2016

# Analysis of Dynamic Stability of Ejector Expansion Refrigeration System

Jianqiang Deng
*School of Chemical Engineering and Technology, Xi'an Jiaotong University, China.*, dengjq@mail.xjtu.edu.cn

Lixing Zheng
*School of Chemical Engineering and Technology, Xi'an Jiaotong University, China.*, zhenglixing@stu.xjtu.edu.cn

Fei Wang
*School of Chemical Engineering and Technology, Xi'an Jiaotong University, China.*, wangfei830@stu.xjtu.edu.cn

# Analysis of dynamic stability of ejector expansion refrigeration system

Jianqiang Deng[1]*, Lixing Zheng[2], Fei Wang[3]

[1] School of Chemical Engineering and Technology, Xi′an Jiaotong University,
Xi′an, Shaanxi, P.R. China
Tel.: +86-029-82663413. Fax: +86-029-82663413. Email: dengjq@mail.xjtu.edu.cn
[2] School of Chemical Engineering and Technology, Xi′an Jiaotong University,
Xi′an, Shaanxi, P.R. China
Tel.: +86-029-82663413. Fax: +86-029-82663413. Email: zhenglixing@stu.xjtu.edu.cn
[3] School of Chemical Engineering and Technology, Xi′an Jiaotong University,
Xi′an, Shaanxi, P.R. China
Tel.: +86-029-82663413. Fax: +86-029-82663413. Email: wangfei830@stu.xjtu.edu.cn

* Corresponding Author

## ABSTRACT

The stable operation of a system is restricted by the intrinsic characteristics, namely harmonious matching among and between structure parameters and operation parameters. In this paper, we put forward an analysis method of the dynamic stability of the refrigeration system based on the First Approximation Theory of Lyapunov Stability Theorem and the evaluation of stability margin. It is carried out by linearizing the governing equations and analyzing the eigenvalue form of coefficient matrix. And the minimum logarithmic decrement is calculated to represent the stability margin. Analyzing the stability of gas cooler confirms the consistency between the mathematical stability and the actual dynamic one. The proposed method is performed in a transcritical $CO_2$ ejector expansion refrigeration system (EERS) to analyze the system dynamic stability. The present results show that, even each component of the system is in the stable state, it cannot guarantee the dynamic stability of the whole system. Moreover, the effect of state parameter on system dynamic stability is investigated. The work supplies a guiding principle on system control and may be extended in more general thermodynamic cycle.

## 1. INTRODUCTION

Stability is one of the basic conditions for the system operation, and none of systems can work normally and reliably under unstable conditions. The characteristics of refrigeration system are highly nonlinear and hysteretic, and its running progress is susceptible to external factors. Instability of the system will result in some issues such as reducing safety factor, shortening lifespan and increasing energy consumption. Therefore, stable operation is the prerequisite for a safe and efficient refrigeration system.

The instability of refrigeration system is mainly embodied in the form of state parameters oscillatory motion and the unstable, the nonequilibrium flow within the system. Liang *et al.* (2010) have summarized the instabilities phenomenon of refrigeration system into two parts: the two-phase flow instability and the control characteristic instability in the system. The two-phase flow instability in the refrigeration system represented the oscillatory motion of mixture-vapor transition point and the instability of throttling device. The control characteristic instability referred to the aspect of control algorithm, which was commonly discussed through the static analysis, frequency domain analysis and dynamic simulation. Tian *et al.* (2002) divided the stability of refrigeration system into three categories: oscillatory motion of mixture-vapor transition point in evaporator, control loop stability in evaporator and thermostatic expansion valve and fixed capacity refrigeration system stability. On the basis of analyzing and correcting the minimum stable superheat line, Chen *et al.* (2008) discussed experimentally the influence of system intrinsic properties on operation stability in variable speed compressor refrigeration system. Lin *et al* .(2012)

adopted the CFD (Computational Fluid Dynamics) technology and combined with more than 200 different cases to study the influences of cooling load changes on the ejector pressure recovery performance in multistage evaporates ejector refrigeration system. The results showed that pressure recovery rate was relatively sensitive to the changes in cooling load of the ejector primary flow and the ejector suction flow. Deng *et al.* (2007) analyzed theoretically the working characteristic of transcritical $CO_2$ ejector expansion refrigeration system (EERS). The EERS has a special equilibrium stability relationship relative to the traditional vapor compression system (VCS), and the system performance and stability were more sensitive to the operating parameter. Xu *et al.* (2010) experimentally studied the system stability of transcritical $CO_2$ EERS. They noted that when the system changed from one steady state to another steady state, it was in an unstable state during the transition period. And the EERS became unstable if the throttle valve opening approached the limit.

However, the present studies mainly focus on the inherent characteristic of two-phase fluid and advanced control strategy, and it is lack of the research on the dynamic stability to evaluate the matching degree among and between structure parameters and operation parameters. Although it is feasible to analyze the stability of system by the transient simulation, the matching of system parameters is very complex and it needs spend a lot of time to calculate and simulate the whole system. By contrast, Lyapunov stability theory can predict the dynamic stability according to the present conditions, without solving the differential equations. The theory mainly involves the *V* Function Method and the First Approximation Theorem. (Ma and Zhou, 2001).

Oliveira *et al.* (2011) developed a multiple Lyapunov *V* function to design the control rules of a switch in vapor compression refrigeration system and ensured the expected steady-state condition in the asymptotic stability of zero solution. The results showed that the switch controller could quickly drive system to the reference point and satisfactorily resist external perturbations. Rasmussen and Larsen (2011) proposed a novel control method of the refrigeration system. The superheat was controlled by the compressor speed, and the cooling capacity was dominated by the refrigerant flow rate. The system stability under above control method have been theoretically analyzed and verified by the Lyapunov equation. Cai and Mijanovic (2012) also employed multiple Lyapunov function to analyze the stability of linear hybrid system, and deduced the linear matrix inequalities to validate the stability of industrial refrigeration. However, the actual engineering system is very complex and it is very difficult to construct higher-order *V* function which always requires sufficient experience, resulting in the development of *V* function method is restricted in the engineering field. For the Lyapunov First Approximation Theorem, the system stability is researched through analyzing the structure of eigenvalues of the linear differential equations. Every eigenvalue $\lambda$ and corresponding eigenvector $x$ define an inherent system mode. The eigenvalue $\lambda$ reflects the response speed of each mode, and the greater its absolute value, the faster attenuation rate or divergence rate after the disturbance. Conversely, the smaller its absolute value, the slower attenuation rate or divergence rate.

Qi and Deng (2008) explored the feasibility of adopting dynamic linear control model to design multi-variable control strategy of a direct expansion air conditioning (DX A/C) system. Through utilizing the mass conservation and the energy conservation, the governing equations were established and linearized near the steady state point to obtain the linear governing equations. Although they found that all eigenvalues of the linear model were negative and that meant the DX A/C system was asymptotically stability, they have not further studied in a theoretical and systematic way. Rasmussen *et al.* (2005) introduced the eigenvalue analysis method of the transcritical $CO_2$ vapor compression refrigeration system. The results showed that the components and the system have various time scale behaviors. In addition, they compared eigenvalues of linear reduced order model with that of full-order model of the system. The reduced model had smaller calculation error compared to the full-order model, and could be used in the system analysis, control strategy design and so on. Alleyne and Rasmussen (2007) calculated the eigenvalues and pointed out that the mode corresponding to the larger eigenvalue was relative to the dominant mode of the system. Thereby, the differential equation could be replaced by the homologous steady state equation, which was in favor of the system order reduction. Their study paid attention to the reduction of system model order and the development of control and diagnostic approaches, no discussion of the system dynamic stability.

In this study, we combine the First Approximation Theorem of Lyapunov Stability Theorem and the calculation of the stability margin to propose a method of dynamic stability to evaluate the matching degree in a system. The analysis method is introduced in detail. The stability of a gas cooler is discussed to explore the consistency between the mathematical method and its own stability attribute. Furthermore, the transcritical $CO_2$ EERS is taken as another example to analyze the effects of the operation conditions on system stability. This method is helpful to improve the dynamic stability and enhance the anti-disturbance performance of the system

## 2. ANALYSIS METHOD OF THE DYNAMIC STABILITY

Herein, we divide the instability of a refrigeration system into two types: the intrinsic instability and the control characteristic instability. The stable operation of a system is restricted by the intrinsic characteristic, namely harmonious matching among and between structure parameters and operation parameters. The analysis method of dynamic stability devotes to predicting intrinsic stability performance and obtaining the stable structure and operation parameters. The First Approximation theorem of Lyapunov stability discusses the structure of eigenvalues of the linear equations, which obtains from the linearization governing equations of thermodynamic developed based on the three conservation laws (continuity, momentum and energy conservation), as shown in Equations (1)–(3).

$$\frac{\partial \rho}{\partial t} = \frac{\dot{m}_{in} - \dot{m}_{ou}}{A\Delta z} \tag{1}$$

$$\frac{\partial \dot{m}}{\partial t} = \frac{P_{in}A_{in} - P_{ou}A_{ou}}{\Delta z} + \frac{\dot{m}_{in}u_{in} - \dot{m}_{ou}u_{ou}}{\Delta z} \tag{2}$$

$$\frac{d(\dot{m}u)}{dt} = \dot{m}_{in}h_{in} - \dot{m}_{ou}h_{ou} \tag{3}$$

On the basis of the relationship among the physical state parameters, the nonlinear mathematical matrix expression of the system is deduced: $Z \cdot \dot{x} = f(x,u)$, where $x$ is the state vector, $u$ is the input vector. The right side of the equation is dealt with by the Taylor series expansion near the steady state solution, and then eliminating the higher order terms obtain the linear differential governing equation, which is: $\dot{x} = A \cdot \delta x + B \cdot \delta u$, where $A=Z^{-1}f_x$.

If all eigenvalues $\lambda$ of the coefficient matrix $A$ are negative, the system is in asymptotic stability of zero solution. It means the state parameter can restore the initial stable condition over time after a disturbance. If all eigenvalues $\lambda$ of the coefficient matrix $A$ are non-positive and there is only one zero eigenvalue, the system is in zero solution stability. In this condition, the state parameter may achieve stability in the limited neighborhood from the initial stable point. The rest is zero solution instability. Under the zero instability, the state parameter may unrestrictedly diverge from the initial steady state until the system instability (Ma and Zhou, 2001).

Moreover, in order to improve the capability of resisting disturbance, it is very necessary to analyze the stability margin. The logarithmic decrement is employed to characterize the stability margin (Wen *et al.*, 2000). Assuming the *j* order eigenvalue is $\delta_j=2\pi\tau_j/\omega_j$, the corresponding logarithmic decrement is $\lambda_j=-\tau_j+i\omega_j$, and the stability margin of the system is represented by the minimum logarithmic decrement, $\delta_{\min}=\min(\delta_j)$. The bigger the minimum logarithmic decrement is, the greater the stability margin gets and the larger the bearable perturbation range obtains.

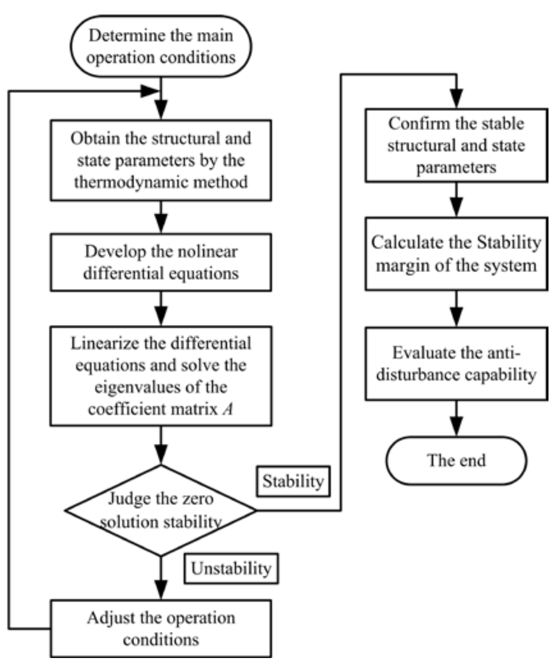


**Figure 1:** Flowchart of dynamic stability analysis method

The specific process of stability analysis method is: firstly, the main operation conditions are determined and then the steady state parameters and structural parameters are obtained by the thermodynamic method. Secondly, developing the nonlinear differential equations of system and then linearize the governing equations. The zero solution stability of the system is analyzed with the steady state parameters. If the system is zero solution instability, it means that the state parameter may unrestricted diverge from the initial stable point and the system operation needs to be adjusted. For the stability solution, it means the stable state and structure parameters are obtained. Thirdly, the stability margins are calculated under the stable state conditions, and then evaluate the anti-disturbance capability. The flowchart of dynamic stability analysis method is showed in Figure 1.

Herein, we extend the Lyapunov First Approximation Theorem and stability margin calculation to the stability analysis of the refrigeration system, rather than focusing on the dynamic performance and control and diagnostic approach like the literatures by Allenyne and Rasmussen (2007). What we are concerned about is whether the eigenvalues are all negative, whether they have positive values, and how many zero values when the eigenvalues are non-positive. In the study of Allenyne and Rasmussen (2007), eigenvalue calculations are used to reduce system model order. What they are interested in is the size of eigenvalue. The larger of the magnitude of eigenvalue is, the faster of the response of its corresponding dynamic mode will be, which will be smaller loss for model reduction. Using the dynamic stability analysis we can predict the motion tends of the thermodynamic system. Compared to the conventional dynamic performance simulation and experimental study, this method is a special theoretical research on system stability with the advantages of time saving, simple and feasible as well as wider applied range.

## 3. CASE STUDY

### 3.1 Gas cooler

Heat exchanger plays an important role in the thermodynamic system. This paper takes the gas cooler for example to discuss the stability (as shown in Figure 2). The governing ordinary differential equations (ODEs) of gas cooler are obtained by integrating the governing partial differential equations (PDEs) of the conservation of refrigerant mass and energy as well as the wall energy (expressed by Equation (4)–(6)), and the results are given in Equation (7), which is the form of $Z \cdot \dot{x} = f(x,u)$, with state vector $x = \left[P_{gc},\ h_{gc},\ T_{gc,w}\right]^T$.

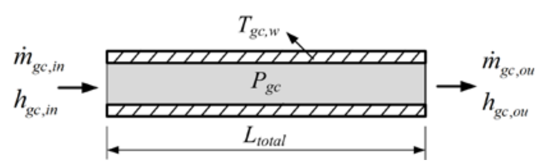


**Figure 2:** Physical model of gas cooler

$$\frac{\partial(\rho_{gc}A_{cs})}{\partial t} + \frac{\partial(\dot{m}_{gc})}{\partial z} = 0 \tag{4}$$

$$\frac{\partial(A_{cs}\rho_{gc}h_{gc} - A_{cs}P_{gc})}{\partial t} + \frac{\partial(\dot{m}_{gc}h_{gc})}{\partial z} = \alpha_i A_i(T_{gc,w} - T_{gc,r}) \tag{5}$$

$$\left(C_p\rho A\right)_{gc,w}\frac{\partial T_{gc,w}}{\partial t} = \alpha_i A_i(T_{gc,r} - T_{gc,w}) + \alpha_o A_o(T_{gc,wa} - T_{gc,w}) \tag{6}$$

$$\begin{bmatrix} z_{11} & z_{12} & 0 \\ z_{21} & z_{22} & 0 \\ 0 & 0 & z_{33} \end{bmatrix}\begin{bmatrix} \dot{P}_{gc} \\ \dot{h}_{gc} \\ \dot{T}_{gc,w} \end{bmatrix} = \begin{bmatrix} \dot{m}_{gc,in} - \dot{m}_{gc,ou} \\ \dot{m}_{gc,in}h_{gc,in} - \dot{m}_{gc,ou}h_{gc,ou} + \alpha_{gc,i}A_{gc,i}(T_{gc,w} - T_{gc,r}) \\ \alpha_{gc,i}A_{gc,i}\left(T_{gc,r} - T_{gc,w}\right) - \alpha_{gc,o}A_{gc,o}\left(T_{gc,w} - T_{gc,wa}\right) \end{bmatrix} \tag{7}$$

Then the right side of the Equation (7) is dealt with by Taylor series expansion and its first order partial derivative $f_x$ is obtained omitting high order terms. Finally, the coefficient matrix $A$ ($A=Z^{-1}f_x$) of linear differential equations is found and expressed as Equation (8). Because the inlet and outlet mass flow rate depend on the upstream and downstream components, the derivatives of the flow rate change are zero, resulting in zero of the second line of matrix $A$. The expression of each element in matrix $Z$ and $f_x$ are described in the literature (Rasmussen, 2002).

$$A=\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ 0 & 0 & 0 \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \tag{8}$$

Observing the structure of the coefficient matrix $A$, it is easy to find that there is a zero eigenvalue, $\lambda_1$=0. According to the relationship between eigenvalues and matrix elements, the remaining eigenvalue could be described in Equations (9) and (10).

$$\lambda_2\cdot\lambda_3=\frac{1}{m_{gc}(C_{p,gc}\rho V)_{gc,w}}\left[\begin{array}{l} 2(\alpha_i A_i)_{gc}\left(\left.\dfrac{\partial h_{gc}}{\partial u_{gc}}\right|_{\rho_{gc}}\right)+2(\alpha_o A_o)_{gc}\left(\left.\dfrac{\partial h_{gc}}{\partial u_{gc}}\right|_{\rho_{gc}}\right)\left(1-\dfrac{\partial T_{gc,wa}}{\partial T_{gc,w}}\right)+ \\ (\alpha_i A_i \alpha_o A_o)_{gc}\left(\left.\dfrac{\partial T_{gc,r}}{\partial u_{gc}}\right|_{\rho_{gc}}\right)\left(1-\dfrac{\partial T_{gc,wa}}{\partial T_{gc,w}}\right) \end{array}\right] \tag{9}$$

$$\lambda_2+\lambda_3=-\left[\begin{array}{l} \dfrac{2\dot{m}_{gc,ou}}{m_{gc}}\left(\left.\dfrac{\partial h_{gc}}{\partial u_{gc}}\right|_{\rho_{gc}}\right)+\dfrac{(\alpha_i A_i)_{gc}}{m_{gc}}\left(\left.\dfrac{\partial T_r}{\partial u_{gc}}\right|_{\rho_{gc}}\right)+\dfrac{(\alpha_i A_i)_{gc}}{(C_{p,gc}\rho V)_{gc,w}}+ \\ \dfrac{(\alpha_o A_o)_{gc}}{(C_{p,gc}\rho V)_{gc,w}}\left(1-\dfrac{\partial T_{gc,wa}}{\partial T_{gc,w}}\right) \end{array}\right] \tag{10}$$

According to the inherent link among the thermodynamic properties, each term in Equation (9) and Equation (10) are greater than zero. Thus the two other eigenvalues satisfy: $\lambda_2\cdot\lambda_3 > 0$, $\lambda_2+\lambda_3 < 0$, which can be deduced to $\lambda_2 < 0$, $\lambda_3 < 0$. Thereby, the result infers that the gas cooler is in zero solution stability after a small perturbation. And this result also shows the consistency between the mathematical stability and the actual stability. In actual thermal process of the gas cooler, when the input parameters are changed because of the small disturbance, there are variations in the output parameters, and finally it will achieve stability. From the above analysis of eigenvalues, it shows that the Lyapunov First Approximation Theorem can obtain consistent results with the actual performance of gas cooler.

### 3.2 Transcritical $CO_2$ EERS

Figure 3 shows the schematic diagram of transcritical $CO_2$ EERS, which composes by two sub-cycles. One is the high pressure cycle includes compressor, gas cooler, ejector and separator. The other one is the low pressure cycle includes the separator, expansions valve, evaporator and ejector. Compared the conventional vapor compression cycle, the modeling of transcritical $CO_2$ EERS needs to integrate the ejector into the system and consider the coupling between two sub-cycles and that within the sub-cycle.

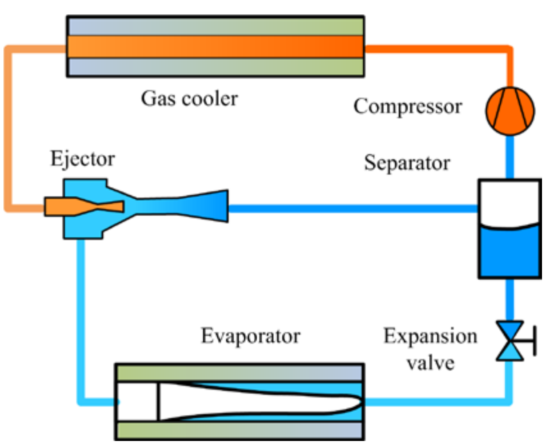


**Figure 3:** Schematic diagram of transcritical $CO_2$ ejector expansion refrigeration system (EERS)

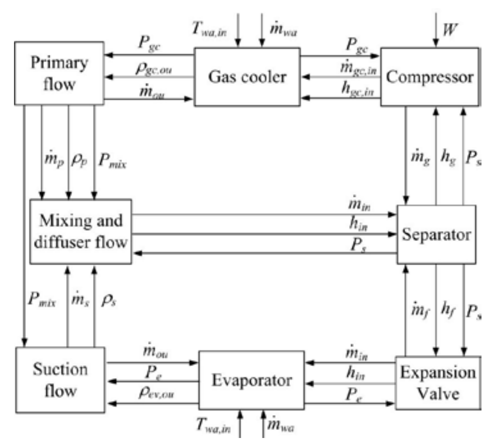


**Figure 4:** Coupling relationship diagram for physical parameters of each components in EERS

In the derivation of nonlinear governing equation, the gas cooler model and the evaporator model are developed based on the widely adopted moving boundary approach, which can capture the dynamics of two-phase fluid and predict the effective position of phase change as well as provide accurate transient predictions against experimental data. The effect of the two-phase flow instability has been included in the stability analysis of evaporator. The heat transfer coefficients of supercritical $CO_2$ and two-phase $CO_2$ as well as water are referred to in the literatures (Hwang, 1997; Dang and Hihara, 2004; Sarkar *et al.*, 2006). The dynamic model of separator is developed based on the continuity equation and the energy conservation equation. The derivation process can be seen in the literature by Rasmussen (2002) and Zheng *et al.* (2015). The compressor and expansion valve are modeled using the lumped parameter method referred to the literatures by Sarkar *et al.* (2006) and Ma *et al.* (2005), respectively. The ejector model is developed referring to Nehdi *et al.* (2007) with the assumption that the primary flows occurs chocked while the suction flow is unimpeded. The coupling relationship among the physical parameters of each component is showed in Figure 4. FORTRAN language is used for coding programs and the data for the properties of $CO_2$ are obtained from REFPROP 7.0 (McLinden, *et al.*, 2002).

The dynamic equations of the system are derived through combining the governing equations of evaporator, gas cooler, separator and ejector. After linearization, the linear differential equation of the system can be obtained, and the coefficient matrix *A* is also acquired. The state parameter *x* of system is shown in Equation (11), where the subscript 1 and 2 represents the two phase region and the superheat region of the evaporator, respectively.

$$x = \left[ L_{e1},\, P_e,\, h_{e,ou},\, T_{e1,w},\, T_{e2,w},\, P_{gc},\, h_{gc},\, T_{gc,w},\, P_{se},\, m_{se},\, \rho_{po},\, \dot{m}_{po},\, \rho_{so},\, \dot{m}_{so},\, \rho_d,\, \dot{m}_d \right]^T \tag{11}$$

The initial operation parameters of the EERS are showed in Table 1. Then the eigenvalues of evaporator $\lambda_1$, gas cooler $\lambda_2$, separator $\lambda_3$, ejector $\lambda_4$ and the whole system $\lambda_5$ are calculated based on the steady state parameters, expressed as Equations (12)-(16), respectively.

**Table 1:** The initial operation parameters of EERS

| | Pressure<br>$P$/MPa | Refrigerant outlet temperature<br>$T$/°C | Inlet water temperature<br>$T$/°C | Outlet water temperature<br>$T$/°C | Ejector Area ratio<br>$\phi$ |
|---|---|---|---|---|---|
| Gas cooler | 10.5 | 36 | 25 | 55 | — |
| Evaporator | 3.9 | 9.3 | 20 | 10 | — |
| Ejector | — | — | — | — | 10 |

$$\lambda_1 = \{-21.82,\ -17.18,\ -2.55,\ -0.60,\ 0\} \tag{12}$$

$$\lambda_2 = \{-0.61,\ -0.0042,\ -0.00041\} \tag{13}$$

$$\lambda_3 = \{-1090.56,\ 0\} \tag{14}$$

$$\lambda_4 = \{-5113.77 \pm 14848.09\mathrm{j},\ -1457.31 \pm 4371.96\mathrm{j},\ -1647.41,\ 0\} \tag{15}$$

$$\lambda_5 = \begin{Bmatrix} -5133.76 \pm 14878.09\mathrm{j},\ -1511.85 \pm 4596.42\mathrm{j},\ -552.95 \pm 2880.39\mathrm{j},\ -1544.41, \\ -17.18,\ -2.48,\ -0.90,\ -0.61,\ -0.16,\ 0.0079,\ -0.0016,\ 0,\ 0 \end{Bmatrix} \tag{16}$$

From $\lambda_1$, $\lambda_3$ and $\lambda_4$, it can be seen that the eigenvalues of evaporator and separator as well as ejector include only one zero, and the remaining real number or real component of plural are all negative, so they are zero solution stability. Due to the eigenvalue $\lambda_2$ all negative, the gas cooler is in the asymptotically stability of zero solution. In contrast with the result of gas cooler in section 3.1, the reason for discordance between this two calculation results is that the primary flow of ejector is choked in the EERS. When the ejector primary flow does not appear chocked, the gas cooler is zero solution stability.

It can be found from the system eigenvalue $\lambda_5$, there are two zero value and a positive value. According to the Lyapunov First Approximation Theorem, if more than one zero eigenvalue or positive value, it means the system instability. Thus the results show that the eigenvalues between the system and the components presents the different dynamic characteristics. It also means that the stability of individual components cannot guarantee the stability of the whole system. There are some risks in determining operation parameters if only according to the thermodynamic common sense, which sometimes will lead a system to dynamic instability.

On the basis of Table 1, the effects of some parameters on the system dynamic stability are further discussed. Figure 5 shows the variation trends of system maximal eigenvalue $\lambda_{max}$ with the change of gas cooler pressure $P_{gc}$ at different gas cooler outlet temperature $T_{gc,ou}$. With the increment of $T_{gc,ou}$, the maximal eigenvalue of system rises in the lower $P_{gc}$. Meanwhile, for a higher or lower gas cooler pressure, there exists positive eigenvalue and may reveal the system instability. Figure 6 shows the system minimum logarithmic decrement $\delta_{min}$ with the change of gas cooler pressure when $T_{gc,ou}$=33°C. In this figure, with the increase of gas cooler pressure, the minimum logarithmic decrement slowly rises under the stable conditions, which means the system stability margin increases. In order to obtain a wider stability operating range in this conditions, it should be better to select $T_{gc,ou}\leq 33$ °C and $9.4\leq P_{gc}\leq 11.3$ MPa. Under this range, even if there are instable phenomena during the dynamic process, the divergent rate is quite small and the system is easy to be controlled.

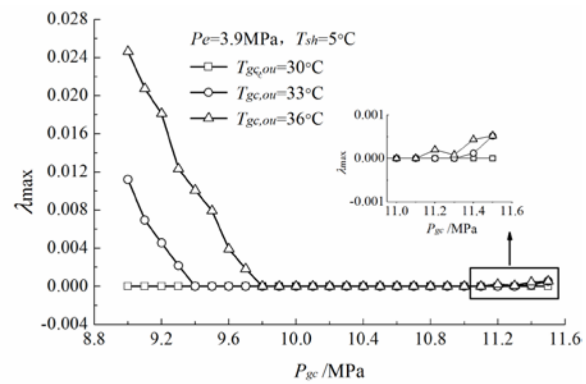


**Figure 5:** Effect of the state parameters of gas cooler on system stability: Maximum eigenvalue

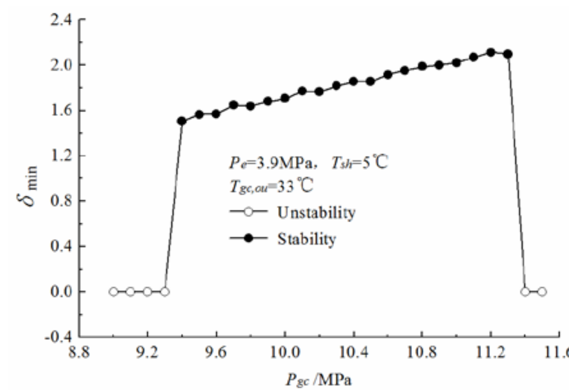


**Figure 6:** Effect of the state parameters of gas cooler on system stability: Minimum logarithmic decrement

## 4. CONCLUSIONS

Combining the First Approximation Theory of Lyapunov Stability Theorem and the stability margin calculation, a dynamic stability analysis method is proposed for the refrigeration system. The inherent link between mathematical stability and actual stability is discussed by the stability of gas cooler. Comparing with actual thermodynamic processes inside the gas cooler, it reveals the consistence between mathematical stability and actual one. Then, the transcritical $CO_2$ ejector expansion refrigeration system (EERS) is taken for example to introduce the analysis method used in the system stability. The matching degree between the components and the whole system are explored. Under a certain conditions, the present result shows that even all components are stable but that cannot guarantee the stability of whole system. In addition, the stability of gas cooler pressure and outlet temperature of gas cooler are investigated. The stability analysis method reflects a harmonious matching level among and between structure parameters and operation parameters, which is expected to be used in more general thermodynamic cycle.

## NOMENCLATURE

| | | |
|---|---|---|
| $A$ | area | (m$^2$) |
| $C_p$ | specific heat | (J kg$^{-1}$ K$^{-1}$) |
| $f$ | continuous function | (–) |
| $h$ | specific enthalpy | (J kg$^{-1}$) |
| $L$ | length | (m) |
| $m$ | fluid reserves | (kg) |
| $\dot{m}$ | mass flow rate | (kg s$^{-1}$) |
| $P$ | pressure | (MPa) |
| $T$ | temperature | (°C) |
| $t$ | time | (s) |
| $V$ | volume | (m$^3$) |
| $x$ | state vector | (–) |
| $Z$ | matrice | (–) |

| | | |
|---|---|---|
| $\Delta z$ | element length | (m) |

**Greek symbols**

| | | |
|---|---|---|
| $\alpha$ | heat transfer coefficient | (W $m^{-2}$ $K^{-1}$) |
| $\gamma$ | void fraction | (–) |
| $\delta$ | logarithmic decrement | (–) |
| $\lambda$ | eigenvalue | (–) |
| $u$ | velocity | (m $s^{-1}$) |
| $\rho$ | density | (kg $m^{-3}$) |

**Subscripts**

| | |
|---|---|
| *c* | compressor |
| *cs* | cross-sectional |
| *d* | diffuser |
| *e* | evaporator |
| *e*1, *e*2 | two-phase region and superheated region in evaporator |
| *ev* | expansion valve |
| *f* | saturated liquid |
| *g* | saturated vapor |
| *gc* | gas cooler |
| *i* | inner |
| *in* | inlet |
| *mix* | mix flow |
| *o* | outer |
| *ou* | outlet |
| *po* | primary flow outlet |
| *r* | refrigerant |
| *se* | separator |
| *sh* | superheat |
| *so* | suction flow outlet |
| *w* | wall |
| *wa* | water |

## REFERENCES

Alleyne A. G., Rasmussen B. P. (2007). Advances in energy systems modeling and control. *Proceedings of the American Control Conference*, New York, pp. 4363–4373.

Cai C. H., Mijanovic S. (2012). LMI–based stability analysis of linear hybrid systems with application to switched control of a refrigeration process. *Asian Journal of Control*, 14(1): 12–22.

Chen Y. M., Deng S. M., Xu X. G., Chan M. Y. (2008). A study on the operational stability of a refrigeration system having a variable speed compressor. *Int. J. Refrig.*, 31(8): 1368–1374.

Dang C. B., Hihara E. (2004). In-tube cooling heat transfer supercritical carbon dioxide, Part1. Experimental measurement. *Int. J. Refrig.*, 27(7), 736–747.

Deng J. Q., Jiang P. X., Lu T., Lu W. (2007). Particular characteristics of transcritical $CO_2$ refrigeration cycle with an ejector. *Appl. Therm. Eng.*, 27(2-3), 381–388.

Hwang Y. (1997). Comprehensive investigation of carbon dioxide refrigeration cycle. Master thesis, University of Maryland, College Park.

Liang N., Shao S. Q., Xu H. B., Tian C. Q. (2010). Instability of refrigeration system-a review. *Energy Convers. Manage.*, 51(11), 2169–2178.

Lin C., Cai W. J., Li Y. Z., Yan J., Hu Y. (2012). Pressure recovery ratio in a variable cooling loads ejector-based multi-evaporator refrigeration system. *Energy*, 44(1), 649–656.

Ma S. W., Zhang C., Chen J. P., Chen Z. J. (2005). Experimental research on refrigerant mass flow coefficient of electronic expansion valve. *Appl. Therm. Eng.*, 25(14–15), 2351–2366.

Ma Z. E., Zhou Y. C. (2001). *Qualitative Theory of Ordinary Differential Equations and Stability Methods*. Beijing, Science Press (in Chinese).

McLinden, M. O., Klein, S. A., Lemmon, E.W., Peskin, A.P. (2002). NIST Reference Fluid Thermodynamics and Transport Properties "REFPROP", National Institute of Standards and Technology. U.S. Department of Commerce, Gaithersburg.

Nehdi E., Kairouani L., Bouzaina M. (2007). Performance analysis of the vapour compression cycle using ejector as an expander. *Int. J. Energy Res.*, 31(4), 364–375.

Oliveira V., Trofino A., Hermes C. J. L. (2011). A switching control strategy for vapor compression refrigeration systems. *Appl. Therm. Eng.*, 31(17–18): 3914–3921.

Qi Q., Deng S. M. (2008). Multivariable control-oriented modeling of a direct expansion (DX) air conditioning (A/C) system. *Int. J. Refrig.*, 31(5), 841–849.

Rasmussen B. P. (2002). Control-oriented modeling of transcritical vapor compression systems, Master thesis. University of Illinois, Urbana-Champaign.

Rasmussen B. P., Alleyne A. G, Musser A. B. (2005). Model-driven system identification of transcritical vapor compression system. *IEEE Trans. Control Syst. Technol.*, 13(3), 444–451.

Rasmussen H. L., Larsen F. S. (2011). Non-linear and adaptive control of a refrigeration system. *IET Control Theory Appl.*, 5(2), 364–378.

Sarkar J., Bhattacharyya S., Gopal M. R. (2006). Simulation of a transcritical $CO_2$ heat pump cycle for simultaneous cooling and heating applications. *Int. J. Refrig.*, 29(5), 735–743.

Tian C. Q., Dou C. P. (2002). The stability of refrigeration systems. *Fluid Machinery*, 30(4): 44–47. (in Chinese)

Wen B. C., Gu J. L., Xia S. P., Wang Z. (2000). Higher rotor dynamics–theory, technology and applications. Beijing: China Machine Press. (in Chinese)

Xu X. X., Tang L. M., Zhu Z. J., Liang L. X., Chen G. M. (2010). Experimental study of working stability in transcritical $CO_2$ compression-ejection system. *Journal of Zhejiang University (engineering science)*, 44(9), 1838–1844 (in Chinese).

Zheng L. X., Deng J. Q., He Y., Jiang P. X. (2015). Dynamic model of a transcritical $CO_2$ ejector expansion refrigeration system. *Int. J. Refrig.*, 60(12), 247–260.

## ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China (Grant No. 51076120).