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Performance Evaluation Criteria and Selection Utility Function for Compact Air-to-Refrigerant Heat Exchangers

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ABSTRACT

The issue of Performance Evaluation Criteria (PEC) for Heat eXchangers (HX) is widely discussed in the literature. However, not often is discussed some deeper meanings of each PEC metric and the consequences they potentially have in the design and selection of HX's when using one or another. There are two main evaluation approaches: energy-based and entropy-based PEC's. The main difference between them is that the first objectively compares the quantities of energy coming in and out of the HX, whilst the second quantifies and qualifies all the factors that affect the overall performance. Furthermore, with the advances in computational capabilities and manufacturing technologies, there is an increasing development of novel HX concepts, and large data sets from multi-objective optimization studies. When the options multiply, the more challenging is to compare and select the best alternative. In this paper, we provide a brief review on the existing PEC from the literature. Additionally, we present a set of PEC that purposely addresses the HX performance in a fundamentally consistent way, however general allowing a fair comparison between different HX's. We compare how different metrics affect the outcomes of an optimization and discuss the results. Lastly, we describe a procedure for translating the optimization results into a Multi-Attribute Utility Function that serves for HX selection method, based on existing methods in the engineering decision-making literature. All the analyses presented in this paper are valid for single-phase HX applications using fixed heat transfer rates and fixed heat capacitance rates.

1. INTRODUCTION

The research on heat transfer augmentation (HTA) relentlessly seeks developing highly compact heat exchangers (CHX) with high performance surfaces. A CHX is the definition of high surface-to-volume ratio (Kays & London, 1984). The definition of high-performance surface, however, is more subject to interpretations, particularly when evaluating a full-sized HX. The sole evaluation of the thermal-hydraulic ratio of a surface do not necessarily portray the broader characteristics in the context of the HX, including overall size, face area and degradation aspects. The literature on HX Performance Evaluation Criteria (PEC) is quite extensive. There are two main approaches to assess the HX PEC: a) energy-based (first law of thermodynamics); b) entropy-based (second law of thermodynamics).

Cowell (1990) revised the main categories within the energy-based PEC. The first, known as "area goodness" factor, is a typical way of evaluating surfaces and HX's, and is simply defined as the ratio of j and f factors (equation 1). The main advantage of such metric is the non-dimensional aspect, which allows one to compare surfaces regardless the geometrical scale, particularly the surface hydraulic diameter.

Although it well represents the surface characteristics, it leads to potential skewed evaluation of the HX or even biasing the search made by an optimizer. The simplified form shows the dependency to the thermal conductance and the inverse of the pressure drop and the square of the minimum free flow area. In other words, this metric can only have some meaning either if the thermal hydraulic ratio is fixed or if the minimum free flow area is fixed. Furthermore, the general knowledge is that this factor is inversely proportional to the Reynolds number, which is not necessarily desired to be relatively low. The reason for this is that the pressure drop and face area (assuming constant flow rate) terms are more sensitive to the variation in velocity than for the thermal conductance (equation 2). If one uses this metric as an optimization objective there is a possibility the optimizer will search for either low-pressure drop and/or small face area designs instead of lower thermal resistance.

$$
\frac{j}{f} = \left[\frac{h}{\rho u_c c_p} \mathbf{Pr}^{2/3}\right] / \left[\frac{2A_c \Delta P}{A_o \rho u_c^2}\right] = K \frac{1}{A_c^2} \frac{NTU}{\Delta P}, \quad K = \frac{\dot{m}^2 \mathbf{Pr}^{2/3}}{2\rho}
$$
(1)

$$
\Delta P \propto u_c^m, m > 1.0; \quad h \propto u_c^n, 0.0 < n < 1.0; \quad A_c \propto u_c^{-1}
$$
 (2)

The second category is the "volume goodness", also described by London (1964) but discussed in other relevant publications including Kays and London (1984), Webb and Kim (2005) and Shah (1978). This category evaluates the dimensioned heat transfer coefficient and pressure drop (in the form of pumping power per surface area) (equation 3). The common observation with regards to these metrics is their dependency to the hydraulic diameter, thus in order for one to make a fair comparison between two or more designs they must have the same hydraulic diameters (Webb & Kim, 2005; Cowell, 1990; Stone, 1996; London, 1964). Additionally, the reduction in pressure drop is usually simpler to obtain instead of improving the heat transfer coefficient, thus normally resulting in large face area designs.

$$
\frac{h}{\dot{W}} = \frac{h}{\Delta P \cdot \dot{V} / A_o} = \left(\frac{c_p \mu}{\text{Pr}^{2/3}} \frac{j \text{Re}}{D_h}\right) / \left(\frac{\mu^3}{2\rho^2} \frac{f \text{Re}^3}{D_h^3}\right) = \frac{2j\rho^2 c_p D_h^2}{f \mu^2 \text{Pr}^{2/3} \text{Re}^2}
$$
(3)

The third main category identified by Cowell (1990) include the 12 scenario design method (Bergles, Bllumenkrantz, & Taborek, 1973) , which are at most limited to one or more fixed parameters, in addition to fixed hydraulic diameters. Such category was not intended to be applied to actual variable geometry HX's, much less comparing multiple HX's with very different surface types.

The fourth category, and Cowell's (1990) own method, account for methods that are either of diffusive interpretation, very particular or by any means extendable to a more general method, or a variation of the previous categories.

In spite of the particular issues and limitations, the common denominator to all energy-based PEC metrics is the premise that the performance degradation is solely due to the hydraulic resistance. When one thinks of degradation, it can be flatly interpreted as the direct energy cost for driving the fluid through the HX. Alternatively, the degradation can be interpreted as everything that can cause a negative impact not only on the overall HX performance but also to a larger control volume including a system of sub components (Shah, 2006). For the second interpretation, the entropybased PEC (or thermodynamic) approach is more appropriate. Additionally, in many cases the entropy generation due to the finite temperature difference is significantly larger than it is for the pressure drop.

McClintock (1951) introduced the concept of irreversibility to HX design, which was later formalized by Bejan (1977) where he defined the concept of Number of Entropy Generation Units (N_S) as an evaluation metric. His work culminated in the idealization of the Entropy Generation Minimization (EGM) for broad applications of finite-size systems and finite-time processes (Bejan et al., 1996).

$$
N_{S} = \frac{\dot{S}_{gen}}{C_{\text{min}}} \tag{4}
$$

According to the literature, it is well established that the tradeoff between energy-based and entropy-based approaches comprises balancing out the HX size and production costs directly for savings in energy degradation (irreversibilities) (Bejan, 1977) further down the process. It is also a common sense that a larger and "more expensive" HX is more thermodynamically efficient (Bejan, 1977), and better heat transfer performance does not lead to minimum entropy generation (Bejan & Pfister, 1980; Seculik & Herman, 1986).

The evolution of computational tools (such as CFD), optimization algorithms, storage capacities, processing speed and manufacturing technologies enable a large number of novel ideas and concepts establishing new frontiers. Unfortunately, while the more novel heat transfer types and shapes are being developed the less clear their consequences are to a full HX design. Furthermore, it is becoming harder to compare and select an optimum HX on a fair basis.

In this paper, we propose the use of a set of comprehensive metrics attempting to address the challenges from the common PEC approaches. We show how the optimization outcomes can be shifted when using different objectives and demonstrate why one metric should be chosen over the other. Lastly, we present a procedure based on Scott and Antonsson (2005) to build a Multi-Attribute Performance Utility Function (MAPUF) and apply it to HX selection.

2. HX EVALUATION CRITERIA

2.1 Performance-Degradation Number

Considering the brief literature review in the previous section, it should be clear that we want to find a metric that, not only carries quantitative and qualitative information regarding the performance and degradation aspects, but also it has to be sufficiently general so one can compare multiple HX types fairly.

Bejan (1982) first studied the relationship between the Number of Entropy Generation Units (Ns) and the Number of Transfer Units (NTU) for a balanced counter flow HX with no pressure drop. He encountered what was called the "entropy generation paradox" when the Ns went to zero for either NTU = 0 or ∞ , but reached a maximum at an intermediate NTU. Shah & Skiepko (2004) interpreted such behavior as the irreversibility tend to zero whenever the heat transfer potential is zero; i.e. at NTU = 0 there is no heat flow thus, from the Second Law, S_{gen} has to be zero for it cannot be negative. When NTU $\rightarrow \infty$ the hot and cold stream temperatures approach to the same value, thus nulling the heat transfer potential. Ogiso (2003) defined the dimensionless "entropy generation index" (N_s/NTU) and showed that the Bejan's paradox can be Shah & Skiepko (2004) since the index is not defined at NTU = 0 or NTU $\rightarrow \infty$.

This metric satisfies the criteria we looked for since it provides the information on the thermal performance (NTU), the degradation factors (Ns) and is non-dimensional. In this paper, we use the inverse and call it the performancedegradation number (equation 5). The reason is merely convenience; to have a number, which higher values are better.

$$
\psi = \frac{NTU}{N_s} \tag{5}
$$

For purposes of this paper, all analysis will focus on the airside. With this assumption, we can use the ideal gas model (equation 6) to calculate ψ using the non-dimensionalized entropy generation in equation (7).

$$
\Delta s = \int \frac{\partial q}{T} + s_{gen} \rightarrow s_{gen} \approx c_p \ln \left(\frac{T_o}{T_i} \right) - \frac{q}{\overline{T}} - R \ln \left(\frac{P_o}{P_i} \right) \tag{6}
$$

$$
N_s = N_{s,\Delta T} + N_{s,\Delta P}, \quad N_{s,\Delta T} = \frac{\dot{S}_{gen,\Delta T}}{C} = \ln\left(\frac{T_o}{T_i}\right) - \frac{NTU\Delta T_{ml}}{\overline{T}}, \quad N_{s,\Delta P} = \frac{S_{gen,\Delta P}}{R} = \ln\left(\frac{P_o + \Delta P}{P_o}\right)
$$
(7)

$$
\psi = \frac{NTU}{\ln\left(\frac{T_o(P_o + \Delta P)}{T_i P_o}\right) - \frac{NTU\Delta T_{ml}}{\overline{T}}} = \frac{NTU}{\ln\left(\frac{T_o(P_o + \Delta P)}{T_i P_o}\right) - \frac{\varepsilon \Delta T_{\max}}{\overline{T}}}
$$
(8)

On equation (8) it is clear that the performance degradation number has on its denominator, in addition to the pressure drop, the finite temperature difference contribution.

2.2 HX Compactness and Face Area

Typically when using EGM or any other entropy-based PEC for designing a HX the trade-off between size and low entropy generation is always an issue. In reality, the larger HX's actually have larger heat transfer surfaces in order to reduce the overall thermal resistance. For conventional surface types and dimensions, larger area will naturally result in larger volumes, thus the reference to the HX size. However, the next generation of HX's is shifting to novel shapes and towards smaller tube sizes, which result in surfaces that are more compact. Additionally, smaller sized surfaces have higher heat transfer coefficients. In other words, these novel HX's have the potential to reduce thermal resistance in a smaller envelope compared to conventional HX's, but not proportionally increasing the surface area once the heat transfer coefficient is higher. Additionally, the term "size" is normally used loosely, i.e. most studies do not qualify what aspect of the size is the most relevant. In many applications, the envelope volume is not much of an issue as long as the design can satisfy potential limitations on tube length, face area and/or aspect ratio. The face area can be more critical since it can affect the cross section of an air duct, size of an equipment casing, or the size of the front of a car. Ultimately, the metrics that better evaluate the geometrical aspects of a HX are the face area and the surface hydraulic diameter (equation 9), since it represents the inverse of compactness.

$$
D_h = 4 A_c / A_o d = 4\sigma V / A_o \tag{9}
$$

3. HX DESIGN

In this paper, we study the design of a 1.0 kW air-to-water HX in cross flow. We investigate two different HX surfaces [\(Figure 1\)](#page-4-0): a) round bare tubes in staggered arrangement with diameters below 2.0mm (RTHX), b) Webbed NURBS tube (NURBS shaped channels connected by a longitudinal web) (WTHX). Here we investigate how the different performance metrics affect the optimization results. For this study, we solve three multi-objective optimization problems targeting minimizing face area and maximizing ψ , j/f, and h/ ΔP , respectively for each problem.

Figure 1: HX surface types: a) RTHX; b) WTHX.

Figure 2: HX Design results: a) RTHX; b) WTHX.

[Figure 2](#page-4-1) presents the optimization results on a compactness vs. face area plot, where the shading indicates the pressure drop and the symbol size the inlet approach temperature. The symbol type indicates the optimization problem from [Table 1.](#page-4-2) On both surface types, the OPT03 resulted in designs with relatively higher-pressure drop and lower approach temperature, as expected. Additionally, for the RTHX both OPT01 and OPT02 resulted in designs with longitudinal pitch larger than the transverse pitch, unlike the conventional tube arrangements. Furthermore, the OPT03 results have satisfactory geometrical characteristics; for the RTHX surface, it resulted in the most compact designs with relatively small face area, whereas for the WTHX it resulted in designs with the smallest face area with relatively high compactness. [Figure 3](#page-5-0) shows how, for this application, the entropy generation due to finite temperature difference is significantly larger than for pressure drop.

4. HX SELECTION

Decision-making is an intrinsic step in any engineering design application. In many cases the ranking, sorting and finally selecting can be straightforward. In many others, however, the number of parameters to be considered and particularly for non-linear problems this process can be quite challenging. In the case of HX design, optimization and selection, there are numerous metrics of interest. Three main categories of metrics can be defined: a) performance/degradation; b) geometry; c) manufacturing, operating and maintenance. The first two were discussed previously in this manuscript, the third, however important, will not be considered for the purposes of this exercise, but the method does not lack any generality by doing it so. The objective is, therefore, to develop a Multi-Attribute Utility Function (MAUPF) that describes the HX according to the utility of each category and use such expression to determine the design with the highest aggregate utility. A common way of describing a utility function for single criteria is by using the exponential function (Herrmann, 2014) (equation 10). This method is only valid for utilities that the maximum is better. For simplicity the utility functions are normalized between bounds a (equation 10) and b (equations 10 and 11). The resulting single attribute utility functions are shown in [Figure 4.](#page-6-0)

$$
u(x_i) = \frac{1 - e^{-\gamma(x_i - a)}}{1 - e^{-\gamma(b - a)}}, \quad x_i = PEC, \quad a \le x_i \le b, \quad 0 \le u(x_i) \le 1
$$
 (10)

It is also important that the definitions of the bounds are well understood. In this paper, we will use the compactness, face area and performance-degradation number as criteria for the HX selection. The bounds for compactness are set to 0 and the ratio of the larger heat transfer area by the minimum volume of all optimum designs. The face area is evaluated by its inverse in order to obtain a crescent value criteria, and it is bounded by 0 and the face area at the maximum velocity evaluated in the design space [\(Table 1\)](#page-4-2). The performance degradation number is bounded by 0 and

the theoretical performance degradation number for an outlet approach temperature of 1°C, minimum inlet approach temperature from the design space [\(Table 1\)](#page-4-2) and no pressure drop.

$$
b_{\psi} = \frac{NTU\left(\Delta T_{out} = 1, \Delta T_{in} = 0.8\Delta T_{in_baseline}\right)}{\ln\left(\frac{\overline{T} - 1}{T_i}\right) - \frac{0.78\Delta T_{in_baseline}}{\overline{T}}}
$$
(11)

The shape factor (γ) can be obtained if a utility value is known. One way to define that is by assuming that for the median criteria values the utility is 0.5. Scott and Antonsson (2005) presented a way of considering the aggregate utility function (MAUPF) based on a weighted L^p-norm expression (equation 12).

$$
U(\underline{x}) = \left(\frac{\sum w_i u(x_i)^p}{\sum w_i}\right)^{1/p}, \quad \text{if } p = 0 \implies U(\underline{x}) = \left(\sum u(x_i)^{w_i}\right)^{1/\sum w_i} \tag{12}
$$

The maximum value of MAUPF will indicate the HX to be selected. Another challenging task is choosing the weights and the compensation parameter (p). The latter will lead to a more compensating solution the further away it is from 0. In some cases there will be no preference regarding the compensating characteristics. For such occasions, one may perform an optimization to maximize MAUPF by varying the weight vector and the compensating parameter. To avoid trivial solutions the weight parameters must be bounded to a minimum positive number. In this paper, we bounded the weight factors in [0.5,1.0] and p in [-500,500].

Table 2: HX Selection MAUPF results.

The single utility functions all have positive shape factor, which translates into valuing more a change in the attribute value when it is near the lowest value. This is a result from the criterion used to determine the shape factor, which is larger than the median of normalized attribute values. Such curves could have negative shape factors by changing that criterion. For both surfaces, the selection parameters resulted in non-compensating solutions since the compensating parameter (p) is very close to 0. The weight factors are of same order of magnitude, which suggests the optimizer was not biased towards any of the criteria.

For Pareto sets with curvature towards the ideal design, the non-compensating solutions are closer to the ideal design than the compensating solutions, as illustrated in [Figure 5.](#page-7-0) Since we have not established any preferences about our attributes, it is expected that the highest utility function would be from a non-compensating solution. Depending on the profile of the Pareto frontier the outcomes of this approach may shift towards any of the compensating solutions.

Although there are still decisions to be made while using this approach, it has translated subjective decisions into objective mathematical formulations. If one changes any of the parameters, it becomes easier to map different types of preferences and narrow down the designs that best suite each. [Figure 6](#page-7-1) shows the selected designs obtained with the above procedure.

Figure 5: Normalized Pareto sets for the RTHX.

Figure 6: Selected HX: a) RTHX; b) WTHX.

5. CONCLUSIONS

This paper presented a comprehensive analysis on Performance Evaluation Criteria (PEC) for HX's. The use of the performance-degradation number (ψ) has proven to be a better metric in terms of reducing the thermal resistance while still minimizing entropy generation. The energy-based PEC's typically yield in higher thermal-hydraulic ratio by reducing hydraulic resistance above all. The optimization using ψ does not necessarily result in the best geometric features, however it leads the optimizer to find a good compromise. When using shape optimization the different metrics have significant impact on the tube shape and, for this study, the ψ lead to much smaller face areas. Additionally, the optimized tube shape designs have better aspect ratios and smaller face area overall. The last part of this manuscript described a procedure to translate the optimization results into mathematical formulations that can be used to find the design with the highest MAUPF. Such procedure can be modified according to decision-maker preferences in terms of criteria and compensating parameter. In summary, this paper presented a robust approach for design, optimization and selection of compact HX's using fundamentally, and practically, superior performance criteria.

NOMENCLATURE

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