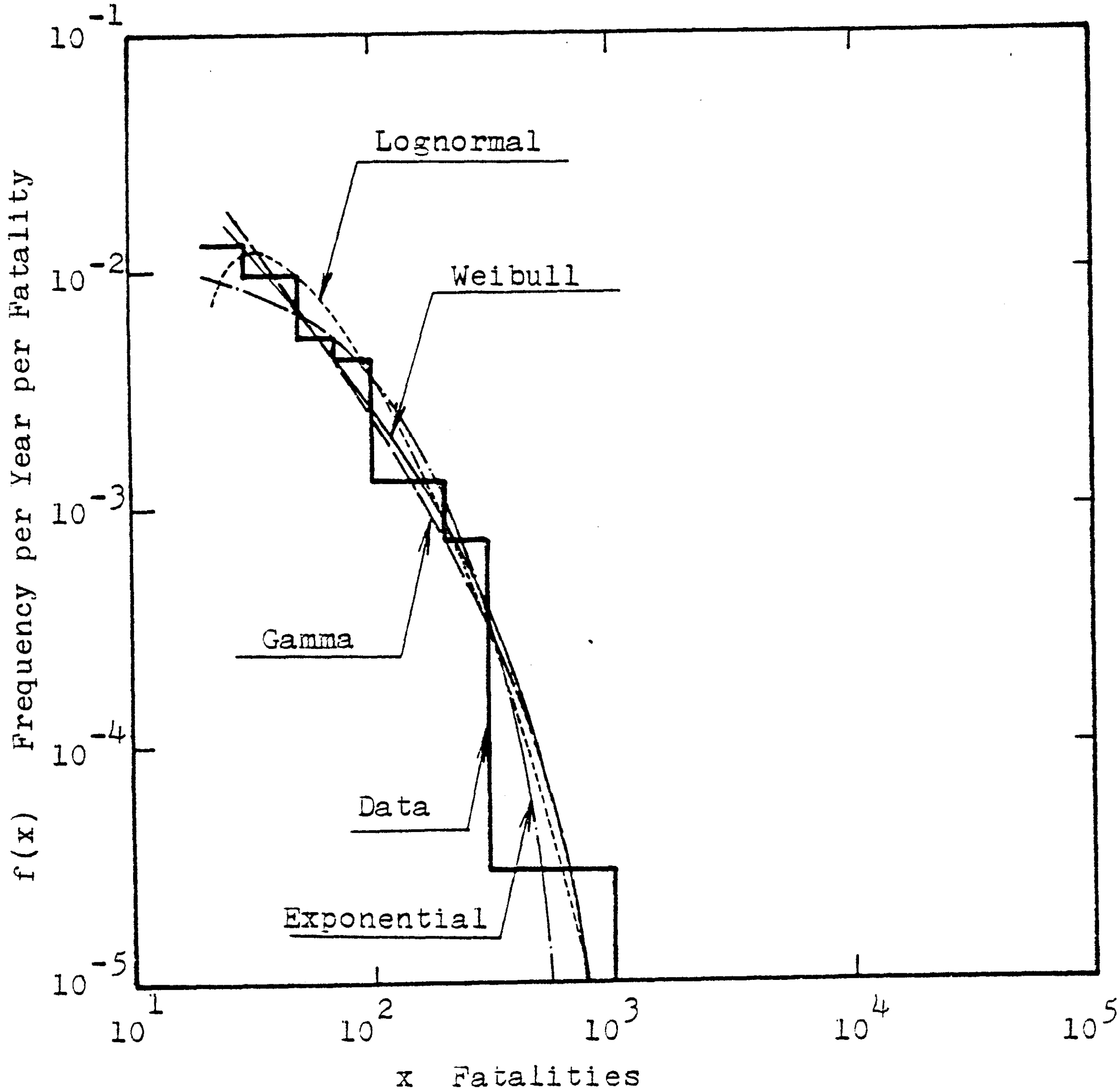

Fig.3.12 Frequency Distribution of Fatalities due to Tornadoes

III.4.5 Dam Failures

The historical records of the fatalities in dam failures are summarized in Ref. 1[1]. Eight fatal incidents were recorded in 84 years. The domain is taken to be greater than zero as was done in the distributions of hurricanes and earthquakes. The normalization constant and the first two risk moments are estimated from the historical data in Ref. 1. The estimates are given in Table 3.6. The results of fitting are given in Table 3.6, Figs. 3.13 and 3.14.

All of the four candidate distributions underestimate the complementary cumulative frequency for the largest consequence, but they are within the confidence bounds of the data. The residual mean square of the gamma distribution is the smallest. The next are the Weibull and the lognormal. The exponential has the largest residual mean square.

[1]See Table 6.12 in the Main Report of WASH-1400 (Ref. 1)

Table 3.6

Estimates of the Parameters of the Fatalities Distribution in Dam Failures

$x_o = 0$, $\alpha = .0952$, $M_1 = 3.48 \times 10^1$, $M_2 = 5.07 \times 10^4$

Candidate Distribution	Estimates of Parameters		Residual Mean Square
Exponential	$\theta = 3.65 \times 10^2$		1.70
Gamma	$\beta = .335$	$\theta = 1.09 \times 10^3$	.37
Weibull	$\beta = .608$	$\eta = 2.47 \times 10^2$	.39
Lognormal	$\mu = 5.21$	$\sigma = 1.38$	.57

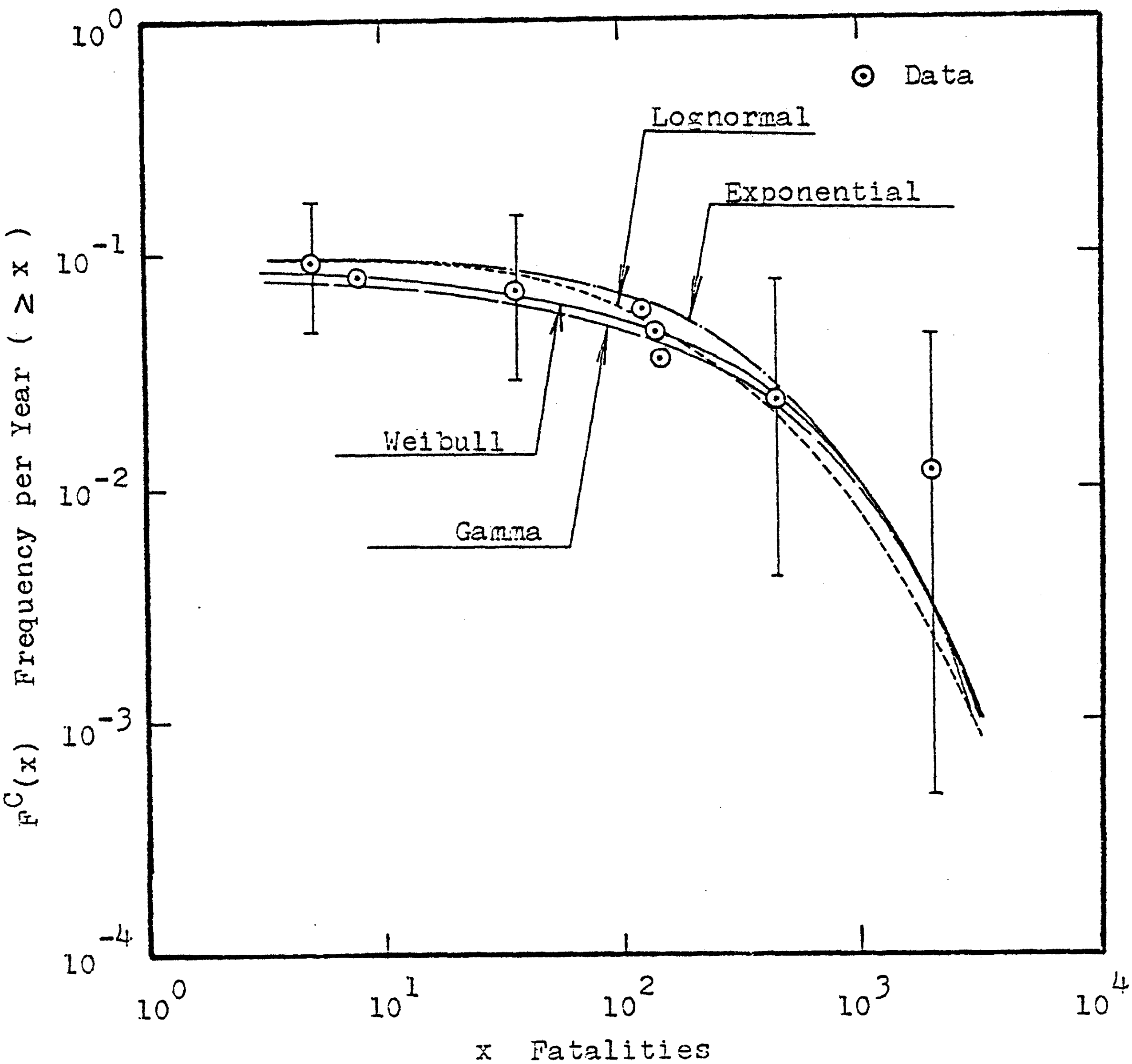

Fig.3.13 Complementary Cumulative Distribution of Fatalities due to Dam Failures

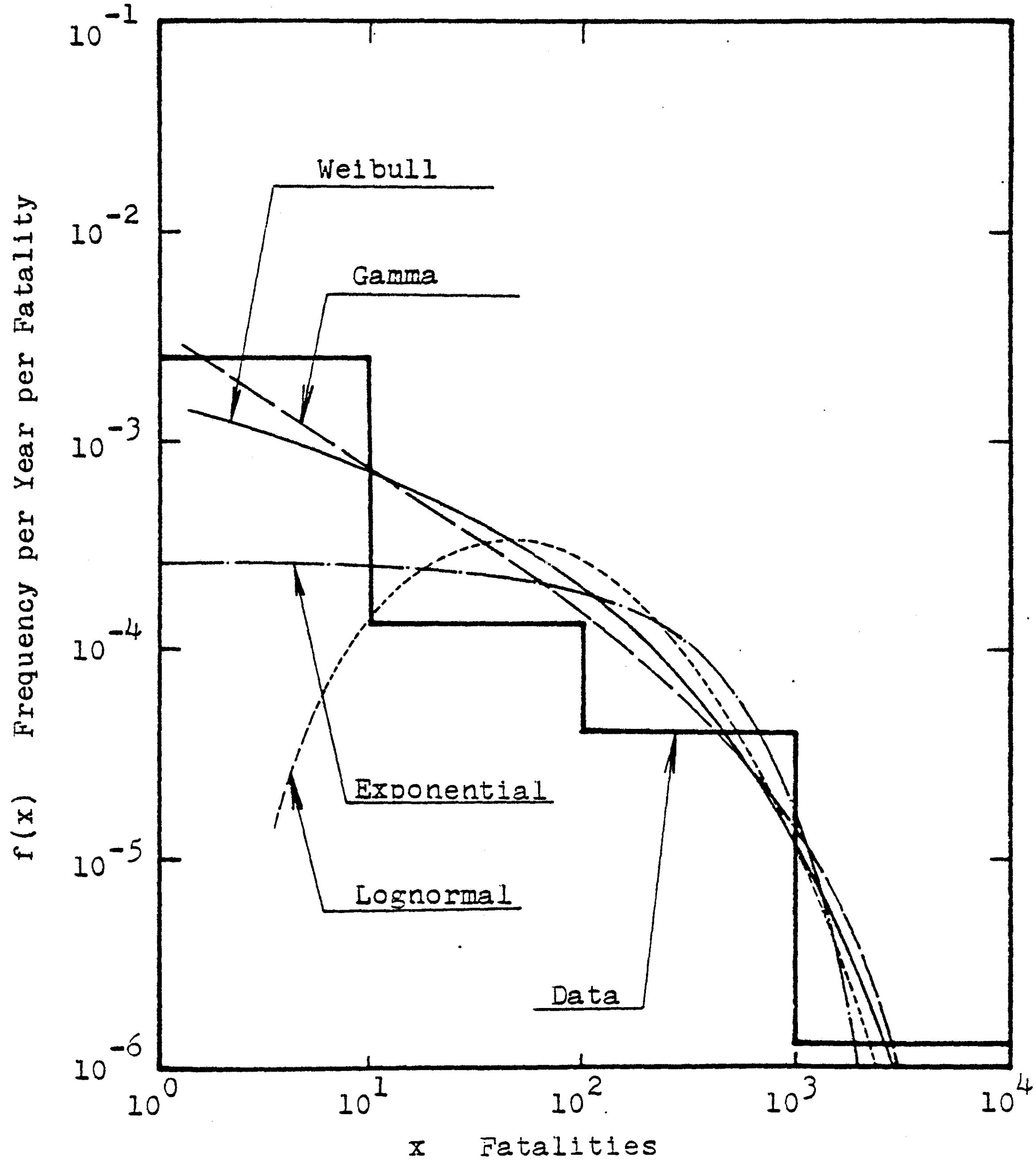

Fig.3.14 Frequency Distribution of Fatalities due to Dam Failures

III.4.6 Summary of Fitting of the Non-nuclear Risk Distributions

In the previous sections the candidate distributions have been examined based on the historical records of hurricanes, earthquakes, tornadoes and dam failures. From the largest deviation of the fitted distribution from the data, the exponential distribution is found to be inadequate to fit the data of hurricanes, earthquakes and tornadoes. The gamma distribution is found to be inadequate to fit the hurricane data. The Weibull and lognormal distributions fit the data within the confidence bounds.

Table 3.7 summarizes the residual mean squares of the fitting. The residual mean squares indicate the order of the adequacy of fitting. The residual mean squares of the Weibull are the smallest for hurricanes, earthquakes and tornadoes. The gamma distribution has the smallest residual mean square in fitting of the data of dam failures.

If a single family of distributions is selected for all of the examined non-nuclear risk distributions, the Weibull is assessed as the distribution which is preferred, because its complementary cumulative distributions are within the 90% confidence bounds of the data and its residual mean squares are the smallest or next to the smallest for all of the studied non-nuclear risk distributions.

Table 3.7

Residual Mean Squares of Fitting of

the Non-nuclear Risk Distributions

Type of Risk	Candidate Distributions			
	Exponential	Gamma	Weibull	Lognormal
Hurricanes	10.9	.31	.11	.21
Earthquakes	2.96	.27	.26	.42
Tornadoes	.66	.11	.086	.093
Dam Failures	1.70	.37	.39	.57

III.5 Fitting of Nuclear Risk Distributions

III.5.1 Sources of the Data

The candidate distributions are now tested by the early fatalities distributions of nuclear reactor accidents. The distributions investigated here are the average of the first 100 commercial nuclear power plants in U.S. and the distributions for two individual sites. The average distribution is derived from the risk estimates of the first 100 nuclear reactors given in the Reactor Safety Study (Ref. 1).

The distributions of the individual sites are calculated in this thesis using the consequence model under the calculation conditions discussed in Section I.4.3. The population distributions used in the individual site calculations are selected from the population distributions of the 68 sites at which the first 100 commercial power plants are located. The selected two sites noted by A and B are the 3rd highest and 3rd lowest respectively when the 68 sites are ranked in a descending order by the cumulative population within 5 miles. The selected two sites can be interpreted as representing the 95% upper and 5% lower bounds of the spectrum of the population distributions. The population distributions of the selected two sites are given in Appendix C. PWR accidents and BWR accidents are calculated seperately in the individual site calculations. Since PWR and BWR accidents have similar early fatalities risk curves, the following combinations are considered to cover the spectra of the population distributions and the reactor types. The calculated cases are PWR accidents at site A and BWR accidents at site B.

The risk distributions and risk moments are calculated by the consequence model. As discussed in Section I.4, the consequence model uses sampling methods in estimating the risk distribution. Let x_i and p_i be the consequence magnitude and the probability of the sample trial (i). The probability p_i assigned to the trial is calculated from the probability of the release, the probability of the wind direction, the probability of the evacuation speed and the number of samples picked from the meteorological records. The complementary cumulative frequency is estimated by the summation of the probabilities of the trials having consequences greater than the specific value as:

$$F^C(x) = \sum_{x_i \geq x} p_i \tag{3.35}$$

The frequency distribution is also estimated from the consequence results by the summation of the probabilities of the trials having consequences within certain intervals.

$$f(x) = \frac{\sum_{x \leq x_i < x+\Delta x} p_i}{\Delta x} = \frac{1}{\Delta x} \{F^C(x+\Delta x) - F^C(x)\} \tag{3.36}$$

For all of the nuclear risk curves, the lower end of the domain is taken to be zero. The first two risk moments about the origin are estimated from the consequence results as:

$$M_1 = \sum_i x_i \cdot p_i \tag{3.37}$$

$$M_2 = \sum_i x_i^2 \cdot p_i \tag{3.38}$$

In the following sections and the chapters about the nuclear risks the risk moments will always be evaluated about the origin. Unless the reference point to evaluate the risk moments is specified, it should be considered to be about the origin. The normalization constant α is estimated by:

$$\alpha = \sum_{x_i > 0} p_i \tag{3.39}$$

The calculated risk distributions have the following two types of errors. One error is due to sampling since the model picks certain number of weather data out of the one year meteorological record. The other type of error is due to the uncertainties of the parameters in the consequence model, such as the probabilities of the occurrences of the releases, the deposition velocities, the dose response relationship, etc.

The sampling error depends on the number of the trials having consequences greater than the specified value. The confidence factors discussed in Section III.4.1 can be applied to determine the magnitude of the sampling errors. From Table 3.1 the probability of the largest consequence has 90% confidence factors of 5 and 1/20. The sampling error is effectively zero for the lower consequences because of the large number of trials having consequences greater than the specified magnitude. Because of the increasing size of the sampling error, the results of the calculation are truncated at the complementary cumulative frequency of 10^{-9}/year for both the average distribution of the 100 reactors and the risk distributions at individual sites, as done in the Reactor Safety Study (Ref. 1).

The uncertainties of the parameters are due to the insufficiency of our knowledge about the parameters. For example, the dose-response relationship (the relationship between the dose to the organs and the fatal fraction of population exposed to the radiation) is not precisely known because of the insufficiency of the available data.

For the average risk curve of the 100 reactors the uncertainties due to the above two causes were estimated in WASH-1400 (Ref. 1) to be represented by factors of 1/4 and 4 on the consequence magnitude and 1/5 and 5 on the probabilities. No estimate of uncertainties has been made for the individual site calculations. It can be expected that the uncertainty bounds of the individual site calculations will be larger than those of the average case because of the smaller number of trials involved in the calculations. However, since the sampling error is small compared to the uncertainties of the parameters except for the largest consequence whose probability is below 10^{-9} per reactor year, it is assumed in this study that the uncertainty bounds of the individual site calculations have comparable magnitudes to those of the average of the 100 reactors.

III.5.2 Average of U.S. 100 Reactors

The total risk of the first 100 commercial nuclear power plants were estimated in the Reactor Safety Study (Ref. 1). The risk curves, the risk moments and the normalization constant are derived from the consequence results obtained in the Reactor Safety Study after dividing the probabilities by 100 to get the average of the 100 reactors. The calculated complementary cumulative distribution of early fatalities is given in Fig. 3.15 by the dots. The calculated distribution is not smooth because of the sampling error. The bands attached to the dots indicate the magnitudes of the uncertainties in the consequence calculation. The calculated frequency distribution is given in Fig. 3.16 as a histogram. The calculated risk moments and normalization constant are given in Table 3.8.

As before, the parameters of the candidate distributions are estimated from the first two risk moments and the normalization constant (Eqs. (3.35) through (3.39)). The estimates and the residual mean squares are given in Table 3.8. The estimated complementary cumulative distributions and the frequency distributions of the candidate parametric distributions are given in Fig. 3.15 and 3.16 respectively.

Fig. 3.15 shows that the exponential distribution overestimates the complementary cumulative frequency (denoted by c.c.f. in the following) in the range of less than 200 fatalities and underestimates it above 200 fatalities. The estimated consequence magnitude at about 10^{-9} per reactor year is smaller than the consequence results by a factor of 5. The gamma distribution underestimates the c.c.f. by a factor of 2 for the range of less than 100 fatalities and overestimates the c.c.f.

Table 3.8 Estimates of the Parameters of the Early Fatalities Distribution of the Average of U.S. 100 Commercial Reactors

$x_0 = 0$, $\alpha = 4.72 \times 10^{-7}$, $M_1 = 4.60 \times 10^{-2}$, $M_2 = 6.45 \times 10^{-2}$

Candidate Distribution	Estimates of Parameters		Residual Mean Square
Exponential	$\theta = 9.75 \times 10^{1}$		47.07
Gamma	$\beta = .0783$	$\theta = 1.30 \times 10^{3}$	.691
Weibull	$\beta = .371$	$\eta = 2.45 \times 10^{1}$	.194
Lognormal	$\mu = 3.31$	$\sigma = 2.62$	.057

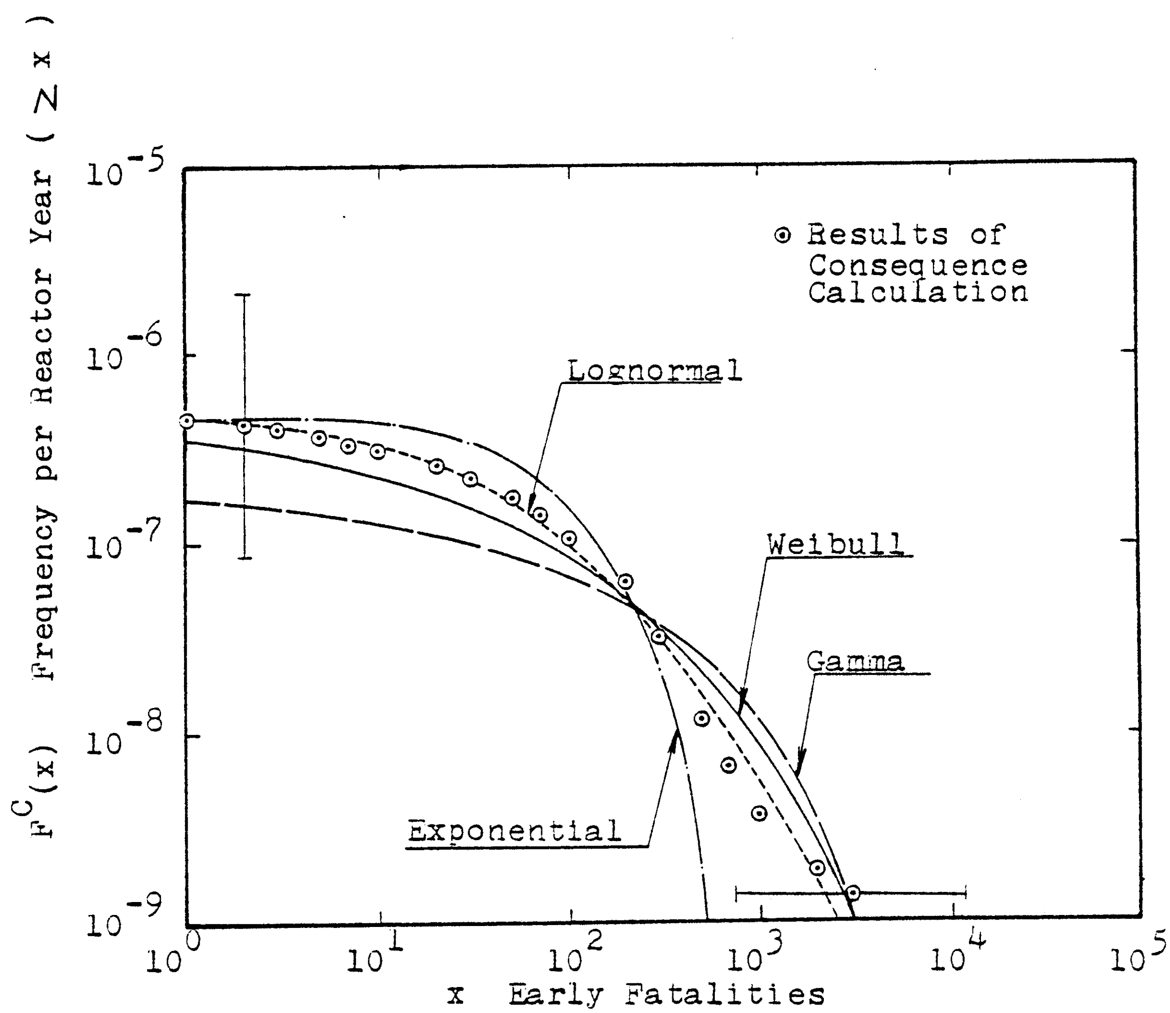

Fig.3.15 Complementary Cumulative Distribution of Early Fatalities in the Average of U.S. 100 Reactors

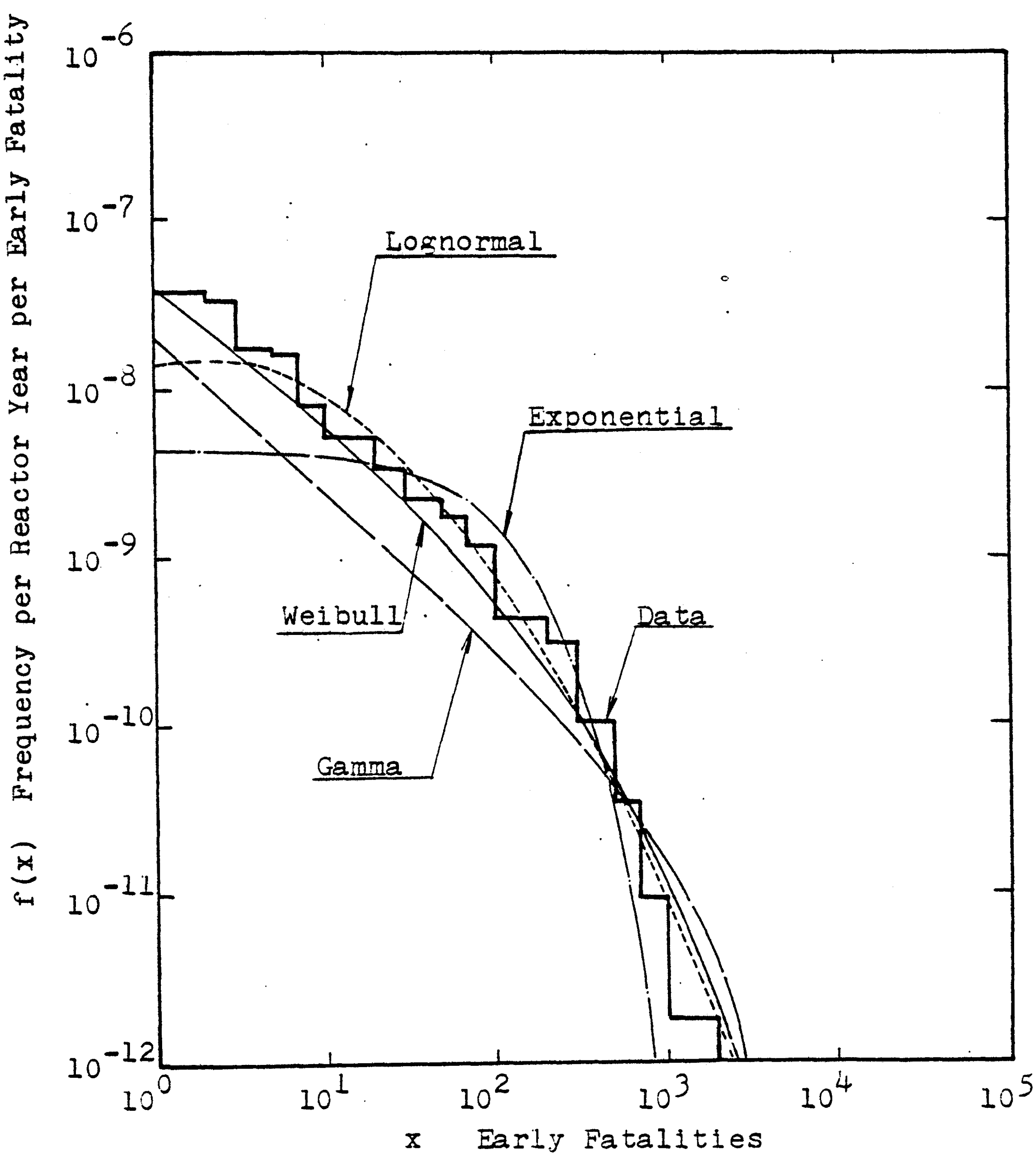

Fig. 3.16 Frequency Distribution of Early Fatalities in the Average of the U.S. 100 Reactors

between 500 and 2000 fatalities. The lognormal distribution appears not to have systematic errors. Except for the exponential distribution, the other three distributions are within the range of the uncertainties of the consequence model. The residual mean square of the lognormal is the smallest in Table 3.8. The Weibull and gamma are the next. The exponential has the largest residual mean square.

III.5.3 PWR Accidents at Site A

The consequence calculation is made in this thesis using the population distribution of Site A in Table C.5 and the release characteristics of PWR accidents in Table C.3. As discussed in Section I.5.3, the obtained consequence distribution is hypothetical because of the assumptions of the meteorological conditions, the plant capacity and the probabilities of the reactor system failures. The assumed conditions are not based on the actual data of the power plant at Site A.

From the consequence calculation, the normalization constant and the first two risk moments are estimated by Eqs. (3.35) through (3.39). The parameters of the candidate functions are estimated in Table 3.9. The estimated candidate distributions are shown in Figs. 3.17 and 3.18 along the calculated distributions by the consequence model. (The calculated distributions are shown by dots in Fig. 3.17 and as a histogram in Fig. 3.18).

Fig. 3.17 shows that the exponential distribution slightly overestimates the c.c.f. in the range between 10 and 500 fatalities and underestimates the c.c.f. in the range greater than 100 fatalities.

Table 3.9 Estimates of Parameters of the Early Fatalities Distribution in PWR Accidents at Site A

$x_0 = 0$, $\alpha = 5.78 \times 10^{-7}$, $M_1 = 2.72 \times 10^{-4}$, $M_2 = 5.77 \times 10^{-1}$

Candidate Distribution	Estimates of Parameters		Residual Mean Square
Exponential	$\theta = 4.61 \times 10^1$		14.28
Gamma	$\beta = .284$	$\theta = 1.66 \times 10^3$	.095
Weibull	$\beta = .570$	$\eta = 2.91 \times 10^2$	.102
Lognormal	$\mu = 5.40$	$\sigma = 1.51$	.195

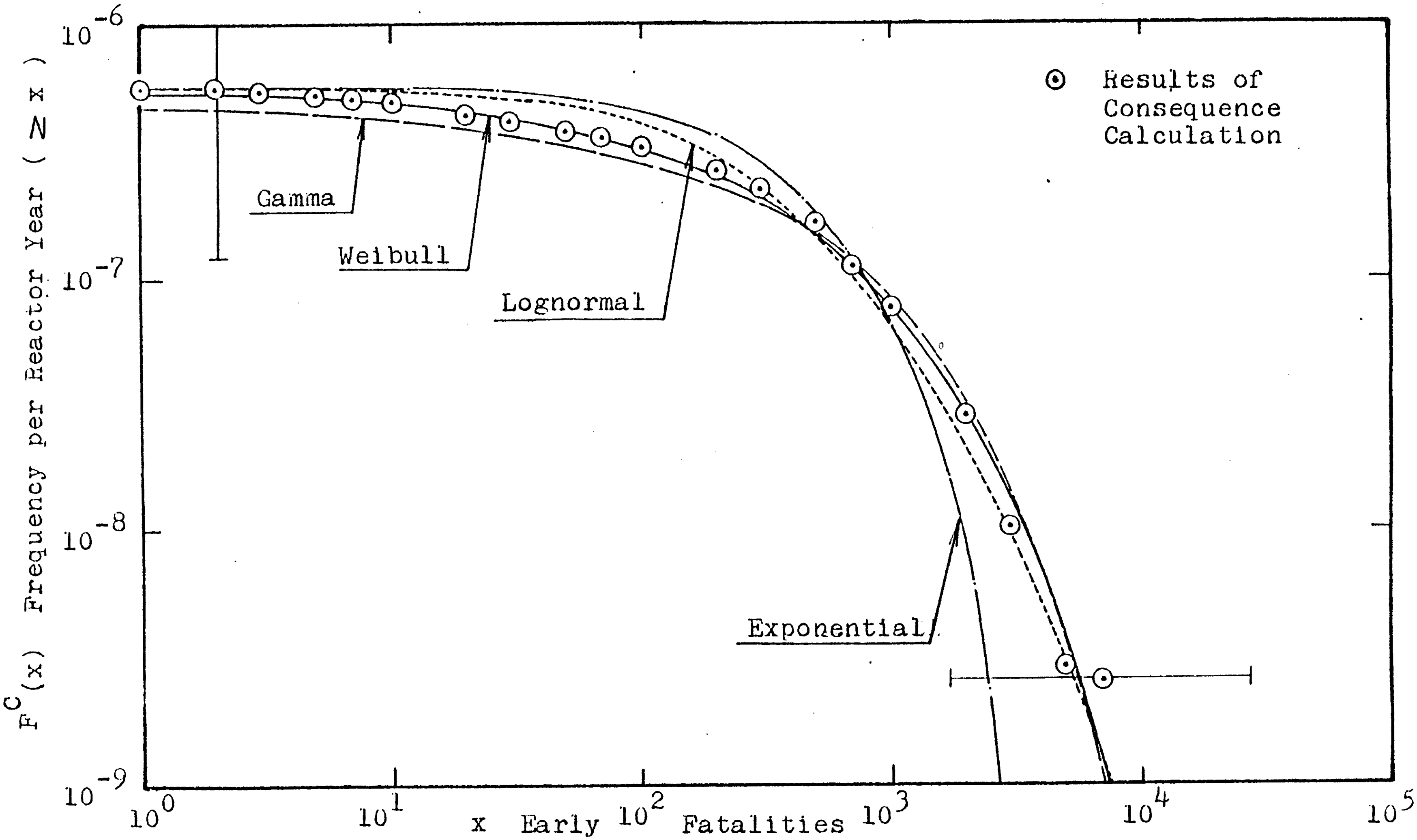

Fig. 3.17 Complementary Cumulative Distribution of Early Fatalities in PWR Accidents at Site A

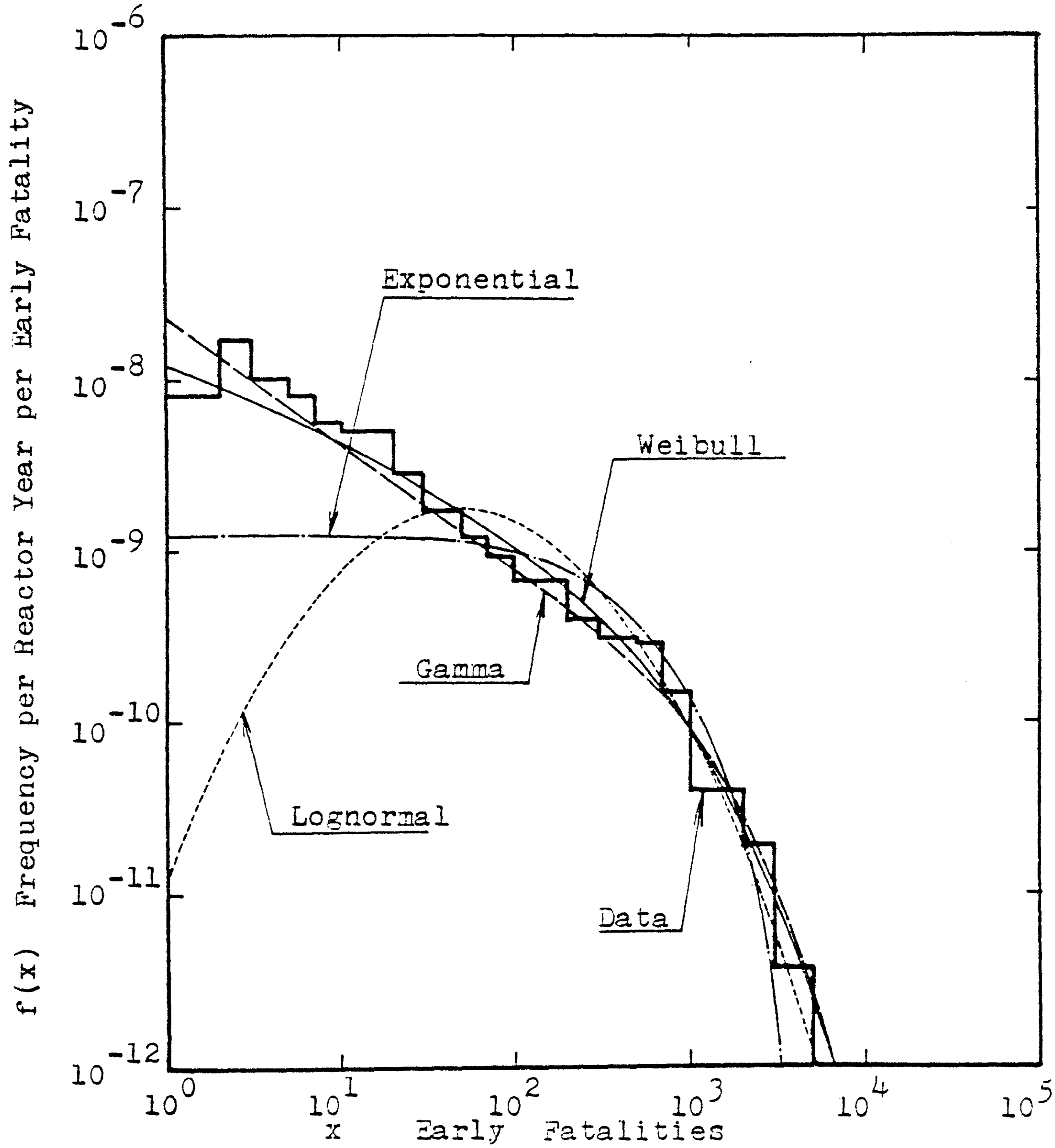

Fig.3.18 Frequency Distribution of Early Fatalities in PWR Accidents at Site A

The lognormal distribution slightly overestimates the c.c.f. in the range between 10 and 200 fatalities and the gamma distribution slightly underestimates it in the range less than 300 fatalities. The Weibull appears not to have systematic errors. The candidate distributions are within the range of the uncertainties of the consequence calculation but the exponential distribution is less favorable than the other three because of the underestimation of the magnitude by a factor of 3 at about 10^{-9} per reactor year. The residual mean square of the gamma distribution is the smallest in Table 3.9. The Weibull and the lognormal are the next. The exponential has the largest residual mean square.

III.5.4 BWR Accidents at Site B

The consequence calculation is made in this thesis using the population distribution of Site B in Table C.6 and the release characteristics of the BWR accidents in Table C.3. The calculated distribution is also hypothetical like the distribution at Site A in the previous section. The results of the fitting are given in Figs. 3.19, 3.20 and Table 3.10.

Fig. 3.19 shows that the exponential distribution slightly overestimates the c.c.f. in the range between 10 and 100 fatalities. The gamma distribution underestimates the c.c.f. for less than 10 fatalities. The lognormal and the Weibull slightly overestimate the c.c.f. in the range between 10 and 50 fatalities. All of the candidate distributions are within the uncertain ranges of the consequence model. The order of preference based on the residual mean squares in Table 3.10 is Weibull, gamma, lognormal and exponential.

Table 3.10 Estimates of Parameters of the Early Fatalities Distribution in BWR Accidents at Site B

$x_0 = 0$, $\alpha = 1.61 \times 10^{-8}$, $M_1 = 9.92 \times 10^{-7}$, $M_2 = 3.46 \times 10^{-4}$

Candidate Distribution	Estimates of Parameters		Residual Mean Square
Exponential	$\theta = 6.17 \times 10^1$		2.15
Gamma	$\beta = .214$	$\theta = 2.87 \times 10^2$	.152
Weibull	$\beta = .513$	$\eta = 3.23 \times 10^1$	.107
Lognormal	$\mu = 3.26$	$\sigma = 1.73$	.186

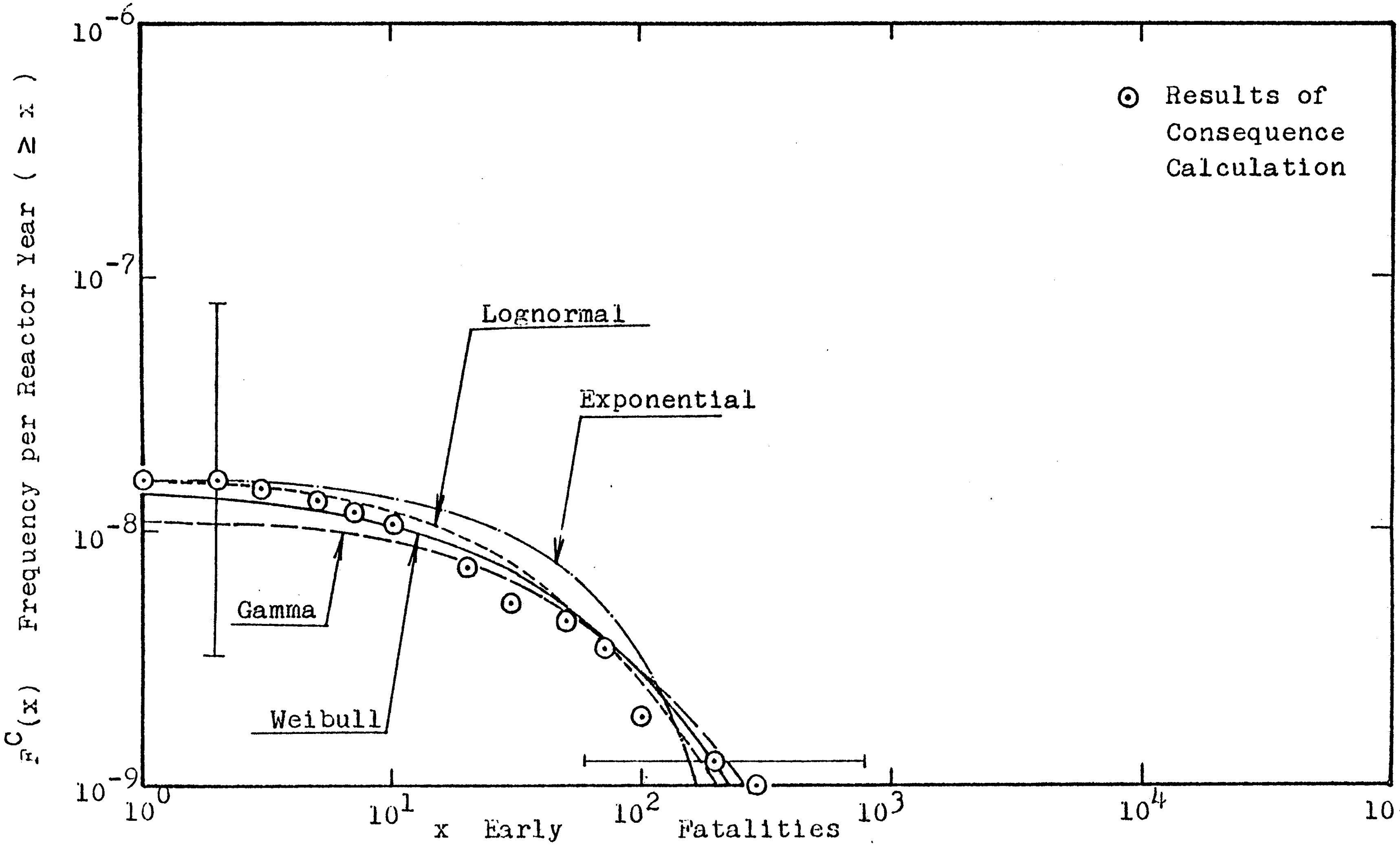

Fig. 3.19 Complementary Cumulative Distribution of Early Fatalities in BWR Accidents at Site B

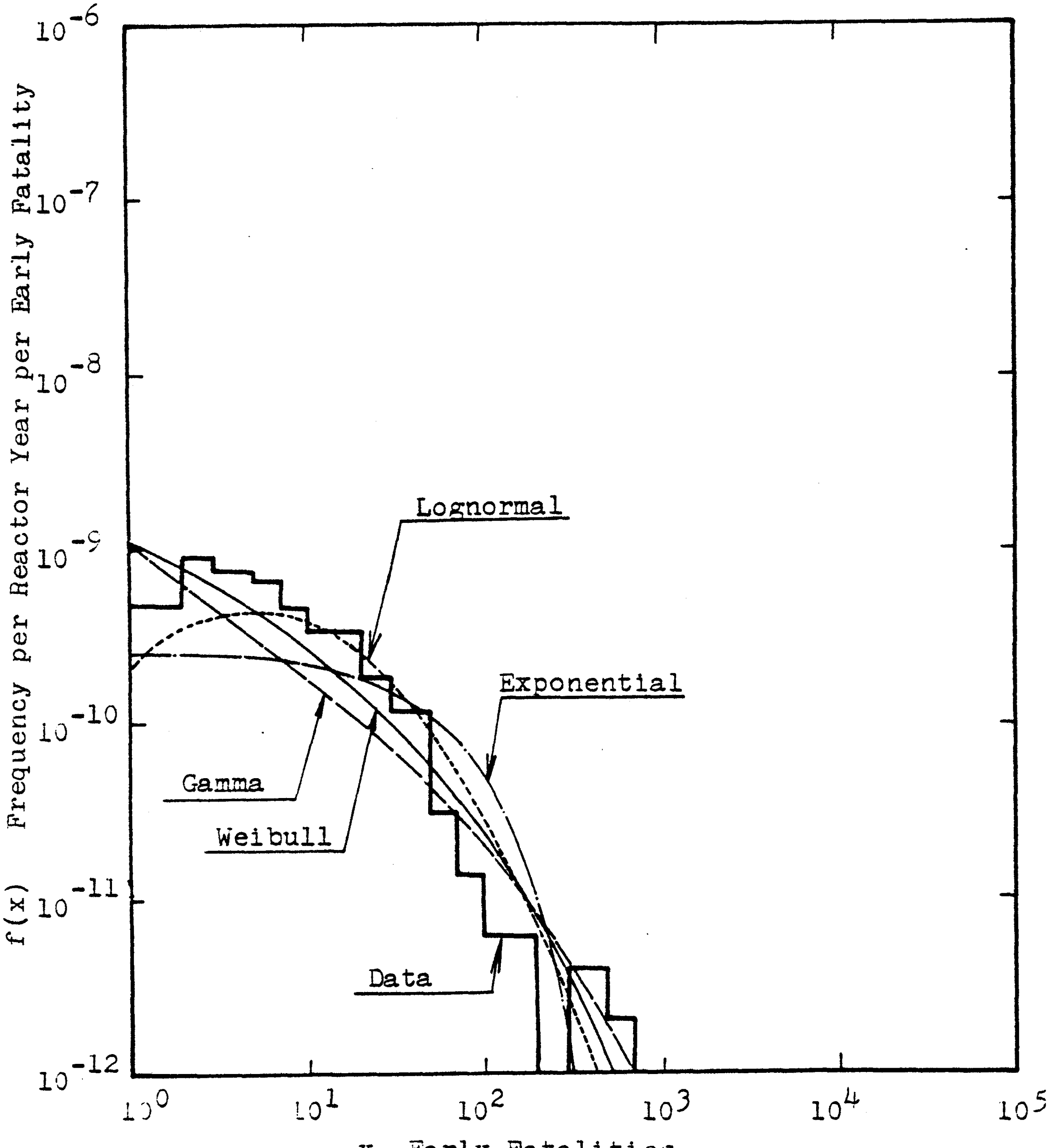

Fig. 3.20 Frequency Distribution of Early Fatalities in BWR Accidents at Site B

III.5.5 Summary of Fitting of Nuclear Risk Distributions

Based on the fittings for nuclear risks, the exponential is found to be inadequate to fit the average distribution of the U.S. 100 reactors. The residual mean squares in Table 3.11 show the order of preference of the renaining candidate distributions. If a single family of distributions is selected for all of the examined risk curves, the Weibull is assessed as being adequate because its residual mean squares are the smallest or the second smallest for all of the examined risk distributions.

III.6 Summary and Conclusions

The approach developed in Chapter II is demonstrated in this chapter to examine the early fatalities distributions of nuclear and non-nuclear risks. Four candidate distributions are studied, exponential, gamma, Weibull and lognormal distributions. They are selected from the considerations of (1) having no mode or at most one mode, (2) positively skewed behaviors (3) different tail behaviors and (4) having only one or two parameters to be estimated. The method of moments is used to estimate the parameters of these distributions.

In order to select a distribution family which adequately describes the fatalities distributions, the historical records of hurricanes, earthquakes, tornadoes and dam failures are examined. The Weibull distribution is assessed to be appropriate as a family of distributions that describe the examined non-nuclear risk distributions. For the calculated nuclear risks from the average of U.S. 100 reactors and from the two individual site calculation results, the Weibull distribution is also assessed to be appropriate. For both nuclear and non-nuclear risks, the Weibull distribution is determined to be the distribution which adequately describes the examined risk curves.

Table 3.11 Residual Mean Squares of Nuclear Risks

	Candidate Risk Models			
Reactor	Exponential	Gamma	Weibull	Lognormal
Average of U.S. Reactors	47.07	.691	.194	.057
PWR at Site A	14.28	.095	.102	.195
BWR at Site B	2.15	.152	.107	.186

CHAPTER IV

BASIS FOR REGRESSION ANALYSIS

IV.1 Introduction

In the preceding two chapters, the fittings of the risk distributions to the parametric distributions were discussed. The next major step in the analysis is to derive the equations that relate the distribution parameters to the basic variables that drive and control the consequences of the nuclear reactor accidents. In this chapter, a general discussion will be made about derivation of the basic variable equations. The application will then be discussed in the following chapters.

IV.2 Derivation of the Basic Variable Equations

IV.2.1 Outline of the Approach

In this study the regression analysis approach is used to relate the distribution parameters to the basic variables. For the purpose of presentation, the approach in the analysis can be represented by six fundamental steps. Such a breakdown represents useful means of giving a perspective on the process, although a simple summary of this kind cannot fully describe all the elements in a complex analysis. The six fundamental steps are:

(1) Identification of the basic driving variables to be studied.

(2) Selection of the dependent variables of the regression equations.

(3) Assembling the data to be used in identifying the relationship between the dependent and basic variables.

(4) Formulation of candidate equations relating the dependent and basic variables.

(5) Estimation of the unknown constants in the equations.

(6) Investigation of the adequacy of the derived equations.

Each step is now discussed in context of a risk analysis of the nuclear reactor accidents.

IV.2.2 Identification of the Basic Variables

The following are some examples of the basic variables that would be of interest in a risk analysis of the nuclear reactor accidents:

(1) Population distribution.

(2) Meteorological condition.

(3) Probabilities and magnitudes of radioactive releases.

(4) Evacuation speed and evacuation area in emergency situations of the reactor accidents.

These variables would be of interest in the following decision making and evaluation studies:

(1) The population distributions and the meteorological conditions would be of interest in selection of sites for nuclear power plants.

(2) The probabilities and magnitudes of radioactive releases would be of interest in evaluation of safety systems in a nuclear power plant involving engineering safety features, operation restrictions and maintenance activities.

(3) The evacuation speed and area would be important in emergency planning.

In the regression analysis, the basic variables to be studied are called "regressor variables." The population distribution and the characteristics of radioactive releases will be studied as regressor variables in the following chapters to demonstrate the regression analysis approach for identifying the dependent and basic regressor variables.

IV.2.3 Selection of the Dependent Variables

The dependent variables can be selected from the risk characteristics or the distribution parameters of the fitted distributions. Since the appropriate family of the parametric distributions has been selected, the other risk characteristics or distribution parameters can be estimated from the selected variables. The following variables can be studied as dependent variables:

(1) Scale factor, shape factor and normalization constant of the fitted parametric distribution.

(2) Risk moments about a specific magnitude of consequence.

(3) Complementary cumulative frequency at a specific magnitude of consequence.

(4) Magnitude of consequence at a specific value of complementary cumulative frequency.

(5) Slope of the tangent of the complementary cumulative distribution at a specific magnitude of consequence.

The selection of the dependent variables is based on the following considerations:

(a) The relationship between the dependent and basic variables can be expressed by fairly simple and straightforward equations.

(b) The selection may depend on the situation being considered in the decision making or evaluation process.

The variables listed above would be of interest in the following situations:

(1) Distribution parameters of the selected parametric distribution: The parameters control the behavior of the distribution. For example, the shape factor β of the Weibull distribution controls the rate of decrease in the tail. The scale factor η of the Weibull distribution represents the magnitude of consequence at a complementary cumulative frequency of α/e, where e is the Euler's constant. The normalization constant represents the frequency that the consequence is greater than the lower end of the domain. When the decision is based on these characteristic quantities, they can be selected as dependent variables.

(2) Risk moments: The first risk moment about the origin will be selected when the decision is based on the expectation of the magnitude of consequence. The second and higher moments about the origin represent the tail behavior of the distribution. When the decision is based on the extreme consequences, the second and higher moments would be of interest.

(3) Complementary cumulative frequency at a specific magnitude of consequence: When the decision is based on the frequency at a specific magnitude (for example, 1000 fatalities), it can be selected as a dependent variable.

(4) Magnitude of consequence at a specific frequency: When the decision is based on the magnitude at a specific complementary cumulative frequency (for example, 10^{-9}/year), it can be selected as a dependent variable.

(5) Slope of the tangent of the complementary cumulative distribution: The slope represents the rate of decrease of the frequency. Specifically the slope at the tail would be selected when the extrapolation of the distribution is of interest to the consequences greater than the largest consequence in the historical records or in the calculation results.

When the scale and shape factors are not selected as dependent variables, they will be estimated from the selected dependent variables. For example, the first two risk moments about the origin and the normalization constant will be selected as dependent variables in Chapter 5. The Weibull parameters β and η can be estimated by Eqs. (3.27) and (3.28). Once the Weibull parameters are estimated, we have an entire distribution and can derive any risk characteristic in terms of the parameters. For example, the magnitude of consequence at a specific complementary cumulative frequency F^c is give by:

$$x = x_0 + \hat{\eta} \cdot [\ln \left(\frac{\alpha}{F^c}\right)]^{1/\hat{\beta}} \qquad (4.1)$$

where $\hat{\beta}$ and $\hat{\eta}$ are the estimates by Eqs. (3.27) and (3.28).

IV.2.4 Assembling of the Data

In the risk analysis the data are generally obtained from the historical records or from the calculational model. The data obtained can be certain risk characteristics or risk distributions. To identify the relation to the basic variables, the data must be obtained for different values of the basic variables. A set of the data used for the analysis is called "data base" in this study.

In this thesis the data base is obtained from the consequence model. For example, in Chapter 5 the first two risk moments and the normalization constant will be calculated by the consequence model for 68 different population distributions. The calculated 68 different sets of the risk moments and the normalization constant will be used in identifying the relationship between the dependent variables and the population distribution.

IV.2.5 Formulation of Candidate Equations

A number of candidate equations with unknown constants are formulated to relate the dependent variables to the regressor variables. Simple and straightforward equations with a small number of unknown constants are desirable. Consider the following two candidate equations:

$$y = h(z_1, z_2, \ldots, z_m | \tau_1, \ldots, \tau_k) + \varepsilon \tag{4.2}$$

$$y = h'(z_1, z_2, \ldots, z_m | \tau_1, \ldots, \tau_k, \tau_{k+1}, \ldots, \tau_{k+\nu}) + \varepsilon' \tag{4.3}$$

where y is the dependent variable and $z_1, \ldots, z_m$ are the regressor variables. τ's are the unknown constants and ε and ε' are the random error variables. Eq. (4.3) has ν additional unknowns compared to Eq. (4.2). Generally Eq. (4.3) with $(k+\nu)$ unknowns predict the value of y more accurately than Eq. (4.2) with k unknowns. But Eq. (4.2) is more desirable than Eq. (4.3) because of its smaller number of unknowns. As a compromise the significance of added ν unknown constants is tested by the partial F-statistic which will be discussed in the following sub-section.

IV.2.6 Estimation of the Unknown Constants

The method of least squares is used to estimate the unknown constants. For example, the unknowns in Eq. (4.2) are estimated by minimizing:

$$\Delta^2 = \sum_{i=1}^{n} [y_i - h(z_{1i},\ldots,z_{mi}|\tau_1,\ldots,\tau_k)]^2 \tag{4.4}$$

where the subscript i refers to the data value prepared in Section IV.2.4 and n is the total number of the sample data.

Having obtained the estimates $\hat{\tau}_1,\ldots,\hat{\tau}_k$, the significance of the derived equations are expressed by the F-value defined by:

$$F = \frac{S_G^2/k}{S_R^2/(n-k-1)} \tag{4.5}$$

where

$$S_G^2 = \sum_i [y_0 - h(z_{1i},\ldots,z_{mi}|\hat{\tau}_1,\ldots,\hat{\tau}_k)]^2 \tag{4.6}$$

$$y_0 = \frac{1}{n}\sum_i y_i \tag{4.7}$$

$$S_R^2 = \sum_i [y_i - h(z_{1i},\ldots,z_{mi}|\hat{\tau}_1,\ldots,\hat{\tau}_k)]^2 \tag{4.8}$$

If the F-value determined by Eq. (4.5) is larger than the F-value at the predetermined significance level with (k,n-k-1) degrees of freedom, the candidate equation Eq. (4.2) is found to be significant to express the variation of the dependent variable of the data. The F-value in Eq. (4.5) is related to the multiple correlation co-efficient ρ_m which is defined by:

$$\rho_m^2 = \frac{S_G^2}{S_G^2 + S_R^2} \tag{4.9}$$

The multiple correlation coefficient also indicates the significance of the regression results.

In the preceding section, the compromise between the accuracy of prediction and the number of unknowns is discussed. Now Eqs. (4.2) and (4.3) are compared. Let $\hat{\tau}'_1,\ldots,\hat{\tau}'_k,\hat{\tau}'_{k+1},\ldots,\hat{\tau}'_{k+\nu}$ be the estimates of the unknowns in Eq. (4.3) determined by the method of least squares. The significance of the added ν unknowns is examined by the partial F-statistic defined as:

$$F' = \frac{[S_R^2 - (S_R^2)']/\nu}{(S_R^2)'/(n-k-\nu-1)} \tag{4.10}$$

where

$$(S_R^2)' = \sum_i [y_i - h'(z_{1i},\ldots,z_{mi}|\hat{\tau}'_1,\ldots,\hat{\tau}'_{k+\nu})]^2 \tag{4.11}$$

If the partial F-value in Eq. (4.10) is smaller than the F-value at the predetermined significance level with $(\nu, n-k-\nu-1)$ degrees of freedom, the added ν unknowns can be eliminated and Eq. (4.2) with k unknowns is found to be adequate.

Stepwise regression technique is a method for determining equations with the minimum number of unknowns without decreasing the accuracy in predicting the variation of the dependent variables. It uses the partial F-tests repeatedly by adding or eliminating the unknown constants (or the regressor variables associated with the unknown constants).

Details of the regression techniques and the tables of the F-distribution are found in Ref-8.

IV.2.7 Test of the Adequacy of the Derived Equations

The following criteria are used to investigate the adequacy of the derived equations:

(1) The F-value in Eq. (4.5) or the multiple correlation coefficient in Eq. (4.9) should be large. This criterion can be

taken to be a relative measure to be used in comparing different possible equations.

(2) The error should not be systematic. When the regression estimates of the dependent variables are plotted versus the data values used for regression, the points should lie closely about the 45 degree line and no tendency is observed to overpredict or underpredict various range of the data.

(3) The fitted risk distribution using the derived basic variable relations will be compared to the data distribution.

(4) Various risk characteristics will also be compared using the basic variable relation to determine the fitted risk characteristics.

IV.3 Summary

The approach for deriving the regression equations is discussed in this chapter. The fundamental elements of the approach are identified as: (1) identification of the basic regressor variables; (2) selection of the dependent variables; (3) assembling of the data; (4) formulation of candidate equations; (5) estimation of the unknown constants; and (6) investigation of the adequacy of the derived equations.

Some of the possible basic variables are identified and two of them will be studied in the following chapters. The dependent variables can be selected from the risk characteristics or the distribution parameters of the fitted distributions. The data used for regression analysis can be obtained from the historical records or the calculational model. In this study they are obtained from the consequence model. The candidate equations with a small number of unknown constants are desired. The unknown constants are estimated by the method of least squares. The significance of adding or eliminating unknown constants can be tested by the partial F-statistic. The adequacy of the derived equations is examined by: (1) F-value or multiple correlation co-efficient; (2) systematic error in prediction of the dependent variables; (3) comparison of the fitted risk distribution to the data distribution; and (4) comparison of the predicted risk characteristics to those calculated by the consequence model.

CHAPTER V

REGRESSION ANALYSIS OF POPULATION DISTRIBUTION

V.1 Introduction

In the previous chapter a general procedure of regression analysis was proposed. The procedure will be demonstrated in this chapter in an example in which the population distribution is a basic variable. Since the population distribution is one of the potentially important factors in decisions on sites for nuclear power plants, the equations relating the risk to the population distribution will provide help in decision on an acceptable population distribution.

The example studied in this chapter is the relationship between the population distribution and the early fatalities distribution of PWR accidents in northeastern valley meteorological condition. But the methods developed in this chapter will be generally applicable to other consequences, other types of reactor accidents and other meteorological conditions.

The discussion in this chapter follows the procedure of regression analysis proposed in the preceding chapter. Section V.2 discusses the population distribution which is the basic variable in this chapter. The selection of the dependent variables is made in Section V.3 and the data base is prepared in Section V.4. The regression model is formulated in Section V.5 and the regression fitting is made in Section V.6. The adequacy of the derived equations is examined in Section V.7. An example of decision making involving siting for a nuclear power plant is given in Section V.8.

V.2 Incorporation of the Population Distribution in a Risk Model

A polar coordinate system is used here to describe the population distribution. The origin is set at the location of the nuclear power plant. The number of people living in $(\Delta r, \Delta\theta)$ at (r,θ) is defined to be:

$$n(r,\theta)\ \Delta r\ \Delta\theta = r \cdot \rho(r,\theta)\ \Delta r \cdot \Delta\theta \qquad (5.1)$$

where $n(r,\theta)$ is the number of people per radian per unit distance and $\rho(r,\theta)$ is the population per unit area.

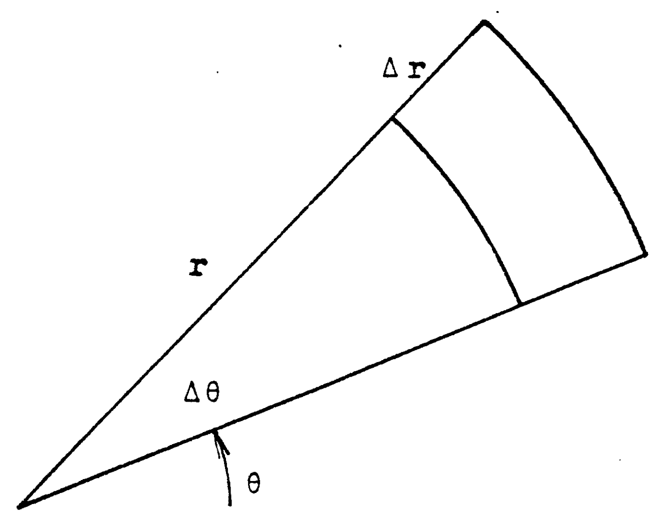

Fig. 5.1 Illustration of the Polar Coordinate Sustem for Describing the Population Distribution

In the consequence computer model, the population distribution is discretized by dividing a circle of 500 miles radius[1] into sixteen 22-1/2 degree sectors and dividing a sector into 34 annular segments. Fig. 5.2 illustrates some of the annular segments in the consequence model. Eq. (5.1) is first integrated over a 22-1/2 degree sector in the direction j.

$$n_j(r) = \int_{\frac{\pi}{8}} n(r,\theta)d\theta \qquad (5.2)$$

where $n_j(r)$ is the population per unit distance at r in a 22-1/2 degree sector in the direction j. Eq. (5.2) is then integrated over r to derive the population in the k-th annular segment from the origin in a sector of the direction j.

$$N_{jk} = \int_{r_{k-\Delta r_{k/2}}}^{r_{k+\Delta r_{k/2}}} n_j(r)dr \qquad (5.3)$$

where r_k is the distance of the midpoint of the k-th segment from the reactor and Δr_k is the width of the annular segment. r_k and Δr_k used in the consequence calculation are listed in Appendix C. The populations in the annular segments are treated as basic regressor variables in this chapter.

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[1]The effects of nuclear reactor accidents on the public beyond 500 miles are considered too small and no calculation is performed beyond 500 miles.

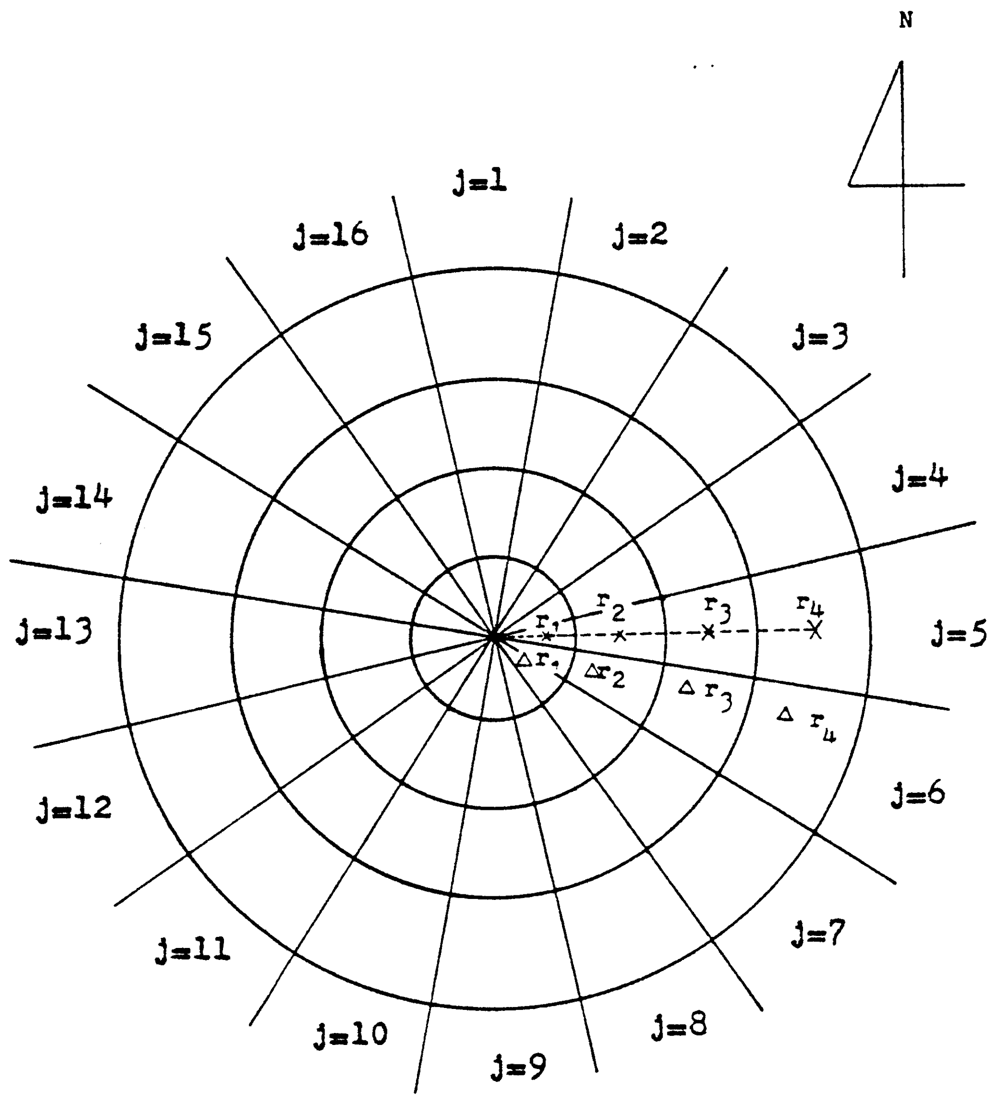

Fig. 5.2 Illustration of the Annular Segments in the Consequence Model

V.3 Selection of the Dependent Variables

The dependent variables can be selected from the risk characteristics or the distribution parameters listed in Section IV.2.2. In this chapter, the first two risk moments and the normalization constant are selected as the dependent variables, since they have been used to derive the fitted Weibull distributions which have been shown to adequately describe the data distributions of consequence vs. frequency. These three variables represent the following behaviors of the distribution. The first risk moment gives the average number of fatalities per unit time. The second risk moment accounts the tail behavior of the distribution. The normalization constant gives the area under the frequency distribution, which is the probability per unit time of consequences being greater than zero.

V.4 The Data Base for Regression Analysis

A total of 68 different population distributions are used for the regression analysis. The populations correspond to the 68 sites where the 100 reactors are now in use or planned to be located. The populations are calculated from the 1970 census bureau data (Ref. 2). As shown in Table 5.1, the 68 populations have a large spread with regard to the cumulative distribution. The populations also cover different patterns as shown in Fig. 5.3. The regression equations derived from these populations should therefore cover the likely variations which might be considered in selection of sites for nuclear power plants.

The first two risk moments and the normalization constant are calculated by the consequence computer model for each of the 68 population distributions assuming PWR accidents and northeastern valley

Table 5.1 Spread of Cumulative Population in the 68 Population Distributions

Radius (miles)	Cumulative Population in a Circle (thousands)		
	Highest Distribution	Average of the Distributions	Lowest Distribution
2	21	1.4	0
5	62	8.7	0
10	207	42	1.4
20	896	214	19
50	16,485	2,073	171
100	23,908	6,973	523
500	108,757	60,302	6,947

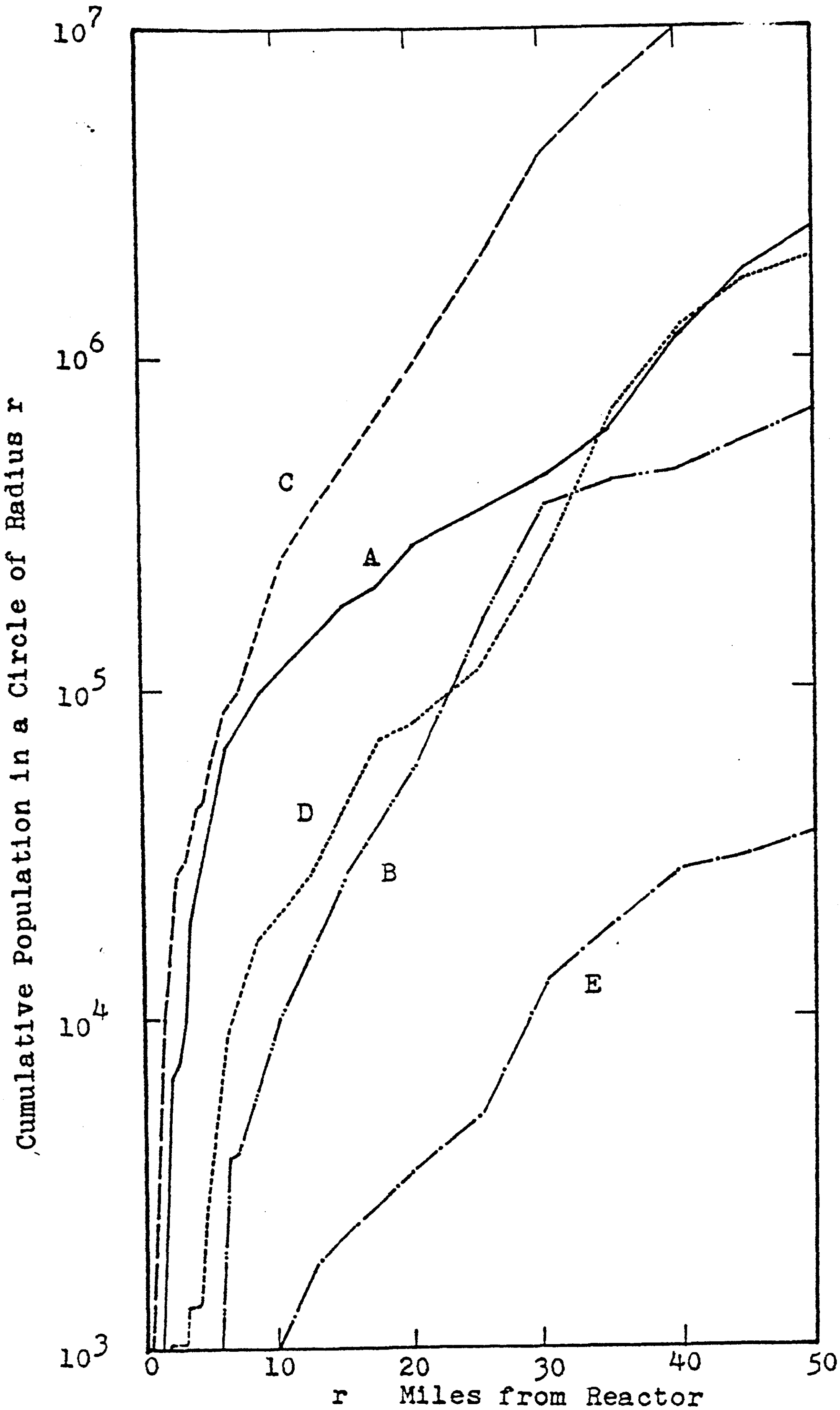

Fig. 5.3 Cumulative Population Distributions of Different Patterns

Note : Sites C,A,D,B,E correspond to the 1st,3rd,35th,65th,68th when 68 sites are ranked based on the populations in 5 miles.

Table 5.2 Results of Consequence Calculations of PWR Accidents for 68 Different Population Distributions

Population Distribution Sample No.	First Risk Moment	Second Risk Moment	Normalization Constant
1	9.15E-05	8.26E-02	3.13E-07
2	2.72E-04	5.77E-01	5.78E-07
3	1.59E-05	7.15E-03	1.60E-07
4	8.87E-07	6.47E-04	1.28E-08
5	7.57E-05	8.62E-02	2.30E-07
6	3.20E-05	1.30E-02	2.82E-07
7	5.70E-05	5.35E-02	2.76E-07
8	5.70E-05	5.35E-02	2.76E-07
9	1.34E-05	1.12E-02	8.74E-08
10	2.94E-05	6.78E-02	7.48E-08
11	3.38E-05	5.66E-02	1.13E-07
12	6.95E-04	2.07E 00	7.09E-07
13	2.94E-05	6.78E-02	7.48E-08
14	1.25E-04	1.53E-01	4.02E-07
15	1.66E-05	1.37E-02	8.71E-08
16	1.21E-04	2.17E-01	3.65E-07
17	3.85E-04	8.95E-01	6.85E-07
18	1.88E-05	9.99E-03	1.26E-07
19	5.61E-05	1.03E-01	1.98E-07
20	1.71E-04	3.85E-01	4.42E-07
21	6.73E-05	7.21E-02	3.11E-07
22	3.87E-05	8.63E-02	1.95E-07
23	7.30E-06	1.92E-03	9.81E-08
24	1.73E-05	1.69E-02	1.09E-07
25	4.82E-05	3.64E-02	1.94E-07
26	1.22E-05	5.14E-03	9.39E-08
27	3.18E-05	5.42E-02	9.22E-08
28	1.73E-05	5.04E-02	3.70E-08
29	8.30E-06	3.57E-03	7.05E-08
30	2.02E-05	9.13E-03	1.98E-07
31	1.01E-04	1.10E-01	3.17E-07
32	1.31E-05	5.98E-03	1.22E-07
33	1.34E-05	9.52E-03	1.21E-07
34	5.95E-05	6.03E-02	2.12E-07
35	9.63E-06	2.83E-03	1.05E-07
36	3.82E-05	1.10E-01	1.37E-07
37	3.81E-05	3.94E-02	2.44E-07
38	2.23E-05	1.11E-02	1.74E-07
39	1.45E-04	2.12E-01	2.47E-07
40	8.45E-06	3.56E-03	8.54E-08
41	5.84E-05	4.42E-02	2.77E-07
42	1.16E-05	3.73E-03	1.19E-07
43	8.77E-05	6.02E-02	5.13E-07
44	4.62E-05	2.81E-02	1.96E-07
45	3.14E-05	4.59E-02	1.97E-07
46	1.91E-05	1.38E-02	1.66E-07
47	1.53E-05	1.21E-02	1.50E-07
48	5.32E-04	1.53E 00	8.59E-07
49	1.22E-04	3.04E-01	3.30E-07
50	3.88E-05	6.02E-02	2.40E-07
51	6.78E-05	6.62E-02	3.01E-07
52	1.14E-04	1.27E-01	3.76E-07
53	2.75E-04	4.70E-01	6.51E-07
54	4.08E-05	3.67E-02	2.65E-07
55	1.98E-05	8.46E-03	1.81E-07
56	4.18E-05	4.32E-02	2.07E-07
57	3.79E-05	3.02E-02	2.12E-07
58	8.55E-06	1.97E-03	2.08E-07
59	2.79E-05	1.59E-02	1.55E-07
60	1.07E-04	1.05E-01	3.49E-07
61	4.37E-05	1.82E-02	2.49E-07
62	6.42E-06	3.20E-03	6.44E-08
63	2.08E-05	1.06E-02	1.17E-07
64	3.29E-06	3.68E-04	9.67E-08
65	4.12E-05	5.21E-02	2.43E-07
66	4.27E-06	6.95E-03	3.55E-08
67	2.77E-05	8.38E-02	6.49E-08
68	7.98E-06	4.74E-03	4.59E-08

meteorological conditions. The results are given in Table 5.2 and will be used as the data base for the regression analysis.

V.5 Formulation of the Regression Model

Having obtained the data base, the next step in the analysis is to formulate a model that relate the dependent variables M_1, M_2 and α to the populations in the annular segments. To keep the model simple and also to make the results applicable to other geometries, the regression coefficients will be expressed as functions of the distance from the reactor. The functions will be called "transfer functions" in this study. Before defining the transfer functions, some of the assumptions and techniques in the consequence model will be discussed because the forms of the transfer functions are dependent on the assumptions and techniques in the consequence model.

V.5.1 Assumptions and Techniques in the Consequence Model

Only the assumptions and techniques related to the definition of the transfer functions are briefly discussed. A full description of the consequence model can be found in Appendix VI of WASH-1400 (Ref.1). The discussion of the effects on the transfer functions will be made in the course of defining the transfer functions. With regard to the assumptions and techniques, the following points are important.

(1) A sampling method is used in the consequence model. One trial consists of one radioactive release, one evacuation speed, one starting time of meteorological conditions (stability, precipitation, and wind speed) and one wind direction.

(2) The variables listed above are considered to be independent of each other. The probability assigned to one trail is therefore a product of the probabilities of the individual events.

$$p_t = p_R \cdot p_V \cdot p_S \cdot p_j \tag{5.4}$$

where

p_t: probability assigned to one trial.

p_R: probability of a release occuring.

p_V: probability of an evacuation speed being realized.

p_S: probability assigned to one starting time of meteorological data. As discussed in Section I.4, if 90 starting times are selected, each of them is assigned with a probability of 1/90.

p_j: probability of the wind blowing in the specific direction.

(3) The shift of the wind direction in the downwind is not explicitly treated. The radioactive plume travels in the direction in which the wind was blowing at the starting time of release. Therefore for one trial the fatalities occur only in one direction.

(4) The frequency distribution of the wind direction is uniform over the 16 directions. The probability p_j in Eq. (5.4) is therefore 1/16. The probability p_t assigned to one trial is thus independent of the specific wind direction.

V.5.2 Definition of Transfer Functions

Consider one trial in which the wind is blowing in the direction j. Let A(r) be the ratio of the fatalities per unit r at r to the population per unit r at r in a 22.5 degree sector for the trial. A(r) is a function of the dose to the critical organs and the area covered by the radioactive plume. It is then dependent on the specific release, evacuation speed and meterological condition of the trial, but it is independent of the wind direction. Since the shift of the wind direction is not considered, the total number of fatalities for the trial is given by:

$$x = \int_r A(r) \cdot n_j(r) \cdot dr \qquad (5.5)$$

The first risk moment is the expectation of x over all trials.

$$M_1 = E[x] \qquad (5.6)$$

where E refers to an expectation over all trials. From Eq. (5.5), M_1 is then given by:

$$M_1 = E\,[\int A(r) \cdot n_j(r)]dr = \int E[A(r) \cdot n_j(r)]dr \qquad (5.7)$$

Since the frequency distribution of the wind direction is uniform,

$$M_1 = \frac{1}{16} \sum_j \int E[A(r)] \cdot n_j(r)dr \qquad (5.8)$$

The first transfer function will therefore be defined as:

$$a(r) = \frac{1}{16} \cdot E[A(r)] \tag{5.9}$$

Then M_1 is expressed as:

$$M_1 = \sum_j \int_r a(r) \cdot n_j(r) \cdot dr \tag{5.10}$$

As M_1 is an annual expected number of fatalities, $a(r)$ is an annual expected number of fatalities per individual at distance r. The quantity $a(r)$ can also be interpreted as a probability of death per reactor year for an individual living at distance r.

The second risk moment is an expectation of x^2.

$$x^2 = [\int_r A(r) \cdot n_j(r)]^2$$

$$= \iint_{rr'} A(r) \cdot A(r') \cdot n_j(r) \cdot n_j(r')drdr \tag{5.11}$$

Then,

$$M_2 = E[x^2] = \frac{1}{16} \sum_j \int_r \int_{r'} E[A(r) \cdot A(r')] \cdot n_j(r) \cdot n_j(r')drdr' \tag{5.12}$$

The second transfer function $b(r,r')$ will be defined as:

$$b(r,r') = \frac{1}{16} E[A(r) \cdot A(r')] \tag{5.13}$$

Then,

$$M_2 = \sum_j \int_r \int_{r'} b(r,r') \cdot n_j(r) \cdot n_j(r')drdr' \tag{5.14}$$

The quantity $b(r,r')$ is the annual expected number of fatalities at r and r' per individual at r and r' arising from the same accident. It also can be interpreted as a probability that an individual at r and r' will both be killed in the same accident.

Finally, the third transfer function $c(r)$ will be defined to relate the normalization constant α with the population distribution. The constant α is the probablity per reactor year for which the fatalities will be greater than zero.

$$\alpha = E[H(x)] \tag{5.15}$$

where

$$H(x) = 1 \qquad \text{for } x > 0$$
$$= 0 \qquad \text{for } x = 0$$

Let d_j be the closest distance at which people live from a reactor in the direction j.

$$\begin{aligned} n_j(r) &= 0 \qquad \text{at } r < d_j \\ &> 0 \qquad \text{at } r = d_j \\ &\geq 0 \qquad \text{at } r > d_j \end{aligned} \tag{5.16}$$

Then,

$$\begin{aligned} x &= \int_0^\infty A(r) \cdot n_j(r)dr \\ &= \int_{d_j}^\infty A(r) \cdot n_j(r)dr \end{aligned} \tag{5.17}$$

Now, α is expressed by:

$$\alpha = E[H(\int_{d_j}^{\infty} A(r) \cdot n_j(r)dr)] \tag{5.18}$$

Since it is difficult to express the expectation of H equation in a simple form, an approximation relating α to the closest distance d_j will be constructed. The third transfer function c(r) is then defined as:

$$\alpha = \sum_j [c(r)]_{r=d_j} \tag{5.19}$$

The adequacy of Eq. (5.19) will be tested by the regression fits.

In the consequence computer model, a circle of 500 miles radius is divided into 16 x 34 annular segments. The key equations of the transfer functions are then expressed in the discrete geometry of the consequence model by the following equations.

$$a(r_k) = \frac{1}{16} E[A(r_k)] \tag{5.20}$$

$$M_1 = \sum_j \sum_k a(r_k) \cdot N_{jk} \tag{5.21}$$

$$b(r_k, r_k') = \frac{1}{16} E[A(r_k) \cdot A(r_k')] \tag{5.22}$$

$$M_2 = \sum_j \sum_k \sum_{k'} b(r_k, r_k') \cdot N_{jk} \cdot N_{jk}' \tag{5.23}$$

$$\alpha = \sum_j [c(r)]_{r=r_{k_{min(j)}}} \tag{5.24}$$

where $k_{min}(j)$ is the closest segment in which the population is greater than zero in the direction j.

The transfer functions a(r), b(r,r') and c(r) are dependent on the type of consequence, the average weather characteristics and the type of releases, but they are independent of the population distribution. The transfer functions for early fatalities in PWR accidents in northeastern valley meteorological conditions are being studied in this chapter.

To keep the model simple, the transfer functions will be expressed in terms of possible parametric functions which will be tested in the regression analysis. The forms of the functions and the constants to be fitted by the regression analysis will be studied in Section V.6.

V.6 Regression Fitting

V.6.1 Methods for Fitting

We want to express the transfer functions as parametric functions with a small number of unknown constants which give adequate fits. Two approaches are studied in order to derive the form and the constants from the consequence calculation. The first approach is to use the data base prepared in Section V.4 for the 68 sample population and derive a(r), b(r,r') and c(r) by Eqs. (5.21), (5.23) and (5.24) using regression analysis. The second approach involves calculating the ratio $A(r_k)$ of the fatalities at r_k to the population in a sector at r_k for each trial from the consequence calculation. Using Eqs. (5.20) and (5.23), the average of $A(r_k)$ and $A(r_k) \cdot A(r_k')$ over all the trials will give

$a(r_k)$ and $b(r_k, r_k')$ respectively. $a(r_k)$ and $b(r_k, r_k')$ can then be fitted to the parametric functions involving the distance r. Though both approaches can give the same results (within fitting errors), each has its own advantages and disadvantages. The two approaches are discussed in more detail in the following subsections.

V.6.1.1 Regression from Data Base of M_1, M_2 and α

This approach uses the data base in Section V.4 and Eqs. (5.21), (5.23) and (5.24). Possible parametric functions are assumed for $a(r)$, $b(r, r')$ and $c(r)$. Let $h_a(r| a_1, a_2, \ldots, a_\nu)$ be the assumed parametric functions of $a(r)$ with unknown constants $a_1, a_2, \ldots, a_\nu$. The estimates of the dependent variable M_1 for the 68 populations are given in Table 6.2. Also the populations in the annular segments N_{jk} are given for the 68 samples. Since the range of the estimates of M_1 cover several orders of magnitude, the regression analysis will be based on the natural logarithmic transformation of M_1.

$$\ell n\, M_1 = \ell n \{ \sum_j \sum_k h_a(r_k | a_1, \ldots, a_\nu) \cdot N_{jk} \} + \varepsilon \qquad (5.25)$$

where ε refers to the random error variable. Using the non-linear regression analysis, the unknown constants $a_1, \ldots, a_\nu$ are estimated by minimizing:

$$\Delta_a^2 = \sum_i [\ell n(M_1)_i - \ell n \{\sum_j \sum_k h_a(r_k | a , \ldots, a_\nu) \cdot (N_{jk})_i\}]^2 \qquad (5.26)$$

where the subscript i refers to the population sample.

The derivation of $b(r, r')$ is similar to $a(r)$. A candidate function $h_b(r, r' | b_1, \ldots, b_\nu')$ is assumed and the unknown constants $b_1, \ldots, b_\nu$ are estimated by minimizing:

$$\Delta_b^2 = \sum_i [\ell n(M_2)_i - \ell n \{\sum_j \sum_k \sum_{k'} h_b(r, r' | b_1, \ldots, b_\nu') \cdot (N_{jk})_i \cdot (N_{jk'})_i\}] \qquad (5.27)$$

Finally let $hc(r | c_1, \ldots, c_\nu'')$ be the candidate function of $c(r)$. The closest segments at which the population are greater than zero are identified for each of the population distributions. The unknown constants are estimated by minimizing:

$$\Delta_c^2 = \sum_i [\ell n\ \alpha_i - \ell n \{ \sum_j h_c(r_{k_{\min(j)}} | c_1, \ldots, c_\nu'')] \qquad (5.28)$$

The above approach has the following advantages:

(1) The number of population distributions used can be arbitrary as long as the number is greater than or equal to the number of unknown constants. The fitting errors can be decreased by increasing the number of population distributions.

(2) Since the dependent variables M_1, M_2 and α are integrated over distance, their estimates from their consequence program have relatively small sampling errors of the trials.

The disadvantages of this approach are:

(1) A sizable amount of computation time can be required to estimate M_1, M_2 and α by the consequence program for a larger number of population distributions. For example,

approximate 10 minutes of CPU time on the IBM 360 were required to prepare the data base in Table V.2.

(2) Since the risk moments do not directly suggest appropriate functional forms of the candidate functions, a number of functional forms may need to be tried to find an adequate fitting form.

V.6.1.2 Use of the Averages of Ratios of Fatalities

The second approach involves having the consequence model calculate the ratio of fatalities at the distance r to the population in a 22-1/2 degree sector at r_k for each trial. These ratios are then averaged over all trials. The ratio $[A_k]_t$ for the specific trial t is calculated by:

$$[A_k]_t = \frac{[(N_f)_{jk}]_t}{N_{jk}} \qquad (5.29)$$

where $[(N_f)_{jk}]_t$ is the number of fatalities in the annular segment (j,k) at the trial t and N_{jk} is the population in the annular segment (j, k). As the wind direction is assumed to be independent of the radioactive release, the starting time for the meteorological conditions and the evacuation speed, $[A_k]_t$ is consequently independent of the wind direction. Furthermore, $[A_k]_t$ can be calculated using one sample population distribution. To avoid the case of N_{jk} = 0, a uniform population distribution is used as a sample population in this study.

$$N_{jk} = \frac{\pi}{8} \cdot \rho_o \cdot r_k \cdot \Delta r_k \tag{5.30}$$

where ρ_o is the population density of the uniform population distribution.

Averaging $[A_k]_t$ over all the trials, the estimate of $a(r_k)$ is obtained as:

$$a_k = \sum_t \cdot [A_k]_t \cdot p_R \cdot p_V \cdot p_S \cdot p_j$$

$$= \frac{1}{16} \sum_t [A_k]_t \cdot p_R \cdot p_V \cdot p_S \tag{5.31}$$

where p's are the probabilities assigned to the individual events in Eq. (5.4).

The estimates of $b(r_k, r_k')$ is also obtained from $[A_k]_t$ as:

$$b_{kk'} = \frac{1}{16} \sum_t [A_k]_t \cdot [A_k']_t \cdot p_R \cdot p_V \cdot p_S \tag{5.32}$$

The quantity $c(r_k)$ is not derivable by this averaging approach since $c(r)$ is defined by Eq. (5.19) which is used to approximate the expectation of H equation in Eq. (5.18). Instead of $c(r)$ another type of approximation can be used.

$$\alpha = E[H(\int_{d_j}^{\infty} A(r) \cdot n_j(r) dr)] \tag{5.33}$$

α is approximated by:

$$\alpha \cong E[H(A(d_j) \cdot n_j(d_j))] \tag{5.34}$$

By definition of d_j,

$$n_j(d_j) > 0 \tag{5.35}$$

Then,

$$\alpha \cong E[H(A(d_j))] \tag{5.36}$$

Since the wind direction distribution is uniform,

$$\alpha \cong \frac{1}{16} \sum_j E[H(A(r))]_{r=d_j} \tag{5.37}$$

Another transfer function $\gamma(r)$ is defined as:

$$\gamma(r) = \frac{1}{16} E[H(A(r))] \tag{5.38}$$

Then,

$$\alpha \cong \sum_j [\gamma(r)]_{r=d_j} \tag{5.39}$$

The estimate of $\gamma(r_k)$ is obtained from the consequence calculation by:

$$\gamma_k = \frac{1}{16} \sum_t [H((A_k)_t)] \cdot p_R \cdot p_V \cdot p_S \tag{5.40}$$

The approximation in Eq. (5.34) assumes that whenever there are fatalities occurring then some of these will most likely occur at the closest distance from the reactor at which people live. Since the complete integral in Eq. (5.33) is approximated by the closest distance in Eq. (5.34), the approximation of Eq. (5.39) can underestimate α. However this appears to be a reasonable approximation and furthermore can be used only to give the functional form of $c(r)$. The estimates of $\gamma(r)$ in Eq. (5.40) will be used to obtain the functional form of $c(r)$. Having obtained the estimates of $a(r_k)$, $b(r_k, r_k')$ and $\gamma(r_k)$, they can then be fitted to the parametric functions. The method of fitting will also involve least squares. Suppose $h_a(r|\ a_1, \ldots, a_\nu)$ is a candidate function of $a(r)$. The unknown constants are then estimated by minimizing:

$$\Delta_a^2 = \sum_{k=1}^{K} [\ell n\ a_k - \ell n\ h_a(r_k|\ a_1, \ldots, a_\nu)]^2 \qquad (5.41)$$

where K is the number of the annular segment in one direction. The natural logarithmic transformation is used in Eq. (5.41) because a varies over several orders of magnitude.

In a similar manner, the unknown constants of the candidate function $h_b(r, r'|\ b_1, \ldots, b_\nu')$ are estimated by minimizing:

$$\Delta_b^2 = \sum_{k=1}^{K} \sum_{k'=1}^{K} [\ell n\ b_{kk'} - \ell n\ h_b(r_k, r_k'|\ b_1, \ldots, b_\nu')] \qquad (5.42)$$

Finally, if $h_\gamma(r| \gamma_1, \ldots, \gamma_\nu''')$ is the candidate function for $\gamma(r)$, the unknown constants $\gamma_1, \ldots, \gamma_\nu'''$ are estimated by minimizing:

$$\Delta_\gamma^2 = \sum_{k=1}^{K} [\ell n\ \gamma_k - \ell n\ h\gamma(r_k|\ \gamma_1, \ldots, \gamma_\nu''')] \tag{5.43}$$

The advantages of this approach are:

(1) The estimates of $a(r_k)$, $b(r_k, \gamma_k')$ and $\gamma(r_k)$ from the consequence program can be plotted to suggest appropriate forms for the candidate functions.

(2) Computation time needed to derive a_k, b_{kk}' and γ_k can be much smaller than that required to estimate the risk moments and the normalization constants for many population distributions.

The disadvantages are the following:

(1) The estimates a_k, b_{kk}' and γ_k can have large sampling errors if smaller number of trias are used in the consequence calculation. The occurrence of precipitation in the plume can especially cause large scattering in the estimates.

(2) $c(r)$ is not derivable by this approach. Instead of $c(r)$, the further approximation involving $\gamma(r)$ is required.

V.6.1.3 Combinations of the Two Approaches

Two approaches for deriving the functional forms and the unknown constants of the candidate functions have been discussed in the preceding two subsections. In this study the two approaches are combined. The method of averaging ratios of fatalities is first used to investigate appropriate forms of the candidate functions. After the candidate functions are selected, the unknown constants are then finally estimated using the risk moments and the normalization constants from the 68 population distribution. This combination approach is used in this study since the regression fits from the 68 population distributions will have the smallest sampling errors and the averaging of ratios involves little computer time to investigate possible candidate functions.

V.6.2 Evaluation of a(r)

Based on Eqs. (5.29) and (5.31), the quantities a_k's are estimated by the consequence program. The final estimates are plotted versus miles from a reactor in Fig. 5.4. The scattering of the data points in Fig. 5.4 is due to sampling error. Fig. 5.4 suggests an exponential function as a candidate function:

$$h_a(r) = a_1 \cdot \exp[-a_2 \cdot r] \tag{5.44}$$

Using the data base in Table 5.2, the constants are now derived by the regression using Eq. (5.25). The derived constants a_1 and a_2 are given in Table 5.3 with their 90% confidence bounds.

In addition to the exponential, the following candidate functions are also tested:

$$h_a(r) = a_1 \cdot r^{-a_2} \tag{5.45}$$

$$h_a(r) = a_1 \cdot \exp[-a_2 \cdot r] + a_3 \cdot \exp[-a_4 \cdot r] \tag{5.46}$$

The constants are also estimated from the data base in Table 5.2 using Eq. (5.25). The derived constants are also given in Table 5.3.

The sums of the residual squares are calculated by:

$$S_R^2 = \sum_i [\ln (M_1)_i - \ln \{ \sum_j \sum_k h_a(r_k) \cdot N_{jk} \}]^2 \tag{5.47}$$

The multiple correlation co-efficients are calculated by Eq. (4.9). The results are also given in Table 5.3.

Eqs. (5.44) and (5.45) are first compared with each other because both have two unknown constants. From Table 5.3, the exponential function Eq. (5.44) has a larger multiple correlation coefficient than the power function Eq. (5.45). Eq. (5.44) is then preferred as an equation with two unknowns. Eq. (5.46) has two additional unknowns compared to Eq. (5.44). The decrease of the residual squares due to the added unknowns is tested by the partial F-value defined by Eq. (4.10):

$$F' = \frac{(1.61 - .656)/2}{(.656)/(68 - 4 - 1)} = 45.8$$

Since the upper 10% F-value with (2,63) degrees of freedom is 2.39, the added two unknowns have a statistically significant effect on the variation of the first risk moment. The derived equations having the forms of Eqs. (5.44) and (5.46) are plotted in Fig. 5.4. Fig. 5.4 shows that the double exponential equation (5.46) fits the consequence result better than the single exponential equation (5.44) in the range of

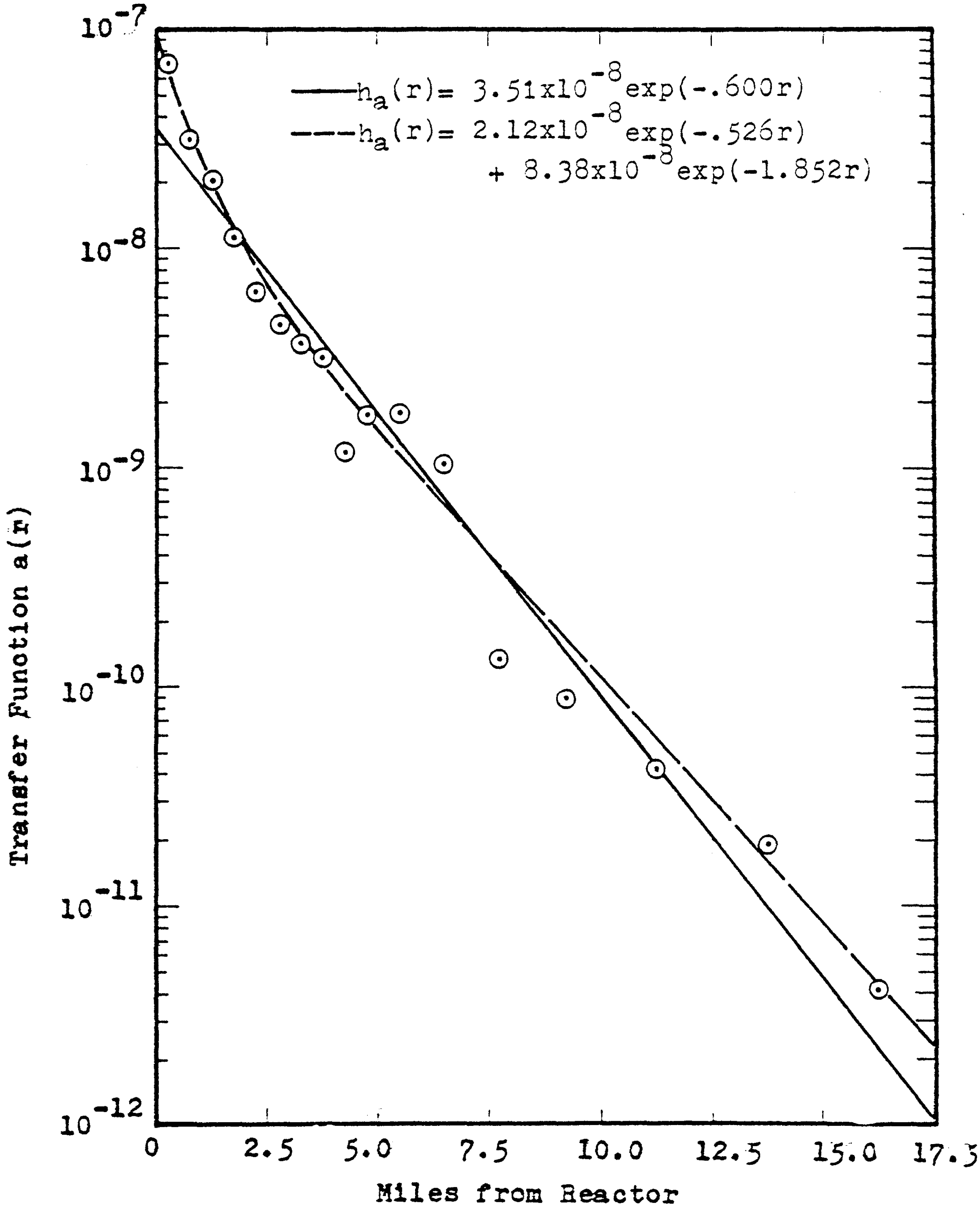

Fig.5.4 Transfer Function a(r) for PWR Accidents

Table 5.3 Estimates of Parameters of a(r) and Sum of Residual Squares

Candidate Function	Estimates of Parameters	90% Confidence Bounds: Upper	90% Confidence Bounds: Lower	Sum of Residual Squares	Multiple Correlation Coefficient	Standard Deviation
$a_1 \cdot \exp(-a_2 \cdot r)$	$a_1 = 3.51 \times 10^{-8}$	3.87×10^{-8}	3.18×10^{-8}	1.61	.992	.155
	$a_2 = .600$	.621	.580			
$a_1 \cdot r^{-a_2}$	$a_1 = 1.86 \times 10^{-8}$	2.27×10^{-8}	1.53×10^{-8}	11.85	.937	.421
	$a_2 = 1.994$	2.105	1.883			
$a_1 \cdot \exp(-a_2 \cdot r) + a_3 \cdot \exp(-a_4 \cdot r)$	$a_1 = 2.12 \times 10^{-8}$	2.51×10^{-8}	1.79×10^{-8}	.656	.997	.099
	$a_2 = .526$	.550	.502			
	$a_3 = 8.38 \times 10^{-8}$	1.06×10^{-7}	6.60×10^{-8}			
	$a_4 = 1.852$	2.198	1.506			

$r < 1$ mile and $r > 12.5$ miles. These two equations will be further examined in Section V.7.

V.6.3 Evaluation of b(r,r')

Based on Eqs. (5.29) and (5.31), the quantities $(b_{kk'})$'s are eliminated by the consequence program. Since $b_{kk'}$ is two-dimensional, the diagonal components (b_{kk}) are plotted in Fig. 5.5(a). The off-diagonal components $(b_{kk'})$ are plotted versus the distance between r and r' for a given value of r in Fig. 5.5(b). Fig. 5.5(a) shows that the diagonal components decrease approximately exponentially. Fig. 5.5(b) shows that the off-diagonal components also decrease approximately exponentially. Since b(r,r') is symmetrical with respect to r and r', the following candidate function is therefore considered.

$$h_b(r,r') = b_1 \cdot \exp[-b_2 \cdot (r+r')] \cdot \exp[-b_3 \cdot |r-r'|] \qquad (5.48)$$

In addition, the following candidate functions are also examined:

$$h_b(r,r') = b_1 \cdot \exp[-b_2 \cdot (r+r')] \cdot \exp[-b_3 \cdot (r-r')^2] \qquad (5.49)$$

$$h_b(r,r') = b_1 \cdot (r)^{-b_2} \cdot (r')^{-b_2} \cdot \exp[-b_3 \cdot |r-r'|] \qquad (5.50)$$

$$h_b(r,r') = \Big\{b_1 \cdot \exp[-b_2 \cdot (r+r')] + b_3 \cdot \exp[-b_4 \cdot (r+r')] \cdot \cdot \exp[-b_5 \cdot |r-r'|] \qquad (5.51)$$

Using the data base in Table 5.2 and Eq. (5.27), the constants of the candidate equations are estimated. The sums of the residual squares and the multiple correlation co-efficients are also calculated. The results are given in Table 5.4.

The multiple correlation co-efficients of Eqs. (5.48) and (5.49) in

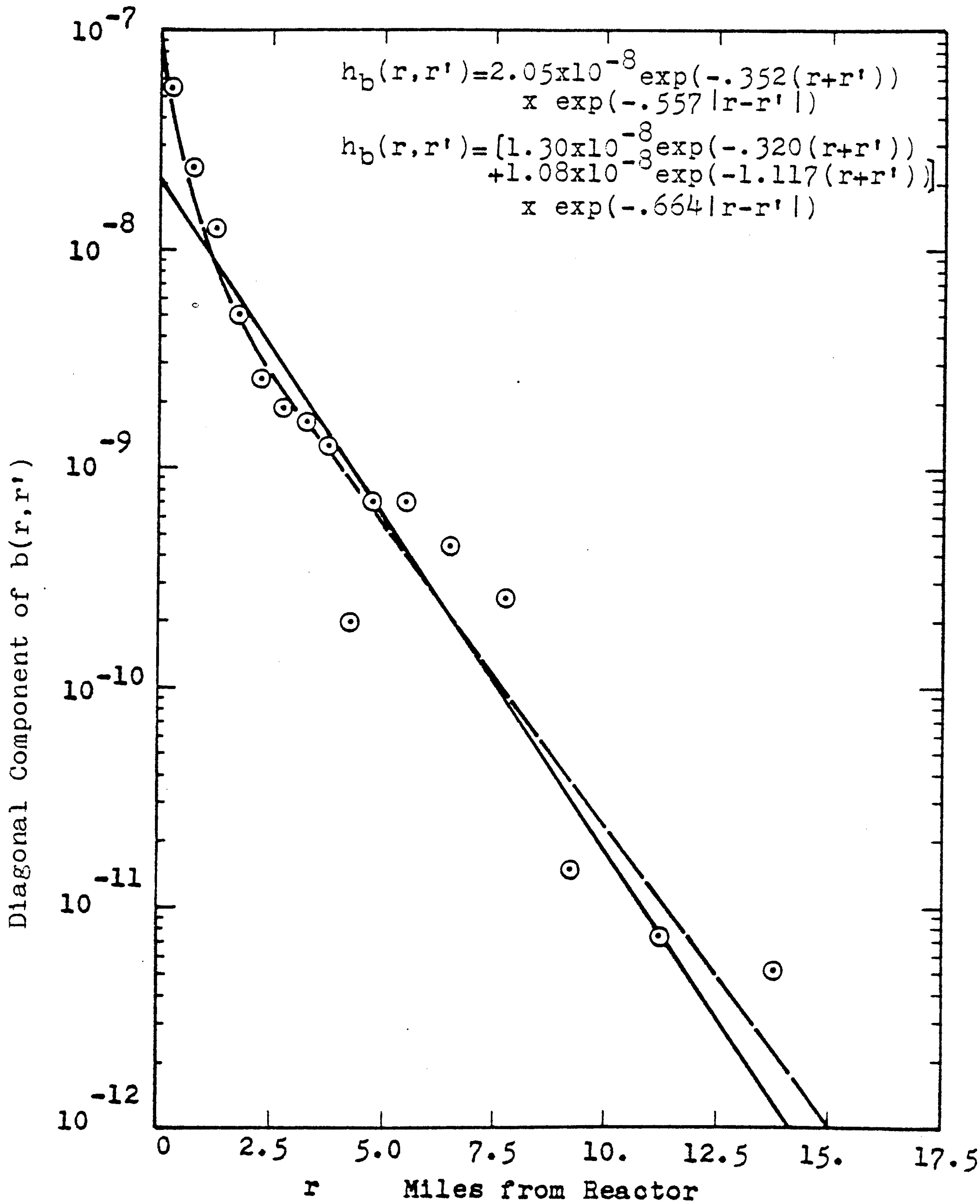

Fig.5.5a Diagonal Component of the Transfer Function b(r,r') for PWR Accidents

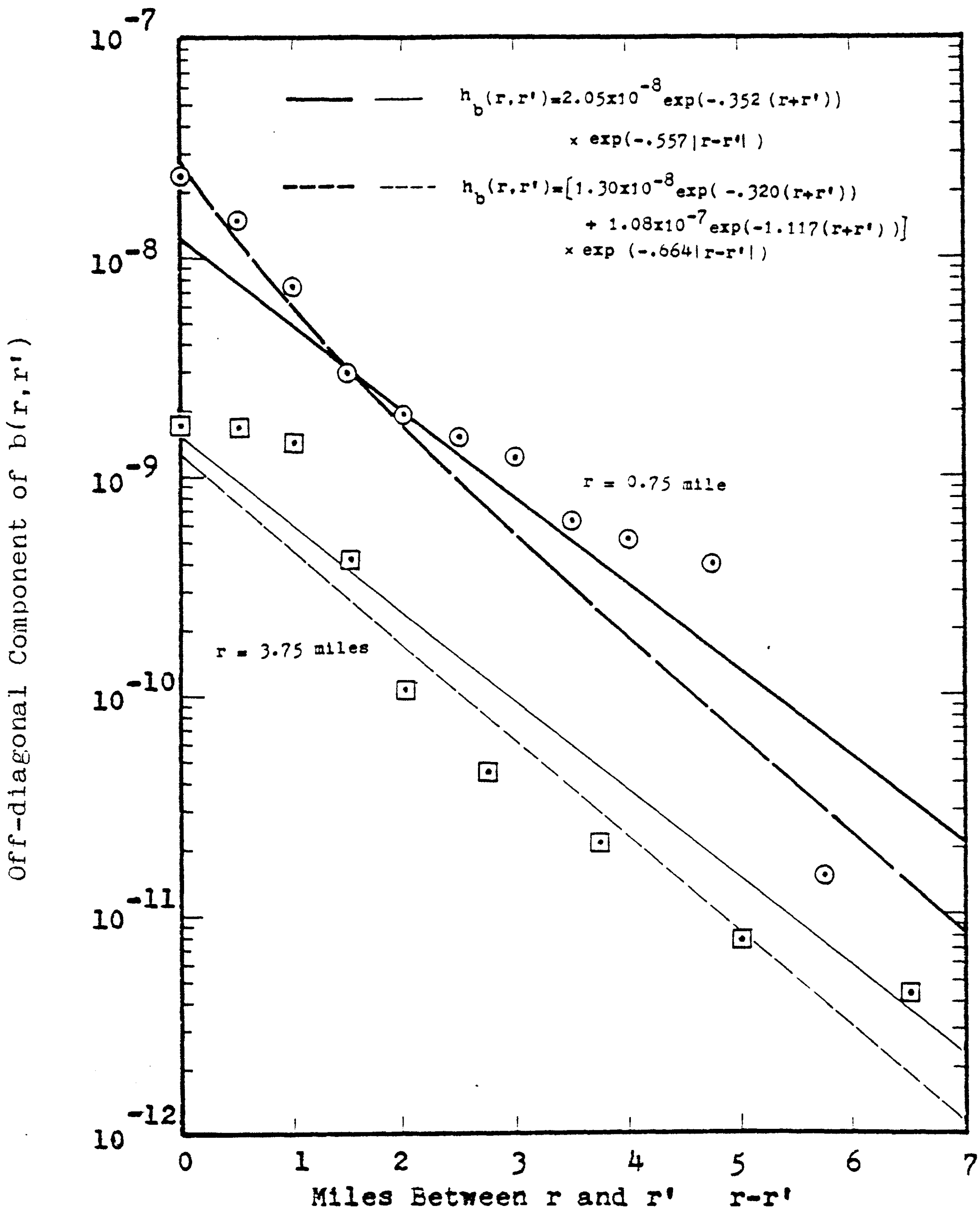

Fig. 5.5b Off-diagonal Component of the Transfer Function b(r,r') for PWR Accidents

Table 5.4 Estimates of Parameters of b(r,r') and Sum of Residual Squares

Candidate Function	Estimates of Parameters	90% Confidence Bounds Upper	Lower	Sum of Residual Squares	Multiple Correlation Coefficient	Standard Deviation
$b_1 \cdot \exp[-b_2 \cdot (r+r')] \cdot \exp[-b_3 \cdot \lvert r-r' \rvert]$	$b_1 = 2.05 \times 10^{-8}$	2.50×10^{-8}	1.68×10^{-8}	5.85	.986	.295
	$b_2 = .352$	.368	.341			
	$b_3 = .557$	.826	.287			
$b_1 \cdot \exp[-b_2 \cdot (r+r')] \cdot \exp[-b_3 \cdot (r-r')^2]$	$b_1 = 2.00 \times 10^{-8}$	2.43×10^{-8}	1.65×10^{-8}	5.92	.985	.297
	$b_2 = .343$	.359	.327			
	$b_3 = .472$	.787	.158			
$b_1 \cdot (r \cdot r')^{-b_2} \exp[-b_3 \cdot \lvert r-r' \rvert]$	$b_1 = 1.38 \times 10^{-8}$	2.05×10^{-8}	9.30×10^{-9}	33.74	.913	.710
	$b_2 = 1.362$	1.462	1.262			
	$b_3 = .515$	1.138	0			
$\{b_1 \cdot \exp[-b_2 \cdot (r+r')] + b_3 \cdot \exp[-b_4 \cdot (r+r')] \times \exp[-b_5 \cdot \lvert r-r' \rvert]$	$b_1 = 1.30 \times 10^{-8}$	1.71×10^{-8}	9.83×10^{-9}	3.79	.991	.238
	$b_2 = .320$	.507	.133			
	$b_3 = 1.08 \times 10^{-7}$	1.85×10^{-7}	6.28×10^{-8}			
	$b_4 = 1.117$	1.459	.775			
	$b_5 = .664$	.933	.395			

Table 5.4 are approximately equal. The difference in the off-diagonal components between Eqs. (5.48) and (5.49) has an insignificant effect on the multiple correlation co-efficient. The power function Eq. (5.50) has a smaller multiple correlation coefficient in Table 5.4. Among the examined equations of three unknown constants, Eq. (5.48) is selected in this study because of its simple form and larger multiple correlation coefficient.

The effect of the added two unknowns in Eq. (5.51) is tested by the partial F-value:

$$F' = \frac{(5.85 - 3.79)/2}{3.79/(68 - 5 - 1)} = 16.88$$

Since the upper 10% F-value with (2,61) degrees of freedom is 2.39, the added two unknowns have a statistically significant effect on the variation of the second risk moment. The derived equations having the forms of Eqs. (5.48) and (5.51) are shown in Figs. 5.5(a) and 5.5(b). Eq. (5.51) fits the consequence results better than Eq. (5.48) in the range of r and $r' < 1$ mile. Eqs. (5.48) and (5.51) will be further examined in Section V.7.

V.6.3 Evaluation of c(r)

Based on Eqs. (5.29) and (5.40), the quantities γ_k's are estimated by the consequence program and the final estimates are plotted in Fig. 5.6. As discussed in Section V.6.1.2, $\gamma(r)$ can underestimate c(r) but it can be expected that c(r) and $\gamma(r)$ can be expressed by the same form of functions. Since Fig. 5.6 suggest an exponential function, an exponential function, an exponential candidate function of c(r) is studied:

$$h_c(r) = c_1 \cdot \exp[-c_2 \cdot r] \tag{5.52}$$

In addition, the following functions are also tested:

$$h_c(r) = c_1 \cdot r^{-c_2} \tag{5.53}$$

$$h_c(r) = c_1 \cdot \exp[-c_2 \cdot r] + c_3 \cdot \exp[-c_4 \cdot r] \tag{5.54}$$

Using the data base in Table 5.2, the constants of the candidate functions are derived. The estimates of the constants, the sums of the residual squares and the multiple correlation coefficients are given in Table 5.5.

The multiple correlation coefficient of the power function Eq. (5.53) is smaller than that of the exponential function Eq. (5.52). The exponential function is then preferred to the power function. The effect of the two additional unknowns in Eq. (5.54) is studied by the partial F-value as:

$$F' = \frac{(.288 - .240)/2}{.240/63} = 6.3$$

Since the upper 10% F-value with (2,63) degrees of freedom is 2.39, the added two unknowns have a statistically significant effect on the variation of the normalization constant. The derived equations (5.52) and (5.54) are compared with the consequence results in Fig. 5.6. Both of the derived equations of c(r) slightly overestimate the plots of γ_k's as discussed in Section V.6.1.2. But the difference between c(r) and γ_k's appears to be small. The double exponential equation (5.54) has a slower rate of decrease than the single exponential equation (5.52) in the range of $r > 10$ miles. Eqs. (5.52) and (5.54) will be further examined in Section V.7.

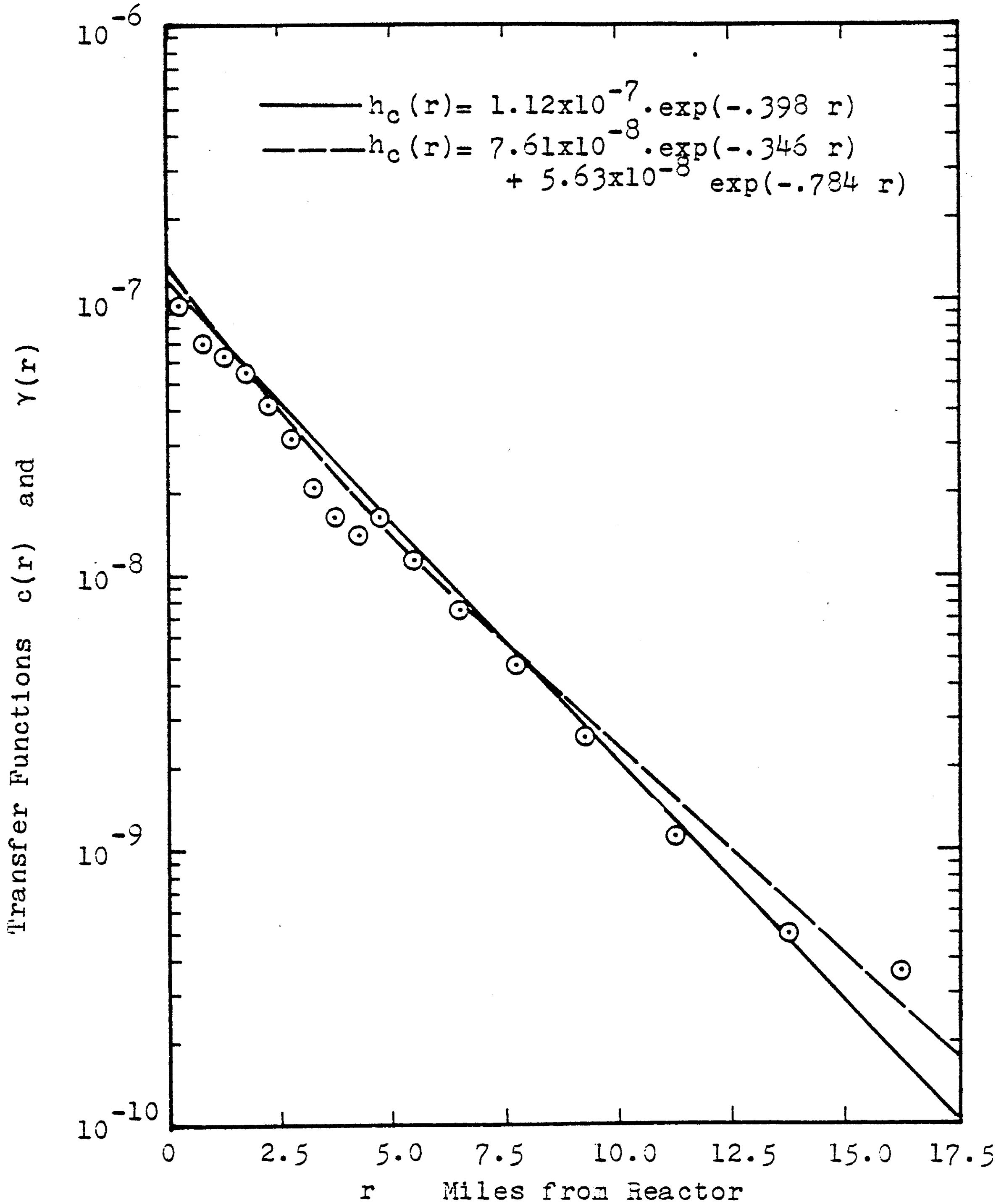

Fig. 5.6 Transfer Functions $c(r)$ and $\gamma(r)$ for PWR Accidents

Note: The lines show the estimates of $c(r)$ and the dots show $\gamma(r)$, which is an approximation of $c(r)$.

Table 5.5 Estimates of Parameters of c(r) and Sum of Residual Squares

Candidate Function	Estimates of Parameters	90% Confidence Bounds Upper	90% Confidence Bounds Lower	Sum of Residual Squares	Multiple Correlation Coefficient	Standard Deviation
$c_1 \cdot \exp[-c_2 \cdot r]$	$c_1 = 1.12 \times 10^{-7}$	1.16×10^{-7}	1.08×10^{-7}	.288	.999	.066
	$c_2 = .398$	.407	.390			
$c_1 \cdot r^{-c_2}$	$c_1 = 7.26 \times 10^{-8}$	8.03×10^{-8}	6.57×10^{-8}	4.30	.979	.253
	$c_2 = 1.124$	1.195	1.053			
$c_1 \cdot \exp[-c_2 \cdot r]$ $+c_3 \cdot \exp[-c_4 \cdot r]$	$c_1 = 7.61 \times 10^{-8}$	1.38×10^{-7}	4.19×10^{-8}	.240	.999	.060
	$c_2 = .346$	.407	.284			
	$c_3 = 5.63 \times 10^{-8}$	1.09×10^{-7}	2.90×10^{-8}			
	$c_4 = .784$	1.315	.253			

V.7 Examination of the Adequacy of the Regression Equations

The adequacy of the regression equations derived in the previous section is investigated with regard to the predicted risk characteristics and predicted distribution behaviors.

V.7.1 Predicted Risk Characteristics

(1) <u>First Risk Moment M_1</u>

The first risk moment is first estimated from the regression results of the single exponential equation (5.44) for each of the 68 sample population distributions. The regression estimate is given by:

$$(M_1)_i = \sum_j \sum_k \hat{a}_1 \cdot \exp[-\hat{a}_2 \cdot r_k] \cdot (N_{jk})_i \qquad i=1,\ldots,68 \tag{5.55}$$

where $\hat{a}_1$ and $\hat{a}_2$ are the derived constants. The estimates by Eq. (5.55) are given in Table 5.6. The estimates are then plotted versus the consequence results in Table 5.2. This plot is shown in Fig. 5.7. If the regression estimates accurately predict the data, the points in Fig. 5.7 should lie about the 45 degree line and no systematic error is observed (i.e., tendencies to overpredict or underpredict various ranges of the data). The largest deviation between the predicted and data first risk moment is a factor of 1.7.

The regression results of the double exponential equation (5.46) are examined in a similar manner. The regression estimates are given by:

Table 5.6 Estimates of the Dependent Variables from the Single Exponential Transfer Functions

$$a(r) = a_1 \cdot \exp(-a_2 \cdot r)$$
$$b(r,r') = b_1 \cdot \exp(-b_2 \cdot (r+r')) \cdot \exp(-b_3 \cdot |r-r'|)$$
$$c(r) = c_1 \cdot \exp(-c_2 \cdot r)$$

Sample No.	M_1	M_2	α
1	9.145E-05	8.605E-02	1.844E-06
2	2.666E-04	7.437E-01	3.145E-06
3	1.499E-05	6.419E-03	6.094E-07
4	9.797E-07	3.887E-04	4.319E-08
5	7.648E-05	7.544E-02	1.453E-06
6	3.749E-05	2.338E-02	1.238E-06
7	5.779E-05	5.922E-02	1.313E-06
8	5.779E-05	5.922E-02	1.313E-06
9	1.691E-05	1.798E-02	3.925E-07
10	2.692E-05	7.133E-02	3.661E-07
11	3.736E-05	6.041E-02	5.971E-07
12	6.866E-04	1.607E 00	5.765E-06
13	2.692E-05	7.133E-02	3.661E-07
14	1.189E-04	1.339E-01	2.296E-06
15	1.672E-05	2.054E-02	3.538E-07
16	1.215E-04	2.511E-01	1.851E-06
17	4.086E-04	1.078E 00	4.376E-06
18	1.956E-05	1.066E-02	6.162E-07
19	5.698E-05	1.101E-01	9.514E-07
20	1.717E-04	4.219E-01	2.272E-06
21	6.496E-05	6.663E-02	1.511E-06
22	3.126E-05	5.084E-02	6.769E-07
23	8.299E-06	2.664E-03	3.870E-07
24	1.636E-05	1.760E-02	4.198E-07
25	4.886E-05	2.218E-02	1.377E-06
26	1.291E-05	7.010E-03	4.209E-07
27	3.880E-05	8.762E-02	5.374E-07
28	1.634E-05	4.762E-02	2.034E-07
29	8.773E-06	4.400E-03	3.063E-07
30	2.108E-05	1.131E-02	7.801E-07
31	1.023E-04	1.041E-01	2.040E-06
32	1.093E-05	4.306E-03	4.610E-07
33	1.278E-05	1.092E-02	4.155E-07
34	5.577E-05	4.717E-02	1.203E-06
35	7.976E-06	2.632E-03	3.761E-07
36	3.462E-05	8.516E-02	5.480E-07
37	3.776E-05	4.748E-02	8.925E-07
38	2.422E-05	1.705E-02	7.393E-07
39	1.454E-04	8.704E-02	2.460E-06
40	6.324E-06	2.335E-03	2.891E-07
41	5.531E-05	4.242E-02	1.400E-06
42	1.225E-05	4.967E-03	4.901E-07
43	8.500E-05	4.904E-02	2.639E-06
44	4.530E-05	2.259E-02	1.272E-06
45	3.380E-05	3.236E-02	8.812E-07
46	1.698E-05	1.072E-02	5.818E-07
47	1.330E-05	9.626E-03	4.652E-07
48	5.337E-04	1.432E 00	5.512E-06
49	1.215E-04	2.990E-01	1.601E-06
50	4.541E-05	1.053E-01	8.309E-07
51	6.649E-05	6.643E-02	1.508E-06
52	1.179E-04	1.160E-01	2.331E-06
53	2.903E-04	5.637E-01	3.878E-06
54	4.123E-05	3.034E-02	1.185E-06
55	2.067E-05	1.284E-02	7.203E-07
56	3.843E-05	2.892E-02	1.017E-06
57	3.992E-05	3.671E-02	1.043E-06
58	7.927E-06	1.393E-03	5.627E-07
59	2.503E-05	1.123E-02	6.252E-07
60	1.168E-04	1.438E-01	2.040E-06
61	4.583E-05	1.460E-02	1.661E-06
62	7.377E-06	3.634E-03	2.713E-07
63	2.175E-05	1.404E-02	6.079E-07
64	3.560E-06	4.278E-04	2.904E-07
65	4.696E-05	8.228E-02	9.735E-07
66	4.570E-06	6.138E-03	1.149E-07
67	2.283E-05	5.838E-02	3.237E-07
68	8.030E-06	5.438E-03	2.405E-07

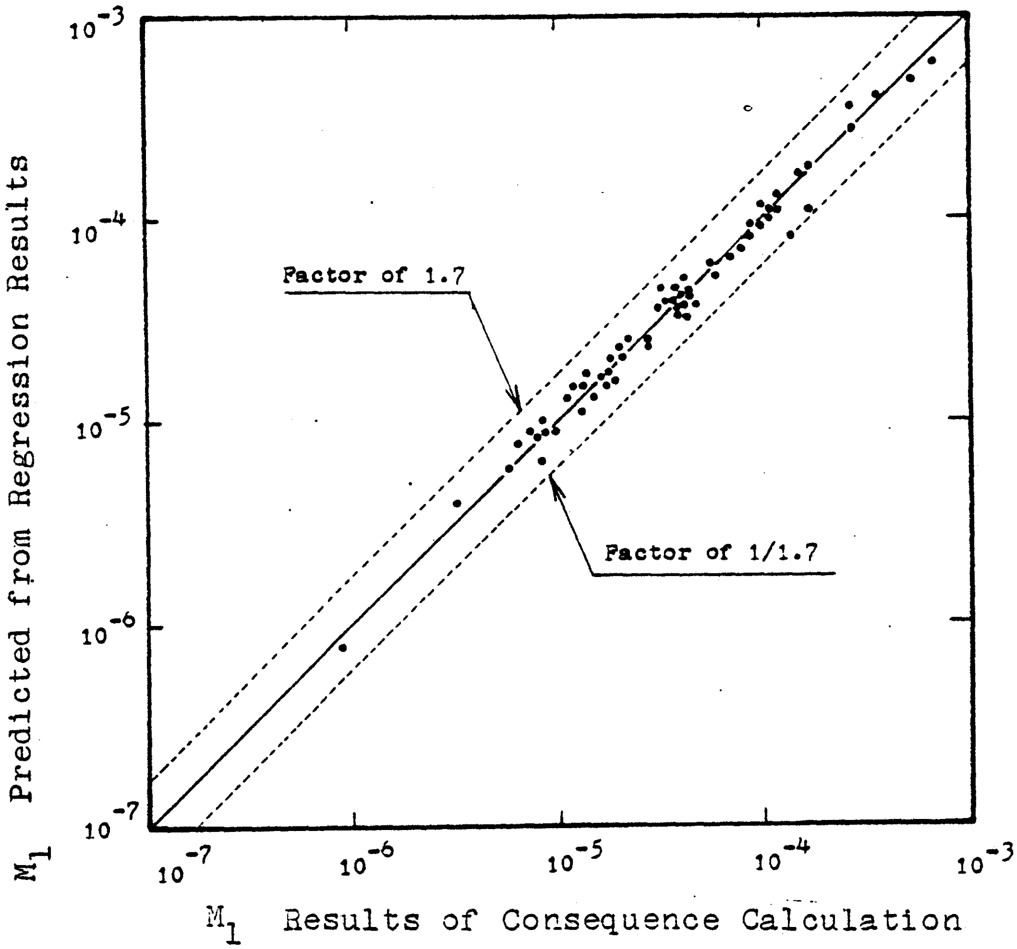

Fig.5.7 Test of the Regression Results of the First Risk Moment for $a(r) = a_1 \cdot \exp(-a_2 \cdot r)$

$$(M_1)_i = \sum_j \sum_k \left\{ \hat{a}_1 \cdot \exp[-\hat{a}_2 \cdot r_k] + \hat{a}_3 \cdot \exp[-\hat{a}_4 \cdot r_k] \right\} \cdot (N_{jk})_i \qquad i=1,\ldots,68 \tag{5.56}$$

where $\hat{a}_1,\ldots,\hat{a}_4$ are the derived constants. The estimates by Eq. (5.56) are given in Table 5.7. The estimates are plotted in Fig. 5.8. The largest deviation between the predicted and the data first moment is a factor of 1.3.

The largest deviation of a factor of 1.7 of the estimates by Eq. (5.55) is judged to be acceptable for risk analysis and decision making considering the uncertainties of the consequence model. If more accuracy is required in the risk analysis, the estimates of the double exponential function by Eq. (5.56) can be used. The distribution behaviors will be examined later in this section based on the estimates by Eq. (5.55).

(2) Second Risk Moment M_2.

The second risk moment is first estimated from the derived regression equation (5.48) for each of the 68 sample population distributions by:

$$(M_2)_i = \sum_j \sum_k \sum_{k'} \hat{b}_1 \cdot \exp[-\hat{b}_2 \cdot (r_k + r_{k'})] \cdot \exp[-\hat{b}_3 \cdot |r_k - r_{k'}|] \cdot (N_{jk})_i \cdot (N_{jk'})_i \qquad i=1,\ldots,68 \tag{5.57}$$

where $\hat{b}_1$, $\hat{b}_2$ and $\hat{b}_3$ are the derived constants. The predicted second risk moments are given in Table 5.6. The plots of the predicted versus the data second risk moments are given in Fig. 5.9. The points in Fig. 5.9 lie about the 45 degree line and the deviations show no systematic error. The largest

Table 5.7 Estimates of the Dependent Variables from the Double Exponential Transfer Functions

$$a(r) = a_1 \cdot \exp(-a_2 \cdot r) + a_3 \cdot \exp(-a_4 \cdot r)$$

$$b(r,r') = (b_1 \cdot \exp(-b_2 \cdot (r+r')) + b_3 \cdot \exp(-b_4 (r+r'))) \cdot \exp(-b_5 \cdot |r-r'|))$$

$$c(r) = c_1 \cdot \exp(c_2 \cdot r) + c_3 \cdot \exp(c_4 \cdot r)$$

Sample No.	M_1	M_2	α
1	9.14E-05	9.21E-02	1.81E-06
2	2.67E-04	6.14E-01	3.33E-06
3	1.50E-05	6.02E-03	6.22E-07
4	9.30E-07	4.98E-04	3.99E-08
5	7.65E-05	7.53E-02	1.45E-06
6	3.75E-05	2.05E-02	1.29E-06
7	5.78E-05	5.84E-02	1.32E-06
8	5.78E-05	5.84E-02	1.32E-06
9	1.69E-05	1.76E-02	3.95E-07
10	2.69E-05	6.58E-02	3.75E-07
11	3.74E-05	6.41E-02	5.87E-07
12	6.87E-04	2.08E 00	5.38E-06
13	2.69E-05	6.58E-02	3.75E-07
14	1.19E-04	1.40E-01	2.27E-06
15	1.67E-05	2.01E-02	3.56E-07
16	1.22E-04	2.39E-01	1.88E-06
17	4.09E-04	9.40E-01	4.55E-06
18	1.96E-05	1.02E-02	6.25E-07
19	5.70E-05	9.64E-02	9.90E-07
20	1.72E-04	3.83E-01	2.34E-06
21	6.50E-05	6.34E-02	1.53E-06
22	3.13E-05	4.44E-02	7.06E-07
23	8.30E-06	2.47E-03	3.96E-07
24	1.64E-05	1.87E-02	4.12E-07
25	4.89E-05	3.65E-02	1.18E-06
26	1.29E-05	6.42E-03	4.33E-07
27	3.86E-05	7.74E-02	5.58E-07
28	1.63E-05	4.88E-02	2.02E-07
29	8.77E-06	3.90E-03	3.18E-07
30	2.11E-05	1.00E-02	8.11E-07
31	1.02E-04	1.17E-01	1.97E-06
32	1.09E-05	4.06E-03	4.70E-07
33	1.28E-05	9.12E-03	4.40E-07
34	5.58E-05	5.63E-02	1.14E-06
35	7.96E-06	2.27E-03	3.94E-07
36	3.46E-05	8.06E-02	5.57E-07
37	3.78E-05	4.83E-02	8.83E-07
38	2.42E-05	1.65E-02	7.48E-07
39	1.45E-04	2.25E-01	1.89E-06
40	6.32E-06	2.24E-03	2.93E-07
41	5.53E-05	4.08E-02	1.42E-06
42	1.22E-05	4.32E-03	5.13E-07
43	8.50E-05	5.91E-02	2.49E-06
44	4.53E-05	2.79E-02	1.19E-06
45	3.38E-05	3.43E-02	8.65E-07
46	1.70E-05	1.22E-02	5.58E-07
47	1.33E-05	1.14E-02	4.41E-07
48	5.34E-04	1.41E 00	5.54E-06
49	1.22E-04	2.84E-01	1.63E-06
50	4.54E-05	1.07E-01	8.27E-07
51	6.65E-05	6.07E-02	1.55E-06
52	1.18E-04	1.42E-01	2.19E-06
53	2.90E-04	4.97E-01	4.02E-06
54	4.12E-05	2.66E-02	1.23E-06
55	2.07E-05	1.09E-02	7.60E-07
56	3.94E-05	3.03E-02	1.00E-06
57	3.99E-05	3.49E-02	1.06E-06
58	7.95E-06	1.31E-03	5.73E-07
59	2.5[illegible]E-05	1.27E-02	7.94E-07
60	1.17E-04	1.38E-01	2.07E-06
61	4.37E-05	1.82E-02	2.49E-07
62	6.42E-06	3.20E-03	6.44E-08
63	2.08E-05	1.06E-02	1.17E-07
64	3.29E-06	3.68E-04	9.67E-08
65	4.12E-05	5.21E-02	2.43E-07
66	4.27E-06	6.95E-03	3.55E-08
67	2.77E-05	8.38E-02	6.49E-08
68	7.98E-06	4.74E-03	4.99E-08

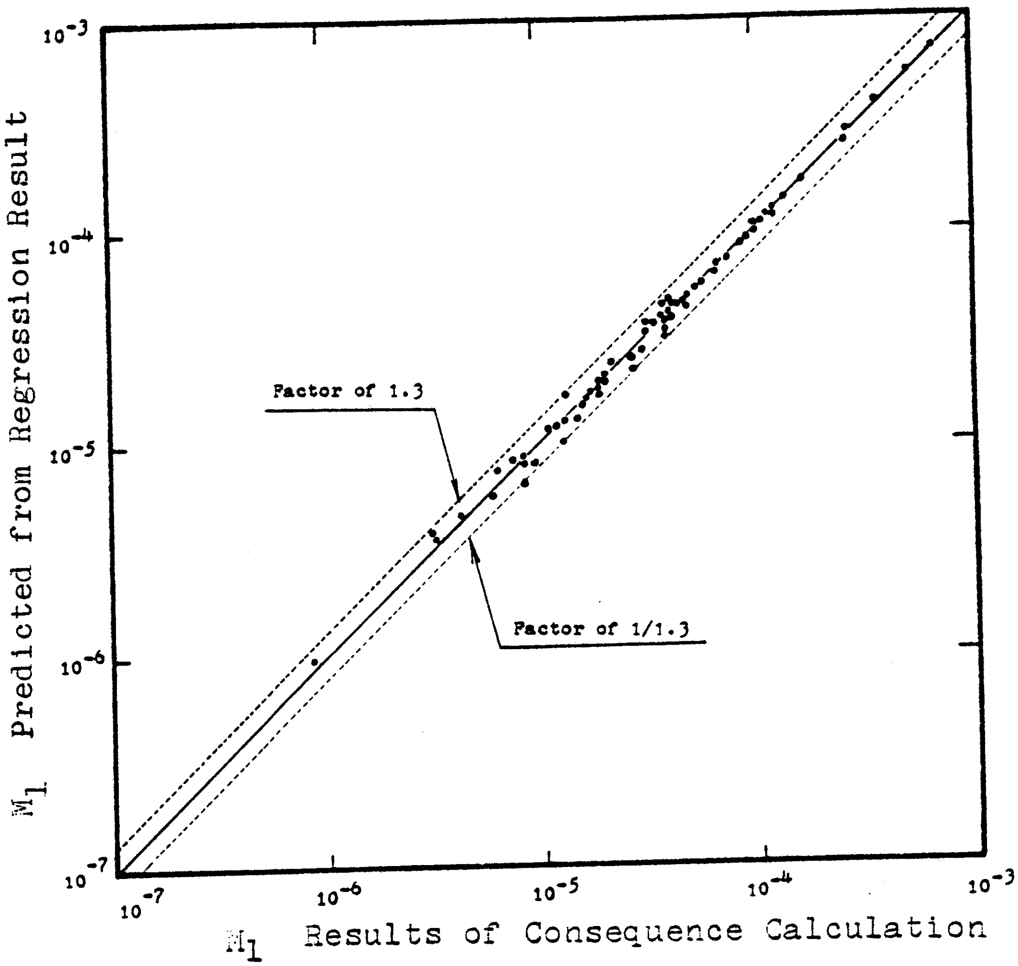

Fig.5.8 Test of the Regression Results of the First Risk Moment for $a(r)=a_1 \cdot \exp(-a_2 \cdot r)+ a_3 \cdot \exp(-a_4 \cdot r)$

deviation between the predicted and data second risk moments is a factor of 2.4.

The regression results of Eq. (5.51) are also examined. The regression estimates are given by:

$$(M_2)_i = \sum_j \sum_k \sum_{k'} \Big\{ \hat{b}_1 \cdot \exp[-\hat{b}_2 \cdot (r_k + r_{k'})] + + \hat{b}_3 \cdot \exp[-\hat{b}_4 \cdot (r_k + r_{k'})] \times \exp[-\hat{b}_5 \cdot |r_k - r_{k'}|] \cdot \cdot (N_{jk})_i \cdot (N_{jk'})_i \quad i=1,\ldots,68 \qquad (5.58)$$

The estimates are shown in Table 5.7 and Fig. 5.10. The largest deviation between the predicted and the data is a factor of 1.9.

The largest deviation of a factor of 2.4 of Eq. (5.57) is judged to be acceptable for risk analysis. The distribution behavior will be studied later in this section based on the second risk moment estimated by Eq. (5.57). If further accuracy is required in the analysis, the estimates by Eq. (5.58) can be used.

(3) Normalization Constant α

The normalization constant is estimated from the derived single exponential equation (5.52) for each of the 68 samples by:

$$\hat{\alpha}_i = \sum_j \hat{c}_1 \, e^{-\hat{c}_2 (d_j)_i} \qquad i=1,\ldots,68 \qquad (5.59)$$

where $\hat{c}_1$ and $\hat{c}_2$ are the derived constants. The results are given in Table 5.6 and Fig. 5.11. The points in Fig. 5.11 do not show any systematic error. The largest deviation is a

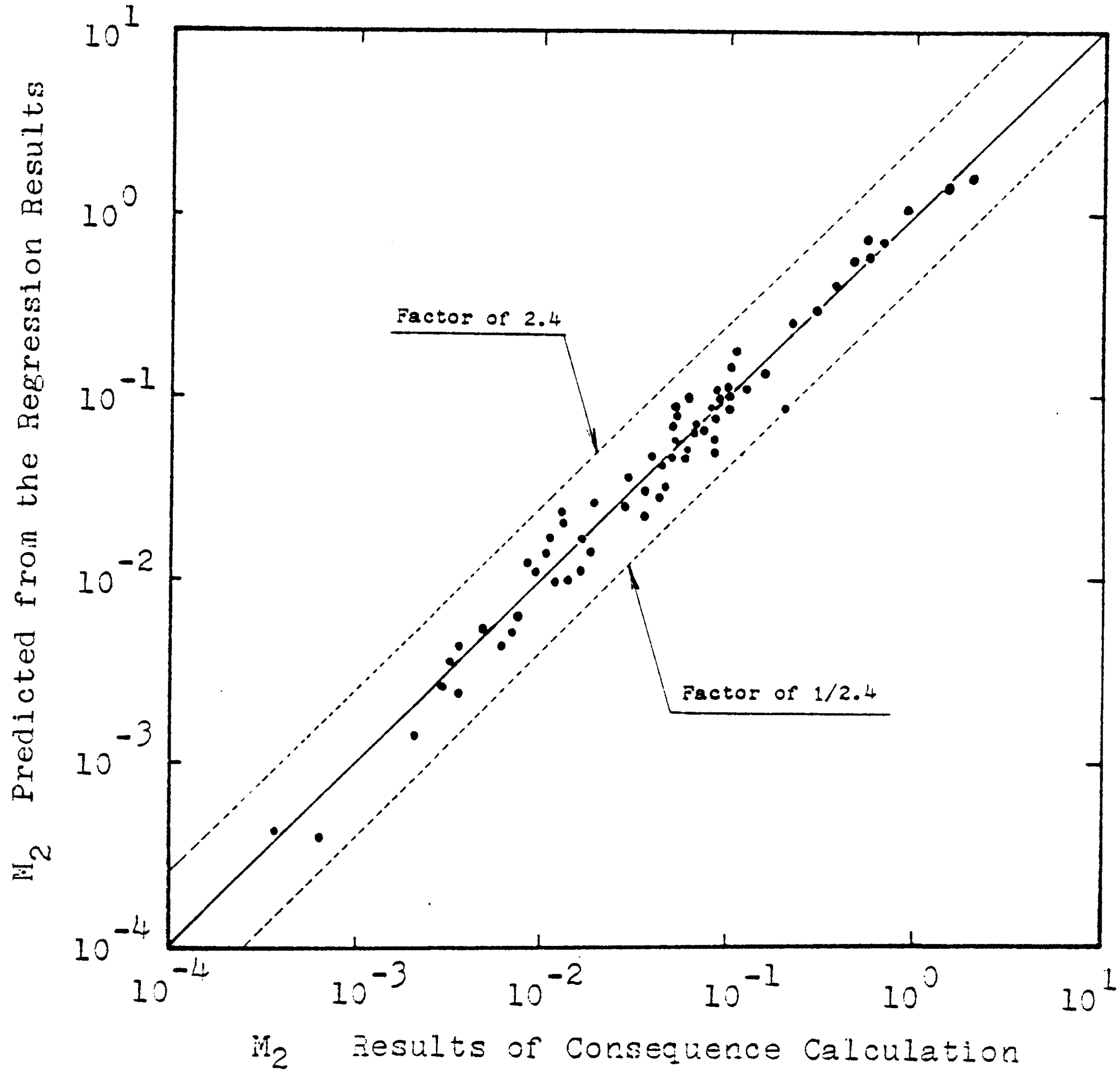

Fig.5.9 Test of the Regression Results of the Second Risk Moment for $b(r,r') = b_1 \cdot \exp(-b_2 \cdot (r+r')) \cdot \exp(-b_3 |r-r'|)$

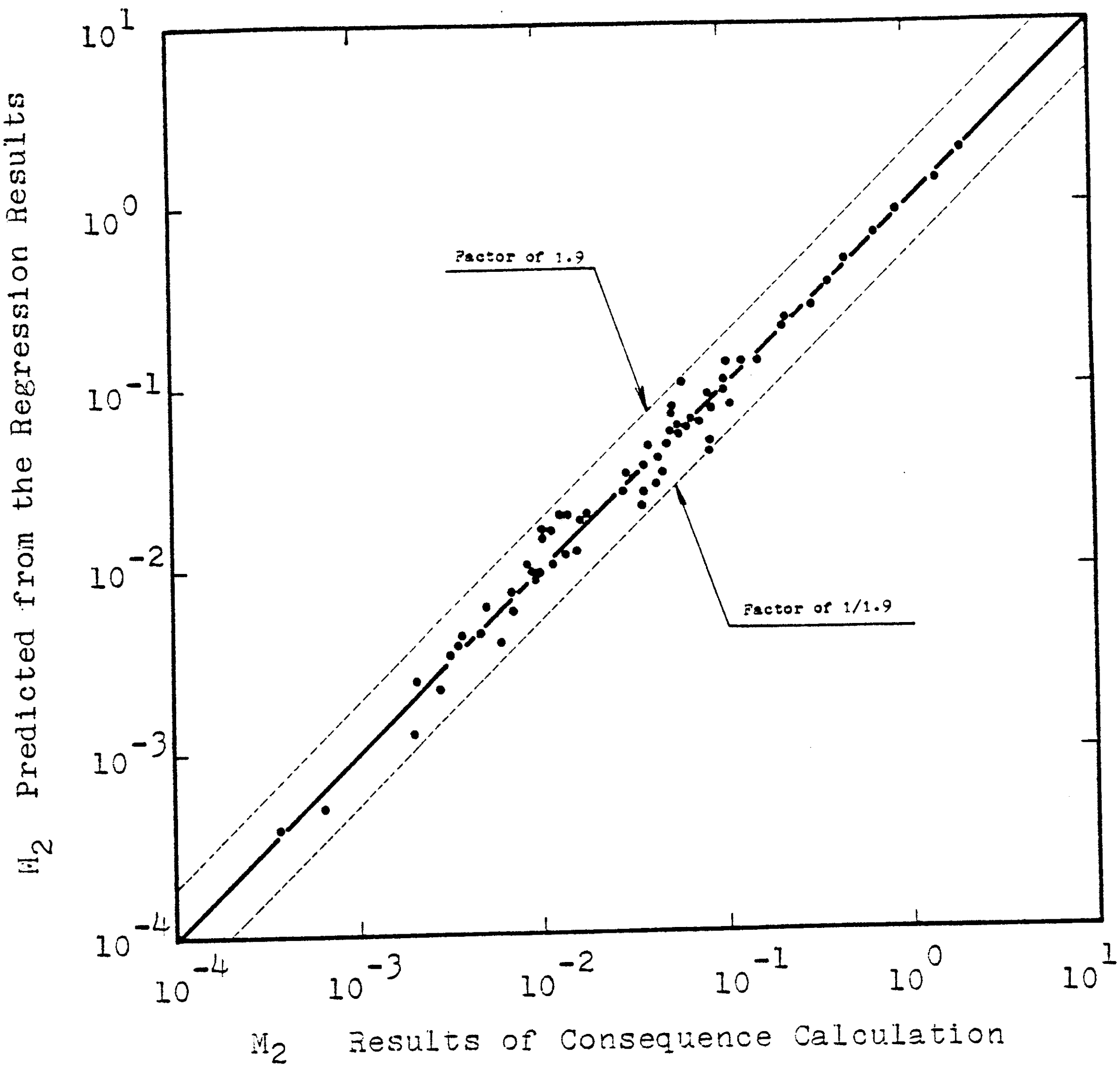

Fig. 5.10 Test of the Regression Results of the Second Risk Moment for

$$b(r,r')=(b_1 \cdot \exp(-b_2 \cdot (r+r'))+b_3 \cdot \exp(-b_4(r+r'))) \exp(-b_5 \cdot |r-r'|)$$

factor of 1.2.

The double exponential equation (5.54) is also examined. The normalization constant is estimated by:

$$\alpha_i = \sum_j \left\{ \hat{c}_1 \cdot \exp\,[-\hat{c}_2 \cdot (d_j)_i] + \hat{c}_3 \cdot \exp\,[-\hat{c}_4 \cdot (d_j)_i] \right.$$

$$i=1,\ldots,68 \qquad (5.60)$$

where $\hat{c}_1,\ldots,\hat{c}_4$ are the derived constants. The estimates are given in Table 5.7 and Fig. 5.12. The largest deviation is a factor of 1.2.

The largest deviation of a factor of 1.2 of the estimates by Eq. (5.59) is judged to be acceptable considering the uncertainties of the consequence model. The distribution behaviors will be examined later in this section based on the estimates by Eq. (5.59). If more accuracy is required in the analysis, the estimates by Eq. (5.60) can be used.

Since no systematic error is observed in the normalization constant and since the deviations between the predicted and data normalization constants are smaller than those of the first and second risk moments, the approximation of Eq. (5.19) relating α to the closest distance d_j at which people live is therefore judged to be adequate for the calculations performed in this study. However it should be noted that this specific example does not prove that the approximation of Eq. (5.19) is valid for other types of consequences and for other types of meteorological models. Careful studies will be required for each different case.

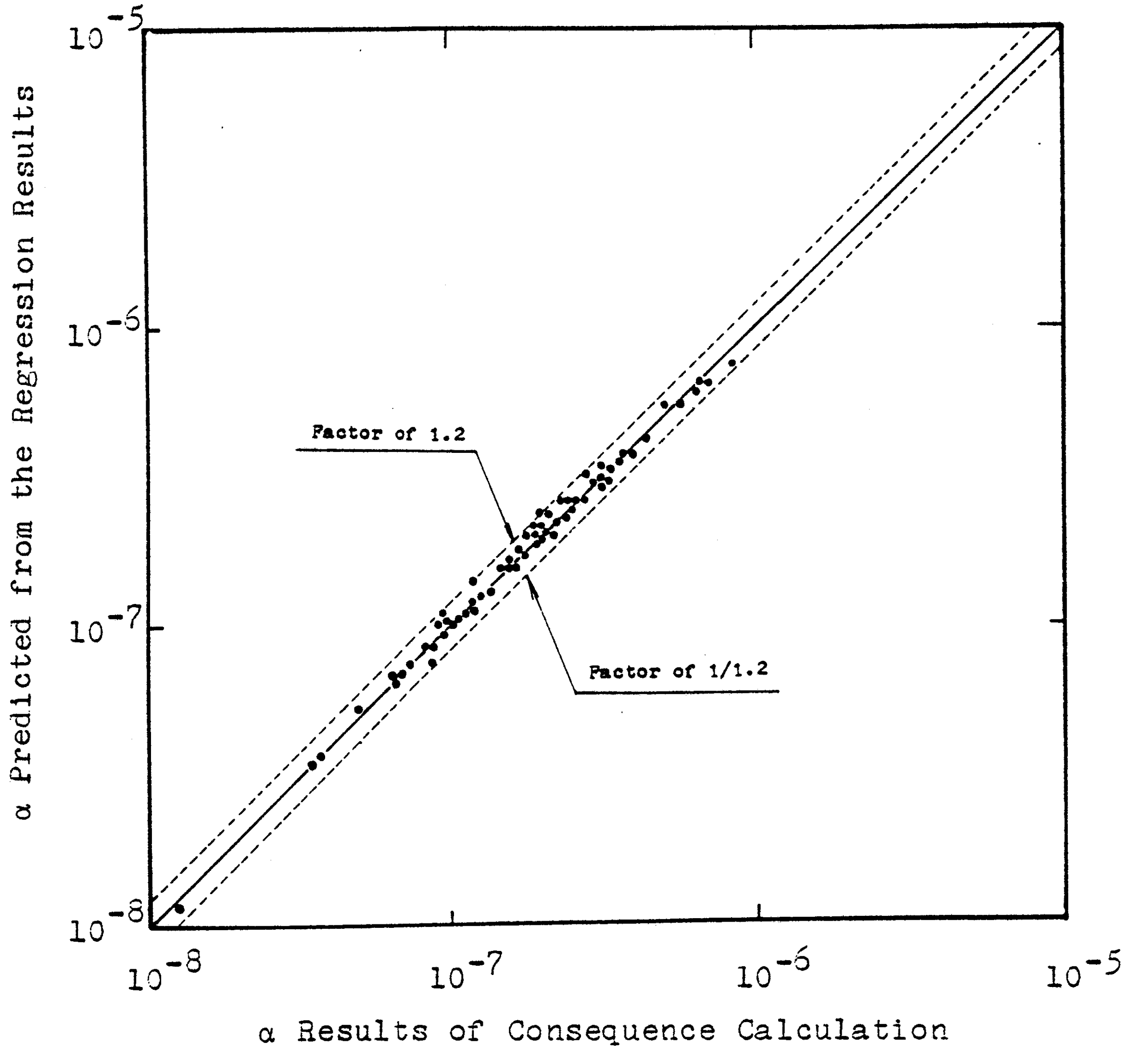

Fig.5.11 Test of the Regression Results of the Normalization Constant for $c(r)=c_1 \cdot \exp(-c_2 \cdot r)$

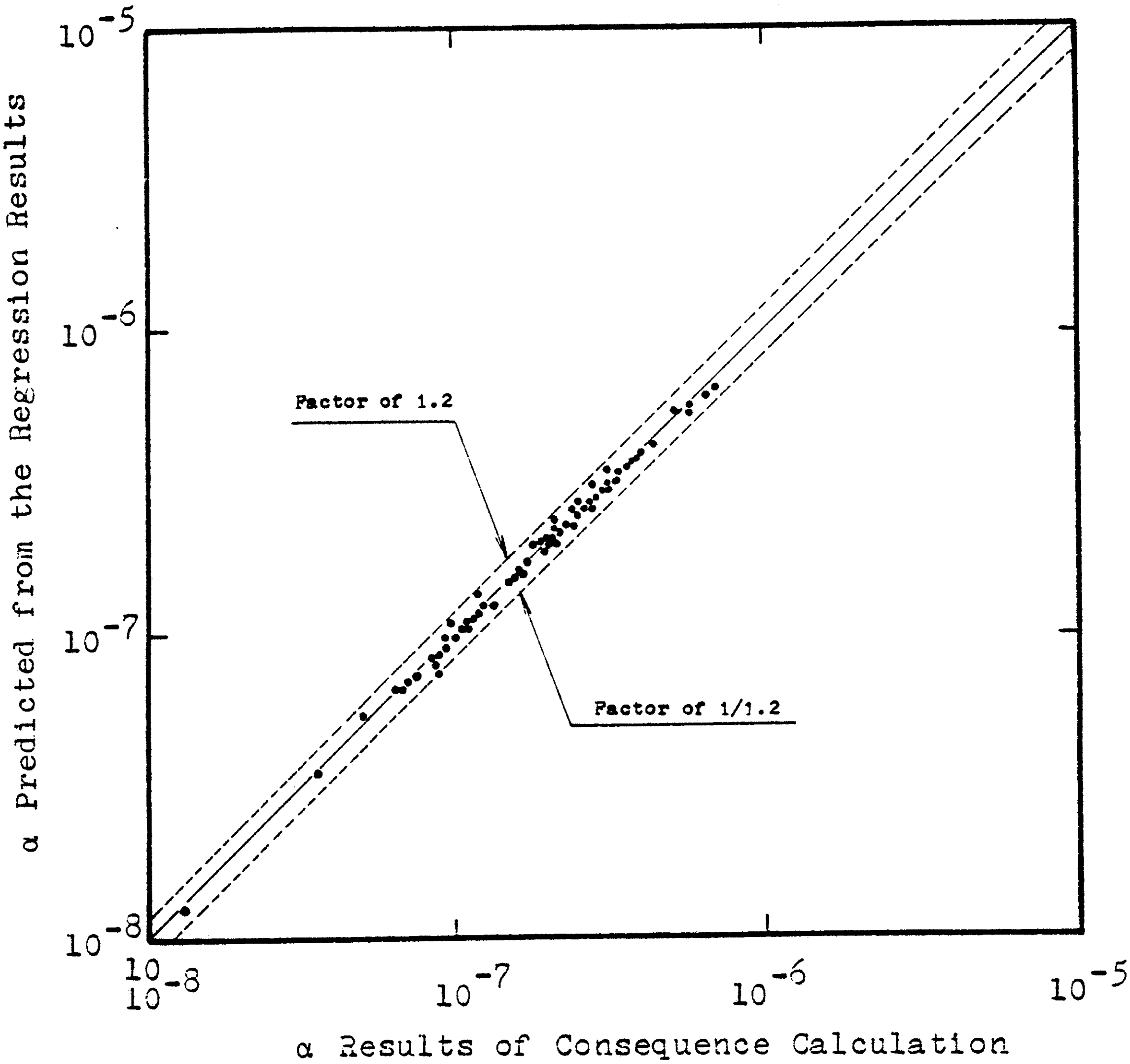

Fig.5.12 Test of Regression Results of the Normalization Constant for

$$c(r) = c_1 \cdot \exp(-c_2 \cdot r) + c_3 \cdot \exp(-c_4 \cdot r)$$

V.7.2 Predicted Distribution Behaviors

The next step in assessing the regression results is to test the combined effects of regression errors on the distribution behaviors. The examined regression results are single exponential equations (5.44), (5.48) and (5.52). The distribution behaviors are predicted by the Weibull distribution, the parameters of which are estimated from the regression results.

(1) Weibull Shape Factor and Scale Factor

The shape factor β and scale factor η are first derived from the regression results of M_1, M_2 and α given in Table 5.6 for each of the 68 samples of the population distributions. Secondly, β and η are then derived from the data values of M_1, M_2 and α given in Table 5.2.

The shape factors from the regression results and the data values are compared in Fig. 5.13. The points lie about the 45 degree line and the deviations do not show systematic error in Fig. 5.13. The largest deviation is 0.14 and 90% of the points are within the bounds of $\pm.08$. The scale factors are similarly compared in Fig. 5.14. The points lie about the 45 degree line and the deviations do not show any systematic error. The largest deviation is a factor of 1.9 and 90% of the points are within factors of 1.4 and 1/1.4.

The deviations of the shape and scale factors are within the uncertainties of the consequence model: further judgement in the acceptability is obtained from the complementary cumulative distributions which are discussed next.

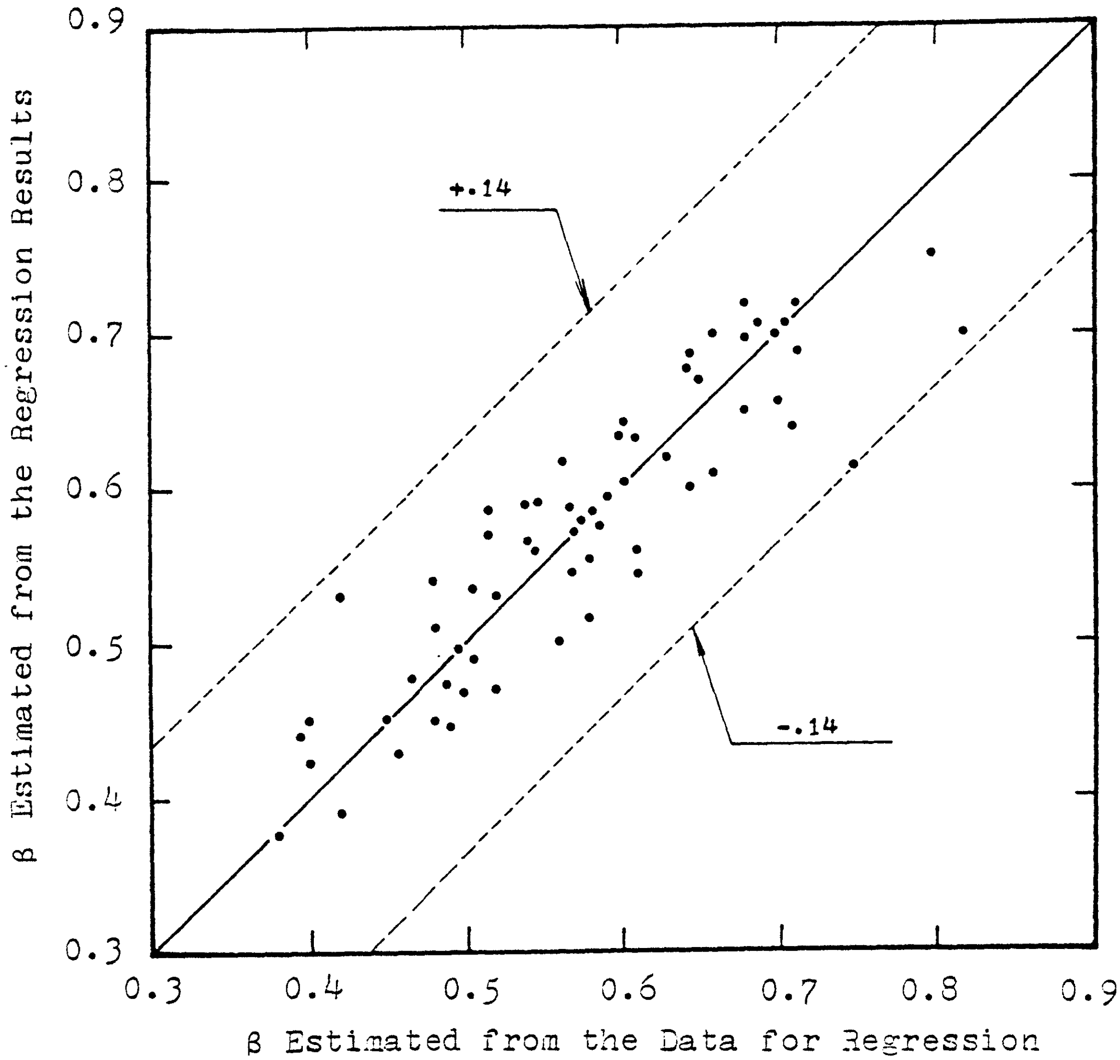

Fig.5.13 Test of the Regression Results for the Weibull Shape Factor

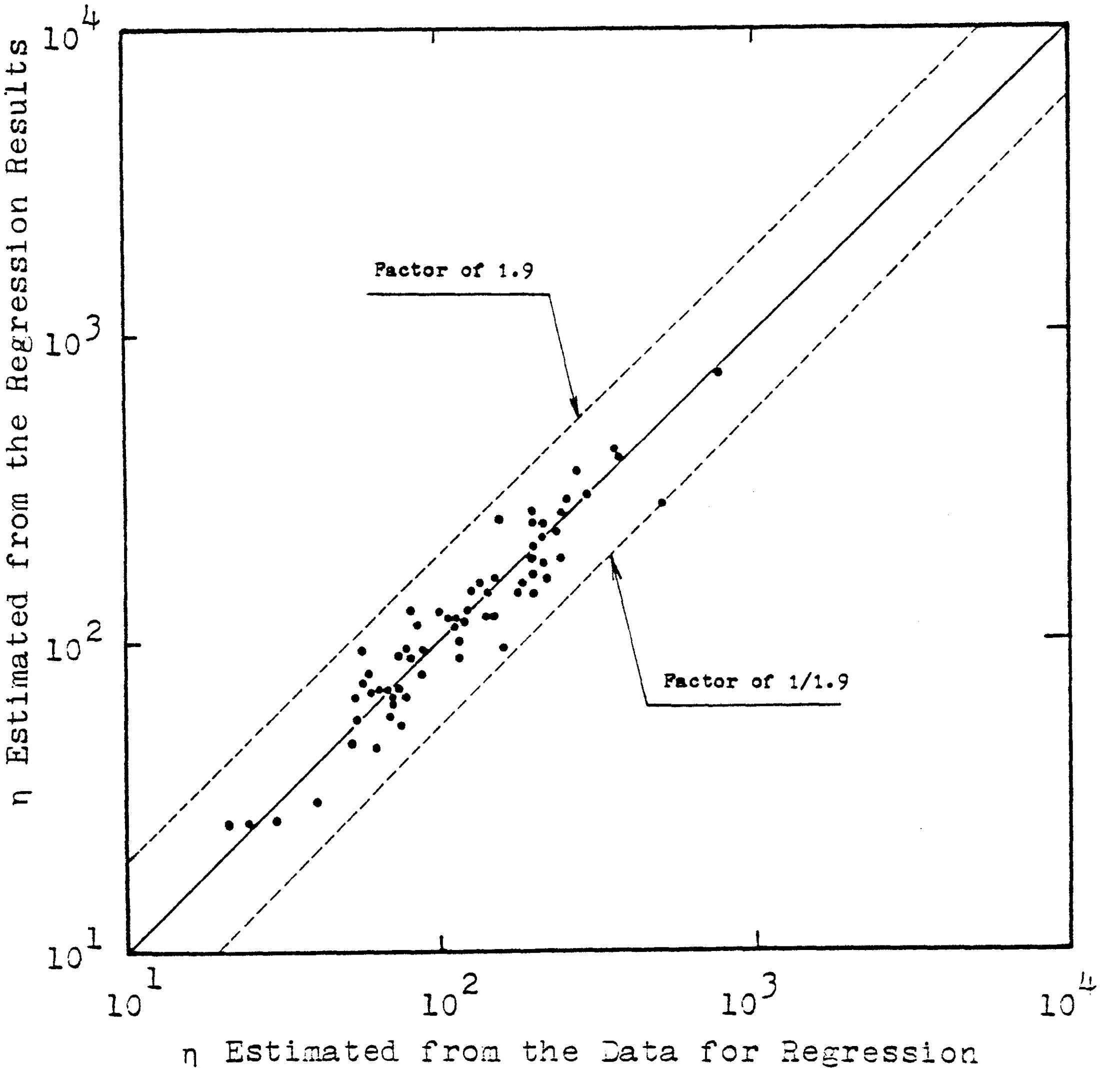

Fig. 5.14 Test of the Regression Results for the Weibull Scale Factor

(2) Complementary Cumulative Distribution

The complementary cumulative distribution is obtained from the shape factor and scale factor estimated from the regression equations for each of the 68 samples, i.e.,

$$F^c(x) = \alpha \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right] \tag{5.61}$$

This derived complementary cumulative distribution is then compared for each of the sample population distributions with the data distribution of consequence vs. frequency calculated by the consequence model. These data curves are obtained directly from the consequence calculation and do not involve fittings to the data values of M_1, M_2 and α. Two of the samples will be specifically discussed here. One is the sample (#63) which gives the largest deviation of β in Fig. 5.13. The other is the sample (#39) which gives the largest deviation of η in Fig. 5.14.

Fig. 5.15 compares the predicted complementary cumulative distribution with the data distribution of site (#63). The predicted distribution underestimates the probabilities between 100 fatalities and 500 by a factor of maximum 1.2 and underestimates the magnitude below 10^{-8}/year by a factor of 1.6. The magnitudes of these errors are smaller than the uncertainty ranges of the consequence model, which were estimated to be factors of 5 and 1/5 on the probabilities and factors of 4 and 1/4 for the consequence magnitude. (See Section III.5.1).

Fig. 5.16 compares the complementary cumulative distribution estimated from the regression equations to the data distribution of site (#39). The estimated distribution underestimates the probabilities between 300 fatalities and 3000 by a factor of 4 at most. The underestimation of the consequence magnitude is maximum a factor of 3 in the same interval. These errors are also within the uncertainty ranges given for the consequence model. For the other samples examined, the complementary cumulative distributions from regression and the data complementary cumulative distributions agree at least as well as for the samples (#39) and (#63).

The samples other than (#39) and (#63) are now examined with regard to the consequence magnitude at a specific complementary cumulative frequency. Since the effects of the errors of β and η on the tail behaviors can be large and the tail behaviors are of importance, the consequence magnitudes at 10^{-9}/year are selected to test the regression fits. The value of 10^{-9}/year is a truncation point in the consequence model, which was determined by the compromise between accuracy and computation time (Ref-1).

The consequence magnitude at 10^{-9}/year is first derived for the 68 samples from β and η estimated by the regression results. The percentile is given by:

$$x_{(10^{-9})} = \eta \cdot [\ln \left(\frac{\alpha}{10^{-9}}\right)]^{1/\beta} \tag{5.62}$$

The consequence magnitude of 10^{-9}/year of the data distribution are then estimated by interpolation of the adjacent two data points below and above 10^{-9}/year.

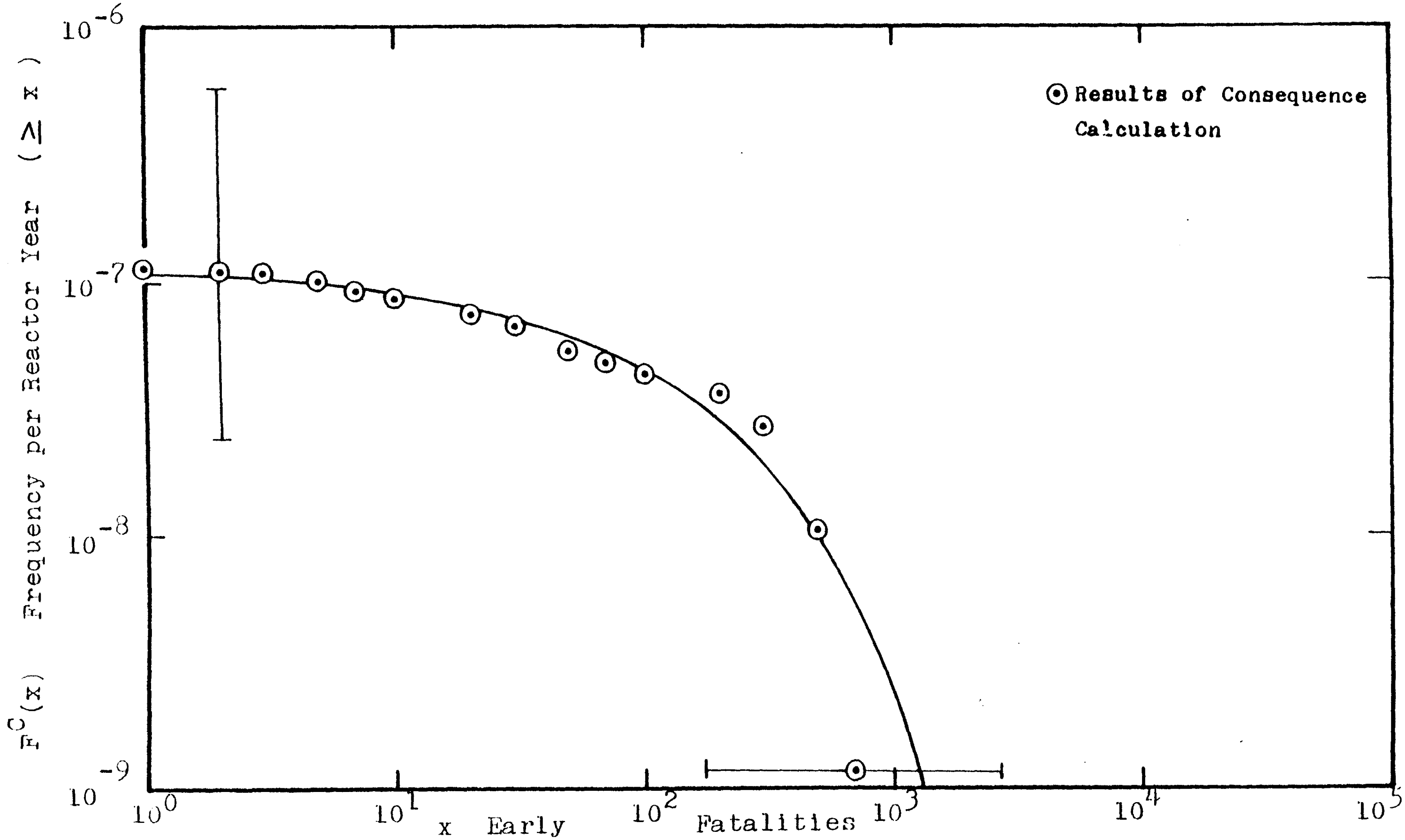

Fig.5.15 Test of the Regression Results for Site #63 of the Largest Deviation in β

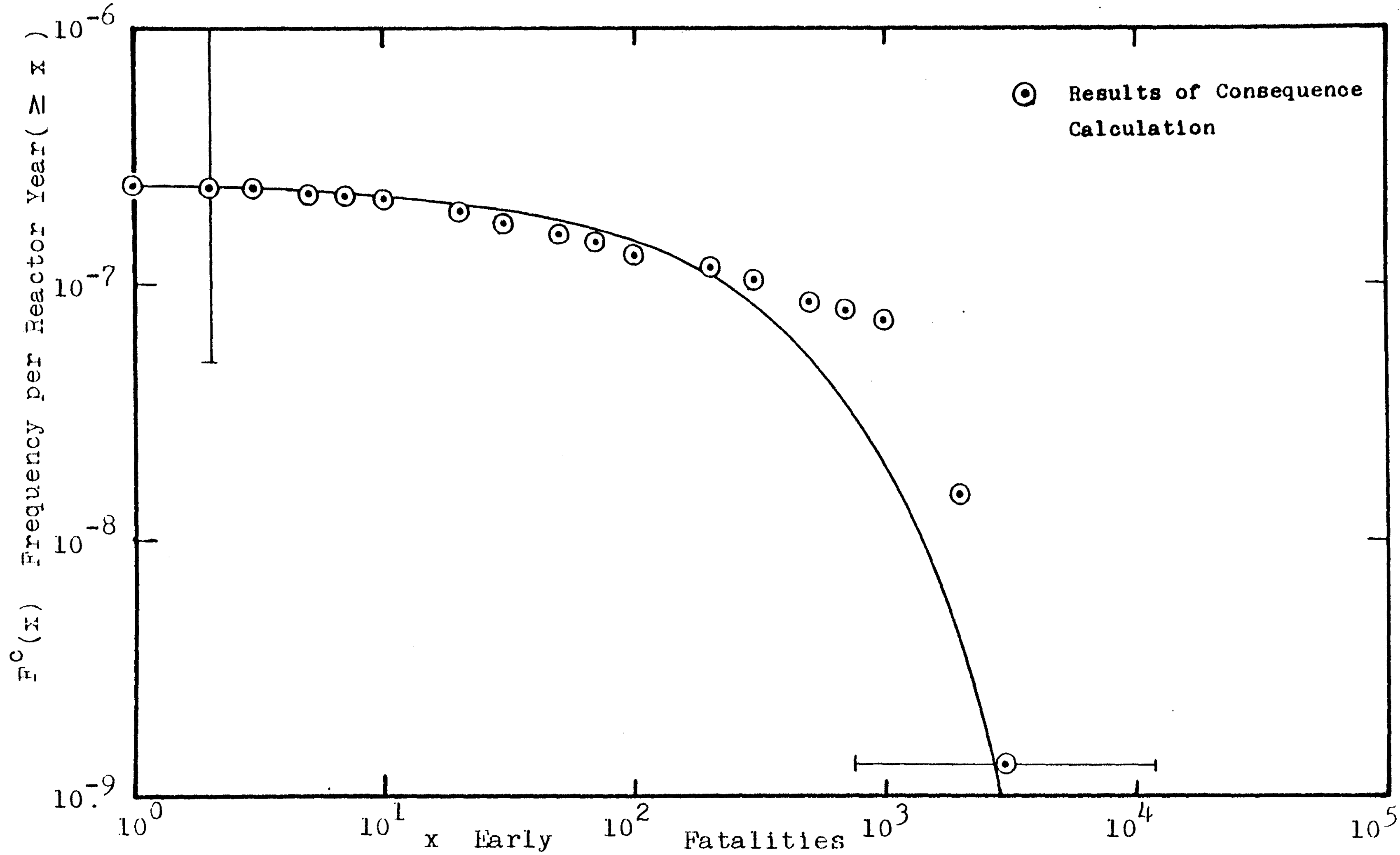

Fig 5.16 Test of the Regression Results for Site #39 of the Largest Deviation in n

$$\ln [x_{(10^{-9})}] = \ln [x_\ell] + \frac{\ln (10^{-9}) - \ln (F_\ell^C)}{\ln (F_h^C) - \ln (F_\ell^C)} \cdot$$
$$\cdot (\ln x_h - \ln x_\ell) \tag{5.63}$$

where the subscripts h and ℓ denote the two adjacent points.

Fig. 5.17 compares the consequence magnitudes estimated from regression to those estimated from the data distributions. The estimates from regression systematically overpredict the estimates of the data distributions. The bias is a factor of 1.2. This error can be due to the fact that the consequence model tend to underestimate the tails of the distributions if sufficient number of trials are not taken. More importantly, the largest deviation is a factor of 2.0, which is smaller than the uncertainty ranges of factors 4 and 1/4 in the consequence model.

V.7.3 Conclusions from the Regression Examinations

The regression results have been examined for their ability to predict the risk characteristics and distribution behaviors. The equations examined were:

$$a(r) = 3.51 \times 10^{-8} \cdot \exp [-.600\, r] \tag{5.64}$$

$$a(r) = 2.12 \times 10^{-8} \exp [-.526\, r] + 8.38 \times 10^{-8} \exp [-1.852\, r] \tag{5.65}$$

$$b(r,r') = 2.05 \times 10^{-8} \exp [-.352\, (r + r')] \cdot \exp [-.557\, |r - r'|] \tag{5.66}$$

$$b(r,r') = \left\{1.30 \times 10^{-8} \exp [-.320\, (r + r')] + + 1.08 \times 10^{-7} \exp [-1.117\, (r + r')]\right\} \cdot \exp [-.664\, |r - r'|] \tag{5.67}$$

$$c(r) = 1.12 \times 10^{-7} \exp [-.398\, r] \tag{5.68}$$

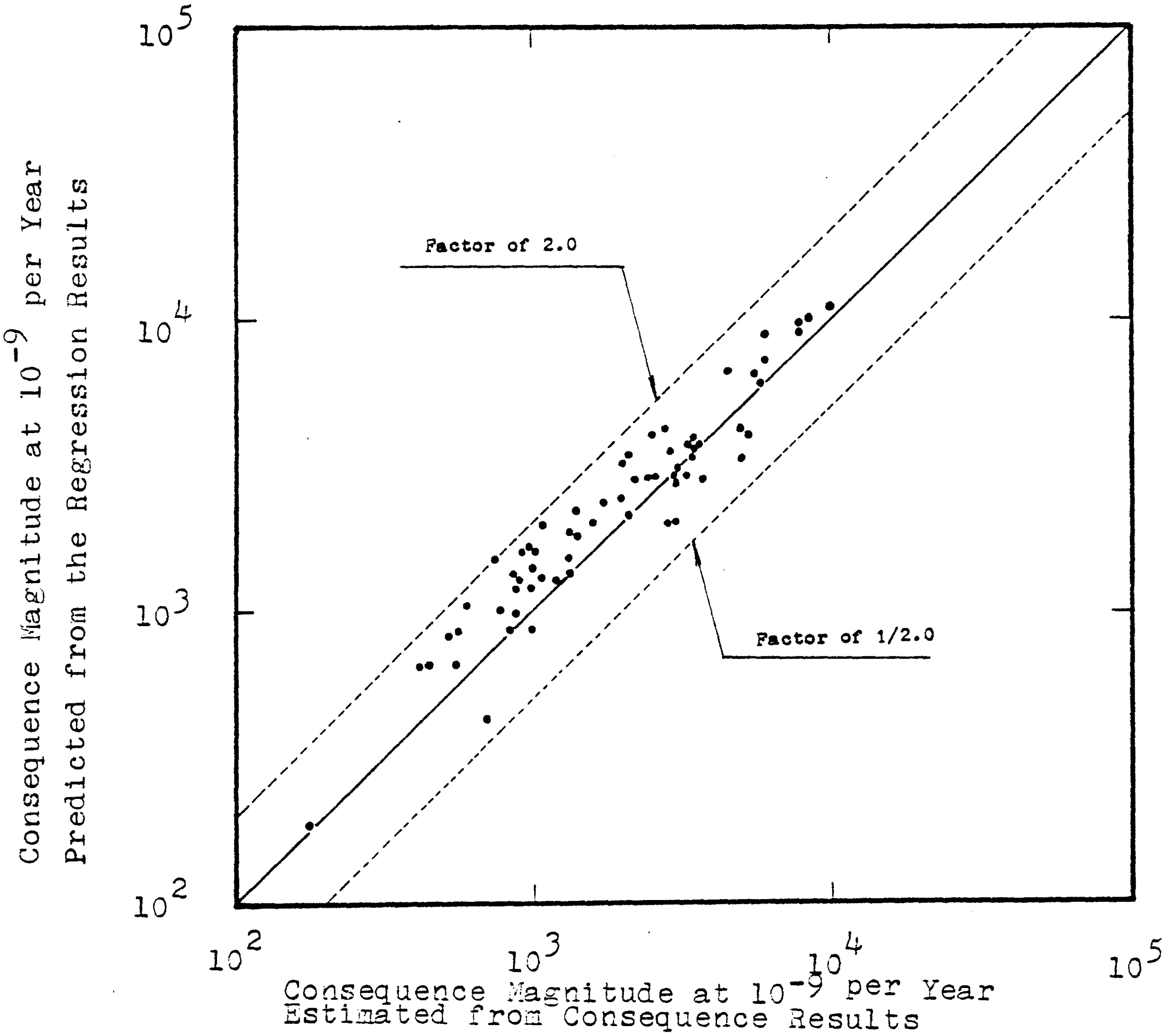

Fig.5.17 Test of the Regression Results for the Consequence Magnitude at 10^{-9} per Year

$$c(r) = 7.61 \times 10^{-8} \exp[-.346\, r] + 5.63 \times 10^{-8} \exp[-.784\, r] \quad (5.69)$$

No systematic errors were observed in the prediction of M_1, M_2 and α. The largest deviations were factors of 1.7 for Eq. (5.64), 1.3 for Eq. (5.65), 2.4 for Eq. (5.66), 1.9 for Eq. (5.67), 1.2 for Eq. (5.68) and 1.2 for Eq. (5.69). The equations (5.64), (5.66) and (5.68) with smaller number of unknowns were judged to be acceptable considering the uncertainties of the consequence model.

The predicted distribution behaviors were then examined for Eqs. (5.64), (5.66) and (5.68). No systematic errors were observed for the prediction of β and η. The largest deviations were 0.14 for β and a factor of 1.9 for η. The complementary cumulative distributions for the two samples which showed the largest deviation for β and η were within the uncertainty ranges of the consequence model. The consequence magnitudes at 10^{-9}/year derived from the regressions overestimate those from the data by a factor of 1.2. This factor is not large and is within the uncertainty ranges of the consequence model.

Based on the above results, the derived equations (5.64), (5.66) and (5.68) were therefore judged to be acceptable for risk analysis and decision making.

V.8 Example of Applications of the Regression Results

Having obtained the regression results, they can then be used to estimate the risk distributions for new situations of different populations without having to rerun the consequence model. Furthermore, because of the explicit relationship of the regression equations (transfer functions), the sensitivity studies and decision making studies are able to be performed in a straightforward manner. The regression results applied to siting will be discussed here.

V.8.1 Application of Regression Results to Siting

The population distribution is one of the important factors in selection of sites for nuclear power plants. An example is given here for the application of the regression results to the siting studies based on an idealized population distribution. The population model considered is a bell-shaped, gaussian distribution illustrated in Fig. 5.18. The population distribution of a particular city or a town is expressed by the bell-shaped model in Fig. 5.18 and the overall population distribution of a site surrounded by numerous cities and towns can be expressed by the series of the bell-shaped population distributions. A city or a town expressed by the bell-shaped model is called a "population group" in this study.

The population distribution of a particular population group is assumed to be symmetric about its center. Let N_T be the total population in the group, R be the distance of its center from the reactor and σ_R be the average deviation from the center. 47% of the total population are living within the radius of σ_R and 90% are living within the radius of $2\sigma_R$. Using the (r,ζ) co-ordinate in Fig. 5.18, the population per unit area at (r,ζ) is expressed by:

$$\rho(r,\zeta) = \frac{N_T}{2\pi\sigma_R^2} \exp\left[-\frac{(r-R)^2}{2\sigma_R^2} - \frac{\zeta^2}{2\sigma_R^2}\right] \tag{5.70}$$

From the regression results, the first risk moment is expressed as:

$$\begin{aligned} M_1 &= \sum_j \int_0^\infty a(r)\cdot n_j(r)\,dr \\ &= \int_0^\infty a(r)\cdot\left\{\sum_j n_j(r)\right\}\cdot dr \end{aligned} \tag{5.71}$$

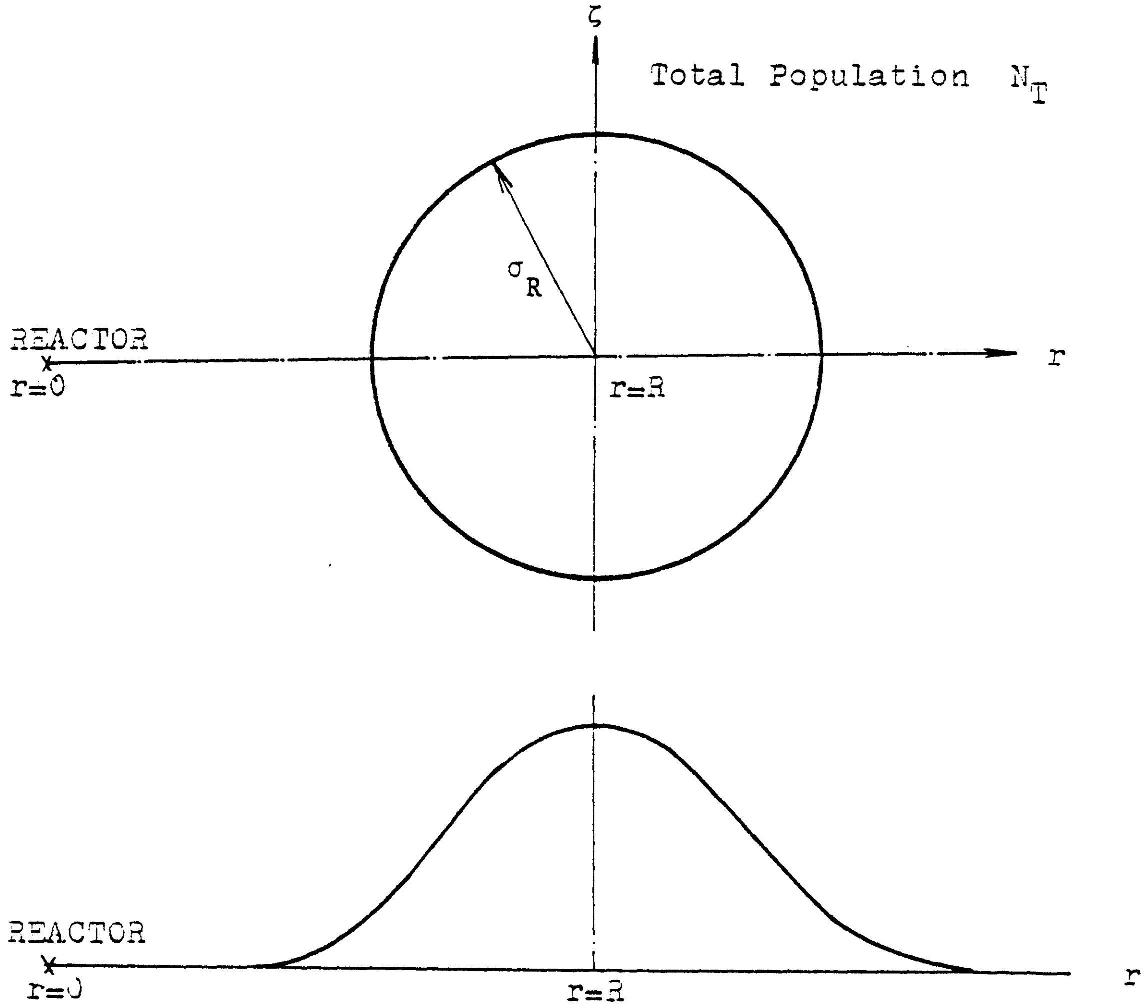

Fig.5.18 Bell-shaped Model of Population Distribution

Let $n_T(r)$ be the population in an annulus per unit r at distance r, i.e.,

$$n_T(r) = \sum_j n_j(r) \tag{5.72}$$

Then,

$$M_1 = \int a(r) \cdot n_T(r) \cdot dr \tag{5.73}$$

Since the regression equation (5.71) is based on the (r,θ) coordinate, an approximation is made here to estimate $n_T(r)$ from $\rho(r,\zeta)$. The integration with respect to θ is approximated by the integration with respect to ζ.

$$n_T(r) = \int_{-\infty}^{\infty} \rho(r,\zeta)\, d\zeta$$

$$= \frac{N_T}{\sqrt{2\pi}\,\sigma_R} \exp\left[-\frac{(r-R)^2}{2\sigma_R^2}\right] \tag{5.74}$$

$n_T(r)$ is also a gaussian distribution. When numerous cities and towns are considered, the overall population distribution is expressed by the series of the gaussian distributions:

$$n_T(r) = \sum_\ell \frac{(N_T)_\ell}{\sqrt{2\pi}\,(\sigma_R)^2_\ell} \exp\left[-\frac{(r-R_\ell)^2}{2(\sigma_R)^2_\ell}\right] \tag{5.75}$$

where the subscript ℓ refers to a specific city or town.

Using the population distribution in Eq. (5.75) and an exponential function for the transfer function a(r), the first risk moment can be estimated to be:

$$M_1 = \int_0^\infty a(r) \cdot n_T(r)\, dr$$

$$= \int_0^\infty a_1 \cdot \exp[-a_2 \cdot r] \cdot \left\{ \sum_\ell \frac{(N_T)_\ell}{\sqrt{2\pi}\,(\sigma_R)_\ell} \exp\left[-\frac{(r - R_\ell)^2}{2(\sigma_R)_\ell^2}\right] \right\} dr$$

$$= \sum_\ell a_1 \cdot (N_T)_\ell \cdot \exp\left[-a_2 \cdot R_\ell + \frac{a_2^2 \cdot (\sigma_R)_\ell^2}{2}\right] \times$$

$$\times \int_0^\infty \frac{1}{\sqrt{2\pi}\,(\sigma_R)_\ell} \exp\left(-\frac{[r - R_\ell + a_2^2 \cdot (\sigma_R)_\ell^2]^2}{2(\sigma_R)_\ell^2}\right) dr \qquad (5.76)$$

The integral in Eq. (5.76) can be approximated by unity under the following conditions:

$$R_\ell > 2(\sigma_R)_\ell + a_2 \cdot (\sigma_R)_\ell^2 \qquad (5.77)$$

The discussion of this approximation is given in Appendix F. Then the first risk moment is finally estimated to be:

$$M_1 = \sum_\ell a_1 \cdot (N_T)_\ell \cdot \exp\left[-a_2 \cdot R_\ell + \frac{a_2^2 \cdot (\sigma_R)_\ell^2}{2}\right] \qquad (5.78)$$

The second risk moment and the normalization constant can be estimated in a similar manner. The estimation of these quantities are also discussed in Appendix F. Having obtained the first two risk moments and the normalization constant, the Weibull parameters can then be estimated by Eqs. (3.25) and (3.26). The comparison of the risk distributions derived from the bell-shaped population model to the results of the consequence calculation is also given in Appendix F.

Using the bell-shaped population model and the regression results, such as Eq. (5.78), the investigation can be made on the contributions of the cities and towns to the risk distribution. Alternatively, given the distances, radii and populations of the cities and towns, the decision making studies on selection of sites for nuclear power plants can be made from the regression results, such as Eq. (5.78) In the following section, a numerical example is given for siting studies.

V.8.2 Numerical Example of Siting

A hypothetical siting problem is discussed here. Though siting problems are generally two-dimensionsal, the situation given here is a one-dimensional case. The two-dimensional problems can be solved by the same approach as in the example here.

The problem is posed as follows:

(1) A nuclear power plant is planned on a line between two large cities A and D in Fig. 5.19. Two towns are located between them. The populations other than the above four are not considered.

(2) The cities and towns have bell-shaped population distributions and their distances, radii and populations are given in Fig. 5.19.

(3) Only the early fatalities are considered. The transfer functions previously derived for PWR accidents in the north-eastern valley meteorological condition are used.

(4) The site is desired to be selected so as to keep the first risk moment less than that for the average of the first 100 commercial power plants, which is 4.6×10^{-5}/reactor year. (See Section III.5.2.)

Set the origin of the axis at the center of the city A as shown in Fig. 5.19. The distance r of a site from the center of the city A is the variable that will be examined. As the site should be between A and D, the constraint is $0 < r < R_D$. The problem then is to estimate the value of r that keeps the first risk moment less than 4.6×10^{-5}/year

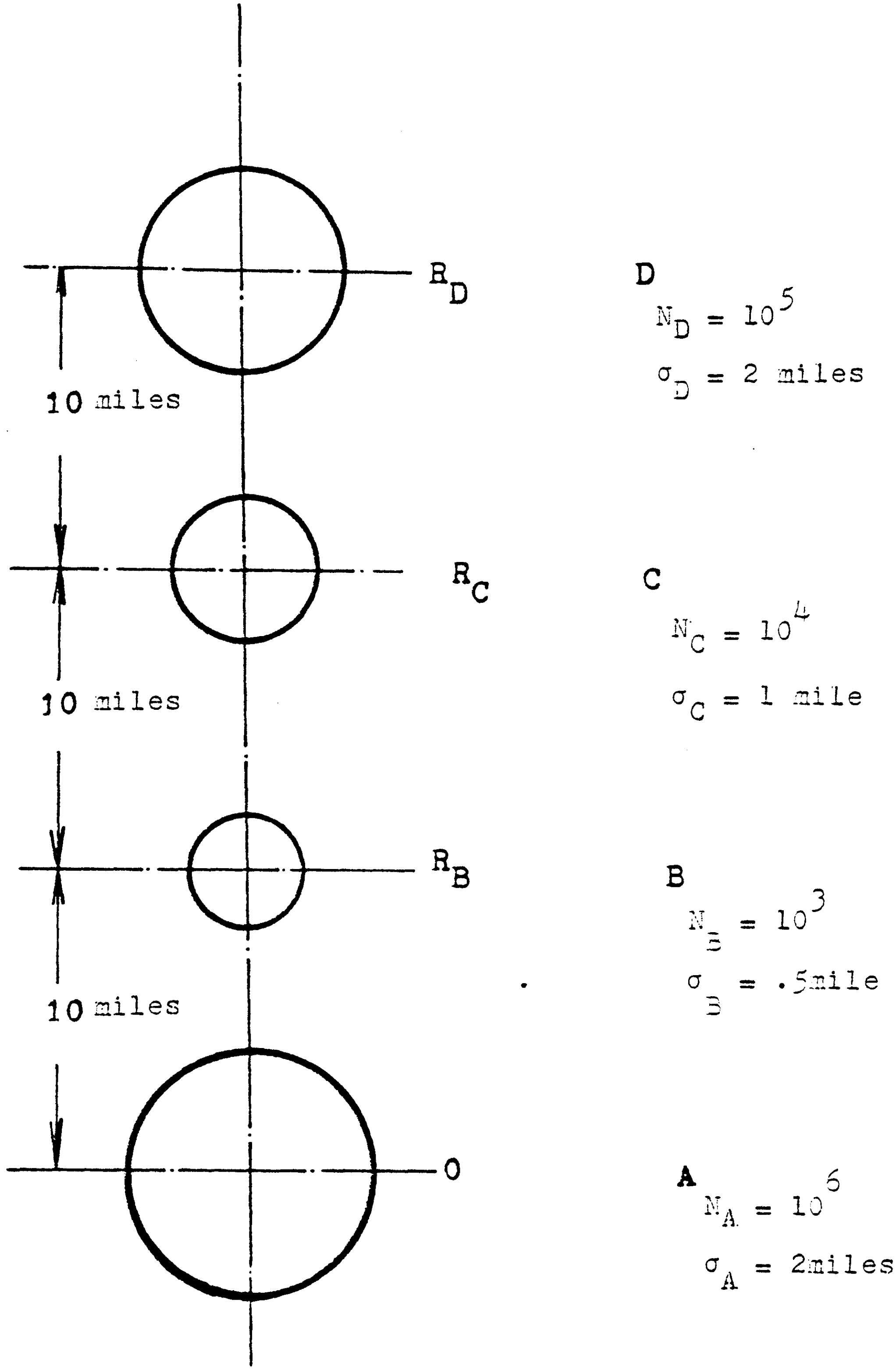

Fig. 5.19 Geometry for the Example Siting Problem

under the constraint of $0 < r < R_D$.

From Eq. (5.78), the first risk moment is estimated as the sum of the contributions of the four population groups:

$$\begin{aligned} M_1 = {} & N_A \cdot a_1 \cdot \exp\left[-a_2 \cdot r + \frac{a_2^2}{2} \cdot \sigma_A\right] \\ & + N_B \cdot a_1 \cdot \exp\left[-a_2 \cdot |r - R_B| + \frac{a_2^2}{2} \cdot \sigma_B\right] \\ & + N_C \cdot a_1 \cdot \exp\left[-a_2 \cdot |r - R_C| + \frac{a_2^2}{2} \cdot \sigma_C\right] \\ & + N_D \cdot a_1 \cdot \exp\left[-a_2 \cdot |r - R_D| + \frac{a_2^2}{2} \cdot \sigma_D\right] \end{aligned} \tag{5.79}$$

Using the numerical values in Fig. 5.19, and the constants of transfer functions estimated previously in Section V.6.2, the first risk moment is calculated and plotted in Fig. 5.20. The solid line in Fig. 5.20 shows the estimate of the first risk moments as a function of the distance from the center of the city A. The dashed lines show the contributions of each population group. From Fig. 5.20, the distances that satisfy the criteria are estimated to be:

$$13 \text{ miles} < r < 16 \text{ miles}$$

The plant can be selected within this area and will satisfy the imposed criteria.

Even though the example given here is highly restrictive, it shows the methods by which the approaches discussed in this study can be used in decision making involving risk. In more realistic situations, the second risk moment and the complementary cumulative distribution can be used to compare with additional risk criterial. Actual population distributions can also be used, perhaps involving numerical techniques and computer evaluations.

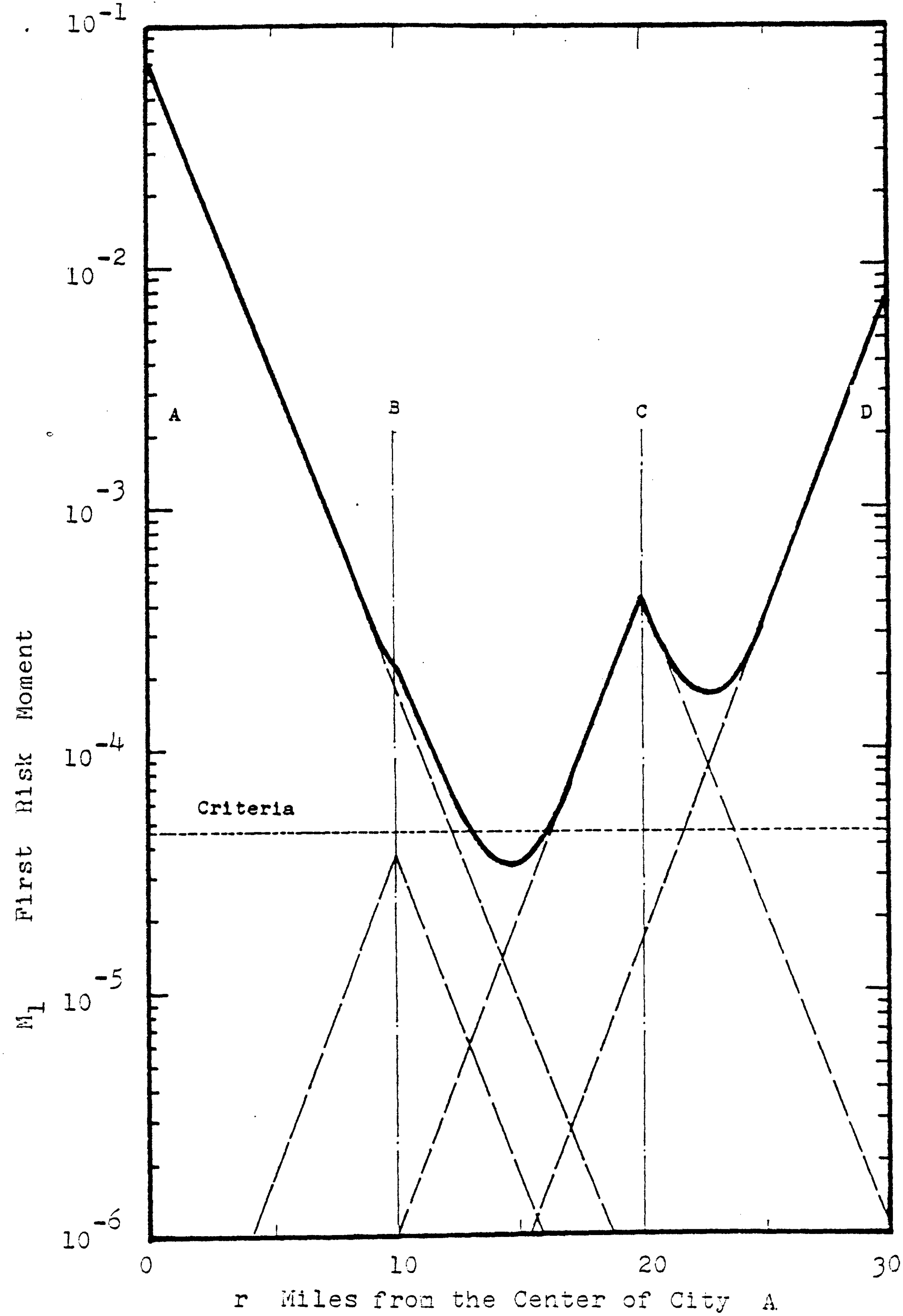

Fig.5.20 Estimate of the First Risk Moment for the Example Siting Problem

V.9 Summary and Conclusions

The regression approach discussed in Chapter 4 was demonstrated in this chapter in which the population distribution was taken to be the basic variable. The early fatalities distribution of PWR accidents in the northeastern valley meteorological condition was used to derive the regression results. In the regression analysis, the first two risk moments and the normalization constant were selected as dependent variables. The data base for the regression analysis was prepared by the consequence computer program using the population distributions of the 68 sites as sample population distributions.

A number of candidate regression equations were studied. The following were judged to be adequate:

$$M_1 = \sum_j \int_r a_1 \exp(-a_2 r)\, n_j(r)\, dr \qquad (5.80)$$

$$M_2 = \sum_j \int_r \int_{r'} b_1 \exp[-b_2(r + r')] \exp[b_3 |r - r'|] \cdot n_j(r)\, n_j(r)\, dr\, dr' \qquad (5.81)$$

$$\alpha = \sum_j [c_1 \exp(-c_2 r)]_{r=d_j} \qquad (5.82)$$

The unknown constants in the equations above were estimated by the nonlinear least squares. The derived equations were tested for the predicted risk characteristics and for the predicted distribution behaviors. No systematic errors were observed for the risk characteristics and for the shape and scale factors of the Weibull distribution. The distributions of consequence vs. frequency derived from the regression equations agreed with the results of the consequence calculation within the uncertainty range of the consequence model.

Having obtained the regression results, they can be applied to new situations for sensitivity studies and decision making investigations. Because of the simple form of the regression equations, the involved calculations are straightforward and do not require consequence code or large computer times. With regard to the new situations, the regression equations were applied to a hypothetical example of decision making involving siting. The location of a site which satisfy the specified criteria was obtained from the regression equations. The example illustrated how the approach of the study can be used in decision making.

CHAPTER VI

REGRESSION ANALYSIS OF RADIOACTIVE RELEASE

VI.1 Introduction

The methods developed in this study will be applied to another evaluation situation in which the probabilities and magnitudes of radioactive releases are taken as the basic variables. The situation considered in this chapter concerns the evaluation of safety systems in nuclear power plants, involving engineering safety features, operation restrictions and maintenance procedures. Safety systems in nuclear power plants are designed to reduce the probabilities of the occurrences of the accidents or alternatively to reduce the magnitudes of the releases to tne environment. The equations relating the risk to the probabilities and magnitudes of the radioactive releases can then provide valuable information for the evaluations of safety systems.

In the Reactor Safety Study, the spectrum of the radioactive releases was expressed by the release categories shown in Table 6.1. These release categories are composites of numerous accident sequences with similar characteristics. PWR accidents are represented by 9 release categories[(1)] and BWR accidents are represented by 5 release categories. In the preceding chapters of this thesis, the consequence calculation has been carried out for each of the release categories and the results have been combined to produce the overall risk from potential nuclear accidents. In this chapter each release category is

[1]Since PWR-1 category is subdivided into PWR-1A and PWR-1B due to the difference of energy release, PWR accidents are effectively represented by 10 release categories.

Table 6.1 Summary of Accidents Involving Core

RELEASE CATEGORY	PROBABILITY per Reactor-Yr	TIME OF RELEASE (Hr)	DURATION OF RELEASE (Hr)	WARNING TIME FOR EVACUATION (Hr)	ELEVATION OF RELEASE (Meters)	CONTAINMENT ENERGY RELEASE (10^6 Btu/Hr)	FRACTION OF CORE INVENTORY RELEASED[a]							
							Xe-Kr	Org. I	I	Cs-Rb	Te-Sb	Ba-Sr	Ru[b]	La[c]
PWR 1	9×10^{-7}	2.5	0.5	1.0	25	520[d]	0.9	6×10^{-3}	0.7	0.4	0.4	0.05	0.4	3×10^{-3}
PWR 2	8×10^{-6}	2.5	0.5	1.0	0	170	0.9	7×10^{-3}	0.7	0.5	0.3	0.06	0.02	4×10^{-3}
PWR 3	4×10^{-6}	5.0	1.5	2.0	0	6	0.8	6×10^{-3}	0.2	0.2	0.3	0.02	0.03	3×10^{-3}
PWR 4	5×10^{-7}	2.0	3.0	2.0	0	1	0.6	2×10^{-3}	0.09	0.04	0.03	5×10^{-3}	3×10^{-3}	4×10^{-4}
PWR 5	7×10^{-7}	2.0	4.0	1.0	0	0.3	0.3	2×10^{-3}	0.03	9×10^{-3}	5×10^{-3}	1×10^{-3}	6×10^{-4}	7×10^{-5}
PWR 6	6×10^{-6}	12.0	10.0	1.0	0	N/A	0.3	2×10^{-3}	8×10^{-4}	8×10^{-4}	1×10^{-3}	9×10^{-5}	7×10^{-5}	1×10^{-5}
PWR 7	4×10^{-5}	10.0	10.0	1.0	0	N/A	6×10^{-3}	2×10^{-5}	2×10^{-5}	1×10^{-5}	2×10^{-5}	1×10^{-6}	1×10^{-6}	2×10^{-7}
PWR 8	4×10^{-5}	0.5	0.5	N/A	0	N/A	2×10^{-3}	5×10^{-6}	1×10^{-4}	5×10^{-4}	1×10^{-6}	1×10^{-8}	0	0
PWR 9	4×10^{-4}	0.5	0.5	N/A	0	N/A	3×10^{-6}	7×10^{-9}	1×10^{-7}	6×10^{-7}	1×10^{-9}	1×10^{-11}	0	0
BWR 1	1×10^{-6}	2.0	2.0	1.5	25	130	1.0	7×10^{-3}	0.40	0.40	0.70	0.05	0.5	5×10^{-3}
BWR 2	6×10^{-6}	30.0	3.0	2.0	0	30	1.0	7×10^{-3}	0.90	0.50	0.30	0.10	0.03	4×10^{-3}
BWR 3	2×10^{-5}	30.0	3.0	2.0	25	20	1.0	7×10^{-3}	0.10	0.10	0.30	0.01	0.02	3×10^{-3}
BWR 4	2×10^{-6}	5.0	2.0	2.0	25	N/A	0.6	7×10^{-4}	8×10^{-4}	5×10^{-3}	4×10^{-3}	6×10^{-4}	6×10^{-4}	1×10^{-4}
BWR 5	1×10^{-4}	3.5	5.0	N/A	150	N/A	5×10^{-4}	2×10^{-9}	6×10^{-11}	4×10^{-9}	8×10^{-12}	8×10^{-14}	0	0

(a) A discussion of the isotopes used in the study is found in Appendix VI. Background on the isotope groups and release mechanisms is found in Appendix VII.

(b) Includes Mo, Rh, Tc, Co.

(c) Includes Nd, Y, Ce, Pr, La, Nb, Am, Cm, Pu, Np, Zr.

(d) A lower energy release rate than this value applies to part of the period over which the radioactivity is being released. The effect of lower energy release rates on consequences is found in Appendix VI.

Note: Reproduced from TABLE 5-1 in Main Report of WASH-1400(Ref-1) treated separately to study the consequences of a specific release.

VI.2 Radioactive Release Variables

The regressor variables are identified from the characteristics of radioactive releases. Though the probability and magnitude are major characteristics of releases, other chacteristics also affect the consequences of radioactive releases, i.e., the time of the release, the duration of the release, the warning time for evacuation, the elevation of the release, and the energy content in the released plume. Table 6.1 shows the characteristics of the release categories of PWR and BWR accidents taken from WASH-1400 (Ref-1). These release data are used to generate the data base for the regression analysis. Each of the variables that characterize the radioactive releases will be discussed in the following subsections.

VI.2.1 Probability of Occurrence

Since the probability of occurrence does not affect the magnitude of consequences, the distribution $f_q(x)$ of consequence vs. frequency for a specific release q is expressed as the product of the probability of occurrence P_q and the conditional distribution $f_q^*(x)$ given the release q occurs.

$$f_q(x) = P_q \cdot f_q^*(x) \tag{6.1}$$

The regression analysis is based on the conditional distribution $f_q^*(x)$ and the probability of occurrence is therefore not included in the regressor variables.

VI.2.2 Time of Release

The time of the release refers to the time interval between the start of the accident and the release of the radioactive materials from the containment building to the atmosphere. The time of the release is used to calculate the initial decay of the radioactivity. Since increasing times reduce the amount of radioactivity released to the environment, the variable is included in the expression for effective source which will be defined in subsection VI.2.7. The time of the release is denoted by (T_r) hours.

VI.2.3 Duration of Release

The duration of the release is the total time during which the radioactive materials are emitted into the atmosphere. The duration is used to make it possible to account for the wind meander in long duration releases. The duration is denoted by (T_d) hours in the following equations.

VI.2.4 Warning Time for Evacuation

The warning time is the time interval between the awareness of impending core melt and the release of radioactive materials from the containment building. A longer warning time allows more time to evacuate the public to areas where the radiation exposure will be smaller or none. This variable is denoted by (T_w) hours in the regression equations.

VI.2.5 Elevation of Release

The elevation of release affects the dispersion pattern of airborne radioactive isotopes in the atmosphere. As the elevation increases, the maximum airborne concentration of radioactivity at the ground level decreases. The variable is denoted by (h) meters in the regression equations.

VI.2.6 Energy Content of Release

When the containment of a reactor breaks, a large amount of energy may be released with the radioactive isotopes in a form of high temperature steam. When the gas is at a high temperature, the radioactive plume will rise due to its buoyancy. The variable is denoted by (E) $10^6 \times$Btu/hr in the regression equations.

VI.2.7 Release Fractions

From the large number of isotopes produced in a reactor, 54 radioisotopes were assessed to be of importance in the Reactor Safety Study. The selection was based on quantities (curies), release fractions, radioactive half-lives, emitted radiation types and chemical characteristics. The 54 selected isotopes were grouped into 8 isotope groups based on their chemical behaviors. The release fractions of the core inventories were determined for the 8 isotope groups as given in Table 6.1.

Two approaches for selection of regressor variables are considered with regard to the release fractions. One is to select the release fractions of the eight isotope groups as the basic regressor variables. The isotope groups which have insignificant effect on the consequence can be eliminated, for example, by the stepwise regression method which was discussed in Section IV.2.6. A second approach is to define one variable which is a weighted sum of the release fractions of the eight isotope groups. In this study, the second approach is selected from the following reasons:

(1) Early fatalities are caused by the combined effects of the doses from the eight isotope groups. The decrease of the release fraction of one isotope group can be compensated by the increases of the releases of the other isotope groups.

(2) The release fractions of the eight groups are correlated with each other. For example, in Table 6.1 the release fractions of all of the eight isotope groups for PWR-9 release category are smaller than those for PWR-1 release category, because similar physical processes underly in the release mechanism for all of the isotope groups.

The weighting factors of the release fractions are derived from the physical consideration of the effects on early fatalities. The factors considered are the inventories in the core, the dose conversion factors and the dose-response factors. Since the early fatalities result essentially from the damage to three organs, the weighting factors are first defined for each organ. The organs considered are bone-marrow, lung and gastrointestinal tract. The weighting factor of isotope group (g) for organ (k) is defined to be:

$$\Omega_g^{(k)} = \sum_{j \text{ in group } g} \frac{I_j \cdot \exp[-\lambda_j \cdot T_r] \cdot C_j^{(k)}}{(LD)_{50}^{(k)}} \qquad (6.2)$$

where

$\Omega_g^{(k)}$ = weighting factor of group (g) for organ (k).

I_j = inventory of isotope (j) in the core [curies].

λ_j = radioactive decay constant of isotope (j) [/hour].

T_r = time of release [hour].

$C_j^{(k)}$ = dose conversion factor of isotope (j) to organ (k) $[rem \cdot m^3/C_i - sec]$

$(LD)_{50}^{(k)}$ = dose to organ (k) lethal to 50% of the exposed population [rem]

When the contribution of the build-up from the parent isotope is significant, the radioactive decay term $\exp[-\lambda_j \cdot T_r]$ is corrected to include the build-up term from the parent isotope.

The dose conversion factor in Eq. (6.2) is the sum of the three modes of exposure, which are inhalation dose, ground shine dose and cloud shine dose.

$$C_j^{(k)} = B \cdot (C_I)_j^{(k)} + s_C \cdot (C_C)_j^{(k)} + s_G \cdot (C_G)_j^{(k)} \cdot (V_d)_j \qquad (6.3)$$

where

B = breathing rate $[m^3/sec]$.

$(C_I)_j^{(k)}$ = inhalation dose conversion factor of isotope (j) to organ (k) $[rem/C_i]$.

s_C = shielding factor for cloud shine dose.

$(C_C)_j^{(k)}$ = cloud shine dose conversion factor of isotope (j) to organ (k) $[rem \cdot m^3/C_i \cdot sec]$.

s_G = shielding factor for ground shine dose.

$(C_G)_j^{(k)}$ = ground shine dose conversion factor of isotope (j) to organ (k) $[rem \cdot m^2/C_i]$.

$(V_d)_j$ = deposition velocity of isotope (j) [m/sec].

The effective source for organ (k) is then defined by the sum of the release fractions weighted by the factors $\Omega_g^{(k)}$.

$$\psi^{(k)} = \sum_g \Omega_g^{(k)} \cdot q_g \qquad (6.4)$$

where

$\psi^{(k)}$ = effective source for organ (k) [sec/m^3].

q_g = release fraction of isotope group (g).

The quantity $\psi^{(k)}$ can be interpreted as being related to the inverse of the atmospheric dispersion factor (χ/Q) at a distance where 50% of the exposed population die due to the damage to the organ (k). The weighting factors $\Omega_g^{(k)}$ are given in Table 6.2. The discussion on the basis for the definition of the weighting factors and the source data used for deriving the values in Table 6.2 are given in Appendix G.

Since the risks resulting from the damage to the three organs are competing with each other, the overall effective source is defined by the maximum value of the ($\psi^{(k)}$)'s of the three organs.

$$\psi = \text{Max}\left\{\psi^{\text{MARROW}},\ \psi^{\text{LUNG}},\ \psi^{\text{G.I.}}\right\} \qquad (6.5)$$

The overall effective source defined above is used in this study as the regressor variable. It is denoted by (ψ) $\times 10^5$ sec/m^3 in the regression equations.

VI.3 Selection of the Dependent Variables

As discussed in Section VI.2.1, the regression analysis is based on the conditional risk distribution given the specific release occurrence. The risk characteristics of the conditional risk distribu-

Table 6.2 Weighting Factors of Isotope Groups for Effective Source

Organ	Isotope Group	Weighting Factor Ω_g
Bone Marrow	Kr - Xe	$5.73 \times 10^3 + 7.90 \times 10^4 \exp[-.20 \cdot Tr]$
	I[(1)]	$7.81 \times 10^5 \exp[-.058 \cdot Tr]$
	Cs - Rb	5.64×10^4
	Te - Sb	2.54×10^5
	Ba - Sr	5.01×10^5
	Ru	2.28×10^5
	La	1.77×10^6
Lung	Kr - Xe	$1.21 \times 10^2 + 1.6 \times 10^3 \exp[-.20 \cdot Tr]$
	I[(1)]	$3.35 \times 10^4 \exp[-.058 \cdot Tr]$
	Cs - Rb	7.43×10^3
	Te - Sb	6.83×10^4
	Ba - Sr	3.32×10^4
	Ru	9.53×10^5
	La	4.28×10^6
G.I. Tract	Kr - Xe	$4.18 \times 10^2 + 8.2 \times 10^3 \exp[-.20 \cdot Tr]$
	I[(1)]	$7.70 \times 10^4 \exp[-.058 \cdot Tr]$
	Cs - Rb	4.08×10^3
	Te - Sb	6.18×10^4
	Ba - Sr	1.69×10^5
	Ru	2.92×10^5
	La	1.53×10^6

[1]Organic iodines and non-organic iodines are included.

tion are defined in a similar manner to those of the overall risk distribution given in Section I.2. For example, the risk moments of the conditional risk distribution about the origin are defined as:

$$M_t^* = \int x^t \cdot f^*(x) \cdot dx \qquad (6.6)$$

where M_t^* is the t-th risk moments of the conditional risk distribution about the origin.

The normalization constant of the conditional risk distribution α^* is similarly defined as:

$$\alpha^* = \int f^*(x) \cdot dx \qquad (6.7)$$

The transfer functions relating the risk moments to the population variables are also re-defined based on the conditional risk distribution as:

$$M_1^* = \sum_j \int_r a^*(r) \cdot n_j(r) \cdot dr \qquad (6.8)$$

$$M_2^* = \sum_j \int_r \int_{r'} b^*(r,r') \cdot n_j(r) \cdot n_j(r') \cdot dr \cdot dr' \qquad (6.9)$$

$$\alpha^* = \sum_j [c^*(r)]_{r=d_j} \qquad (6.10)$$

The dependent variables of the regression analysis can be selected from the risk characteristics of the conditional risk distribution. In this chapter the transfer functions are again fitted to the parametric functions of the distance r and the constants of the fitted functions are used as dependent variables. The constants are now treated as being functions of the release characteristics. The advantage of the constants of the transfer functions is their independence of the specific population distribution. Therefore the results of the regression analysis are applicable to any population distribution.

VI.4 The Data Base for Regression Analysis

VI.4.1 Input Conditions

The release categories of PWR and BWR accidents in Table 6.1 are used as samples of radioactive releases for the regression analysis. A consequence calculation is made for each of the release categories using the northeastern valley meteorological condition and the radioactive inventories of a 3200 MW/th power plant. Early fatalities occur only in eight out of the fifteen release categories. Since eight samples are not sufficient as the data base for the analysis, an additional 20 cases are calculated by changing one regressor variable at a time in the consequence program. The input conditions of the additional calculations are given in Table 6.3. The total 28 cases of calculation are performed. It should be noted that the probabilities of occurrence are assumed to be unity in the calculations in Table 6.3 since the regression analysis is based on the distribution of consequence vs. conditional probability given the accident occurrence.

VI.4.2 Derivation of the Constants of the Transfer Functions

The methods discussed in Section V.6.1 are used to derive the forms and the constants of the transfer functions. Figs. 6.1 through 6.4 show the consequence calculation results for BWR-1, BWR-2 and BWR-3 release categories. The following candidate functions are considered for these curves. They are the same functions that were considered for the PWR accidents in Chapter V.

$$a^*(r) = a_1 \cdot \exp\,[-a_2 \cdot r] \qquad (6.11)$$

Table 6.3 Conditions of Additional Consequence Calculations for Regression Analysis

Case No.	Time of Release (hr)	Duration of Release (hr)	Warning Time for Evacuation (hr)	Elevation of Release (m)	Energy Release (10^6 Btu/hr)	Release Fractions			
						I	Ru	Te	others[1]
1	2.0	0.5	1.5	25	300	.4	.5	.7	BWR-1
2	2.0	0.5	1.5	25	30	.4	.5	.7	BWR-1
3	2.0	0.5	1.5	25	6	.4	.5	.7	BWR-1
4	30.0	3.0	2.0	10	6	.9	.03	.3	BWR-2
5	2.0	0.5	.5	25	130	.4	.5	.7	BWR-1
6	2.0	0.5	1.0	25	130	.4	.5	.7	BWR-1
7	2.0	0.5	2.0	25	130	.4	.5	.7	BWR-1
8	2.0	0.5	3.0	25	130	.4	.5	.7	BWR-1
9	2.5	0.5	2.0	25	520	.7	.4	.4	PWR-1
10	2.5	0.5	3.0	25	520	.7	.4	.4	PWR-1
11	2.5	0.5	2.0	25	20	.7	.4	.4	PWR-1
12	2.5	0.5	3.0	25	20	.7	.4	.4	PWR-1
13	2.0	1.5	1.5	25	130	.4	.5	.7	BWR-1
14	2.0	3.0	1.5	25	130	.4	.5	.7	BWR-1
15	2.0	0.5	1.5	25	130	.1	.5	.7	BWR-1
16	2.0	0.5	1.5	25	130	1.0	.5	.7	BWR-1
17	2.0	0.5	1.5	25	130	.4	.1	.7	BWR-1
18	2.0	0.5	1.5	25	130	.4	1.0	.7	BWR-1
19	2.0	0.5	1.5	10	130	.4	.5	.7	BWR-1
20	2.0	0.5	1.5	1	130	.4	.5	.7	BWR-1

[1]The release fractions of the other isotopes are the same as in the release categories given here.

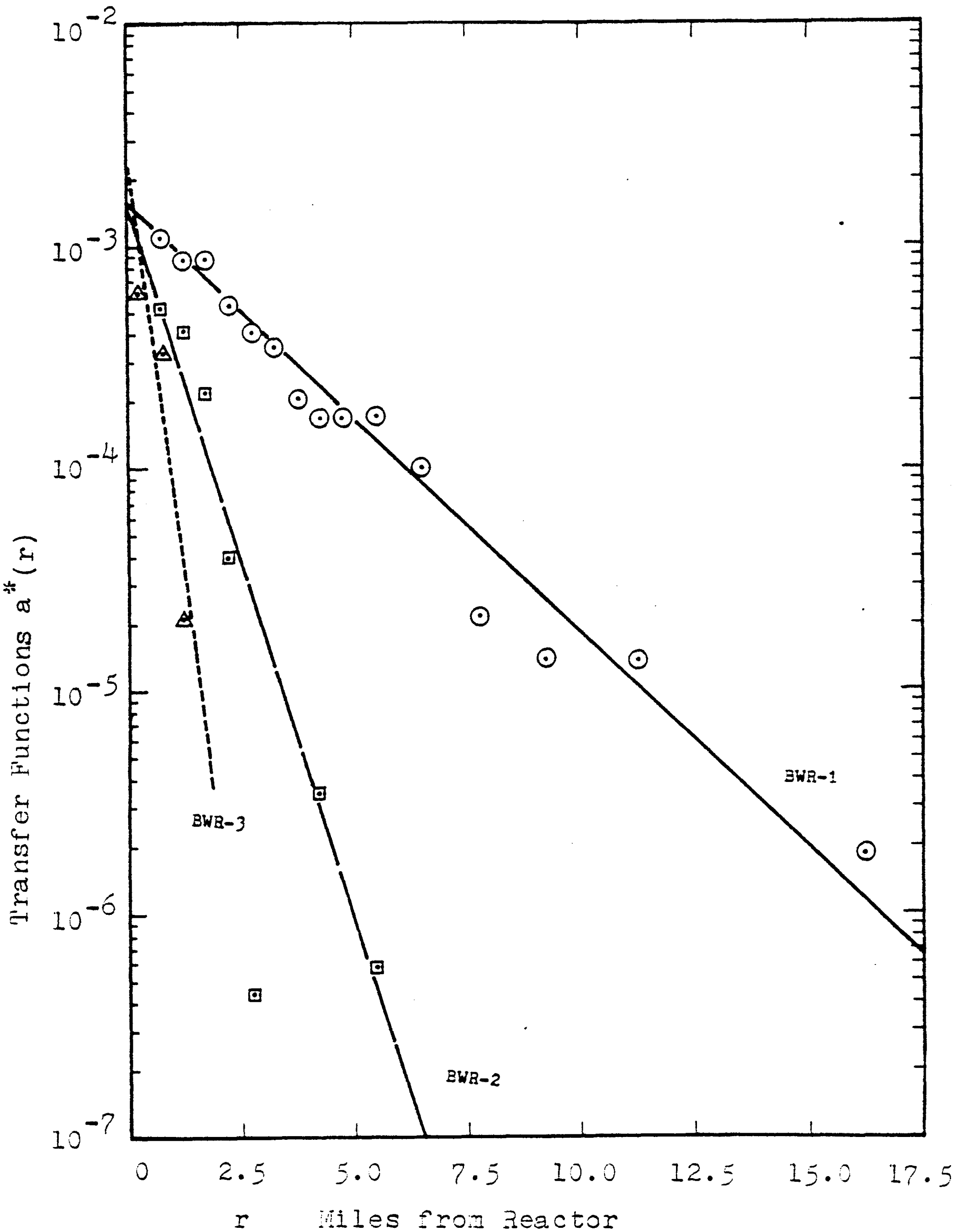

Fig. 6.1 Transfer Functions $a^*(r)$ for the BWR Release Categories.

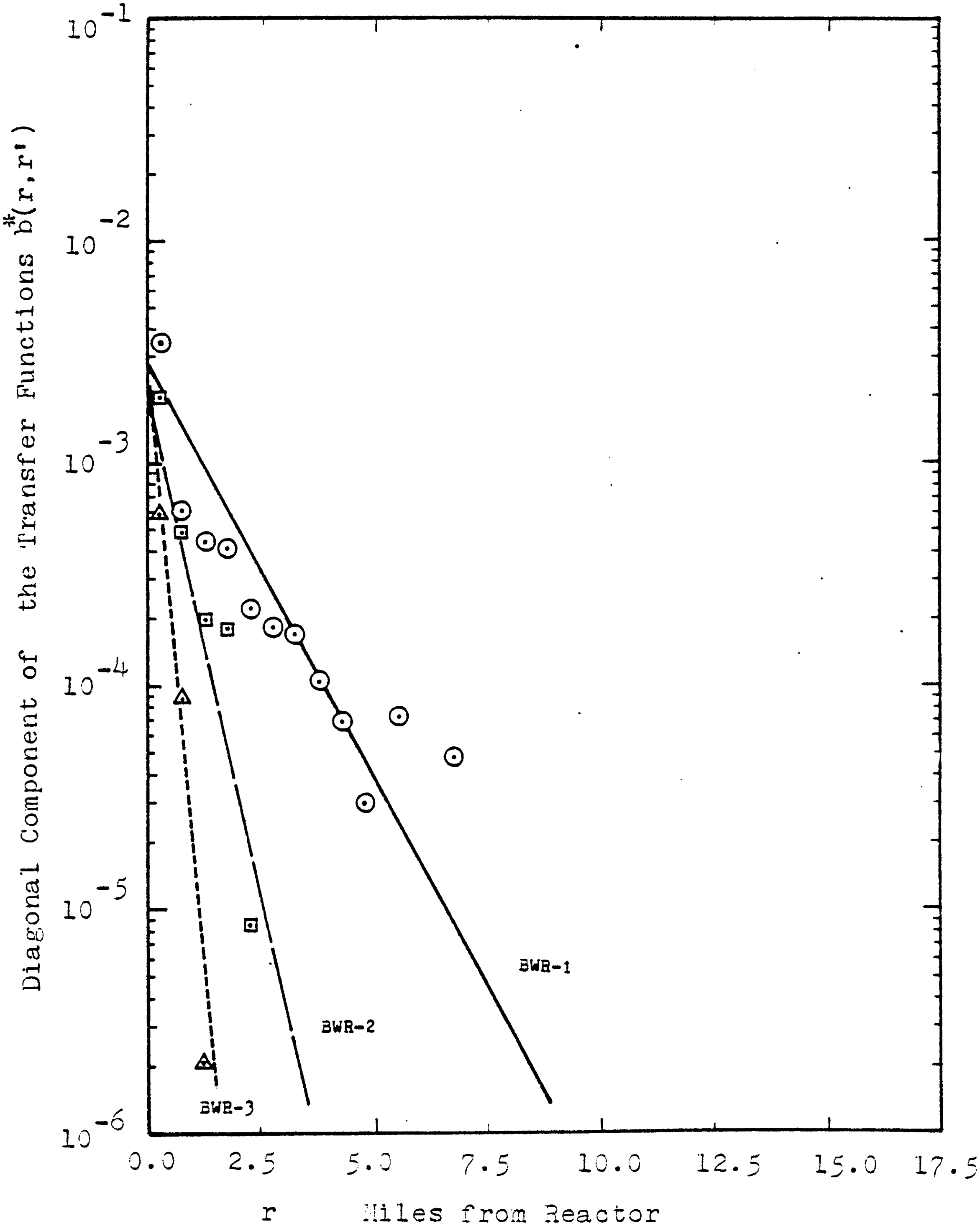

Fig.6.2 Diagonal Components of the Transfer Functions $b^*(r,r')$ for BWR Release Categories

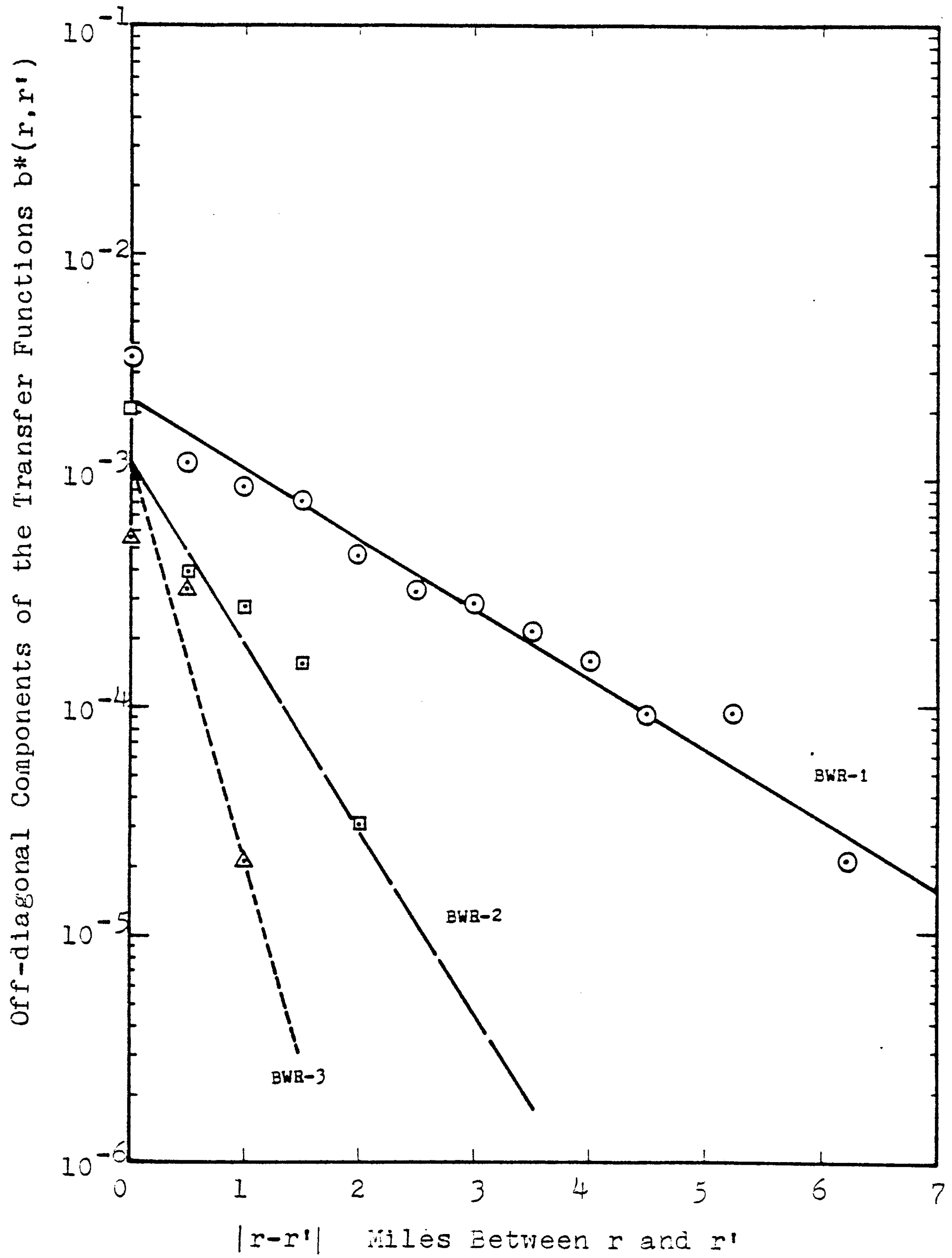

Fig.6.3 Off-diagonal Components of the Transfer Functions b*(r,r') at r=0.25 mile for the BWR Release Categories

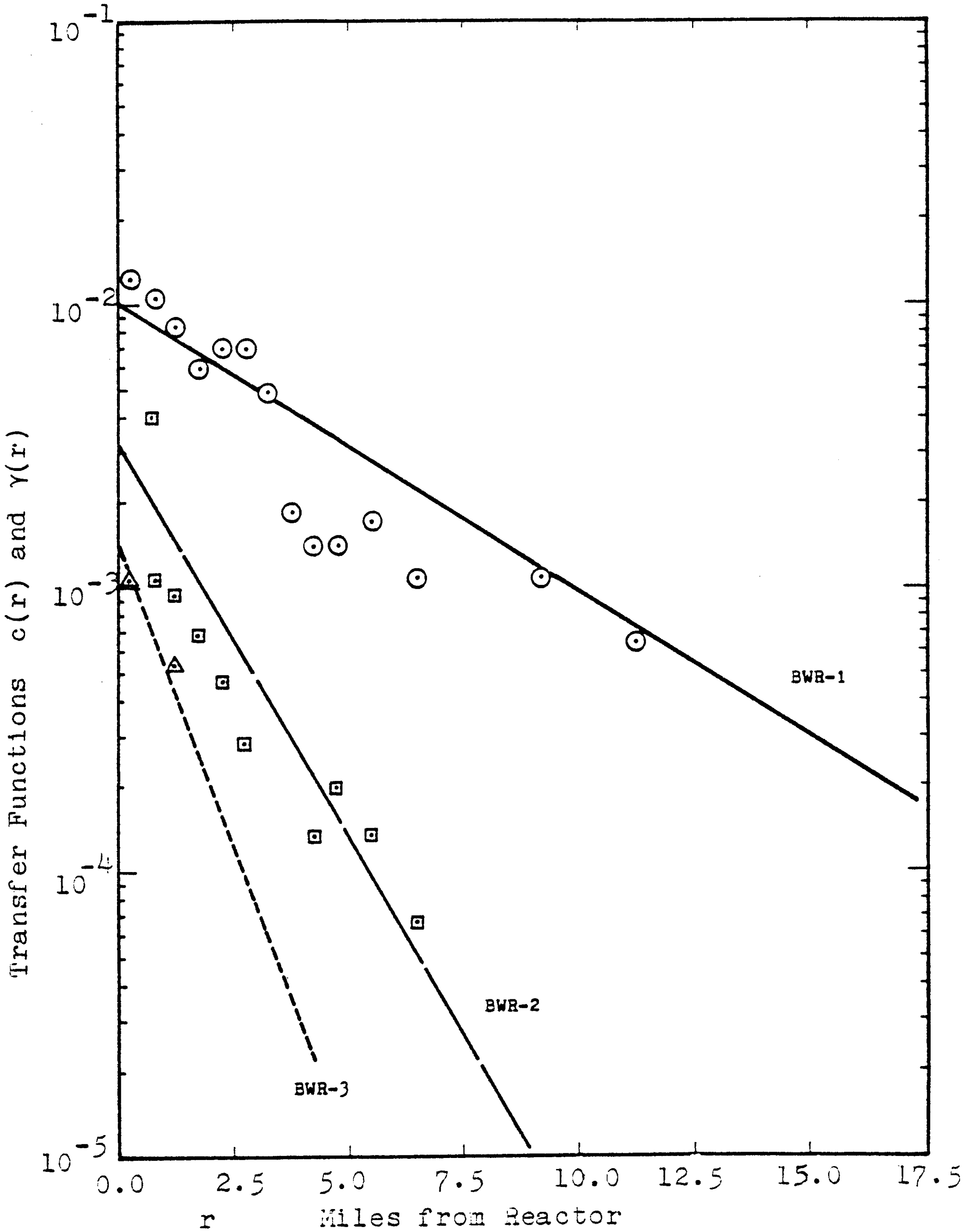

Fig.6.4 Transfer Functions $c^*(r)$ and $\gamma^*(r)$ for the BWR Release Categories

Note: The lines show the estimates of $c^*(r)$.
The dots show $\gamma^*(r)$, which are approximations of $c^*(r)$.

Table 6.4 Estimates of a_1 and a_2 as the Data Base for the Regression of the Release Variables

Calculation Case		a_1	a_2
PWR - 1A		9.13×10^{-3}	.437
PWR - 1B		1.85×10^{-3}	.562
PWR - 2		1.73×10^{-3}	.512
PWR - 3		1.40×10^{-2}	1.600
PWR - 4		3.38×10^{-2}	3.390
BWR - 1		1.66×10^{-3}	.451
BWR - 2		3.30×10^{-3}	1.66
BWR - 3		3.50×10^{-3}	3.76
Additional Cases[(1)]:	1	1.57×10^{-3}	.504
	2	5.75×10^{-3}	.403
	3	1.87×10^{-2}	.397
	4	1.21×10^{-2}	1.401
	5	2.11×10^{-3}	.468
	6	1.87×10^{-3}	.458
	7	1.47×10^{-3}	.460
	8	1.28×10^{-3}	.449
	9	1.49×10^{-3}	.546
	10	1.29×10^{-3}	.535
	11	7.77×10^{-3}	.442
	12	6.93×10^{-3}	.432
	13	2.15×10^{-3}	.618
	14	2.48×10^{-3}	.737
	15	1.63×10^{-3}	.522
	16	1.55×10^{-3}	.301
	17	2.23×10^{-3}	.661
	18	2.29×10^{-3}	.323
	19	3.10×10^{-3}	.489
	20	3.22×10^{-3}	.505

[1] Corresponding to the calculation case number in Table 6.3.

$$b^*(r,r') = b_1 \cdot \exp\,[-b_2 \cdot (r+r')] \cdot \exp\,[-b_3 \cdot |r-r'|] \quad (6.12)$$

$$c^*(r) = c_1 \cdot \exp\,[-c_2 \cdot r] \quad (6.13)$$

For the other releases, the same exponential functions are considered. The estimates of a_1 and a_2 for the 28 cases are given in Table 6.4. The estimates of the other constants b_1, b_2, b_3, c_1 and c_2 are given in Appendix H. The estimates of the constants are used as the data base for the regression analysis.

VI.5 Formulation of the Regression Model

The next step is to select candidate equations that relate the dependent variables a_1, a_2, b_1, b_2, b_3, c_1 and c_2 to the regressor variables discussed in Section VI.3. The analysis of the dependent variable a_1 is discussed in detail. The results of the other regressions are mostly briefly presented.

The following points are considered in the selection of the candidate questions:

(1) The values of the dependent variables a_1, a_2, b_1, b_2, b_3, c_1 and c_2 are positive.

(2) The equations should have as few unknown constants as possible which still adequately fit the distributions of the dependent variables.

(3) The equations with smaller sum of the residual squares and no significant systematic error are desirable.

The relation of the dependent variables and each of the regressor variables is studied first. Table 6.5 shows the correlation co-efficient between the dependent variable a_1 and each of the regressor variables. Linear and natural logarithmic transformations are investigated. To have smaller sum of the residual squares, the transformation that gives the largest correlation co-efficient is preferred. Except for the elevation term (h), the natural logarithmic transformations of the regressor variables give larger correlation co-efficients than the linear transformations. Even for the elevation term, the difference of the correlation co-efficients between the two transformations of the regressor variable (h) is less than 0.1. To keep the model as simple as possible, natural logarithmic forms are selected for all of the regressor variables. Since the dependent variable a_1 should be positive, the following regression model is considered:

$$\ln a_1 = k_{01} + k_{11} \cdot \ln h + k_{21} \cdot \ln T_w + k_{31} \cdot \ln T_d + + k_{41} \cdot \ln E + k_{51} \cdot \ln \psi + \varepsilon_1 \quad (6.14)$$

where $k_{01}, \ldots, k_{51}$ are constants to be derived and ε_1 is the random error variable. Eq. (6.14) does not include the interaction terms. Possible interactions will be tested later.

The candidate equations of the other dependent variables are selected in a similar process. The following equations are thus considered in this study:

$$\ln a_2 = k_{02} + k_{12} \cdot \ln h + k_{22} \cdot \ln T_w + k_{32} \cdot \ln T_d + + k_{42} \cdot \ln E + k_{52} \cdot \ln \psi + \varepsilon_2 \quad (6.15)$$

$$\ln b_1 = k_{03} + k_{13} \cdot \ln h + k_{23} \cdot \ln T_w + k_{33} \cdot \ln T_d + + k_{43} \cdot \ln E + k_{53} \cdot \ln \psi + \varepsilon_3 \quad (6.16)$$

Table 6.5 Correlation Coefficients of a_1 and Regressor Variables

Dependent Variable	Regressor Variable	Correlation Coefficient
a_1	h	-.466
	ln h	-.391
ln a_1	h	-.496
	ln h	-.453
a_1	T_w	.107
	ln T_w	.153
ln a_1	T_w	.071
	ln T_w	.126
a_1	T_d	.380
	ln T_d	.382
ln a_1	T_d	.362
	ln T_d	.371
a_1	E	-.445
	ln E	-.837
ln a_1	E	-.597
	ln E	-.882
a_1	ψ	-.501
	ln ψ	-.569
ln a_1	ψ	-.466
	ln ψ	-.492

(Note): h = elevation of release (m)

T_w = warning time for evacuation (hour)

T_d = duration of release (hour)

E = energy release (10^6 Btu/hr)

ψ = effective source (10^5 m^3/sec)

$$\ln b_2 = k_{04} + k_{14} \cdot \ln h + k_{24} \cdot \ln T_w + k_{34} \cdot \ln T_d + + k_{44} \cdot \ln E + k_{54} \cdot \ln \psi + \varepsilon_4 \tag{6.17}$$

$$\ln b_3 = k_{05} + k_{15} \cdot \ln h + k_{25} \cdot \ln T_w + k_{35} \cdot \ln T_d + + k_{45} \cdot \ln E + k_{55} \cdot \ln \psi + \varepsilon_5 \tag{6.18}$$

$$\ln c_1 = k_{06} + k_{16} \cdot \ln h + k_{26} \cdot \ln T_w + k_{36} \cdot \ln T_d + + k_{46} \cdot \ln E + k_{56} \cdot \ln \psi + \varepsilon_6 \tag{6.19}$$

$$\ln c_2 = k_{07} + k_{17} \cdot \ln h + k_{27} \cdot \ln T_w + k_{37} \cdot \ln T_d + + k_{47} \cdot \ln E + k_{57} \cdot \ln \psi + \varepsilon_7 \tag{6.20}$$

where k's are unknown constants and ε's are random error variables.

VI.6 Derivation of the Constants of the Regression Equations

In the previous population regressions a small number of unknowns were involved. Because of the larger number of terms in the regression equations considered here, stepwise regression analysis is used to eliminate the terms which have insignificant effect on the variation of the dependent variables. In the stepwise regression, a partial F-statistic is used to eliminate the terms of insignificant effects, as discussed in Section IV.2.6. The linear multiple regression program in the DCRT Mathematical and Statistical Package of National Institute of Health (Ref-9) is used to calculate the F-values. An upper 10% level is selected as the criterion of elimination of the insignificant terms. Table 6.6 shows the process of elimination in the regression equation of $(\ln a_1)$. The calculated F-value of the warning time term $(\ln T_w)$ is smaller than the upper 10% F-value with (1,22) degrees of freedom. The term $(\ln T_w)$ can then be eliminated. Then the equation without $(\ln T_w)$ is tested.

$$\ln a_1 = k_{01} + k_{11} \cdot \ln h + k_{31} \cdot \ln T_d + k_{41} \cdot \ln E + k_{51} \cdot \ln \psi + \varepsilon_1 \tag{6.21}$$

The partial F-value is calculated again. Similarly, the term $(\ln T_d)$ can also be eliminated. The elimination process is terminated when the partial F-values for the remaining variables are larger than the 10% level. For example, the partial F-value of $(\ln \psi)$ shown in Table 6.6 is larger and hence is not eliminated. Additional t-tests are also made, as shown in Table 6.7, to help assure that the remaining terms cannot be eliminated.

From the stepwise regression, the final derived equation of $\ln a_1$ is thus:

$$\ln a_1 = -2.56 - .53 \ln E - .46 \ln h - .40 \ln \psi \tag{6.22}$$

Interaction terms are then considered by adding the product terms to Eq. (6.22). For example, to consider the interaction of $(\ln h)$ and $(\ln \psi)$ the following equation is studied:

$$\ln a_1 = k'_{01} + k'_{11} \cdot \ln E + k'_{21} \cdot \ln h + k'_{31} \cdot \ln \psi + {} + k'_{41} \cdot \ln \psi \cdot \ln h + \varepsilon'_1 \tag{6.23}$$

where $k'_{01}, \ldots, k'_{41}$ are constants and ε'_1 is the random error variable. Partial F-tests are made again with regard to the product term and are eliminated as shown in Table 6.8.

The significance of the final regression analysis is also tested by the F-value given in Table 6.9, which is related to the multiple

Table 6.6 Partial F-tests for the Elimination of Insignificant Regressor Variables for $\ln a_1$

Eliminated Regressor Variable	Difference of Residual Squares by Elimination	Mean of Residual Squares	Partial F-value	F-value at 10% level (Degrees of Freedom)	
$\ln T_w$	.027	.106	.26	2.95	(1,22)
$\ln T_d$	.033	.102	.32	2.94	(1,23)
$\ln \psi$	.751	.099	7.59	2.93	(1,24)

Table 6.7 Results of t-tests of the Remaining Regressor Variables

Regressor Variable	Regression Coefficient	Standard Deviation of Regression Coefficient	t-value[1]
ln E	-.596	.053	-11.3
ln h	-.456	.091	-4.99
$\ln \psi$	.403	.147	2.75

[1] t=1.31 at 10% level with 24 degrees of freedom. If the absolute value of t is smaller than 1.31, the regression variable can be eliminated.

Table 6.8 Partial F-test of Interaction Terms

Interaction Term Studied	Sum of Square Attributable to the Interaction Term	Mean Square of Deviation from Regression	F-value (Degrees of Freedom)
$(\ln E) \cdot (\ln h)$	.003	.103	.03
$(\ln h) \cdot (\ln \psi)$	.034	.102	.333
$(\ln \psi) \cdot (\ln E)$	.050	.101	.496

(Note): F-value is 2.94 at upper 10% significance level with degrees of freedom of (1,23).

Table 6.9 Analysis of Variance of Regression Analysis of ln a_1

	Degrees of Freedom	Sum of Squares	Mean Squares	F-value
Attributable to Regression Analysis	3	21.09	7.03	70.8
Deviation from Regression Analysis	24	2.38	.099	
Total	27	23.47		

Intercept	-2.56
Multiple Correlation	.948
Standard Error of Estimate	.315

correlation co-efficient. As the F-value at upper 0.1% significance level with (3,24) degrees of freedom is 7.55, the F-value of 70.8 in Table 6.9 shows that the regression equation (6.22) is statistically significant.

The final regression results of a_1 are therefore:

$$a_1 = 7.73 \times 10^{-2} \cdot E^{-.53} \cdot h^{-.46} \cdot \psi^{.40} \tag{6.24}$$

The 90% confidence bounds on a_1 are estimated by $e^{1.645\,s}$ and $e^{-1.645\,s}$, where s is the standard deviation of $\ln a_1$ and is equal to 0.315.

Similar analyses are made for the other dependent variables. The regression result of a_2 is given in Table 6.10 and the results of b_1, b_2, b_3, c_1 and c_2 are summarized in Appendix H. The final equations obtained are:

$$a_2 = 2.93 \cdot t_d^{.23} \cdot E^{.059} \cdot \psi^{-.98} \tag{6.25}$$

$$b_1 = 4.16 \times 10^{-2} \cdot h^{-.27} \cdot E^{-.39} \tag{6.26}$$

$$b_2 = 1.75 \cdot h^{.043} \cdot t_d^{.19} \cdot E^{.12} \cdot \psi^{-.99} \tag{6.27}$$

$$b_3 = 1.45 \cdot \psi^{-.52} \tag{6.28}$$

$$c_1 = 8.63 \times 10^{-2} \cdot h^{-.37} \cdot t_d^{-.65} \cdot E^{-.65} \cdot \psi^{.93} \tag{6.29}$$

$$c_2 = 2.43 \cdot h^{-.080} \cdot \psi^{-1.02} \tag{6.30}$$

VI.7 Investigation of the Adequacy of the Regression Results

The regression results of a_1 and a_2 are tested individually and collectively as follows. The examination of the other dependent variables is given in Appendix H.

Table 6.10 Regression Analysis of a_2

Dependent Variable	Regressor Variable	Regression Coefficient	Standard Error of Regression Coefficient	t-value
$\ln a_2$	$\ln t_d$	.233	.0453	5.2
	$\ln E$	.059	.0177	3.7
	$\ln \psi$	-.980	.066	-14.9

Intercept	1.074
Multiple Correlation	.988
Standard Error of Estimate	.106
F-value	323.1

(0.1% F-value for 3 and 24 degrees of freedom is 7.55)

VI.7.1 Examination of Individual Results

The quantity a_1 is estimated from the regression results Eq. (6.24) for each of the 28 samples of the radioactive releases and is compared with the data in Table 6.5. The estimates and data are plotted in Fig. 6.5. If the regression estimates accurately predict the data, the points in Fig. 6.5 should lie closely about the 45 degree line. As observed the points do lie about the 45 degree line and no systematic error is observed (i.e., tendencies to overpredict or underpredict various ranges of data). The quantity a_2 is similarly examined in Fig. 6.6 and no systematic error is observed.

Vi.7.2 Examination of the Combined Regression Results

The quantities a_1 and a_2 are constants of the transfer function a(r). Possible combined errors are examined by estimating the first risk moments of the sample population distributions using a_1 and a_2 derived by the regression. The first risk moment is estimated from the regression results by:

$$(M_1^*)_{i,q} = \sum_j \sum_k (a_1)_q \cdot \exp[-(a_2)_q \cdot r_k] \cdot (N_{jk})_i \qquad (6.31)$$

where $(M_1^*)_{i,q}$ is the estimate of the first risk moment of the conditional distribution at site i for the release q. $(a_1)_q$ and $(a_2)_q$ are the constants of the transfer function for the release q estimated from the regression results. $(N_{jk})_i$ is the population in the k-th annular segment in the direction j at the site i. The estimates of the first risk moment is compared with the results of the consequence calculation. The population distributions Site A and Site B are used to evaluate the adequacy of the regression. The results in Fig. 6.7 do not show systematic error and the largest error is a factor of 1.7. In Chapter V

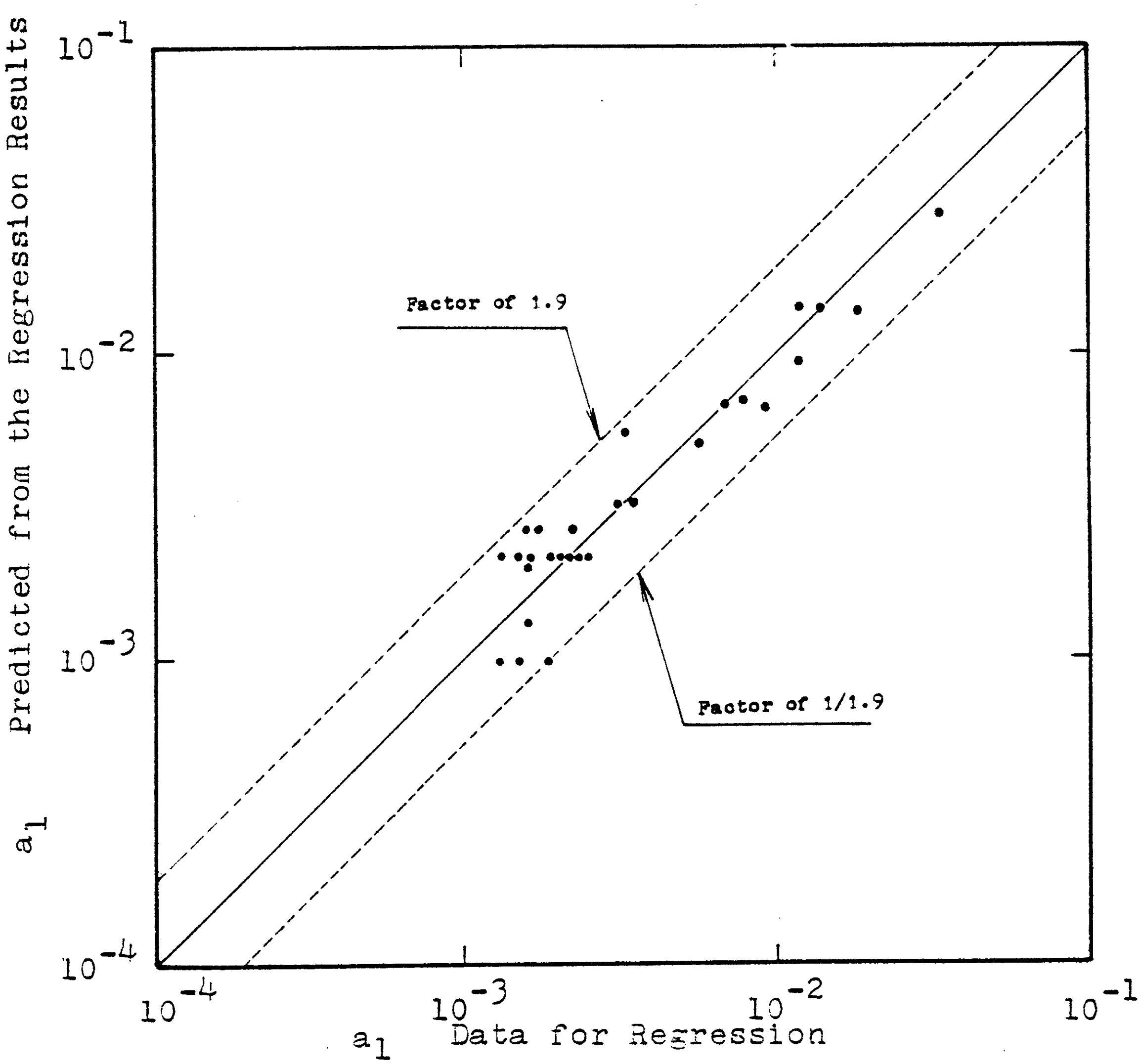

Fig.6.5 Test of the Regression Results of a_1

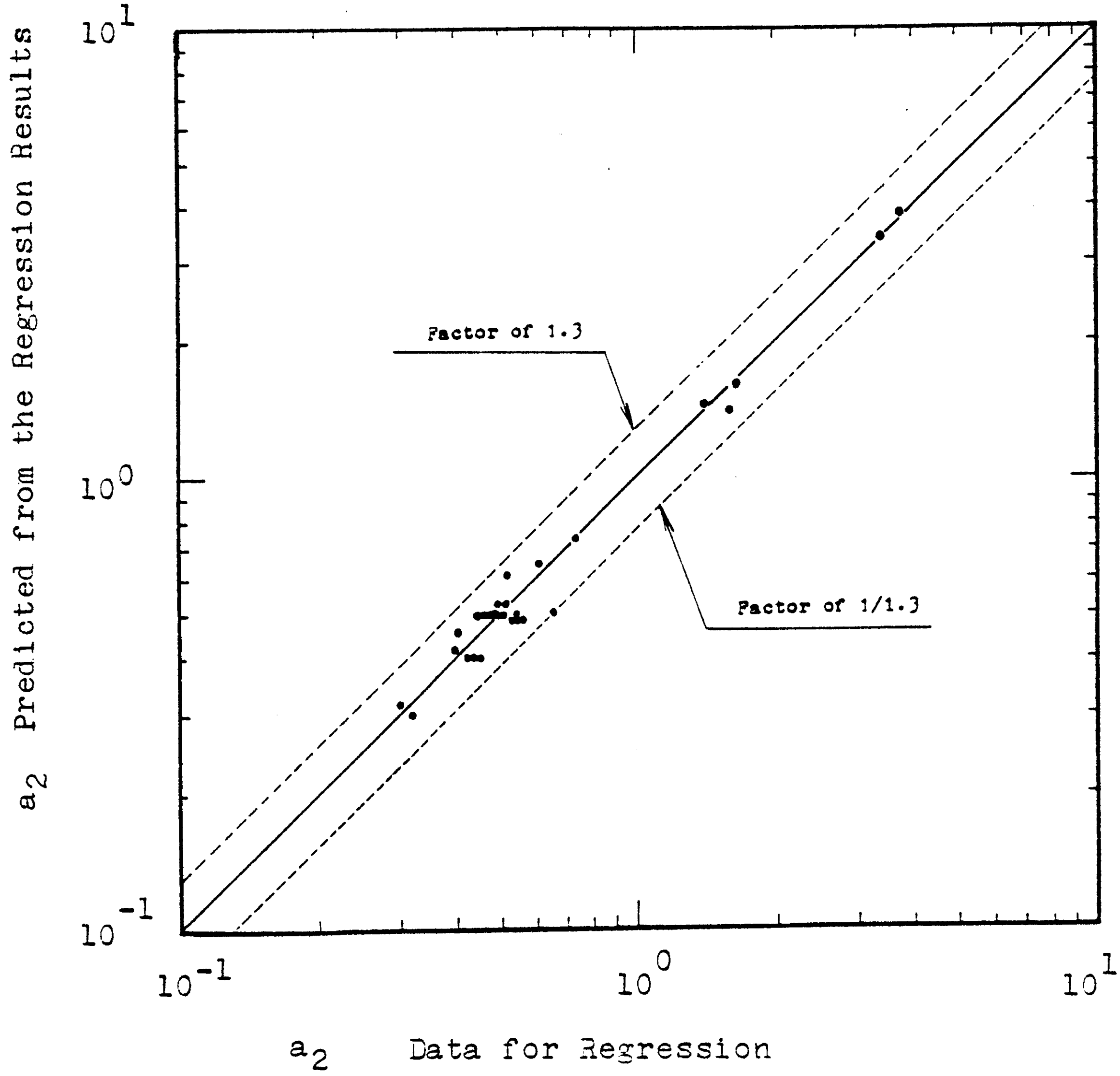

Fig.6.6 Test of the Regression Results of a_2

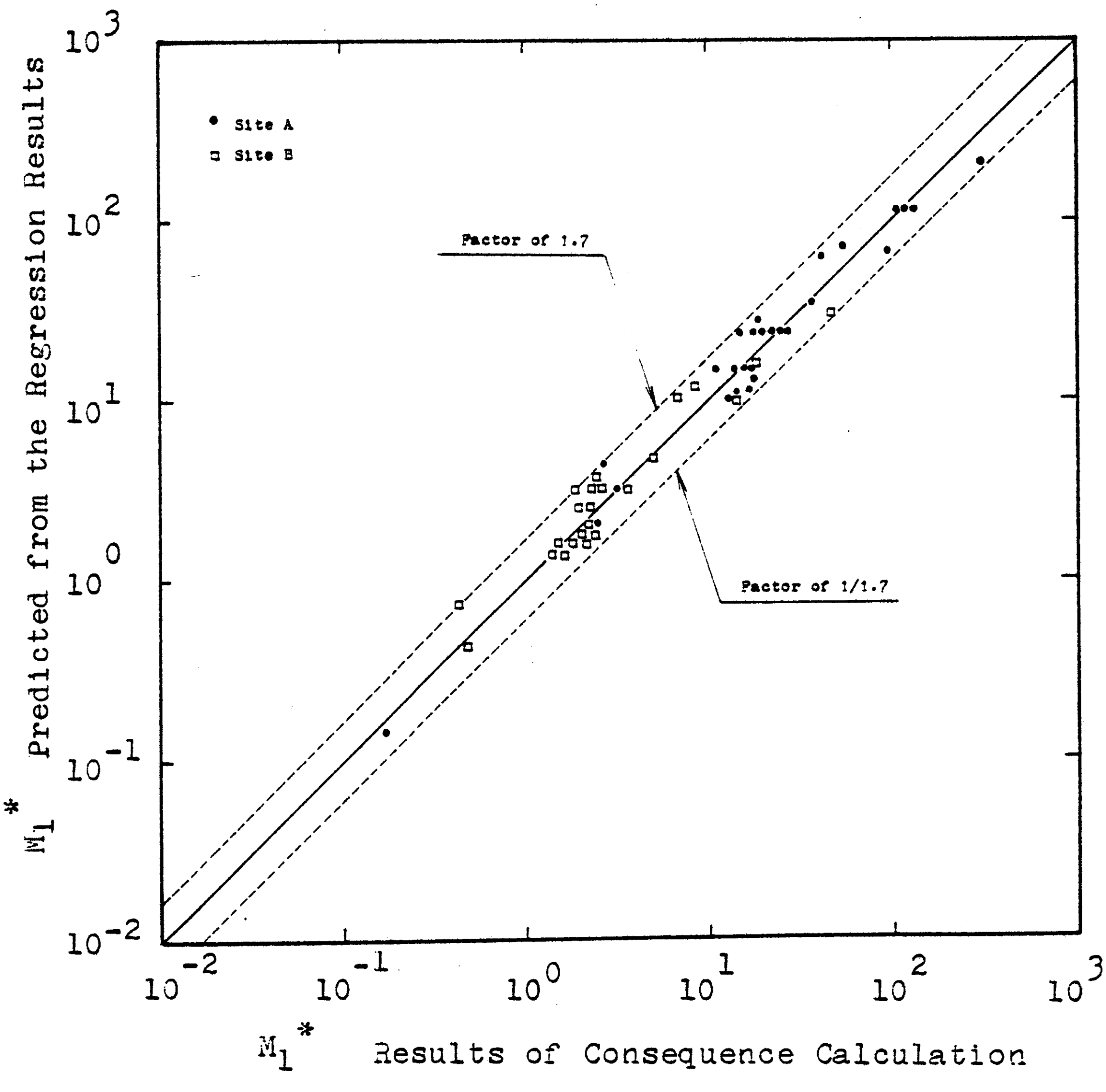

Fig.6.7 Comparison of the Estimated First Risk Moment from Regression with the Consequence Results

the largest error observed in Fig. 5.6 was a factor of 1.7 and was found to be within the uncertainty bounds of the consequence model. Therefore the error in Fig. 6.7 can also be concluded within the error bounds of the consequence model.

Similar examinations are made for the regression results Eqs. (6.26) through (6.30) in Appendix H. The results are found to be adequate.

VI.8 Example of Possible Applications of the Regression Results

Having obtained the regression results, they can then be used for estimating the consequences of radioactive releases of different characteristics without having to rerun the consequence program. For example, in the Reactor Safety Study, numerous accident sequences obtained by the event tree analysis are grouped into the release categories in Table 6.1. Using the regression results, the first two risk moments and the normalization constant for each of the accident sequences in the release category can be estimated without rerunning the consequence program. Because of the explicit relationship of the regression equations, sensitivity studies and decision making studies are also able to be carried out in a straightforward manner. The regression results applied to an evaluation of the safety systems in a nuclear power plant will be particularly discussed here.

VI.8.1 Evaluation of the Safety Systems

The safety systems in a nuclear power plant include engineering safety features, operation restrictions and maintenance activities. They are designed to reduce the risk of the reactor accidents by reducing the probabilities of the occurrences or alternatively by reducing the magnitudes of radioactive releases to the environment.

To present the application of the regression results to the evaluation of safety systems, a particular accident sequence q is considered. The distribution of consequence versus probability for the accident sequence is given by Eq. (6.1) as:

$$f_q(x) = P_q \cdot f_q^*(x) \tag{6.32}$$

where P_q is the probability estimated for the accident sequence q and $f_q^*(x)$ is the conditional distribution given the accident occurrence.

The regression results allow the first two risk moments and the normalization constant of the conditional distribution $f_q^*(x)$ to be estimated from the release characteristics of the accident sequence, which involves the release fractions of the core inventories, the elevation of the release, the energy content of the release, the time of the release, the duration of the release and the warning time for evacuation. For example, the constants of the transfer function $a^*(r)$ are estimated from the release characteristics by Eqs. (6.24) and (6.25) as:

$$(a_1)_q = 7.73 \times 10^{-2} \cdot (E)_q^{-.53} \cdot (h)_q^{-.46} \cdot (\psi)_q^{.40} \tag{6.33}$$

$$(a_2)_q = 2.93 \cdot (T_d)_q^{.23} \cdot (E)_q^{.059} \cdot (\psi)_q^{-.98} \tag{6.34}$$

Given a population distribution, the first risk moment of the conditional distribution given the accident occurrence is estimated by:

$$(M_1^*)_q = \sum_j \sum_k (a_1)_q \cdot \exp\left[-(a_2)_q \cdot r_k\right] \cdot N_{jk} \tag{6.35}$$

The first risk moment of the unconditional distribution is then given by:

$$(M_1)_q = P_q \cdot \sum_j \sum_k (a_1)_q \cdot \exp[-(a_2)_q \cdot r_k] \cdot N_{jk} \tag{6.36}$$

The second risk moment and the normalization constant of $f_q(x)$ are estimated in a similar manner.

If the safety systems are designed to reduce the probability of occurrence P_q, the effects of the systems can be evaluated from the regression results, such as Eq. (6.36), because the probability term P_q is separated from the effects of the other release characteristics. Given the population distribution and the risk moments of the conditional distribution, criteria can be considered for the probability of the occurrence P_q which give the acceptable risk characteristics.

If the safety systems are designed to reduce the magnitude of the release, the effect of the decrease of the magnitude can be estimated from the regression results, such as Eqs. (6.33), (6.34) and (6.36). A numerical example is given in the following subsection about the evaluation of a hypothetical iodine removal system.

The regression results furthermore allow trade-off studiss to be considered between the population distribution, the probability of occurrence and the magnitude of the release. For example, the objective to obtain the acceptable first risk moment in Eq. (6.36) can be achieved by selecting a site of low population or by adding or improving the safety systems, which reduces the probability of occurrence or the magnitude of release. Such trade-off studies can be straightforwardly made from the regression results.

VI.8.2 Numerical Example of Application of the Regression Results

A hypothetical iodine removal system is studied to demonstrate the application of the regression results to the evaluation of the safety systems. The problem is to express the decrease of the first risk moment in terms of the iodine removal efficiency under the following assumptions:

(1) The release characteristics considered are similar to those of a PWR-2 release category shown in Table 6.1 when no iodine is removed by the system considered.

(2) Only the release fraction of the iodine is affected by the system and the other release characteristics are unchanged by the system.

(3) The population distribution at Site A shown in the Appendix C is used.

(4) Only early fatalities are considered. The regression results derived in this chapter are then applied, which are based on the northeastern valley meteorological condition and radioactive inventories of a 3200 MW-th plant.

Let ω be the iodine removal efficiency of the considered system. As 70% of the iodine inventory in the core is released when no iodine is removed by the system considered, the release fraction of the iodine at the removal efficiency ω is given by:

$$q_I(\omega) = 0.70 \quad (1 - \omega) \tag{6.37}$$

The effective source term is calculated by Eqs. (6.2) and (6.4) from the iodine release fraction in Eq. (6.37), the release fractions of the other isotope groups in Table 6.1 and the weighting factors in Table 6.2. The calculated effective sources for the three organs are given in Fig. 6.8 as a function of the removal efficiency. Fig. 6.8 shows that the effective source to the bone marrow is dominant over

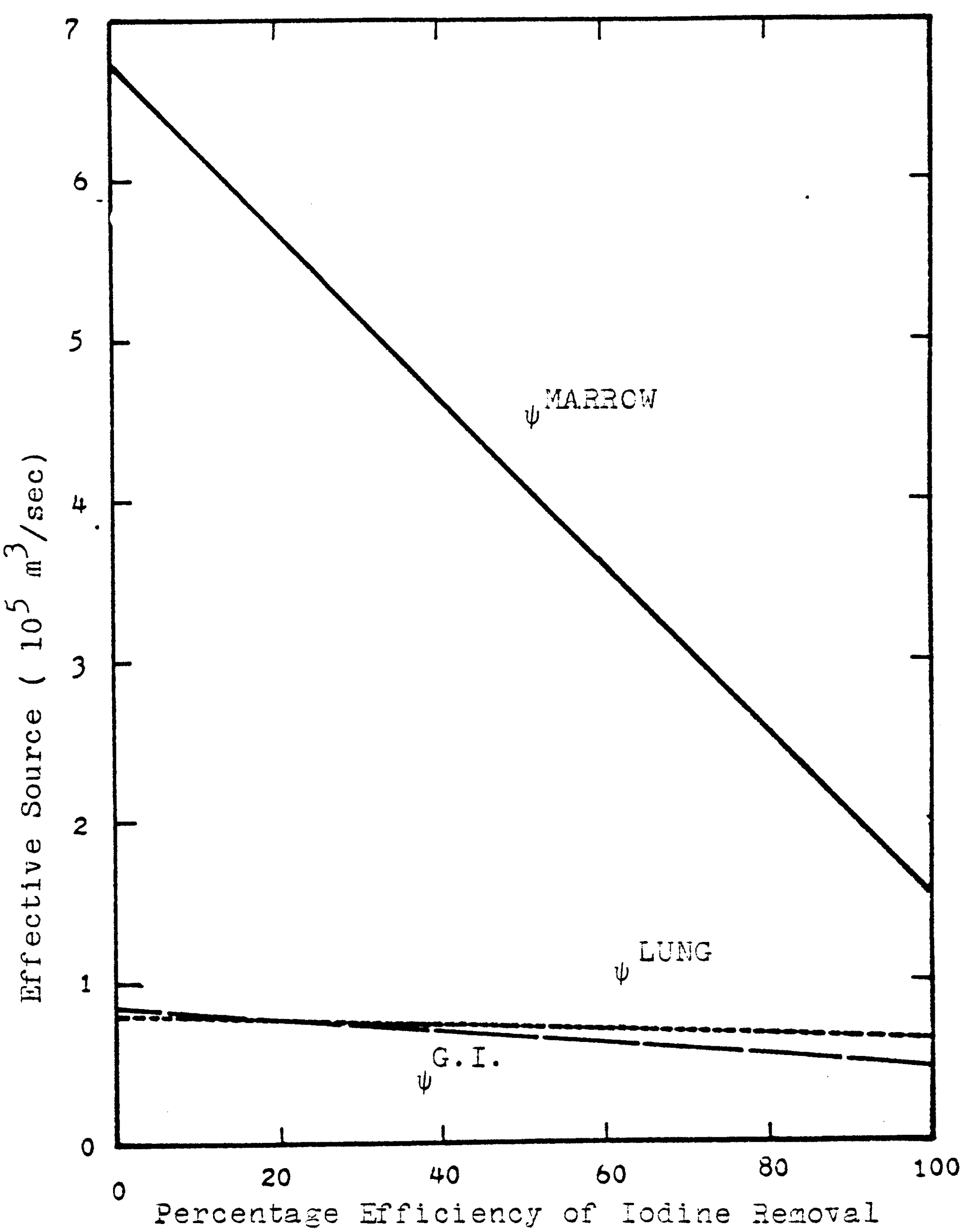

Fig.6.8 Decrease of the Effective Source by the Removal of Iodine

those of the other two organs. The effective source for the bone marrow is therefore selected as the overall effective source term.

The constants a_1 and a_2 of the transfer function $a^*(r)$ are estimated by Eq. (6.33) and (6.34) from the overall effective source term in Fig. 6.8 and the other release characteristics of the PWR-2 release category in Table 6.1. The estimated constants a_1 and a_2 are given in Fig. 6.9 as a function of the iodine removal efficiency ω. From the population distribution at Site A, the constants a_1 and a_2 in Fig. 6.9 and the probability of occurrence of 8×10^{-6} per reactor year (PWR-2 release in Table 6.1), the first risk moment is estimated by Eq. (6.44). The result is given in Fig. 6.10 as a function of the removal efficiency. Finally, the decrease of the first risk moment by the iodine removal system is also shown in Fig. 6.10 as a function of the removal efficiency.

Fig. 6.10 can be used to evaluate the decrease of the first risk moment when data in the iodine removal efficiency of the system are available. Alternatively, Fig. 6.10 can be used to calculate the required iodine removal efficiency of the system to obtain the acceptable first risk moment.

VI.9 Summary and Conclusions

The regression approach discussed in Chapter IV was demonstrated in this chapter in which the release characteristics was taken to be the basic variable. The early fatalities distribution in the northeastern valley meteorological condition was used to derive the regression results. The regressor variables are the warning time for evacuation, the duration of the release, the energy content in the released

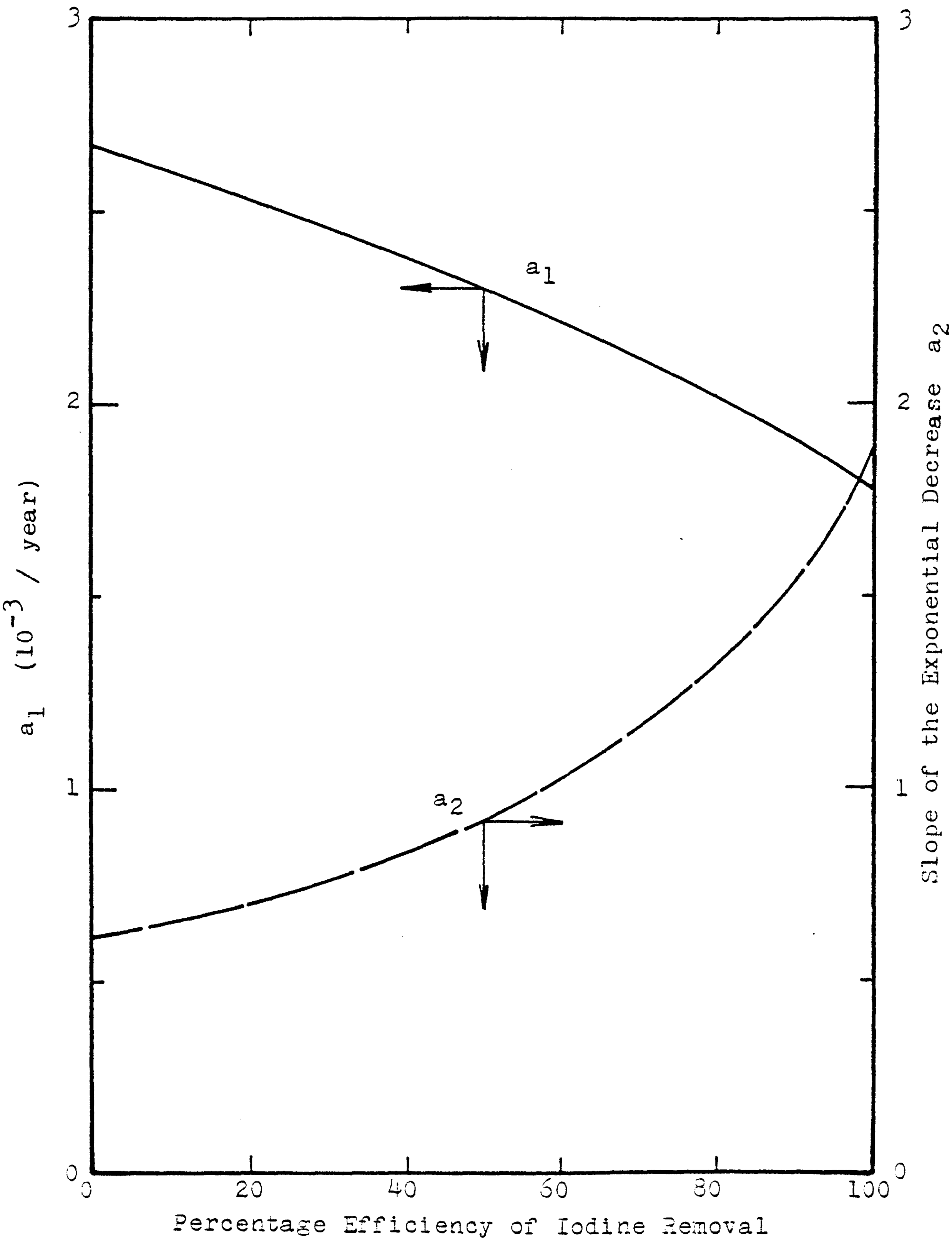

Fig. 6.9 Effect of the Iodine Removal on the Constants of the Transfer Function $a^*(r)$

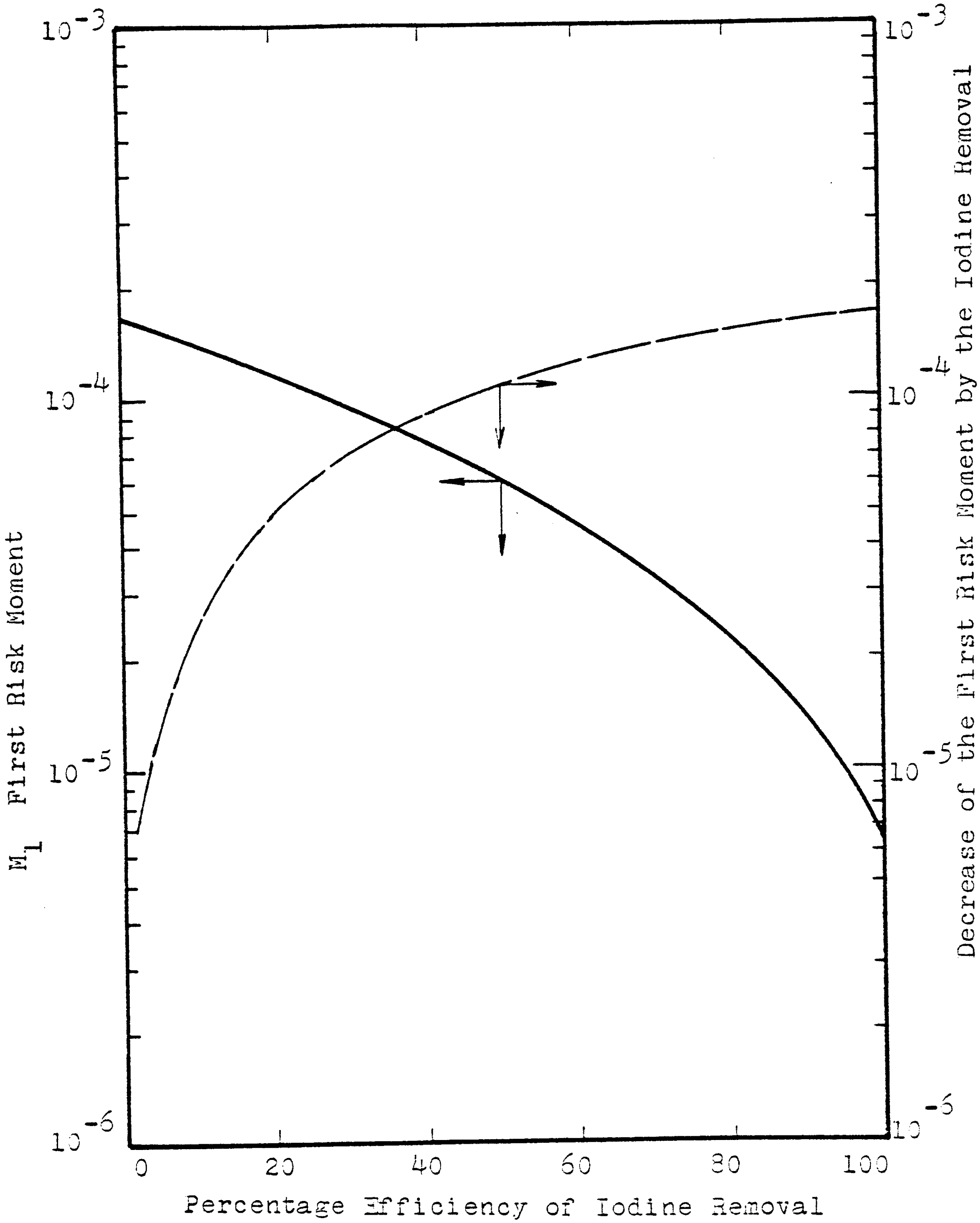

Fig.6.10 Effect of the Iodine Removal on the First Risk Moment

plume and the effective source which is a weighted sum of the release fractions. The probability of the occurrence was not taken as a regressor variable by considering the conditional distribution of early fatalities given the accident occurrence. The constants of the transfer functions discussed in the preceding chapter were taken to be the dependent variables.

The lognormal equations, such as given below, were tested.

$$\ln a_1 = k_{01} + k_{11} \cdot \ln h + k_{21} \cdot \ln T_w + k_{31} \cdot \ln T_d + \\ + k_{41} \cdot \ln E + k_{51} \cdot \ln \psi + \varepsilon_1 \qquad (6.38)$$

The terms that have insignificant effects on the variation of the dependent variables were eliminated by the partial F-test. The final equations obtained are:

$$a_1 = 7.73 \times 10^{-2} \cdot E^{-.53} \cdot h^{-.46} \cdot {}^{.40} \qquad (6.39)$$

$$a_2 = 2.93 \cdot T_d^{.23} \cdot E^{.059} \cdot \psi^{-.98} \qquad (6.40)$$

$$b_1 = 4.16 \times 10^{-2} \cdot h^{-.27} \cdot E^{-.39} \qquad (6.41)$$

$$b_2 = 1.75 \cdot h^{.043} \cdot T_d^{.19} \cdot E^{.12} \cdot \psi^{-.99} \qquad (6.42)$$

$$b_3 = 1.45 \cdot \psi^{-.52} \qquad (6.43)$$

$$c_1 = 8.63 \times 10^{-2} \cdot h^{-.37} \cdot T_d^{-.65} \cdot E^{-.65} \cdot \psi^{.93} \qquad (6.44)$$

$$c_2 = 2.43 \cdot h^{-.080} \cdot \psi^{-1.02} \qquad (6.45)$$

Systematic errors were not observed for prediction of the dependent variables and the estimates of the risk characteristics M_1, M_2 and α were found to be within the uncertainty range of the consequence model.

Having obtained the regression results, they can be applied to new situations for sensitivity studies and decision making investigations. Because of the simple form of the regression equations, the involved calculations are straightforward and do not require the consequence codes or large computation time. The regression results were applied to an example of evaluation of a hypothetical iodine removal system. The decrease of the first risk moment was finally expressed as a function of the iodine removal efficiency of the system. The example illustrates how the regression results can be used in evaluation and decision making.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

VII.1 Summary and Conclusion

The objective of this thesis is to develop a methodology for deriving a set of explicit equations which relate the public risk in potential nuclear accidents to the basic variables which determine the consequences of the accidents. The equations give insight into the physical relationships which are involved in the accident risks. Once the equations are derived, they can be used for sensitivity analyses and decision making studies without the need of complex computer programs.

The methodology developed in this study consists of two steps. The first step involves describing the consequence versus frequency curve in terms of a parametric distribution having a small number of parameters. The second step involves relating the parameters to the basic driving variables.

A general approach for fitting the consequence versus frequency distributions to the parametric distributions consists of three fundamental steps. These steps are selection of the candidate parametric distributions, estimation of the unknown parameters and determination of adequate fits. The selection of the candidate parametric distributions is based on the properties of the risk distributions including the domain of the independent variables, number of modes, skewness, and tail behavior. The method of moments and the method of least squares are discussed as means of estimating the unknown constants. Criteria of adequate fits are based on the largest deviation of the fits, systematic errors in the fits and residual mean squares.

The developed approach is demonstrated for the examples of fatality distributions of nuclear and non-nuclear risks. Four candidate distributions are examined: exponential, gamma, Weibull and lognormal distributions. For these examples, the method of moments is used to estimate the unknown parameters. In order to select a distribution family which adequately describes the fatalities distributions, the historical records of hurricanes, earthquakes, tornadoes and dam failures are examined. The calculated risk curves of nuclear reactor accidents are also examined for different population distributions and different types of the accidents. Based on these examinations, the Weibull distribution is determined to be the distribution which adequately describes all these various risk curves. The estimates of the Weibull parameters for the examined curves are summarized in Table 7.1. The lower end of the domain x_0, the normalization constant α, the risk moments about x_0 are determined from the historical data or from the results of consequence calculation. The Weibull shape parameter β and scale parameter η are determined from the first two risk moments, allowing simple and efficient estimation to be performed.

For the second step in the methodology, relating the distribution parameters to the basic driving variables, regression techniques are used in this study. The regression approach consists of 6 fundamental steps. These fundamental steps are: (1) identification of regressor variables, i.e., the basic driving variables to be considered; (2) selection of the dependent variables; (3) assembling the data to be used in the regression; (4) formulation of candidate regression equations which express the relationship between the dependent and regressor variables; (5) estimation of the unknown constants in the regression equations by the method of least squares; and (6) testing of the adequacy of the derived equations.

The regression analysis approach is demonstrated in two examples. One example uses the population distribution as the basic regressor variable. Three exponential functions (called "transfer functions") are derived which relate the first two risk moments and the normalization constant to the population distribution. Table 7.2 gives the transfer function results which are determined in this study. An application of the derived equations is demonstrated for an example of selection of a site for a nuclear power plant.

The regression approach is demonstrated for another example in which the characteristics of the radioactive releases are treated as the basic regressor variables. The dependent variables are taken to be the constants of the transfer functions determined in the preceding analysis of the population distribution. The lognormal equations which are determined are given in Table 7.3. The derived equations are applied to the evaluation of a hypothetical iodine removal system.

In conclusion, the methodology proposed in this study is found to be appropriate in deriving explicit equations which relate the risk to basic driving variables. The derived equations are fairly simple and straightforward, which allows for simple and straightforward applications to decision making studies and other calculations and evaluations.

VII.2 Recommendations

The methodology proposed in this study is one attempt at deter-

Table 7.1 Estimates of the Parameters of the Weibull Distribution

$$F^c(x) = \alpha \cdot \exp\left[-\left(\frac{x - x_0}{\eta}\right)^{\beta}\right] \qquad f(x) = \alpha \cdot \frac{\beta}{\eta} \cdot \left(\frac{x - x_0}{\eta}\right)^{\beta-1} \cdot \exp\left[-\left(\frac{x - x_0}{\eta}\right)^{\beta}\right]$$

Events	x_0 (1)	α (1/year) (2)	M_1	M_2	β	η
Hurricanes	0	6.30×10^{-1}	1.27×10^{2}	5.64×10^{5}	.387	7.48×10^{1}
Earthquakes	0	1.64×10^{-1}	1.53×10^{1}	8.13×10^{3}	.511	4.84×10^{1}
Tornadoes	20	8.10×10^{-1}	6.62×10^{1}	1.67×10^{4}	.708	6.53×10^{1}
Dam Failures	0	9.52×10^{-2}	3.48×10^{1}	5.07×10^{4}	.608	2.47×10^{2}
Average of U.S. Reactors	0	4.72×10^{-7}	4.60×10^{-5}	6.45×10^{-2}	.371	2.45×10^{1}
PWR Accidents at Site A	0	5.78×10^{-7}	2.72×10^{-4}	5.77×10^{-1}	.570	2.91×10^{2}
BWR Accidents at Site B	0	1.61×10^{-8}	9.92×10^{-7}	3.46×10^{-4}	.513	3.23×10^{1}

[1] x_0 is determined from the smallest consequence in the data.

[2] α is determined from the number of events having consequences greater than x_0.

Table 7.2 Transfer Function Results of PWR Accidents in Northeastern Valley Meteorological Conditions.

Dependent Variable	Transfer Equations	Constants
First Risk Moment	$M_1 = \sum_j \int a_1 \cdot \exp(-a_2 \cdot r) \cdot n_j(r) \cdot dr$	$a_1 = 3.51 \times 10^{-8}$ $a_2 = .600$
Second Risk Moment	$M_2 = \sum_j \iint b_1 \cdot \exp[-b_2 \cdot (r+r')] \cdot \exp[-b_3 \cdot \lvert r-r' \rvert] \cdot n_j(r) \cdot n_j(r') \cdot dr \cdot dr'$	$b_1 = 2.05 \times 10^{-8}$ $b_2 = .352$ $b_3 = .557$
Normalization Constant	$\alpha = \sum_j c_1 \cdot \exp[-c_2 \cdot d_j]$	$c_1 = 1.79 \times 10^{-6}$ $c_2 = .398$

(Note): (1) $n_j(r)$ is the population per unit distance at r in a 22½ degree sector of the direction j.

(2) d_j is the minimum distance at which people live from a reactor in the direction j.

Table 7.3 Summary of the Regression Results of the Radioactive Releases[1]

Transfer Functions[2]	Regression Equations[3]
$M_1 = P \cdot \sum_j \int_r a_1 \cdot \exp(-a_2 \cdot r) \cdot n_j(r)\, dr$	$a_1 = 7.73 \times 10^{-2} \cdot E^{-.53} \cdot h^{-.46} \cdot \psi^{.40}$ $a_2 = 2.93 \times t_d^{.23} \cdot E^{.059} \cdot \psi^{-.98}$
$M_2 = P \cdot \sum_j \int_r \int_{r'} b_1 \cdot \exp[-b_2 \cdot (r+r')] \cdot \exp[-b_3 \cdot \lvert r-r' \rvert]\, n_j(r)\, n_j(r')\, dr\, dr'$	$b_1 = 4.16 \times 10^{-2} \cdot h^{-.27} \cdot E^{-.39}$ $b_2 = 1.75 \cdot h^{.043} \cdot t_d^{.19} \cdot E^{-.99}$ $b_3 = 1.45 \cdot \psi^{-.52}$
$\alpha = P \cdot \sum_j c_1 \exp[-c_2 \cdot d_j]$	$c_1 = 8.63 \times 10^{-2} \cdot h^{-.37} \cdot t_d^{-.65} \cdot E^{.93}$ $c_2 = 2.43 \cdot h^{-.080} \cdot \psi^{-1.02}$

[1] The northeastern valley meteorological conditions are assumed.

[2] P = probability of the occurrence (1/year).

[3] t_d = duration of the release (hours), E = energy content in the plume (10^6 Btu/hr), h = elevation of the release (meters), ψ = effective source (10^5 m^3/sec).

mining basic relationships which can be used in risk evaluations and decision making situations involving risks. The methodology is demonstrated for only one type of consequence (early fatalities), one meteorological condition (northeastern valley sites) and two sets of basic variables (population distribution and radioactive releases). Further studies will be required to develop broader results, such as considering other types of consequences, other meteorological conditions, and other basic variables. As consequence models and computer programs change, the regression relationships will also need to be reevaluated to determine updated results.

The methodology may also be applicable to evaluation of non-nuclear risks, such as dam failures. Relating the risks to the basic variables of interest may provide help in decision making and risk evaluations in these situations. Further studies are recommended to determine the feasibility of applying the methodology to these different situations.

With regard to the more detailed recommendations, the following studies are specifically recommended:

<u>Chapter II and Chapter III</u>

(1) Only two general fitting techniques were discussed in this study. However a large number of other techniques have been developed and the most appropriate technique may depend on the candidate parametric distributions. For example, a linear estimator of the Weibull distribution with the logarithmic transformations of dependent and independent variables is discussed in Ref-6. Further studies are recommended to test other techniques of fitting parametric distributions to the consequence versus frequency risk distributions.

(2) Four candidate distributions were examined to fit the risk curves. Other distributions should also be investigated to determine their feasibility and particular advantages and disadvantages.

Chapter V

(3) The transfer function c(r) that relates the normalization constant with the closest distance at which people live to the reactor was defined in Section V.5 as an approximation of the expectation of the H equation. In the example case of the fatalities distribution of PWR accidents, the error of this approximation was found to be within the uncertainty range of the consequence model. However in other situations, this approximation may not be appropriate. Therefore, further studies are required to define the transfer function that relates the normalization constant with the population distribution for a wider variety of consequences.

Chapter VI

(4) The effective source was defined for early fatalities in Section VI.2 because the interaction effects of the release fractions of various isotope groups are not simple. For other types of consequences, however, one or two isotope groups may have dominant effects on the magnitude of consequence. For example, the property damage may be dominated by the release fraction of the Cs group. In these cases, the selection of the release fractions as regressor variables may be appropriate. Further studies are recommended in

studying basic regressor variables for the analysis of a wide variety of consequences.

APPENDIX A

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APPENDIX B

NOMENCLATURE

Since this thesis is related to various different fields, such as statistics, meteorology, health physics, etc., it is sometimes difficult to achieve a consistency about the notation. The nomenclature is thus given here for each chapter.

Chapter I

F	number of events per unit time
F^c	complementary cumulative frequency (number of events per unit time)
f	frequency per unit time per unit consequence
M	risk moment
t	order of the risk moment
x	consequence magnitude
x_a, x_b	integration interval
ξ	reference magnitude for the evaluation of the risk moment

Chapter II

E	expectation
$e_1, \ldots, e_n$	random error variables
f	frequency distribution
F^c	complementary cumulative distribution
$\tilde{F}^c_i$	complementary cumulative frequency of the data i
G	candidate parametric function
i	subscript denoting the data to be fitted

k	number of parameters
M	risk moment of the candidate distribution
$\tilde{M}$	risk moment estimated from the data to be fitted
m	order of the risk moment
n	number of the data
s^2	residual mean square of the estimated equation
x	independent variable
x_i	observed value of the independent variable
Y	random variable
Y_i, y_i	observed value of Y
Δ^2	residual mean square to be minimized for the method of least squares
ξ	reference point for the evaluation of the risk moments
σ_Y	standard deviation of the random variable Y
$\tau_1,\ldots,\tau_k$	parameters of the candidate function
$\hat{\tau}_1,\ldots,\hat{\tau}_k$	estimates of the parameters

Chapter III

F^c	complementary cumulative frequency
f	frequency distribution (number of events per unit time per unit consequence)
$\bar{f}$	normalized density distribution (number of events per unit consequence)
M_1	first risk moment about the lower end of the domain
M_2	second risk moment about the lower end of the domain
M_m	m-th risk moment about the lower end of the domain
m	order of the risk moment
p	probability assigned to the sample data or the trial in the consequence calculation

T	time period in which the historical records are available for non-nuclear risks
x	magnitude of consequence
x_0	lower end of the domain of x
Δx	interval of the consequence magnitude for the calculation of the frequency distribution from the historical records or from the consequence results
α	normalization constant
β	shape factor of the gamma distribution or the Weibull distribution
Γ	Gamma function
η	scale factor of the Weibull distribution
θ	scale factor of the exponential distribution or the gamma distribution
κ	number of historical observations having consequences greater than the specified magnitude
$\Delta\kappa$	number of historical observations having consequences in the certain range of the magnitude Δx
μ	mean of the normal variate (ln x) of the lognormal distribution
ξ	reference magnitude for evaluation of the risk moment
σ	standard deviation of the normal variate (ln x) of the lognormal distribution

Chapter IV

F	F-value for the evaluation of the significance of the regression equation
F'	partial F-value for the evaluation of the significance of the added unknown constants
F^c	complementary cumulative frequency
h	candidate regression equation
k	number of parameters
m	number of regressor variables

n	number of the data for regression
S_G^2	sum of squares attributable to regression
$S_R^2, S_R^{2'}$	sum of residual squares
x	magnitude of consequence
x_0	lower end of the domain of x
y	dependent variable
y_0	average of y-values of the data
z	regressor variable
α	normalization constant
β	shape factor of the Weibull distribution
Δ^2	sum of residual squares to be minimized in the regression analysis
$\varepsilon, \varepsilon'$	random error variables
η	scale factor of the Weibull distribution
ν	number of added unknowns in the regression equation
ρ_m	multiple correlation coefficient
τ	unknown constants in the candidate equations
$\hat{\tau}$	estimates of τ by regression

Chapter V

A	ratio of the fatalities to the population for a specific trial
A_k	ratio of the fatalities to the population in the k-th annular segment for a specific trial
a	transfer function that relates the first risk moment to the population distribution
$a_1, \ldots, a_\nu$	unknown constants of the candidate function of a(r)
$\hat{a}_1, \ldots, \hat{a}_\nu$	estimates of $a_1, \ldots, a_\nu$ by regression
a_k	the average of A_k over all the trials

b	transfer function that relates the second risk moment to the population distribution
$b_1,\ldots,b_{\nu'}$	unknown constants of the candidate function of $b(r,r')$
$\hat{b}_1,\ldots,\hat{b}_{\nu'}$	estimates of $b_1,\ldots,b_{\nu'}$ by regression
$b_{kk'}$	the average of $[A_k \cdot A_{k'}]$ over all the trials
c	transfer function that relates the normalization constant α to the closest distance of population from the reactor
$c_1,\ldots,c_{\nu''}$	unknown constants of the candidate function of $c(r)$
$c_1,\ldots,c_{\nu''}$	estimates of $c_1,\ldots,c_{\nu''}$ by regression
d	closest distance at which people live from a reactor
E	expectation over the trials
F^c	complementary cumulative frequency
F^c_h, F^c_ℓ	complementary cumulative frequencies of the two adjacent data points in the consequence results below and above 10^{-9}/year
F'	partial F-statistic for the evaluation of the added unknowns
H	unit step function
h_a	candidate function of $a(r)$
h_b	candidate function of $b(r,r')$
h_c	candidate function of $c(r)$
h_γ	candidate function of $\gamma(r)$
i	subscript denoting the sample data
j	subscript denoting the wind direction
K	number of segments considered in the consequence model
k,k'	subscript denoting the segment
k_{min}	the closest segment at which people live
ℓ	subscript denoting the population group in the bell-shaped population model
M_1	first risk moment about the lower end of the domain

M_2	second risk moment about the lower end of the domain
N	population in an annular segment
N_f	fatalities in an annular segment
N_T	total population in a population group in the bell-shaped population model
n	population per unit distance per radian
n_j	population per unit distance in a 22.5 degree section in the direction j
n_T	population per unit distance in an annulus of unit width
p_j	probability of the wind blowing to the direction j
p_R	probability of occurrence of release
p_S	probability assigned to a specific sample of the weather data
p_t	probability assigned to a specific trial
p_v	probability assigned to a specific evacuation speed
R	distance from the origin to the center of the bell-shaped population group
R_B, R_C, R_D	distance from the origin to the center of the population groups B, C and D respectively
r	distance from the origin
r_k	distance from the origin to the center of the annular segment
Δr_k	width of the k-th annular segment
S_R	sum of residual squares
t	subscript denoting the trial
x	magnitude of consequence
x_h, x_ℓ	consequence magnitudes of the two adjacent points in the consequence results below and above the complementary cumulative frequency of 10^{-9}/year
α	normalization constant
β	shape factor of the Weibull distribution

γ	transfer function which approximately relates the normalization constant to the closest distance at which people live from a reactor
$\gamma_1,\ldots,\gamma_{\nu'''}$	unknown constants of the candidate function of $\gamma(r)$
$\Delta_a^2,\Delta_b^2,\Delta_c^2,\Delta_\gamma^2$	sum of residual squares to be minimized in the regression approach
ε,ε'	random error variables
ζ	coordinate axis perpendicular to r
η	scale factor of the Weibull distribution
θ	angular coordinate in the polar coordinate system
ν,ν',ν'',ν'''	number of unknown constants in the candidate functions
ρ	population per unit area
$\sigma_A,\sigma_B,\sigma_C,\sigma_D$	average deviation of the population in the bell-shaped population groups A, B, C and D
σ_R	average deviation of the population in a bell-shaped population model

<u>Chapter VI</u>

a^*	transfer function relating the condition first risk moment M_1^* to the population distribution
a_1,a_2	parameters of the exponential function to fit $a^*(r)$
B	breathing rate
b^*	transfer function relating the conditional second risk moment M_2^* to the population distribution
b_1,b_2,b_3	parameters of the exponent function to fit $b^*(r,r')$
C	dose conversion factor involving three modes of exposure
C_C	dose conversion factor for cloud shine dose
C_G	dose conversion factor for ground shine dose
C_I	dose conversion factor for inhalation dose
c^*	transfer function relating the conditional normalization constant α^* to the closest distance at which people live

c_1, c_2	parameters of the exponential function to fit $c^*(r)$
d	closest distance at which people live from the reactor
E	energy content in the released plume
f	frequency distribution
f^*	conditional frequency distribution given the release occurrence
g	subscript denoting the isotope groups for the evaluation of the release fractions
h	elevation of radioactive release
I	inventory of radioactivity in a reactor core
j	subscript denoting the isotope
k	subscript denoting the organ in a body
$k_{01}, k_{02}, \ldots, k_{57}$; $k'_{01}, k'_{02}, \ldots, k'_{57}$	constants in the regression equations
$(LD)_{50}$	a dose that causes deaths to 50% of the exposed population
M_1^*	first risk moment of the conditional distribution $f^*(x)$ given the accident occurrence about the lower end of the domain
M_2^*	second risk moment of the conditional distribution $f^*(x)$ given the accident occurrence about the lower end of the domain
M_t^*	t-th risk moment of the conditional distribution given the accident occurrence
N	population in an annular segment
n_j	population per unit distance in a 22.5 degree sector in the direction j
P	probability of occurrence of release
Q	released amount of radioactivity
q	subscript denoting a specific release
q_g	release fraction of the isotope group g
q_I	release fraction of iodine

r	distance from the origin
s	standard deviation of the estimate of the dependent variable
s_C	cloud shine shielding factor
s_G	ground shine shielding factor
T_d	duration of the release
T_r	time of the release
T_w	warning time for evacuation
t	subscript denoting the order of the risk moment
V_d	deposition velocity
x	consequence magnitude
α^*	normalization constant of the conditional frequency distribution $f^*(x)$ given the accident occurrence
$\varepsilon_1,\ldots,\varepsilon_7$	random error variables
λ	radioactive decay constant
χ	ground level airborne concentration of radioactivity
ψ	effective source
Ω	weighting factor for effective source
ω	iodine removal efficiency

Chapter VII

a	transfer function relating the first risk moment to the population distribution
a_1,a_2	parameters of the exponential function fitted to $a(r)$
b	transfer function relating the second risk moment to the population distribution
b_1,b_2,b_3	parameters of the exponential function fitted to $b(r,r')$
c	transfer function relating the normalization constant to the closest distance at which people live
c_1,c_2	parameters of the exponential function fitted to $c(r)$

d	closest distance at which people live from a reactor
E	energy content of the released plume
F^c	complementary cumulative frequency
h	elevation of release
j	subscript denoting the direction
k, k'	subscript denoting the segment
M_1	first risk moment about the lower end of the domain
M_2	second risk moment about the lower end of the domain
n_j	population per unit distance in a 22.5 degree sector in the direction j
P	probability of occurrence of release
r	distance from the origin
T_d	duration of release
x	magnitude of consequence
x_0	lower end of the domain of x
α	normalization constant
β	shape factor of the Weibull distribution
η	scale factor of the Weibull distribution
ψ	effective source

APPENDIX C

INPUT DATA FOR CONSEQUENCE CALCULATION OF INDIVIDUAL SITES

Major input data for the consequence calculation of the individual sites are summarized in this appendix. Since most of the input data for the individual site calculations are the same as those for the calculation of the first 100 commercial nuclear power plants performed in the Reactor Safety Study, only the data of specific importance in the individual site calculations are given in this appendix. The input data which are not given here are found in Appendix VI of WASH-1400 (Ref-1.

The characteristics of the northeastern valley meteorological condition are given in Table C.1. All of the calculations of the individual sites in this study are based on this meteorological condition.

The inventories of the radioactive isotopes in Table C.2 are used in this study. The inventories in Table C.2 were calculated in the Reactor Safety Study, assuming a 3200 MW-th PWR core at a time of just prior to refueling after the operation at a constant specific power density of 40 MW/kg U. BWRs have approximately the same inventories as PWRs.

The release characteristics of PWR and BWR accidents are given in Table C.3. The calculation results in Chapter III and Chapter V are based on the overall risks from the release categories in Table C.3. In Chapter VI, these release categories provide the data base for the regression analysis.

The geometry for the population distribution used in the consequence code is given in Table C.4. The population distributions of

Site A and Site B are given in Tables C.5 and C.6, respectively, as examples of the population distributions used in this study. They correspond to the 3rd highest and 3rd lowest, respectively, when the 68 sites considered in this study are ranked in a descending order based on the cumulative populations within 5 miles. These population distributions are used in Chapters III and VI and Appendix F as the examples for demonstrating the methodologies.

Table C.1 Joint Frequency Distribution for Thermal Stability, Windspeed, and Rain for Northeastern Valley Meteorological Condition

Thermal Stability	Rain	Wind Speed (m/s) 0 - 1	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	> 7	Summation
		%	%	%	%	%	%	%	%	%
A	No rain	1.83	2.93	2.69	2.52	2.17	1.42	1.14	1.54	
	Rain	0.09	0.06	0.03	0.05	0.00	0.01	0.00	0.01	16.50
B	No rain	0.49	0.34	0.26	0.21	0.25	0.15	0.06	0.15	
	Rain	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.92
C	No rain	0.51	0.49	0.49	0.31	0.34	0.27	0.23	0.40	
	Rain	0.01	0.05	0.02	0.00	0.00	0.00	0.00	0.01	3.13
D	No rain	6.06	4.52	4.84	3.54	2.68	2.33	1.22	2.57	
	Rain	0.37	0.51	0.41	0.19	0.10	0.10	0.10	0.23	29.87
E	No rain	7.61	5.64	4.77	3.45	2.45	1.18	0.88	1.70	
	Rain	0.53	0.54	0.57	0.39	0.16	0.06	0.06	0.17	30.15
F	No rain	8.11	4.86	2.51	1.21	0.50	0.31	0.15	0.17	
	Rain	0.25	0.10	0.09	0.08	0.07	0.01	0.01	0.00	18.44
Summation		25.87	20.05	16.61	11.85	8.85	5.83	3.85	7.10	100.00

(Note): From Table VI-5-2A in Appendix VI of WASH-1400 (Ref-1).

TABLE C.2 Initial Activity of Radionuclides in the Nuclear Reactor Core at the Time of Hypothetical Accident

No.	Radionuclide	Radioactive Inventory Source (curies x 10^8)	Half-Life (days)
1	Cobalt-58	0.0078	71.0
2	Cobalt-60	0.0029	1,920
3	Krypton-85	0.0056	3,950
4	Krypton-85m	0.24	0.183
5	Krypton-87	0.47	0.0528
6	Krypton-88	0.68	0.117
7	Rubidium-86	0.00026	18.7
8	Strontium-89	0.94	52.1
9	Strontium-90	0.037	11,030
10	Strontium-91	1.1	0.403
11	Yttrium-90	0.039	2.67
12	Yttrium-91	1.2	59.0
13	Zirconium-95	1.5	65.2
14	Zirconium-97	1.5	0.71
15	Niobium-95	1.5	35.0
16	Molybdenum-99	1.6	2.8
17	Technetium-99m	1.4	0.25
18	Ruthenium-103	1.1	39.5
19	Ruthenium-105	0.72	0.185
20	Ruthenium-106	0.25	366
21	Rhodium-105	0.49	1.50
22	Tellurium-127	0.059	0.391
23	Tellurium-127m	0.011	109
24	Tellurium-129	0.31	0.048
25	Tellurium-129m	0.053	0.340
26	Tellurium-131m	0.13	1.25
27	Tellurium-132	1.2	3.25
28	Antimony-127	0.061	3.88
29	Antimony-129	0.33	0.179
30	Iodine-131	0.85	8.05
31	Iodine-132	1.2	0.0958
32	Iodine-133	1.7	0.875
33	Iodine-134	1.9	0.0366
34	Iodine-135	1.5	0.280
35	Xenon-133	1.7	5.28
36	Xenon-135	0.34	0.384
37	Cesium-134	0.075	750
38	Cesium-136	0.030	13.0
39	Cesium-137	0.047	11,000
40	Barium-140	1.6	12.8
41	Lanthanum-140	1.6	1.67
42	Cerium-141	1.5	32.3
43	Cerium-143	1.3	1.38
44	Cerium-144	0.85	284
45	Praseodymium-143	1.3	13.7
46	Neodymium-147	0.60	11.1
47	Neptunium-239	16.4	2.35
48	Plutonium-238	0.00057	32,500
49	Plutonium-239	0.00021	8.9×10^6
50	Plutonium-240	0.00021	2.4×10^6
51	Plutonium-241	0.034	5,350
52	Americium-241	0.000017	1.5×10^5
53	Curium-242	0.0050	163
54	Curium-244	0.00023	6,630

Note: From TABLE VI 3-1 in Appendix VI of WASH-1400(Ref-1)

Table C.3 Release Characteristics of PWR and BWR Accidents

RELEASE CATEGORY	PROBABILITY per Reactor-Yr	TIME OF RELEASE (Hr)	DURATION OF RELEASE (Hr)	WARNING TIME FOR EVACUATION (Hr)	ELEVATION OF RELEASE (Meters)	CONTAINMENT ENERGY RELEASE (10^6 Btu/Hr)	FRACTION OF CORE INVENTORY RELEASED[a] Xe-Kr	Org. I	I	Cs-Rb	Te-Sb	Ba-Sr	Ru[b]	La[c]
PWR 1	$9x10^{-7}$	2.5	0.5	1.0	25	520[d]	0.9	$6x10^{-3}$	0.7	0.4	0.4	0.05	0.4	$3x10^{-3}$
PWR 2	$8x10^{-6}$	2.5	0.5	1.0	0	170	0.9	$7x10^{-3}$	0.7	0.5	0.3	0.06	0.02	$4x10^{-3}$
PWR 3	$4x10^{-6}$	5.0	1.5	2.0	0	6	0.8	$6x10^{-3}$	0.2	0.2	0.3	0.02	0.03	$3x10^{-3}$
PWR 4	$5x10^{-7}$	2.0	3.0	2.0	0	1	0.6	$2x10^{-3}$	0.09	0.04	0.03	$5x10^{-3}$	$3x10^{-3}$	$4x10^{-4}$
PWR 5	$7x10^{-7}$	2.0	4.0	1.0	0	0.3	0.3	$2x10^{-3}$	0.03	$9x10^{-3}$	$5x10^{-3}$	$1x10^{-3}$	$6x10^{-4}$	$7x10^{-5}$
PWR 6	$6x10^{-6}$	12.0	10.0	1.0	0	N/A	0.3	$2x10^{-3}$	$8x10^{-4}$	$8x10^{-4}$	$1x10^{-3}$	$9x10^{-5}$	$7x10^{-5}$	$1x10^{-5}$
PWR 7	$4x10^{-5}$	10.0	10.0	1.0	0	N/A	$6x10^{-3}$	$2x10^{-5}$	$2x10^{-5}$	$1x10^{-5}$	$2x10^{-5}$	$1x10^{-6}$	$1x10^{-6}$	$2x10^{-7}$
PWR 8	$4x10^{-5}$	0.5	0.5	N/A	0	N/A	$2x10^{-3}$	$5x10^{-6}$	$1x10^{-4}$	$5x10^{-4}$	$1x10^{-6}$	$1x10^{-8}$	0	0
PWR 9	$4x10^{-4}$	0.5	0.5	N/A	0	N/A	$3x10^{-6}$	$7x10^{-9}$	$1x10^{-7}$	$6x10^{-7}$	$1x10^{-9}$	$1x10^{-11}$	0	0
BWR 1	$1x10^{-6}$	2.0	2.0	1.5	25	130	1.0	$7x10^{-3}$	0.40	0.40	0.70	0.05	0.5	$5x10^{-3}$
BWR 2	$6x10^{-6}$	30.0	3.0	2.0	0	30	1.0	$7x10^{-3}$	0.90	0.50	0.30	0.10	0.03	$4x10^{-3}$
BWR 3	$2x10^{-5}$	30.0	3.0	2.0	25	20	1.0	$7x10^{-3}$	0.10	0.10	0.30	0.01	0.02	$3x10^{-3}$
BWR 4	$2x10^{-6}$	5.0	2.0	2.0	25	N/A	0.6	$7x10^{-4}$	$8x10^{-4}$	$5x10^{-3}$	$4x10^{-3}$	$6x10^{-4}$	$6x10^{-4}$	$1x10^{-4}$
BWR 5	$1x10^{-4}$	3.5	5.0	N/A	150	N/A	$5x10^{-4}$	$2x10^{-9}$	$6x10^{-11}$	$4x10^{-9}$	$8x10^{-12}$	$8x10^{-14}$	0	0

(a) A discussion of the isotopes used in the study is found in Appendix VI. Background on the isotope groups and release mechanisms is found in Appendix VII.

(b) Includes Mo, Rh, Tc, Co.

(c) Includes Nd, Y, Ce, Pr, La, Nb, Am, Cm, Pu, Np, Zr.

(d) A lower energy release rate than this value applies to part of the period over which the radioactivity is being released. The effect of lower energy release rates on consequences is found in Appendix VI.

Note: From Table 5-1 in the Main Report of WASH-1400 (Ref-1)

Table C.4 Geometry for the Population Distribution in the Consequence Model

Segment Number	Distance to Midpoint (miles)	Width of Segment Δr_k (miles)	Outer Radius (miles)
1	.25	.5	0.5
2	.75	.5	1.0
3	1.25	.5	1.5
4	1.75	.5	2.0
5	2.25	.5	2.5
6	2.75	.5	3.0
7	3.25	.5	3.5
8	3.75	.5	4.0
9	4.25	.5	4.5
10	4.75	.5	5.0
11	5.50	1.0	6.0
12	6.50	1.0	7.0
13	7.75	1.5	8.5
14	9.25	1.5	10.0
15	11.25	2.5	12.5
16	13.75	2.5	15.0
17	16.25	2.5	15.0
18	18.75	2.5	20.0
19	22.5	5.0	25.0
20	27.5	5.0	30.0
21	32.5	5.0	35.0
22	37.5	5.0	40.0
23	42.5	5.0	45.0
24	47.5	5.0	50.0
25	52.5	5.0	55.0
26	57.5	5.0	60.0
27	62.5	5.0	65.0
28	67.5	5.0	70.0
29	77.5	15.0	85.0
30	92.5	15.0	100.0
31	125.	50.0	150.0
32	175.	50.0	200.0
33	275.	150.0	350.0
34	425.	150.0	500.0

Table C.5 Population Distribution of Site A

OUTER RADIUS (MILES)	N	NNE	NE	ENE	E	ESE	SE	SSE	S	SSW	SW	WSW	W	WNW	NW	NNW
	******	******	******	******	******	******	******	DIRECTION	******	******	******	******	******	******	******	******
0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.0	0	0	517	0	0	0	0	0	0	0	0	0	0	0	0	0
1.5	0	0	0	0	1394	0	0	0	0	0	0	0	0	0	0	0
2.0	1420	0	0	0	0	0	0	0	0	0	0	0	569	1518	0	1335
2.5	0	856	0	0	0	0	0	0	0	0	777	0	0	0	0	0
3.0	0	43	0	0	0	0	0	0	0	0	0	0	0	0	0	1239
3.5	823	1019	2396	0	1542	0	0	0	0	0	0	0	941	1679	977	0
4.0	0	0	2867	5689	1179	0	0	0	0	0	0	0	0	0	0	968
4.5	0	0	5799	798	1020	0	0	0	0	0	0	0	0	0	0	0
5.0	0	652	6775	889	0	0	0	0	0	0	0	0	0	0	0	0
6.0	434	2726	7278	6175	782	0	0	0	0	0	0	399	1042	0	1547	0
7.0	915	858	4592	2204	0	0	0	0	0	0	0	0	0	0	0	0
8.5	0	4256	7938	3566	759	0	0	0	0	0	0	0	1139	2384	0	1088
10.0	2261	7599	364	1940	1600	462	0	0	0	0	0	45	2168	0	0	550
12.5	3541	6208	2030	9518	522	0	0	0	0	787	0	1134	5121	3957	0	0
15.0	3021	10909	3052	3205	1413	0	0	0	0	0	0	0	3457	3289	1020	1453
17.5	1688	26107	0	3373	1893	0	0	0	0	0	832	0	2311	2460	0	488
20.0	1787	7754	1211	15319	4089	0	0	0	886	24	3784	0	9029	2805	3168	4504
25.0	2770	10065	2486	6318	1687	0	1096	696	6530	4509	4692	0	9365	4610	5858	3685
30.0	21879	6800	1452	4621	963	0	0	0	1970	4416	6247	0	13674	13305	11742	9268
35.0	19621	11528	1476	5755	545	489	0	0	0	8304	4202	0	34394	36574	40040	12437
40.0	6610	14936	7371	22436	2999	0	0	0	0	6514	13068	0	178010	83520	156970	69793
45.0	6182	11700	55024	41517	0	0	0	0	0	865	14037	48	132151	59762	322301	42205
50.0	7970	10741	121117	72295	0	0	0	0	0	0	14988	15548	112374	172335	60077	48224
55.0	24863	35130	321289	53334	0	0	0	0	0	0	32641	32222	217120	43473	30237	58590
60.0	15838	26930	205818	109403	0	0	0	0	0	0	73599	54202	99506	16039	30356	187016
65.0	27124	50616	67253	41739	0	0	0	0	0	0	68147	100013	51920	13529	26717	175737
70.0	24593	203493	86026	132811	0	0	0	0	0	0	95166	220161	123614	16902	2410	84775
85.0	19121	354533	500129	62444	11307	0	0	0	0	0	141653	1090240	183464	43838	15469	90053
100.0	78311	478713	1686529	78092	1481	0	0	0	0	0	0	5815452	454492	163701	92020	54689
150.0	156574	884875	389710	38010	3774	0	0	0	0	0	476429	8233114	417610	184469	800860	133377
200.0	109916	330907	0	0	0	0	0	0	0	0	593538	5159235	707930	194843	289926	137918
350.0	156070	244386	819	0	0	0	0	0	0	0	1414621	6884925	1254929	2440634	419873	313433
500.0	0	0	0	0	0	0	0	0	0	165363	2855303	1465250	7580584	1325597	0	0

Table C.6 Population Distribution of Site B

OUTER RADIUS (MILES)	N	NNE	NE	ENE	E	ESE	SE	SSE	S	SSW	SW	WSW	W	WNW	NW	NNW
	DIRECTION															
0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4.5	0	0	0	0	0	0	0	0	87	0	0	0	0	0	0	0
5.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6.0	1396	0	0	0	0	1250	0	0	0	0	0	0	915	0	0	0
7.0	0	0	0	0	0	0	0	0	0	0	0	264	0	0	0	0
8.5	0	0	0	0	0	0	0	0	0	0	1024	0	0	0	0	0
10.0	0	0	0	1358	0	0	0	1029	0	1303	240	0	555	845	0	0
12.5	0	0	0	0	1457	0	0	0	2401	0	720	0	0	0	0	0
15.0	1112	0	1020	5723	0	1326	0	0	0	0	1076	1505	785	0	0	0
17.5	0	0	0	844	0	0	0	2073	0	0	1067	0	6792	538	461	0
20.0	0	0	340	0	0	1134	716	5214	2023	0	0	965	4613	452	0	0
25.0	1040	912	0	2117	3433	2280	49557	21187	3124	2862	558	971	1836	1201	2226	1031
30.0	4546	7585	2185	639	850	4482	142541	32266	1808	5307	921	1008	0	4281	1306	829
35.0	3225	4226	3270	851	3246	1841	14461	2769	2430	4858	5725	466	1415	9390	1139	16076
40.0	473	1899	2670	1020	2310	883	2933	1813	617	1613	1293	1653	2930	6307	3663	1654
45.0	2378	7157	21842	5785	12451	450	4882	3998	2339	1491	4011	3982	2511	15274	2869	3532
50.0	9160	41979	7235	4312	5423	3375	5112	3463	2952	1062	3769	5476	25979	891	5818	5894
55.0	4984	11168	4725	3199	3334	15282	2403	2330	3237	7397	3534	3809	9807	7425	14667	6995
60.0	9046	9470	4082	3924	3164	14833	1897	7613	2135	27118	0	2703	4640	6880	73877	31089
65.0	21370	37443	15888	4767	8190	38280	4076	25449	7934	19384	10425	3619	15285	9285	28138	8565
70.0	96446	176732	15226	2218	9966	5824	5462	6355	5966	35117	48355	669	3398	41898	32737	16944
85.0	75129	175670	25076	24004	48351	19418	21406	14176	14363	32871	91338	8517	48286	180558	71267	51179
100.0	84410	121590	37974	50060	59293	32545	12044	19270	15311	14005	24007	13037	31294	59540	39494	17099
150.0	222715	546872	257343	309093	172937	103035	296588	87053	64769	71130	84538	122658	193617	68705	233763	162022
200.0	179717	285104	655395	268252	162641	3310	0	1302	221572	99781	317577	973929	726125	148461	486811	390124
350.0	1138357	913554	1495084	1429043	171310	0	0	0	895335	509457	659040	1035683	1867137	1563248	952202	1034402
500.0	7626447	3120437	6465520	1285	0	0	0	0	2568382	111300	519240	1094509	1655700	1187267	3337521	5305572

APPENDIX D

TABLES FOR ESTIMATION OF THE WEIBULL PARAMETERS FROM THE MOMENTS

For the convenience of the calculation of the Gamma functions in estimating the Weibull parameters, the following quantities are given as functions of β in the range $.1 \leq \beta < 1.1$.

Table D.1: $[\Gamma(1+\frac{1}{\beta})]^2/\Gamma(1+\frac{2}{\beta})$

Table D.2: $\Gamma(1+\frac{1}{\beta})$

Table D.3: $\Gamma(1+\frac{2}{\beta})$

Table D.1 Table for Estimation of Weibull Parameters $\left[\Gamma(1+\frac{1}{\beta})\right]^2 / \Gamma(1+\frac{2}{\beta})$

β	0.0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.10	5.413E-06	6.179E-06	7.035E-06	7.989E-06	9.050E-06	1.023E-05	1.153E-05	1.297E-05	1.456E-05	1.630E-05
0.11	1.822E-05	2.032E-05	2.262E-05	2.513E-05	2.787E-05	3.084E-05	3.408E-05	3.759E-05	4.138E-05	4.549E-05
0.12	4.993E-05	5.471E-05	5.986E-05	6.539E-05	7.133E-05	7.770E-05	8.452E-05	9.182E-05	9.962E-05	1.079E-04
0.13	1.168E-04	1.262E-04	1.363E-04	1.469E-04	1.582E-04	1.702E-04	1.829E-04	1.964E-04	2.106E-04	2.255E-04
0.14	2.414E-04	2.580E-04	2.756E-04	2.941E-04	3.135E-04	3.339E-04	3.553E-04	3.778E-04	4.013E-04	4.260E-04
0.15	4.518E-04	4.788E-04	5.070E-04	5.364E-04	5.671E-04	5.992E-04	6.326E-04	6.674E-04	7.036E-04	7.412E-04
0.16	7.804E-04	8.211E-04	8.633E-04	9.072E-04	9.527E-04	9.998E-04	1.049E-03	1.099E-03	1.152E-03	1.206E-03
0.17	1.262E-03	1.320E-03	1.380E-03	1.442E-03	1.505E-03	1.571E-03	1.639E-03	1.709E-03	1.781E-03	1.855E-03
0.18	1.932E-03	2.010E-03	2.091E-03	2.174E-03	2.260E-03	2.348E-03	2.438E-03	2.531E-03	2.626E-03	2.723E-03
0.19	2.823E-03	2.926E-03	3.031E-03	3.139E-03	3.249E-03	3.362E-03	3.478E-03	3.596E-03	3.718E-03	3.842E-03
0.20	3.968E-03	4.098E-03	4.230E-03	4.365E-03	4.503E-03	4.644E-03	4.788E-03	4.935E-03	5.085E-03	5.238E-03
0.21	5.394E-03	5.553E-03	5.715E-03	5.880E-03	6.048E-03	6.219E-03	6.393E-03	6.571E-03	6.752E-03	6.936E-03
0.22	7.123E-03	7.313E-03	7.507E-03	7.703E-03	7.903E-03	8.107E-03	8.313E-03	8.523E-03	8.737E-03	8.953E-03
0.23	9.173E-03	9.396E-03	9.623E-03	9.853E-03	1.009E-02	1.032E-02	1.056E-02	1.081E-02	1.105E-02	1.130E-02
0.24	1.156E-02	1.182E-02	1.208E-02	1.234E-02	1.261E-02	1.288E-02	1.315E-02	1.343E-02	1.371E-02	1.400E-02
0.25	1.429E-02	1.458E-02	1.487E-02	1.517E-02	1.547E-02	1.578E-02	1.609E-02	1.640E-02	1.672E-02	1.704E-02
0.26	1.736E-02	1.769E-02	1.802E-02	1.835E-02	1.869E-02	1.903E-02	1.937E-02	1.972E-02	2.007E-02	2.042E-02
0.27	2.078E-02	2.114E-02	2.151E-02	2.187E-02	2.224E-02	2.262E-02	2.300E-02	2.338E-02	2.376E-02	2.415E-02
0.28	2.454E-02	2.494E-02	2.534E-02	2.574E-02	2.614E-02	2.655E-02	2.696E-02	2.738E-02	2.780E-02	2.822E-02
0.29	2.864E-02	2.907E-02	2.950E-02	2.993E-02	3.037E-02	3.081E-02	3.126E-02	3.170E-02	3.216E-02	3.261E-02
0.30	3.307E-02	3.353E-02	3.399E-02	3.445E-02	3.492E-02	3.540E-02	3.587E-02	3.635E-02	3.683E-02	3.732E-02
0.31	3.780E-02	3.829E-02	3.879E-02	3.928E-02	3.978E-02	4.029E-02	4.079E-02	4.130E-02	4.181E-02	4.232E-02
0.32	4.284E-02	4.336E-02	4.388E-02	4.441E-02	4.494E-02	4.547E-02	4.600E-02	4.654E-02	4.708E-02	4.762E-02
0.33	4.816E-02	4.871E-02	4.926E-02	4.981E-02	5.037E-02	5.093E-02	5.149E-02	5.205E-02	5.262E-02	5.319E-02
0.34	5.376E-02	5.433E-02	5.491E-02	5.549E-02	5.607E-02	5.665E-02	5.724E-02	5.782E-02	5.842E-02	5.901E-02
0.35	5.960E-02	6.020E-02	6.080E-02	6.140E-02	6.201E-02	6.262E-02	6.323E-02	6.384E-02	6.445E-02	6.507E-02
0.36	6.569E-02	6.631E-02	6.693E-02	6.755E-02	6.818E-02	6.881E-02	6.944E-02	7.008E-02	7.071E-02	7.135E-02
0.37	7.199E-02	7.263E-02	7.327E-02	7.392E-02	7.457E-02	7.522E-02	7.587E-02	7.652E-02	7.718E-02	7.783E-02
0.38	7.849E-02	7.915E-02	7.982E-02	8.048E-02	8.115E-02	8.181E-02	8.248E-02	8.316E-02	8.383E-02	8.450E-02
0.39	8.518E-02	8.586E-02	8.654E-02	8.722E-02	8.790E-02	8.859E-02	8.928E-02	8.996E-02	9.065E-02	9.135E-02
0.40	9.204E-02	9.273E-02	9.343E-02	9.413E-02	9.483E-02	9.553E-02	9.623E-02	9.693E-02	9.764E-02	9.834E-02
0.41	9.905E-02	9.976E-02	1.005E-01	1.012E-01	1.019E-01	1.026E-01	1.033E-01	1.040E-01	1.048E-01	1.055E-01
0.42	1.062E-01	1.069E-01	1.076E-01	1.084E-01	1.091E-01	1.098E-01	1.105E-01	1.113E-01	1.120E-01	1.127E-01
0.43	1.135E-01	1.142E-01	1.149E-01	1.157E-01	1.164E-01	1.171E-01	1.179E-01	1.186E-01	1.194E-01	1.201E-01
0.44	1.208E-01	1.216E-01	1.223E-01	1.231E-01	1.238E-01	1.246E-01	1.253E-01	1.261E-01	1.268E-01	1.276E-01
0.45	1.283E-01	1.291E-01	1.298E-01	1.306E-01	1.313E-01	1.321E-01	1.328E-01	1.336E-01	1.344E-01	1.351E-01
0.46	1.359E-01	1.366E-01	1.374E-01	1.382E-01	1.389E-01	1.397E-01	1.404E-01	1.412E-01	1.420E-01	1.427E-01
0.47	1.435E-01	1.443E-01	1.450E-01	1.458E-01	1.466E-01	1.473E-01	1.481E-01	1.489E-01	1.496E-01	1.504E-01
0.48	1.512E-01	1.519E-01	1.527E-01	1.535E-01	1.543E-01	1.550E-01	1.558E-01	1.566E-01	1.574E-01	1.581E-01
0.49	1.589E-01	1.597E-01	1.605E-01	1.612E-01	1.620E-01	1.628E-01	1.636E-01	1.643E-01	1.651E-01	1.659E-01
0.50	1.667E-01	1.674E-01	1.682E-01	1.690E-01	1.698E-01	1.706E-01	1.713E-01	1.721E-01	1.729E-01	1.737E-01
0.51	1.745E-01	1.752E-01	1.760E-01	1.768E-01	1.776E-01	1.784E-01	1.791E-01	1.799E-01	1.807E-01	1.815E-01
0.52	1.823E-01	1.830E-01	1.838E-01	1.846E-01	1.854E-01	1.862E-01	1.870E-01	1.877E-01	1.885E-01	1.893E-01
0.53	1.901E-01	1.909E-01	1.916E-01	1.924E-01	1.932E-01	1.940E-01	1.948E-01	1.956E-01	1.963E-01	1.971E-01
0.54	1.979E-01	1.987E-01	1.995E-01	2.002E-01	2.010E-01	2.018E-01	2.026E-01	2.034E-01	2.041E-01	2.049E-01
0.55	2.057E-01	2.065E-01	2.073E-01	2.080E-01	2.088E-01	2.096E-01	2.104E-01	2.112E-01	2.119E-01	2.127E-01
0.56	2.135E-01	2.143E-01	2.151E-01	2.158E-01	2.166E-01	2.174E-01	2.182E-01	2.190E-01	2.197E-01	2.205E-01
0.57	2.213E-01	2.221E-01	2.228E-01	2.236E-01	2.244E-01	2.252E-01	2.259E-01	2.267E-01	2.275E-01	2.283E-01
0.58	2.290E-01	2.298E-01	2.306E-01	2.314E-01	2.321E-01	2.329E-01	2.337E-01	2.344E-01	2.352E-01	2.360E-01
0.59	2.368E-01	2.375E-01	2.383E-01	2.391E-01	2.398E-01	2.406E-01	2.414E-01	2.421E-01	2.429E-01	2.437E-01

Table D.1 (continued) $\left[\Gamma(1+\frac{1}{\beta})\right]^2/\Gamma(1+\frac{2}{\beta})$

β	0.0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.60	2.445E-01	2.452E-01	2.460E-01	2.468E-01	2.475E-01	2.483E-01	2.490E-01	2.498E-01	2.506E-01	2.513E-01
0.61	2.521E-01	2.529E-01	2.536E-01	2.544E-01	2.551E-01	2.559E-01	2.567E-01	2.574E-01	2.582E-01	2.589E-01
0.62	2.597E-01	2.605E-01	2.612E-01	2.620E-01	2.627E-01	2.635E-01	2.642E-01	2.650E-01	2.658E-01	2.665E-01
0.63	2.673E-01	2.680E-01	2.688E-01	2.695E-01	2.703E-01	2.710E-01	2.718E-01	2.725E-01	2.733E-01	2.740E-01
0.64	2.748E-01	2.755E-01	2.763E-01	2.770E-01	2.778E-01	2.785E-01	2.793E-01	2.800E-01	2.807E-01	2.815E-01
0.65	2.822E-01	2.830E-01	2.837E-01	2.845E-01	2.852E-01	2.859E-01	2.867E-01	2.874E-01	2.881E-01	2.889E-01
0.66	2.896E-01	2.904E-01	2.911E-01	2.918E-01	2.926E-01	2.933E-01	2.940E-01	2.948E-01	2.955E-01	2.962E-01
0.67	2.970E-01	2.977E-01	2.984E-01	2.992E-01	2.999E-01	3.006E-01	3.013E-01	3.021E-01	3.028E-01	3.035E-01
0.68	3.042E-01	3.050E-01	3.057E-01	3.064E-01	3.071E-01	3.079E-01	3.086E-01	3.093E-01	3.100E-01	3.107E-01
0.69	3.115E-01	3.122E-01	3.129E-01	3.136E-01	3.143E-01	3.150E-01	3.158E-01	3.165E-01	3.172E-01	3.179E-01
0.70	3.186E-01	3.193E-01	3.200E-01	3.207E-01	3.214E-01	3.222E-01	3.229E-01	3.236E-01	3.243E-01	3.250E-01
0.71	3.257E-01	3.264E-01	3.271E-01	3.278E-01	3.285E-01	3.292E-01	3.299E-01	3.306E-01	3.313E-01	3.320E-01
0.72	3.327E-01	3.334E-01	3.341E-01	3.348E-01	3.355E-01	3.362E-01	3.369E-01	3.376E-01	3.383E-01	3.390E-01
0.73	3.396E-01	3.403E-01	3.410E-01	3.417E-01	3.424E-01	3.431E-01	3.438E-01	3.445E-01	3.452E-01	3.458E-01
0.74	3.465E-01	3.472E-01	3.479E-01	3.486E-01	3.493E-01	3.499E-01	3.506E-01	3.513E-01	3.520E-01	3.527E-01
0.75	3.533E-01	3.540E-01	3.547E-01	3.554E-01	3.560E-01	3.567E-01	3.574E-01	3.580E-01	3.587E-01	3.594E-01
0.76	3.601E-01	3.607E-01	3.614E-01	3.621E-01	3.627E-01	3.634E-01	3.641E-01	3.647E-01	3.654E-01	3.661E-01
0.77	3.667E-01	3.674E-01	3.680E-01	3.687E-01	3.694E-01	3.700E-01	3.707E-01	3.713E-01	3.720E-01	3.727E-01
0.78	3.733E-01	3.740E-01	3.746E-01	3.753E-01	3.759E-01	3.766E-01	3.772E-01	3.779E-01	3.785E-01	3.792E-01
0.79	3.798E-01	3.805E-01	3.811E-01	3.818E-01	3.824E-01	3.831E-01	3.837E-01	3.843E-01	3.850E-01	3.856E-01
0.80	3.863E-01	3.869E-01	3.875E-01	3.882E-01	3.888E-01	3.895E-01	3.901E-01	3.907E-01	3.914E-01	3.920E-01
0.81	3.926E-01	3.933E-01	3.939E-01	3.945E-01	3.952E-01	3.958E-01	3.964E-01	3.970E-01	3.977E-01	3.983E-01
0.82	3.989E-01	3.996E-01	4.002E-01	4.008E-01	4.014E-01	4.020E-01	4.027E-01	4.033E-01	4.039E-01	4.045E-01
0.83	4.051E-01	4.058E-01	4.064E-01	4.070E-01	4.076E-01	4.082E-01	4.088E-01	4.095E-01	4.101E-01	4.107E-01
0.84	4.113E-01	4.119E-01	4.125E-01	4.131E-01	4.137E-01	4.143E-01	4.150E-01	4.156E-01	4.162E-01	4.168E-01
0.85	4.174E-01	4.180E-01	4.186E-01	4.192E-01	4.198E-01	4.204E-01	4.210E-01	4.216E-01	4.222E-01	4.228E-01
0.86	4.234E-01	4.240E-01	4.246E-01	4.252E-01	4.258E-01	4.263E-01	4.269E-01	4.275E-01	4.281E-01	4.287E-01
0.87	4.293E-01	4.299E-01	4.305E-01	4.311E-01	4.317E-01	4.322E-01	4.328E-01	4.334E-01	4.340E-01	4.346E-01
0.88	4.352E-01	4.357E-01	4.363E-01	4.369E-01	4.375E-01	4.381E-01	4.386E-01	4.392E-01	4.398E-01	4.404E-01
0.89	4.409E-01	4.415E-01	4.421E-01	4.427E-01	4.432E-01	4.438E-01	4.444E-01	4.450E-01	4.455E-01	4.461E-01
0.90	4.467E-01	4.472E-01	4.478E-01	4.484E-01	4.489E-01	4.495E-01	4.501E-01	4.506E-01	4.512E-01	4.517E-01
0.91	4.523E-01	4.529E-01	4.534E-01	4.540E-01	4.545E-01	4.551E-01	4.557E-01	4.562E-01	4.568E-01	4.573E-01
0.92	4.579E-01	4.584E-01	4.590E-01	4.595E-01	4.601E-01	4.606E-01	4.612E-01	4.617E-01	4.623E-01	4.628E-01
0.93	4.634E-01	4.639E-01	4.645E-01	4.650E-01	4.656E-01	4.661E-01	4.666E-01	4.672E-01	4.677E-01	4.683E-01
0.94	4.688E-01	4.694E-01	4.699E-01	4.704E-01	4.710E-01	4.715E-01	4.720E-01	4.726E-01	4.731E-01	4.736E-01
0.95	4.742E-01	4.747E-01	4.752E-01	4.758E-01	4.763E-01	4.768E-01	4.774E-01	4.779E-01	4.784E-01	4.789E-01
0.96	4.795E-01	4.800E-01	4.805E-01	4.811E-01	4.816E-01	4.821E-01	4.826E-01	4.831E-01	4.837E-01	4.842E-01
0.97	4.847E-01	4.852E-01	4.857E-01	4.863E-01	4.868E-01	4.873E-01	4.878E-01	4.883E-01	4.888E-01	4.894E-01
0.98	4.899E-01	4.904E-01	4.909E-01	4.914E-01	4.919E-01	4.924E-01	4.929E-01	4.934E-01	4.940E-01	4.945E-01
0.99	4.950E-01	4.955E-01	4.960E-01	4.965E-01	4.970E-01	4.975E-01	4.980E-01	4.985E-01	4.990E-01	4.995E-01
1.00	5.000E-01	5.005E-01	5.010E-01	5.015E-01	5.020E-01	5.025E-01	5.030E-01	5.035E-01	5.040E-01	5.045E-01
1.01	5.050E-01	5.055E-01	5.060E-01	5.064E-01	5.069E-01	5.074E-01	5.079E-01	5.084E-01	5.089E-01	5.094E-01
1.02	5.099E-01	5.104E-01	5.108E-01	5.113E-01	5.118E-01	5.123E-01	5.128E-01	5.133E-01	5.137E-01	5.142E-01
1.03	5.147E-01	5.152E-01	5.157E-01	5.162E-01	5.166E-01	5.171E-01	5.176E-01	5.181E-01	5.185E-01	5.190E-01
1.04	5.195E-01	5.200E-01	5.204E-01	5.209E-01	5.214E-01	5.219E-01	5.223E-01	5.228E-01	5.233E-01	5.237E-01
1.05	5.242E-01	5.247E-01	5.251E-01	5.256E-01	5.261E-01	5.265E-01	5.270E-01	5.275E-01	5.279E-01	5.284E-01
1.06	5.289E-01	5.293E-01	5.298E-01	5.303E-01	5.307E-01	5.312E-01	5.316E-01	5.321E-01	5.325E-01	5.330E-01
1.07	5.335E-01	5.339E-01	5.344E-01	5.348E-01	5.353E-01	5.357E-01	5.362E-01	5.365E-01	5.371E-01	5.375E-01
1.08	5.380E-01	5.384E-01	5.389E-01	5.393E-01	5.398E-01	5.402E-01	5.407E-01	5.411E-01	5.416E-01	5.420E-01
1.09	5.425E-01	5.429E-01	5.434E-01	5.438E-01	5.443E-01	5.447E-01	5.451E-01	5.456E-01	5.460E-01	5.465E-01

Table D.2 Table for Estimation of Weibull Parameters $\Gamma(1+\frac{1}{\beta})$

β	0.0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.10	3.629E 06	2.876E 06	2.292E 06	1.837E 06	1.479E 06	1.197E 06	9.736E 05	7.955E 05	6.529E 05	5.382E 05
0.11	4.455E 05	3.703E 05	3.091E 05	2.589E 05	2.177E 05	1.838E 05	1.557E 05	1.323E 05	1.128E 05	9.652E 04
0.12	8.283E 04	7.131E 04	6.157E 04	5.332E 04	4.630E 04	4.032E 04	3.520E 04	3.082E 04	2.705E 04	2.380E 04
0.13	2.099E 04	1.855E 04	1.644E 04	1.459E 04	1.299E 04	1.158E 04	1.035E 04	9.267E 03	8.315E 03	7.474E 03
0.14	6.731E 03	6.073E 03	5.489E 03	4.970E 03	4.507E 03	4.094E 03	3.726E 03	3.395E 03	3.099E 03	2.833E 03
0.15	2.594E 03	2.378E 03	2.183E 03	2.007E 03	1.848E 03	1.703E 03	1.572E 03	1.453E 03	1.345E 03	1.246E 03
0.16	1.155E 03	1.073E 03	9.975E 02	9.283E 02	8.650E 02	8.069E 02	7.533E 02	7.040E 02	6.587E 02	6.169E 02
0.17	5.782E 02	5.426E 02	5.095E 02	4.790E 02	4.506E 02	4.243E 02	3.999E 02	3.772E 02	3.561E 02	3.364E 02
0.18	3.181E 02	3.010E 02	2.851E 02	2.702E 02	2.562E 02	2.432E 02	2.309E 02	2.195E 02	2.087E 02	1.986E 02
0.19	1.892E 02	1.803E 02	1.719E 02	1.640E 02	1.566E 02	1.496E 02	1.430E 02	1.357E 02	1.308E 02	1.253E 02
0.20	1.200E 02	1.150E 02	1.103E 02	1.058E 02	1.016E 02	9.759E 01	9.378E 01	9.016E 01	8.673E 01	8.346E 01
0.21	8.036E 01	7.740E 01	7.459E 01	7.191E 01	6.936E 01	6.693E 01	6.461E 01	6.239E 01	6.028E 01	5.826E 01
0.22	5.633E 01	5.449E 01	5.272E 01	5.104E 01	4.942E 01	4.788E 01	4.640E 01	4.493E 01	4.361E 01	4.231E 01
0.23	4.106E 01	3.986E 01	3.870E 01	3.759E 01	3.653E 01	3.551E 01	3.452E 01	3.357E 01	3.266E 01	3.179E 01
0.24	3.094E 01	3.013E 01	2.935E 01	2.859E 01	2.786E 01	2.716E 01	2.648E 01	2.583E 01	2.520E 01	2.459E 01
0.25	2.400E 01	2.343E 01	2.288E 01	2.235E 01	2.184E 01	2.134E 01	2.086E 01	2.039E 01	1.994E 01	1.951E 01
0.26	1.909E 01	1.868E 01	1.828E 01	1.790E 01	1.753E 01	1.716E 01	1.681E 01	1.647E 01	1.614E 01	1.582E 01
0.27	1.551E 01	1.521E 01	1.492E 01	1.463E 01	1.436E 01	1.409E 01	1.383E 01	1.357E 01	1.333E 01	1.309E 01
0.28	1.285E 01	1.263E 01	1.240E 01	1.219E 01	1.198E 01	1.177E 01	1.158E 01	1.138E 01	1.119E 01	1.101E 01
0.29	1.083E 01	1.065E 01	1.048E 01	1.032E 01	1.015E 01	9.997E 00	9.842E 00	9.691E 00	9.544E 00	9.401E 00
0.30	9.261E 00	9.124E 00	8.990E 00	8.859E 00	8.732E 00	8.607E 00	8.485E 00	8.365E 00	8.250E 00	8.136E 00
0.31	8.024E 00	7.916E 00	7.809E 00	7.705E 00	7.603E 00	7.503E 00	7.406E 00	7.310E 00	7.217E 00	7.125E 00
0.32	7.035E 00	6.948E 00	6.862E 00	6.778E 00	6.695E 00	6.614E 00	6.535E 00	6.457E 00	6.381E 00	6.307E 00
0.33	6.234E 00	6.162E 00	6.092E 00	6.023E 00	5.955E 00	5.889E 00	5.824E 00	5.760E 00	5.697E 00	5.636E 00
0.34	5.575E 00	5.516E 00	5.458E 00	5.401E 00	5.345E 00	5.290E 00	5.236E 00	5.183E 00	5.131E 00	5.079E 00
0.35	5.029E 00	4.980E 00	4.931E 00	4.883E 00	4.836E 00	4.790E 00	4.745E 00	4.700E 00	4.657E 00	4.614E 00
0.36	4.571E 00	4.530E 00	4.489E 00	4.448E 00	4.409E 00	4.370E 00	4.331E 00	4.294E 00	4.256E 00	4.220E 00
0.37	4.184E 00	4.148E 00	4.114E 00	4.079E 00	4.045E 00	4.012E 00	3.979E 00	3.947E 00	3.915E 00	3.884E 00
0.38	3.853E 00	3.823E 00	3.793E 00	3.764E 00	3.735E 00	3.706E 00	3.678E 00	3.650E 00	3.623E 00	3.596E 00
0.39	3.569E 00	3.543E 00	3.517E 00	3.492E 00	3.467E 00	3.442E 00	3.418E 00	3.394E 00	3.370E 00	3.346E 00
0.40	3.323E 00	3.301E 00	3.278E 00	3.256E 00	3.234E 00	3.213E 00	3.191E 00	3.170E 00	3.150E 00	3.129E 00
0.41	3.109E 00	3.089E 00	3.070E 00	3.050E 00	3.031E 00	3.012E 00	2.994E 00	2.975E 00	2.957E 00	2.939E 00
0.42	2.921E 00	2.904E 00	2.886E 00	2.869E 00	2.853E 00	2.836E 00	2.820E 00	2.803E 00	2.787E 00	2.771E 00
0.43	2.756E 00	2.740E 00	2.725E 00	2.710E 00	2.695E 00	2.680E 00	2.666E 00	2.651E 00	2.637E 00	2.623E 00
0.44	2.609E 00	2.595E 00	2.582E 00	2.568E 00	2.555E 00	2.542E 00	2.529E 00	2.516E 00	2.504E 00	2.491E 00
0.45	2.479E 00	2.466E 00	2.454E 00	2.442E 00	2.430E 00	2.419E 00	2.407E 00	2.396E 00	2.384E 00	2.373E 00
0.46	2.362E 00	2.351E 00	2.340E 00	2.329E 00	2.319E 00	2.308E 00	2.298E 00	2.287E 00	2.277E 00	2.267E 00
0.47	2.257E 00	2.247E 00	2.238E 00	2.228E 00	2.218E 00	2.209E 00	2.199E 00	2.190E 00	2.181E 00	2.172E 00
0.48	2.163E 00	2.154E 00	2.145E 00	2.136E 00	2.128E 00	2.119E 00	2.111E 00	2.102E 00	2.094E 00	2.086E 00
0.49	2.077E 00	2.069E 00	2.061E 00	2.053E 00	2.046E 00	2.038E 00	2.030E 00	2.022E 00	2.015E 00	2.007E 00
0.50	2.000E 00	1.993E 00	1.985E 00	1.978E 00	1.971E 00	1.964E 00	1.957E 00	1.950E 00	1.943E 00	1.936E 00
0.51	1.930E 00	1.923E 00	1.916E 00	1.910E 00	1.903E 00	1.897E 00	1.890E 00	1.884E 00	1.878E 00	1.871E 00
0.52	1.865E 00	1.859E 00	1.853E 00	1.847E 00	1.841E 00	1.835E 00	1.829E 00	1.823E 00	1.818E 00	1.812E 00
0.53	1.806E 00	1.801E 00	1.795E 00	1.790E 00	1.784E 00	1.779E 00	1.773E 00	1.768E 00	1.763E 00	1.757E 00
0.54	1.752E 00	1.747E 00	1.742E 00	1.737E 00	1.732E 00	1.727E 00	1.722E 00	1.717E 00	1.712E 00	1.707E 00
0.55	1.702E 00	1.698E 00	1.693E 00	1.688E 00	1.684E 00	1.679E 00	1.674E 00	1.670E 00	1.665E 00	1.661E 00
0.56	1.657E 00	1.652E 00	1.648E 00	1.643E 00	1.639E 00	1.635E 00	1.631E 00	1.627E 00	1.622E 00	1.618E 00
0.57	1.614E 00	1.610E 00	1.606E 00	1.602E 00	1.598E 00	1.594E 00	1.590E 00	1.586E 00	1.583E 00	1.579E 00
0.58	1.575E 00	1.571E 00	1.567E 00	1.564E 00	1.560E 00	1.556E 00	1.553E 00	1.549E 00	1.546E 00	1.542E 00
0.59	1.538E 00	1.535E 00	1.531E 00	1.528E 00	1.525E 00	1.521E 00	1.518E 00	1.514E 00	1.511E 00	1.508E 00

Table D.2 (continued) $\Gamma(1+\frac{1}{\beta})$

β	0.0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.60	1.505E 00	1.501E 00	1.498E 00	1.495E 00	1.492E 00	1.489E 00	1.485E 00	1.482E 00	1.479E 00	1.476E 00
0.61	1.473E 00	1.470E 00	1.467E 00	1.464E 00	1.461E 00	1.458E 00	1.455E 00	1.452E 00	1.449E 00	1.446E 00
0.62	1.444E 00	1.441E 00	1.438E 00	1.435E 00	1.432E 00	1.430E 00	1.427E 00	1.424E 00	1.421E 00	1.419E 00
0.63	1.416E 00	1.413E 00	1.411E 00	1.408E 00	1.406E 00	1.403E 00	1.400E 00	1.398E 00	1.395E 00	1.393E 00
0.64	1.390E 00	1.388E 00	1.385E 00	1.383E 00	1.381E 00	1.378E 00	1.376E 00	1.373E 00	1.371E 00	1.369E 00
0.65	1.366E 00	1.364E 00	1.362E 00	1.359E 00	1.357E 00	1.355E 00	1.353E 00	1.350E 00	1.348E 00	1.346E 00
0.66	1.344E 00	1.341E 00	1.339E 00	1.337E 00	1.335E 00	1.333E 00	1.331E 00	1.329E 00	1.327E 00	1.324E 00
0.67	1.322E 00	1.320E 00	1.318E 00	1.316E 00	1.314E 00	1.312E 00	1.310E 00	1.308E 00	1.306E 00	1.304E 00
0.68	1.302E 00	1.300E 00	1.299E 00	1.297E 00	1.295E 00	1.293E 00	1.291E 00	1.289E 00	1.287E 00	1.285E 00
0.69	1.284E 00	1.282E 00	1.280E 00	1.278E 00	1.276E 00	1.275E 00	1.273E 00	1.271E 00	1.269E 00	1.268E 00
0.70	1.266E 00	1.264E 00	1.262E 00	1.261E 00	1.259E 00	1.257E 00	1.256E 00	1.254E 00	1.252E 00	1.251E 00
0.71	1.249E 00	1.247E 00	1.246E 00	1.244E 00	1.243E 00	1.241E 00	1.239E 00	1.238E 00	1.236E 00	1.235E 00
0.72	1.233E 00	1.232E 00	1.230E 00	1.229E 00	1.227E 00	1.226E 00	1.224E 00	1.223E 00	1.221E 00	1.220E 00
0.73	1.218E 00	1.217E 00	1.215E 00	1.214E 00	1.212E 00	1.211E 00	1.210E 00	1.208E 00	1.207E 00	1.205E 00
0.74	1.204E 00	1.203E 00	1.201E 00	1.200E 00	1.199E 00	1.197E 00	1.196E 00	1.195E 00	1.193E 00	1.192E 00
0.75	1.191E 00	1.189E 00	1.188E 00	1.187E 00	1.185E 00	1.184E 00	1.183E 00	1.182E 00	1.180E 00	1.179E 00
0.76	1.178E 00	1.177E 00	1.175E 00	1.174E 00	1.173E 00	1.172E 00	1.171E 00	1.169E 00	1.168E 00	1.167E 00
0.77	1.166E 00	1.165E 00	1.163E 00	1.162E 00	1.161E 00	1.160E 00	1.159E 00	1.158E 00	1.157E 00	1.155E 00
0.78	1.154E 00	1.153E 00	1.152E 00	1.151E 00	1.150E 00	1.149E 00	1.148E 00	1.147E 00	1.146E 00	1.144E 00
0.79	1.143E 00	1.142E 00	1.141E 00	1.140E 00	1.139E 00	1.138E 00	1.137E 00	1.136E 00	1.135E 00	1.134E 00
0.80	1.133E 00	1.132E 00	1.131E 00	1.130E 00	1.129E 00	1.128E 00	1.127E 00	1.126E 00	1.125E 00	1.124E 00
0.81	1.123E 00	1.122E 00	1.121E 00	1.120E 00	1.119E 00	1.118E 00	1.117E 00	1.116E 00	1.116E 00	1.115E 00
0.82	1.114E 00	1.113E 00	1.112E 00	1.111E 00	1.110E 00	1.109E 00	1.108E 00	1.107E 00	1.106E 00	1.106E 00
0.83	1.105E 00	1.104E 00	1.103E 00	1.102E 00	1.101E 00	1.100E 00	1.100E 00	1.099E 00	1.098E 00	1.097E 00
0.84	1.096E 00	1.095E 00	1.094E 00	1.094E 00	1.093E 00	1.092E 00	1.091E 00	1.090E 00	1.090E 00	1.089E 00
0.85	1.088E 00	1.087E 00	1.086E 00	1.086E 00	1.085E 00	1.084E 00	1.083E 00	1.082E 00	1.082E 00	1.081E 00
0.86	1.080E 00	1.079E 00	1.079E 00	1.078E 00	1.077E 00	1.076E 00	1.076E 00	1.075E 00	1.074E 00	1.073E 00
0.87	1.073E 00	1.072E 00	1.071E 00	1.071E 00	1.070E 00	1.069E 00	1.068E 00	1.068E 00	1.067E 00	1.066E 00
0.88	1.066E 00	1.065E 00	1.064E 00	1.063E 00	1.063E 00	1.062E 00	1.061E 00	1.061E 00	1.060E 00	1.059E 00
0.89	1.059E 00	1.058E 00	1.057E 00	1.057E 00	1.056E 00	1.055E 00	1.055E 00	1.054E 00	1.053E 00	1.053E 00
0.90	1.052E 00	1.052E 00	1.051E 00	1.050E 00	1.050E 00	1.049E 00	1.048E 00	1.048E 00	1.047E 00	1.047E 00
0.91	1.046E 00	1.045E 00	1.045E 00	1.044E 00	1.043E 00	1.043E 00	1.042E 00	1.042E 00	1.041E 00	1.041E 00
0.92	1.040E 00	1.039E 00	1.039E 00	1.038E 00	1.038E 00	1.037E 00	1.036E 00	1.036E 00	1.035E 00	1.035E 00
0.93	1.034E 00	1.034E 00	1.033E 00	1.033E 00	1.032E 00	1.031E 00	1.031E 00	1.030E 00	1.030E 00	1.029E 00
0.94	1.029E 00	1.028E 00	1.028E 00	1.027E 00	1.027E 00	1.026E 00	1.025E 00	1.025E 00	1.024E 00	1.024E 00
0.95	1.023E 00	1.023E 00	1.022E 00	1.022E 00	1.021E 00	1.021E 00	1.020E 00	1.020E 00	1.019E 00	1.019E 00
0.96	1.018E 00	1.018E 00	1.017E 00	1.017E 00	1.016E 00	1.016E 00	1.015E 00	1.015E 00	1.014E 00	1.014E 00
0.97	1.013E 00	1.013E 00	1.013E 00	1.012E 00	1.012E 00	1.011E 00	1.011E 00	1.010E 00	1.010E 00	1.009E 00
0.98	1.009E 00	1.008E 00	1.008E 00	1.007E 00	1.007E 00	1.007E 00	1.006E 00	1.006E 00	1.005E 00	1.005E 00
0.99	1.004E 00	1.004E 00	1.003E 00	1.003E 00	1.003E 00	1.002E 00	1.002E 00	1.001E 00	1.001E 00	1.000E 00
1.00	1.000E 00	9.996E-01	9.992E-01	9.987E-01	9.983E-01	9.979E-01	9.975E-01	9.971E-01	9.967E-01	9.963E-01
1.01	9.959E-01	9.954E-01	9.950E-01	9.946E-01	9.942E-01	9.938E-01	9.934E-01	9.930E-01	9.927E-01	9.923E-01
1.02	9.919E-01	9.915E-01	9.911E-01	9.907E-01	9.903E-01	9.899E-01	9.895E-01	9.892E-01	9.888E-01	9.884E-01
1.03	9.880E-01	9.877E-01	9.873E-01	9.869E-01	9.865E-01	9.862E-01	9.858E-01	9.854E-01	9.851E-01	9.847E-01
1.04	9.843E-01	9.840E-01	9.836E-01	9.833E-01	9.829E-01	9.826E-01	9.822E-01	9.818E-01	9.815E-01	9.811E-01
1.05	9.808E-01	9.804E-01	9.801E-01	9.798E-01	9.794E-01	9.791E-01	9.787E-01	9.784E-01	9.780E-01	9.777E-01
1.06	9.774E-01	9.770E-01	9.767E-01	9.764E-01	9.760E-01	9.757E-01	9.754E-01	9.751E-01	9.747E-01	9.744E-01
1.07	9.741E-01	9.738E-01	9.734E-01	9.731E-01	9.728E-01	9.725E-01	9.722E-01	9.719E-01	9.715E-01	9.712E-01
1.08	9.709E-01	9.706E-01	9.703E-01	9.700E-01	9.697E-01	9.694E-01	9.691E-01	9.688E-01	9.685E-01	9.682E-01
1.09	9.679E-01	9.676E-01	9.673E-01	9.670E-01	9.667E-01	9.664E-01	9.661E-01	9.658E-01	9.655E-01	9.652E-01

Table D.3 Table for Estimation of Weibull Parameters $\Gamma(1+\frac{2}{\beta})$

β	0.0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	.008	0.009
0.10	2.433E 18	1.339E 18	7.470E 17	4.223E 17	2.418E 17	1.401E 17	8.220E 16	4.878E 16	2.928E 16	1.776E 16
0.11	1.089E 16	6.748E 15	4.222E 15	2.668E 15	1.701E 15	1.[illegible]95E 15	7.11[illegible]E 14	4.656E 14	3.[illegible]75E 14	2.[illegible]48E 14
0.12	1.374E 14	9.295E 13	6.334E 13	4.348E 13	3.006E 13	2.092E 13	1.466E 13	1.034E 13	7.343E 12	5.246E 12
0.13	3.771E 12	2.726E 12	1.982E 12	1.450E 12	1.066E 12	7.880E 11	5.856E 11	4.374E 11	3.283E 11	2.477E 11
0.14	1.877E 11	1.429E 11	1.093E 11	8.398E 10	6.480E 10	5.021E 10	3.906E 10	3.051E 10	2.393E 10	1.884E 10
0.15	1.489E 10	1.181E 10	9.401E 09	7.510E 09	6.020E 09	4.842E 09	3.908E 09	3.164E 09	2.569E 09	2.093E 09
0.16	1.711E 09	1.402E 09	1.152E 09	9.500E 08	7.853E 08	6.51[illegible]E 08	5.411E 08	4.5[illegible]9E 08	3.767E 08	3.[illegible]55E 08
0.17	2.650E 08	2.230E 08	1.882E 08	1.591E 08	1.349E 08	1.146E 08	9.759E 07	8.327E 07	7.121E 07	6.1[illegible]2E 07
0.18	5.239E 07	4.508E 07	3.886E 07	3.357E 07	2.905E 07	2.518E 07	2.188E 07	1.904E 07	1.659E 07	1.449E 07
0.19	1.267E 07	1.111E 07	9.747E 06	8.568E 06	7.544E 06	6.653E 06	5.876E 06	5.197E 06	4.604E 06	4.085E 06
0.20	3.629E 06	3.229E 06	2.876E 06	2.566E 06	2.292E 06	2.051E 06	1.837E 06	1.647E 06	1.479E 06	1.330E 06
0.21	1.197E 06	1.079E 06	9.736E 05	8.795E 05	7.955E 05	7.202E 05	6.529E 05	5.924E 05	5.382E 05	4.894E 05
0.22	4.455E 05	4.060E 05	3.703E 05	3.381E 05	3.091E 05	2.827E 05	2.589E 05	2.373E 05	2.177E 05	1.999E 05
0.23	1.838E 05	1.691E 05	1.557E 05	1.434E 05	1.323E 05	1.221E 05	1.128E 05	1.043E 05	9.652E 04	8.936E 04
0.24	8.283E 04	7.683E 04	7.131E 04	6.624E 04	6.157E 04	5.728E 04	5.332E 04	4.967E 04	4.630E 04	4.319E 04
0.25	4.032E 04	3.756E 04	3.52[illegible]E 04	3.293E 04	3.082E 04	2.886E 04	2.705E 04	2.536E 04	2.[illegible]80E 04	2.234E 04
0.26	2.099E 04	1.972E 04	1.855E 04	1.746E 04	1.644E 04	1.548E 04	1.459E 04	1.376E 04	1.299E 04	1.226E 04
0.27	1.158E 04	1.095E 04	1.035E 04	9.791E 03	9.267E 03	8.776E 03	8.315E 03	7.881E 03	7.474E 03	7.[illegible]91E 03
0.28	6.731E 03	6.392E 03	6.073E 03	5.772E 03	5.489E 03	5.222E 03	4.969E 03	4.732E 03	4.507E 03	4.295E 03
0.29	4.094E 03	3.905E 03	3.726E 03	3.556E 03	3.395E 03	3.243E 03	3.099E 03	2.962E 03	2.833E 03	2.710E 03
0.30	2.594E 03	2.483E 03	2.378E 03	2.278E 03	2.183E 03	2.[illegible]93E 03	2.007E 03	1.925E 03	1.848E 03	1.774E 03
0.31	1.703E 03	1.636E 03	1.572E 03	1.511E 03	1.453E 03	1.397E 03	1.345E 03	1.294E 03	1.246E 03	1.199E 03
0.32	1.155E 03	1.113E 03	1.07[illegible]E 03	1.034E 03	9.975E 02	9.622E 02	9.283E 02	8.960E 02	8.650E 02	8.353E 02
0.33	8.068E 02	7.795E 02	7.533E 02	7.281E 02	7.040E 02	6.8[illegible]9E 02	6.587E 02	6.374E 02	6.169E 02	5.972E 02
0.34	5.782E 02	5.600E 02	5.426E 02	5.257E 02	5.095E 02	4.940E 02	4.79[illegible]E 02	4.645E 02	4.5[illegible]6E 02	4.372E 02
0.35	4.243E 02	4.119E 02	3.999E 02	3.884E 02	3.772E 02	3.665E 02	3.561E 02	3.461E 02	3.364E 02	3.271E 02
0.36	3.181E 02	3.094E 02	3.010E 02	2.929E 02	2.851E 02	2.775E 02	2.702E 02	2.631E 02	2.562E 02	2.496E 02
0.37	2.432E 02	2.369E 02	2.309E 02	2.251E 02	2.195E 02	2.140E 02	2.087E 02	2.036E 02	1.986E 02	1.938E 02
0.38	1.892E 02	1.846E 02	1.803E 02	1.76[illegible]E 02	1.719E 02	1.679E 02	1.640E 02	1.602E 02	1.566E 02	1.530E 02
0.39	1.496E 02	1.462E 02	1.43[illegible]E 02	1.398E 02	1.367E 02	1.337E 02	1.308E 02	1.280E 02	1.253E 02	1.226E 02
0.40	1.200E 02	1.175E 02	1.15[illegible]E 02	1.126E 02	1.1[illegible]3E 02	1.080E 02	1.058E 02	1.037E 02	1.[illegible]16E 02	9.957E 01
0.41	9.759E 01	9.566E 01	9.378E 01	9.195E 01	9.016E 01	8.842E 01	8.673E 01	8.508E 01	8.346E 01	8.189E 01
0.42	8.036E 01	7.886E 01	7.740E 01	7.598E 01	7.459E 01	7.324E 01	7.191E 01	7.062E 01	6.936E 01	6.813E 01
0.43	6.693E 01	6.575E 01	6.461E 01	6.349E 01	6.239E 01	6.132E 01	6.028E 01	5.926E 01	5.826E 01	5.728E 01
0.44	5.633E 01	5.540E 01	5.449E 01	5.360E 01	5.272E 01	5.187E 01	5.104E 01	5.022E 01	4.942E 01	4.864E 01
0.45	4.788E 01	4.713E 01	4.64[illegible]E 01	4.568E 01	4.498E 01	4.429E 01	4.361E 01	4.296E 01	4.231E 01	4.166E 01
0.46	4.106E 01	4.045E 01	3.986E 01	3.927E 01	3.870E 01	3.814E 01	3.759E 01	3.706E 01	3.653E 01	3.6[illegible]1E 01
0.47	3.551E 01	3.501E 01	3.452E 01	3.404E 01	3.357E 01	3.311E 01	3.266E 01	3.222E 01	3.179E 01	3.136E 01
0.48	3.094E 01	3.053E 01	3.013E 01	2.973E 01	2.935E 01	2.896E 01	2.859E 01	2.822E 01	2.786E 01	2.751E 01
0.49	2.716E 01	2.682E 01	2.648E 01	2.615E 01	2.583E 01	2.551E 01	2.520E 01	2.489E 01	2.459E 01	2.429E 01
0.50	2.400E 01	2.371E 01	2.343E 01	2.315E 01	2.288E 01	2.261E 01	2.235E 01	2.209E 01	2.184E 01	2.159E 01
0.51	2.134E 01	2.110E 01	2.086E 01	2.063E 01	2.039E 01	2.[illegible]17E 01	1.994E 01	1.973E 01	1.951E 01	1.93[illegible]E 01
0.52	1.909E 01	1.888E 01	1.868E 01	1.848E 01	1.828E 01	1.809E 01	1.79[illegible]E 01	1.771E 01	1.753E 01	1.734E 01
0.53	1.716E 01	1.699E 01	1.681E 01	1.664E 01	1.647E 01	1.631E 01	1.614E 01	1.598E 01	1.582E 01	1.567E 01
0.54	1.551E 01	1.536E 01	1.521E 01	1.506E 01	1.492E 01	1.478E 01	1.463E 01	1.450E 01	1.436E 01	1.422E 01
0.55	1.409E 01	1.396E 01	1.383E 01	1.370E 01	1.357E 01	1.345E 01	1.333E 01	1.321E 01	1.309E 01	1.297E 01
0.56	1.285E 01	1.274E 01	1.263E 01	1.251E 01	1.240E 01	1.230E 01	1.219E 01	1.208E 01	1.198E 01	1.188E 01
0.57	1.177E 01	1.167E 01	1.158E 01	1.148E 01	1.138E 01	1.129E 01	1.119E 01	1.11[illegible]E 01	1.111E 01	1.092E 01
0.58	1.083E 01	1.074E 01	1.065E 01	1.057E 01	1.048E 01	1.040E 01	1.032E 01	1.024E 01	1.015E 01	1.008E 01
0.59	9.997E 00	9.919E 00	9.842E 00	9.766E 00	9.691E 00	9.617E 00	9.544E 00	9.472E 00	9.4[illegible]1E 00	9.33[illegible]E 00

Table D.3 (continued) $\Gamma(1+\frac{2}{\beta})$

β	0.0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.60	9.261E 00	9.192E 00	9.124E 00	9.056E 00	8.990E 00	8.924E 00	8.859E 00	8.795E 00	8.732E 00	8.669E 00
0.61	8.607E 00	8.546E 00	8.485E 00	8.425E 00	8.366E 00	8.307E 00	8.250E 00	8.192E 00	8.136E 00	8.080E 00
0.62	8.024E 00	7.970E 00	7.916E 00	7.862E 00	7.809E 00	7.757E 00	7.705E 00	7.654E 00	7.603E 00	7.553E 00
0.63	7.503E 00	7.454E 00	7.406E 00	7.358E 00	7.310E 00	7.263E 00	7.217E 00	7.171E 00	7.125E 00	7.080E 00
0.64	7.035E 00	6.991E 00	6.948E 00	6.904E 00	6.862E 00	6.819E 00	6.778E 00	6.736E 00	6.695E 00	6.654E 00
0.65	6.614E 00	6.574E 00	6.535E 00	6.496E 00	6.457E 00	6.419E 00	6.381E 00	6.344E 00	6.307E 00	6.27[illegible]E 00
0.66	6.234E 00	6.198E 00	6.162E 00	6.127E 00	6.092E 00	6.057E 00	6.023E 00	5.989E 00	5.955E 00	5.922E 00
0.67	5.889E 00	5.856E 00	5.824E 00	5.792E 00	5.760E 00	5.728E 00	5.697E 00	5.666E 00	5.636E 00	5.605E 00
0.68	5.575E 00	5.546E 00	5.516E 00	5.487E 00	5.458E 00	5.429E 00	5.401E 00	5.373E 00	5.345E 00	5.317E 00
0.69	5.290E 00	5.263E 00	5.236E 00	5.209E 00	5.183E 00	5.157E 00	5.131E 00	5.105E 00	5.079E 00	5.054E 00
0.70	5.029E 00	5.004E 00	4.983E 00	4.955E 00	4.931E 00	4.907E 00	4.883E 00	4.860E 00	4.836E 00	4.813E 00
0.71	4.790E 00	4.768E 00	4.745E 00	4.723E 00	4.700E 00	4.678E 00	4.657E 00	4.635E 00	4.614E 00	4.592E 00
0.72	4.571E 00	4.550E 00	4.530E 00	4.509E 00	4.489E 00	4.468E 00	4.448E 00	4.428E 00	4.409E 00	4.389E 00
0.73	4.370E 00	4.350E 00	4.331E 00	4.312E 00	4.294E 00	4.275E 00	4.256E 00	4.238E 00	4.220E 00	4.2[illegible]2E 00
0.74	4.184E 00	4.166E 00	4.148E 00	4.131E 00	4.114E 00	4.096E 00	4.079E 00	4.062E 00	4.045E 00	4.029E 00
0.75	4.012E 00	3.996E 00	3.979E 00	3.963E 00	3.947E 00	3.931E 00	3.915E 00	3.900E 00	3.884E 00	3.869E 00
0.76	3.853E 00	3.838E 00	3.823E 00	3.808E 00	3.793E 00	3.778E 00	3.764E 00	3.749E 00	3.735E 00	3.72[illegible]E 00
0.77	3.706E 00	3.692E 00	3.678E 00	3.664E 00	3.650E 00	3.636E 00	3.623E 00	3.609E 00	3.596E 00	3.583E 00
0.78	3.569E 00	3.556E 00	3.543E 00	3.530E 00	3.517E 00	3.504E 00	3.492E 00	3.479E 00	3.467E 00	3.454E 00
0.79	3.442E 00	3.430E 00	3.418E 00	3.406E 00	3.394E 00	3.382E 00	3.37[illegible]E 00	3.358E 00	3.346E 00	3.335E 00
0.80	3.323E 00	3.312E 00	3.301E 00	3.289E 00	3.278E 00	3.267E 00	3.256E 00	3.245E 00	3.234E 00	3.223E 00
0.81	3.213E 00	3.202E 00	3.191E 00	3.181E 00	3.170E 00	3.160E 00	3.150E 00	3.139E 00	3.129E 00	3.119E 00
0.82	3.109E 00	3.099E 00	3.089E 00	3.079E 00	3.070E 00	3.[illegible]60E 00	3.050E 00	3.041E 00	3.031E 00	3.022E 00
0.83	3.012E 00	3.003E 00	2.994E 00	2.984E 00	2.975E 00	2.966E 00	2.957E 00	2.948E 00	2.939E 00	2.93[illegible]E 00
0.84	2.921E 00	2.912E 00	2.904E 00	2.895E 00	2.886E 00	2.878E 00	2.869E 00	2.861E 00	2.853E 00	2.844E 00
0.85	2.836E 00	2.828E 00	2.82[illegible]E 00	2.811E 00	2.803E 00	2.795E 00	2.787E 00	2.779E 00	2.771E 00	2.764E 00
0.86	2.756E 00	2.748E 00	2.74[illegible]E 00	2.733E 00	2.725E 00	2.717E 00	2.710E 00	2.702E 00	2.695E 00	2.688E 00
0.87	2.680E 00	2.673E 00	2.666E 00	2.658E 00	2.651E 00	2.644E 00	2.637E 00	2.630E 00	2.623E 00	2.616E 00
0.88	2.609E 00	2.602E 00	2.595E 00	2.589E 00	2.582E 00	2.575E 00	2.568E 00	2.562E 00	2.555E 00	2.549E 00
0.89	2.542E 00	2.535E 00	2.529E 00	2.523E 00	2.516E 00	2.510E 00	2.504E 00	2.497E 00	2.491E 00	2.485E 00
0.90	2.479E 00	2.472E 00	2.466E 00	2.46[illegible]E 00	2.454E 00	2.448E 00	2.442E 00	2.436E 00	2.430E 00	2.424E 00
0.91	2.419E 00	2.413E 00	2.407E 00	2.401E 00	2.396E 00	2.390E 00	2.384E 00	2.379E 00	2.373E 00	2.367E 00
0.92	2.362E 00	2.356E 00	2.351E 00	2.345E 00	2.340E 00	2.335E 00	2.329E 00	2.324E 00	2.319E 00	2.313E 00
0.93	2.308E 00	2.303E 00	2.298E 00	2.293E 00	2.287E 00	2.282E 00	2.277E 00	2.272E 00	2.267E 00	2.262E 00
0.94	2.257E 00	2.252E 00	2.247E 00	2.242E 00	2.238E 00	2.233E 00	2.228E 00	2.223E 00	2.218E 00	2.214E 00
0.95	2.209E 00	2.204E 00	2.199E 00	2.195E 00	2.190E 00	2.185E 00	2.181E 00	2.176E 00	2.172E 00	2.167E 00
0.96	2.163E 00	2.158E 00	2.154E 00	2.149E 00	2.145E 00	2.141E 00	2.136E 00	2.132E 00	2.128E 00	2.123E 00
0.97	2.119E 00	2.115E 00	2.111E 00	2.106E 00	2.102E 00	2.098E 00	2.094E 00	2.090E 00	2.086E 00	2.082E 00
0.98	2.077E 00	2.073E 00	2.069E 00	2.065E 00	2.061E 00	2.057E 00	2.053E 00	2.049E 00	2.046E 00	2.042E 00
0.99	2.038E 00	2.034E 00	2.030E 00	2.026E 00	2.022E 00	2.019E 00	2.015E 00	2.011E 00	2.[illegible]7E 00	2.0[illegible]4E 00
1.00	2.000E 00	1.996E 00	1.993E 00	1.989E 00	1.985E 00	1.982E 00	1.978E 00	1.975E 00	1.971E 00	1.967E 00
1.01	1.964E 00	1.960E 00	1.957E 00	1.953E 00	1.950E 00	1.947E 00	1.943E 00	1.940E 00	1.936E 00	1.933E 00
1.02	1.930E 00	1.926E 00	1.923E 00	1.919E 00	1.916E 00	1.913E 00	1.910E 00	1.906E 00	1.903E 00	1.9[illegible]E 00
1.03	1.897E 00	1.893E 00	1.89[illegible]E 00	1.887E 00	1.884E 00	1.881E 00	1.878E 00	1.874E 00	1.871E 00	1.868E 00
1.04	1.865E 00	1.862E 00	1.859E 00	1.856E 00	1.853E 00	1.850E 00	1.847E 00	1.844E 00	1.841E 00	1.838E 00
1.05	1.835E 00	1.832E 00	1.829E 00	1.826E 00	1.823E 00	1.820E 00	1.818E 00	1.815E 00	1.812E 00	1.809E 00
1.06	1.806E 00	1.803E 00	1.801E 00	1.798E 00	1.795E 00	1.792E 00	1.790E 00	1.787E 00	1.784E 00	1.781E 00
1.07	1.779E 00	1.776E 00	1.773E 00	1.771E 00	1.768E 00	1.765E 00	1.763E 00	1.760E 00	1.757E 00	1.755E 00
1.08	1.752E 00	1.750E 00	1.747E 00	1.744E 00	1.742E 00	1.739E 00	1.737E 00	1.734E 00	1.732E 00	1.729E 00
1.09	1.727E 00	1.724E 00	1.722E 00	1.719E 00	1.717E 00	1.714E 00	1.712E 00	1.71[illegible]E 00	1.707E 00	1.7[illegible]5E 00

APPENDIX E

COMPARISON OF FITTING TECHNIQUES

E.1 Introduction

Two fitting techniques were discussed in Section 2.3, the method of moments and the method of least squares. The method of moments was used in Chapter III to examine the fatalities distributions of nuclear risks and non-nuclear risks. In this appendix, the two methods are compared with regard to the residual mean squares and the estimates of the parameters. The comparisons are based on the Weibull distribution. The data distributions examined are the early fatalities distributions of hurricanes, average of U.S. reactors and PWR accidents at Site A.

E.2 Fitting Techniques

E.2.1 Method of Moments

The method of moments was used in Chapter III to estimate the parameters. In the Weibull distribution, the estimates of the shape factor β and the scale factor η are obtained by solving the following equations:

$$\frac{[\Gamma(1+\frac{1}{\beta})]^2}{\Gamma(1+\frac{2}{\beta})} = \frac{M_1^2}{M_2 \cdot \alpha} \qquad \text{(E.1)}$$

$$\eta = \frac{M_1}{\alpha \cdot \Gamma(1+\frac{1}{\beta})} \qquad \text{(E.2)}$$

where

M_1 = the first risk moment.

M_2 = the second risk moment.

α = the normalization constant.

$\Gamma(\cdot)$ = the Gamma function.

E.2.2 Method of Least Squares

The shape factor and the scale factor are estimated by minimizing:

$$\Delta^2 = \frac{1}{n-2} \sum_{i=1}^{n} \left\{ \ln \tilde{F}_i^c - \ln \left[\alpha \cdot \exp \left\{ - \left(\frac{x_i}{\eta} \right)^{\beta} \right\} \right] \right\}^2 \qquad \text{(E.3)}$$

where

$\tilde{F}_i^c$ = complementary cumulative frequency assigned to the data i.

x_i = magnitude of the consequence of the data i.

n = total number of the data.

The natural logarithm is used in the least squares because the fractional errors of the frequencies have comparable magnitudes rather than the absolute errors of the frequencies. The non-linear least-squares program in the DCRT Mathematical and Statistical Package of National Institute of Health (Ref-9) is used. The initial values for the iterative calculation in the method of least squares are obtained from the results by the method of moments. The number of iterations required are from 6 to 8 to reach the convergence level of 10^{-4}.

E.3 Basis for Comparison

The two fitting techniques are compared on the following basis:

(1) In Chapter III the Weibull distribution was found to be

within the error bounds of the data distribution when the parameters were estimated by the method of moments. The method of least squares is examined to determine if it satisfies the same criterion.

(2) The residual mean squares for the two methods are compared.

$$s = \frac{1}{n-2} \sum_{i} [\tilde{F}_i^c - \ln \left\{ \alpha \cdot \exp \left[- \left(\frac{x}{\hat{\eta}} \right)^{\hat{\beta}} \right] \right\}]^2 \tag{E.4}$$

where $\hat{\beta}$ and $\hat{\eta}$ are the estimates of the Weibull parameters.

(3) The estimates of the risk moments are obtained from the least-squares estimates of the parameters.

$$\hat{M}_1 = \alpha \cdot \hat{\eta} \cdot \Gamma(1 + \frac{1}{\hat{\beta}}) \tag{E.5}$$

$$\hat{M}_2 = \alpha \cdot \hat{\eta}^2 \cdot \Gamma(1 + \frac{2}{\hat{\beta}}) \tag{E.6}$$

where $\hat{M}_1$ and $\hat{M}_2$ are the estimates of the first two risk moments. The fitting errors of the risk moments are examined in the method of least squares. In the method of moments the estimates of the risk moments by Eqs. (E.5) and (E.6) are equal to the data values.

For the nuclear curves, the fitting errors are also compared with the regression errors in the regression analysis of the population distribution. When the fitting errors are smaller than the regression errors, the selection of the fitting techniques does not significantly affect the investigation of the relationship between the risk distributions and the population distribution variables.

E.4 Comparison of Fitting Techniques

The early fatalities distributions of hurricanes, average of U.S.

100 commercial reactors and PWR accidents at Site A are examined in the following sections.

E.4.1 Hurricanes

The normalization constant and the lower end of the domain were determined in Section III.4.3 as:

$$\alpha = .63/\text{year}$$

$$x_0 = 0$$

The residual mean square, the estimates of the parameters and the risk moments by the two fitting techniques are given in Table E.1. The complementary cumulative distributions derived from the estimates of the parameters are shown in Fig. E.1 along with the data. The bands attached to the data points are the 90% confidence bounds.

Fig. E.1 shows that the Weibull distributions by the two techniques are both within the 90% confidence bounds of the data. The method of least squares gives somewhat higher probability for the largest consequence. The method of moments gives somewhat higher probability values in the region of medium and low consequences. The residual mean square of the least-squares fitting is smaller by a factor of 1.8 than that of the method of moments. Since the method of least squares gives slower rate of decrease in the tail, the estimates of the risk moments are somewhat larger than those of the method of moments, which are the data value. In conclusion, the selection of the fitting techniques is judged not to have significant effect.

E.4.2 Average of U.S. Reactors

The normalization constant and the lower end of the domain were determined in Section III.5.3 as:

$$\alpha = 4.72 \times 10^{-7}/\text{reactor year}$$

$$x_0 = 0$$

The results of the fittings are given in Table E.1 and Fig. E.2. The uncertainty ranges of the data are represented by factors of 5 and 1/5 on the probability and by factors of 4 and 1/4 on the magnitude. Fig. E.2 shows that both of the fitted distributions are within the uncertainty ranges of the data. The residual mean square of the method of least squares is smaller than that of the moment fitting by approximately 15%. The risk moments estimated by the least-squares fitting are smaller than those of the data values and the moments fitting. The differences are a factor of approximately 0.9 for the first risk moment and a factor of approximately 0.7 for the second risk moment. Since the 90% error bounds in the regression analysis in Chapter V were factors of 1.3 and 1/1.3 for the first risk moment and factor of 1.6 and 1/1.6 for the second risk moment, the selection of the fitting techniques is judged not to have significant effect in this study.

E.4.3 PWR Accidents at Site A

The normalization constant and the lower end of the domain were determined in Section III.5.4 as:

$$\alpha = 5.78 \times 10^{-7}/\text{reactor year}$$

$$x_0 = 0$$

The results of the fittings are given in Table E.1 and Fig. E.3. The uncertainties of the data are represented by factors of 5 and 1/5 on the probability, and 4 and 1/4 on the magnitude. Fig. E.3 shows that both of the fitted curves are within the uncertainty ranges of the data. The residual mean square by the least squares method is smaller than that of the moment method by approximately 20%. The differences of the risk moments between the two methods are a factor of 1.05 for the first risk moment and a factor of 0.95 for the second risk moment. These errors are within the 90% error bounds of the regression analysis performed in Chapter V and are judged not to have significant effects in the regression results.

E.5 Conclusion

The Weibull fittings determined by the two methods are within the uncertainty ranges of the data for all of the examined curves. The method of least squares gives smaller residual mean square than the method of moments, however the differences are less than a factor of 2 for the examined events. The errors of the estimates of the risk moments by the method of least squares are within the 90% error bounds in the regression analysis in Chapter V.

In conclusion, the selection of the fitting techniques is judged not to have significant effects on the analysis in this study.

Table E.1 Comparison of the Fitting Techniques in the Fatalities Distributions

Type of Risk	Variables Compared		Fitting Techniques: Method of Moments	Fitting Techniques: Method of Least Squares
Hurricanes	Residual Mean Square		.107	.060
	Shape Factor	β	.387	.301
	Scale Factor	η	7.48×10^{1}	5.18×10^{1}
	Risk Moments	M_1	1.72×10^{2}	2.96×10^{2}
		M_2	5.64×10^{5}	4.12×10^{6}
Average of U.S. Reactors	Residual Mean Square		.194	.170
	Shape Factor	β	.371	.380
	Scale Factor	η	2.45×10^{1}	2.33×10^{1}
	Risk Moments	M_1	4.60×10^{-5}	4.06×10^{-5}
		M_2	6.45×10^{-2}	4.65×10^{-2}
PWR Accidents at Site A	Residual Mean Square		.102	.081
	Shape Factor	β	.570	.616
	Scale Factor	η	2.91×10^{2}	3.40×10^{2}
	Risk Moments	M_1	2.72×10^{-4}	2.85×10^{-4}
		M_2	5.77×10^{-1}	5.51×10^{-1}

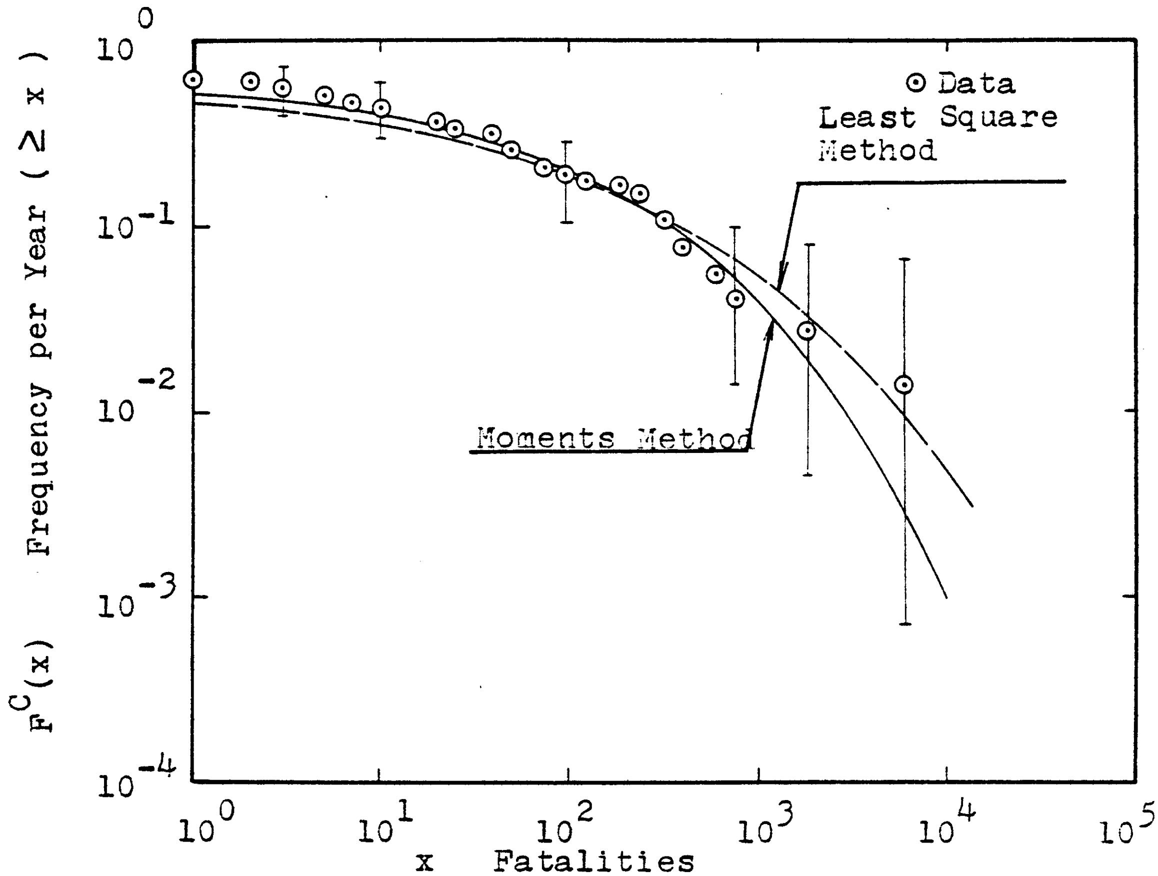

Fig. E.1 Comparison of the fitting Techniques in the Fatalities Distribution of Hurricanes

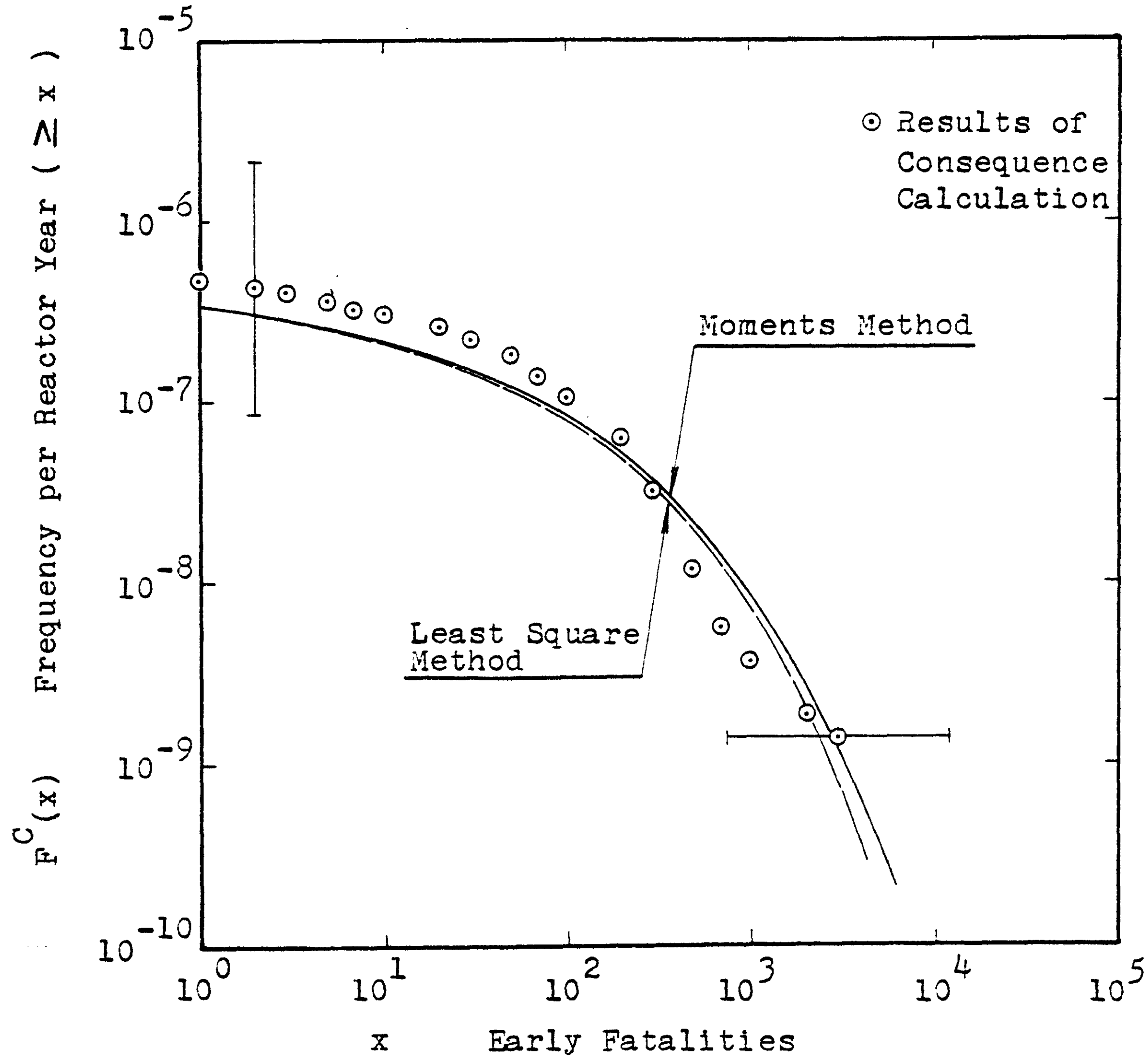

Fig. E.2 Comparison of the Fitting Techniques in the Early Fatalities Distribution of the Average of U.S. 100 Reactors

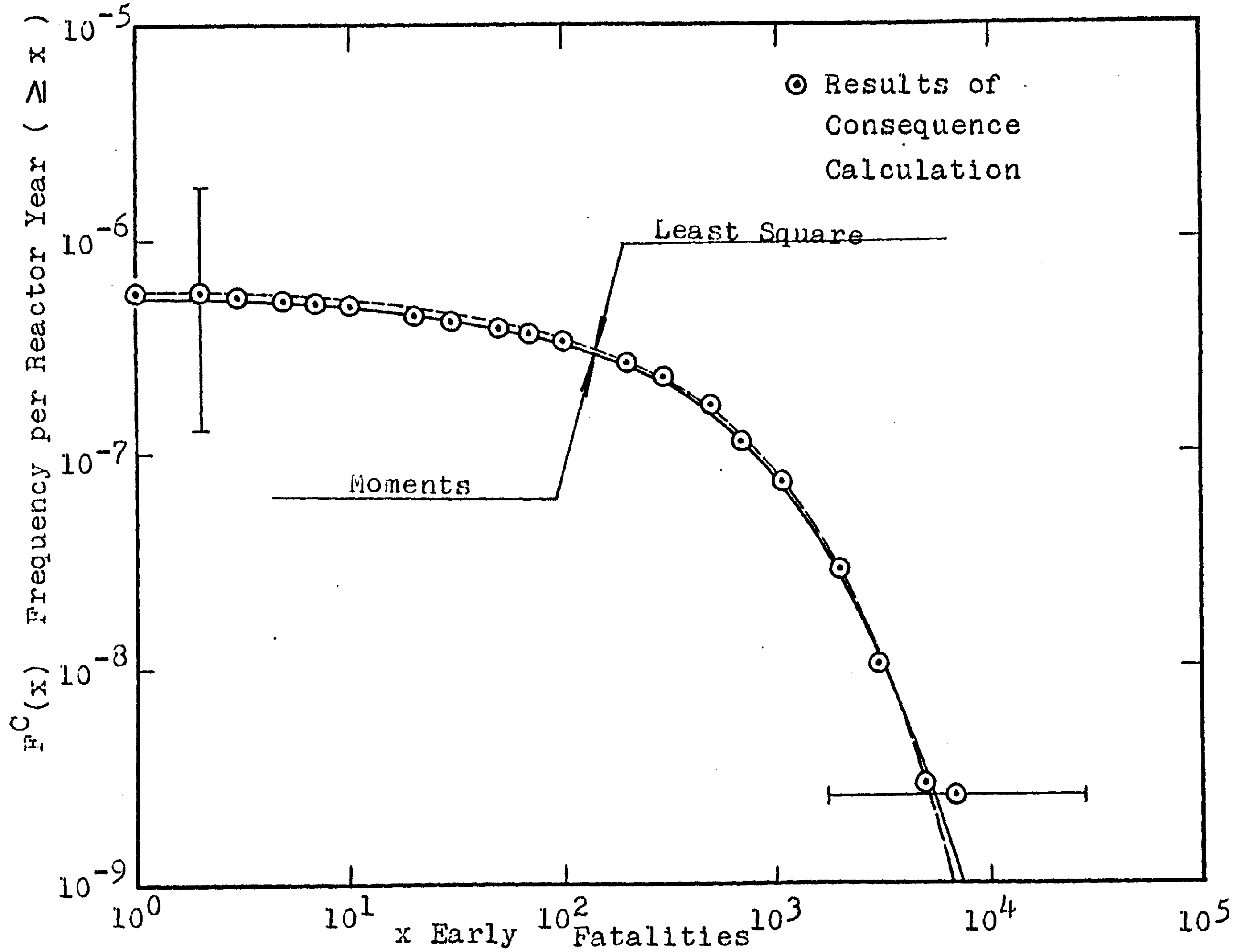

Fig.E.3 Comparison of the Fitting Techniques in the Early Fatalities Distribution of PWR Accidents at Site A

APPENDIX F

BELL-SHAPED POPULATION MODEL

F.1 Introduction

The bell-shaped or gaussian population distribution discussed in Section V.8 is discussed again here in more detail. A numerical example is also given to show the applicability of the model. The bell-shaped population model allows the evaluation of the risk of nuclear reactor accidents to be performed for each of the cities and towns surrounding the nuclear power plants.

F.2 Bell-Shaped Population Model

The population distribution of a city or a town is idealized by a bell-shaped population model shown in Figure F.1. The population distribution is symmetric about its center. Its total population is N_T, the distance of the center from a reactor is R and 90% of the total population are living in a radius of $2\sigma_R$. Now consider the (r,ζ) coordinate in Fig. F.1. The population per unit area at (r,ζ) is expressed as

$$\rho(r,\zeta) = \frac{N_T}{2\pi\sigma_R^2} \cdot \exp\left(-\frac{(r-R)^2}{2\sigma_R^2} - \frac{\zeta^2}{2\sigma_R^2}\right) \tag{F.1}$$

Since the regression equations in Chapter V are based on the (r,θ) co-ordinate, an approximation is made based on the assumption that a city or a town is in a $22\frac{1}{2}$ degree sector, i.e.,

$$2\sigma_R < \frac{\pi}{8} \cdot R \tag{F.2}$$

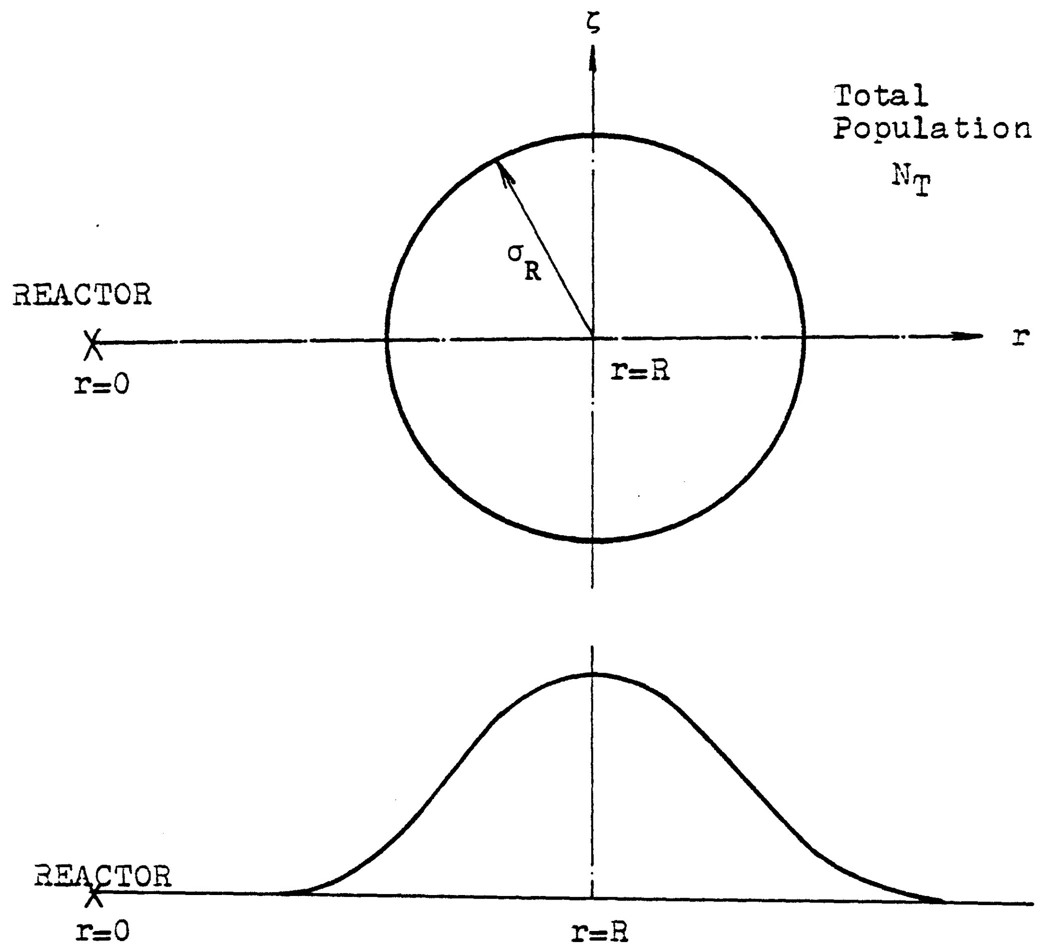

Fig. F.1 Illustration for Bell-shaped Population Model

Based on this assumption, the population per unit distance at r in a $22\frac{1}{2}$ degree sector is approximated by:

$$n_j(r) \approx \int_{-\infty}^{\infty} \rho(r,\zeta)\, d\zeta = \frac{N_T}{\sqrt{2\pi}\,\sigma_R} \exp\left(-\frac{(r-R)^2}{2\sigma_R^2}\right) \qquad \text{(F.3)}$$

The population distribution in a $22\frac{1}{2}$ degree sector is also expressed by a gaussian distribution. When the city or town is large enough to cover a number of sectors, the populations of the city or town are divided into separate population groups, each of which can be expressed by a gaussian distribution with respect to r.

The bell-shaped population model is thus applied to each of the cities and towns surrounding the nuclear power plant. The population distribution in a $22\frac{1}{2}$ degree sector is expressed by the series of the bell-shaped distributions as:

$$n_j(r) = \sum_{\ell=1}^{L_j} \frac{(N_T)_\ell}{\sqrt{2\pi}\,(\sigma_R)_\ell} \cdot \exp\left[-\frac{(r-R_\ell)^2}{2(\sigma_R)_\ell^2}\right] \qquad \text{(F.4)}$$

where the subscript ℓ refers to each of the population groups involving cities and towns. L_j is the total number of the population groups in the direction j.

F.3 Estimation of the Risk Moments

Using the transfer functions derived in Chapter V, the risk moments are estimated for the bell-shaped population distribution. The transfer functions used here are:

$$a(r) = a_1 \cdot \exp\left[-a_2 \cdot r\right] \qquad \text{(F.5)}$$

$$b(r,r') = b_1 \cdot \exp\left[-b_2 \cdot (r+r')\right] \cdot \exp\left[-b_3 \cdot |r-r'|\right] \qquad \text{(F.6)}$$

$$c(r) = c_1 \cdot \exp\left[-c_2 \cdot r\right] \qquad \text{(F.7)}$$

The first risk moment is estimated by:

$$N_1 = \sum_j \int_0^\infty a(r) \cdot n_j(r)\, dr$$

$$= \sum_j \int_0^\infty a_1 \cdot \exp\left[-a_2 \cdot r\right] \cdot \left\{ \sum_{\ell=1}^{L_j} \frac{(N_T)_\ell}{\sqrt{2\pi}(\sigma_R)_\ell} \cdot \exp\left(-\frac{(r-R_\ell)^2}{2\sigma_R^2}\right) \right\} dr$$

$$= \sum_j \sum_{\ell=1}^{L_j} (N_T)_\ell \cdot a_1 \cdot \exp\left(-a_2 R_\ell + \frac{a_2^2 \cdot (\sigma_R)_\ell^2}{2}\right) \times$$

$$\times \int_0^\infty \frac{1}{\sqrt{2\pi}(\sigma_R)_\ell} \cdot \exp\left(-\frac{\left\{r - R_\ell + a_2 \cdot (\sigma_R)_\ell^2\right\}^2}{2(\sigma_R)_\ell^2}\right) dr \qquad \text{(F.8)}$$

The integral in Eq. (F.8) is rewritten as:

$$\int_0^\infty \frac{1}{\sqrt{2\pi}\ (\sigma_R)_\ell} \exp\left(\frac{\left\{r - R_\ell + a_2 \cdot (\sigma_R)_\ell^2\right\}^2}{2 \cdot (\sigma_R)_\ell^2}\right) dr$$

$$= \int_{-R + a_2 \cdot (\sigma_R)_\ell^2}^{\infty} \frac{1}{\sqrt{2\pi}\ (\sigma_R)_\ell} \exp\left(-\frac{\xi^2}{2 \cdot (\sigma_R)_\ell^2}\right) d\xi \qquad \text{(F.9)}$$

where $\xi = r - R_\ell + a_2 \cdot (\sigma_R)_\ell^2$. The approximation is made here based on the assumption as:

$$-R_\ell + a_2 \cdot (\sigma_R)_\ell^2 < -2(\sigma_R)_\ell \qquad \text{(F.10)}$$

Then the integration range in Eq. (F.9) is from less than $-2(\sigma_R)$ to infinity. Therefore the integral in Eq. (F.9) is greater than .97, which is approximately unity. Then Eq. (F.8) is approximately expressed as:

$$M_1 = \sum_j \sum_{\ell=1}^{L_j} (N_T)_\ell \cdot a_1 \cdot \exp\left(-a_2 \cdot R_\ell + \frac{a_2^2 \cdot (\sigma_R)_\ell^2}{2}\right) \qquad \text{(F.11)}$$

The second risk moment M_2 is calculated to be:

$$M_2 = \sum_j \int_0^\infty \int_0^\infty b_1 \cdot \exp\,[-b_2 \cdot (r + r')]\ \exp\,[-b_3 \cdot |r - r'|]\ n_j(r) \cdot$$

$$\cdot\, n_j(r) \cdot dr \cdot dr' \qquad \text{(F.12)}$$

The term $\exp\,[-b_3 \cdot |r - r'|]$ in this equation indicates the simultaneous occurrence of deaths at r and r'. The term decreases by an order of magnitude when the interval between r and r' is more than $2.3/b_3 \approx 2/b_3$. The approximation can be made of calculating the second risk moment for each population group separately if the distance between the two adjacent population groups is more than $2/b_3$.

$$\left| [R_{\ell+1} - 2(\sigma_R)_{\ell+1}] - [R_\ell + 2(\sigma_R)_\ell] \right| > 2/b_3 \qquad \text{(F.13)}$$

where the subscripts ℓ and $(\ell+1)$ refer to the adjacent population groups and the population outside the radius of $2\sigma_R$ are ignored.

Then the risk moment M is calculated to be:

$$M_2 = \sum_j \sum_{\ell=1}^{L_j} \int_0^\infty \int_0^\infty b_1 \cdot \exp\,[-b_2 \cdot (r + r')] \cdot \exp\,[-b_3 \cdot |r - r'|]$$

$$\times \frac{(N_T)_\ell^2}{2\pi(\sigma_R)_\ell^2}\ \exp\left[-\frac{(r - R\ )_\ell^2}{2(\sigma_R)_\ell^2}\right] \cdot \exp\left[-\frac{(r' - R\ )_\ell^2}{2(\sigma_R)_\ell^2}\right] dr \cdot dr' \qquad \text{(F.14)}$$

Eq. (F.14) still requires a numerical integration. Further approximation is made here. For a small town whose radius $2\sigma_R$ is smaller than $1/b_3$, the term $\exp\,[-b_3 \cdot |r - r'|]$ is approximated by 1.

$$2\sigma_R < 1/b_3 \qquad \text{(F.15)}$$

Then the interpretations of r and r' can be separated and the second risk moment becomes:

$$M = \sum_{j} \sum_{\ell=1}^{L_j} (N_T)_\ell^2 \cdot \exp\left[-2 \cdot b_2 \cdot R_\ell + b_2^2 \cdot (\sigma_R)_\ell^2\right] \qquad (F.16)$$

Finally, the normalization constant is calculated from the distance to the closest town or city.

$$\alpha = \sum_{j} c_1 \cdot \exp\left[-c_2 \cdot d_j\right] \qquad (F.17)$$

$$d_j = R_{1j} - 2 \cdot (\sigma_R)_{1j} \qquad (F.18)$$

where R_{1j} and $(\sigma_R)_{1j}$ are the center distance and deviation, respectively, of the closest population group in the direction j. The populations outside the radius of $2 \cdot \sigma_R$ are ignored in Eq. (F.18)

Once M_1, M_2 and α are obtained, the scale factor and the shape factor of the Weibull distribution can be obtained using Eqs. (3.27) and (3.28) in Chapter III. The entire risk distribution can then be derived.

The constraints of the derived equations are discussed here. In estimating the first risk moment by Eq. (F.11), the following constraints should be considered:

(1) The population group is in a $22\frac{1}{2}$ degree sector. (Eq. (F.2))

$$2(\sigma_R)_\ell < \frac{\pi}{8} \cdot R_\ell \qquad (F.19)$$

(2) From Eq. (F.10),

$$R_\ell > a_2 \cdot (\sigma_R)_\ell^2 + 2 \cdot (\sigma_R)_\ell \qquad (F.20)$$

The first constraint Eq. (F.19) can be removed in the estimation of the first risk moment. Let $n_T(r)$ be the total population per unit r at r from the reactor.

$$n_T(r) = \sum_j n_j(r) \tag{F.21}$$

Then the first risk moment is estimated by:

$$M_1 = \int_0^\infty a(r) \cdot n_T(r) \cdot dr \tag{F.22}$$

In the integration over ζ in Eq. (F.1) to calculate the total population per unit distance $n_T(r)$, the assumption (F.2) is not required. However the integration over θ is still approximated by the integration over ζ. When the distance R is greater than $2(\sigma_R)$, the error of the approximation is small. Therefore the constraint of Eq. (F.20) is sufficient. The total population $n_T(r)$ is also expressed by the series of the bell-shaped population distributions. The first risk moment can then be estimated from the following equation without considering the directions:

$$M_1 = \sum_\ell (N_T)_\ell \cdot a_1 \cdot \exp\left[-a_2 \cdot R_\ell + \frac{a_2^2 \cdot (\sigma_R)^2}{2}\right] \tag{F.23}$$

The constraint of this equation is:

$$R_\ell > a_2 \cdot (\sigma_R)_\ell^2 + 2 \cdot (\sigma_R)_\ell^2 \tag{F.24}$$

In estimating the second risk moment by Eq. (F.16), the following constraints should be considered:

(1) $2 \cdot (\sigma_R)_\ell < \frac{\pi}{8} \cdot R_\ell$ (F.25)

(2) $\left|[R_{\ell+1} - 2 \cdot (\sigma_R)_{\ell+1}] - [R_\ell + 2 \cdot (\sigma_R)_\ell]\right| > 2/b_3$ (F.26)

(3) $2 \cdot \sigma_R < 1/b_3$ (F.27)

In Eq. (F.16), the assumption of (F.25) cannot be removed.

In deriving the normalization constant by Eqs. (F.17) and (F.18), the assumption of (F.25) is necessary.

F.4 Application of Bell-Shaped Model to Site A

The adequacy of the bell-shaped population model will be studied by the population distribution of Site A. The population per unit distance in a 22.5 degree sector are fitted by the series of the bell-shaped distributions given by Eq. (F.4). The method for deriving the constants of the bell-shaped model is discussed first.

F.4.1 Derivation of Constants of Bell-Shaped Model

The population data in the annular segments given in Appendix C are used to derive the constants of the bell-shaped model. The first step in the derivation is to separate the population distribution into a series of the population groups. Fig. F.2 shows the population per mile in each of the 16 directions around Site A as a function of distance from the reactor. The population groups are identified by the peaks in Fig. F.2. The neighbouring groups are bunched into one group when their peaks are within 1 mile distance. A total of 44 population groups are identified within 20 miles from the reactor.

The next step is to fit each population group by a bell-shaped model. For presentation, an example in Fig. F.3 is considered. The population in the segments are denoted by ν and the central distances of the segments from the reactor are denoted by r in Fig. F.3. The population in the segments in Fig. F.3 are assumed to belong to one population group. The total population in the group is given by:

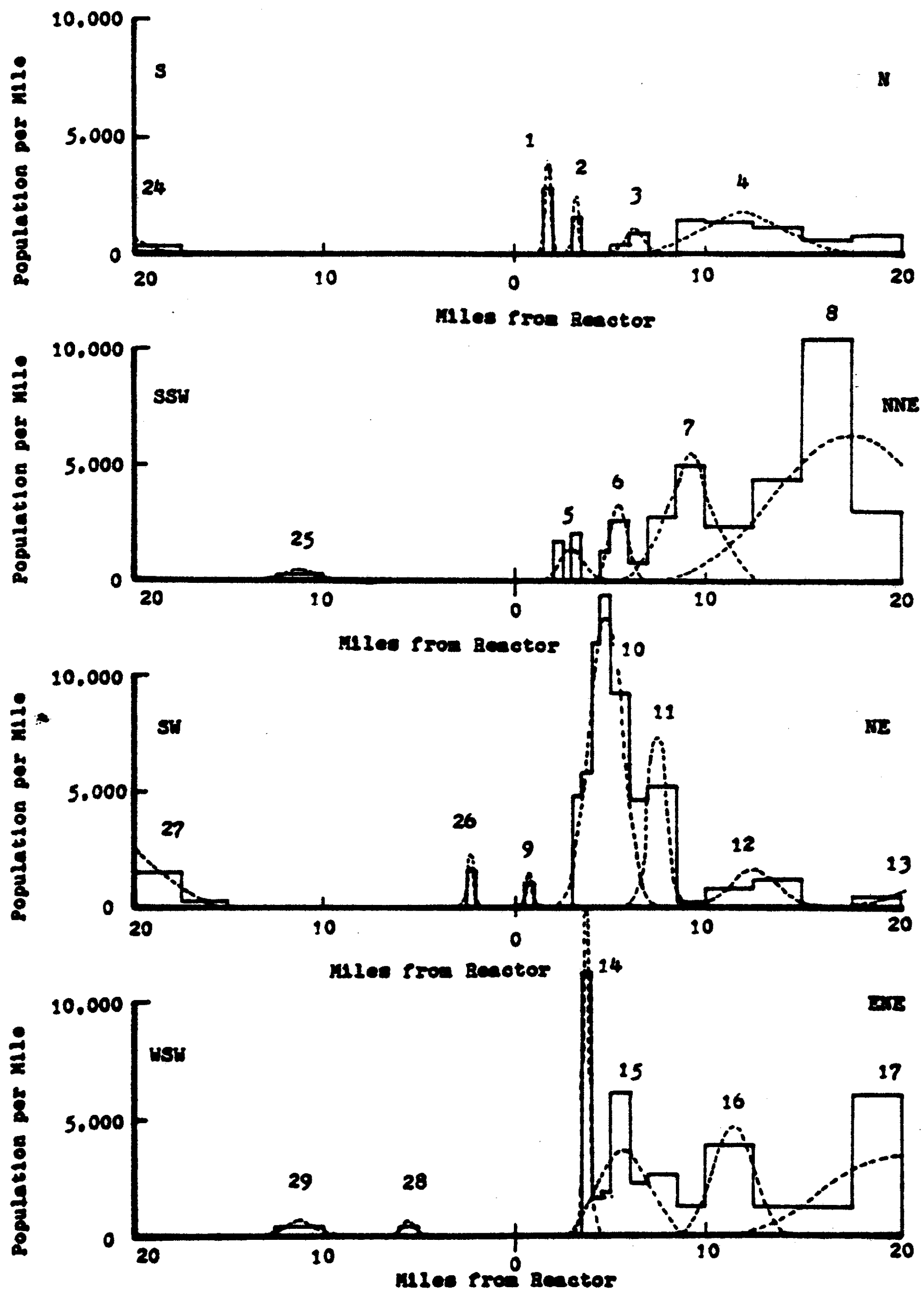

Fig.F.2 Population in a 22.5 Degree Sector Around Site A

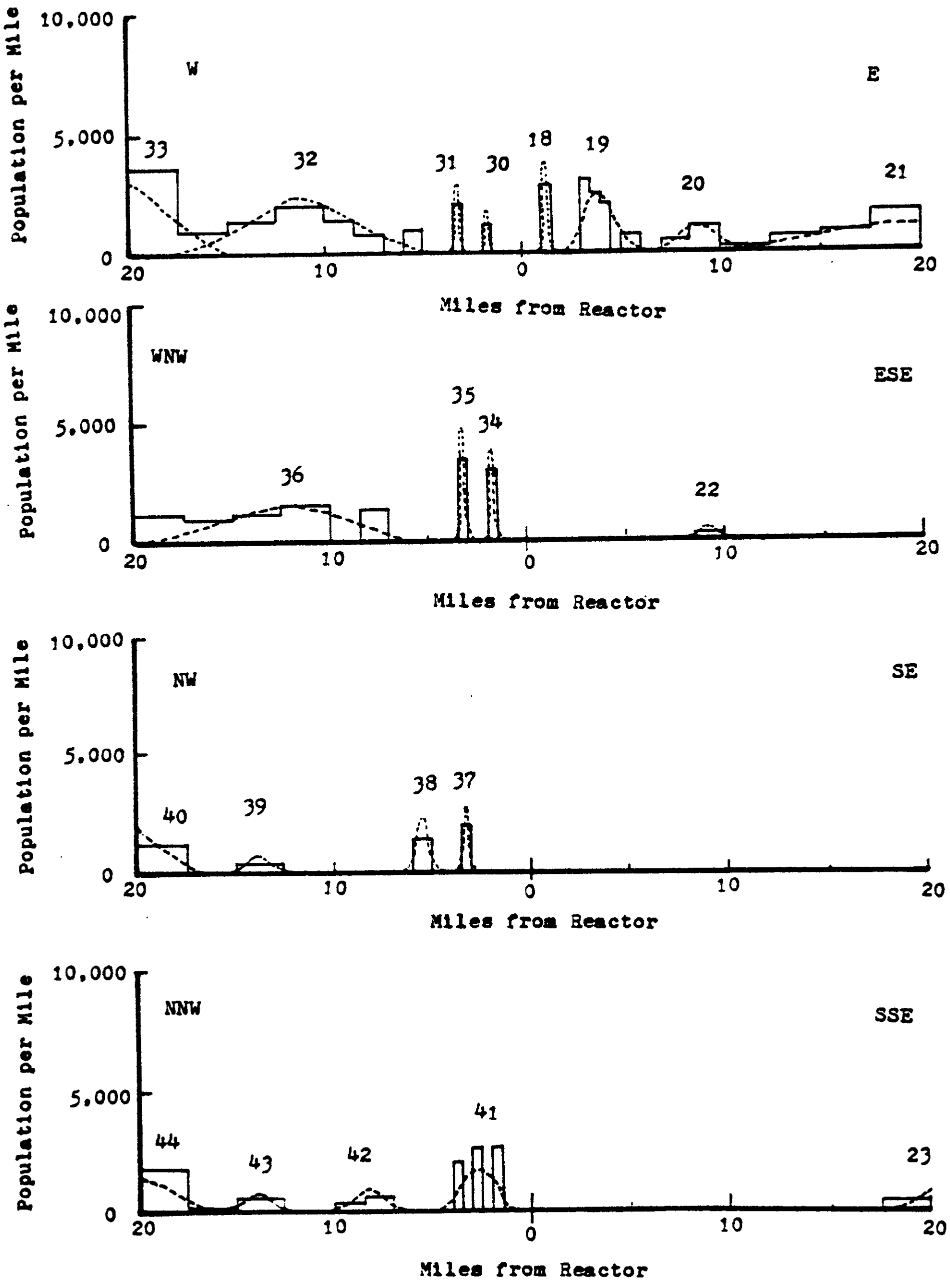

Fig. F.2 (continued)

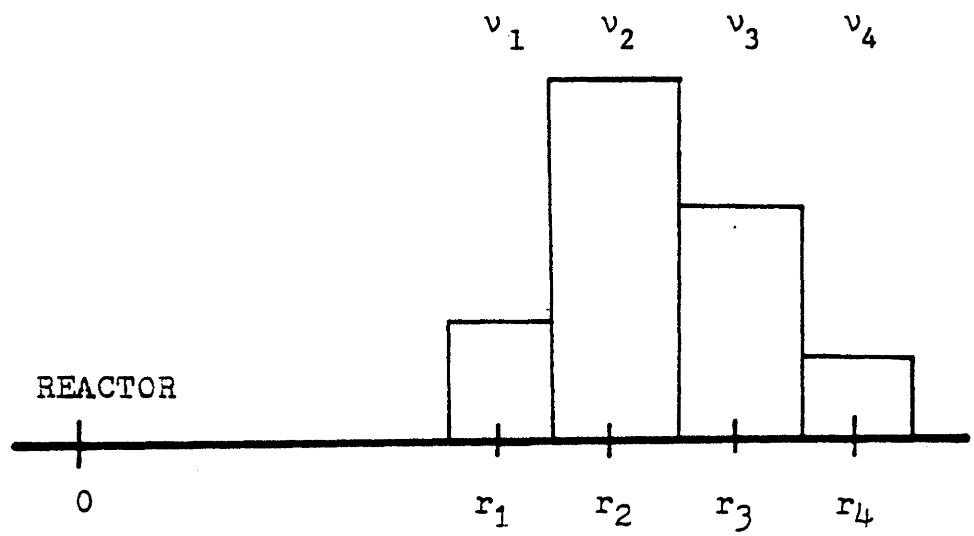

Fig. F.3 Illustration for Derivation of Parameters of the Bell-shaped Population Model

Table F.1 Constants of Bell-Shaped Model and First Two Risk Moments for Site A

Direction	Index	Population N	Miles from Reactor R	Radius σ_R (miles)	Risk Moments M_1		M_2	
N	1	1,420	1.8	.25	1.7	10^{-5}	1.2	10^{-2}
	2	823	3.3	.25	4.1	10^{-6}	1.4	10^{-3}
	3	1,349	6.2	.46	1.2	10^{-6}	5.0	10^{-4}
	4	9,667	12.0	2.1	5.7	10^{-7}	7.2	10^{-4}
NNE	5	1,918	2.8	.49	1.3	10^{-5}	1.1	10^{-2}
	6	3,807	5.5	.46	5.2	10^{-6}	6.3	10^{-3}
	7	15,340	9.2	1.28	3.0	10^{-6}	9.1	10^{-3}
	8	61,340	17.5	3.9	5.3	10^{-7}	2.3	10^{-3}
NE	9	517	.8	.25	1.2	10^{-5}	3.1	10^{-3}
	10	27,410	4.8	.87	6.4	10^{-5}	5.8	10^{-1}
	11	10,420	7.5	.57	4.3	10^{-6}	1.2	10^{-2}
	12	5,264	12.6	1.4	1.3	10^{-7}	1.0	10^{-4}
	13	4,423	22.3	2.8	1.0	10^{-9}	1.6	10^{-7}
ENE	14	6,088	3.8	1.23	2.2	10^{-5}	6.3	10^{-2}
	15	14,203	5.8	1.59	2.4	10^{-5}	9.5	10^{-2}
	16	12,090	11.4	1.06	5.5	10^{-7}	1.1	10^{-3}
	17	28,920	19.7	3.23	4.9	10^{-8}	5.9	10^{-5}
E	18	1,394	1.3	.25	2.3	10^{-5}	1.6	10^{-2}
	19	4,523	4.0	.78	1.6	10^{-5}	2.7	10^{-2}
	20	2,620	9.0	1.0	4.9	10^{-7}	2.8	10^{-4}
	21	10,580	19.2	4.4	5.2	10^{-8}	3.4	10^{-5}
ESE	22	4,620	9.3	.25	6.4	10^{-8}	6.4	10^{-4}
SE								
SSE	23	696	22.5	.8	3.8	10^{-11}	1.4	10^{-9}

(continued)

Table F.1

(continued)

Direction	Index	Population N	Miles from Reactor R	Radius σ_R (miles)	Risk Moments M_1	M_2
S	24	9,386	23.2	2.5	8.9×10^{-10}	3.2×10^{-7}
SSW	25	787	11.3	.42	3.4×10^{-8}	4.6×10^{-6}
SW	26	777	2.3	.25	7.1×10^{-6}	2.5×10^{-3}
	27	6,962	19.7	2.1	4.0×10^{-9}	1.6×10^{-6}
WSW	28	399	5.5	.5	5.2×10^{-7}	7.0×10^{-5}
	29	1,180	11.2	.38	5.2×10^{-8}	1.1×10^{-5}
W	30	569	1.8	.25	7.0×10^{-6}	1.9×10^{-3}
	31	941	3.3	.25	4.7×10^{-6}	1.8×10^{-3}
	32	14,082	11.3	2.3	1.5×10^{-6}	2.7×10^{-3}
	33	14,870	19.7	2.0	1.9×10^{-9}	7.1×10^{-6}
WNW	34	1,518	1.8	.25	1.9×10^{-5}	1.3×10^{-2}
	35	1,679	3.3	.25	8.4×10^{-6}	5.7×10^{-3}
	36	12,090	12.3	2.86	1.2×10^{-6}	1.4×10^{-3}
NW	37	977	3.3	.25	4.9×10^{-6}	1.9×10^{-3}
	38	1,547	5.5	.50	2.2×10^{-6}	1.1×10^{-3}
	39	1,020	13.8	.42	9.7×10^{-9}	1.3×10^{-6}
	40	615,300	40.7	5.1	5.3×10^{-11}	7.0×10^{-8}
NNW	41	3,542	2.6	.80	2.9×10^{-5}	4.5×10^{-2}
	42	1,638	8.3	.71	4.4×10^{-7}	1.7×10^{-4}
	43	1,697	14.1	.88	1.4×10^{-8}	3.2×10^{-6}
	44	6,591	19.7	1.80	3.0×10^{-9}	1.3×10^{-6}
Total of the Risk Moments					3.00×10^{-4}	9.2×10^{-1}
Results of the Consequence Calculation					2.72×10^{-4}	5.8×10^{-1}

(Note): The first risk moments are estimated from the constants of the transfer function of PWR accidents in the northeastern valley weather condition.

$$N_T = \sum_k \nu_k \tag{F.28}$$

The distance of the center of the group from the reactor is estimated by:

$$R = \frac{\sum_k \nu_k r_k}{N} \tag{F.29}$$

The deviation from the center of the population group is estimated by:

$$\sigma_R = \sqrt{\frac{\Sigma\ \nu_k (r_k - R)^2}{N_T}} \tag{F.30}$$

When the tails of the two population groups are overlapping in one segment, half of the population in the segment is assigned to each of the population groups.

The constants of the population groups are estimated for Site A and are given in Table F.1.

F.4.2 Estimation of Risk Moments at Site A

The first two risk moments for the population group ℓ are estimated by:

$$(M_1)_\ell = a_1 \cdot N_\ell \cdot \exp\ [-a_2 R_\ell + \frac{a_2^2}{2} \sigma_\ell^2\ ] \tag{F.31}$$

$$(M_2)_\ell = b_1 \cdot N_\ell^2 \cdot \exp\ [-2b_2 R_\ell + b_2^2 \sigma_\ell^2\ ] \tag{F.32}$$

where a_1, a_2, b_1 and b_2 are the constants of the transfer functions discussed in Chapter 5. The numerical values of the constants for PWR accidents in the northeastern valley weather condition are used. The results are given in Table F.1. The summations of the risk moments of the population groups give the total risk moments of Site A. The estimates are compared with the results of the consequence calculation.

The first risk moment from the bell-shaped model is overestimated by approximately 10% and the second risk moment is overestimated by approximately 60%.

The normalization constant α is estimated by Eqs. (F.17) and (F.18). Using the numerical values of PWR accidents, α is estimated as

$$\alpha = 5.57 \times 10^{-7}/\text{reactor year}$$

The results of the consequence calculation is:

$$\tilde{\alpha} = 5.78 \times 10^{-7}/\text{reactor year}$$

The difference of these estimates of the normalization constant is less than 4%. The distribution of consequence vs. frequency is estimated by Eqs. (3.27) and (3.28) from M_1, M_2 and α of the bell-shaped population model and compared to the results of the consequence calculation in Fig. F.4. The distribution estimated by the bell-shaped population model is within the uncertainty range of the consequence model discussed in Section III.5.2. Therefore the bell-shaped population model is judged to be adequate to describe the population distribution.

Some insight about siting for nuclear power plants can be obtained from Table F.1. A contribution of 20% to the first risk moment and 60% to the second risk moment comes from the population group with 27,410 population at 4.8 miles in the northeast direction. The existence of this population group has a dominant contribution to the tail behavior of the curve. Approximately 50% of the first risk moment comes from numerous small towns with the populations between 500 and 5000 located within 4 miles from the reactor. The existence of these towns have dominant effects on the main body of the curve. The large city of 61,340 at 17.5 miles in NNE and the metropolitan area of 615,300 at 40 miles in NW have insignificant contribution to the risk moments (less than 1%), since the probability of early fatality decreases sharply as the distance from a reactor increases. In this specific example, small towns and a city of 27,000 within 5 miles have dominant contributions to the risk distribution.

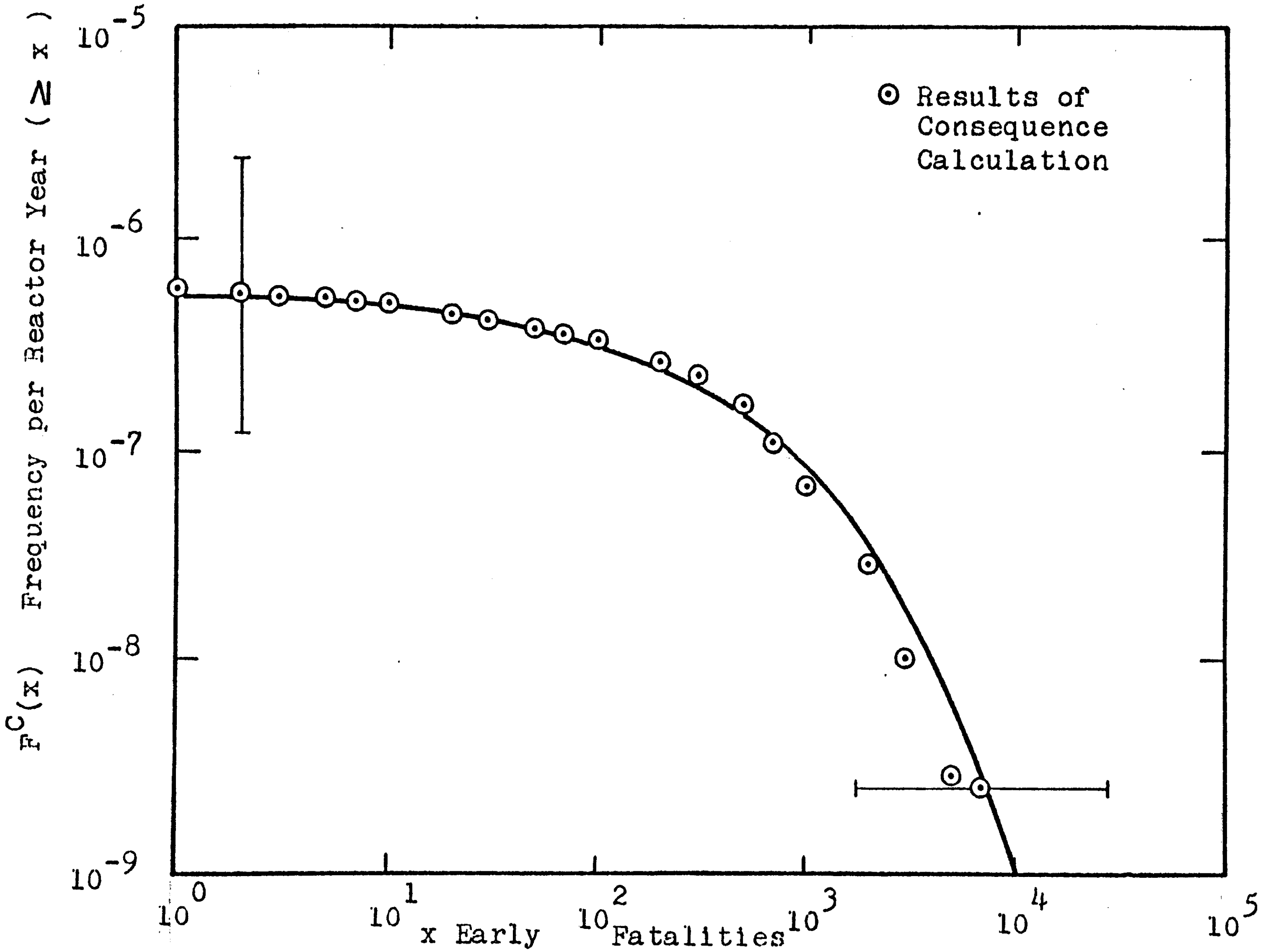

Fig.F.4 Comparison of the Estimates from the Bell-Shaped Population Model to the Results of the Consequence Calculation (Site A)

APPENDIX G

EFFECTIVE SOURCE

G.1 Introduction

In the regression analysis of radioactive releases, a concept of the effective source was introduced to combine the release fractions of the eight isotope groups into one variable. The reasons for introducing the effective source were the following:

(1) Early fatalities are caused by the combined effects of the doses from the eight isotope groups.

(2) The release fractions of the eight isotope groups are correlated with each other because similar physical processes underlie in the release mechanisms for all of the isotope groups.

The effective source was defined as a weighted sum of the release fractions of the eight isotope groups. The weighting factors were derived from the inventories of the radioisotopes, the dose conversion factors and the dose-response relationship. In this appendix, the rationale of derivation of the weighting factors and the source data of the numerical values are discussed.

G.2 Derivation of the Effective Source

In the consequence model, 54 important radioactive isotopes are considered. The 54 isotopes are grouped into 8 isotope groups and the release fractions are estimated for the eight isotope groups. From the inventory of the isotope (j) and the release fraction of the isotope group (g) to which the isotope (j) belongs, the amount of the isotope

(j) released into the environment is given by:

$$Q_j = I_j \cdot q_{g(j)} \cdot \exp\,[-\lambda_j \cdot T_r] \qquad \text{(G.1)}$$

where Q_j = released amount of the isotope (j). [Ci]

I_j = inventory of the isotope (j) in the reactor core. [Ci]

$q_{g(j)}$ = release fraction of the isotope group (g) to which the isotope (j) belongs.

λ_j = radioactive decay constant of the isotope (j). [/hour]

T_r = time of release. [hour]

The exponential term in Eq. (G.1) accounts for radioactive decay before the release. When the build-up from the radioactive decay of the parent isotope is significant, the following term is added to Eq. (G.1):

$$I_p \cdot q_{g'(p)} \cdot \frac{\exp\,[-\lambda_p \cdot T_r] - \exp\,[-\lambda_j \cdot T_r]}{\lambda_j - \lambda_p} \qquad \text{(G.2)}$$

where the subscript (p) refers to the parent isotope of the isotope (j) and g'(p) is the isotope group to which the parent isotope (p) belongs.

From the gaussian dispersion model used in the consequence model, the ground level airborne concentration at the distance r from the reactor is given by:

$$\chi_j(r) = \frac{Q_j}{2\pi\,\sigma_y\,\sigma_z\,u}\ \exp\left(-\frac{h^2}{2\sigma^2}\right) \qquad \text{(G.3)}$$

where $\chi_j(r)$ = ground level airborne concentration of the isotope (j) at the distance r from the reactor. [Ci·sec/m^3]

σ_y, σ_z = dispersion parameters. [m]

u = wind speed. [m/s]

h = elevation of the release. [m]

Since the early fatalities are expected in a close area from the reactor, the radioactive decay after the release is ignored in deriving the effective source.

From the concentrations of the radioactivities the health effects are calculated. Among the various organs in a human body, three organs are particularly critical in causing early fatalities. They are bone marrow, lung and gastrointestinal tract. The dose to these organs consist of three modes of exposure. They are inhalation dose, cloud shine dose and ground shine dose. The inhalation dose to the organ (k) from the isotope (j) is calculated from the airborne concentration:

$$(D_I)_j^{(k)}(r) = B \cdot (C_I)_j^{(k)} \cdot \chi_j(r) \qquad (G.4)$$

where $(D_I)_j^{(k)}(r)$ = inhalation dose to organ (k) from the isotope (j) at the distance r. [rem]

B = breathing rate. [m^3/sec]

$(C_I)_j^{(k)}$ = inhalation dose conversion factor of the isotope (j) to the organ (k). [rem/Ci]

Similarly, the cloud shine dose is determined by:

$$(D_C)_j^{(k)}(r) = s_C \cdot (C_C)_j^{(k)} \cdot \chi_j(r) \cdot \phi \qquad (G.5)$$

where $(D_C)_j^{(k)}(r)$ = cloud shine dose to the organ k from the isotope

(j) at the distance r. [rem]

s_C = cloud shine shielding factor.

$(C_C)_j^{(k)}$ = cloud shine dose conversion factor of the isotope (j) to the organ (k). [$rem \cdot m^3/Ci \cdot sec$]

ϕ = correction factor for the finite cloud.

Practically the correction factor for the finite cloud is close to unity where early fatalities are expected. It will be ignored in the following calculation.

The ground shine dose is proportional to the radioactivity deposited on the ground as:

$$(D_G)_j^{(k)}(r) = s_G \cdot (C_G)_j^{(k)} \cdot G_j(r) \tag{G.6}$$

where $(D_G)_j^{(k)}$ = ground shine dose to the organ (k) from the isotope (j) at the distance r. [rem]

s_G = ground shine shielding factor.

$(C_G)_j^{(k)}$ = ground shine dose conversion factor of the isotope (j) to the organ (k). [$rem \cdot m^2/Ci \cdot sec$]

$G_j(r)$ = concentration of the isotope (j) desposited on the ground. [Ci/m^2]

In a case without rain, the ground concentration is proportional to the ground level airborne concentration as:

$$G_j(r) = \chi_j(r) \cdot (V_d)_j \tag{G.7}$$

where $(V_d)_j$ is a deposition velocity of the isotope (j).

The total dose to the organ (k) is a sum over all the isotopes and over the three modes of exposure:

$$D_T^{(k)}(r) = \sum_j \left[(D_I)_j^{(k)}(r) + (D_C)_j^{(k)}(r) + (D_G)_j^{(k)}(r) \right] \qquad (G.8)$$

From Eqs. (G.4) through (G.8), the total exposure is determined as:

$$\begin{aligned} D_T^{(k)}(r) &= \sum_j \left[B \cdot (C_I)_j^{(k)} \cdot \chi_j(r) + s_C \cdot (C_C)_j^{(k)} \cdot \chi_j(r) + \right. \\ &\quad \left. + s_G \cdot (C_G)_j^{(k)} \cdot (V_d)_j \cdot \chi_j(r) \right] \\ &= \sum_j \left[B \cdot (C_I)_j^{(k)} + s_C \cdot (C_C)_j^{(k)} + s_G \cdot (C_G)_j^{(k)} \cdot (V_d)_j \right] \chi_j(r) \end{aligned} \qquad (G.9)$$

Inserting Eqs. (G.1) and (G.2) into Eq. (G.9):

$$\begin{aligned} D_T^{(k)}(r) &= \sum_j \left\{ [B \cdot (C_I)_j^{(k)} + s_C \cdot (C_C)_j^{(k)} + s_G \cdot (C_G)_j^{(k)} \cdot (V_d)_j \times \right. \\ &\quad \left. \times I_j \cdot q_{g(j)} \cdot \exp(-\lambda_j \cdot T_r) \right\} \frac{1}{2\pi \sigma_y \sigma_z u} \exp\left(-\frac{h^2}{2\sigma^2} \right) \end{aligned} \qquad (G.10)$$

The risks resulting from the damages to the three organs compete with each other, but practically one of them has a dominant effect on early fatalities. That is the dose to the bone marrow. To assure the dominance of the bone marrow dose over the doses to the other two organs, the doses are normalized by the dose-response relationship. As the mortality criteria are often stated in terms of the dose that would be lethal to 50% of the exposed population (denoted by LD_{50}), the doses are normalized as:

$$E^{(k)}(r) = \frac{D_T^{(k)}(r)}{(LD)_{50}^{(k)}} \tag{G.11}$$

where $E^{(k)}(r)$ = normalized dose to the organ (k) at the distance r.

$(LD)_{50}^{(k)}$ = 50% lethal dose to the organ k.

The organ that has the largest value of $E^{(k)}(r)$ has a dominant contribution in causing fatalities. Inserting Eq. (G.10) into Eq. (G.11) and rewriting the summation over isotopes in two steps of summation, one over the isotopes in each of the isotope groups and then over the eight isotope groups, the normalized dose to the organ (k) is given by:

$$E^{(k)}(r) = \frac{1}{2\pi\cdot\sigma_y\cdot\sigma_z\cdot u}\exp\left(-\frac{h^2}{2\sigma^2}\right)\times\left\{\sum_g q_g \times \times \frac{\sum\limits_{j\ \mathrm{in}\ g}\left[B\cdot(C_I)_j^{(k)}+s_C\cdot(C_C)_j^{(k)}+s_G\cdot(C_G)_j^{(k)}\cdot(V_d)_j\right]\cdot I_j\cdot e^{-\lambda_j\cdot T_R}}{(LD_{50})^{(k)}}\right\} \tag{G.12}$$

The weighting factors of the isotope groups are defined as:

$$\Omega_g^{(k)} = \frac{\sum\limits_{j\ \mathrm{in}\ g}\left[B\cdot(C_I)_j^{(k)}+s_C\cdot(C_C)_j^{(k)}+s_G\cdot(C_G)_j^{(k)}\cdot(V_d)_j\right]\cdot I_j\cdot e^{-\lambda_j\cdot T_R}}{(LD_{50})^{(k)}} \tag{G.13}$$

Then, the effective source $\psi^{(k)}$ is defined as:

$$\psi^{(k)} = \sum_g q_g \cdot \Omega_g^{(k)} \tag{G.14}$$

$\Omega_g^{(k)}$ and $\psi^{(k)}$ are independent of the distance r. The normalized dose $E^{(k)}(r)$ is simply rewritten as:

$$E^{(k)}(r) = \frac{1}{2\pi \cdot \sigma_y \cdot \sigma_z \cdot u} \exp\left(-\frac{h^2}{2\sigma_z^2}\right) \cdot \psi^{(k)} \tag{G.15}$$

Then

$$\psi^{(k)} = 2\pi \cdot \sigma_y \cdot \sigma_z \cdot u \cdot \exp\left(+\frac{h^2}{2\sigma_z^2}\right) \cdot E^{(k)}(r) \tag{G.16}$$

At the distance that the dose to the organ (k) is equal to $LD_{50}^{(k)}$, $E^{(k)}(r) = 1$. Therefore, $\psi^{(k)}$ is interpreted as the inverse of the dispersion factor $\frac{1}{2\pi \cdot \sigma_y \cdot \sigma_z \cdot u} \exp\left(-\frac{h^2}{2\sigma_z^2}\right)$ at the distance where 50% of the exposed population are lethal due to the damage to the organ (k).

The organ that has a dominant effect on causing early fatalities can be identified by comparing $\psi^{(k)}$'s. Then the overall effective source is defined as:

$$\psi = \text{Max}\left\{ \psi^{\text{MARROW}}, \psi^{\text{LUNG}}, \psi^{\text{G.I.}} \right\} \tag{G.17}$$

Practically in most of the release categories,

$$\psi = \psi^{\text{MARROW}} \tag{G.18}$$

G.3 Source of Data for Deriving Weighting Factors

The weighting factor was defined in the previous section as:

$$\Omega_g^{(k)} = \frac{\sum_{j \text{ in } g} \left[B \cdot (C_I)_j^{(k)} + s_C \cdot (C_C)_j^{(k)} + s_G \cdot (C_G)_j^{(k)} \cdot (V_d)_j \right] \cdot I_j \cdot e^{-\lambda_j T_R}}{(LD_{50})^{(k)}} \tag{G.19}$$

The data for estimating $\Omega_g^{(k)}$ are given in Tables G.1 through G.5. Table G.1 gives the radioactive inventory I_j and the half-life. The decay constant λ_j is derived from the half-life by:

$$\lambda_j = \frac{.693}{(T_{1/2})_j} \tag{G.20}$$

where $(T_{1/2})_j$ is the half-life of the isotope (j). Tables G.2 through G.4 summarize the dose conversion factors. Table G.5 summarizes the miscellaneous data in Eq. (G.19).

G.4 Numerical Values of Weighting Factors

The weighting factors derived from Eq. (G.19) are shown as functions of the time of the release (T_r) in Figs. G.1 through G.3. These figures show that the changes of the effective source in the range of 1 hr $\leq T_r \leq$ 30 hrs are small except for iodines and noble gases. The effects of the time on the weighting factors of iodines and noble gases are accounted for by assuming the equivalent half-lives for these groups. The effective half-lives are determined from Fig. G.1 through G.3. The results in Table G.6 are used to determine the effective source in Chapter VI.

TABLE G.1 Initial Activity of Radionuclides in the Nuclear Reactor Core at the Time of Hypothetical Accident

No.	Radionuclide	Radioactive Inventory Source (curies x 10^8)	Half-Life (days)
1	Cobalt-58	0.0078	71.0
2	Cobalt-60	0.0029	1,920
3	Krypton-85	0.0056	3,950
4	Krypton-85m	0.24	0.183
5	Krypton-87	0.47	0.0528
6	Krypton-88	0.68	0.117
7	Rubidium-86	0.00026	18.7
8	Strontium-89	0.94	52.1
9	Strontium-90	0.037	11,030
10	Strontium-91	1.1	0.403
11	Yttrium-90	0.039	2.67
12	Yttrium-91	1.2	59.0
13	Zirconium-95	1.5	65.2
14	Zirconium-97	1.5	0.71
15	Niobium-95	1.5	35.0
16	Molybdenum-99	1.6	2.8
17	Technetium-99m	1.4	0.25
18	Ruthenium-103	1.1	39.5
19	Ruthenium-105	0.72	0.185
20	Ruthenium-106	0.25	366
21	Rhodium-105	0.49	1.50
22	Tellurium-127	0.059	0.391
23	Tellurium-127m	0.011	109
24	Tellurium-129	0.31	0.048
25	Tellurium-129m	0.053	0.340
26	Tellurium-131m	0.13	1.25
27	Tellurium-132	1.2	3.25
28	Antimony-127	0.061	3.88
29	Antimony-129	0.33	0.179
30	Iodine-131	0.85	8.05
31	Iodine-132	1.2	0.0958
32	Iodine-133	1.7	0.875
33	Iodine-134	1.9	0.0366
34	Iodine-135	1.5	0.280
35	Xenon-133	1.7	5.28
36	Xenon-135	0.34	0.384
37	Cesium-134	0.075	750
38	Cesium-136	0.030	13.0
39	Cesium-137	0.047	11,000
40	Barium-140	1.6	12.8
41	Lanthanum-140	1.6	1.67
42	Cerium-141	1.5	32.3
43	Cerium-143	1.3	1.38
44	Cerium-144	0.85	284
45	Praseodymium-143	1.3	13.7
46	Neodymium-147	0.60	11.1
47	Neptunium-239	16.4	2.35
48	Plutonium-238	0.00057	32,500
49	Plutonium-239	0.00021	8.9×10^6
50	Plutonium-240	0.00021	2.4×10^6
51	Plutonium-241	0.034	5,350
52	Americium-241	0.000017	1.5×10^5
53	Curium-242	0.0050	163
54	Curium-244	0.00023	6,630

Note: From TABLE VI 3-1 in Appendix VI of WASH-1400(ref.1)

Table G.2 Dose Conversion Factor for Bone Marrow

ISOTOPE	INHALATION (REM/CI)	CLOUD SHINE (REM-SEC/M**3)	GROUND SHINE (REM-SEC/M**2)
CO-58	7.95E 02	2.40E-01	6.15E 01
CO-60	2.00E 03	6.31E-01	1.48E 02
KR-85	6.10E-01	5.78E-04	1.48E-01
KR-85M	3.90E-01	5.50E-02	7.85E 00
KR-87	1.30E 00	1.92E-01	9.65E 00
KR-88	3.10E 00	4.83E-01	5.95E 01
RB-86	3.25E 03	2.27E-02	5.45E 01
SR-89	3.35E 03	0.0	0.0
SR-90	6.10E 03	0.0	0.0
SR-91	2.15E 02	1.93E-01	5.10E 01
Y-90	4.70E 02	0.0	0.0
Y-91	1.43E 03	6.39E-04	1.50E-01
ZR-95	6.70E 02	1.87E-01	4.73E 01
ZR-97	1.90E 02	4.72E-02	3.35E 01
NB-95	5.75E 02	1.83E-01	4.58E 01
MO-99	1.25E 02	4.44E-02	1.56E 01
TC-99M	1.10E 01	5.42E-02	3.70E 00
RU-103	4.05E 02	1.36E-01	3.58E 01
RU-105	2.40E 01	2.21E-01	3.22E 01
RU-106	4.40E 02	5.22E-02	1.33E 01
RH-105	2.30E 01	2.74E-02	6.40E 00
TE-127	3.90E 00	1.16E-03	2.27E-01
TE-127M	1.82E 02	1.79E-03	2.04E 00
TE-129	1.10E 00	1.81E-02	1.21E 00
TE-129M	3.75E 02	9.92E-03	7.10E 00
TE-131M	3.00E 02	3.56E-01	3.55E 01
TE-132	9.4[illegible]E 02	7.31E-02	1.09E 02
SB-127	3.10E 02	1.84E-01	4.53E 01
SB-129	4.60E 01	2.97E-01	4.13E 01
I-131	1.50E 02	1.08E-01	2.73E 01
I-132	5.00E 01	5.89E-01	5.65E 01
I-133	9.35E 01	1.83E-01	4.06E 01
I-134	2.00E 01	5.89E-01	2.28E 01
I-135	9.10E 01	4.42E-01	9.00E 01
XE-133	1.60E 00	1.59E-02	6.55E 00
XE-135	2.10E 00	8.47E-02	1.58E 01
CS-134	4.95E 03	4.03E-01	1.02E 02
CS-136	3.55E 03	5.42E-01	1.32E 02
CS-137	3.25E 03	1.49E-01	3.78E 01
BA-140	2.10E 03	5.61E-02	2.50E 01
LA-140	6.70E 02	6.06E-01	1.31E 02
CE-141	1.13E 02	3.22E-02	9.25E 00
CE-143	9.55E 01	9.36E-02	2.41E 01
CE-144	2.35E 02	7.61E-03	3.92E 00
PR-143	1.78E 01	0.0	0.0
ND-147	1.40E 02	4.39E-02	1.24E 01
NP-239	6.20E 01	4.97E-02	1.74E 01
PU-238	1.71E 02	4.25E-05	1.19E-01
PU-239	1.59E 02	2.17E-05	5.95E-02
PU-240	1.64E 02	3.89E-05	1.09E-01
PU-241	4.20E-02	8.53E-10	5.15E-06
AM-241	2.65E 02	9.33E-03	7.00E 00
CM-242	2.03E 02	3.89E-05	1.02E-01
CM-244	2.01E 02	2.81E-03	1.62E 00

Table G.3 Dose Conversion Factor for Lung

ISOTOPE	INHALATION (REM/CI)	CLOUD SHINE (REM-SEC/M**3)	GROUND SHINE (REM-SEC/M**2)
CO-58	5.20E 04	2.01E-01	5.15E 01
CO-60	2.50E 05	5.67E-01	1.33E 02
KR-85	1.80E-01	4.47E-04	1.15E-01
KR-85M	2.10E-01	3.22E-02	4.61E 00
KR-87	9.60E-01	1.72E-01	8.65E 00
KR-88	2.00E 00	4.47E-01	5.55E 01
RB-86	1.40E 04	1.94E-02	4.63E 00
SR-89	7.80E 03	0.0	0.0
SR-90	1.40E 04	0.0	0.0
SR-91	4.20E 03	1.60E-01	4.13E 01
Y-90	3.30E 04	0.0	0.0
Y-91	1.90E 05	5.94E-04	1.40E-01
ZR-95	1.10E 05	1.52E-01	3.86E 01
ZR-97	1.50E 04	4.00E-02	6.55E 01
NB-95	3.00E 04	1.56E-01	3.91E 01
MO-99	1.60E 04	3.42E-02	1.09E 01
TC-99M	8.90E 01	2.54E-02	4.06E 00
RU-103	5.20E 04	1.05E-01	2.76E 01
RU-105	2.20E 03	1.67E-01	2.43E 01
RU-106	1.60E 06	4.06E-02	1.03E 01
RH-105	3.60E 03	1.61E-02	3.76E 00
TE-127	1.60E 03	8.78E-04	1.71E-01
TE-127M	1.10E 05	5.61E-04	6.70E-01
TE-129	5.60E 02	1.35E-02	9.05E-01
TE-129M	1.50E 05	6.97E-03	5.10E 00
TE-131M	1.10E 04	2.94E-01	6.95E 01
TE-132	3.00E 04	4.19E-02	3.45E 01
SB-127	2.50E 04	1.43E-01	3.53E 01
SB-129	3.20E 03	2.53E-01	3.49E 01
I-131	2.40E 03	8.22E-02	2.08E 01
I-132	1.00E 03	4.83E-01	4.61E 01
I-133	3.10E 03	1.46E-01	3.25E 01
I-134	5.60E 02	5.00E-01	1.93E 01
I-135	2.50E 03	4.07E-01	7.00E 01
XE-133	4.10E-01	6.97E-03	2.88E 00
XE-135	9.40E-01	5.06E-02	9.45E 00
CS-134	3.40E 04	3.28E-01	8.30E 01
CS-136	8.20E 03	4.44E-01	1.08E 02
CS-137	2.50E 04	1.15E-01	2.92E 01
BA-140	6.30E 03	4.14E-02	1.98E 01
LA-140	1.60E 04	5.39E-01	1.17E 02
CE-141	6.10E 04	1.50E-02	3.82E 00
CE-143	1.30E 04	6.08E-02	1.57E 01
CE-144	1.40E 06	3.44E-03	2.49E 00
PR-143	4.90E 04	0.0	0.0
ND-147	3.70E 04	2.78E-02	7.85E 00
NP-239	9.20E 03	2.65E-02	9.35E 00
PU-238	7.00E 07	9.58E-06	2.70E-02
PU-239	6.50E 07	5.42E-06	1.48E-02
PU-240	6.60E 07	9.17E-06	2.57E-02
PU-241	2.20E 04	2.94E-10	1.76E-06
AM-241	7.00E 07	3.22E-03	2.42E 00
CM-242	5.50E 07	8.31E-06	2.18E-02
CM-244	3.80E 06	1.07E-03	6.20E-01

Table G.4 Dose Conversion Factor for Gastrointestinal Tract

ISOTOPE	INHALATION (REM/CI)	CLOUD SHINE (REM-SEC/M**3)	GROUND SHINE (REM-SEC/M**2)
CO-58	4.28E 03	1.42E-01	3.63E 01
CO-60	1.01E 04	4.61E-01	1.09E 02
KR-85	6.12E-02	3.42E-04	9.80E-02
KR-85M	9.46E-02	2.02E-02	2.88E 00
KR-87	8.10E-01	1.43E-01	7.20E 00
KR-88	2.07E 00	3.83E-01	4.75E 01
RB-86	1.40E 03	1.33E-02	3.19E 00
SR-89	6.86E 03	0.0	0.0
SR-90	9.10E 03	0.0	0.0
SR-91	1.43E 03	1.21E-01	3.12E 01
Y-90	2.60E 04	0.0	0.0
Y-91	2.30E 04	5.39E-04	1.27E-01
ZR-95	3.92E 03	1.10E-01	2.79E 01
ZR-97	1.10E 04	3.17E-02	5.05E 01
NB-95	3.11E 03	1.07E-01	2.69E 01
MO-99	7.40E 03	2.52E-02	7.80E 00
TC-99M	6.93E 01	1.58E-02	2.53E 00
RU-103	2.00E 03	8.03E-02	2.11E 01
RU-105	4.92E 02	1.25E-01	1.81E 01
RU-106	8.88E 04	3.08E-02	7.85E 00
RH-105	4.08E 02	1.01E-02	2.35E 00
TE-127	1.79E 02	6.64E-04	1.29E-01
TE-127M	3.57E 03	1.38E-04	1.93E-01
TE-129	2.56E 00	9.92E-03	6.65E-01
TE-129M	1.50E 04	5.06E-03	3.73E 00
TE-131M	5.35E 03	2.14E-01	5.05E 01
TE-132	3.30E 03	2.61E-02	6.25E 01
SB-127	9.10E 03	1.06E-01	2.63E 01
SB-129	4.86E 02	1.85E-01	2.56E 01
I-131	7.26E 01	6.22E-02	1.57E 01
I-132	4.44E 01	3.64E-01	3.46E 01
I-133	1.52E 02	1.13E-01	2.51E 01
I-134	1.72E 01	3.61E-01	1.40E 01
I-135	1.08E 02	3.33E-01	5.80E 01
XE-133	1.30E-01	3.97E-03	1.64E 00
XE-135	4.85E-01	3.19E-02	6.00E 00
CS-134	2.10E 03	2.41E-01	6.10E 01
CS-136	2.32E 03	3.14E-01	7.65E 01
CS-137	9.12E 02	8.78E-02	2.23E 01
BA-140	7.84E 03	3.08E-02	1.56E 01
LA-140	1.07E 04	4.58E-01	9.90E 01
CE-141	1.20E 03	9.17E-03	2.34E 00
CE-143	5.76E 03	4.06E-02	1.04E 01
CE-144	8.52E 04	2.06E-03	1.84E 00
PR-143	6.75E 03	0.0	0.0
ND-147	3.99E 03	1.97E-02	5.55E 00
NP-239	1.04E 03	1.66E-02	5.85E 00
PU-238	5.00E 03	1.41E-05	3.98E-02
PU-239	4.60E 03	6.03E-06	1.64E-02
PU-240	4.70E 03	1.27E-05	3.55E-02
PU-241	0.0	1.64E-10	9.90E-07
AM-241	5.20E 03	1.80E-03	1.35E 00
CM-242	5.50E 03	1.31E-05	3.46E-02
CM-244	5.20E 03	6.29E-04	3.63E-01

Table G.5 Miscellaneous Data for Deriving Weighting Factors

Parameter	Value
Breathing rate B[1]	2.66×10^{-4} m /sec
Shielding factors	
Ground shine dose[2], s_g	.50
Cloud shine dose[3], s_c	1.0
Deposition velocity[4]	
Iodine vapor and particles	10^{-2} m/s
Noble gas	0 m/s
50% lethal dose[5]	
Bone Marrow	510 rem
Lung	20,000 rem
Gastrointestinal Tract	3,500 rem
Time of release	See Table 6.1

(Note): (1) From Section 8.2.3 in Appendix VI of WASH-1400 (Ref-1).
(2) From Table VI 11-9 in Appendix VI of WASH-1400 (Ref-1).
(3) From Table VI 11-7 in Appendix VI of WASH-1400 (Ref-1).
(4) From Section 6.3.1 in Appendix VI of WASH-1400 (Ref-1).
(5) From Fig. VI 9-1, VI 9-2, VI 9-3 in Appendix VI of WASH-1400 (Ref-1).

Table G.6 Weighting Factors of Isotope Groups for Effective Source

Organ	Isotope Group	Weighting Factor Ω_g
Bone Marrow	Kr - Xe	$5.73 \times 10^3 + 7.90 \times 10^4 \exp[-.20 \cdot Tr]$
	I[1]	$7.81 \times 10^5 \exp[-.058 \cdot Tr]$
	Cs - Rb	5.64×10^4
	Te - Sb	2.54×10^5
	Ba - Sr	5.01×10^5
	Ru	2.28×10^5
	La	1.77×10^6
Lung	Kr - Xe	$1.21 \times 10^2 + 1.6 \times 10^3 \exp[-.20 \cdot Tr]$
	I[1]	$3.35 \times 10^4 \exp[-.058 \cdot Tr]$
	Cs - Rb	7.43×10^3
	Te - Sb	6.83×10^4
	Ba - Sr	3.22×10^4
	Ru	9.53×10^5
	La	4.28×10^6
G.I. Tract	Kr - Xe	$4.18 \times 10^2 + 8.2 \times 10^3 \exp[-.20 \cdot Tr]$
	I[1]	$7.70 \times 10^4 \exp[-.058 \cdot Tr]$
	Cs - Rb	4.08×10^3
	Te - Sb	6.18×10^4
	Ba - Sr	1.69×10^5
	Ru	2.92×10^5
	La	1.53×10^6

[1] Organic iodines and inorganic iodines are included.

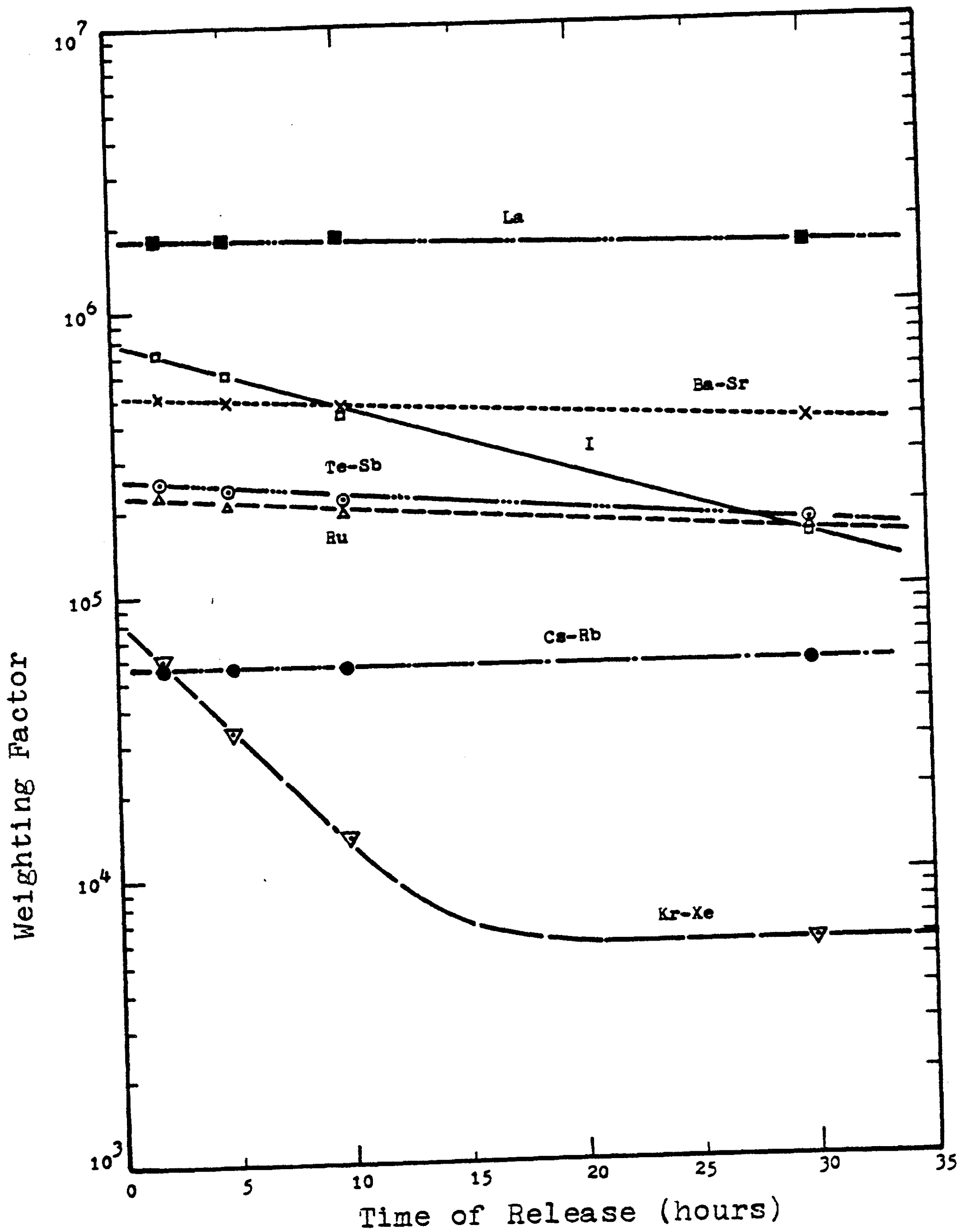

Fig. G.1 Weighting Factor for Bone Marrow

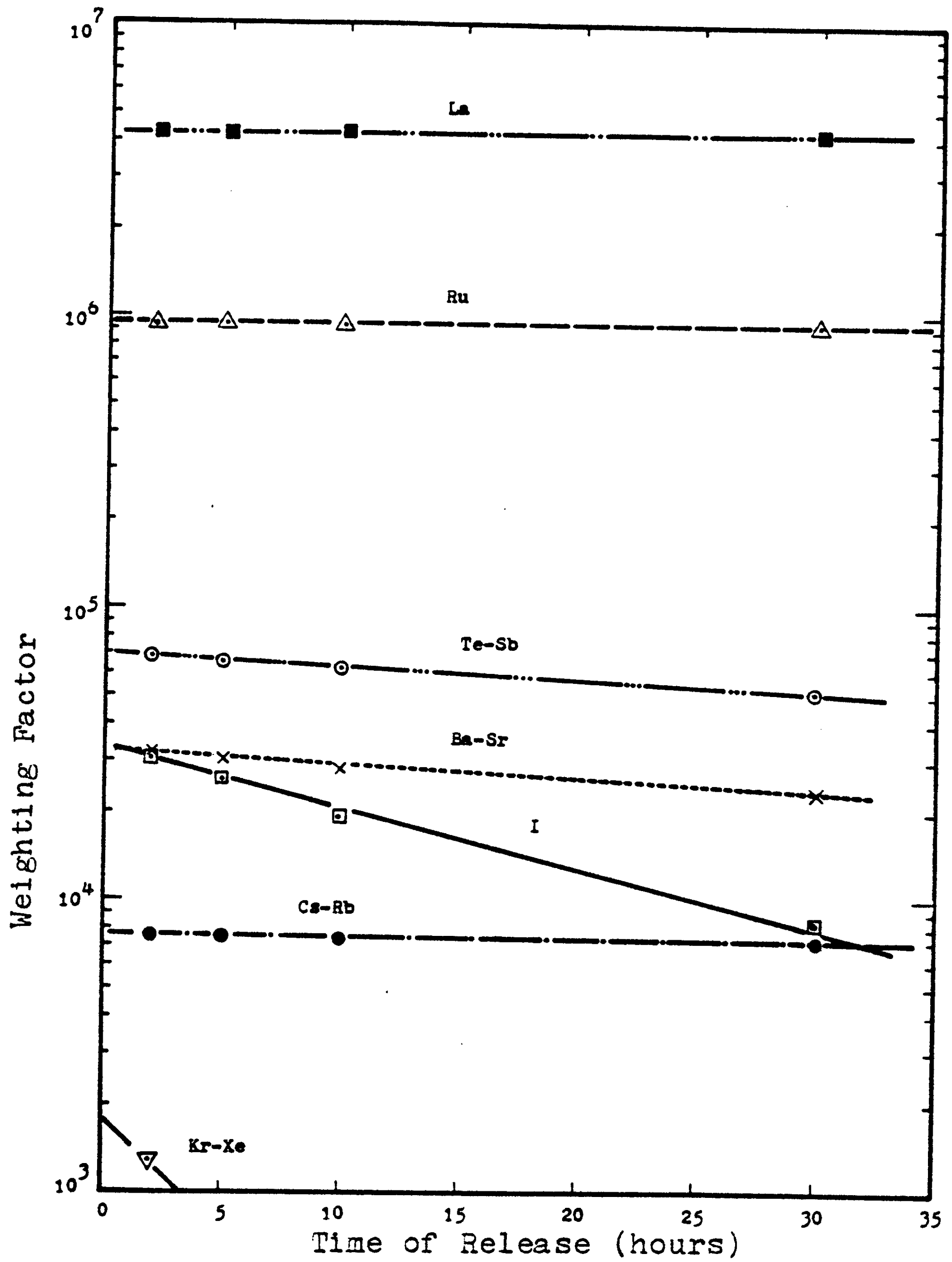

Fig. G.2 Weighting Factor for Lung

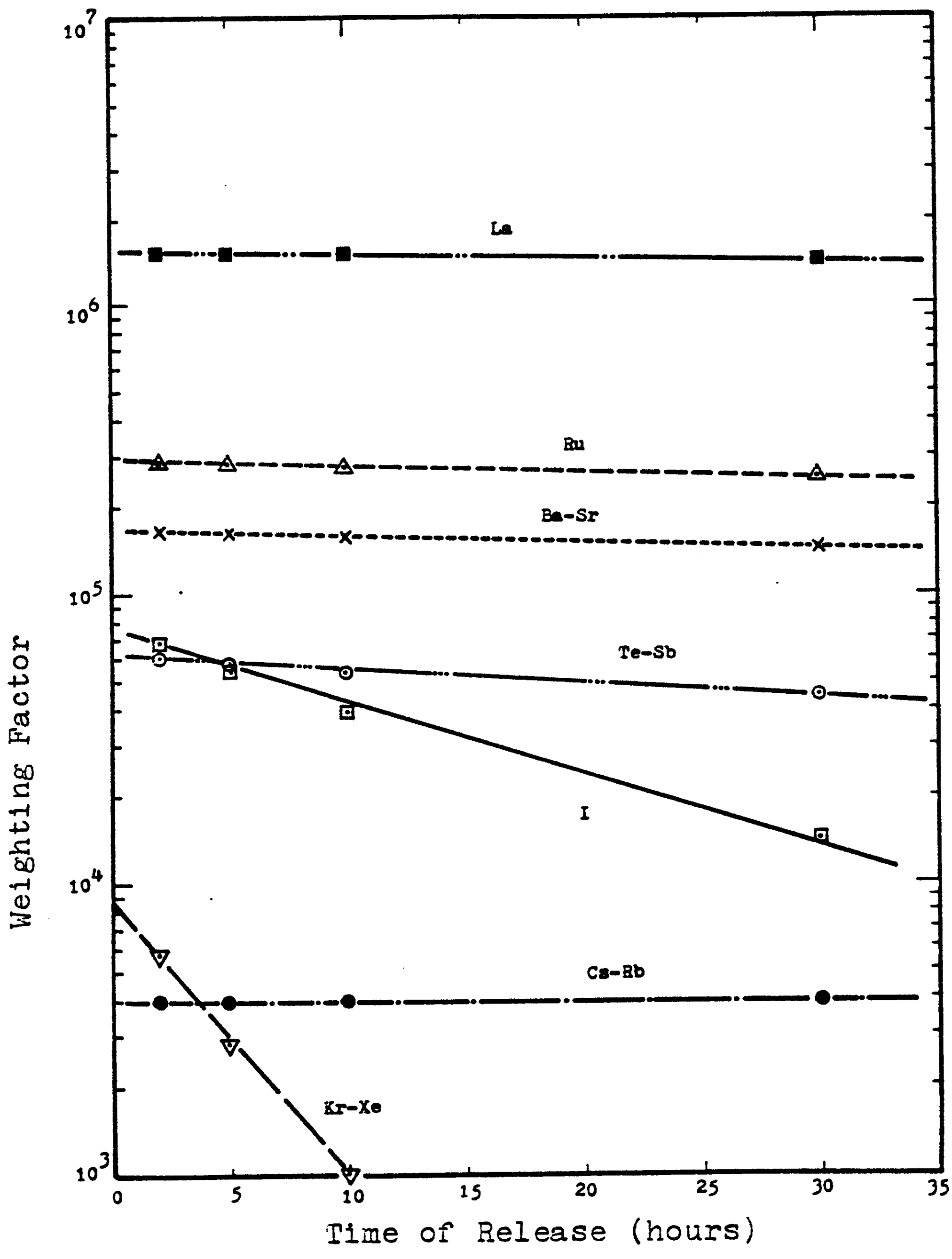

Fig. G.3 Weighting Factor for Gastrointestinal Tract

APPENDIX H

REGRESSION RESULTS OF THE CONSTANTS OF b(r,r') AND c(r) WITH REGARD TO RELEASE CHARACTERISTICS

The regression fittings of the constants of the transfer functions b(r,r') and c(r) are made in the same way as the analysis of a_1 and a_2 in Section VI.6. The results are summarized in the following tables and figures:

Table H.1 Data Base for Regression of b_1, b_2 and b_3

Table H.2 Data Base for Regression of c_1 and c_2

Table H.3 Regression Result of b_1

Table H.4 Regression Result of b_2

Table H.5 Regression Result of b_3

Table H.6 Regression Result of c_1

Table H.7 Regression Result of c_2

Fig. H.1 Test for Adequacy of Regression of b_1

Fig. H.2 Test for Adequacy of Regression of b_2

Fig. H.3 Test for Adequacy of Regression of b_3

Fig. H.4 Test for Adequacy of Regression of c_1

Fig. H.5 Test for Adequacy of Regression of c_2

Fig. H.6 Examination of Combined Result of b(r,r')

Fig. H.7 Examination of Combined Result of c(r)

Table H.1 Data Base for Regression of b_1, b_2 and b_3

Calculation Case		b_1	b_2	b_3
PWR - 1A		5.68×10^{-3}	.333	.443
PWR - 1B		2.27×10^{-3}	.550	.572
PWR - 2		1.72×10^{-3}	.431	.454
PWR - 3		1.40×10^{-2}	1.09	1.32
PWR - 4		3.40×10^{-2}	2.28	1.58
BWR - 1		2.78×10^{-3}	.432	.558
BWR - 2		2.83×10^{-3}	1.100	.801
BWR - 3		6.40×10^{-3}	2.820	1.050
Additional Cases(1):	1	3.18×10^{-3}	.476	.588
	2	3.14×10^{-3}	.309	.489
	3	1.09×10^{-2}	.297	.291
	4	1.59×10^{-2}	1.10	.900
	5	4.22×10^{-3}	.453	.605
	6	3.77×10^{-3}	.466	.582
	7	2.43×10^{-3}	.435	.520
	8	3.14×10^{-3}	.505	.491
	9	2.15×10^{-3}	.509	.541
	10	1.92×10^{-3}	.502	.540
	11	5.33×10^{-3}	.339	.434
	12	5.13×10^{-3}	.339	.428
	13	3.48×10^{-3}	.578	.542
	14	4.05×10^{-3}	.590	.534
	15	1.62×10^{-3}	.468	.559
	16	2.58×10^{-3}	.311	.512
	17	2.78×10^{-3}	.536	.730
	18	1.97×10^{-3}	.289	.592
	19	3.54×10^{-3}	.242	.481
	20	8.40×10^{-3}	.396	.480

[1] Corresponding to the calculation case number in Table 6.3.

Table H.2 Data Base for Regression of c_1 and c_2

Calculation Case		c_1	c_2
PWR - 1A		5.27×10^{-2}	.243
PWR - 1B		7.63×10^{-3}	.297
PWR - 2		1.17×10^{-2}	.437
PWR - 3		1.39×10^{-2}	.714
PWR - 4		3.06×10^{-2}	2.23
BWR - 1		9.68×10^{-3}	.230
BWR - 2		3.29×10^{-3}	.649
BWR - 3		2.50×10^{-3}	1.460
Additional Cases(1):	1	5.06×10^{-3}	.236
	2	2.60×10^{-2}	.214
	3	5.38×10^{-2}	.191
	4	1.59×10^{-2}	.723
	5	1.05×10^{-2}	.242
	6	1.05×10^{-2}	.235
	7	8.61×10^{-3}	.229
	8	7.11×10^{-3}	.211
	9	6.71×10^{-3}	.296
	10	5.73×10^{-3}	.295
	11	4.47×10^{-2}	.252
	12	3.77×10^{-2}	.240
	13	4.12×10^{-3}	.232
	14	3.90×10^{-3}	.249
	15	1.11×10^{-2}	.269
	16	1.27×10^{-2}	.244
	17	7.16×10^{-3}	.294
	18	2.64×10^{-2}	.138
	19	2.55×10^{-2}	.287
	20	4.32×10^{-2}	.282

[1] Corresponding to the calculation case number in Table 6.3.

Table H.3 Regression Analysis of b_1

Dependent Variable	Regressor Variable	Regression Coefficient	Standard Deviation of Regression Coefficient	t-value
ln b_1	ln h	-.266	.097	-2.73
	ln E	-.387	.043	-8.90

Intercept	-3.18
Multiple Correlation	0.893
Standard error of estimate	0.341
F-value	49.5

(0.1% F-value for 2 and 25 degrees of freedom is 9.22)

Table H.4 Regression Analysis of b_2

Dependent Variable	Regressor Variable	Regression Coefficient	Standard Deviation of Regression Coefficient	t-value
$\ln b_2$	$\ln h$	.043	.0319	1.34[1]
	$\ln T_d$	.192	.0472	4.1
	$\ln E$	.116	.0184	6.3
	$\ln \psi$	-.990	.0692	-14.2

Intercept	.559
Multiple Correlation	.984
Standard Error of Estimate	.110
F-value	176.3

(0.1% F-value for 4 and 23 degrees of freedom is 6.69)

[1] t-value at 10% significance level with 23 degrees of freedom is 1.32. The term ($\ln h$) is marginally significant.

Table H.5 Regression Analysis of b_3

Dependent Variable	Regressor Variable	Regression Coefficient	Standard Error of Regression Coefficient	t-value
$\ln b_3$	$\ln \psi$	-.515	.070	-7.31

Intercept	.372
Multiple correlation	.820
Standard error of estimate	.205
F-value	53.48

(0.1% F-value for 1 and 26 degrees of freedom is 13.7)

(Note): The t-value of (ln E) is 1.29, while the upper 10% t-value with 26 degrees of freedom is 1.32. It is eliminated in this study.

Table H.6 Regression Analysis of c_l

Dependent Variable	Regressor Variable	Regression Coefficient	Standard Deviation of Regression Coefficient	Computed t-value
ln c_l	ln h	-.374	.140	-2.68
	ln T_d	-.652	.207	-3.16
	ln E	-.653	.081	-8.11
	ln ψ	.928	.303	3.06

Intercept	-2.45
Multiple correlation	.888
Standard error of estimate	.481
F-value	21.44

(0.1% F-value for 4 and 23 degrees of freedom is 6.69)

Table H.7 Regression Analysis of c_2

Dependent Variable	Regressor Variable	Regression Coefficient	Standard Error of Regression Coefficient	t-value
ln c_2	ln h	-.0801	.0564	1.42
	ln ψ	-1.02	.0690	-14.8

Intercept	.886
Multiple correlation	.953
Standard error of estimate	.194
F-value	123.9

(0.1% F-value for 2 and 25 degrees of freedom is 9.22)

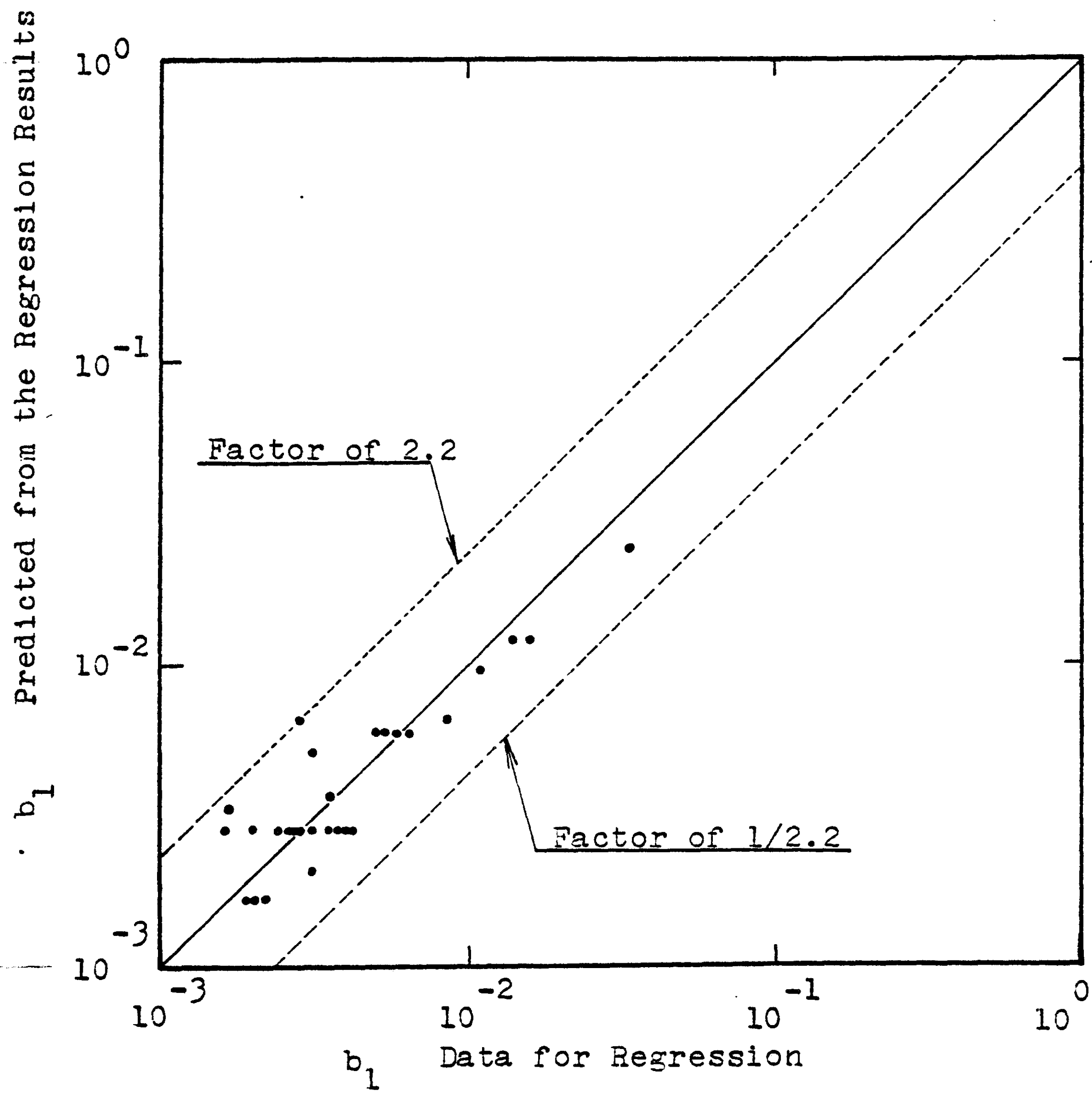

Fig.H.1 Test of the Regression Results of b_1

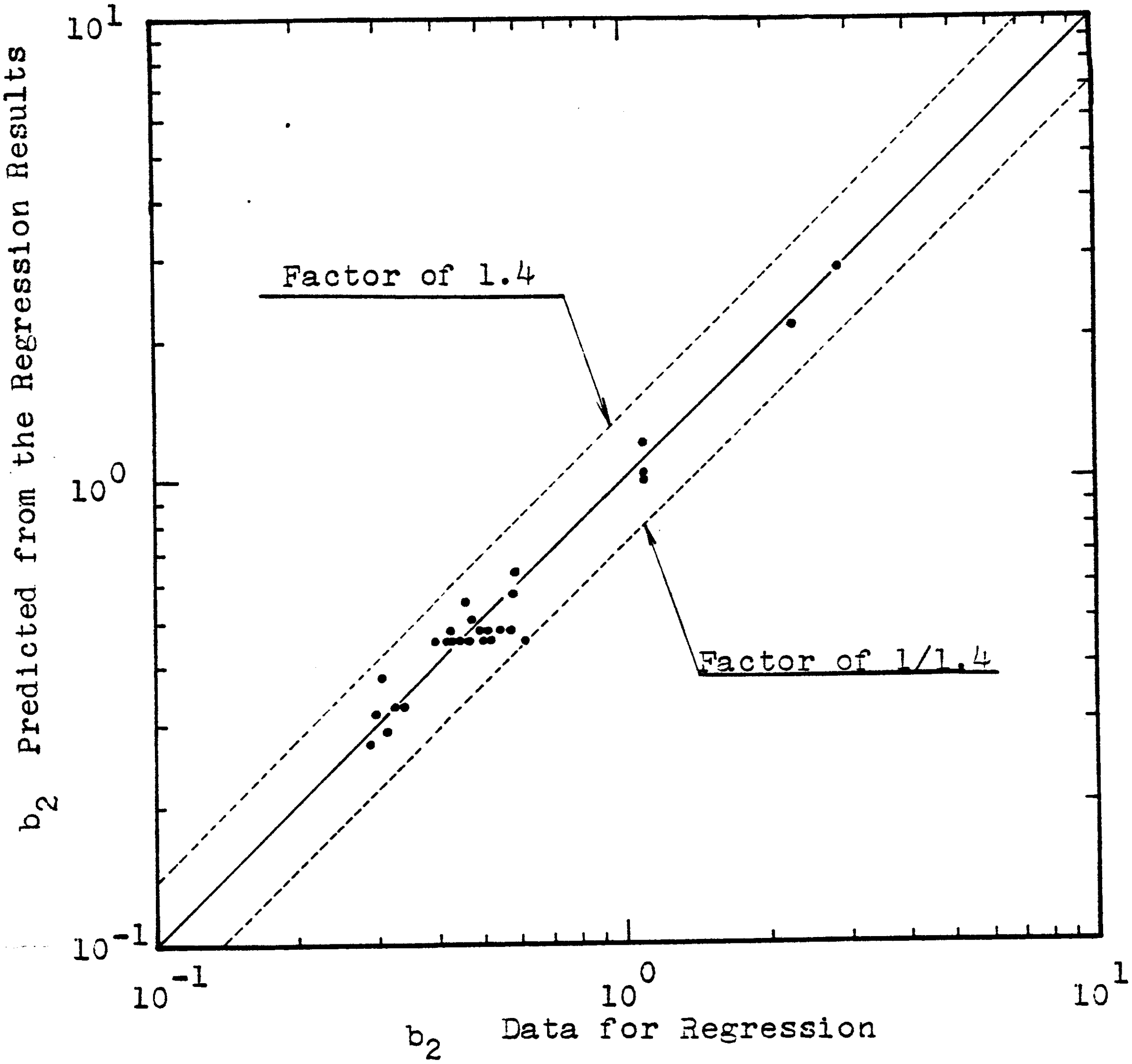

Fig.H.2 Test of the Regression Results of b_2

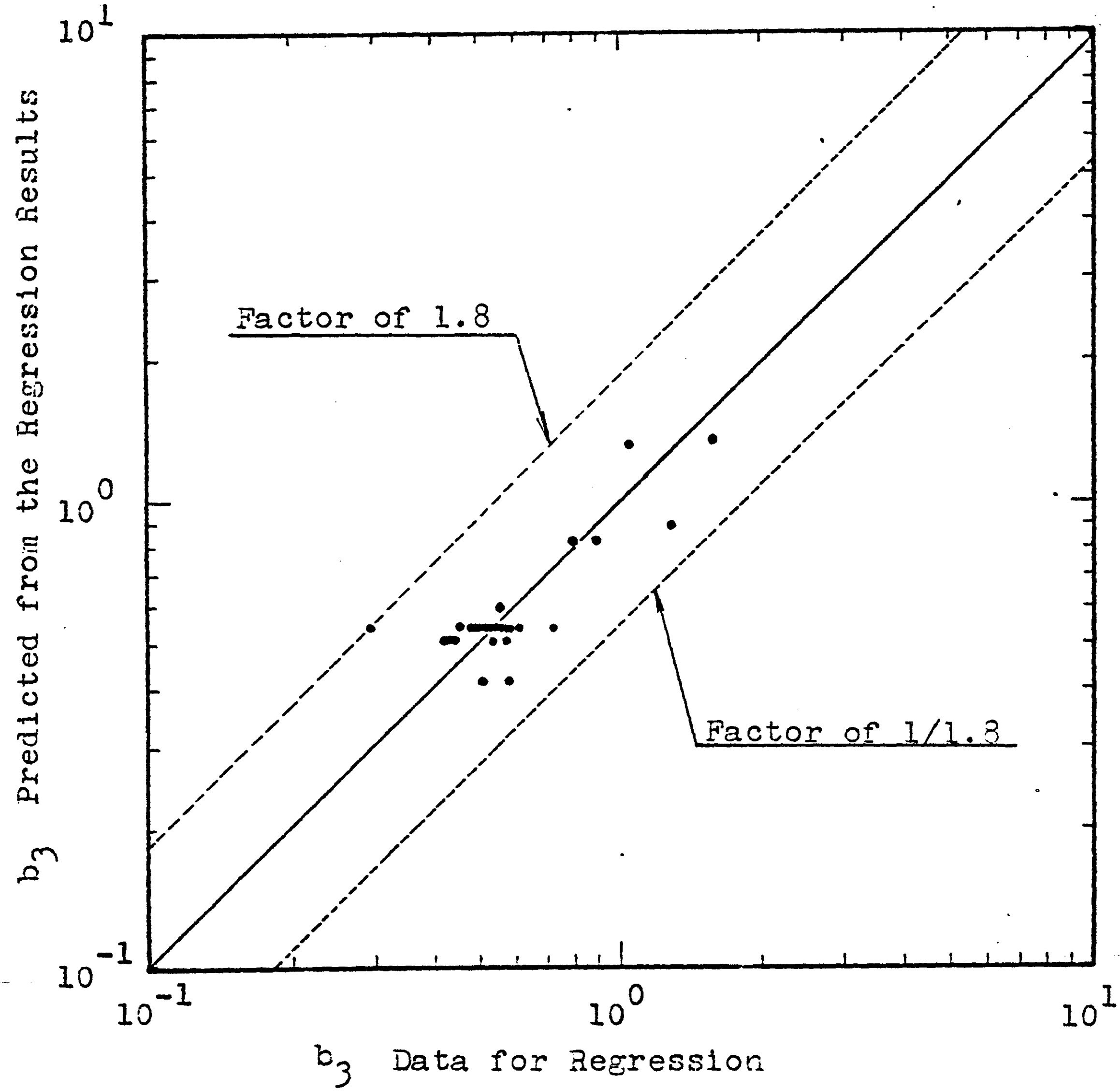

Fig.H.3 Test of the Regression Results of b_3

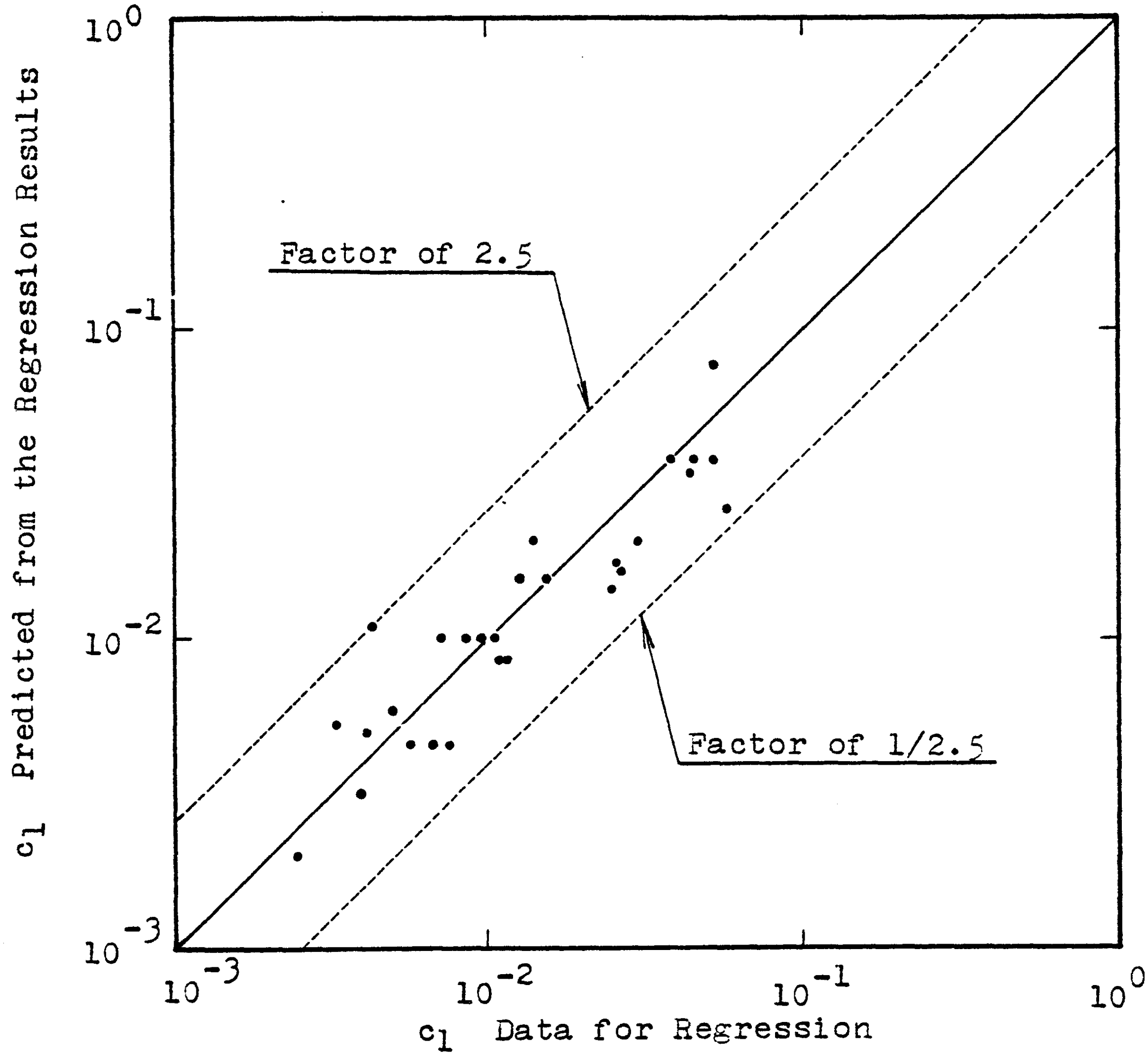

Fig. H.4 Test of the Regression Results of c_1

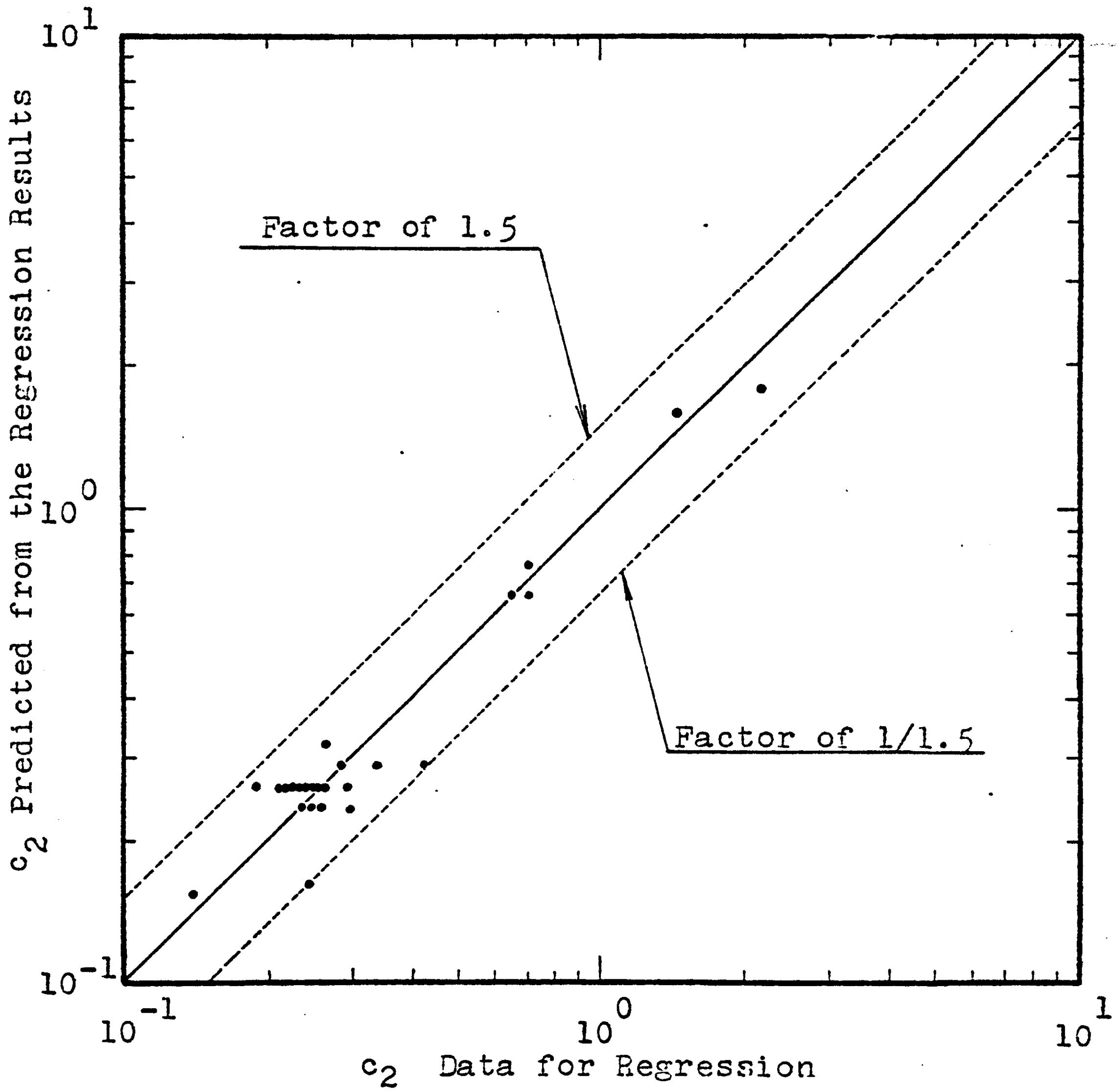

Fig. H.5 Test of the Regression Results of c_2

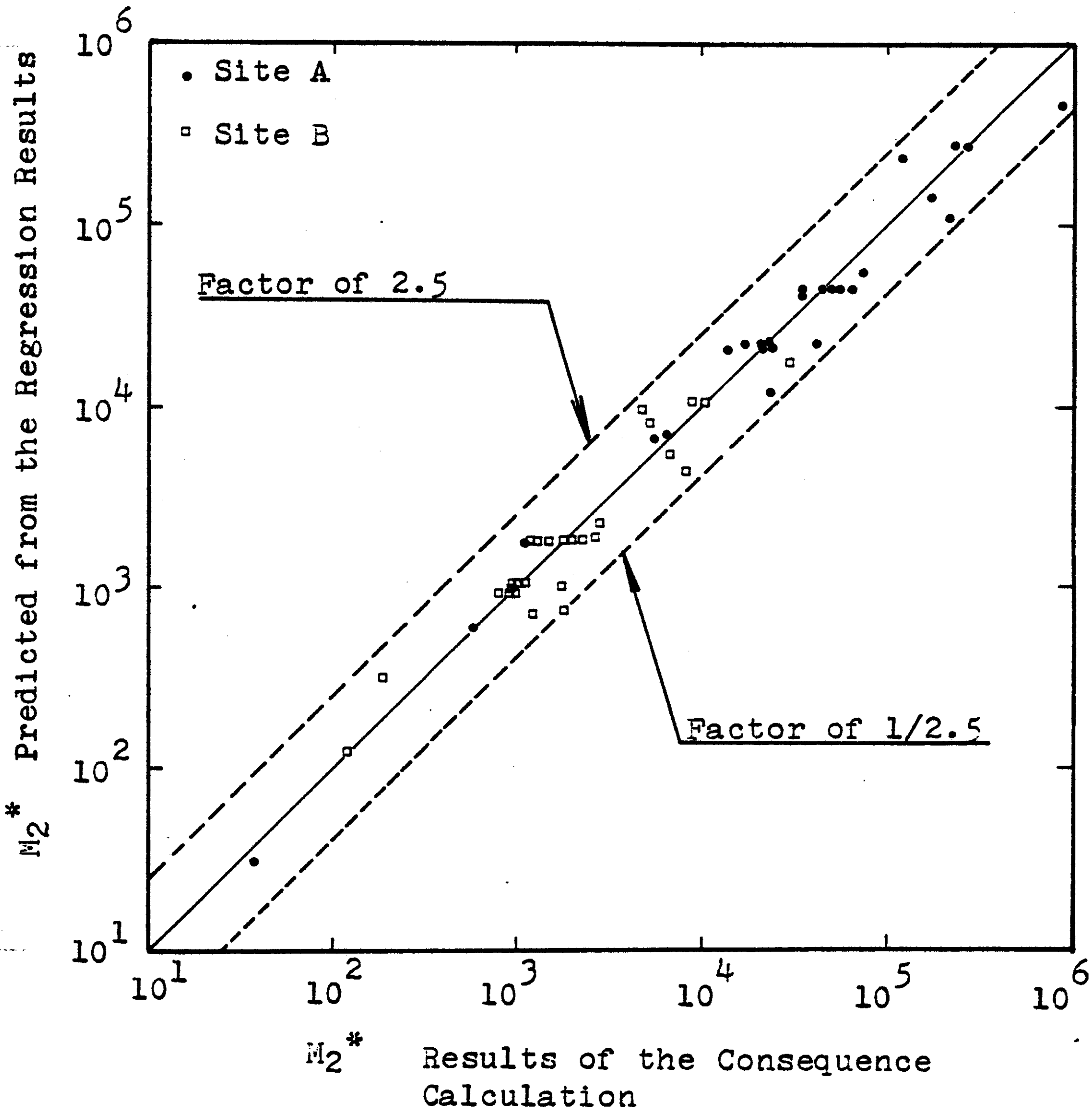

Fig.H.6 Comparison of the Estimated Second Risk Moment from the Regression with the Consequnce Results

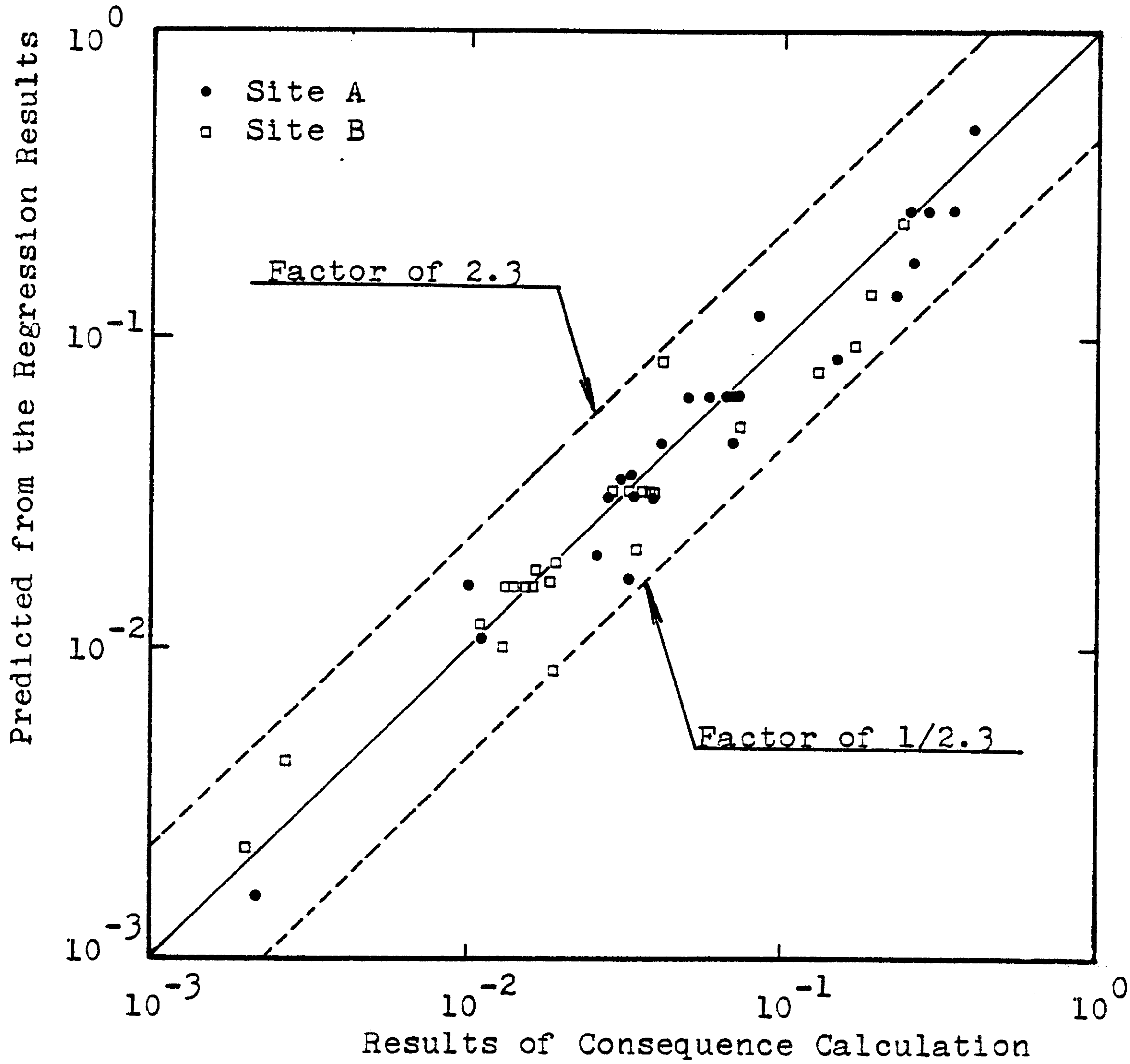

Table H.7 Comparison of the Estimated Normalization Constant from Regression with the Consequence Results

Note: The largest deviation of a factor of 2.3 is larger than the regression errors in Chapter 5 (1.2), but still within the uncertainty ranges of the consequence model.

BIBLIOGRAPHICAL NOTE

The author was born on January 10, 1948, in Genkai town, Saga prefecture in Japan. The Genkai Nuclear Power Plant (2 PWR's) is 5 miles away from his birthplace. The author's father, Giro Maekawa, received a Ph.D. degree in 1948 from the Chemical Engineering Department of the University of Kyushu on "Improvements of the Method for Recovery of Aluminum from Low-grade Ore".

In 1957 his family moved to Kamakura-city, where he spent his junior and senior high school life.

His college life in the University of Tokyo was in the midst of the stormy years of the Students Revolution. He received his B.E. (kogaku-shi) from the University of Tokyo in 1970. The thesis for the degree was on "Radiation-Induced Graftpolymerization of Tetrafluoroethylene on Polyethylene".

After his graduation, he worked in the Nuclear Fuel Department of Furukawa Electric Inc. He joined the joint project for the design of the Gd_2O_3 - UO_2 fuel test assembly between Furukawa Electric Inc. and Japan Atomic Energy Research Institute.

After the two years' working experience, he returned to the University of Tokyo as a Master candidate in 1972. He joined the Environmental Research Group supervised by Prof. Y. Yamatomo and Prof. R. Kiyose. His contribution to the group was a development of an atmospheric dispersion model for the dose estimation of the routine release from a spent-fuel reprocessing plant. Also in 1972 he worked with Mr. Y. Naito in Japan Atomic Energy Research Institute on a development of a three-dimensional neutron diffusion program based on a flux synthesis method.

He was admitted as a Master candidate by the Nuclear Engineering Dept. of MIT in 1973. His M.S. thesis on the analysis of safeguard system against theft was supervised by Prof. N. C. Rasmussen and sponsored by the MIT-Harvard joint committee on Arms Control.

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