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A Multi-objective Approach to Optimal Battery Storage in The Presence of Demand Charges

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ABSTRACT

We propose a multi-objective optimization algorithm for optimal energy storage by residential customers using Li-Ion batteries. Our goal is to quantify the benefits of optimal energy storage to solar customers whose electricity bills consist of both Time of Use charges ($/kWh, with different rates for on-peak and off-peak hours) and demand charges ($/kW, proportional to the peak rate of consumption in a month). We first define our energy storage optimization problem as minimization of the monthly electricity bill subject to certain constraints on the energy level and the charging/discharging rate of the battery, while accounting for battery's degradation due to cycling and depth of discharge. We solve this problem by constructing a sequence of parameterized multi-objective dynamic programs whose sets of non-dominated solutions are guaranteed to contain an optimal solution to our energy storage problem. Unlike the standard formulation of our energy storage problem, each of the parameterized optimization problems satisfy the principle of optimality - hence can be solved using standard dynamic programming algorithms. Our numerical case studies on a wide range of load profiles show that in the presence of demand charges, optimal energy storage using the existing residential batteries can reduce the monthly electricity bill by up to 52% relative to the case where no energy storage is used.

1.INTRODUCTION

One concern of electric utilities is that rapid integration of renewables such as distributed solar generation may change customers' consumption behavior in ways that current generating units cannot accommodate for these changes. The demand peaks are of particular concern to regulated public utilities as these utilities are required to maintain reserve generating capacity as determined by the demand peak. According to a recent report by Arizona Public Services (APS (2014)), the demand peak in Arizona is projected to increase by 40% over a period of next 15 years (See Fig. 1). In order to cover the cost of increasing the generating capacity and influence the consumption behavior of rational customers, Arizona's utilities Salt River Project (SRP) and APS have recently incorporated *demand charges* in some of their pricing plans. Demand charges are proportional to the maximum rate at which the electricity is consumed by the customer during the on-peak hours within a month. Thus, in this new form of pricing, the amount paid by the customer within a month is

$$J(q) := 30\, p_{\text{on}} \int_{T_{\text{on}}} q(t)dt + 30\, p_{\text{off}} \int_{T_{\text{off}}} q(t)dt + p_d \sup_{t \in T_{\text{on}}} q(t), \tag{1}$$

where $q(t)$ is the power supplied by the utility company. The first two terms in (1) are Time of Use (ToU) charges, where p_{on} ($/kWh) and p_{off} ($/kWh) are the on-peak and off-peak prices. The third term is the monthly demand charge, where p_d ($/kW) is called the demand price. In (1), T_{on} and T_{off} denote the on-peak and off-peak periods respectively.

One approach for rational customers to reduce their consumption peaks (as a response to the integration of demand charges) is energy storage (Dunn and Tarascon (1997)). Several papers have studied optimal use of battery storage for residential customers (Bozchalui et al. (2012); Boaro et al. (2013); Huang and Liu (2013); Wei et al. (2015)). Majority of the existing papers use Model Predictive Control (Giorgio et al. (2012)), dynamic programming-based algorithms (Boaro et al. (2013); Huang and Liu (2013)) or neural networks (Wei et al. (2015)) to find optimal storage programs under real-time or ToU pricing. Li et al. (2011) have coupled the problem of optimal scheduling of battery and other controlled appliances with a social welfare optimization problem at the utility level to find optimal battery schedules and optimal real-time prices. Although optimal battery scheduling is a well-developed research area, to the best of our knowledge, there has been no studies on optimal scheduling of residential-sized batteries in the presence of demand charges.

From the mathematical standpoint, the presence of demand charges poses a significant challenge in solving the underlying optimization problem. In particular, because of the existence of the term $\sup q(t)$ (a *non-separable* term with respect to time) in the electricity bill defined in (1), the problem $\min J(q)$ violates the principle of optimality (Bellman and Dreyfus (1962)). The principle of optimality provides sufficient conditions for optimal control algorithms such as dynamic programming and Hamiltonian-based algorithms to converge to an optimal solution. While works such as Oudalov et al. (2007) and Rowe et al. (2014) have included some forms of demand charges in their storage optimization problems, they have not addressed the problem of inseparability of the objective function and violation of the principle of optimality.

In this paper, we establish a provably convergent algorithm for optimal battery scheduling for residential customers, under a combined ToU and demand pricing plan. We achieve this in two steps: First, we replace $\sup q(t)$ in the objective function with an l_p approximation of $q(t)$ for some large p. Then, we construct a sequence of multi-objective optimization problems (similar to Li and Haimes (1991, 1987)) whose sets of non-dominated solutions contain an optimal solution to the residential battery optimization as p tends to infinity. The objective functions of these multi-objective problems are separable with respect to time - hence can be minimized using the existing dynamic programming algorithms. In a number of numerical case studies, we applied our algorithm to the residential battery optimization problem for a wide range of customer's load, while considering the effects of solar generation and battery's degradation over 5 years. We used one of the existing residential-sized battery in the market as the source of energy storage. Our numerical analysis shows that optimal energy storage can reduce the monthly electricity bill by up to 52% using SRP's pricing plan (SRP (2015)). Moreover, we observed that under a combined ToU and demand pricing, optimal energy storage is most effective (in terms of saving over 5 years) for those customers who have the highest monthly peak load within our range of analysis.

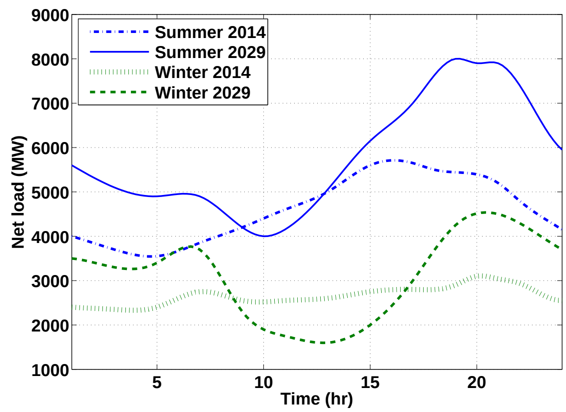

Figure 1: Net retail load for typical summer and winter days in Arizona for years 2014 and 2029 (projected)

2.OPTIMAL ENERGY STORAGE PROBLEM

In this section, we first define a model describing the amount of energy stored in the battery and the battery degradation due to cycling and depth of discharge. We then use this model to define our residential battery storage optimization problem under a mixed ToU and demand pricing plan.

2.1 A Model for Battery Storage and Degradation

To model the amount of energy stored in the battery, we use the difference equation

$$e(d,k+1) = e(d,k) + \eta\, u(d,k)\Delta t, \tag{2}$$

where $e(d,k)$ denotes the amount of energy stored in the battery at time $t = k\,\Delta t$ of day d, $u(d,k)$ denotes the charging/discharging (+/−) rate at time $t = k\,\Delta t$ of day d, and η denotes the charging efficiency. We denote the initial capacity of the battery by C_I, meaning that the initial energy level $e(0,0)$ is bounded by C_I. Moreover, we denote the maximum charging and discharging rates by $\bar{u}$ and $\underline{u}$, meaning that $u(k) \in [\underline{u}, \bar{u}]$ for all k.

We model the capacity of the battery at day d and time-step k as

$$c_b(d,k) = (1 - \beta(d,k))C_I, \tag{3}$$

where $\beta(d,k)$ denotes the battery's degradation rate. We model the degradation rate as the sum of cycling degradation and depth of discharge degradation:

$$\beta(d,k) = \underbrace{\alpha\, n(d,k)}_{\text{cycling degradation}} + \underbrace{\sum_{k=0}^{T} \gamma(d,k)}_{\text{depth of discharge degradation}}. \tag{4}$$

In (4), $n(d,k)$ is the total number of cycles completed till time-step k of day d. We calculate the number of cycles recursively as

$$n(d,k+1) = n(d,k) + \frac{\eta\,\Delta t\,|u(d,k)|}{2\,c_b(d,k)}. \tag{5}$$

In (4), $\gamma(d,k)$ is the depth of discharge degradation at time-step k of day d. We use a model similar to the Li-Ion battery model by Yoshida et al. (2016) to calculate $\gamma(d,k)$ as

$$\gamma(d,k) = \begin{cases} X\,\zeta(d,k) + Y & \zeta(d,k) \geq 0.7 \\ 0 & \text{otherwise} \end{cases}, \tag{6}$$

where $\zeta(d,k)$ is the depth of discharge at time-step k and is calculated as

$$\zeta(d,k) = \frac{c_b(d,k) - e(d,k)}{c_b(d,k)}.$$

In this paper, we consider one of the existing residential-sized batteries in the market as the battery used by the residential customer. As declared by the manufacturer, this battery degrades to 80% of its initial capacity $C_I = 10$ kWh after completing 5000 round-trip cycles. To capture the 20% drop in the capacity, we set the parameters α, X and Y in (4) and (6) to 1×10^{-4}, 5.8×10^{-6} and -4×10^{-6} respectively.

2.2 Problem Statement: Residential Battery Storage Optimization

We define our battery storage optimization problem as minimizing the monthly electricity bill subject to constraints on the battery's energy-level and charging/discharging rates. To define the electricity charges within a month, we divide each day into two off-peak periods from 12 AM to t_{on} and t_{off} to 12 AM. Let us denote the electricity price during the of-peak periods by p_{off} (\$/kWh). The on-peak period starts at t_{on} and ends at t_{off}. We denote the electricity price during this period by p_{on} (\$/kWh). In addition to the on-peak and off-peak charges, we assume that customers are charged proportional to their maximum rate of consumption during the on-peak hours in a month. We denote the proportionality ratio by p_d (\$/kW) and call it the *demand price*. Given the prices $p_{\text{off}}, p_{\text{on}}$ and p_d, the ToU charge in a month is calculated as

$$J_t(q, p_{\text{off}}, p_{\text{on}}) = \sum_{d=1}^{N=30} \left(p_{\text{off}} \sum_{k=0}^{t_{\text{on}}-1} q(d,k)\Delta t + p_{\text{on}} \sum_{k=t_{\text{on}}}^{t_{\text{off}}-1} q(d,k)\Delta t + p_{\text{off}} \sum_{k=t_{\text{off}}}^{24/\Delta t} q(d,k)\Delta t \right),$$

and the demand charge is calculated as

$$J_d(q, p_d) = p_d \sup_{\substack{k \in \{t_{\text{on}}, \cdots, t_{\text{off}}-1\} \\ d \in \{1, \cdots, 30\}}} q(d,k), \tag{7}$$

where $q(d,k)$ is the power supplied by the grid at time-step k of day d.

We now define the problem of optimal residential battery storage for a month as

$$
\begin{aligned}
&\min_{\substack{u(d,k),\beta(d,k)\in\mathbb{R}\\ e(d,k),n(d,k)\in\mathbb{R}}} \quad J_t(q,p_{\text{off}},p_{\text{on}}) + J_d(q,p_d) \qquad \text{subject to} \\
&q(d,k) = q_a(d,k) - q_s(d,k) + u(d,k) && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k \\
&e(d,k+1) = e(d,k) + \eta\, u(d,k)\Delta t && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k \\
&e(d,0) = e\left(d-1,\frac{24}{\Delta t}\right) + \eta\, u\left(d-1,\frac{24}{\Delta t}\right)\Delta t && \text{for } d\in\Phi_d \\
&0 \le e(d,k) \le (1-\beta(d,k))C_I && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k \\
&\beta(d,k) = \alpha\, n(d,k) + \sum_{k=0}^{T}\gamma(d,k) && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k \\
&n(d,k+1) = n(d,k) + \frac{\eta\,\Delta t\,|u(d,k)|}{2\,c_b(d,k)} && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k \\
&\underline{u} \le u(d,k) \le \bar{u} && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k \\
&e(0,0) = e_0,\ n(0,0) = 0, && (8)
\end{aligned}
$$

where we define $\Phi_d := \{1,\cdots,N\}$ and $\Phi_k := \{0,\cdots,24/\Delta t\}$. The second line of Problem (8) constrains the sum of the power, u, given to the battery and the power, q_a, consumed by the appliances to be equal to the total power supplied by the grid and the solar Photovoltaics. Lines 3 and 4 enforce the change in the battery's energy level to be proportional to the power given to or take from the battery. Line 5 constrains the energy level of the battery to stay below the battery's capacity. Lines 6 and 7 model the battery's degradation. Finally, Line 8 constrains the charging/discharging rate to stay within a pre-specified bound determined by the manufacturer.

3.SOLUTION METHODOLOGY: MULTI-OBJECTIVE DYNAMIC PROGRAMMING

The residential battery optimization defined in (8) is an instance of the general optimal control problem

$$
\begin{aligned}
J^\star(z) := \min_{u_k\in\mathbb{R}^m, x_k\in\mathbb{R}^n} \quad & J(x_0,u_0,\cdots,x_{N-1},u_{N-1},x_N) \\
\text{subject to} \quad & x_{k+1} = f(x_k,u_k), x_0 = z && \text{for } k=0,\cdots,N \\
& x_k \in X && \text{for } k=1,\cdots,N \\
& u_k \in U && \text{for } k=0,\cdots,N-1, \qquad (9)
\end{aligned}
$$

where $z\in\mathbb{R}^n$ is a given initial state, and $f:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^n$, $X\subset\mathbb{R}^n$ and $U\subset\mathbb{R}^m$ are known. It can be shown that if the objective function, J, of Problem (9) is *time-separable*, i.e., there exist maps $\psi_k:\mathbb{R}^n\times\mathbb{R}^m\times\mathbb{R}\to\mathbb{R}$ for $k=0,\cdots,N-1$ and $\psi_N:\mathbb{R}^n\to\mathbb{R}$ such that

$$
J(x_0,u_0,\cdots,x_{N-1},u_{N-1},x_N) = \psi_0\Bigg(x_0,u_0,\psi_1\bigg(x_1,u_1,\psi_2\Big(\cdots,\psi_{N-1}\big(x_{N-1},u_{N-1},\psi_N(x_N)\big)\Big)\bigg)\Bigg), \qquad (10)
$$

for all $u_k\in\mathbb{R}^m$, where $x_k = f(x_{k-1},u_{k-1})$ for $k=1,\cdots,N$, then one can apply dynamic programming to Problem (9) to find $J^\star(z)$ for any $z\in X$. Dynamic programming uses the principle of optimality (Bellman and Dreyfus (1962)) to reduce Problem (9) to a finite sequence of optimization problems indexed by the time-steps $k=0,\cdots,N$ (also known as Bellman's recursive formula):

$$
\begin{aligned}
&V_k(z) = \min_{v\in U}\big(g(z,v) + V_{k+1}(f(z,v))\big) \text{ for } z\in X \text{ and } k\in\Phi_k\backslash N \\
&V_N(z) = h(z) \qquad \text{for all } z\in X. && (11)
\end{aligned}
$$

It can be shown that $J(z) = V_0(z)$. Moreover, at each time-step k and state z, the minimizer v in (11) is an optimal control. Unfortunately, the objective function of Problem (8) is not time-separable. Specifically, the supremum function in J_d as defined in (7), can not be represented in the Form (10). Moreover, it can be shown that the objective function of Problem (8) violates the principle of optimality. As a result, dynamic programming will not necessarily converge to an optimal solution. In this section, we resolve this issue in two steps. First, we approximate the supremum function in J_d using the l_p-norm of $q(d,k)$, i.e.,

$$\sup_{\substack{k\in\{t_{\text{on}},\cdots,t_{\text{off}-1}\}\\ d\in\{1,\cdots,30\}}} q(d,k) \approx \sqrt[p]{\sum_{d=1}^{30}\sum_{k=t_{\text{on}}}^{t_{\text{off}}-1} q(d,k)^p}$$

for some large $p\in\mathbb{N}$. We then use Theorem 1 to define a multi-objective optimization problem whose set of *non-dominated* solutions contains an optimal solution to Problem (9) when $p\to\infty$. Before presenting the theorem, we define the set of non-dominated solutions as follows.

Definition 1 *Consider the multi-objective optimization problem*

$$\begin{aligned}
\min_{u_k\in\mathbb{R}^m, x_k\in\mathbb{R}^n}\quad & \left[D_1(x_0,u_0,\cdots,u_{N-1},x_{N-1},x_N),\cdots,D_M(x_0,u_0,\cdots,u_{N-1},x_{N-1},x_N)\right] && \\
\text{subject to}\quad & x_{k+1}=f(x_k,u_k), x_0=z && \text{for } k=0,\cdots,N\\
& x_k\in X && \text{for } k=1,\cdots,N\\
& u_k\in U && \text{for } k=0,\cdots,N-1,
\end{aligned}$$

for some $z\in\mathbb{R}^n, X\subset\mathbb{R}^n, U\subset\mathbb{R}^m$, where the maps $D_i:\mathbb{R}^n\times\mathbb{R}^m\times\cdots\times\mathbb{R}^n\times\mathbb{R}^m\times\mathbb{R}^n\to\mathbb{R}$ are known. A solution $(x_0,u_0^\star,\cdots,u_{N-1}^\star,x_{N-1}^\star,x_N^\star)$ to the multi-objective problem is called non-dominated if there exists no other feasible $(x_0,u_0,\cdots,u_{N-1},x_{N-1},x_N)$ such that $D_i(x_0,u_0,\cdots,u_{N-1},x_{N-1},x_N)\le D_i(x_0,u_0^\star,\cdots,u_{N-1}^\star,x_{N-1}^\star,x_N^\star)$ for $i=1,\cdots,M$, with strict inequality for at least one i.

We now present the main theorem.

Theorem 1 *Suppose there exist maps $f:\mathbb{R}^n\times\mathbb{R}^m$,*

$$G:\underbrace{\mathbb{R}\times\cdots\times\mathbb{R}}_{M\text{-times}}\to\mathbb{R}\quad\text{and}\quad D_i:\underbrace{(\mathbb{R}^n\times\mathbb{R}^m)\times\cdots\times(\mathbb{R}^n\times\mathbb{R}^m)}_{(N-1)\text{-times}}\times\mathbb{R}^n\to\mathbb{R}$$

for $i=1,\cdots,M$ such that:

I. *(**Time-separability of** D_i) For each $i\in\{1,\cdots,M\}$, there exist maps $\psi_{i,j}:\mathbb{R}^n\times\mathbb{R}^m\times\mathbb{R}\to\mathbb{R}$ for $j=1,\cdots,N-1$ and $\psi_{i,N}:\mathbb{R}^n\to\mathbb{R}$ such that*

$$D_i(x_0,u_0,\cdots,x_{N-1},u_{N-1},x_N)=\psi_{i,0}\Big(x_0,u_0,\psi_{i,1}\Big(x_1,u_1,\psi_{i,2}\Big(\cdots,\psi_{i,N-1}\big(x_{N-1},u_{N-1},\psi_{i,N}(x_N)\big)\Big)\Big)\Big)\quad(12)$$

for all $u_k\in\mathbb{R}^m$, where $x_k=f(x_{k-1},u_{k-1})$, $k=1,\cdots,N$;

II. *(**Backward monotonicity** D_i) For each $i\in\{1,\cdots,M\}$ and any $j\in\{1,\cdots,N\}$, if $\psi_{i,j}$ in (12) satisfies*

$$\begin{aligned}
&\psi_{i,j}\Big(x_j,u_j,\psi_{i,j+1}\Big(x_{j+1},u_{j+1},\psi_{i,j+2}\Big(\cdots,\psi_{i,N-1}\big(x_{N-1},u_{N-1},\psi_{i,N}(x_N)\big)\Big)\Big)\Big)\\
&\qquad> \psi_{i,j}\Big(x_j,v_j,\psi_{i,j+1}\Big(y_{j+1},v_{j+1},\psi_{i,j+2}\Big(\cdots,\psi_{i,N-1}\big(y_{N-1},v_{N-1},\psi_{i,N}(y_N)\big)\Big)\Big)\Big)
\end{aligned}$$

for all $x_j\in\mathbb{R}^n$ and for some $(u_j,\cdots,u_{N-1})$ and $(v_j,\cdots,v_{N-1})$, where $x_k=f(x_{k-1},u_{k-1})$ for $k=j+1,\cdots,N$ and $y_k=f(y_{k-1},v_{k-1})$ for $k=j+1,\cdots,N$ with $y_j=x_j$, then

$$\begin{aligned}
&\psi_{i,j-1}\Big(x_{j-1},u_{j-1},\psi_{i,j}\Big(x_j,u_j,\psi_{i,j+1}\Big(x_{j+1},u_{j+1},\psi_{i,j+2}\Big(\cdots,\psi_{i,N-1}\big(x_{N-1},u_{N-1},\psi_{i,N}(x_N)\big)\Big)\Big)\Big)\Big)\\
&\qquad> \psi_{i,j-1}\Big(x_{j-1},u_{j-1},\Big(x_j,v_j,\psi_{i,j+1}\Big(y_{j+1},v_{j+1},\psi_{i,j+2}\Big(\cdots,\psi_{i,N-1}\big(y_{N-1},v_{N-1},\psi_{i,N}(y_N)\big)\Big)\Big)\Big)\Big)
\end{aligned}$$

for all $x_{j-1}\in\mathbb{R}^n$, $u_{j-1}\in\mathbb{R}^m$;

*III. (**Monotonicity of** J) G, D_i and the objective function J in Problem* (9) *satisfy*

$$J(x_0, u_0, \cdots, x_{N-1}, u_{N-1}, x_N) = G\Big(D_1\big(x_0, u_0, \cdots, x_{N-1}, u_{N-1}, X_N\big), \cdots, D_M\big(x_0, u_0, \cdots, x_{N-1}, u_{N-1}, X_N\big)\Big), \tag{13}$$

$$\text{where } \frac{\partial G(D_1, \cdots, D_M)}{\partial D_i} > 0 \quad \text{for } i = 1, \cdots, M.$$

Then, an optimal solution to Problem (9) *is a non-dominated solution of the multi-objective problem*

$$\begin{array}{lll} \min\limits_{u_k \in \mathbb{R}^m, x_k \in \mathbb{R}^n} & \big[D_1(x_0, u_0, \cdots, u_{N-1}, x_{N-1}, x_N), \cdots, D_M(x_0, u_0, \cdots, u_{N-1}, x_{N-1}, x_N)\big] & \\ \textit{subject to} & x_{k+1} = f(x_k, u_k), x_0 = z & \textit{for } k = 0, \cdots, N \\ & x_k \in X & \textit{for } k = 1, \cdots, N \\ & u_k \in U & \textit{for } k = 0, \cdots, N-1. \end{array}$$

See Li and Haimes (1987) for a proof. To apply Theorem 1 to our residential battery optimization problem, defined in (8), let us start by defining the maps G and D_i in theorem 1 as

$$D_1\big(q(1,0), \cdots, q(30, 24/\Delta t)\big) := \sum_{d=1}^{30} \left(p_{\text{off}} \sum_{k=0}^{t_{\text{on}}-1} q(d,k)\Delta t + p_{\text{on}} \sum_{k=t_{\text{on}}}^{t_{\text{off}}-1} q(d,k)\Delta t + p_{\text{off}} \sum_{k=t_{\text{off}}}^{24/\Delta t} q(d,k)\Delta t \right), \tag{14}$$

$$D_2\big(q(1, t_{\text{on}}), \cdots, q(30, t_{\text{off}}-1)\big) := p_d^p \sum_{d=1}^{30} \sum_{k=t_{\text{on}}}^{t_{\text{off}}-1} q(d,k)^p \tag{15}$$

and

$$G(D_1, D_2) := D_1 + \sqrt[p]{D_2}, \tag{16}$$

where recall that $\sqrt[p]{D_2}$ is the l_p-norm approximation of the monthly demand charge, J_d, as defined in (7). Moreover, let us define $x := [q, e, n]$ and $v := [u, \beta]$, where recall that q is the power supplied by the utility company, e is the energy stored in the battery, n is the number of cycles, u is the power given to or taken from the battery and β is the battery's degradation rate. Then, Condition I of Theorem 1 holds if we define

$$\psi_{1,N}(x) := p_{\text{off}}\, \Delta t\, x_1,$$

$$\psi_{1,j}(x, u, \psi_{1,j+1}(y, v)) := \begin{cases} p_{\text{on}}\, \Delta t\, x_1 + \psi_{1,j+1}(y, v, \psi_{1,j+2}) & \text{if } j \in \Gamma \\ p_{\text{off}}\, \Delta t\, x_1 + \psi_{1,j+1}(y, v, \psi_{1,j+2}) & \text{if } j \in \{0, \cdots, N\} \backslash \Gamma \end{cases} \tag{17}$$

and

$$\psi_{2,j}(x, u, \psi_{1,j+1}(y, v)) := \begin{cases} p_d^p\, x_1^p + \psi_{2,j+1}(y, v, \psi_{2,j+2}) & \text{if } j \in \Gamma \\ 0 & \text{if } j \in \{0, \cdots, N\} \backslash \Gamma, \end{cases} \tag{18}$$

where $N = 30 \times 24/\Delta t$ and Γ is defined as $\Gamma := \bigcup_{i=1}^{30} \{i \cdot t_{\text{on}}, \cdots, i\,(t_{\text{off}} - 1)\}$. Condition II follows immediately by using $\psi_{i,j}$ as defined in (17) and (18). Finally, D_1, D_2 and G defined in (14), (15) and (16) satisfy Condition III:

$$\frac{\partial G(D_1, D_2)}{\partial D_1} = 1 > 0, \quad \frac{\partial G(D_1, D_2)}{\partial D_2} = \frac{1}{p} D_2^{\frac{1-p}{p}} > 0$$

if we choose p to be even and

$$\sup\big\{|q(1, \cdots, t_{\text{on}})|, \cdots, |q(30, t_{\text{off}}-1)|\big\} = \sup\big\{q(1, \cdots, t_{\text{on}}), \cdots, q(30, t_{\text{off}}-1)\big\} \geq 0.$$

This corresponds to the typical case where the consumption peak during the on-peak period is positive and is greater than or equal to the maximum power sent back to the grid during the on-peak period. Since all the conditions of Theorem 1 hold, the multi-objective optimization problem

$$
\begin{aligned}
&\min_{\substack{u(d,k),\beta(d,k)\in\mathbb{R}\\ e(d,k),n(d,k)\in\mathbb{R}}} \quad \left[D_1\Big(q(1,0),\cdots,q\Big(30,\frac{24}{\Delta t}\Big)\Big), D_2\Big(q(1,0),\cdots,q\Big(30,\frac{24}{\Delta t}\Big)\Big)\right]\\
&\text{subject to}\\
&q(d,k) = q_a(d,k) - q_s(d,k) + u(d,k) && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k\\
&e(d,k+1) = e(d,k) + \eta\, u(d,k)\Delta t && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k\\
&e(d,0) = e\Big(d-1,\frac{24}{\Delta t}\Big) + \eta\, u\Big(d-1,\frac{24}{\Delta t}\Big)\Delta t && \text{for } d\in\Phi_d\\
&0 \le e(d,k) \le (1-\beta(d,k))C_I && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k\\
&\beta(d,k) = \alpha\, n(d,k) + \sum_{k=0}^{T}\gamma(d,k) && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k\\
&n(d,k+1) = n(d,k) + \frac{\eta\,\Delta t\,|u(d,k)|}{2\,c_b(d,k)} && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k\\
&\underline{u} \le u(d,k) \le \bar{u} && \text{for } d\in\Phi_d \text{ and } k\in\Phi_k\\
&e(0,0) = e_0,\; n(0,0) = 0, && (19)
\end{aligned}
$$

yields an optimal solution to our residential battery optimization problem defined in (8). Various algorithms (e.g., ε-*constraint* by Miettinen (2012) and *Envelope* by Li and Haimes (1987)) can be applied to Problem (19) to compute the set of non-dominated solutions. In this paper, we choose to use the *linear scalarization* approach in Boyd and Vandenberghe (2004). In this approach, the set of non-dominated solutions to Problem (19) is the parameterized set of solutions to

$$
\min \;\; D_1\big(q(1,0),\cdots,q(30,24/\Delta t)\big) + \lambda\, D_2\big(q(1,0),\cdots,q(30,24/\Delta t)\big) \tag{20}
$$

subject to the constraints of Problem (19), where λ is a positive parameter. Fortunately, the objective function of Problem (20) is time-separable - implying that applying dynamic programming for each $\lambda > 0$ yields a non-dominated solution to the multi-objective Problem (19).

4.NUMERICAL CASE STUDIES

In this section, we apply our multi-objective approach, described in Section 3, to our residential battery optimization problem, defined in Section 2, using a wide range of load profiles and two pricing plans. The results are optimal charging/discharging rates as a functions of time, minimum electricity bills, and the amounts which the customers save from optimal energy storage in each scenario.

We consider three load profiles q_{a1}, q_{a2} and q_{a3} (corresponding to small, medium and large houses) with 5.66 kW, 8.09 kW and 12.14 kW as their maximum values over the summer and 3.89 kW, 5.55 kW and 8.32 kW as their maximum values over the winter. These load profiles were synthesized based on the measured load of a group of Salt River Project's (SRP) retail customers in Arizona. As for the electricity pricing, we use SRP's E-22 plan and a modified version of E-27 P plan(SRP (2015)). The first plan only includes ToU charges, whereas the second plan charges include both ToU and demand charges. The on-peak period for E-22 plan consist of those hours from 4 PM to 7 PM. The on-peak period of plan E-27 P is from 1 PM to 8 PM for the months May to October. From November to April, the on-peak period of plan E-27 P consist of those hours from 5 AM to 9 AM and from 5 PM to 9 PM. The electricity prices for all the seasons are presented in Table 1. To synthesize a solar generation profile for each day, we created a random generator which uses a time-varying mean and standard deviation of solar generation for a typical summer day and a time-varying mean and standard deviation for a typical winter day. The means and standard deviations for summer and winter were calculated based on the solar radiation data measured at Equestrian Manor station in Scottsdale, AZ for the entire months July and January. In our scenarios, we consider Tesla's Powerwall battery as the source of energy storage. The two existing models of Powerwall battery possess C_I =7 kWh and C_I =10 kWh as storage capacities, $\bar{u}$ = 3.3 kW as the maximum charging/discharging rate and η = 0.92 as the charging/discharging efficiency.

Table 1: SRP's pricing plans (SRP (2015)): E-22 (ToU) and modified E-27 P (ToU & demand). Summer: May, June, Sep. & Oct. Summer peak: July & Aug. Winter: Nov. through April.

Pricing plan	Off-peak price ($/kWh)	On-peak price ($/kWh)	Demand price ($/kW)
E-22 (Summer)	0.0840	0.3033	0
E-22 (Summer peak)	0.0864	0.3588	0
E-22 (Winter)	0.0758	0.1215	0
E-27 P (Summer)	0.0371	0.0486	14.63
E-27 P (Summer peak)	0.0423	0.0633	17.82
E-27 P (Winter)	0.0390	0.0430	5.68

4.1 Case Study I: Analysis of The Benefits of Optimal Energy Storage Over a Month

We applied our multi-objective method to the battery optimization problem defined in (8), except that we reduced the 30-day period ($N = 30$) in Problem (8) to 3 Summer peak days in order to simplify the presentation of results. Accordingly, we scaled the demand price in Table 1 by a factor of $\frac{1}{10}$, i.e., $p_d = \frac{1}{10} \times 17.82$ \$/kW. In Table 2, we have presented the resulting ToU and demand charges for 3 customers with loads q_{a1}, q_{a2} and q_{a3}, and for pricing plans E-22 and modified E-27 P. Assuming that the three-day simulation is repeated ten times, we calculated the monthly bill by multiplying the sum of 3-day charges by ten. For comparison, we have also included the same results for the case where no battery storage is used. For all the cases associated with modified E-27 P pricing, we solved Problem (20) for 500 values of the weighting parameter λ in (20), using time-step $\Delta t = 0.5$ hour and l_{20}-norm approximation for the supremum function in (7). The resulting non-dominated solutions corresponding to the load q_{a2} are shown in Fig. 2. A Matlab implementation of our method calculated the 500 non-dominated solutions in 14.11 minutes using a Core i7 machine with 16 GB of RAM. From Table 2 we observe that in the presence of demand charges (as in modified E-27 P pricing), the customer with the largest house (heavy load) achieves the greatest saving from optimal energy storage, whereas in the absence of demand charges (E-22 pricing), savings from optimal storage are independent of load. In Fig. 3, we have shown the resulting optimal charging/discharging rates and optimal energy storage corresponding to the load q_{a2} for plans E-27 P and E-22 respectively. Fig. 3 (Left) shows that in the presence of demand charges, optimal strategy for battery storage involves precise peak shaving (power going into the battery decreases in response to an increase in appliances load) during the on-peak hours. In the absence of demand charges, optimal strategy is straightforward: Apply maximum discharging rate during the on-peak hours (see Fig. 3 (Right)).

4.2 Case Study II: Long-term Analysis of The Benefits of Optimal Energy Storage

To quantify the long-term benefits of optimal energy storage for residential customers, and assessing battery's degradation under optimal operation, we applied our method to Problem (8) for the duration of 5 years. To reduce the computation, we used the appliances load of 3 representative days for each month, assuming that the same load is repeated 10 times during the month. We accounted for the battery's degradation during the entire month by multiplying the degradation rate β (defined in (4)) by a factor of 10. The resulting total bills (for 5 years) corresponding to using q_{a1}, q_{a2} and q_{a3} as the load profiles and modified E-27 P as the pricing plan are shown in Table 3. For comparison, in Table 3, we have also included the bills corresponding to no battery storage. Moreover, we have included the cost of operating the battery for 5 years as $(C_I - C_F)/C_I \times \$3750$ considering that \$3750 is the market price of Tesla's Powerwall battery with $C_I = 10$ kWh of capacity. C_F denotes

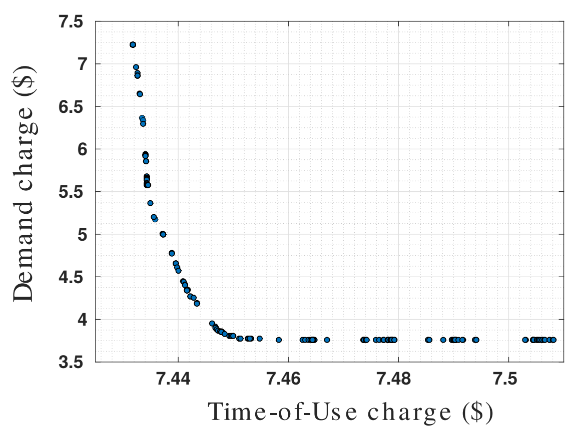

Figure 2: Non-dominated solutions for the multi-objective battery storage optimization problem using medium load profile q_{a2}

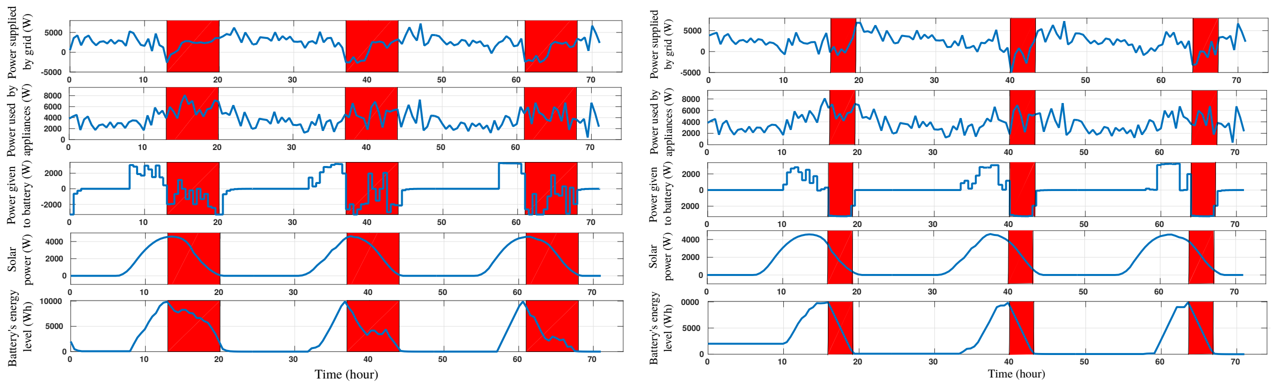

Figure 3: Optimal charging/discharging rates and energy storage during three Summer peak days for q_{2a} appliances load and E-27 P (Left), E-22 (Right) pricing plans

Table 2: ToU & demand charges for 3 days using Summer peak plans E-27 P (top) and E-22 (bottom), and light, medium & heavy loads: q_{a1}, q_{a2} & q_{a3}. (Left: with battery - Right: no battery)

Load profile	ToU charge($)	Demand charge($)	Monthly bill ($)	Monthly saving ($)
q_{a1} (light)	3.30 - 4.03	1.37 - 5.86	46.70 - 98.95	52.25
q_{a2} (medium)	7.45 - 8.20	3.78 - 9.49	112.30 - 176.91	64.61
q_{a3} (heavy)	14.39 - 15.15	9.95 - 16.38	243.43 - 315.04	71.61

Load profile	ToU charge($)	Demand charge($)	Monthly bill ($)	Monthly saving ($)
q_{a1} (light)	2.38 - 11.26	0	23.88 - 112.64	88.76
q_{a2} (medium)	13.59 - 22.47	0	135.94 - 224.71	88.77
q_{a3} (heavy)	32.27 - 41.14	0	322.71 - 411.47	88.76

the remaining capacity after 5 years. We have also calculated the total saving from the battery (see Table 3) as $B_b - (C_b + B_n)$, where B_b denotes the electricity bill when optimal energy storage is used, B_n denotes the electricity bill when no energy storage is used, and C_b denotes the cost of operating the battery. From the table we observe that similar to our analysis in Section 4.1, the customer with the highest load (q_{a3}) achieves the greatest benefit from optimal energy storage. Specifically, the customer with load q_{a3} gains 21% and 32% greater benefits relative to the customers with q_{a2} and q_{a1} respectively.

5.CONCLUSIONS

We addressed the problem of optimal energy storage for residential customers who are charged for both total energy consumed and peak rate of consumption (demand) during a month. Because of the presence of demand charges, this problem violates the principle of optimality - a sufficient condition for Hamiltonian-based algorithms and dynamic programming to converge to an optimal solution. We approached this problem by defining a sequence of multi-objective dynamic programs, indexed by p, whose sets of non-dominated solutions are guaranteed to contain an optimal solution to the original problem as $p \to \infty$. Since the principle of optimality holds for each multi-objective problem in the sequence, standard policy iteration and linear scalarization can be used to find sub-optimal non-dominated solutions for each p. Our numerical case studies over a wide range of customers' load profiles show that optimal energy storage using the existing residential batteries and solar generation can reduce the monthly electricity bill by up to 52%, in the presence of demand charges. Moreover, our long-term numerical analysis over the load range, 5.6 kW to 12.1 kW, shows that in the presence of demand charges and battery degradation, a customer with twice the peak load gains 32% higher benefit from optimal energy storage over 5 years.

Table 3: Total bill (for 5 years), final capacity of battery, cost to battery & saving from storage using light, medium and heavy load: q_{a1}, q_{a2} & q_{a3}. (Left: with battery - Right: no battery)

Load profile	Total bill ($)	Final Capacity (kWh)	Cost to Battery ($)	Total saving ($)
q_{a1} (light)	1599.13 - 2352.73	9186.1	305.21	448.39
q_{a2} (medium)	3493.93 - 4318.25	9098.3	338.13	486.19
q_{a3} (heavy)	6730.27 - 7657.36	9103.2	336.37	590.72

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