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Donghun Kim
Purdue University, United States of America, kim1077@purdue.edu

Sugirdhalakshmi Ramaraj
Purdue University, United States of America, sramaraj@purdue.edu

James Braun
Purdue University, United States of America, jbraun@purdue.edu

Computationally Efficient Modeling Approach for Evaporator Performance under Frost Conditions

Donghun Kim[1]*, Sugirdhalakshmi Ramaraj[1], James E. Braun[1]

[1]Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University
West Lafayette, IN, U.S.
kim1077@purdue.edu
sramaraj@purdue.edu
jbraun@purdue.edu

* Corresponding Author

ABSTRACT

Growth of a frost layer on an evaporator surface due to low evaporator temperature as well as moisture contained in surrounding air deteriorates performance of a refrigeration system significantly and requires significant energy for defrost. Many studies have been performed to model the heat and mass transfer phenomena in an attempt to have insight and accurate prediction. However, many models form nonlinear algebraic differential equations and thereby it is computationally demanding to include them into a typical building energy simulation environment for a cooler or freezer consisting of an enclosure, refrigeration equipment, defrost elements, and controls. Computationally efficient but reasonably accurate models are needed in order to evaluate overall system performance. The objective of this paper is to introduce a modeling approach to overcome the problem. A numerical solution strategy based on an enthalpy-based reformulation and linearization method will be presented. Comparisons of a proposed and detailed model results are provided.

1. INTRODUCTION

Frost accumulation on a cold evaporator surface occurs when moist air flows over the surface. It could significantly deteriorate performance of a refrigeration unit due to the increase of thermal resistance of frost and decrease of air flow rate associated with the frost built-up. A number of models have been developed for accurately predicting the frost growth on refrigeration system evaporators. Jones and Parker (1975) developed a one-dimensional model to predict dynamic responses of a frost layer on a cold plate. The model is based on the assumption that the amount of water vapor being frozen is uniform in a frost layer and they arrived at a nonlinear algebraic equation that has to be solved at each time step. Kondepudi and O'Neal (1993) developed a finned-tube heat exchanger model under frosting conditions. An energy transfer coefficient, which is a generalization of an overall heat transfer coefficient for dry conditions, was introduced and the ε-NTU method was applied. Lee et. al. (1997) developed a model for a flat plate based on the assumption of a homogenous reaction of water vapor in a frost layer. Analytic solutions for distributions of temperature and water vapor density in a frost layer were derived. A total heat flux and frost surface temperature are calculated iteratively with an air side heat transfer equation. Tso et. al. (2005), Yang et. al. (2006) and Kim et. al. (2009) applied the homogenous reaction model to finned tube heat exchangers.

Many models, even for one-dimensional heat transfer, form nonlinear algebraic differential equations and thereby iterative numerical methods are needed for each time step and control volume. Therefore, it is computationally demanding to include them within a typical building energy simulation environment for a cooler or freezer consisting of an enclosure, refrigeration equipment, defrost elements, and controls. Computationally efficient but reasonably accurate models are needed in order to evaluate overall system performance.

This paper aims at developing a numerically efficient modeling approach to overcome this problem. It is based on an enthalpy-based reformulation and linearization method. Section 2 describes issues related to the high computational

load in detail. Section 3 and Section 4 describe a baseline model and our modeling approach, respectively. Comparisons of the proposed and reference model results are provided in Section 5.

2. MODELING APPROACHES AND ISSUES

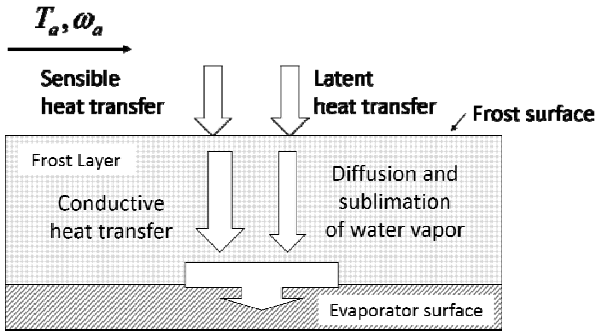

Figure 1: Heat and mass transfer phenomena on a cold evaporator surface

Fig. 1 shows a schematic of the physical phenomena occurring at a cold evaporator surface at a given time. It is assumed that the evaporator surface temperature is below the freezing point, so that frost has accumulated. As moist air flows with temperature T_a and humidity w_a, both heat and water vapor are transferred from the air to the frost surface. A portion of the transferred water vapor passes through the frost surface and contributes to the densification of the frost layer. The remainder accumulates on the frost surface and increases the frost layer thickness. As water vapor diffuses through a porous structure of the frost layer, it contacts and is converted to ice crystals while emitting the heat of sublimation to the neighboring surfaces. Therefore, total heat transferred to the evaporator surface is due to both the conductive heat transfer through the existing frost layer and the total heat of sublimation generated in the frost layer due to frost growth.

Because the total heat transfer is not simply driven by the difference between the evaporator and frost surface temperatures due to the internal diffusion of water vapor, one cannot simply utilize the concept of overall heat transfer coefficient. In general, the inclusion of a diffusion equation to the calculation of the frost surface temperature and total heat flux leads to a set of nonlinear algebraic equations, and thereby an iterative numerical solution technique, e.g. Newton-Raphson method, is needed (Jones and Parker (1975), Lee et. al. (1997), Seker et. al. (2004)).

Computational demands using these approaches are significant because;

- At a given time, an iteration is required for each elementary control volume since the air temperature and humidity change along the air flow direction, and accordingly frost density and thickness vary.
- At a given control volume, an iteration is required for each time step because the frost layer thickness changes.

It is difficult to use an evaporator model that has multiple control volumes and relies on an iterative solver when the model is to be integrated within an overall refrigeration system model. This is because a system model typically requires other iterations to solve equations for all components (i.e. evaporator, condenser, compressor and expansion valve) and the high computational load of the evaporator model becomes a big burden. As a result, this type of model is infeasible for assessing overall system performance over any significant period time, such as for assessment of alternative defrost strategies. In general, this type of model is not appropriate for integration within building simulation tools, such as TRNSYS (Klein et. al. (1979)) and EnergyPlus (Crawley et. al. (2001)), which typically perform simulations over several months or a year. In addition, the solution to this type of model may not be robust under all conditions and could lead to numerical problems that don't allow completion of a longer-term simulation.

Therefore, a computationally efficient but reasonably accurate modeling approach for simulating evaporator performance under frosting conditions is extremely important.

3. A DESCRIPTION OF A REFERENCE MODEL

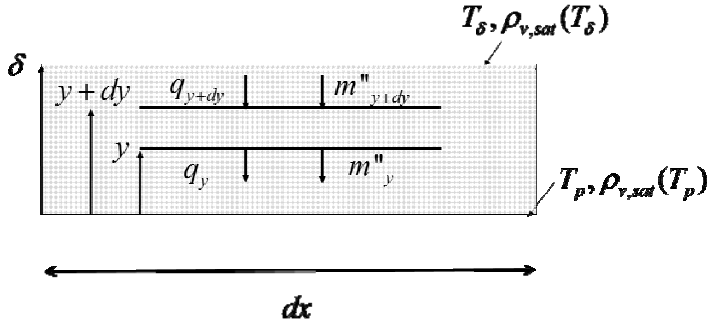

Figure 2: Schematic diagram in a frost layer

For simplicity in understanding the basic modeling approach, consider a flat plate where the temperature is fixed as T_p. We chose Lee's model (Lee et. al. (1997)) as a reference model that is used to develop and benchmark a simplified model described in the next section. The reference model that describes the water vapor absorption phenomena in a frost layer is analogous to a homogeneous first-order chemical reaction model. Fig. 2 shows a control volume of width *dx* where mass and energy balances are applied and where y measures the distance from the plate surface into the frost layer. The following assumptions are employed in the model

- frost surface temperature in each control volume is lower than freezing point
- heat and mass transfer within the frost layer are one dimensional and quasi-steady
- water vapor is saturated at the frost surface and within the frost layer (thermal equilibrium)
- thermal conductivity of the frost layer varies only with frost density
- diffusion coefficient of water vapor is uniform
- uniform frost density
- the amount of water vapor sublimated is proportional to the local water vapor density

The governing equations and boundary conditions for a given control volume of width *dx* over a time step *Δt* are

$$D\frac{d^2\rho_v}{dy^2} = \kappa\rho_v, \quad -k_{fr}\frac{d^2T}{dy^2} = h_{sub}\kappa_r\rho_v \tag{1}$$

and

$$\frac{d\rho_v}{dy}(0) = 0, \;\; \rho_v(0) = \rho_{v,sat}(T_p), \;\; k_{fr}\frac{dT}{dy}(\delta) = q_{tot} - h_{sub}D\frac{d\rho_v}{dy}(\delta), \;\; T(0) = T_p, \tag{2}$$

where the total energy heat flux from the air to the frost layer is given by

$$q_{tot} = \mathrm{h}(T_a - T_\delta) + h_{sub}\mathrm{h}_d(\omega_a - \omega_{v,sat}(T_\delta)). \tag{3}$$

Corresponding solutions for Eqn. (1) to (3) are (Lee et. al. (1997))

$$\rho_v(y) = \rho_{v,sat}(T_p)\cosh(my) \tag{4}$$

$$T(y) = T_p + \frac{q_{tot}}{k_{fr}} y - \frac{\kappa h_{sub} \rho_{v,sat}(T_p)}{k_{fr} m^2} (\cosh(my) - 1) \tag{5}$$

where $m = \sqrt{\kappa / D}$.

The reaction coefficient (or absorption coefficient) can be calculated by imposing a boundary condition, i.e. $\rho_v(0) = \rho_{v,sat}(T_p)$, as follows.

$$\kappa = \frac{D}{\delta^2} [\cosh^{-1}(\frac{\rho_{v,sat}(T_\delta)}{\rho_{v,sat}(T_p)})]^2. \tag{6}$$

From Eqn. (5),

$$T_\delta = T_p + \frac{q_{tot}}{k_{fr}} \delta - \frac{\kappa h_{sub} \rho_{v,sat}(T_p)}{k_{fr} m^2} (\cosh(m\delta) - 1). \tag{7}$$

Note that, to calculate the temperature at the surface of the frost layer, i.e. T_δ, based on a local inlet air condition (T_a, ω_a) and a plate temperature T_p, Eqn. (3), (6) and (7) must be solved iteratively using psychrometric relations. Once T_δ is obtained, the mass flux of water vapor from the air to frost surface and densification rate are obtained by the equations of the air-side mass transfer and $D \frac{d\rho_v}{dy}(\delta)$, respectively. Using those, the frost thickness and density are updated for the next time step. The air temperature and humidity at the exit of the control volume are calculated using energy and water vapor mass balances. The exit condition becomes the inlet condition for the next control volume. The above procedure is repeated for a predefined simulation period.

4. SIMPLIFIED MODEL DEVELOPMENT

A numerical strategy to avoid iteration consists of the following two main steps.

1. Reformulate Eqn. (3) with an approximation such that q_{tot} relies on a difference in enthalpy rather than that of temperature and humidity.
2. Reformulate and linearize Eqn. (7).

The main idea for the first step comes from the enthalpy potential approach developed to analyze cooling towers, cooling coils and desiccant wheels where sensible and latent heat transfer occur simultaneously (Mitchell and Braun (2013), McQuiston (1982)). We define the following quantity for moist air.

$$\eta(T, \omega) \coloneqq h(T, \omega) + (h_{sub} - h_v^{ref})\omega \quad \forall T, \omega \quad . \tag{8}$$

where h represents the specific enthalpy of moist air used in developing the psychrometric chart (ASHRAE (2001)). More precisely, $h(T, \omega) = (C_{p,a} + C_{P,v}\omega)T + h_v^{ref}\omega$ and h_v^{ref} is the specific enthalpy of saturated water vapor at 0 (^{o}C), i.e. 2501 (kJ/kg). η is also a specific enthalpy for moist air but having the heat of sublimation of water vapor as an enthalpy reference rather than h_v^{ref} .

When the Lewis number is unity, Eqn. (3) can be approximated as follows using the boundary layer analogy.

$$q_{tot} \approx \mathrm{h}_d[(C_{p,a} + C_{p,v}\omega_a)T_a - (C_{p,a} + C_{p,v}\omega_{sat}(T_\delta))T_\delta + h_{sub}(\omega_a - \omega_{sat}(T_\delta))]. \tag{9}$$

This approximation is valid with negligible error if the Lewis number is close to one. Therefore,

$$\begin{aligned} q_{tot} &\approx \mathrm{h}_d[h(T_a,\omega_a) - h_{v,sat}(T_\delta) + (h_{sub} - h_v^{ref})(\omega_a - \omega_{sat}(T_\delta))] \\ &= \mathrm{h}_d[\eta(T_a,\omega_a) - \eta_{v,sat}(T_\delta)] \end{aligned}. \tag{10}$$

The second step starts from reformulation of Eqn. (7). By plugging Eqn. (4) into (7) and applying the mean value theorem, one can obtain the following expression.

$$q_{tot} = \frac{k_{fr} + h_{sub} D \frac{d\rho_{v,sat}}{dT}(T^{**})}{\delta} \times \frac{\eta_{v,sat}(T_\delta) - \eta_{v,sat}(T_p)}{\frac{d\eta_{v,sat}}{dT}(T^*)}, \tag{11}$$

for some $T^*, T^{**} (\in \{T_p, T_\delta\})$.

By approximating the two derivatives in Eqn. (11), the expression for heat flux becomes

$$q_{tot} \approx \frac{k_{eq,fr}}{C^* \delta}(\eta_{v,sat}(T_\delta) - \eta_{v,sat}(T_p)). \tag{12}$$

where

$$k_{eq,fr} = k_{fr} + h_{sub} D\left(\frac{\rho_{v,sat}(T_\delta^-) - \rho_{v,sat}(T_p^-)}{T_\delta^- - T_p^-}\right)$$

$$C^* = \frac{\eta_{v,sat}(T_\delta^-) - \eta_{v,sat}(T_p^-)}{T_\delta^- - T_p^-}$$

and (T_δ^-, T_p^-) are temperatures of the surface of the frost layer and the plate at the previous time step. For this calculation, the saturation curve of $\eta_{v,sat}$ is obtained using psychrometric relations (ASHRAE (2001)) along with Eqn. (8).

Since the forms of Eqn. (10) and (12) provide a way to construct a thermal network, one can solve $q_{tot}, \eta_{v,sat}(T_\delta)$ without any iterations as follows.

$$q_{tot} = \frac{\eta(T_a,\omega_a) - \eta_{v,sat}(T_p)}{R_a^* + R_{fr}^*} \tag{13}$$

and

$$\eta_{v,sat}(T_\delta) = \frac{R_a^*}{R_a^* + R_{fr}^*}\eta_{v,sat}(T_p) + \frac{R_{fr}^*}{R_a^* + R_{fr}^*}\eta(T_a,\omega_a). \tag{14}$$

where $\mathrm{R}_a^* = 1/\mathrm{h}_d$ and $R_{fr}^* = C^*\delta / k_{eq,fr}$. T_δ is determined from a correlation for $\eta_{v,sat}(T_\delta)$. Once q_{tot}, T_δ are calculated at each time step, other quantities, e.g. the densification rate for a frost layer and T_a, ω_a for the next downstream control volume can be easily calculated as described in Section 3.

5. RESULTS AND DISCUSSION

The evaluation of the proposed simplified model, i.e. Eqn. (13) and (14), is performed by comparing the predicted frost thickness and frost surface temperature with those of the reference model and experimental data available from

literature (Lee et. al. (1997)). In the test, the temperature of a flat plate, air temperature, humidity and velocity were regulated. For more detailed descriptions of the experimental setup, see the reference paper.

For each control volume and time step, the reference model, i.e. Eqn. (3), (6) and (7), was solved using *fsolve* in Matlab, which is a nonlinear equation solver, with its *default* numerical termination criteria; the maximum iteration number is 400 and tolerance on a function value is 10^{-6}. Frost density and thickness were updated at each time step based on a solution for frost surface temperature. CoolProp (Bell et. al., 2013) was used to calculate properties of dry and moist air, and correlations for the diffusion coefficient, thermal conductivity of frost layer, and air side heat transfer coefficient presented in the reference paper were applied. A 10 sec time step was employed. The same process was applied to the simplified model except for the set of equations to be solved. In other words, Eqn. (13) and (14) were computed, and then Eqn. (6) was used to get the absorption coefficient for the proposed model.

Fig. 3 shows comparisons of frost layer thickness and surface temperature under a frost condition; Air velocity, air temperature, air humidity and plate temperature are 1 (m/s), 25 (^{o}C), 70 (%) and 15 (^{o}C), respectively. There are some differences in predictions between the reference and simplified model resulting from simplifying assumptions. However, the discrepancies are in a reasonable range (up to 7% relative error) and it is evident that the simplified model captures the most significant dynamics of frost accumulation with a reasonable accuracy even compared to measurements.

Table 3 compares simulation times for the two models for the 2-hr simulation period. The simplified model reduces the computation time by about a factor of 15 compared to the reference model.

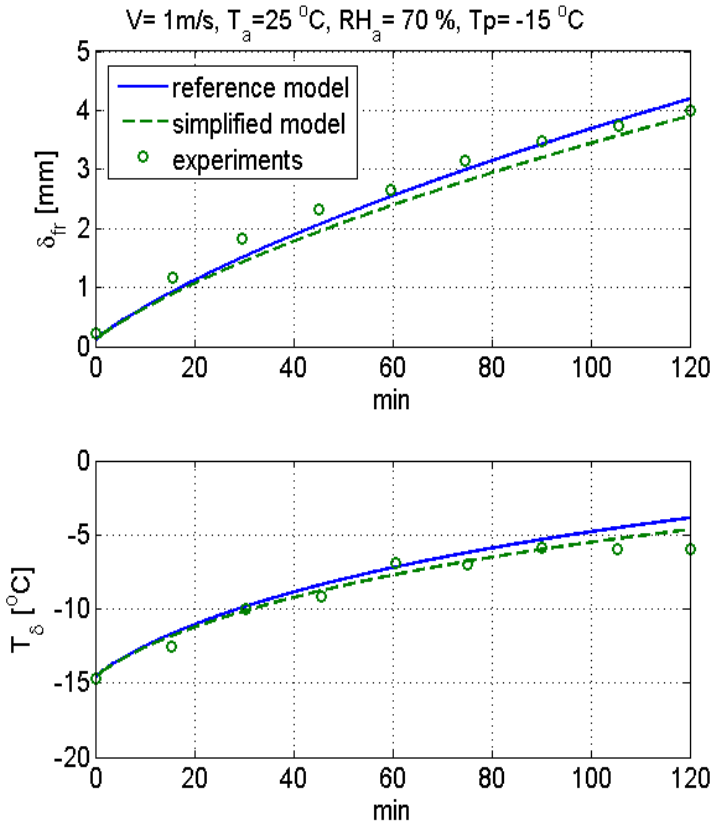

Figure 3: Comparisons of the simplified model with a reference model and experimental data

Table 1: Comparison of simulation times

	Reference Model	**Present Model**
Computation time (sec)	24.36	1.55

Of course, if a different baseline model was selected, the computation time savings could differ significantly. Indeed, many models have been developed to describe performance of evaporators under frosting conditions. They may be classified into two groups based on the inclusion of a diffusion equation at the stage of calculating the frost surface temperature and total heat flux. The first approach solves coupled differential equations of energy and water vapor mass for a frost layer, and couples the solutions to air side energy and mass balances. This approach leads to a set of nonlinear algebraic equations, and thereby an iterative numerical solution technique is needed. The baseline model in this paper is an example. Although the simplified model was derived from a model that adopts a first-order homogenous chemical reaction to describe heat and mass transfer in a frost layer, it can be shown that Eqn. (12) is generally valid for other models in this group. Therefore, similar computational savings are expected when comparing to this group of models.

For the second approach, a diffusion equation is not included at the stage of getting the frost surface temperature and total heat flux, but is included in the next stage of calculating the densification rate. This becomes possible either by neglecting the diffusion of water vapor in a frost layer or by constructing an empirical correlation for an effective thermal conductivity taking account of the internal diffusion of water vapor. Compared to this approach, there may be no advantages of the simplified model in terms of computing time. However, the proposed simplified modeling approach is still attractive because it is linear and simple, and does not rely on an arguable correlation[1].

6. CONCLUSIONS AND FUTURE WORK

A numerically efficient model that avoids iterations in predicting the performance of evaporators under frosting conditions was derived and demonstrated. The simplified model was developed based on reformulation of a more detailed reference model based on an enthalpy potential and linear approximation method. The proposed model is 15 times faster than the reference model but with a reasonable accuracy. The simple model structure provides a natural way to construct a thermal network and hence allows utilization of the ε-NTU method for further computation savings.

Although the method has been derived from one particular reference model, it is valid for other types of models for frost build-up. Future work will compare results of this modeling approach with different frost-layer models for finned-tube exchangers.

[1] Most correlations for an effective thermal conductivity are valid for a limited range of operations and the effective thermal conductivity is indeed affected by various frosting factors (Hayashi et. al. (1977), Yang et. al. (2006)).

NOMENCLATURE

T	Temperature	(^{o}K)
T_δ	surface temperature of a frost layer for a control volume at each time step	(^{o}K)
ω	humidity ratio	(kg/kg-air)
δ	frost thickness at a time	(m)
q	heat flux	(W/m^2)
q_{tot}	total heat flux, sensible and latent	(W/m^2)
m''	mass flux	(kg/m^2-s)
ρ	density	(kg/m^3)
k	thermal conductivity	(W/m-K)
$k_{eq,fr}$	equivalent thermal conductivity of a frost layer	(W/m-K)
κ	absorption rate	(1/sec)
D	diffusion coefficient	(m^2/s)
h	a specific enthalpy of moist air used in developing the psychrometric chart (ASHRAE)	(J/kg)
h_v^{ref}	a reference specific enthalpy of water vapor	(J/kg)
η	a specific enthalpy of water vapor	(J/kg)
h	convective heat transfer coefficient	(W/m^2-K)
h_d	$h_d = h_m / \rho_a$	(m/s)
h_m	convective mass transfer coefficient	(kg/m^2-s)
C_p	specific heat	(J/kg-K)
R^*	thermal resistance w.r.t. enthalpy potential difference	(m^2-s/kg)

Subscript

a	air
fr	frost
p	plate
v	vapor
v,sat	saturated vapor
sub	sublimation

REFERENCES

ASHRAE (2001). Fundamentals. *American Society of Heating, Refrigerating and Air Conditioning Engineers, Atlanta*, *111*.

Bell, I. H., Quoilin, S., Wronski, J., & Lemort, V. (2013). CoolProp: An open-source reference-quality thermophysical property library. In *ASME ORC 2nd International Seminar on ORC Power Systems*.

Crawley, D. B., Lawrie, L. K., Winkelmann, F. C., Buhl, W. F., Huang, Y. J., Pedersen, C. O., ... & Glazer, J. (2001). EnergyPlus: creating a new-generation building energy simulation program. *Energy and buildings*, *33*(4), 319-331.

Hayashi, Y., Aoki, A., Adachi, S., & Hori, K. (1977). Study of frost properties correlating with frost formation types. *Journal of heat transfer*, *99*(2), 239-245.

Jones, B. W., & Parker, J. D. (1975). Frost formation with varying environmental parameters. *Journal of Heat Transfer*, *97*(2), 255-259.

Kim, J. S., Lee, K. S., & Yook, S. J. (2009). Frost behavior on a fin considering the heat conduction of heat exchanger fins. *International Journal of Heat and Mass Transfer*, *52*(11), 2581-2588.

Kondepudi, S. N., & O'Neal, D. L. (1993). Performance of finned-tube heat exchangers under frosting conditions: I. Simulation model. *International Journal of Refrigeration*, *16*(3), 175-180.

Lee, K. S., Kim, W. S., & Lee, T. H. (1997). A one-dimensional model for frost formation on a cold flat surface. *International Journal of Heat and Mass Transfer*, *40*(18), 4359-4365.

McQuiston, F. C., & Parker, J. D. (1982). Heating, ventilating, and air conditioning: analysis and design.

Mitchell, J. W., & Braun, J. E. (2013). *Principles of heating, ventilation, and air conditioning in buildings*. Wiley.

Seker, D., Karatas, H., & Egrican, N. (2004). Frost formation on fin-and-tube heat exchangers. Part I—Modeling of frost formation on fin-and-tube heat exchangers. *International Journal of Refrigeration*, *27*(4), 367-374.

Tso, C. P., Cheng, Y. C., & Lai, A. C. K. (2006). An improved model for predicting performance of finned tube heat exchanger under frosting condition, with frost thickness variation along fin. *Applied Thermal Engineering*, *26*(1), 111-120.

Klein, S. A., & University of Wisconsin--Madison. Solar Energy Laboratory (1979). *TRNSYS, a transient system simulation program*. Solar Energy Laborataory, University of Wisconsin--Madison.

Yang, D. K., & Lee, K. S. (2005). Modeling of frosting behavior on a cold plate. *International journal of refrigeration*, *28*(3), 396-402.

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