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A Bayesian Approach for Learning and Predicting Personal Thermal Preference

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ABSTRACT

Typical thermal control systems automated based on the use of “widely acceptable” thermal comfort metrics cannot achieve high levels of occupant satisfaction and productivity since individual occupants prefer different thermal conditions. The objective of this study is to develop environmental control systems that provide personalized indoor environments by learning their occupants and being self-tuned. Towards this goal, this paper presents a new methodology, based on Bayesian formalism, to learn and predict individual occupant’s thermal preference without developing different models for each occupant. We develop a generalized thermal preference model in which our key assumption, “Different people prefer different thermal conditions” is explicitly encoded. The concept of clustering people based on a hidden variable which represents each individual’s thermal preference characteristic is introduced. Also, we exploited equations in the Predicted Mean Vote (PMV) model as physical knowledge in order to facilitate modeling combined effects of various factors on thermal preference. Parameters in the equations are re-estimated based on the field data. The results show evidence of the existence of multi-clusters in people with respect to thermal preference.

1. INTRODUCTION

Heating and cooling systems in buildings have been automated based on the use of “widely acceptable” thermal comfort metrics and simple heuristic rules. However, field studies show that individual occupants prefer different thermal conditions (Brager & de Dear, 1998; de Dear & Brager, 1998; Fanger, 1967, 1970). As a result, typical thermal control systems cannot achieve high levels of occupant satisfaction and productivity. Moreover, because of the conservative control settings designed for “widely acceptable” conditions, there is high probability of energy waste (Hwang et al., 2009; Nicol & Humphreys, 2009). Studies have recognized that the aforementioned problems can be resolved by incorporating building occupants in sensing and control frameworks, and tuning systems based on individual preferences to achieve personalized indoor environments (Auffenberg et al., 2015; Feldmeier & Paradiso, 2010; Murakami et al., 2007). Learning occupant’s preference is an essential part of this innovative concept.

Several studies have been conducted to develop methods for learning individual occupant’s thermal preference (Daum et al., 2011; Erickson & Cerpa, 2012; Ghahramani et al., 2014; Ghahramani et al., 2015; Guillemain & Molteni, 2002; Jazizadeh et al., 2014). Although the studies have demonstrated the feasibility of learning occupants’ thermal preference and have shown that occupants’ thermal satisfaction has been improved by controlling HVAC systems based on it, there are two significant problems associated with the implementation of these methods in real buildings. First, learning requires a long-term data collection, since the rate of data collection is limited by the fact that occupants should not be exposed to potentially uncomfortable conditions for long time. Second, in order to avoid overfitting, the limited data availability imposes model structures that may be too simple to describe the human preference. Therefore, the majority of existing models consider only the effect of air temperature and ignore other parameters, e.g., local

airflow, mean radiant temperature that could be important. Although (Humphreys & Nicol, 2002) reported that a prediction made with the air temperature was not inferior to using a complex model in typical buildings, if the other environmental factors either do not highly co-vary with air temperature or not remain constant, the simple model may not provide reliable predictions. For example, in perimeter zones, occupants are affected by both solar radiation and long-wave radiation which may vary significantly according to the sky condition and the position of the sun (La Gennusa et al., 2007). Also, in cases that the HVAC system controls not only air temperature but also other parameters in order to create comfortable thermal environments (e.g., radiant heating/cooling systems or local systems exploiting the effect of increased air velocity), the simple air-temperature based model cannot predict occupant's thermal preference.

In order to overcome these limitations, a new method has been designed for learning and predicting individual occupant's thermal preference based on a Bayesian approach.

2. METHODOLOGY

2.1 Model Structure

Collecting enough data to train a model from an individual occupant is challenging especially if the model structure is complex and many parameters in the model have to be estimated. Therefore, we designed a method to develop a generalized thermal preference model based on a large dataset collected from various people and use the model for learning and predicting an individual occupant's preference by explicitly encoding our key assumption: "Different people prefer different thermal condition" in the model instead of developing different models for each occupant. In order to develop the model, first, a graph representing connections between occupant's thermal preference and related factors (i.e. environmental, human, contextual, hidden features, and occupant's behaviors) has been developed based on our knowledge and beliefs. Fig. 1 shows a simplified version of the graph. Grey and white nodes correspond to observed and hidden random variables, respectively. The arrows of the graph represent conditional probabilistic relationships between the nodes. An important point is that individual occupant's thermal preference characteristic, which is an unobserved variable, is included in the graph denoting our aforementioned key assumption. This study assumes that occupant's thermal preference is mainly governed by an overall energy balance condition and the individual occupant's thermal preference characteristic (bold elements in Fig. 1) for simplicity.

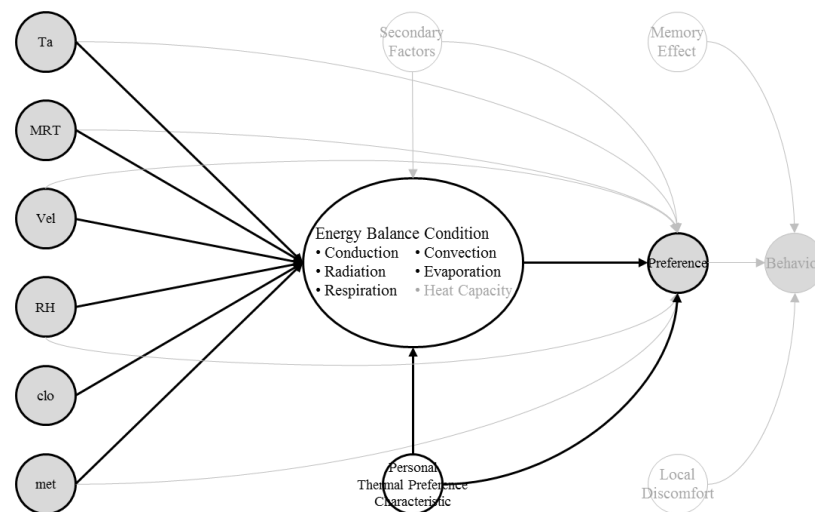


Figure 1: A graph representing our knowledge and beliefs regarding occupants' thermal preference

The graph corresponds to a decomposition of the joint probability density of all the random variables (observed and hidden) (Eq. 1):

$$P(y, E, z, \mathbf{x}) = P(y | E, z)P(E | z, \mathbf{x})P(z)P(\mathbf{x}) \quad (1)$$

where y denotes occupant's thermal preference, E denotes energy balance condition, z denotes thermal preference characteristic of an individual occupant and \mathbf{x} denotes input variables (i.e. air temperature, MRT, air velocity, relative humidity, clo, met). In this study, y , i.e. occupant's thermal preference, is discretized to take only three values: "want warmer", "no change", "want cooler". Also, by assuming that there is an unknown number K of possible clusters of people who have similar z , i.e. thermal preference characteristic, z is discretized to take K values. In order to quantify the energy balance condition, imbalance between the actual heat flow from the body and the heat flow required for optimum comfort is used.

Eq. 1 represents that the two sub-parts for $P(y|E, z)$ and $P(E|z, \mathbf{x})$ are required in the model. $P(E|z, \mathbf{x})$ predicts the energy balance condition and since the energy transfer processes depend on the combined effects of the input variables, modeling this condition with classical regression methods is challenging and requires estimating many parameters for which we don't have prior knowledge and a large dataset. This problem can be resolved by introducing physical knowledge and results of previous studies in the model and estimating fewer parameters with our prior knowledge. In this study, equations in PMV model developed by Fanger to predict occupant's thermal sensation are used (Fanger, 1970). Although previous studies have shown that occupant's thermal sensation and preference are different (Van Hoof, 2008), and the PMV model with its parameters estimated based on the laboratory study by Fanger cannot explain occupant's thermal preference in real buildings (Barlow & Fiala, 2007; Corgnati et al., 2007; Wagner et al., 2007), it has significant advantages as it calculates the energy balance condition by combining the effects of major environmental factors based on physical mechanisms examined in previous research. Therefore, the equations are introduced in our model as a physical knowledge. However, 15 major parameters (Table 1) are re-estimated since they were originally estimated to predict thermal sensation based on data collected from chamber experiments. Moreover, since six of the major parameters ($\xi_{1:6}$, in Table 1) may relate to the thermal preference characteristic z , the model is designed to allow each cluster to have its own values for these six parameters. $P(y|E, z)$ maps the energy balance condition to occupant's thermal preference, and since y is ordered discrete variable, ordered logit function is used for the mapping (McCullagh, 1980). The model allows each cluster to have its own values for parameters in ordered logit function for the same reasons described above.

The aforementioned parts are assembled into one model and the parameters are estimated using a large dataset along with the hidden cluster value of each occupant in the dataset (representing individual occupant's thermal preference characteristic) with a fully Bayesian approach. The rationale behind our modeling choice is related to its inherent advantages: it allows encoding and testing our prior knowledge and beliefs about the relationships of the various variables; it can easily account for hidden (unobserved) variables; and it can seamlessly combine data from heterogeneous sources (Jaynes, 2003). Eq. 2 and Eq. 3 show the posterior density and likelihood of the modeling problem respectively. In order to develop the model in fully Bayesian approach, the parameters and the hidden cluster values should be sampled from the posterior distribution. However, since the posterior distribution is intractable analytically, MCMC (Markov Chain Monte Carlo) is used for the estimation. MCMC provides samples from the posterior distribution for each parameter, and it allows quantifying the uncertainty of the model and its prediction. In this study, we used Python package PyMC and select adaptive Metropolis-hasting algorithm for MCMC sampling (Fonnesbeck et al., 2015).

Detailed information for the parameters and prior distribution for each parameter in the Bayesian estimation is presented in Table 1 and Fig. 2 shows the overall model structure.

$$P(z_{1:D}, \mathbf{w}, \boldsymbol{\theta}, \boldsymbol{\xi} | \mathbf{x}_{1:D}, y_{1:D}) = \frac{P(y_{1:D} | \mathbf{x}_{1:D}, z_{1:D}, \mathbf{w}, \boldsymbol{\theta}, \boldsymbol{\xi})P(z_{1:D})P(\mathbf{w}, \boldsymbol{\theta})P(\boldsymbol{\xi})}{P(\mathbf{x}_{1:D}, y_{1:D})} \quad (2)$$

$$\begin{aligned}
P(y_{1:D} | \mathbf{x}_{1:D}, z_{1:D}, \mathbf{w}, \boldsymbol{\theta}, \boldsymbol{\xi}) &= \prod_{d=1}^D P(y_d | \mathbf{x}_d, z_d, \mathbf{w}, \boldsymbol{\theta}, \boldsymbol{\xi}), \\
P(y_d | \mathbf{x}_d, z_d, \mathbf{w}, \boldsymbol{\theta}, \boldsymbol{\xi}) &= \prod_{i=1}^{n_d} P(y_i | \mathbf{x}_i, z_d, \mathbf{w}, \boldsymbol{\theta}, \boldsymbol{\xi}), \\
P(y_i \leq j | \mathbf{x}_i, z_d, \mathbf{w}, \boldsymbol{\theta}, \boldsymbol{\xi}) &= \phi_{z_d, j} = \frac{1}{1 + \exp(\mathbf{w}_{z_d} E(\mathbf{x}_i, \boldsymbol{\xi}_{z_d}) - \theta_{z_d, j})}, \quad j = 0, \dots, 2 \\
P(y_i = j | \mathbf{x}_i, z_d, \mathbf{w}, \boldsymbol{\theta}, \boldsymbol{\xi}) &= \begin{cases} \phi_{z_d, j} - \phi_{z_d, j-1}, & j = 1, 2 \\ \phi_{z_d, j}, & j = 0 \end{cases}
\end{aligned} \tag{3}$$

D : The number of people

d : Occupant ID

n_d : The number of data from occupant d

K : The number of clusters

\mathbf{x} : Input data

y : Output data (0: want warmer, 1: no change, 2: want cooler)

z : Hidden cluster value (1, ..., K)

E : Energy balance condition calculated by PMV equations with input \mathbf{x} and parameters $\boldsymbol{\xi}$

\mathbf{w} : Set of parameters in ordered logit function ($K \times 2$ matrix)

$\boldsymbol{\theta}$: Set of parameters in ordered logit function ($K \times 3$ matrix)

$\boldsymbol{\xi}$: Set of parameters in PMV equations ($K \times 15$ matrix)

Table 1: Estimated parameters and prior distributions in Bayesian estimation

Parameter	Related Element	Prior Distribution			
$\mathbf{w}, \boldsymbol{\theta}$	Cluster Specific	Ordered logistic function	Normal	Mean: 0	Std.: 100
ξ_1	Cluster Specific	Balance between the actual heat flow from the body and the heat flow required for optimum comfort	Exp.	λ : 3.300	
ξ_2			Normal	Mean: -0.036	Std.: 0.013
ξ_3			Exp.	λ : 35.71	
ξ_4	Cluster Specific	Optimal skin temperature making human comfortable	Normal	Mean: 308.7	Std.: 0.658
ξ_5			Normal	Mean: 0.028	Std.: 0.002
ξ_6	Cluster Specific	Heat emission through sweating	Normal	Mean: 0.42	Std.: 0.099
ξ_7	Shared	Forced convective heat transfer	Normal	Mean: 12.1	Std.: 1.316
ξ_8		Natural convective heat transfer	Normal	Mean: 2.38	Std.: 0.263
ξ_9			Exp.	λ : 1.0	
ξ_{10}	Shared	Evaporative heat transfer from the skin	Normal	Mean: 5733	Std.: 131.6
ξ_{11}			Normal	Mean: 6.99	Std.: 1.656
ξ_{12}			Exp.	λ : 2.0	
ξ_{13}	Shared	Clothing area factor estimation based on clo	Exp.	λ : 0.775	
ξ_{14}			Exp.	λ : 1.818	
ξ_{15}			Exp.	λ : 1.550	

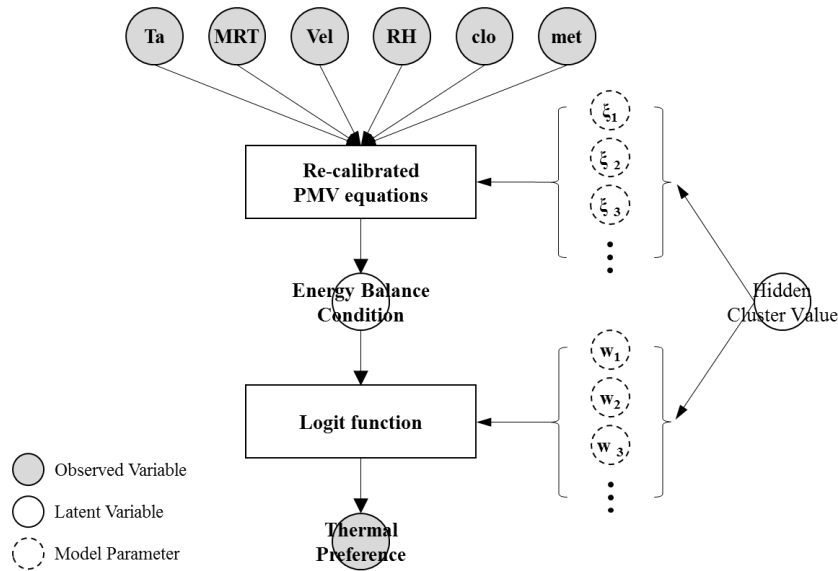


Figure 2: Overall model structure

2.2 Individual Occupant’s Thermal Preference

Exactly the same process is used to learn the thermal preference of a new individual occupant. When observations have been collected for the occupant, they are added into the large dataset, and the model is updated based on the new dataset. During this process, $P(z_{new} = k | \mathbf{x}_{1:D}, y_{1:D}, \mathbf{x}_{new}, y_{new}), k = 1, \dots, K$, which denotes the probability of the occupant being in cluster k is estimated. Then, occupant’s thermal preference under certain condition can be predicted by a mixture of sub-models for each cluster with the set of probabilities following Eq. 4.

$$P(y_{inference} = j | \mathbf{x}_{inference}, \mathbf{x}_{1:D}, y_{1:D}, \mathbf{x}_{new}, y_{new}, \mathbf{w}, \boldsymbol{\theta}, \boldsymbol{\xi}) = \sum_{k=1}^K p(y_{inference} = j | \mathbf{x}_{inference}, \mathbf{x}_{1:D}, y_{1:D}, \mathbf{x}_{new}, y_{new}, \mathbf{w}, \boldsymbol{\theta}, \boldsymbol{\xi}, z_{new} = k) P(z_{new} = k | \mathbf{x}_{1:D}, y_{1:D}, \mathbf{x}_{new}, y_{new}), \quad j = 0, \dots, 2 \quad (4)$$

The overall process for learning and predicting individual occupant’s thermal preference is shown in Fig. 3. First, a large dataset collected from various people with which our prior knowledge, beliefs, and hypotheses can be tested is prepared. Then, a cluster model for general people dealing with observed data and unobserved hidden variables are developed. With the dataset and the model, a new individual occupant’s thermal preference can be learned and prediction problem can be solved.

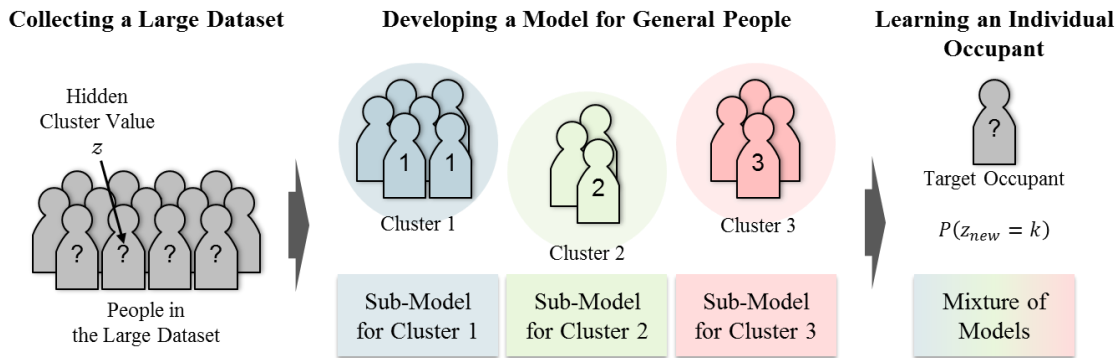


Figure 3: Overall process for learning and predicting individual occupant’s preference

2.3 Data

A large dataset regarding occupant's thermal comfort was collected by ASHRAE through RP-884 (de Dear et al., 1997). This dataset consists of physical data (e.g., indoor and outdoor air temperature, MRT, air velocity, etc.), human attributes (e.g., age, gender, etc.), survey data (e.g., thermal preference, sensation, acceptability). In this study, a subset of this dataset with data collected from HVAC conditioned office buildings in North America is used to evaluate the feasibility of our method. The dataset includes 3,070 observations collected from 1,248 occupants and supports the rationale for re-calibrating the parameters in PMV equations instead of using the original PMV model. Fig. 4 shows the thermal preference distribution with respect to the original PMV values. As shown in the figure, some people do not prefer a warmer condition even though the PMV value is quite low. Moreover, a non-negligible percentage of the occupants prefer cooler conditions with PMV of less than -1. In other words, the PMV model with its original parameters cannot predict or explain occupant's thermal preference in real buildings.

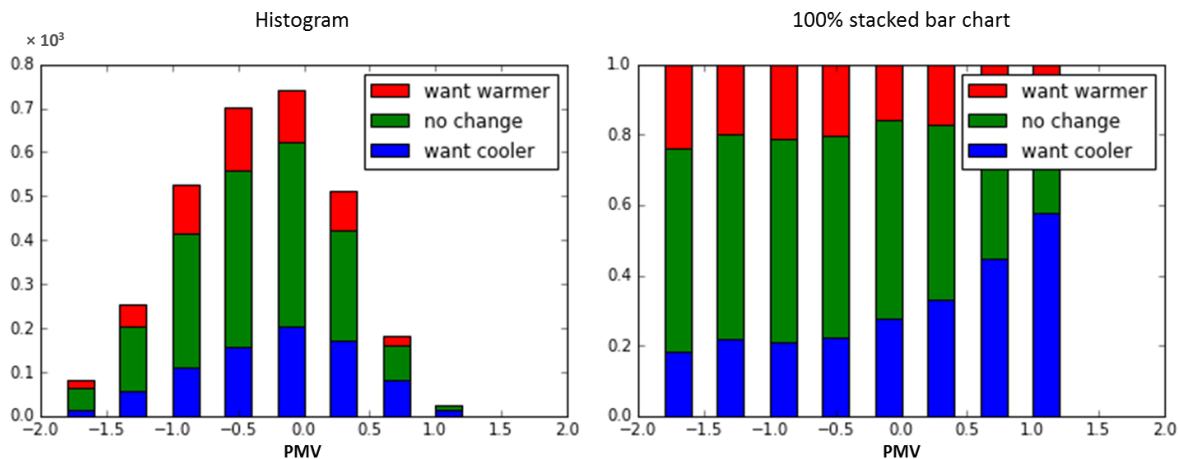


Figure 4: Thermal preference distribution with respect to PMV (ASHRAE RP-884)

3. RESULTS

3.1 Single Cluster Model

Since the thermal preference characteristic is a hidden variable, the number of clusters for the characteristic is also unknown. Therefore, the optimal number of clusters K for this general model should be identified with the large dataset before using this method to learn and predict individual occupant's preference. In order to determine the optimal number of clusters, models which have different number of clusters should be developed and evaluated. First, a model having a single cluster was developed, i.e. modeling without clustering. Fig. 5 shows estimated parameters (histograms) and Fanger's original values (red vertical line) in the equations. As shown in the figure, most of estimated parameters are different from Fanger's values.

Fig. 6 (a) shows the probability distribution of an occupant being in each preference class (i.e., "want warmer", "no change", "want cooler") with respect to the air temperature variation, where MRT is equal to the air temperature, air velocity is 0.1m/s, relative humidity is 50%, met is 1.2, clo is 0.65. In this figure, since each sample from MCMC is equally probable to be a model, the expected probabilities and the associated 95% credible intervals for each set of inputs are calculated with the sample sets, and plotted as solid lines and shaded areas respectively. Since the number of data in both lower and higher temperature ranges are insufficient, the width of the shaded area in these ranges are wider than that of the middle range.

Fig. 6 (b) and (c) show the probability distribution for different air velocity (0.2 m/s) and clo (1.0) respectively. In accordance with general knowledge about thermal comfort, the shape of the probability curves changes. For example, curves move to the right with increased air velocity, i.e. people prefer warmer condition, and curves move to the left with higher clo, i.e. people prefer cooler condition.

However, although the calculated probability distributions follow reasonable trends, since the model is developed with a dataset collected from various people and there are many contradicting observations, the shape of distributions is quite smooth, and the maximum probability of an occupant preferring no change is only around 0.6. As a result, although this model could be used for explaining general thermal preference of people, it cannot be used for learning and predicting individual occupant's thermal preference.

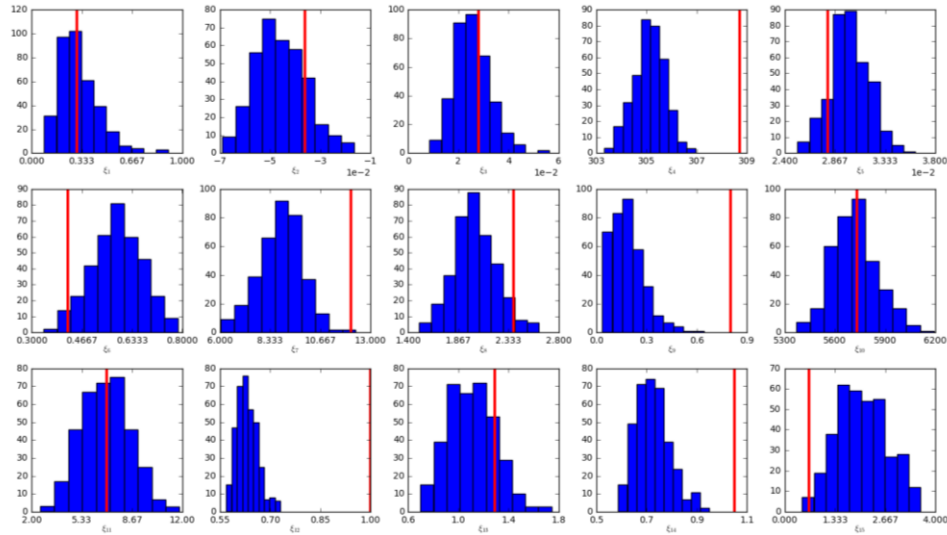


Figure 5: Estimated parameters (histograms) and Fanger's original values (red vertical line)

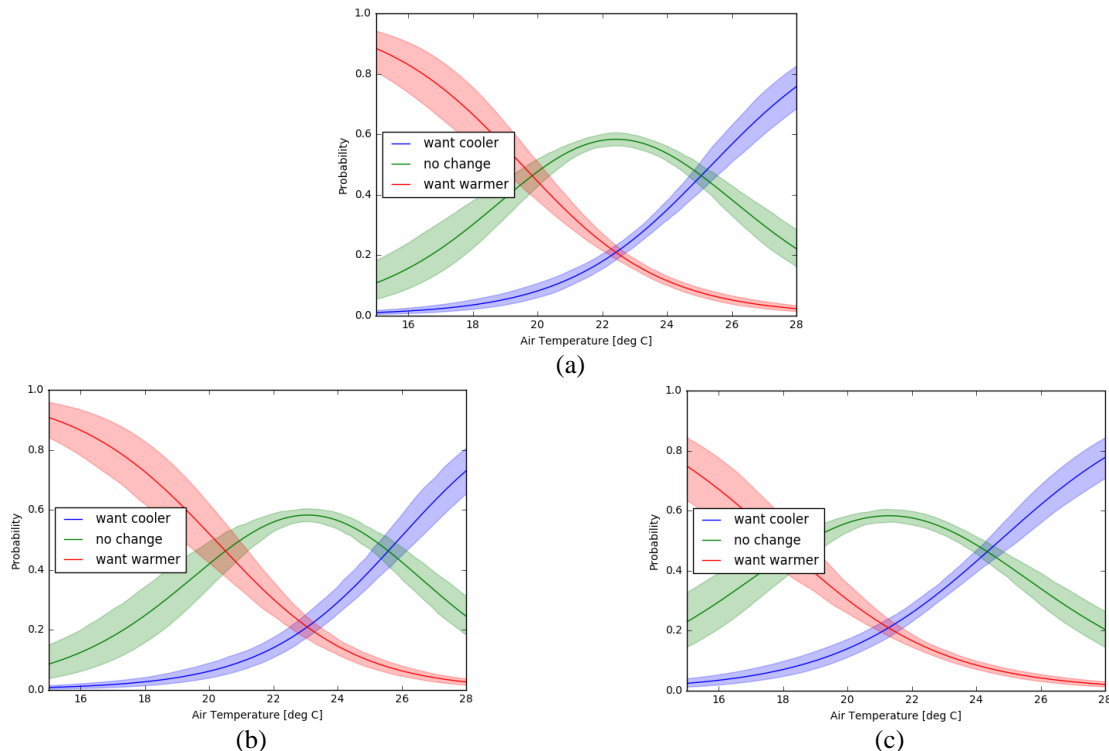


Figure 6: Probability distribution of an occupant being in each preference class with respect to air temperature change (MRT is equal to air temperature)
 (a) air velocity = 0.1m/s, relative humidity = 50%, met = 1.2, clo = 0.65
 (b) air velocity = 0.2m/s, relative humidity = 50%, met = 1.2, clo = 0.65
 (c) air velocity = 0.1m/s, relative humidity = 50%, met = 1.2, clo = 1.0

3.2 Two Clusters Model

A model which has two clusters (i.e. $K = 2$) was developed. Since developing a full model including PMV equations is computationally expensive, a simpler model was developed for a preliminary study. The simpler model is an ordered logit model of which input is only air temperature. Note that the full model is being developed, and the result will be reported.

Fig. 7 shows MCMC samples for the parameters. The blue histogram and crosses are samples for one of the clusters and the green ones are samples for the other cluster. As shown in the figure, the blue crosses and the green circles for the parameter \mathbf{w} and θ are clearly distinguished. This provides evidence that there is a high probability of classifying two or more clusters of people with respect to thermal preference characteristic.

Fig. 8 shows the estimated hidden cluster value for each occupant in the dataset. Blue and red pixels denote each cluster respectively, each row of pixels in the figure is a set of samples sampled together from one MCMC iteration and the x-axis denotes each occupant. Therefore, each column means probability of the occupant being in cluster k (i.e. $P(z_d = k | \mathbf{x}_{1:D}, y_{1:D})$, $k = 1$ or 2). Note that the order of occupants was sorted by the probability of the occupant being in cluster 1, $P(z_d = 1 | \mathbf{x}_{1:D}, y_{1:D})$, in order to visualize the results more clearly. As shown in the figure, occupants on the left side of the figure are highly probable to be in the cluster 1, and occupants on the opposite side are highly probable to be in the cluster 2. This provides more evidence that there are two clusters of people having different thermal preference characteristic. If there was only one cluster, the results would show that most of the occupants have the same cluster value z . In case of occupants in the middle, since $P(z_d = k | \mathbf{x}_{1:D}, y_{1:D})$ is not highly biased, it can be explained that the occupants are between the two clusters or there are not observations to decide in which cluster the occupants belong too. For the occupants in the dataset, their thermal preference can be predicted by a mixture of sub-models for each cluster with $P(z_d = k | \mathbf{x}_{1:D}, y_{1:D})$ visualized in the figure following Eq. 4.

Fig. 9 shows the probability distribution of an occupant being in each preference class with respect to the air temperature variation for each cluster. As shown in the figures, sub-models for a cluster of people preferring cooler condition and another cluster preferring warmer condition were developed respectively. Since the sub-models are developed for each cluster, the shape of distributions becomes sharper compared to that of the single cluster model and the maximum probabilities of an occupant preferring no change are higher than 0.6. These are good characteristics in terms of predicting occupant's thermal preference.

The results from the two clusters model imply that there is a high probability of existence of multi-clusters in people with respect to thermal preference characteristic.

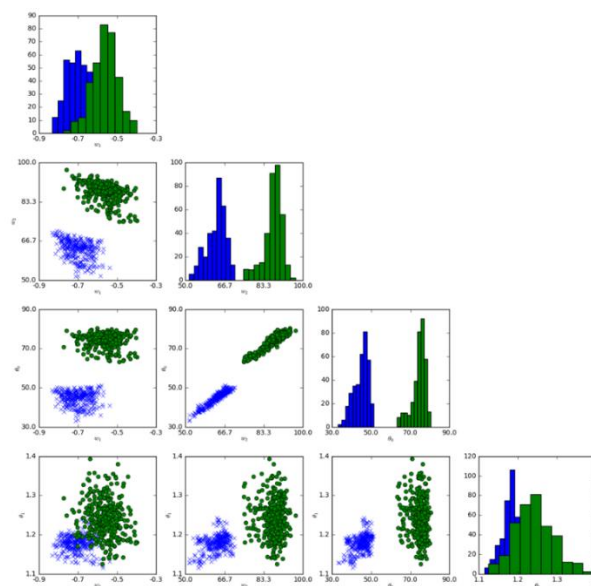


Figure 7: MCMC samples for parameters

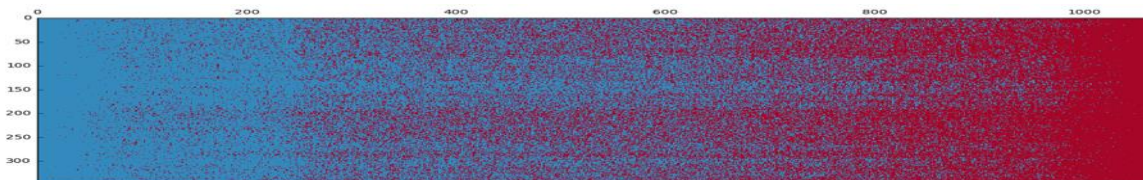


Figure 8: Estimated hidden cluster value for each occupant in the dataset

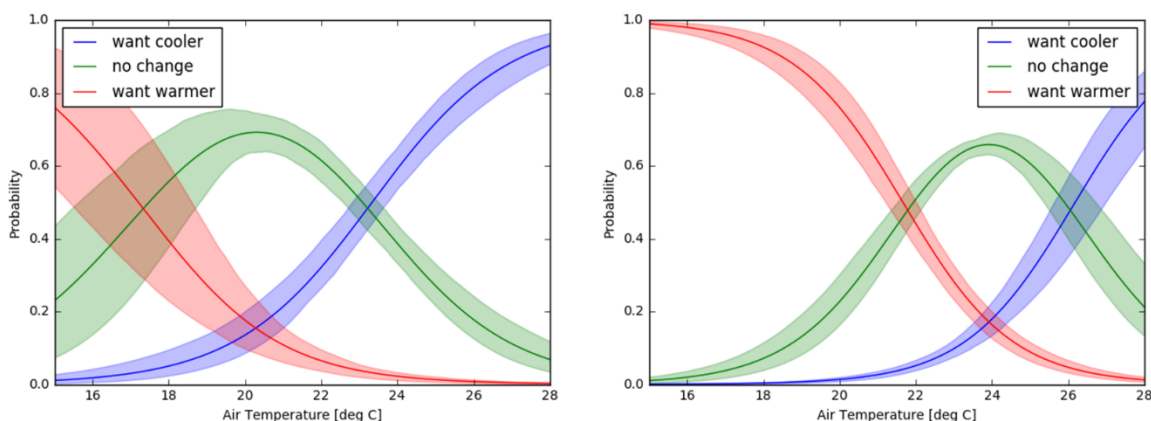


Figure 9: Probability distribution of an occupant being in each preference class with respect to air temperature change for each cluster.

4. DISCUSSION AND CONCLUSIONS

In this paper, a new method for learning and predicting individual occupant's thermal preference was introduced. The results show that the probabilities calculated by the models are in accordance with the general knowledge, and there is a high probability of classifying two or more clusters of people with respect to thermal preference characteristic. As a result, the two cluster model is more promising than the single cluster model considering our modeling purpose, i.e. learning and predicting individual occupant's thermal preference.

In this paper, a single cluster model and a two cluster model were qualitatively evaluated and compared with each other, however, the models should be also evaluated and compared in an objective and quantitative way, which will be presented in forthcoming publications by the authors. Also, the optimal number of clusters will be identified by testing different models and consequently, the hidden cluster value of a new occupant will be predicted.

Since the authors believe that occupant's thermal preference is affected by other factors, additional relationships and variables will be introduced in the general model and tested. Also, carefully designed experiments will be conducted in the future to enrich the dataset for this specific purpose. Since the Bayesian approach can seamlessly combine data from heterogeneous sources, all the data would be exploited for developing a general thermal preference model.

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