



The University of Chicago Booth School of Business

Working Paper No. 09-16

Econometrics of the Basu Asymmetric Timeliness Coefficient and Accounting Conservatism

Ray Ball

University of Chicago Booth School of Business

S.P. Kothari

MIT Sloan School of Management

Valeri Nikolaev

University of Chicago Booth School of Business

This paper also can be downloaded without charge from the
Social Science Research Network Electronic Paper Collection:
<http://ssrn.com/abstract=999710>

**Econometrics of the Basu Asymmetric Timeliness Coefficient
and Accounting Conservatism**

Ray Ball

University of Chicago Booth School of Business
5807 South Woodlawn Avenue, Chicago, IL 60637
(773) 834-5941
ray.ball@chicagobooth.edu

S.P. Kothari

MIT Sloan School of Management
30 Memorial Drive, E60-382 Cambridge, MA 02142-1347
(617) 253-0994
kothari@mit.edu

Valeri V. Nikolaev

University of Chicago Booth School of Business
5807 South Woodlawn Avenue, Chicago, IL 60637
(773) 834-4116
valeri.nikolaev@chicagobooth.edu

January 3, 2013

We acknowledge the helpful comments of Sudipta Basu, John Core, Andrei Kovrijnykh, Jerry Zimmerman, Phil Berger (editor), two anonymous referees, and workshop participants at the University of Chicago.

Econometrics of the Basu Asymmetric Timeliness Coefficient and Accounting Conservatism

Abstract

A substantial literature investigates conditional conservatism, defined as asymmetric accounting recognition of economic shocks (“news”), and how it depends on various market, political and institutional variables. Studies typically assume the Basu (1997) asymmetric timeliness coefficient (the incremental slope on negative returns in a piecewise-linear regression of accounting income on stock returns) is a valid conditional conservatism measure. We analyze the measure’s validity, in the context of a model with accounting income incorporating different types of information with different lags, and with noise. We demonstrate that the asymmetric timeliness coefficient varies with firm characteristics affecting their information environments, such as the length of the firm’s operating and investment cycles, and its degree of diversification. We particularly examine one characteristic, the extent to which “unbooked” information (such as revised expectations about rents and growth options) is independent of other information, and discuss the conditions under which a proxy for this characteristic is the market-to-book ratio. We also conclude that much criticism of the Basu regression misconstrues researchers’ objectives.

Keywords: Conditional conservatism; Timely loss recognition; Basu model; Returns-earnings regressions; Earnings response coefficients; Market-to-book ratio.

Econometrics of the Basu Asymmetric Timeliness Coefficient and Accounting Conservatism

1. Introduction

Conservatism has been a central accounting principle for centuries (Watts and Zimmerman, 1986; Basu, 1997; Watts, 2003a). Basu (1997, p. 7) defines conservatism as “the accountant’s tendency to require a higher degree of verification to recognize good news as gains than to recognize bad news as losses,” a definition that is consistent with the adage “anticipate no profits but anticipate all losses.” In an efficient market, stock return reflects all new public information, and thus is a valid proxy for economic shocks to value. Then, in a piecewise-linear regression of accounting income on fiscal-period stock return, the incremental coefficient on negative return (the proxy for negative shocks, or “bad news”) is assumed to be a valid measure of asymmetrically timely loss recognition.¹ Basu predicts and finds that the incremental coefficient indeed is positive, indicating timelier incorporation of negative economic shocks than positive shocks.²

Under this definition of conservatism, how accounting income incorporates shocks to firm value depends on their sign, so Ball and Shivakumar (2005) and Beaver and Ryan (2005) term it *conditional* conservatism. This contrasts with defining conservatism as reporting unconditionally low earnings or book value of equity. The key difference between these concepts is that conditional conservatism carries new information.³ Basu’s formulation of conservatism in this fashion was an important breakthrough in our understanding of financial reporting rules and practices.

In a comparatively short period of time, the Basu (1997) piecewise-linear regression of accounting income on stock return has become one of the principal models of the financial

¹ We use the term “validity” as shorthand for “construct validity,” which Peter (1981, p. 134) notes “generally is used to refer to the vertical correspondence between a construct which is at an unobservable, conceptual level and a purported measure of it which is at an operational level.” See the classic Cook and Campbell (1979) text and its following editions.

² The model and its origins are described in Basu (2009).

³ Basu (2005, Section 2), Ball, Kothari, and Robin (2000, fn. 15) and Ball and Shivakumar (2005, 2008).

accounting literature. The range and importance of these applications is testimony to the pervasiveness of conservatism as a property of financial reporting, and also to researchers' confidence in the validity of their estimates of it.⁴ Furthermore, these studies regularly report differences in conditional conservatism that are consistent with plausible hypotheses, which provides added confidence in the validity of the estimates.

Nevertheless, researchers have assumed the Basu regression produces valid measures of conditional conservatism due largely to its intuitive appeal, without rigorous analysis.⁵ The need for formal econometric analysis is heightened by claims that the Basu asymmetric timeliness coefficient is not a valid measure of conservatism (Dietrich et al., 2007), that it is unduly affected by variables such as the book-to-market ratio (Pae et al., 2005; Givoly et al., 2007; Roychowdhury and Watts, 2007), and that Basu coefficients should be avoided by researchers (Patatoukas and Thomas, 2011).

We build our analysis on a model of the relation between accounting income and economic income that captures the salient properties of income recognition as it is practiced. We then use the model to derive and analyze the Basu earnings asymmetric timeliness coefficient. The model addresses shocks to firm value that arise from different types of information, and consequently our analysis is conducted entirely in the context of "news" and its incorporation in earnings. In particular, we ignore variation in expected returns across both time and firms. This framework allows a formal derivation of the Basu measure of conservatism, and also improves our understanding of how that measure interacts with characteristics of a firm's information environment.

The model distinguishes four components of information that financial reporting rules and practices cause to be incorporated in accounting income at different points in time. One information

⁴ As of 24 December 2012, Basu (1997) has 1914 citations in Google Scholar and 319 citations in the Social Sciences Citation Index, making it one of the most highly referenced papers in the modern accounting literature.

⁵ Basu (1997) offers no formal analysis. Increasingly formal but nevertheless limited analyses are offered in Ball, Kothari and Robin (2000), Pope and Walker (1999), Ryan and Zarowin (2003), and Beaver and Ryan (2005).

component is incorporated contemporaneously. The second information component is incorporated contemporaneously or with a lag, depending on its sign or magnitude. This earnings component is the source of conditional conservatism, which arises because negative news about future cash flows is subject to a lower accounting verification threshold than positive news. The third information component in our model always is incorporated with a lag, such as news about “unbooked” rents or growth options. The first three information components are assumed to be reflected in security prices, even when they are not recognized in current earnings. The model also has accounting income incorporating “noise,” for example due to random errors in counting inventory or in valuing accounts receivable, that reverses over time. We believe this model incorporates the salient properties of accounting recognition rules and practices.

The primary result in this paper is that the Basu regression provides econometrically valid estimates of conditional conservatism. In particular, we show that, holding other things constant, the Basu regression identifies conditional conservatism only when it exists. The model also allows us to pursue the secondary goal of this paper, which is to show how conditional conservatism is a function of the relative importance of the various information components. Consequently, we are able to formalize and extend the results in Roychowdhury and Watts (2007) on the relation between conditional conservatism and market-to-book ratios. We also address the Dietrich et al. (2007) claim that return endogeneity and sample truncation lead to biased Basu regression estimates.

Section 2 outlines the model of the income-return relation that incorporates salient properties of accounting income. Section 3 then proceeds with a formal analysis of the Basu regression in the context of that model. Section 4 analyzes the relation between timeliness and firm characteristics, the role of the market-to-book ratio, and other empirical implications. Section 5 addresses return endogeneity and sample truncation. A short summary and our conclusions appear in section 6.

2. A framework for interpreting regressions of earnings on returns

We develop a model of the relation between accounting and economic incomes that incorporates realistic and intuitive assumptions about how accounting rules and practices incorporate different types of information about economic value into accounting income, depending on properties of the information. In subsequent sections we use this model as a framework to analyze the validity of Basu regression measures of asymmetrically timely recognition of economic gains and losses.

2.1 A model of accounting income recognition

The origins of the following model can be traced back to Ball and Brown (1968), Beaver et al. (1980), Fama (1990), Kothari and Sloan (1992), Basu (1997), Kothari (2001), Beaver and Ryan (2005), Roychowdhury and Watts (2007), and others. We begin by assuming that capital markets are informationally efficient.⁶ We focus on shocks to firm value, ignoring variation in expected returns, the implicit assumption being that variation in expected returns either does not occur or is controlled for as discussed in Ball et al. (2013).

While informationally efficient returns reflect all publicly available information in a timely fashion, we assume that accounting rules and practices emphasize verifiability, objectivity, and conservatism, as reflected for example in the historical cost principle and revenue recognition rules, and hence accounting income in any period incorporates some but not all the information that becomes publicly available during the period. The consequence is that accounting income incorporates some information with a lag: that is, “prices lead earnings.”⁷

Information in our model can be made public via earnings or any other information channel. We do not implicitly or explicitly assume that accounting systems and earnings announcements have

⁶ Studies using the Basu model typically use annual returns to coincide with the periodicity of reported earnings and the settlement period in many contracts based on earnings (notably, debt, supply and compensation agreements). As argued in Ball et al. (2000), annual horizons substantially mitigate concerns about the informational efficiency of security prices.

⁷ That prices lead earnings is clear from the graphs in Ball and Brown (1968), Foster et al. (1984) and Bernard and Thomas (1990), and from the longer-horizon studies of Beaver, Lambert and Morse (1980) and Kothari and Sloan (1992).

no role in price formation. We assume only that, to the extent the market responds to reported earnings, it understands its components. In particular, the market distinguishes the components that reflect information made public in prior periods from those that reflect current-period information.

In line with the above assumptions, the total revision in security price (i.e., stock return) comprises three components, which are incorporated in accounting income differently:

$$R_t = x_t + y_t + g_t \quad (2.1)$$

$$I_t = x_t + w_t y_t + (1 - w_{t-1})y_{t-1} + g_{t-1} + \epsilon_t - \epsilon_{t-1} \quad (2.2)$$

where the subscripts t and $t-1$ refer to time periods, and:

R_t = total unexpected security return;⁸

x_t = portion of the total unexpected return R_t that invariably is contemporaneously captured in accounting income, I_t ;

y_t = portion of the total unexpected return R_t that is not contemporaneously captured in I_t unless required by conservative accounting;

g_t = portion of the total unexpected return R_t that never is contemporaneously captured in I_t , but always is incorporated with a lag;

I_t = accounting income;

w_t = is an indicator variable that takes the value of one when conservative accounting rules and practices lead to recognition of y in the current period; and

ϵ_t = “noise” in accounting earnings that reverses in the next period.

The unexpected return components x_t , y_t , and g_t are stationary and time-independent random variables (Bachelier 1900, Samuelson 1965, Fama, 1970; Campbell et al., 1997) whose variances are denoted by σ_x^2 , σ_y^2 , and σ_g^2 , respectively, and:

$$\text{corr}(x_t, y_t) = \rho_{xy} > 0, \text{ corr}(x_t, g_t) = \rho_{xg} > 0, \text{ and } \text{corr}(y_t, g_t) = \rho_{yg} > 0 \quad (2.3)$$

Accounting-induced noise ϵ_t has variance σ_ϵ^2 and is assumed independent of return components.

⁸ We ignore dividends, the implicit assumption being Miller-Modigliani dividend irrelevance.

To keep the analysis tractable, we make the following *linearity assumption*. We assume that the information components x , y , and g can be expressed as linear functions of each other (e.g., $x = f(y, g) + e$ where $f(.,.)$ is linear and e is an uncorrelated residual), which we operationalize by assuming that the variance-covariance matrix of x , y , and g is independent of the sign of news. This implies that $\text{cov}(x, y|x > 0) = \text{cov}(x, y|x < 0)$; or more generally, $\text{cov}(x, y|R > 0) = \text{cov}(x, y|R < 0)$ and $\text{var}(x|R > 0) = \text{var}(x|R < 0)$, so that $\text{corr}(x, y|R > 0) = \text{corr}(x, y|R < 0)$.⁹ This assumption allows us to focus on the effect of conservatism and ignore other potential sources of non-linearity in the return-earnings relation, such as an arbitrary non-linear relation between the information components x and y or between x and g .¹⁰ To the extent such non-linearities are present, they comprise a confounding factor (akin to an omitted variables problem) that needs to be understood and controlled for when estimating conditional conservatism.

2.2 An interpretation of the model in terms of accounting practices

In this model, financial reporting rules and practices lead to accounting income always contemporaneously incorporating the information component x_t , regardless of its sign or magnitude (i.e., without conditional conservatism). The intuition is that this component represents the least costly source of information to verify, and thus it invariably is recognized in the same period as it affects returns. This type of information could include current-period news about current-period cash flow, such as learning the actual cash realizations of current period revenues and expenses in comparison with their expectations. It also could include news about future cash flows that is verifiable at low cost, and that is incorporated symmetrically in accounting income via working capital accruals. Symmetrically low-cost verification typically applies to current operating-cycle information, such as cash receipts and payments, and the quantities of accounts receivable, accounts

⁹ In contrast, the distribution of earnings generally will be left skewed, not symmetric, due to conditional conservatism.

¹⁰ Equations 2.1 and 2.2 show that these would induce non-linearity in the relation between I and R . As discussed in Section 3.1, the shape of the return distribution per se does not invalidate the Basu (1997) measure.

payable and inventories. For example, accruals are used to adjust current-period cash flow for both increases and decreases in the quantity of closing inventory relative to opening inventory, because they are approximately equally low in cost to verify. Similarly, cash collections from customers are adjusted for both increases and decreases in accounts receivable. These working capital accruals incorporate into accounting income current information about future cash flows. For example, other things equal an increase in closing inventory is information that less cash will be spent on purchasing inventory in future periods, and it is verifiable at low cost regardless of its sign. In some limited circumstances, symmetrically low-cost verification can apply to long-cycle information as well, an example being index funds, in which gains and losses on long term investments in traded stocks are symmetrically low-cost to verify and in practice are both accounted on a daily basis.

The second component of stock return, y_t , is incorporated in accounting income either contemporaneously or with a lag, depending on the accounting operator w_t . The intuition here is that y_t represents information that is costly to verify, and more cost is incurred in verifying negative than positive news. Consequently the verification threshold is lower for negative news (Basu 1997, page 4), and this information component is incorporated asymmetrically. Such information could include the current-period revision in the expectation of unrealized future-period cash flows from “booked” assets, including long term assets in place and purchased intangible assets such as patents and goodwill. It could include information about the marketability of current assets such as inventory. This component also could include news about the present value of future-period cash outflows, such as lawsuit settlements. Using accrual accounting to bring forward shocks to expected future cash flows is known as timely gain and loss recognition. Conditional conservatism implies that w is more likely to be triggered by bad news (adverse shocks to expected future cash flows) than good news, and hence that loss recognition generally is timelier than gain recognition.

In the event that timely recognition is not triggered and thus the return component y_t is not contemporaneously incorporated in income, it is incorporated with a lag, the intuition being that some revisions in expectations of future cash flows are not reflected in accounting income until the actual cash flow realizations occur. Conditional conservatism implies that incorporation with a lag is more likely for good news than bad. The effect of asymmetrically delayed incorporation of information is that, in a two-period model, the total effect on current period income is $w_t y_t + (1 - w_{t-1})y_{t-1}$.

In practice, the extent of the timely recognition asymmetry is determined by a number of factors. An extensive literature studies determinants that include economic incentives, debt and compensation contracting, governance, GAAP, regulation, and taxes. Since our objective is not to provide an equilibrium model of the extent of conditional conservatism, but rather to investigate the validity of the Basu (1997) measure used in this literature, our model takes the extent of the asymmetry as an exogenous given, and studies the properties of its measurement. In our setting, timely recognition is triggered when y is below an exogenous threshold c .¹¹

The third component of stock return, g_{t-1} , invariably is incorporated in accounting income with a lag. One source of this return component is revisions in the value of growth options or unbooked intangibles (Beaver and Ryan 2005). This information component is similar in nature to “rents” in Roychowdhury and Watts (2007). Because growth options by definition are not “booked” as assets on balance sheets, these shocks to firm value are symmetrically incorporated in earnings with a lag, when the associated cash flows are (or are not) realized.

The model also has accounting income incorporating uncorrelated “noise” that reverses over time. An important source of this earnings component would be imperfect accounting accruals. For example, miscounting inventory affects current and future earnings with opposing signs. Other examples are errors in estimating uncollectible accounts receivable, errors in forecasting deferred tax

¹¹ Alternatively, timely loss recognition can occur with probability p which can be a function of y , i.e., $\Pr(w=1|y) = p(y)$.

liabilities, the effects of using historical-cost interest rates on debt, and errors in estimating assets' useful lives. Because accounting errors reverse over time, in our two-period model the error term is reversed out in the following period. For tractability we assume accounting error is uncorrelated with return components, and thus we ignore non-random "earnings management," including "smoothing," that plausibly is negatively correlated with returns.¹²

In this formulation, y_{t-1} and g_{t-1} are the two sources of delayed recognition. They generate the anticipated or stale component of earnings whose value consequences were previously reflected in returns. They therefore are uncorrelated with current period return R_t , which is influenced only by information arriving contemporaneously. We view this lagged recognition of some components of stock return as a natural feature of the income recognition process in accounting, which can be measured and investigated (for example, as a function of firm characteristics, managers' incentives, or countries' economic and political institutions).

While y_{t-1} and g_{t-1} are assumed to be uncorrelated with x_t , y_t , and g_t , and, therefore, with R_t , there are economic reasons to expect a positive correlation among x_t , y_t , and g_t . Recall that all three components are a consequence of economic news affecting investors' cash flow expectations. However, only the news generating the stock price growth rate component x_t , and possibly also y_t (i.e., when $w_t = 1$), is incorporated in accounting income contemporaneously, whereas news generating the return component g_t finds its way into accounting income in the following period.

The earnings process modeled in Equation (2.2) is intended to capture the important properties of asymmetric accounting recognition rules and practices, and their effect on how new information is incorporated in reported earnings. We next use this framework to investigate the properties of the Basu measure and develop empirical implications.

¹² Knowledge of ϵ_{t-1} would help predict earnings, but not returns. We assume the market unravels the current accounting error ϵ_t and does not react to it. This assumption can be relaxed without altering our conclusions.

3. Econometrics of the Basu Regression

3.1 Representing Timeliness as the Conditional Expectation of Earnings

The Basu (1997) model is used to measure the (asymmetric) timeliness of earnings with respect to economic news, where timeliness is the extent to which a dollar of unexpected economic income is reflected, on average in accounting income over a certain period. Over a firm's life (or a sufficiently long period), and assuming "clean surplus" accounting, each dollar of stock returns translates into a dollar of earnings. Over shorter time periods (e.g., years), however, there is no one-to-one mapping between earnings and returns and thus they can be thought of (and modeled) as random variables sharing a joint probability distribution $f(R_t, I_t)$. Statistically, earnings *timeliness* is thus the *expectation* of earnings conditional on return news R_t , or $E(I_t | R_t)$. Note that this conditional expectation is not a point estimate but is a function $\hat{\mu}(\cdot)$ of R_t . The conservative nature of accounting suggests the shape of this function is non-linear, which the Basu (1997) model accommodates by specifying a pricewise linear regression of earnings I_t on returns R_t .

To see if the least squares regression of earning on returns achieves the objective of measuring earnings timeliness, consider the following (general) model specification:

$$I_t = \mu(R_t) + \varepsilon_t \quad (3.1)$$

where $\mu(\cdot)$ is an arbitrary function, and where I_t and R_t are observed contemporaneously. A well-known result is that:

$$E(I_t | R_t) = \arg \min_{\mu(R)} E(I_t - \mu(R_t))^2 \equiv \hat{\mu}(R_t) \quad (3.2)$$

It follows directly from result (3.2) that, for a correctly specified functional form, the least squares estimation of (3.1) yields an unbiased conditional expectation. This is true irrespective of the distributional assumptions made with respect to I_t and R_t . Thus, a regression of earnings on returns fulfills its research objective of representing timeliness.

3.2 Simple case with no accounting asymmetry

We now use the framework from Section 2 to examine a baseline case with no accounting asymmetry (i.e., $w_t = 0$), and hence where financial reporting incorporates all news of type y with a lag regardless of its sign, in the same fashion as g_t . Consider the econometric properties of regressing accounting income on security return:

$$I_t = \alpha + \beta R_t + \varepsilon_t \quad (3.3)$$

In this case, the slope coefficient reflects the extent to which accounting income contemporaneously captures “good” and “bad” news equally. Let $\hat{\beta}$ denote the OLS estimate, such that:

$$\begin{aligned} \text{plim } \hat{\beta} &= \text{cov}(I_t, R_t) / \text{var}(R_t) \\ &= \text{cov}(x_t + y_{t-1} + g_{t-1} + \varepsilon_t - \varepsilon_{t-1}, x_t + y_t + g_t) / \text{var}(x_t + y_t + g_t) \end{aligned} \quad (3.4)$$

The lagged income components, y_{t-1} and g_{t-1} , do not correlate with x_t , y_t , and g_t , so in the symmetric accounting case:

$$\text{plim } \hat{\beta} = (\text{var}(x_t) + \text{cov}(x_t, y_t + g_t)) / (\text{var}(x_t) + 2\text{cov}(x_t, y_t + g_t) + \text{var}(y_t + g_t)). \quad (3.5)$$

In equation (3.5), the estimated slope coefficient from a regression of earnings on contemporaneous returns increases in the timeliness of earnings, $\text{var}(x_t) / \text{var}(y_t + g_t)$, which is the ratio of the information in returns that accounting income incorporates in a timely fashion to the information it incorporates with a lag. Equivalently, as timeliness increases the contribution to current accounting income I_t of stale news, y_{t-1} and g_{t-1} , is relatively lower.

In one extreme, accounting income is 100% timely with respect to economic income if $\text{var}(y_t)$ and $\text{var}(g_t)$ both are zero, in which case $\hat{\beta}$ converges to one and accounting income perfectly correlates with stock returns. This extreme is approximated by unlevered firms investing in actively-traded securities that are marked to market, such as mutual funds. In the other extreme, accounting income is completely untimely for firms consisting entirely of growth options, as approximated by

start-up firms investing almost entirely in intellectual property. In this case, $\text{var}(x_t)$ and $\text{var}(y_t)$ both are zero, $\hat{\beta}$ converges to zero, and accounting income is uncorrelated with stock return.

3.3 Asymmetric timeliness

We now consider the more general case with asymmetric timeliness, where the function $E(I | R)$ is expected to be non-linear due to accounting conservatism. We demonstrate that conservatism implies that a piecewise linear regression model following Basu (1997) will correctly identify the asymmetry in earnings timeliness. In terms of our model (2), we now consider a case where financial reporting incorporates the information component y without a lag whenever it is below a threshold (i.e., $w_t > 0$), but otherwise with a lag.

The Basu (1997) asymmetric timeliness coefficient is estimated from the regression model:

$$I_t = \alpha_1 + \alpha_2 D_t + \beta_1 R_t + \beta_2 D_t R_t + \varepsilon_t \quad (3.6)$$

where $D_t = 0$ if $R_t \geq 0$, $D_t = 1$ if $R_t < 0$, and β_2 is the asymmetric timeliness coefficient. β_2 then is the incremental coefficient on negative return (the proxy for negative economic income), and is predicted to be positive because conditionally conservative accounting incorporates negative economic income into accounting income sooner than it incorporates positive economic income.

Let $\hat{\beta}_2$ denote the OLS estimate of β_2 from equation (3.6). Then we have:

$$\text{plim} \hat{\beta}_2 = \gamma_2 - \gamma_1, \quad (3.7)$$

$$\text{where } \gamma_1 = \frac{\text{cov}(I_t, R_t | R_t \geq 0)}{\text{var}(R_t | R_t \geq 0)} \text{ and } \gamma_2 = \frac{\text{cov}(I_t, R_t | R_t < 0)}{\text{var}(R_t | R_t < 0)}.$$

Timely loss recognition in accounting is not triggered by all types of economic loss. Our model (2) represents this by assuming symmetrically timely recognition for the verifiable information component x_t , asymmetrically timely recognition of the information component y that is triggered only when it falls below a threshold value such as zero, and symmetrically untimely recognition of

the information component g_t . These individual components are unobservable to the researcher, so the Basu regression model (3.6) conditions the regression slope on total return R_t , which is observable. Because total return incorporates information components that are not recognized asymmetrically, there is a question as to whether the Basu regression provides a valid measure of conditional conservatism. We now show that the incremental regression coefficient $\beta_2 = 0$ if earnings is not conditionally conservative, and $\beta_2 > 0$ if earnings is conditionally conservative.

Our analysis incorporates what we believe to be a realistic scenario, that x_t , y_t , and g_t are positively correlated. The algebra under that assumption is tedious, so we relegate derivations to the Appendix.

3.3.1 Basu regression coefficients with conditionally conservative accounting. We next derive the estimate for the β_2 coefficient under conditionally conservative accounting, and then show that it behaves in a predictable fashion as a function of the firm's information environment.

We begin by considering a simpler case in which the information component g is zero (this is relaxed in Section 4 below). Then:

$$\gamma_1 = \frac{\text{cov}(x_t + w_t y_t + (1 - w_{t-1})y_{t-1} + \varepsilon_t - \varepsilon_{t-1}, x_t + y_t \mid R_t \geq 0)}{\text{var}(R_t \mid R_t \geq 0)} \quad (3.8)$$

$$= \frac{\text{cov}(x_t, x_t + y_t \mid R_t \geq 0) + \text{cov}(w_t y_t, x_t + y_t \mid R_t \geq 0)}{\text{var}(R_t \mid R_t \geq 0)} \quad (3.9)$$

Equation (3.9) follows from the assumption that all shocks to stock returns (and thus total return) are independent over time. Similarly:

$$\gamma_2 = \frac{\text{cov}(x_t, x_t + y_t \mid R_t < 0) + \text{cov}(w_t y_t, x_t + y_t \mid R_t < 0)}{\text{var}(R_t \mid R_t < 0)} \quad (3.10)$$

The assumption that the variance-covariance matrix of the three information components in returns is independent of the sign of economic news implies that $\text{var}(R_t \mid R_t \geq 0) = \text{var}(R_t \mid R_t < 0)$, and we have:

$$\text{plim}\hat{\beta}_2 = \frac{\text{cov}(w_t y_t, x_t + y_t | R_t < 0) - \text{cov}(w_t y_t, x_t + y_t | R_t \geq 0)}{\text{var}(R_t | R_t \geq 0)} \quad (3.11)$$

Henceforth we drop the subscript t , and all variables occur at time t unless explicitly subscripted $t-1$.

We model conditional conservatism as recognition of y in the current period only when it is sufficiently “bad news.” We assume $w_t = 1$ if $y_t < c$ and zero otherwise, where c is a threshold below which current period recognition occurs. In a limiting case where $c = +\infty$, timely recognition of y always occurs, there is symmetric recognition of good and bad news, and thus, $\text{plim}\hat{\beta}_2 = 0$.

Symmetry also exists in the limiting case of $c = -\infty$, where timely recognition of y never occurs. The set up then degenerates to $w = 0$, as analyzed in section 3.2.

We set the threshold c to zero in line with the adage “anticipate no profits, but anticipate all losses” and to reconcile with the empirical formulation of the Basu (1997) regression. In practice, c could be expected to be negative and close to zero. To the best of our knowledge, this assumption is not restrictive and does not bias the results.

To show that $\hat{\beta}_2$ is positive under these assumptions, it suffices to show that:

$$\Delta \equiv \text{cov}(wy, x + y | R < 0) - \text{cov}(wy, x + y | R \geq 0) > 0 \quad (3.12)$$

Equation (3.12) can be expressed as:

$$\begin{aligned} \Delta = & \underbrace{\int_{-\infty}^0 \int_{-\infty}^{-y} y^2 \varphi(x, y | R < 0) dx dy}_{\Theta_1} - \underbrace{\int_{-\infty}^0 \int_{-y}^{+\infty} y^2 \varphi(x, y | R \geq 0) dx dy}_{\Theta_2} + \\ & + \underbrace{\int_{-\infty}^0 \int_{-\infty}^{-y} xy \varphi(x, y | R < 0) dx dy}_{\Theta_3} - \underbrace{\int_{-\infty}^0 \int_{-y}^{+\infty} xy \varphi(x, y | R \geq 0) dx dy}_{\Theta_4} + E(x + y | R \geq 0)E(y | y < 0) \end{aligned} \quad (3.13)$$

where $\varphi(\dots)$ is a bivariate probability density function. This equation results from expanding equation (3.12) to separate its x and y components and expressing it in the form of expectations. The last term follows conveniently from the linearity assumption that the variance-covariance matrix of x

is y is independent of the news sign, which implies that $E(x + y | R \geq 0) = -E(x + y | R < 0)$ and that $E(wy | R < 0) + E(wy | R \geq 0) = E(y | y < 0)$.

To explicitly evaluate the integrals, we further introduce the assumption that $\varphi(\cdot, \cdot)$ follows

Normality. Under this assumption we have (see Appendix for derivation¹³):

$$\Delta = 2\pi^{-1} \sigma_y \sigma_x (1 - \rho_{xy}^2)^{0.5} (1 + \alpha \arctan(\alpha) - (1 + \alpha^2)^{0.5}), \quad (3.14)$$

where $\alpha = \frac{\sigma_y + \sigma_x \rho_{xy}^2}{(1 - \rho_{xy}^2)^{0.5} \sigma_x} \in (0, +\infty)$.

It further can be shown that for any positive α , the expression for Δ is positive (one way to see this is to note that $\Delta(0) = 0$ and that first derivative of Δ is positive over the interval $(0, +\infty)$).¹⁴ This result implies the Basu incremental coefficient on negative returns reflects the existence of conditional conservatism in accounting income.

3.3.2 Basu regression coefficients in the absence of conditional conservatism. As a validity check, consider the asymmetric timeliness coefficient in the absence of conditional conservatism. In this case, accountants symmetrically do not recognize any of the information in y_t in the current period t , regardless of how negative, i.e. $w_t = 0$ for all y_t . Under the linearity assumption, the equation (3.11) implies $\text{plim } \hat{\beta}_2 = 0$. Thus, under the null hypothesis of no conditional conservatism, the Basu coefficient is zero. Indeed, this should be the case if the model is well-specified and the coefficient is a valid measure of conditional conservatism.

3.3.3 Basu regression coefficient validity. We conclude that the Basu asymmetric timeliness coefficient $\hat{\beta}_2$ is positive in the presence of conditional conservatism, and zero in the absence of

¹³ The Appendix provides derivation for a more general case where the information component g also is present. The result below is thus a special case that obtains when setting $g=0$ or, in other words, setting $z=x$.

¹⁴ Taking the second derivative indicates that the first derivative is increasing over the interval $(0, \sqrt{3}]$ and is decreasing over the interval $[\sqrt{3}, +\infty)$, but asymptotes to a positive constant.

conditional conservatism, consistent with it being a valid estimator. This result contradicts the claims of Dietrich et al. (2007), to which we return below. Further, the coefficient is not driven by a single structural parameter but rather is a function of conditional accounting conservatism and other attributes of the information environment, including information about growth options (Roychowdhury and Watts, 2007), which we discuss in more detail in the following section.

4. Conditional conservatism, firm characteristics and empirical implications

In this section, we demonstrate that the asymmetric timeliness coefficient varies with firm characteristics affecting their information environments, such as the length of the operating and investment cycle, and the degree of diversification. We particularly examine one characteristic, the relative amount of symmetrically “unbooked” information (notably, revised expectations about rents or growth options) and the extent to which it is correlated with other information components. We then discuss the conditions under which a proxy for this characteristic is the market-to-book ratio, formalizing and extending the analysis in Roychowdhury and Watts (2007), among others.

4.1 The role of information about growth options and rents

Consider the case where the variance of the information component g in economic income is non-zero. Current-period accounting income then includes a component that is a function of past shocks to the firm’s growth options. In our two-period model, equation (2.2) has shocks to growth options showing up in accounting income with a simple one-period lag.

$$\text{Define } z \equiv x + g, \sigma_z^2 \equiv \text{var}(z), \text{ and } \rho \equiv \text{corr}(z, y) = \frac{\text{cov}(y, x) + \text{cov}(y, g)}{\sqrt{\text{var}(y) \text{var}(x + g)}}. \quad (4.1)$$

The Basu asymmetric timeliness coefficient then is given by (see the Appendix for the derivation):

$$\begin{aligned} \text{plim} \hat{\beta}_2 &= \frac{\text{cov}(wy, x+y+g | R < 0) - \text{cov}(wy, x+y+g | R \geq 0)}{\text{var}(R | R \geq 0)} \\ &= \frac{2\sigma_y \sigma_z \sqrt{1-\rho^2}}{(\pi-2) \text{var}(R)} \left(1 + \alpha_1 \arctan(\alpha_1) - \sqrt{1+\alpha_1^2} \right) > 0 \end{aligned} \quad (4.2)$$

where $\alpha_1 = \frac{\sigma_y + \sigma_z \rho}{(1-\rho^2)^{0.5} \sigma_z} \in (0, +\infty)$.

Several limiting cases are of interest. To avoid cumbersome notation, here we loosely refer to $\text{plim} \hat{\beta}_2$ as β_2 . As the correlation coefficient between z and y approaches 1, we have:

$$\lim_{\rho \rightarrow 1} \beta_2 = \frac{\sigma_y}{\sigma_y + \sigma_z} \quad (4.3)$$

This result is intuitive and suggests the asymmetric timeliness coefficient depends on the variance of y relative to the total variance of returns, as one would expect when the components are perfectly correlated.¹⁵ As one also would expect, $\lim_{\sigma_y \rightarrow \infty} \beta_2 = 1$, and $\lim_{\sigma_y \rightarrow 0} \beta_2 = 0$. In the first case the variation in y subsumes all other components, while in the second there is no role of conditionally conservative accounting. Finally, $\lim_{\sigma_z \rightarrow \infty} \beta_2 = 0$ and $\lim_{\sigma_z \rightarrow 0} \beta_2 = 1$, which mirror the previous results and also are intuitive. The limiting cases suggest the Basu specification captures the extent of accounting conservatism in a meaningful and intuitive way.

We now show that the Basu asymmetric timeliness coefficient declines in the variance of growth options. The derivative of β_2 with respect to σ_z is:

$$\frac{\partial \beta_2}{\partial \sigma_z} = \frac{2\sigma_y \sqrt{1-\rho^2}}{(\pi-2)(\sigma_y^2 + 2\rho\sigma_y\sigma_z + \sigma_z^2)^2} \left(\Upsilon(\sigma_y - \sigma_z - \sigma_y^2 - \rho\sigma_z\sigma_y) - \arctan(\alpha) \sigma_y \sigma_z \sqrt{1-\rho^2} \right), \quad (4.4)$$

where $\Upsilon = 1 + \alpha \arctan(\alpha) - \sqrt{1+\alpha^2} > 0$. This expression is *negative* for economically meaningful values of the parameters and the expression for β_2 is symmetric in σ_x and σ_g . Since σ_z is increasing

¹⁵ Equation (4.2) seems to suggest that β_2 approaches zero as ρ approaches 1. However, α_1 then tends to plus infinity.

in both σ_x and σ_g , it follows that, holding the correlation coefficient ρ constant, β_2 is decreasing in σ_x and σ_g . This result is intuitive, in that the x and g components of revision in price are treated symmetrically in financial reporting (fully incorporated and fully ignored, respectively), whereas the y component is treated asymmetrically (incorporated only if below a threshold).

Figure 1 graphs the asymmetric timeliness coefficient as a function of σ_g and σ_y . Other parameters are fixed at the following levels: $\rho_{xz} = \rho_{xg} = 0.3$ and $\sigma_x = 0.2$.¹⁶ As expected, the coefficient increases in the variance of y (reaching 1 in the limit), as timely loss recognition becomes more important. Figure 1 also illustrates that asymmetrically timely loss recognition decreases in the variance of g (information about growth options), reaching zero in the limit. This result also is intuitive. Shocks to growth expectations are not captured contemporaneously in accounting income, so in a regression of accounting income on stock returns their variability σ_g dampens the coefficient on returns. These results confirm the original conjecture of Ball, Kothari and Robin (2000, p. 48) that asymmetric earnings timeliness is a function of the amount of *news* about “unbooked” growth options relative to news from other sources.

[Figure 1 about here.]

4.2 Effect of correlation among components of earnings innovations.

The literature is silent on how asymmetric timeliness is affected by complementarities or correlations among the information components that drive returns but that are reflected in earnings in different periods. We use equation (4.2) to plot β_2 as a function of the correlations ρ_{xg} and ρ_{yg} (we do not separately consider ρ_{yx} since it is easy to see that its effect is the same as that of ρ_{yg}).

[Figure 2 about here.]

¹⁶ Analogous results obtain when other parameters are fixed at different levels, including correlation coefficients of zero. For illustration, we report the results only under one set of parameters.

Figure 2 presents results when other parameters are fixed at the levels: $\rho_{xy} = 0.3$ and $\sigma_x = \sigma_y = \sigma_g = 0.2$. As the figure shows, asymmetric timeliness increases in ρ_{yg} , the correlation between y and g , and it decreases in ρ_{xg} , the correlation between x and g . Intuitively, the first result obtains because in our model y exhibits timely recognition, but the researcher observes total return R_t , which also contains other information components. As the correlation between the return components y and g strengthens, the Basu model's asymmetric timeliness coefficient increases. Similarly, the second result obtains because variation in the sum of the symmetrically-recognized x and g components becomes more pronounced due to their covariability, and the conditional conservatism asymmetry becomes less salient. Thus, the correlations and interactions between different information components are expected to have an effect on timely loss recognition empirically.

4.3. Earnings timeliness and market-to-book.

In this subsection, we discuss the relation between the (beginning-of-period) market-to-book ratio (m/b) and asymmetric timeliness, which has been the subject of some controversy in the literature. A number of studies (e.g., Pae et al. 2005, Givoly et al. 2007) show that this relation is negative and, to the extent m/b is also a measure of conservatism, these studies suggest the Basu measure is biased. Roychowdhury and Watts (2007) provide the most recent and comprehensive treatment on this subject by proposing a theoretical framework that explains the inverse relation of asymmetric timeliness and m/b (see also Beaver and Ryan, 2005). We start by summarizing their arguments and then explain how we complement and extend their analysis.

Roychowdhury and Watts (2007) decompose equity value into separable assets and rents. Separable assets include unverifiable assets in excess of cost, verifiable assets in excess of cost, and assets at cost (net of depreciation and impairments). Rents stand for above-competitive future returns and may (but need not) include growth options, synergies, monopoly power, etc. They argue that

accounting excludes the measurement of rents or their changes in the financial statements in part because they are unverifiable. Further, they maintain that conservatism is characterized by the degree of understatement of net separable assets on the balance sheet, not due to the omission of rents from the financial statements. Consequently, in the context of the Basu regression model, Roychowdhury and Watts (2007) view rents and changes in rents as measurement error arising from using stock prices and stock returns, respectively, to infer properties of balance sheet and income statement information. Because rents are reflected in the numerator of m/b , econometrically m/b serves as a proxy for the measurement error in returns, the independent variable in the Basu regression model, which in turn attenuates the Basu measure of conservatism.¹⁷

We start by noting an important conceptual point of distinction of our model. Viewed from the perspective of timeliness, news about rents (in our notation, g) is not measurement error because rents eventually are converted into net assets (ultimately, cash) and go through the income statement. By the time the conversion takes place, the stock price already has reflected g . Therefore, the matching between stock returns and earnings is a matter of *time*, or leads and lags. Given that the objective of Basu model is to measure timeliness, g is a natural information component that affects overall and asymmetric timeliness (as Figure 1 indicates). Since m/b is likely correlated with the relative variance of g , the negative relation between m/b and conditional conservatism is a natural property of accounting earnings in our model.¹⁸

Our model advances the analysis in Roychowdhury and Watts (2007) in three ways. First, our model shows that the effect of m/b on asymmetric timeliness is more complex than first appears.

The analyses in subsection 4.1 and subsection 4.2 demonstrate that the Basu coefficient is not

¹⁷ A similar conclusion follows from Beaver and Ryan's (2005) simulation that alters intangibles intensity, changes in which are viewed as noise.

¹⁸ Whether one calls this a "measurement error" problem or a natural property of accounting earnings is at one level a semantic issue. Nevertheless, this issue is important as the appropriate econometric solutions differ, depending on the context (as discussed further below).

determined by the amount of rents (or growth options) relative to other assets per se, but by the relative amounts of *new information* about them and how this information is correlated with other information: i.e., it is a second moment effect. For example, consider a firm such as a regulated utility where shocks to the value of asymmetrically booked assets (y in our notation) are negligible, i.e., $\sigma_y \rightarrow 0$. At this extreme, all information about firm value is incorporated symmetrically in earnings, so here too the incremental Basu slope is zero, independent of m/b . More generally, a higher m/b could imply a simultaneous increase in the variance of g and a decrease in the variance of x . As equation (4.4) and Figure 1 indicate, the effect on asymmetric timeliness is ambiguous in such case. Further, while Roychowdhury and Watts (2007) assume that rents are independent of other information types (p. 11), we show that the correlation between shocks to assets in place and growth options can affect the relation between earnings timeliness and market-to-book ratios. In particular, Figure 2 indicates that, if higher m/b also leads to a higher correlation between y and g , the link of m/b and asymmetric timeliness is not necessarily negative.

Second, Roychowdhury and Watts (2007) argue that asymmetric timeliness should become more pronounced over longer earnings horizons because rents convert to separable assets and thus their negative effect on the Basu measure diminishes. Our analysis generalizes this argument, but also suggests that the effect of horizon is not always clear. Intuitively, as the return period is extended back, current-period earnings reflect more of the growth options at the beginning of the period. Further, shocks to growth options are more likely to flow through the financials over longer horizons. One can think of this as increasing the variance of x or y relative to the variance of g . On the one hand, to the extent that a longer horizon means that g can be treated as y , i.e., the relative variance of y widens, asymmetric timeliness increases. On the other hand, over an even longer horizon x can replace g , in which case the relative variance of x widens and asymmetric timeliness decreases

(intuitively, over sufficiently long horizons earnings approximate returns and hence the asymmetry should disappear).

Finally, our model implies that there are important non-linearities and interactions in the relation between asymmetric timeliness and the variances of x , y , and g (and hence m/b), as illustrated in Figure 1.

An important related question is whether the validity of the Basu asymmetric timeliness coefficient as a proxy for conditional conservatism is threatened because in fact Basu coefficients and market-to-book ratios are negatively correlated.¹⁹ Indeed, the negative correlation has lent credence to criticisms that the Basu asymmetric timeliness coefficient is flawed (Dietrich et al., 2007; Patatoukas and Thomas, 2011). Our analysis suggests otherwise, as it demonstrates that Basu coefficients derive from an interaction between asymmetrically conservative accounting rules and practices and the underlying economic characteristics of the firm. Depending on one's research objectives, controlling for firm characteristics may or may not be appropriate. As an example, consider Nikolaev's (2010) hypothesis that financial covenants are effective in distressed situations only if there is timely recognition of losses in earnings, and evidence that financial covenants in public debt contracts indeed are correlated with firms' Basu coefficients. For this research question, it is not essential whether the size of the Basu coefficient derives from firm characteristics, from accounting, or from the way they intersect.²⁰

4.4 Other determinants of conditional conservatism.

The result that asymmetric timeliness is decreasing in σ_x is intuitive in our setup and presents several additional empirical predictions. For example, companies with short operating cycles, short

¹⁹ Gigler and Hemmer (2001), Richardson and Tinaikar (2004), Beaver and Ryan (2005), Price (2005), Pae, Thornton, and Welker (2005), Givoly, Hayn, and Natarajan (2007), and Roychowdhury and Watts (2007).

²⁰ Ball et al. (2008) raise the related issue of whether firm characteristics truly are exogenous in cross-country research on the effects of accounting rules and practices, or whether they are determined jointly.

investment cycles, short asset maturity, or companies with a substantial realized component of economic income, are expected to exhibit lower levels of timely loss recognition. This reconciles with the evidence of Khan and Watts (2009), and also with their prediction that conditional conservatism increases in environmental uncertainty, which we expect to be inversely related to the variance of x and positively related to the variance of y (due to different verification requirements for earnings components that are more uncertain and more costly to verify).

Finally, we note that asymmetric timeliness is not affected by σ_e^2 , the amount of accounting-induced noise. This is a direct consequence of accounting income being specified as the dependent variable (as well as noise being assumed independent of all return components). The implication is that, while asymmetric timeliness is a valid property of accounting income, it is by no means a complete measure of accounting quality. In particular, it does not reflect random accruals errors, such as errors in recording inventory receivables and payables.

5. Return Endogeneity and Sample Truncation

Dietrich et al. (2007), henceforth DMR, argue that the Basu regression specification is misspecified for two reasons: earnings cause returns; and conditioning on the sign of the returns induces sample truncation bias. Both claims are based on a misconception that the objective of a Basu specification is to estimate structural parameters in the model (DMR, p. 100):

$$R_t = \gamma I_t + \eta_t \tag{5.1}$$

where the error term η_t is independent of I_t , and γ is known as the “earnings response coefficient.”²¹

DMR show that OLS slope coefficient from a “reverse” regression of earnings I on returns R is a biased estimator of the inverse of earnings response coefficient, $1/\gamma$, because the error term in the

²¹ Because this model suppresses conditional conservatism, the variable I in this section is different from the variable I in our analyses above. In particular, DMR’s (9.3) and derivative equations cannot be substituted into our earlier equations.

reverse model is not independent of R_t . This well-known result is irrelevant if the research objective is to measure how accounting income incorporates the information in stock return, i.e., to estimate the functional shape of the conditional expectation $E(I|R)$, in which case (5.1) is the incorrect specification. As shown in section 3.1, least squares regression achieves that research objective without bias.

Second, DMR (p. 102) argue that conditioning the regression of earnings on returns upon the sign of returns induces “sample truncation bias.” Specifically, DMR argue that the conditions $E(I\eta | R > 0) \neq 0$ and $E(I\eta | R < 0) \neq 0$ imply a truncation bias in a Basu regression. However, these conditions only are relevant in a regression of returns on earnings, as in their (5.1).²² The relevant condition for a Basu association study regression is $E(R\xi | R > 0) = 0$, where $\xi \equiv I - E(I|R)$. Indeed, $E(R_t\xi_t | R_t > 0) = E(E(R_t\xi_t | R_t) | R_t > 0) = E(R_tE(\xi_t | R_t) | R_t > 0) = 0$. The effect of sample truncation on the least squares outcome for equation (3.1) can be expressed as:

$$\begin{aligned} E(I | R, \text{truncation rule}) &= E(E(I | R) + \xi | R, \text{truncation rule}) = \\ &= E(I | R) + E(\xi | \text{truncation rule}) \end{aligned} \quad (5.4)$$

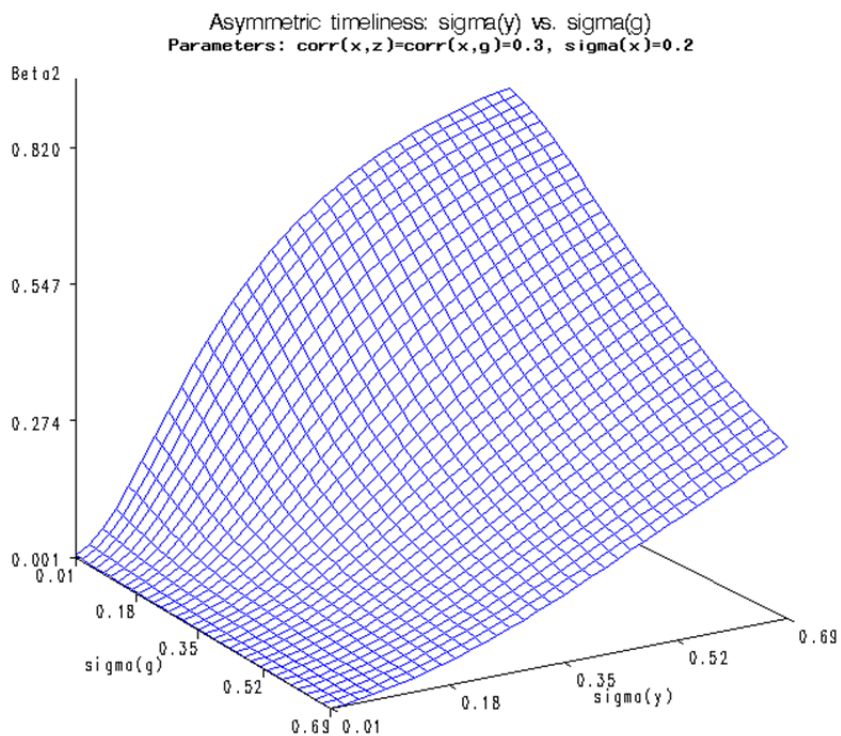
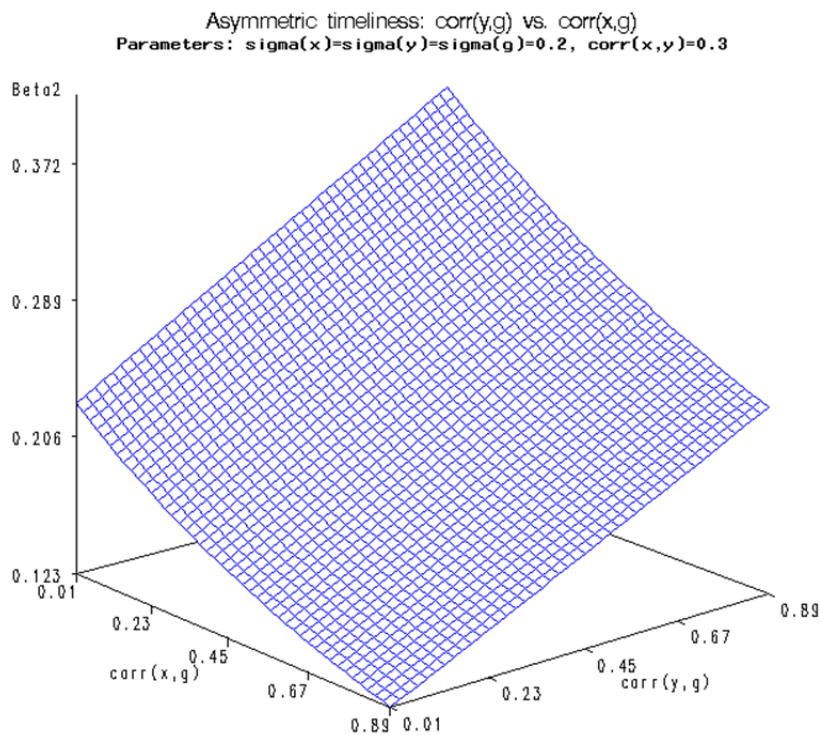
This implies that truncation bias arises when $E(\xi | \text{truncation rule}) \neq 0$. However, truncating the sample on R (or a function of R) does not lead to a bias as, using the law of iterated expectations, $E(\xi | R < 0) = E(E(\xi | R) | R < 0) = 0$. Consequently, when the research objective is to estimate the functional shape of the conditional expectation $E(I|R)$, return is the correct independent variable, and conditioning on it does not induce bias.

²² Truncation is figuratively depicted incorrectly in DMR (Figure 2), where the regression of earnings on returns minimizes the horizontal (squared) deviations from the regression line, *not the vertical deviations*. After one notes this, it immediately becomes clear from the graph that no truncation bias is present in a Basu regression. DMR also offer a “simulation” which they claim should not exhibit non-zero Basu coefficients. Ball et al. (2013, fn 16) disprove this claim.

6. Summary and Conclusions

We present a model of accounting income that is based on salient properties of income recognition in accounting, and use it to analyze the econometrics of the Basu asymmetric timeliness coefficient. The analysis addresses conceptual and econometric challenges to the coefficient's validity as a measure of conditional conservatism. We show that the Basu measure is unbiased under the null hypothesis of zero asymmetry, and that under the alternative hypothesis it captures conditional conservatism as formulated in our model.

We also demonstrate that conditional conservatism is a function of a variety of firm characteristics. Notably, we formalize the intuition and extend the analysis in Roychowdhury and Watts (2007), demonstrating econometrically that a negative relation between market-to-book ratios and asymmetric timeliness coefficients is expected, but is more complex than previously thought. The market-to-book ratio does not *per se* determine asymmetric timeliness; its effect arises because it is correlated with the relative amount of total price revision associated with revisions in "unbooked" components such as growth option expectations. We also discuss the effect of firm characteristics such as operating and investment cycle length, asset maturity, the correlation between the components of returns (hence the degree of diversification) and the length of the fiscal period studied. An important issue that context-specific research design must consider is whether it is appropriate to control for such firm characteristics.

Figure 1: β_2 coefficient as a function of σ_g and σ_y Figure 2: β_2 coefficient as a function of ρ_{xg} and ρ_{yg} 

Appendix A

Analytical derivation of asymmetric timeliness: The case of correlated x , y , and g .

We derive the asymmetric timeliness coefficient for a general case where all three components of return, x , y , and g , are considered. The cases when some of these components are omitted follow immediately. We omit subscripts t when all random variables are contemporaneous.

We start by noting that:

$$\begin{aligned}
 \gamma_1 &= \frac{\text{cov}(x_t + w_t y_t + (1 - w_{t-1}) y_{t-1} + g_{t-1} + \varepsilon_t - \varepsilon_{t-1}, x_t + y_t + g_t \mid R_t \geq 0)}{\text{var}(R \mid R \geq 0)} \\
 &= \frac{\text{cov}(x_t + w_t y_t, x_t + y_t + g_t \mid R_t \geq 0)}{\text{var}(R \mid R \geq 0)} \\
 &= \frac{\text{cov}(x, x + y + g \mid R \geq 0) + \text{cov}(w y, x + y + g \mid R \geq 0)}{\text{var}(R \mid R \geq 0)}
 \end{aligned} \tag{A1}$$

$$\text{And } \gamma_2 = \frac{\text{cov}(x, x + y + g \mid R < 0) + \text{cov}(w y, x + y + g \mid R < 0)}{\text{var}(R \mid R < 0)} \tag{A2}$$

Exploiting symmetry we have:

$$\begin{aligned}
 \text{plim } \hat{\beta}_2 &= \gamma_2 - \gamma_1 = \\
 &= \frac{\text{cov}(w y, x + y + g \mid R < 0)}{\text{var}(R \mid R < 0)} - \frac{\text{cov}(w y, x + y + g \mid R \geq 0)}{\text{var}(R \mid R \geq 0)} \\
 &= \frac{\text{cov}(w y, x + y + g \mid R < 0) - \text{cov}(w y, x + y + g \mid R \geq 0)}{\text{var}(R \mid R \geq 0)}
 \end{aligned} \tag{A3}$$

where $w = 1(y < 0)$ is an indicator variable taking a value of one in case $y < 0$ and zero otherwise.

Define $z \equiv x + g$, $\sigma_z^2 \equiv \text{var}(z)$, and

$$\rho \equiv \text{corr}(z, y) = \frac{\text{cov}(y, x) + \text{cov}(y, g)}{\sqrt{\text{var}(y) \text{var}(x + g)}} = \frac{\rho_{xy} \sigma_x + \rho_{yg} \sigma_g}{\sqrt{\sigma_x^2 + 2\rho_{xg} \sigma_x \sigma_g + \sigma_g^2}}.$$

To show that is $\beta_2 = 0$ positive it suffices to show that $\Delta > 0$, where:

$$\Delta \equiv \text{cov}(w y, z + y \mid R < 0) - \text{cov}(w y, z + y \mid R > 0) > 0.$$

Note further that:

$$\begin{aligned}
\Delta &= E(wy^2 + wyz | R < 0) - E(wy^2 + wyz | R \geq 0) \\
&= -E(wy | R < 0)E(z + y | R < 0) + E(wy | R \geq 0)E(z + y | R \geq 0) = \\
&= \int_{-\infty}^0 \int_{-\infty}^{-y} (y^2 + zy)\varphi(z, y | R < 0) dz dy - \int_{-\infty}^0 \int_{-y}^{+\infty} (y^2 + zy)\varphi(z, y | R \geq 0) dz dy \\
&+ E(z + y | R \geq 0) \underbrace{\int_{-\infty}^0 \int_{-\infty}^{+\infty} y\varphi(z, y | y < 0) dz dy}_{\equiv \Theta} = \\
&= \underbrace{\int_{-\infty}^0 \int_{-\infty}^{-y} y^2\varphi(z, y | R < 0) dz dy}_{\equiv \Theta_1} - \underbrace{\int_{-\infty}^0 \int_{-y}^{+\infty} y^2\varphi(z, y | R \geq 0) dz dy}_{\equiv \Theta_2} + \\
&+ \underbrace{\int_{-\infty}^0 \int_{-\infty}^{-y} zy\varphi(z, y | R < 0) dz dy}_{\equiv \Theta_3} - \underbrace{\int_{-\infty}^0 \int_{-y}^{+\infty} zy\varphi(z, y | R \geq 0) dz dy}_{\equiv \Theta_4} + E(z + y | R \geq 0) \underbrace{\int_{-\infty}^0 \int_{-\infty}^{+\infty} y\varphi(z, y | y < 0) dz dy}_{\equiv \Theta}
\end{aligned} \tag{A4}$$

where $\varphi(z, y) = \frac{1}{2\pi\sigma_z\sigma_y(1-\rho^2)^{0.5}} \exp\left(-\frac{1}{2(1-\rho^2)}\left(\frac{z^2}{\sigma_z^2} - \frac{2\rho zy}{\sigma_z\sigma_y} + \frac{y^2}{\sigma_y^2}\right)\right)$ is bivariate normal

$$\text{density and } E(z + y | R \geq 0) = E(R | R \geq 0) = \frac{\varphi(0)}{1 - \Phi(0)} \sqrt{\text{var}(R)} = \frac{2\sqrt{\sigma_z^2 + 2\rho\sigma_z\sigma_y + \sigma_y^2}}{\sqrt{2\pi}}.$$

The integrals can be evaluated explicitly under the normality assumption, first noting that symmetry implies $\Pr(R < 0) = 0.5$.

$$\begin{aligned}
\Theta &= \frac{1}{2\pi\sigma_z\sigma_y(1-\rho^2)^{0.5} \Pr(R < 0)} \int_{-\infty}^0 \int_{-\infty}^{\infty} y \exp\left(-\frac{1}{2(1-\rho^2)}\left(\frac{z^2}{\sigma_z^2} - \frac{2\rho zy}{\sigma_z\sigma_y} + \frac{y^2}{\sigma_y^2}\right)\right) dz dy \\
&= \frac{1}{\pi\sigma_z(1-\rho^2)^{0.5}} \int_{-\infty}^0 \int_{-\infty}^{\infty} \frac{y}{\sigma_y} \exp\left(-\frac{1}{2(1-\rho^2)}\left((1-\rho^2)\left(\frac{\sigma_y z - \rho\sigma_z y}{(1-\rho^2)^{0.5}\sigma_z\sigma_y}\right)^2 + (1-\rho^2)\frac{y^2}{\sigma_y^2}\right)\right) dz dy \tag{A5} \\
&= \left\langle u = \frac{y}{\sigma_y}, v = \frac{\sigma_y z - \rho\sigma_z y}{(1-\rho^2)^{0.5}\sigma_z\sigma_y}, dy = \sigma_y du, dz = \sigma_z(1-\rho^2)^{0.5} dv \right\rangle \\
&= \frac{\sigma_y}{\pi} \int_{-\infty}^0 u \exp\left(-\frac{1}{2}u^2\right) du \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}v^2\right) dv = -\frac{2\sigma_y}{\sqrt{2\pi}}
\end{aligned}$$

Further, making the same transformation, and integrating by parts it can be shown that:

$$\begin{aligned}
\Theta_1 &= \frac{\sigma_y}{\pi\sigma_z(1-\rho^2)^{0.5}} \int_{-\infty}^0 \int_{-\infty}^{-y} \frac{y^2}{\sigma_y^2} \exp\left(-\frac{1}{2(1-\rho^2)}\left((1-\rho^2)\left(\frac{\sigma_y z - \rho\sigma_z y}{(1-\rho^2)^{0.5}\sigma_z\sigma_y}\right)^2 + (1-\rho^2)\frac{y^2}{\sigma_y^2}\right)\right) dz dy \\
&= \left\langle u = \frac{y}{\sigma_y}, v = \frac{\sigma_y z - \rho\sigma_z y}{(1-\rho^2)^{0.5}\sigma_z\sigma_y}, dy = \sigma_y du, dz = \sigma_z(1-\rho^2)^{0.5} dv, -\frac{\sigma_y + \sigma_z\rho}{(1-\rho^2)^{0.5}\sigma_z} u \equiv -\alpha u \right\rangle \\
&= \frac{\sigma_y^2}{\pi} \int_{-\infty}^0 \int_{-\infty}^{-\alpha u} u^2 \exp\left(-\frac{1}{2}(v^2 + u^2)\right) dv du = \frac{\sigma_y^2}{\pi} \left[\pi - \omega + \frac{\alpha}{1+\alpha^2} \right]
\end{aligned} \tag{A6}$$

where $\omega = \pi/2 - \arctan(\alpha)$, and $\alpha = \frac{\sigma_y + \sigma_z\rho}{(1-\rho^2)^{0.5}\sigma_z} \in (0, +\infty)$.

By analogy (making the same substitution as above) we have:

$$\Theta_2 = \frac{\sigma_y^2}{\pi} \int_{-\infty}^0 \int_{-\alpha u}^{\infty} u^2 \exp\left(-\frac{1}{2}(v^2 + u^2)\right) dv du = \frac{\sigma_y^2}{\pi} \left[\omega - \frac{\alpha}{1+\alpha^2} \right] \tag{A7}$$

Using integration by parts, it can be further shown that:

$$\begin{aligned}
\Theta_3 &= \frac{1}{\pi\sigma_z\sigma_y(1-\rho^2)^{0.5}} \int_{-\infty}^0 \int_{-\infty}^{-y} zy \exp\left(-\frac{1}{2(1-\rho^2)}\left(\frac{z^2}{\sigma_z^2} - 2\rho\frac{zy}{\sigma_z\sigma_y} + \frac{\rho^2 y^2}{\sigma_y^2} + (1-\rho^2)\frac{y^2}{\sigma_y^2}\right)\right) dz dy \\
&= \left\langle u = \frac{y}{\sigma_y}, v = \frac{\sigma_y z - \rho\sigma_z y}{(1-\rho^2)^{0.5}\sigma_z\sigma_y}, dy = \sigma_y du, dz = \sigma_z(1-\rho^2)^{0.5} dv, -\frac{\sigma_y + \sigma_z\rho}{(1-\rho^2)^{0.5}\sigma_z} u \equiv -\alpha u \right\rangle \\
&= \frac{\sigma_z\sigma_y}{\pi} \int_{-\infty}^0 \int_{-\infty}^{-\alpha u} u(v\sqrt{1-\rho^2} + \rho u) \exp\left(-\frac{1}{2}(v^2 + u^2)\right) dv du \\
&= \frac{\sigma_z\sigma_y}{\pi} \left[\sqrt{1-\rho^2} \frac{1}{1+\alpha^2} + \rho \left(\pi - \omega + \frac{\alpha}{1+\alpha^2} \right) \right],
\end{aligned} \tag{A8}$$

and, by analogy,

$$\begin{aligned}
\Theta_4 &= \frac{\sigma_z\sigma_y}{\pi} \int_{-\infty}^0 \int_{-\alpha u}^{\infty} u(v\sqrt{1-\rho^2} + \rho u) \exp\left(-\frac{1}{2}(v^2 + u^2)\right) dv du \\
&= \frac{\sigma_z\sigma_y}{\pi} \left[-\sqrt{1-\rho^2} \frac{1}{1+\alpha^2} + \rho \left(\omega - \frac{\alpha}{1+\alpha^2} \right) \right].
\end{aligned} \tag{A9}$$

Now we can use these results to evaluate the expression for Δ :

$$\begin{aligned}
\Delta &= \int_{-\infty}^0 \int_{-\infty}^{-y} (y^2 + zy)f(z, y)dzdy - \int_{-\infty}^0 \int_{-y}^{+\infty} (y^2 + zy)f(z, y)dzdy + E(z + y | R > 0) \int_{-\infty}^0 \int_{-\infty}^{+\infty} y f(x, y)dzdy \\
&= \Theta_1 - \Theta_2 + \Theta_3 - \Theta_4 + \Theta \frac{2\sqrt{\sigma_z^2 + 2\rho\sigma_z\sigma_y + \sigma_y^2}}{\sqrt{2\pi}} = \frac{\sigma_y^2}{\pi} \left[\pi - \omega + \frac{\alpha}{1 + \alpha^2} \right] - \frac{\sigma_y^2}{\pi} \left[\omega - \frac{\alpha}{1 + \alpha^2} \right] \\
&+ \frac{\sigma_z\sigma_y}{\pi} \left[\sqrt{1 - \rho^2} \frac{1}{1 + \alpha^2} + \rho \left(\pi - \omega + \frac{\alpha}{1 + \alpha^2} \right) \right] - \frac{\sigma_z\sigma_y}{\pi} \left[-\sqrt{1 - \rho^2} \frac{1}{1 + \alpha^2} + \rho \left(\omega - \frac{\alpha}{1 + \alpha^2} \right) \right] \\
&- \frac{2\sigma_y}{\sqrt{2\pi}} \frac{2\sqrt{\sigma_z^2 + 2\rho\sigma_z\sigma_y + \sigma_y^2}}{\sqrt{2\pi}}.
\end{aligned} \tag{A10}$$

Rearranging this expression yields:

$$\begin{aligned}
\Delta &= \frac{\sigma_y}{\pi} \left((\sigma_y + \rho\sigma_z) \left(\pi - 2\omega + \frac{2\alpha}{1 + \alpha^2} \right) + \frac{2\sigma_z\sqrt{1 - \rho^2}}{1 + \alpha^2} - 2\sigma_z\sqrt{1 - \rho^2}\sqrt{1 + \alpha^2} \right) \\
&= \frac{2\sigma_y\sigma_z\sqrt{1 - \rho^2}}{\pi} \left(1 + \alpha \arctan(\alpha) - \sqrt{1 + \alpha^2} \right) > 0.
\end{aligned} \tag{A11}$$

Now the asymmetric timeliness coefficient can be computed as follows:

$$\beta_2 = \frac{\Delta}{\text{var}(R | R > 0)} = \frac{2\sigma_y\sigma_z\sqrt{1 - \rho^2}}{(\pi - 2)\text{var}(R)} \left(1 + \alpha \arctan(\alpha) - \sqrt{1 + \alpha^2} \right) > 0 \tag{A12}$$

where we use the result that $\text{var}(z + y | R > 0) = \text{var}(R) - E(R | R > 0)^2 = \text{var}(R) \left(1 - \frac{2}{\pi} \right)$.

References

- Ali, A., Hwang, L., Trombley, M., 2003, Arbitrage risk and the book-to-market anomaly, *Journal of Financial Economics* 69, 355–373.
- Bachelier, L., 1900, Théorie de la spéculation, *Annales Scientifiques de l'École Normale Supérieure* 3, 21-86.
- Ball, R., Brown, P., 1968, An empirical evaluation of accounting income numbers, *Journal of Accounting Research* 6, 159-177.
- Ball, R., Kothari, S., Nikolaev, V., 2013. On estimating conditional conservatism, forthcoming in *The Accounting Review*.
- Ball, R., Kothari, S., Robin, A., 2000, The effect of international institutional factors on properties of accounting earnings, *Journal of Accounting & Economics* 29, 1-51.
- Ball, R., Robin, A., Sadka, G., 2008, Is financial reporting shaped by equity markets or by debt markets? An international study of timeliness and conservatism, *Review of Accounting Studies* 13, 168–205.
- Ball, R., Shivakumar, L., 2005, Earnings quality in U.K. private firms: comparative loss recognition timeliness, *Journal of Accounting & Economics* 39, 83-128.
- Ball, R., Shivakumar, L., 2008, Earnings quality at initial public offerings: Managerial opportunism, or public-firm conservatism? *Journal of Accounting & Economics* 45, 324-349.
- Basu, S., 1997, The conservatism principle and the asymmetric timeliness of earnings, *Journal of Accounting & Economics* 24, 3-37.
- Basu, S., 2005, Discussion of “Conditional and unconditional conservatism: Concepts and modeling,” *Review of Accounting Studies* 10, 311–321.
- Basu, S., 2009, Conservatism research: Historical development and future prospects, *China Journal of Accounting Research* 2, 1-20.
- Beaver, W., Lambert, R., Morse, D., 1980, The information content of security prices, *Journal of Accounting & Economics* 2, 3-28.
- Beaver, W., Ryan, S., 2005, Conditional and unconditional conservatism: Concepts and modeling, *Review of Accounting Studies* 10, 269-276.
- Bernard, V., Thomas, J., 1990, Evidence that stock prices do not fully reflect the implications of current earnings for future earnings, *Journal of Accounting and Economics* 13, 305-340.
- Brealey, R., Myers, S., Allen, F., 2006, *Principles of Corporate Finance*, 8th edition, McGraw-Hill Irwin.
- Campbell, J., Lo, A., MacKinlay, C., 1997, *The Econometrics of Financial Markets*. Princeton University Press, Princeton, NJ.
- Collins, Daniel W., S.P. Kothari, 1989, An analysis of intertemporal and cross-sectional determinants of earnings response coefficients, *Journal of Accounting and Economics*, 11, 143-181.
- Cook, T. D. and Campbell, D.T., 1979, *Quasi- Experimentation: Design and Analysis Issues for Field Settings*. Chicago: Rand-McNally.

- Dietrich, D., Muller, K., Riedl, E., 2007, Asymmetric timeliness tests of accounting conservatism, *Review of Accounting Studies* 12, 95-124.
- Easton, Peter E., and Mark E. Zmijewski, 1989, Cross-sectional variation in the stock market response to accounting earnings announcements, *Journal of Accounting and Economics* 11, 117-141.
- Fama, E., 1970, Efficient capital markets: A review of theory and empirical work, *Journal of Finance* 25, 383-417.
- Fama, E., 1990, Stock returns, expected returns, and real activity, *Journal of Finance* 45, 1089-1108.
- Foster, G., Olsen, C., Shevlin, T., 1984, Earnings releases, anomalies, and the behavior of securities returns, *The Accounting Review* 59, 574-603.
- Gigler, F., Hemmer, T., 2001, Conservatism, optimal disclosure policy, and the timeliness of financial reports, *The Accounting Review* 76, 471-493.
- Givoly, D., Hayn, C., Natarajan, A., 2007, Measuring reporting conservatism, *The Accounting Review* 82, 65-106.
- Khan, M., Watts, R., 2009, Estimation and empirical properties of a firm-year measure of accounting conservatism, *Journal of Accounting & Economics* 48, 132-150.
- Kothari, S., 2001, Capital markets research in accounting, *Journal of Accounting & Economics* 31, 105-231.
- Kothari, S., Sloan, R., 1992, Information in prices about future earnings, *Journal of Accounting & Economics* 15, 143-171.
- Lewellen, J., 1999, The time-series relations among expected return, risk, and book-to-market, *Journal of Financial Economics* 54, 5-43.
- Nikolaev, V., 2010. Debt covenants and accounting conservatism, *Journal of Accounting Research*, 48, 54-89.
- Pae, J., Thornton, D., Welker, M., 2005, The link between accounting conservatism and the price-to-book ratio, *Contemporary Accounting Research* 22, 693-717.
- Patatoukas, P. N., and J. K. Thomas, 2011. More evidence of bias in the differential timeliness measure of conditional conservatism. *The Accounting Review* 86, 765-1793.
- Peter, J. P., 1981, Construct Validity: A review of basic issues and marketing practices, *Journal of Marketing Research* 18, 133-145.
- Pope, P., Walker, M., 1999, International differences in the timeliness, conservatism and classification of earnings, *Journal of Accounting Research* 37, 53-87.
- Price, R., 2005, Accounting conservatism and the asymmetry in current and lagged returns, working paper, Rice University.
- Richardson, G., Tinaikar, S., 2004, Accounting based valuation models: What have we learned? *Accounting and Finance* 44, 223-255.
- Ross, S., Westerfield, R., Jaffe, J., 2005, *Corporate Finance*, Seventh edition, McGraw-Hill Irwin.
- Roychowdhury, S., Watts, R., 2007, Asymmetric timeliness of earnings, market-to-book and conservatism in financial reporting, *Journal of Accounting & Economics* 44, 2-31.

- Ryan, S.G., Zarowin, P., 2003, Why has the contemporaneous linear returns-earnings relation declined? *The Accounting Review* 78, 523-553.
- Samuelson, P. A., 1965, Proof that properly anticipated prices fluctuate randomly, *Industrial Management Review* 6, 41-49.
- Watts, R., 2003a, Conservatism in accounting Part I: Explanations and implications, *Accounting Horizons* 17, 207-221.
- Watts, R., Zimmerman, J., 1986, *Positive Accounting Theory*, Prentice-Hall.