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Theoretical and Experimental Analysis of Scroll Expander

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ABSTRACT

In this article, both theoretical and experimental analysis on the performance of a new scroll expander is presented and discussed. Various working chamber volumes from suction to discharge are defined. Thereafter, the evolution and derivative of the working chamber volume with respect to the orbiting angle are discussed. In order to investigate the performance of scroll expander, a detail thermodynamic modeling based on energy and mass balances is established. Radial and flank leakage, heat transfer between the working fluid, scroll wraps and plates are considered in the thermodynamic modeling. Volume, pressure, mass flow of working chamber and power are investigated by solving the thermodynamic modeling. A simple experimental rig for scroll expander is set up. From the comparison of the simulated and measured data, it can be seen that the expander model predicts the output power very well (within 5%). So the proposed mathematical modeling can accurately describe all the suction, expansion and discharge processes for scroll expander.

1. INTRODUCTION

In recent years, protecting environment and reducing energy consumption become two major issues for the human being. Many researchers concentrate on the low-grade energy recovery technique in order to economize energy sources. The small scale energy recovery system makes important contribution to energy crisis and environment pollution which is normally used in the vehicle and mobile equipment. As a kind of high efficient positive machine, scroll compressor has many advantages such as simple structure, high-efficiency, low noise, high reliability, low vibration, light weight and small size compared with other types of compressors. It is becoming popular and widely used in refrigeration, air-conditioning, various kinds of gas compression and pressurized pump products, etc (Peng Bin *et al.*, 2016). Scroll expander could be used as the energy recovery machine in the small energy recovery system. In recent years, scroll expander is applied in the low-temperature waste heat recovery system, fuel cell system and carbon dioxide refrigeration system. There are a lot of researches on experimental evaluations and numerical simulations for the performance of scroll expanders (Liu Guangbin *et al.*, 2013. Song P. *et al.*, 2015. Tarique M. A. *et al.*, 2014. Lemort V. *et al.*, 2009. Quoilin S. *et al.*, 2010. Zhu J. *et al.*, 2016. Wu Z. *et al.*, 2015. Quoilin S., 2011). However, for scroll expander, it is worthwhile to get more comprehensive understanding on the various aspects of its working process using the detail mathematical model and quantitative analysis in order to increase the accuracy of simulation and performance prediction. In this paper, effort has been made to present in detail inner working process of scroll expander. This research is expected to provide an insight to understand the qualitative and quantitative characteristics of the working process of scroll expander. So this study develops a new scroll expander and presents

a comprehensive simulation model that predicts the performance of a scroll expander. A simple experimental campaign is also conducted to measure expander performance and validate the mathematical model.

2. GEOMETRIC MODEL

The goal of the geometry model is to achieve expressions for the volumes of the different expander chambers as a function of the orbiting angle. The change of the working chamber volume with respect to the orbiting angle is calculated according to one single chamber from suction to discharge. Figure 1 is the structure of scroll expander. The basic parameters of the scroll expander are listed in Table 1.

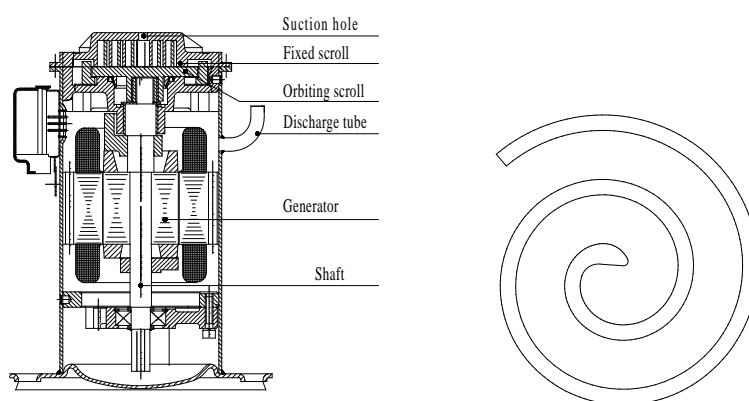


Figure 1: Scroll expander

Table 1: The basic parameters of the scroll expander

Parameters	Value
Base circle radius r_b	2.387mm
Pitch P	15mm
Thickness t	3.6mm
Height h	20mm
Ending angle of the scroll profile φ_e	27.25rad
Radius of big modification arc R_{O1}	4.13mm
Radius of small modification arc R_{O2}	1.02mm
Modification line length L	5.71mm

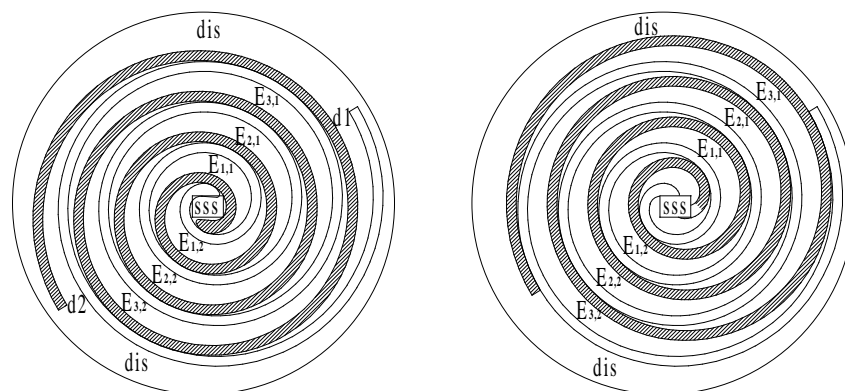


Figure 2: The definition of expansion chambers

Figure 2 is the definition of working chambers. The working chambers $E_{1,1}$, $E_{1,2}$, $E_{2,1}$, $E_{2,2}$, $E_{3,1}$ and $E_{3,2}$ are expansion chambers. The working chambers d_1 and d_2 are discharge chambers. The working chamber $_{sss}$ is suction chamber. The base line method is used to calculate the volume of scroll expander (Peng Bin *et al.*, 2015).

The working chamber volumes $V_{E_{1,1}}(\theta)$ and $V_{E_{1,2}}(\theta)$ can be calculated by:

$$V_{E_{1,1}}(\theta) = h\pi r_b R_{or} (2\theta - (\varphi_{i0} - \varphi_{o0} - \pi)) \quad 0 \leq \theta < 2\pi \quad (1)$$

$$V_{E_{1,2}}(\theta) = V_{E_{1,1}}(\theta) \quad (2)$$

The working chamber volumes $V_{E_{2,1}}(\theta)$ and $V_{E_{2,2}}(\theta)$ can be calculated by:

$$V_{E_{2,1}}(\theta) = h\pi r_b R_{or} (2(2\pi - \theta) - (\varphi_{i0} - \varphi_{o0} - \pi)) \quad 0 \leq \theta < 2\pi \quad (3)$$

$$V_{E_{2,2}}(\theta) = V_{E_{2,1}}(\theta) \quad (4)$$

The working chamber volumes $V_{E_{3,1}}(\theta)$ and $V_{E_{3,2}}(\theta)$ can be calculated by:

$$V_{E_{3,1}}(\theta) = h\pi r_b R_{or} (2(4\pi - \theta) - (\varphi_{i0} - \varphi_{o0} - \pi)) \quad 0 \leq \theta < 2\pi \quad (5)$$

$$V_{E_{3,2}}(\theta) = V_{E_{3,1}}(\theta) \quad (6)$$

The working chamber volumes $V_{d_1}(\theta)$ and $V_{d_2}(\theta)$ can be calculated by:

$$V_{d_1}(\theta) = hr_b R_{or} ((2\varphi_e - 2\theta)\varphi_e - (\varphi_e - \theta)^2 - (\varphi_e - \theta)(\varphi_{i0} + \varphi_{o0} + \pi) + 2(1 - \cos(\varphi_e - \theta)) - 2(\varphi_e - \pi)\sin(\varphi_e - \theta)\frac{\pi}{4}\sin(2(\varphi_e - \theta))) \quad \varphi_e - 2\pi \leq \theta < \varphi_e \quad (7)$$

$$V_{d_2}(\theta) = V_{d_1}(\theta) \quad (8)$$

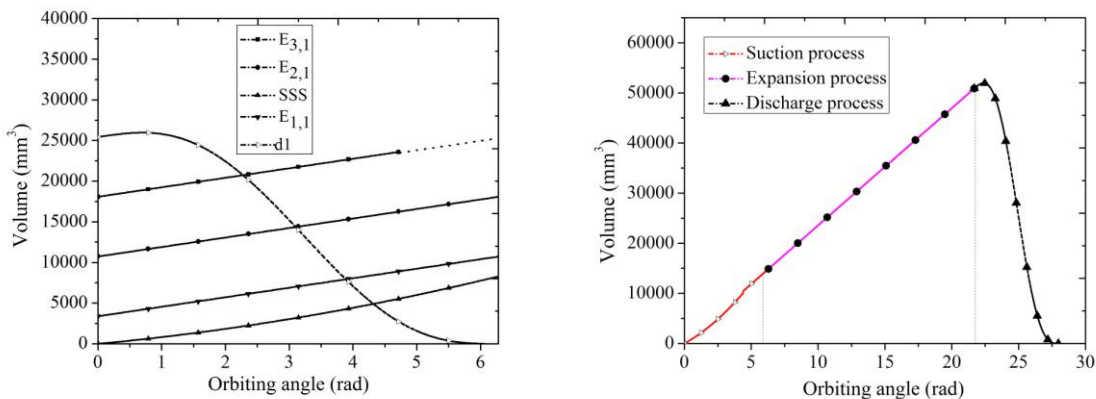
The working chamber volume $V_{_{sss}}(\theta)$ can be calculated by:

$$V_s(\theta) = hr_b R_{or} \theta(\theta - \varphi_{i0} - \varphi_{o0} + 3\pi) \quad 0 \leq \theta < 2\pi \quad (9)$$

The inner volume ratio v_e of scroll expander can be calculated by:

$$v_e = \frac{2\varphi_e - \varphi_{i0} + \varphi_{o0} - 3\pi}{5\pi - \varphi_{i0} + \varphi_{o0}} \quad (10)$$

Figure 3 is the change of the working chamber volume with respect to the orbiting angle. It can be seen that the volume increases in suction process. As expansion volume increases, temperature and pressure decrease. The expansion chambers change into discharge chambers at discharge angle. As the discharge process ends, one entire suction-expansion-discharge process finished.



(a) The working chambers volume

(b) The expansion process

Figure 3: The change of the working chamber volume with respect to the orbiting angle

3. THERMODYNAMIC MODEL

3.1 Basic equations

Change of temperature, mass and pressure in each expansion chamber with respect to orbiting angle θ can be calculated from the first law of thermodynamics (Equation (11)) for an open control volume in conjunction with an equation of state (13), (14) and the mass balance (Equation (15)). The change of the gas (N_2) temperature with respect to θ can be written as (Halm N.H, 1997. Chen Yu, 2000)

$$\frac{dT}{d\theta} = \frac{1}{mC_v} \left\{ -T \left(\frac{\partial p}{\partial T} \right)_v \left[\frac{dV}{d\theta} - \frac{v}{\omega} (m_{in} - m_{out}) \right] - \sum \frac{m_{in}}{\omega} (h - h_{in}) + \frac{Q}{\omega} \right\} \quad (11)$$

$$\omega = \frac{d\theta}{dt} \quad (12)$$

$$\frac{dp}{dT} = \frac{R}{v}, \quad \frac{dp}{dv} = -\frac{RT}{v^2} \quad (13)$$

$$dh = c_p dT, \quad c_p = 1.11 - 0.48 \left(\frac{T}{1000} \right) + 0.96 \left(\frac{T}{1000} \right)^2 - 0.42 \left(\frac{T}{1000} \right)^3 \quad (14)$$

$$\frac{dm}{d\theta} = \sum \frac{m_{in}}{\theta} + \frac{m_{out}}{\theta} \quad (15)$$

3.2 Suction gas mass flow

The suction gas mass flow rate is calculated using the flow equation for isentropic flow of a compressible ideal gas, corrected by a flow factor ψ (Fox RW and McDonald AT, 1992)

$$\dot{m} = \psi A_s \sqrt{2 p_h \rho_h} \sqrt{\frac{\gamma}{\gamma-1} \left[\left(\frac{p_l}{p_h} \right)^{\frac{2}{\gamma}} - \left(\frac{p_l}{p_h} \right)^{\frac{\gamma+1}{\gamma}} \right]} \quad (16)$$

This flow is restricted by a critical pressure ratio p_l / p_h for choked flow conditions.

$$\left(\frac{p_l}{p_h} \right)_{crit} = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \quad (17)$$

3.3 Heat transfer model

The expansion process in scroll expander is achieved in pockets between the fixed scroll and the orbiting scroll. The efficiency of scroll expander depends significantly on the convective heat transfer and the leakage between the scrolls and working fluid. Therefore, the accurate understanding of the inside physical condition of gas that is influenced by the convective heat transfer with scrolls, is very important to correctly analyze the expansion process in scroll expander. Heat transfer can occur at several locations in a scroll expander because of the different temperature of ambient, shell, fluid, scroll wraps and plates. Heating during the expansion process causes the expansion to go farther away from the ideal isentropic expansion process.

3.3.1 Heat transfer during suction and discharge

The heat transfer coefficient h_c can be calculated by the following relations (Incropera FP and Dewitt DP, 1996)

$$h_c = 0.023 \frac{\kappa}{d_p} Re^{0.8} Pr^{0.4} \quad (18)$$

$$Re = \frac{4\dot{m}}{\pi d_p \mu} \quad (19)$$

$$Pr = \frac{\mu c_p}{\kappa} \quad (20)$$

The outlet temperature $T_{s,o}$ of the gas can be calculated by

$$T_{s,o} = T_{pipe} - (T_{pipe} - T_i) \exp \left(-\frac{\pi d_p L_p h_c}{\dot{m} c_p} \right) \quad (21)$$

The heat flow rate from the pipe to the fluid \dot{Q}_{pipe} is given by

$$\dot{Q}_{pipe} = \dot{m}c_p(T_{pipe} - T_{s,i}) \left[1 - \exp\left(-\frac{\pi d_p L_p h_c}{\dot{m}c_p}\right) \right] \quad (22)$$

which is evaluated for both the inlet and outlet paths of the expander.

3.3.2 Scrolls heat transfer

As the fluid is expanded by the scroll expander, it experiences heat transfer from the scrolls, the bottom and top plate. In order to account for these losses, the heat transfer process needs to be modeled and incorporated into the expansion process model.

The correlation h_{cm} for the spiral heat exchanger is as follows (Kakac S and Shah RK, 1987. Rajavel R and Saravanan K, 2008. K Jang and S Jeong, 2006)

$$h_{cm} = 0.023 \frac{\kappa}{D_{ef}} Re^{0.8} Pr^{0.4} \left(1 + 1.77 \frac{D_{ef}}{R_{aver}} \right) (1 + 8.48 [1 - \exp(-5.35St)]) \quad (23)$$

The hydraulic diameter D_{ef} is defined as

$$D_{ef} = \frac{4V}{A} \quad (24)$$

The average radius R_{aver} is defined in this model as

$$R_{aver} = a \left[\frac{(\varphi_k - \pi/2) + (\varphi_{k-1} - \pi/2)}{2} \right] \quad (25)$$

The temperature distribution of the scroll is non-uniform. It is assumed that the temperature along the scroll wraps is linear with the generating angle φ between two generating angles φ_1 at the center of the scroll and generating angle φ_2 at the outer edge of the scrolls. The average temperature of scrolls is at the generating angle of $\frac{\varphi_1 + \varphi_2}{2}$ (middle of the scrolls). Then, the temperature distribution in the scrolls can be expressed as (Chen Yu, 2000. Rajavel R and Saravanan K, 2008. K Jang and S Jeong, 2006)

$$T(\varphi) = \frac{\Delta T}{\Delta \varphi} (\varphi - \varphi_1) + T_{s1} \quad (26)$$

The heat exchange rate $\dot{Q}_{scrolls}$ from the scroll walls/plates to the gas in any chamber can now be calculated by an integral method according to the following equation

$$\dot{Q}_{scrolls} = h_c \int_A [T(\varphi) - T(k, j)] dA \quad (27)$$

3.4 Leakage model

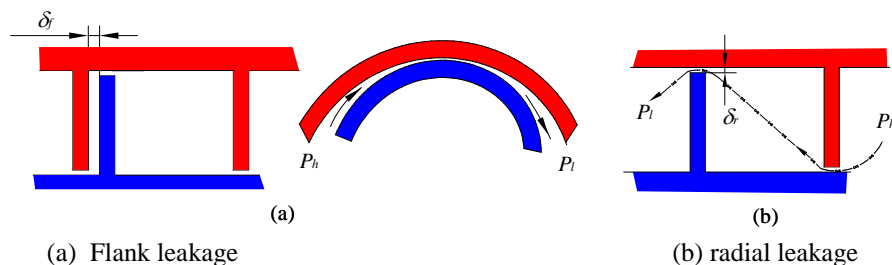


Figure 4: Flank and radial leakage

During expansion process, the pressure difference of adjacent chambers leads to leakage between in the high and low pressure chambers. There are two different paths for leakage in a scroll expander. One is the path that is formed by a gap between the flanks of the two scrolls and is called flank leakage. Another path is formed by a gap between the bottom or the top plates and the scrolls. This kind of leakage is called radial leakage. The two kinds of leakage are illustrated in Figure 4. Flank leakage and radial leakage account for losses during the expansion process as they decrease the volumetric efficiency significantly. Leakages also account for a decrease in specific expansion work

rate because gas leaking from high pressure regions back into low pressure regions and needs to be re-expanded. In order to calculate the leakage rate from a higher pressure chamber to one with low pressure, the width of the flow path needs to be evaluated (Halm N.H, 1997).

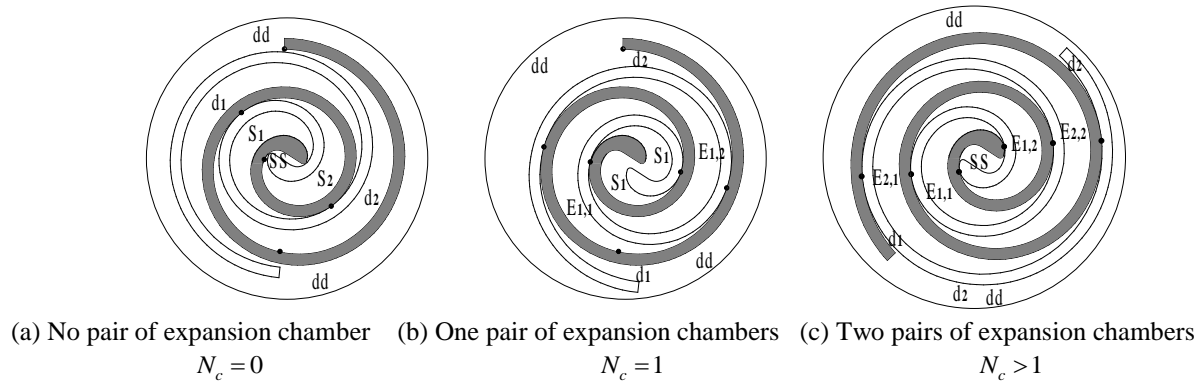


Figure 5: Three cases for expansion chambers

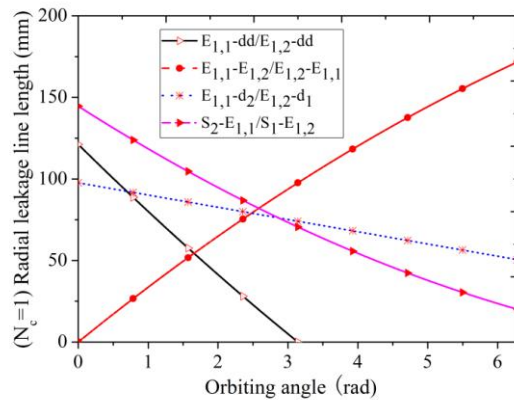


Figure 6: Evolution of the radial leakage line length with respect to the orbiting angle

For the flank leakage, the gas flow area A_f is

$$A_f = h\delta_f \tag{28}$$

According to the scroll expander manufacturer, the flank gap size δ_f can be represented as a linear function of the pressure ratio.

$$\delta_f = -9.615 \times 10^{-5} \left(\frac{p_h}{p_l} - 1.67 \right) + 20 \times 10^{-6} \tag{29}$$

For the radial leakage, the gas flow area A_r is

$$A_r = \delta_r L_r \tag{30}$$

The effect of the expansion ratio on the radial gap size is much less. It is assumed however, that the radial gap increases with increasing expansion ratio. The gap size δ_r is also represented by a linear function.

$$\delta_r = 1.1 \times 10^{-6} \left(\frac{p_h}{p_l} - 1.67 \right) + 10^{-6} \tag{31}$$

Figure 5 shows three cases for expansion chambers. Table 2 is generating angle of the radial leakage for different cases. The leakage line length L_r is

$$L_r = r_b \left(\frac{1}{2} (\varphi_{\max}^2 - \varphi_{\min}^2) - \varphi_0 (\varphi_{\max} - \varphi_{\min}) \right) \tag{32}$$

φ_{\max} and φ_{\min} are displayed in the table 2.

Table 2: Generating angle of the radial leakage

N_c	The chambers of leakage	φ_{\max}	φ_{\min}
$N_c = 0$	$s_2 - d_1 / s_1 - d_2$	φ_{ie}	$\max(\varphi_{ie} - \theta, \varphi_{ie} - \pi)$
	$s_1 - s_2 / s_2 - s_1$	$\varphi_{ie} - \pi$	$\min(\varphi_{ie} - \theta, \varphi_{ie} - \pi)$
$N_c = 1$	$E_{1,1} - dd / E_{1,2} - dd$	$\max(\varphi_{ie} - \theta, \varphi_{ie} - \pi)$	$\varphi_{ie} - \pi$
	$E_{1,1} - E_{1,2} / E_{1,2} - E_{1,1}$	$\max(\varphi_{ie} - \pi - \theta, \varphi_{ie} - \pi)$	$\varphi_{ie} - \theta - \pi$
	$E_{1,1} - d_2 / E_{1,2} - d_1$	$\varphi_{ie} - \theta - \pi$	$\varphi_{ie} - \theta - 2\pi$
	$s_2 - E_{1,1} / s_1 - E_{1,2}$	$\varphi_{ie} - \theta - 2\pi$	φ_{is}
$N_c > 1$	$E_{2,1} - dd / E_{2,2} - dd$	$\max(\varphi_{ie} - \theta, \varphi_{ie} - \pi)$	$\varphi_{ie} - \pi$
	$E_{2,1} - d_2 / E_{2,2} - d_1$	$\max(\varphi_{ie} - \pi - \theta, \varphi_{ie} - \pi)$	$\varphi_{ie} - \theta - \pi$
	$E_{2,1} - E_{2,2} / E_{2,2} - E_{2,1}$	$\varphi_{ie} - \theta - \pi$	$\varphi_{ie} - \theta - 2\pi$
	$E_{\alpha-1,1} - E_{\alpha,2} / E_{\alpha-1,2} - E_{\alpha,1}$	$\varphi_{ie} - \theta - 2\pi(\alpha - 1)$	$\varphi_{ie} - \theta - 2\pi(\alpha - 1) - \pi$
	$E_{\alpha,1} - E_{\alpha,2} / E_{\alpha,2} - E_{\alpha,1}$	$\varphi_{ie} - \theta - 2\pi(\alpha - 1) - \pi$	$\varphi_{ie} - \theta - 2\pi\alpha$
	$s_2 - E_{1,1} / s_1 - E_{1,2}$	$\varphi_{ie} - \theta - 2\pi N_c$	φ_{is}

3.5 Output characteristics of scroll expander

The working process of scroll expander is unsteady state. The parameters change with the orbiting angle. As the shaft rotates, the gas sucked into the expansion chamber is expanded. In the scroll expander structure, the orbiting scroll can move between the fixed scroll and the frame, the gas forces and moments acting on it are finally transferred to the bearings installed in the orbiting scroll and the frame. The tangential and axial gas forces also account for an increase frictional loss. The output torque of scroll expander is mainly caused by the tangential gas force. The main gas forces and moments are showed in Figure 7.

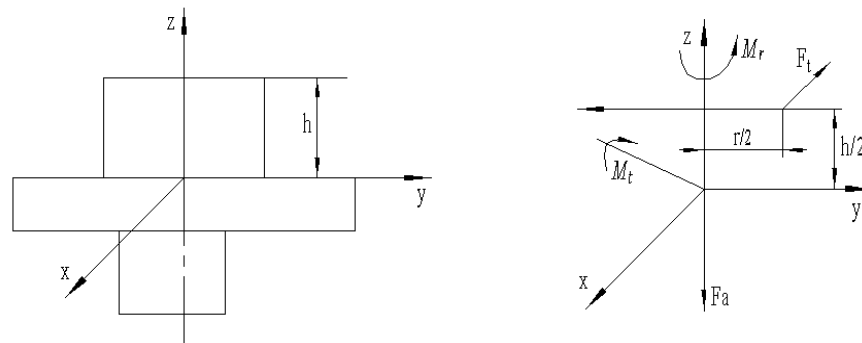


Figure 7: Gas forces and moments acting on the orbiting scroll

The tangential gas force $F_t(\theta)$ can be computed by

$$F_t(\theta) = \sum_{i=1}^N P \left(2i - \frac{2\pi - \theta}{\pi} \right) h (p_i - p_{i+1}) \tag{33}$$

The tangential gas moment $M_t(\theta)$ can be computed by

$$M_t(\theta) = F_t(\theta)R_{or} = \sum_{i=1}^N P(2i - \frac{2\pi - \theta}{\pi})h(p_i - p_{i+1})R_{or} \quad (34)$$

The output power $W(\theta)$ is given by

$$W(\theta) = M_t(\theta)\omega\eta_m \quad (35)$$

The mechanical efficient η_m is given by (Park Y. C. *et al.*, 2002)

$$\eta_m = 0.868 + 0.0048f - 4.4444 \times 10^{-5} f^2 \quad (36)$$

4. PERFORMANCE ANALYSIS

4.1 Simulation analysis

The detail thermodynamic model has been implemented into FORTRAN and Matlab computer code. This code calculates the pressure and mass in each of the working chambers as a function of the orbiting angle for an entire revolution. It also calculates the power consumption of the scroll expander for a specified operating condition. Figure 8 is the flowchart for the implementation of the thermodynamic model.

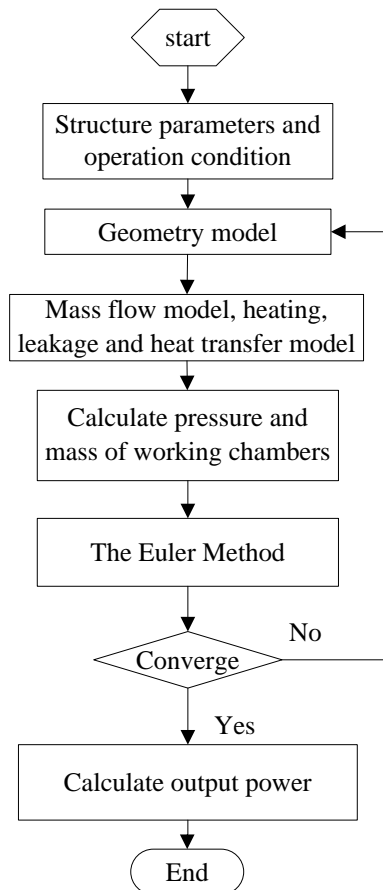
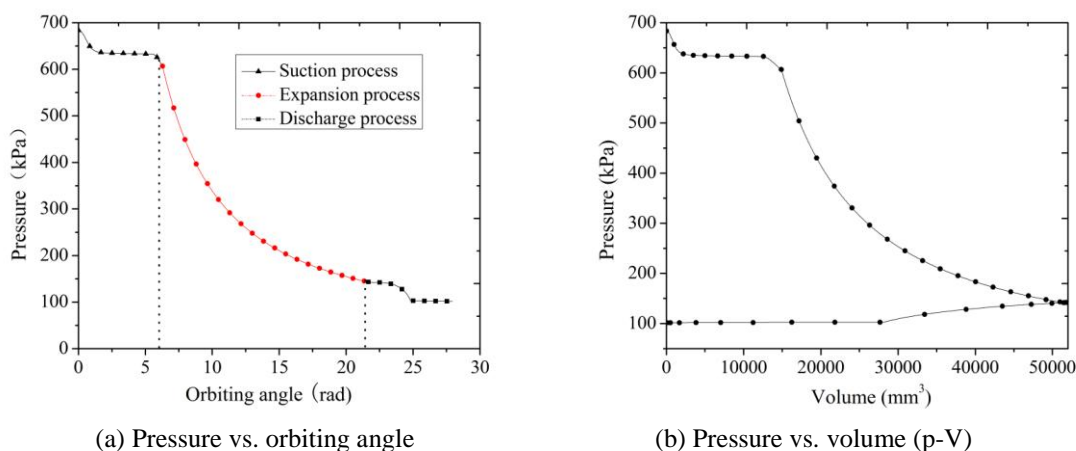


Figure 8: Flowchart of thermodynamic model

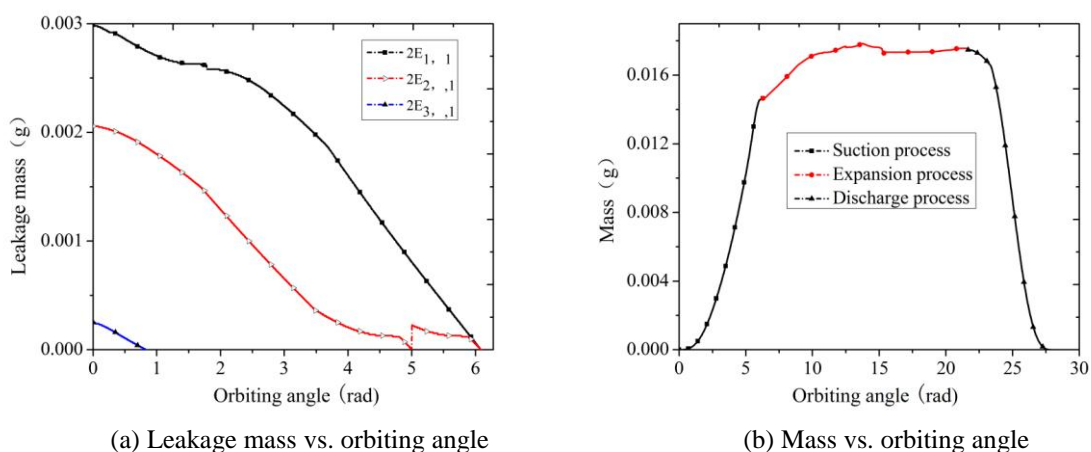
Figure 9 is the change of pressure with respect to the orbiting angle and volume. The suction pressure is 683.5kPa and speed is 2500r/min. Due to throttling of suction hole, the pressure has slightly decreased during the suction process. The suction pressure decreases about 77kPa before it enters the expansion chamber. During the expansion stage, the change of pressure is from 606kPa to 143kPa. When the last expansion chamber opens to the discharge chamber the discharge process begins. Because of big discharge hole, during the discharge process the pressure

slightly drops, the change of pressure is from 143kPa to 100kPa. During the expansion process, the pressure decreases with the increase of working chamber volume and it changes more and more gently in Figure 9(b).



(a) Pressure vs. orbiting angle

(b) Pressure vs. volume (p-V)

Figure 9: The change of pressure with respect to the orbiting angle and volume

(a) Leakage mass vs. orbiting angle

(b) Mass vs. orbiting angle

Figure 10: The change of leakage mass and working chamber mass with respect to the orbiting angle

Figure 10 is the change of leakage mass and working chamber mass with respect to the orbiting angle. The mass first increases until the suction chamber is closed. During the expansion process, the mass first changes sharply and then goes slowly. In the discharge process, the mass starts to decrease until the end of discharge process.

4.2 Model validation

A hermetic scroll expander is used to measure performance and validate model in a simple test rig as seen in Figure 11. The high pressure nitrogen enters the suction chamber through reducing valve, control valve and flow meter. Control valve is used to control suction pressure. The scroll expander is driven by the compressed nitrogen. Electric generator transforms mechanical energy into electricity. The voltage, current and power can be gotten from data acquisition system. Figure 12 is the change of performance with respect to the suction pressure. When the suction pressure reached 1000kPa, the test output power is almost 1242W in Figure 12(b). The absolute error of the output power is from 16W to 45W. By comparing the calculating results with the test results it is proved that the proposed thermodynamic model can simulate the working process of scroll expander accurately.

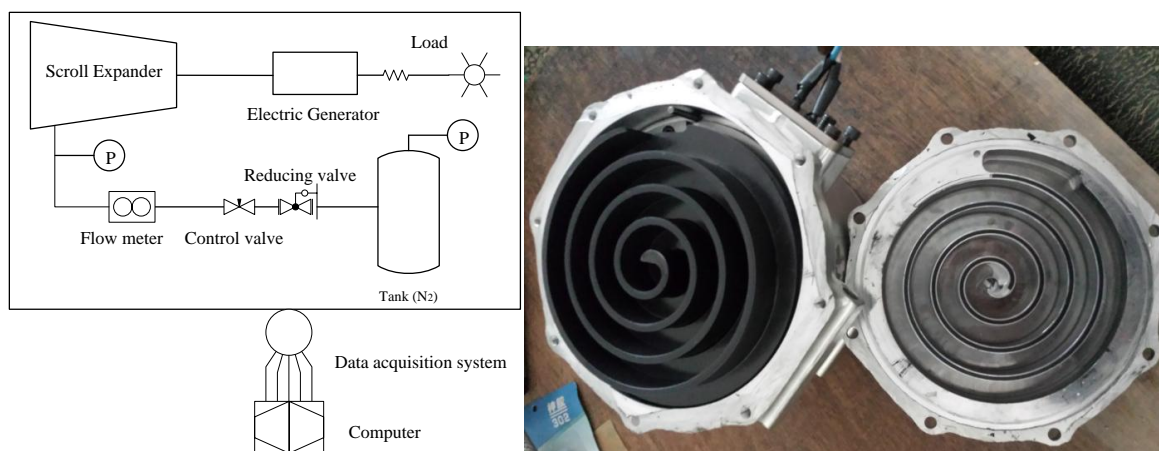


Figure 11: The test rig

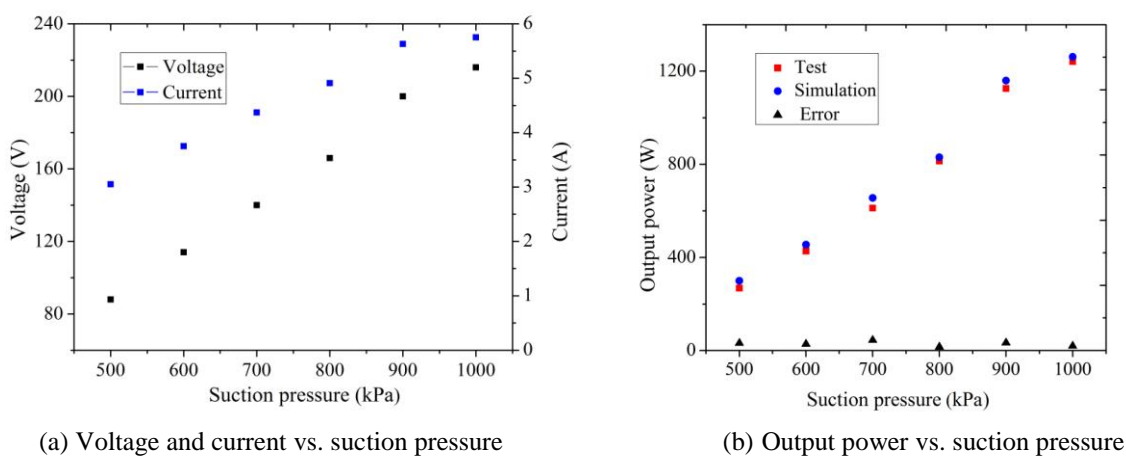


Figure 12: The change of performance with respect to the suction pressure

5. CONCLUSIONS

- (1) A geometry model of scroll expander is presented. The evolution of the working chamber volume with respect to the orbiting angle has been derived.
- (2) Based on energy and mass conservation equations, the thermodynamic modeling of scroll expander is set up. The mass and pressure of gas in working chambers are determined by solving the detail thermodynamic modeling of scroll expander using the Euler explicit method.
- (3) The developed scroll expander prototype is used to validate the thermodynamic modeling. The simulation results obtained from the thermodynamic modeling agree very well with experimental results (within 5%).
- (4) As future work a more detailed analysis will be done with the mathematical model. And an ORC system will be set up to test different operational conditions of scroll expanders and working fluids.

NOMENCLATURE

φ_{i0}	generating angle of inner scroll	(rad)
φ_{o0}	generating angle of outer scroll	(rad)
R_{or}	turning radius	(mm)
T	temperature	(K)

m	gas mass	(kg)
C_v	specific heat at constant specific volume	(-)
p	gas pressure	(kPa)
v	specific volume	(m ³ /kg)
ω	angular speed of expander shaft	(rad/s)
\dot{m}_in	mass flow rate flowing into control volume	(kg)
\dot{m}_out	mass flow rate flowing out of control volume	(kg)
h	gas specific enthalpy of the control volume	(kJ/kg)
h_in	gas specific enthalpy flowing into the control volume	(kJ/kg)
\dot{Q}	heat flow rate flowing into the control volume	(kW)
c_p	gas specific heat ratio	(kJ/kg-K)
p_h	pressure in the high pressure side	(kPa)
A_s	area of the suction chamber opening	(mm ²)
p_l	pressure in the low pressure side	(kPa)
Re	Reynolds number	(-)
T_{pipe}	temperature of the pipe	(K)
d_p	diameter of the pipe	(mm)
Pr	Prandtl number	(-)
St	Strouhal number	(-)
γ	specific heat ratio	(-)
ρ_h	density of the gas in the high pressure side	(kg/m ³)
μ	viscosity	(Pa-s)
κ	conductivity	(kW/m-k)
L_p	length of the pipe	(mm)
T_{s1}	temperature at the generating angle ϕ_1	(K)
$T(k, j)$	temperature of the gas in the k -th chamber at the angle θ_j	(K)
A	chamber bounding area	(mm ²)
T_{gas}	temperature of the gas in the scroll expander	(K)
dA	heat transfer area of scroll wrap and plate	(mm ²)
f	the frequency of scroll expander	(Hz)

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