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Strategic flexibility

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STRATEGIC FLEXIBILITY

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To my family...

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ABSTRACT

Kim, KiHyung Ph.D., Purdue University, December 2014. Strategic Flexibility. Major Professor: Abhijit Deshmukh.

A flexible system is defined as one that can change the entity's stance, capability or status reacting to a change of the entity's environment. Flexibility has gathered the attention of academic researchers and industry practitioners as an efficient approach to cope with today's volatile environment. As the environments become more unpredictable and volatile, it is imperative for a flexible system to respond quickly to a change in its circumstance. How much flexibility is embedded into the system also has a critical impact on the long-term effectiveness of the flexible system. Moreover, this research focuses on the strategic environment where a decision maker's behavior influences other decision makers' and vice versa.

The primary objectives of this dissertation are developing a concrete framework for designing a flexible system by considering the exercise delay as a measure of flexibility and investigating the rational behaviors of decision makers who operate flexible systems under strategic environments. The general approach employed to develop the theoretical models for this dissertation includes optimal control theory, non-linear optimization, stochastic differential equation and game theory.

The first part of this research studies the optimal decisions on a flexible system with exercise delay within stochastic environments by postulating two level decisions, operational level and design level decisions. The operational level problem is modeled as a delayed optimal stopping time problem, and this research provides a comprehensive profile of the optimal operational policies according to the parameters representing the market conditions and characteristics of the alternative and designed features of the flexible system. In addition, the profile elucidates the interdependence

between the operational level decision and the design level decision separating the entire domain of the design problem into sub-regions. This research effort finds that the design problem is decomposable with well-behaved non-linear optimization problems, and provides illustrative examples to show the usefulness of the developed framework.

The second part of this research concentrates on strategic environments which force a decision maker to cope with both exogenous uncertainty and endogenous interactions among decision makers. As the strategic environment, a duopoly market share competition is postulated where the total market profit is regarded as the underlying uncertainty. The player retaining an exclusive patent is regarded as a player competing in the market with a flexible system that does not have exercise delay, and the other competitor is interpreted as a player operating a flexible system with exercise delay. The open loop and closed loop information structures are considered for each model. The results showed that the open loop equilibrium is unique dominant strategy equilibrium. An interesting implication of the open loop equilibria is that the profitability of the flexible option decides the role of its owner in the duopoly market competition. This research finds that the closed loop equilibrium has two distinctive forms. When the asymmetry of exercise delay is large, the closed loop equilibrium is identical to the open loop equilibrium. On the other hand, if the asymmetry provides only a small enough advantage to the player who has a flexible option without exercise delay, the rational behaviors of the players are complicated in the closed loop equilibrium. The first insight from the closed loop equilibrium with large asymmetry is that the closed loop information structure hastens the execution of flexible options, and it results in lower payoffs to both of the players. Second, the role of each player is determined not only by the characteristics of the flexible options but also by the value of stochastic factor. Third, even the player with a competitive disadvantage from the asymmetry has a positive chance to be the leader of the market.

This research contributes to the area within industrial engineering and operations research by improving the current theoretical achievement of flexibility. The accomplishments of this work provides insights to various domains those would benefit from enhanced flexibility in the decision making process.

1. INTRODUCTION

1.1 Motivation

Flexibility has gathered the attention of academic researchers and industry practitioners as an efficient approach to cope with future uncertainty. A flexible system is defined as one that can change the entity's stance, capability or status reacting to a change of the entity's environment. Needless to say, future uncertainty is an important factor that decision makers have to deal with, and it becomes even more vital in today's volatile environment. Since a flexible system can take the best action within its available options, it is regarded as an effective approach to manage future uncertainty. As the environments become more unpredictable and volatile, it is imperative for a flexible system to respond quickly to a change of its circumstance. How much flexibility is embedded into the flexible system also has critical impact on the long-term effectiveness of the flexible system. Moreover, it is necessary to scrutinize the environment of a system to utilize the benefit from flexibility. Among the environmental characteristics, this research focuses on the strategic environment where a decision maker's behavior influences other decision makers'. Since the interactions among decision makers cause a different type of risk from that which the classical flexibility approach deals with, it is essential to consider the strategic environment appropriately.

The usefulness of flexibility under stochastic environments becomes clear, when it is compared to the traditional theory. According to Dixit and Pindyck [1], the orthodox theory that is mainly based on Net Present Value (NPV) approach has drawbacks in complex and volatile environments. They pointed out that the traditional approach is valid for a reversible decision or a "now or never" type decision. Because NPV is based on the estimated future uncertainty, estimation risks are inherent in the traditional approach. In other words, the estimation may not describe the future states appropriately, and it can cause an unsatisfactory decision under unstable situations. The traditional approach implicitly assumes that a decision maker can revise the decision that is turned out to be grounded in unsatisfactory estimation. However, many of decisions are irreversible in reality. When the decision is irreversible, decision makers prefer flexible decision making that enables to postpone the decision until the underlying uncertainty is resolved enough to avoid the estimation risk. Although, the traditional theory is appropriate for the situations when a decision must be made now or the opportunity to take the action disappears forever, the approach is not adequate for flexible decision making. Because of these reasons, flexibility is increasingly turned to as an effective approach to dealing with uncertainty in systems.

This research extends current research of flexibility by considering exercise delay that does not get enough attention from decision makers. The extent of flexibility can be measured in a number of different ways. For example, the overall cost to change the capacity of a system [2], the extend of possible choices [3], and the states that a flexible system is efficient in [4] are suggested measures of flexibility. This research focuses on the time delay between different system states, or the time required to flex the system. The author believes how quickly a system can implement a flexible decision is an essential measure of any system that is considered flexible. After all, given enough time and resources, any system can be considered flexible. With this in mind, the delay between the time a decision is made and the time that decision takes effect becomes a useful notion of flexibility, which is called "exercise delay." Systems with greater flexibility will have shorter exercise delay, whereas less flexible systems will have greater delays. How United Colors of Benetton acquired its competitive advantage in the fashion industry is a well-known real world example showing the importance of exercise delay [5]. Since customers' preferences to color is volatile and difficult to predict, Benetton employed the "Knit now, Dye later" policy. Benetton produces clothes without colors and dyes pre-produced clothes when the new season's popular color fashions become apparent. Because of the ability adapting Benetton's items for the change of customers' preference quickly, it was popular in the 1980s and the 1990s. This research regards exercise delay as an essential component of a flexible system.

The efficiency of a flexible system is determined by not only how well the system is operated but also by how much flexibility is embedded in the system. For instance, suppose that a man is stranded on a desolate island with a Swiss Army Knife and canned foods. Unless the knife is equipped with a can opener, it is not very useful for his survival. This hypothetical example illustrates that the flexible system may not be effective no matter how well the system is operated, if it is ill-designed. Since the design of a flexible system has been studied mainly in the view point of capability change, the literature lacks a formal approach to studying the optimal design or level of flexibility with respect to the exercise delay.

Insufficient consideration of the strategic environment can cause dire consequences for the system; especially when the other decision makers are uncooperative. A decision maker may overestimate the value of a flexible system and not operate it optimally, if the strategic environments are not considered appropriately. The examples showing the importance of strategic environment are easily found in competitive industries. Google paid 12.5 billion dollars to acquire Motorola in 2012, and sold the company to Lenovo at 2.91 billion dollars except for the majority of Motorola's patents. After spending about 10 billion dollars, Google retains only Motorola's patents, and many of the patents has not been implemented yet. The retained patents provide flexibility to Google, because customers' demands are hard to predict. If the patents enable Google to satisfy a new demand, the company is able to manage the stochastic environment which comes from customers' volatile demands. However, whether the patents are worth of 10 billion dollars is controversial. Samsung is one of the biggest competitors against Google in the mobile phone market. Since Samsung has been engaged in many patent infringement suits, it aggressively invests in the research and development of alternative technologies. If Google underestimated the

R&D efforts of competitors, the deal with Motorola regarding the patents could be overestimated.

The strategic environments affect not only the evaluation of a flexible system but also the operation of it. In the context of launching a new product without competition, a company may want to postpone introducing the new product to utilize the benefits from the existing production line. Suppose that the workers are so accustomed to the current production process that the company saves costs by learning curve effect, and the new product may encroach on the profit of the current product. Then the company has incentive to postpone launching a new product until the customers' demand of the existing product is exhausted. Alternatively, presume that the market is competitive, and the first mover's advantage exists. Then the company has incentive to preoccupy the new product market by introducing the new product earlier than its competitor, even if it costs the benefit from the current production mode. In terms of flexible system management, it suggests the competition hasten the execution time of flexible option.

This research is motivated to develop an concrete framework for an optimal design of a flexible system that has exercise delay and to derive and analyze the equilibrium of interactions between decision makers who operate flexible systems.

1.2 Overview of Research Goals

The primary objectives of this dissertation are to develop a concrete framework of flexible system design considering exercise delay and to analyze the rational behavior of system operators who manage a flexible system and interact with each other. The increasing attention that has been directed toward managing the impact of uncertainties in flexible systems with exercise delay has not yet provided a framework for the system design problem. It is a cornerstone of design task how the designed flexible system is operated, and the operational decision is based on the designed features. To the best of author's knowledge, the current literature has not yet provided a systematic framework for the design problem considering the interdependency of design and operational level decision. Providing a well-established flexible system design framework is one of the main goals of this dissertation.

Strategic environments compel a decision maker to consider both the uncertainty that is not controlled by any of decision makers and the risk caused by other players. Considering the endogenous interdependency among decision makers and investigating decision makers' rational behavior in an equilibrium are the other main goals of this research effort.

These research goals are accomplished through the development of the theoretical models of a flexible system and the solutions under stochastic environments and strategic environments.

1.2.1 Flexible System with Exercising Delay

This research considers an irreversible decision under a stochastic environment. A system designer builds a flexible system that enables the operator of the system to postpone the irreversible decision until the uncertainty is resolved enough to ensure the irreversible decision is effective. This dissertation postulates there is an exercise delay between the decision and implementation of the flexible alternative, and once the decision is made, it cannot be revoked. Since the traditional approach is not adequate in this situation, this research effort utilizes the findings in optimal control theory and develops a systematic framework for designing the flexible system.

This research provides the following benefits to the flexible system managements regarding exercise delay as the measure of flexibility under stochastic environments.

- *Deriving the optimal operational policy of a flexible system with exercise delay*: A comprehensive profile of optimal operational policies according to the parameters those represent market conditions and the flexible system configurations is beneficial to system operators.

- *Analyzing the effect of exercise delay on the optimal operational policy*: During the exercise delay, the system is exposed to additional uncertainty which is avoidable when there is no exercise delay. Therefore the exercise delay affects the optimal operational decision. Analyzing the effect, this research aims to provide insights into the characteristics of a flexible system design problem.

- *Developing a concrete method to find the optimal design of a flexible system*: By investigating the mutual effects between design and operations, this research effort elucidates the structure of design problem and suggests an appropriate approach to solve the design problem. The suggested framework for design problem is desirable to be solved with usual non-linear optimization methods.

To achieve these research goals, Chapter 3 of this dissertation develops theoretical models and illustrative examples for two distinctive flexible systems. The model in Section 3.3 considers only one flexible alternative is available for a flexible system in terms of capability. It postulates that the operational task is to determine the optimal time to initiate the change of system, and the design task is to embed the optimal exercise delay into the flexible system. Section 3.4 models a flexible system whose exercise delay and the level of capability change are determined in the design phase.

1.2.2 Strategic Flexibility

This research expands the assumption of stochastic environments to strategic environments by including the interactions between decision makers. As the application context, this research models a market share competition in a duopoly market where the total market profit is stochastic. One player in the market retains an exclusive patent, and the other player has to complete a research and development project to compete with the player with patent in the new product market. Since this dissertation regards how fast a flexible system responds to the change of environment, the developed model belongs to a stochastic preemption game.

The results of this dissertation provide the following advantages to the flexible system managements under strategic environments.

- *Improving the construction of strategy space of option exercise games*: For deterministic preemption games, how to describe the players strategy is well established with rigorous mathematical background [6]. However, it is still a matter of study in stochastic preemption games. Reflecting the characteristics of stochastic game, this research suggests the strategy space based on both the history and the current position of stochastic factor.

- *Finding the open loop equilibrium of strategic flexibility*: In the open loop model, players cannot observe their opponents' actions during the game [7]. Therefore no player updates his or her strategy after the beginning of the game. By deriving the equilibrium of the model, the rational behavior of the players provides insight to the management of flexible system. Moreover, the equilibrium of this model illustrates how the patent plays a role as a entrance barrier when the information about the competitors' behavior is restricted.

- *Finding the closed loop equilibrium of strategic flexibility*: In the closed loop model, players are able to observe their competitors' actions and to update their own strategies based on the observed actions [7]. Although a comprehensive and concise description of players' strategy is required to derive closed loop equilibrium, a satisfactory construction of the strategy space is not yet reported for stochastic preemption games. This research aims to provide a strategy space that describes the players' strategy appropriately, and to derive the closed loop equilibrium illustrating the rational behavior of the players under the closed loop information structure. Finding insights into the rational decisions by comparing the open loop equilibrium and the closed loop equilibrium is an additional purpose of this research effort.

Chapter 4 is devoted to accomplishing these research objectives. Section 4.3 models the duopoly market share competition, where a player operates a flexible system without exercise delay, and the other player manages a flexible system with a fixed exercise delay. The model in Section 4.4 considers the competition where the exercise delay is a random variable.

1.3 Organization of the Dissertation

The remainder of this dissertation is organized as follows. Chapter 2 reviews the research conducted in the area of flexibility. This chapter emphasizes the evolution of flexibility concept, measure of flexibility, evaluation methods and strategic flexibility.

Chapter 3 introduces two models of flexible systems with exercise delay, and provides the optimal solutions of operational decisions and frameworks for system design problems. The first model assumes that there is only one alternative in terms of the level of capability change, and this assumption is relaxed in the second model to accommodate an infinitely many alternatives case. The results of this chapter provide the closed form solution of the operational level problem that covers the cases of expanding capability, reducing capability and even terminating the operation. Based on the analytical solution, the effect of exercise delay upon the optimal operational decision is analyzed. Moreover, the results clarify a structure of design problem and indicate the difficulties of design problem caused by the interdependence between design and operational level decisions. A decomposition method is developed to solve the design problem with usual non-linear optimization methods. Illustrative examples in the context of a renewable energy utility are included .

Chapter 4 investigates the strategic flexibility in the context of duopoly market share competition. This chapter assumes a company in the market retains an exclusive patent and the other has to conduct Research and Development (R&D) project to compete against the firm with the patent. This research models the exclusive patent as a flexibility without exercise delay and R&D opportunity as that with exercise delay. The first model assumes that the duration and costs of R&D are known as fixed numbers, and the second model considers a stochastic duration and costs case. By deriving the open loop and closed loop equilibria and comparing them, this dissertation provides insights into the rational behaviors of option exercise game players.

This dissertation is concluded in Chapter 5 by presenting conclusions, research contributions and potential extension areas for future research. The author provides appendices containing the proofs of theorems, detailed calculations, numerical solutions of illustrative examples. The bibliography of referenced work is at the end of this dissertation.

2. RELATED WORK

2.1 The Concept of Flexibility

Research in the area of flexibility has over 70 years of history. Stigler [8] published the seminal paper which explicitly used the terminology “flexibility.” In the research, Stigler regarded a system having a flat average cost curve as a flexible system. Marschak and Nelson [9] disputed the concept of flexibility suggested by Stigler. They conceptualized flexibility in terms of marginal cost rather than average cost. Hart [10] emphasized decision postponement as the core component of flexibility. With flexibility, a decision maker can postpone an irreversible decision until future uncertainty is resolved enough. Feibleman and Friend [11] defined flexibility in an organization context as “the capacity of an organization to suffer limited change, without severe disorganization.”

In fact, flexibility is so important that there are entire journals and several excellent literature reviews devoted to the subject; however the concept of flexibility is still vague and ambiguous. Buzacott and Mandelbaum stated the following in their recent review paper [3]:

> “As exciting and useful as the concept of ‘flexibility’ seems, there is no common agreement on how to define or implement the concept and it has been very problematic to get coordination between theoretical academic understanding of flexibility and industrial practice.”

Saleh et al. [4] pointed out that one source of ambiguity is due to confusion among similar terminologies such as robustness, adaptability and agility. Even though the discussion is restricted to flexibility, the ambiguity does not disappear because flexibility concept highly depends on particular managerial situations or context of the system [12].

To elucidate the concept of flexibility, Section 2.1.1 provides the conceptual development in the manufacturing area. Manufacturing flexibility is an extensively studied area of flexibility thanks to the concrete flexible systems, such as flexible manufacturing systems (FMS), group technology and modular manufacturing. The findings in manufacturing flexibility provide a theoretical benchmark to other areas. In the early conceptual studies of manufacturing flexibility, identifying the types of flexibility to improve manufacturing flexibility was the main subject of study.

Later research focused on abstracting the dimensions of flexibility from the identified types of flexibility. Since the flexibility dimension framework is applicable to other areas, it extended to other areas, such as intellectual technology, supply chain, and business process. Section 2.1.2 summarizes the dimensions found in the manufacturing context and the extended dimensions in other fields.

2.1.1 Types of Flexibility in Manufacturing Flexibility

Browne et al. [13] established the taxonomy of 8 flexibility types, and many following conceptual studies in manufacturing flexibility accept the types of flexibility approach. Among the review papers, Sethi and Sethi [14] is often cited. By reviewing over 202 research articles, they provided 3 levels of flexibility: component or basic flexibilities, system flexibilities, and aggregate flexibilities, and defined 11 flexibilities. Vokurka and O'Leary-Kelly [15] improved Sethi and Sethi's framework including empirical research. They defined 15 types of flexibility and identified four important exogenous variables of manufacturing flexibility; strategies, organizational attributes, technology and environmental factors. Other review papers, such as Gupta and Goyal [16], Sarker et al. [17] De Toni and Tonchia [18], Shewchuk and Moodie [19], Parker and Wirth [20], Beach et al. [21] and Bengtsson [22], and Koste et al. [23] accepted the types of flexibility analysis. The author summarizes the widely accepted definitions of flexibility types and the studies supporting the importance of the type.

1. Machine Flexibility: When a machine can complete various types of operations the machine is flexible. Lim [24] studied machine flexibility with FMS designs with empirical data. The research found two bottle neck processes of machine flexibility; automated fixture assembly and mounting. Jaikumar [25] researched how to improve machine flexibility in group technologies.

2. Material Handling Flexibility: If materials, such as raw materials and parts, can be placed in proper positions efficiently, the system has material handling flexibility. Gupta and Somers [2] asserted that there are three activities, loading and unloading of parts, inter-machine transportation, and storage of parts, to improve material handling flexibility. Material handling flexibility improves machine usage and process efficiency.

3. Operation Flexibility: Operation flexibility means the ability to produce a product in various ways. This definition is provided in Browne et al. [13], and Sethi and Sethi [14]. Modular process with standardized components is a typical example of operational flexible system [26].

4. Automation Flexibility: Automated or computerized manufacturing systems are flexible, since the manufacturing process can react to the realization of uncertainty easily. Parthasarthy and Sethi [27] defined intensity of flexible automation, and analyzed its relationship with the performance of the manufacturing system. The empirical studies showed that flexible thinking is required at strategic level. Gebauer and Scharl [28] studied automation flexibility in the context of web business process.

5. Labor Flexibility: Vokuka and O'Leary-Kelly [15] defined labor flexibility as "the range of tasks that an operator can perform within the manufacturing system." Labor flexibility is emphasized in the series of Slack's studies [29–31].

Gerwin [32] studied the labor flexibility in the context of computer-aided manufacturing. He also emphasized the worker's skill on general purpose machines over specialized equipment. Parthasarthy and Sethi [27] found that flexible automated manufacturing system requires more skillful workers and that project team is an appropriate organization of flexible automation.

6. Part-Mix Flexibility: If a system is able to produce the set of parts without major modifications, the system is part-mix flexible. Although Sethi and Sethi called this flexibility "process flexibility," this dissertation uses the candid terminology, part-mix flexibility, because process flexibility has a broader meaning in recent research [33, 34]. Avonts and Van Wassenhove [35] clarified the concept of part-mix flexibility in flexible manufacturing system (FMS) context, and modeled FMS decision making problem as a queuing network problem. Gerwin [36] introduced a similar concept, mix flexibility, as "The processing at any one time of a mix of different parts loosely related to each other." Tomlin and Wang [37] studied supply chain design problem as a mix flexibility optimization problem. They compared performances of four types of system; Single-source dedicated, Single-source flexible, Dual-source dedicated and Dual-source flexible systems.

7. Routing Flexibility: When a system has routing flexibility, the system can produce a part with various routes. Vokuka and O'Leary-Kelly [15] defined the routing flexibility as "the number of alternative paths a part can take through the system in order to be completed." These flexibility definitions concur with the definitions of Browne et al. [13] and Gerwin [36]. Rossi and Dini [38] investigated job-shop scheduling of systems with routing flexibility. They used an ant colony optimization method to utilize the routing flexible system and showed that the proposed algorithm is superior to genetic algorithm. Caprihan and Wadhwa [39] studied the performance of a flexible manufacturing system with routing flexibility with simulation. They found that increasing routing flexibil-

ity does not guarantee the improvement of performance. This result implied that there exists an optimal level of flexibility.

8. Product Flexibility: When existing parts of a system can be replaced with new parts without spending significant amounts of time and costs, the system has product flexibility. Gerwin and Tarondeau [40] refers product flexibility to changeover and modification flexibility, and asserted that firms can response to change of customers' volatile preferences with the flexibility. Palani et al. [41] supported the results of [40] with empirical findings.

9. Design Flexibility: With design flexibility, a system is able to introduce a new product with short amount of time and low costs. Robb Dixon [42] emphasized introducing new product with empirical research. Kouvelis et al. [43] and Kouvelis [44] focused on long-term problems in flexible manufacturing systems such as design and planning problems. They classified the problems in terms of decision level and planning horizons such as long term, medium term and short term. Palani et al. [41] found that modularization, especially designing module, enhances product flexibility.

10. Delivery Flexibility: Slack [31] defined delivery flexibility as "the extent to which delivery dates can be brought forward" and "the time taken to reorganize the manufacturing system so as to replan for the new delivery date." Vokuka and O'Leary-Kelly [15] interpreted Slack's definition as "the ability of the system to respond to changes in delivery requests." Sabri and Beamon [45] analyzed the effect of delivery flexibility on supply chain.

11. Volume Flexibility: Volume flexibility is the ability to adjust output levels according to the change of environment. Vokuka and O'Leary-Kelly [15] defined the measure of volume flexibility as "the Range of output levels that a firm can economically produce products." Jack and Raturi [46] conducted three case studies, and found the drivers and sources of volume flexibility. They classified the sources of volume flexibility into four categories; internal, external,

short-term and long-term sources. The results showed that volume flexibility increases financial performance and delivery performance of firms. Short term sources and internal long-term sources were verified as the sources of volume flexibility through statistical analysis.

12. Expansion Flexibility: If the capacity of a manufacturing system is able to be adjusted easily, the manufacturing system has expansion flexibility. Gupta and Somers [2] defined expansion flexibility as "the extent of overall effort needed to increase the capacity." Karsak and Özogul [47, 48] evaluated the value of expansion flexibility based on a real option approach.

13. Program Flexibility: Sethi and Sethi [14] defined program flexibility as "the ability of the system to run virtually untended for a long enough period," and Vokuka and O'Leary-Kelly [15] stated that "program flexibility reduces the overall manufacturing time by decreasing set-up times." Gupta and Somers [2] verified the importance of program flexibility by factor analysis on data from top level managers.

14. Production Flexibility: When a manufacturing system can produce a new product without adding major capital equipment, the system is production flexible. Gupta and Somers [2] found three factors of determining production flexibility; variety and versatility of available machines, flexibility of material handling systems, and the factory information and control systems.

15. Market Flexibility: Market flexibility is a broad concept of flexibility. Sethi and Sethi [14] defined it as "the ease with which the manufacturing system can adapt to a changing market environment." This flexibility enables the firm to cope with volatile environments and competitors' behavior [2].

2.1.2 Dimensions of Flexibility

Researchers have abstracted the essence of the types of flexibility. The dimensions of manufacturing flexibility provided broader understanding about flexibility rather than focusing on concrete activities. Since the dimensions of manufacturing flexibility are generally applicable, the research of flexibility in other fields employed flexibility dimension analysis.

Recently, Buzacott and Mandelbaum provided am exemplary literature review, including a comprehensive framework for understanding flexibility, and emphasized dimensions of flexibility [3, Section 2.4]. Parker and Wirth [20] suggested the relationships between types and dimensions of manufacturing flexibility. They analyzed manufacturing flexibility with six categories; system vs. machine, action vs. state, static vs. dynamic, range vs. response, potential vs. actual, and short, medium and long term. Shewchuk and Moodie [19] employed a part of the dimensions in their classification.

The research of flexibility dimension analysis has been extended to other fields such as information technology, business process and supply chain management. As a consequence of extension, interdisciplinary study about flexibility has been conducted. Golden and Powell's work [49] is a survey paper of flexibility in information technology field. Schonenberg et al. [34] reviewed flexibility studies in the view point of business process. Saleh et al.'s work provides an interdisciplinary literature review for flexibility [50].

1. Level of flexibility: Gerwin [36] mentioned the level of flexibility, and it is improved in his following work [51]. Suri and Whitney [52] and Kouvelis [44] classified the level of flexibility into three levels. The top level decision is called strategic decisions, and it contains the decisions about part family selection and system capacity. The second level decision includes batching and resource allocation decisions. Scheduling, dispatching, tool management and system monitoring decisions belong to the third decision level. Sethi and Sethi [14] provided

three levels of flexibility in their framework of flexibility; component or basic, system, and aggregate flexibility. Component or basic level flexibility includes machine, material handling, and operation flexibility. System flexibilities consist of process, routing, product, volume and expansion flexibility. Gupta [53] classified flexibility according to the magnitude of changes that flexible choice provides. He asserted machine, cell, plant and corporate levels of flexibility. De Toni and Tonchia [18] emphasized vertical or hierarchical classification of flexibility and summarized the studies related to the level of flexibility. They analyzed flexibility in four levels; plant and machine, production function and work department level, product line level and global level of the firm. Parker and Wirth [20] call this dimension as system vs. machine dimension. Buzacott and Mandelbaum [3] stated that the level of flexibility defines the boundary of decision problems and systems.

In supply chain management, the level of flexibility is extended beyond the firm's level. Mair [54] added the corporation's network level of flexibility to micro level and factory level by studying Honda's case. Stevenson and Spring [55] also emphasized the network, outside of a firm level dimension, as a dimension of flexibility. Although Parker and Wirth [20] called this dimension the system vs. machine dimension, the research of supply chain management field suggest "level of flexibility" is more appropriate name of this dimension. Therefore, this dissertation calls it level of flexibility.

2. Prior, Action, and State Flexibility: Buzacott and Mandelbaum [3, Chapter 2] provided the definitions of prior, action and state flexibility as following.

   - Prior Flexibility: The variety of initial actions or decisions that decision makers can take
   - State Flexibility: The system with state flexibility is able to manage exogenous uncertainty by being effective under any environmental outcome and thus coping with the stochastic environmental change. Suppose that

an alternative is effective for one environment and is not effective for all the others, than the alternative is state inflexible. On the other hand, if a choice is effective for many situations, it is state flexible.

- Action Flexibility: The system equipped with action flexibility is able to respond to resolved uncertainty by taking effective recourse action.

3. Decision Epoch: Carlsson [56] stated that "Static flexibility refers to the ability to deal with foreseeable changes (i.e., risk), such as fluctuations in breakdowns in the production process" and "dynamic flexibility refers to the ability to deal with uncertainty in the form of unpredictable events, such as new ideas, new products, new types of competitors, etc." For dynamic flexibility, Buzacott and Mandelbaum distinguish two periods cases and continuous time cases [3, Section 2.4.6]. Two periods decision making problem is the simplest case of finite decision epoch. Therefore this dissertation suggests four decision epochs; single, finite, countable and continuous decision epoch.

4. Range and Response: Slack [30] recommended decision makers analyze flexibility in terms of range and response dimensions. Range of flexibility refers to the extent of alternatives that a decision maker can choose. On the other hand, response of flexibility expresses how fast the alternative can be implemented. Buzacott and Mandelbaum include this dimension in "ease of change" [3, Section 2.4.3].

5. Potential and Actual Flexibility: Browne et al. [13] discussed the dimensions of potential and actual flexibility. Potential flexibility means the flexibility which exists but is utilized only when it is needed. Actual flexibility refers to the flexibility which is utilized regardless of the environmental status.

6. Decision Horizon: Gershwin et al. [57] provided three levels of decision horizon as following;

   - Long Term: Investment and initial design decisions

- Medium Term: Design and planning decisions
- Short Term: Real time control

Gupta and Buzacott [58] decomposed "changes" with time scale according to Gershwin's decision horizon.

7. Uncertainty: Groote [59] suggested a general framework about the flexibility of production processes. The research asserted three main properties of flexibility in terms of environment allocation, operations strategy and strategic interfaces. Especially, it classified the application area by the number of uncertainty, one-dimensional applications and multidimensional applications. Buzacott and Mandelbaum stated not only the property of uncertainty but also the interaction among decision makers [3, Section 2.4.4]. They emphasized that interaction has different characteristics from exogenous uncertainty. Exogenous uncertainty is not influenced by a decision maker's decision. However, when there is interaction, a decision maker's decision affects the environment.

2.2 The Measure of Flexibility

The measure of flexibility represents how much a system is flexible. It is not surprising that the measure of flexibility is not clear either, since the concept of flexibility has not matured yet. While various measures of flexibility have been suggested, they can be classified into four categories; the amount of cost to change, the number of feasible alternatives, the extent of uncertain states in that the purpose of system is achieved, and others.

The first approach asserts that the cost to accomplish the desired change measures the flexibility of system. For example, Gupta and Somers [2] measures expansion flexibility with the overall cost to increase system's capacity. When the alternatives of flexible systems are fixed, the cost base approach is appropriate. However, it is not sufficient to measure flexibility with only the cost to accomplish the change when there are a number of possible choices.

The second approach measures flexibility with the extent of possible choices. For instance, Mandelbaum and Buzacott [60] suggests the remaining choices available in the subsequent period as the measure of flexibility in the context of decision theory. This approach has disadvantages that useless alternatives are counted in measuring flexibility. If a choice would be never used, the alternative does not contribute to enhance the performance of the flexible system.

The third measure of flexibility is based on the states that a flexible system is efficient. Gupta and Rosenhead [61], which is believed the first research to provide the measure of flexibility [50], mentioned the following:

> Rather than try to identify the best alternatives, we can ensure that our early investment decisions permit the achievement of as many end-states as possible. Subsequent stages of the investment plan are left to be determined at later dates, when more recent information is available $\cdots$. In the context of our discussion, flexibility of a decision must be measured in terms of the number of end states which remain as open options. [61, Page B20-B21].

Subsequent studies support the open state based approach, since it provides useful insight to the value of flexibility [50].

The entropy of flexible systems is suggested as a measure of flexibility [16, 62, 63]. According to Kumar [62], this measure has firm axiomatic foundations. The properties and potential of entropy based measure is still the subject of study.

2.3 The Value of Flexibility

Various methods for valuing flexible systems have been developed. The most popular approach is the real options approach. Since flexibility has similar properties as financial options, researchers employ the findings in financial options to evaluate flexibility. This is called the real options approach. Other than the real options approach, a number of methods are able to assess the value of flexibility according to the context, for instance, dynamic programming, optimal control theory and expanded net present value methods. Because of the variety of the methods, this dissertation focuses on the important dimensions of the value of flexibility rather than the concrete valuation methods.

2.3.1 Real Options Approach

Real options approach is the application of financial option theory to non-financial investments. Financial options are basically contingent claims that can be made only if attractive outcomes occur. Because flexible decision making and financial options have this similarity, the methods to evaluate financial options are applicable to valuing flexible system. Financial options theory is an extensive field, including the Nobel Prize winning contributions of Black and Scholes [64] and Merton [65]. They derived a closed-form solution to the value of a European call option. This method extended to real assets which have similar properties with the European options [66]. Dixit and Pindyck [1] and Trigeorgis [67] provided well-structured accounts of both the theory and applications of real options. Bengtsson [22] reviewed flexibility and real options, based on the classification of Sethi and Sethi [14] and investigated the ability of real options methodologies to model flexibilities such as routing, volume, expansion, shut-down, abandonment and deferment.

Before examining specific real options approachs to flexibility, the author remarks that two critical assumptions of financial options theory should be carefully treated in applying the results to real decision problems. The assumptions are the risk attitude of the decision-maker and the completeness of markets. In the financial realm, risk preferences can often be ignored via market completeness, however, in general decision environments, outcomes cannot be replicated with tradable assets, and risk preferences must be taken into account [1]. The comparable assumptions of flexibility evaluation are risk neutral decision makers and known payoff functions.

1. Option to Defer: Real option provides rights to wait until future uncertainty is resolved enough to make an appropriate decision. Tiltman [68] applied real option theory to pricing and deciding the optimal time to develop vacant urban land. Ingersoll and Ross [69] pointed out that most of investment projects have similar properties with financial options, and the value of waiting to invest can be found through option valuation models. McDonald and Siegel [70] investigated the optimal timing to invest an irreversible project when the value of project follows a continuous stochastic process, geometric Brownian motion. They derived the closed form solution of the optimal timing and the value of project.

2. Time-to-Build: When a project includes multiple stages, the option theory is applicable to each stage. Majd and Pindyck [71] studied staged construction investments with adjustments in response to resolved uncertainty by considering the effects of the time-to-build and opportunity cost on the investment decision. Carr [72] researched sequential exchange opportunities using option pricing theory.

3. Option to Alter Operating Scale: If the facing uncertainty turns to be favorable, the firm can expand the volume of production. On the contrary, if the environment of the firm is hostile, the firm can shut down or contract the operation. Brennan and Schwartz [73] evaluated a flexibility copper mine system applying real option theory. They found the optimal policies to develop operate and abandon the mine considering fluctuating copper price which follows geometric Brownian motion. Trigeorgis and Mason [74] and Pindyck [75] are other examples which belong to this category of real option.

4. Option to Abandon: If the environment is extremely unfavorable, a decision maker can abandon an ongoing project permanently. The value of a project

which can be abandoned and the optimal time of abandonment are the main subjects of option to abandon. McDonald and Siegel [76] analyzed the value of abandon option under the assumption of geometric Brownian motion. They considered risk-neutral and risk-averse decision makers, and conducted sensitivity analysis of the optimal policy and the value of option. Myers and Majd [77] caught the similarity between the option to abandon and American put option on a stock which pays dividend. Using an American put option evaluation method, they found the value of option to abandon and optimal strategy to exercise abandonment option.

5. Option to Switch: If a firm has product mix flexibility or process flexibility, managers of the firm can change the current state of the firm to another available state. Kulatilaka [78] modeled a flexible industrial steam boiler which can choose its fuel between oil and natural gas. Kulatilaka and Trigeorgis [79] developed a general framework to evaluate switch option.

2.3.2 Dimensions of Valuing Flexibility

Other than real options approach, Ross et al. [80] presented a framework for defining the "changeability" of a system, which encompasses flexibility, adaptability, scalability, and robustness. Ross et al. present their framework as a basis for design, analysis, and evaluation of engineered systems. While the emphasis of this literature has surrounded defining, measuring, and best practices for designing flexible systems [81], there is comparatively little work on techniques for valuing flexibility, and their assumptions, trade-offs, and abilities. Notable works in this area include [82–84]. Neely [82] and Nilchiani and Hastings [83] approached the valuation problem from different domains, such as R&D and space systems, but each consider net present value, decision analysis and real options as approaches for valuing projects and systems. They arrived at similar conclusions that the classification of uncertainty can differentiate systems by which approaches are most appropriate. These works commonly point out there are system characteristics affecting the evaluation methods. The author recapitulates the important dimensions of evaluating flexibility.

1. The Number of Decision Epochs: In the simplest case, there is a single point in time when a decision can be made, for example, European style financial options. In this case, closed-form solutions for the value of the flexibility are often available. For example, when the value of the optional asset follows the log-normal distribution and other standard market assumptions hold, the Black-Scholes formulas give the value of the option. More generally, there may be multiple, but finitely many decision epochs. This is the case for systems with recurring decision opportunities and fixed lifespans. If the life of the system is indefinite or approximately infinite, a system with discrete decision epochs can be modeled as having countably many epochs. In the extreme, systems can be modeled with uncountably many decision epochs if decisions can be made continuously. In financial options applications, distinctions between the numbers of decision epochs can clearly be seen in the distinctions between European, Bermudan, and American style options. In general, systems with more decision epochs require more sophisticated formulations.

2. The Number of Alternatives: By definition, any decision epoch has at least two alternatives. Systems with minimal flexibility, two alternatives at exactly one decision epoch, are rarely encountered in reality, and are structurally equivalent to European-style financial options. Although it is often the case, or assumed for convenience, systems need not have the same number of alternatives per decision epoch. Systems with just two alternatives per epoch are often well-behaved, with state thresholds delineating optimality regions for the two alternatives. Operational decisions with a range of discrete or continuous alternatives, e.g., inventory management of a continuous commodity can have finitely many, countably many, or uncountably many alternatives per epoch. As the number of alternatives per decision epoch increases, there is greater need to

express the relationship between the costs, benefits, and constraints of the system and the alternatives through mathematical functions. *Ceteris paribus*, the value of flexibility is always non-decreasing in the number of total alternatives the system operator faces over the lifespan of the system. Generally speaking, as the number of alternatives increases, solving for the optimal operational or control strategies becomes increasingly difficult.

3. The Characterization of Uncertainty: The underlying uncertainty plays an important role in the modeling, formulation, and techniques which are appropriate for valuing flexibility. Increasing the number of uncertain factors modeled significantly hampers the prospect of analytical solutions, and increases the computational burden of numerical and simulation-based solutions. The level of precision needed to characterize uncertainty parallels the timing of the decision epochs. There is no need to model the stochastic process of uncertain variables at a greater level of detail than can be utilized by the decision-maker. When detailed information on the stochastic process governing a random variable can always be reduced to the so-called calibrating distribution of the random variable at the decision epochs analytically or numerically [85]. When limited information about uncertain variables is available, e.g., moments or data on the distribution function, the maximum entropy principle can be used to estimate the distribution of the uncertainty. When modeling uncertainties, the assumptions of stationarity (probabilities are invariant to time shifts) and independence (multiple realizations of uncertainty provides no information about each other) are assumptions that allow for stronger formulations and solution methods.

4. The Decision Maker's Objective and Constraints: The typical decision maker's goals are to maintain a given capability while minimizing cost, maximize the rewards from a fixed cost, or most generally, maximize the net value of benefits less costs. Moreover, when there are constraints that limit the operation of a flexible system, it affects the value of flexibility.

2.4 Strategic Flexibility

Strategic flexibility is the intersection of game theory and traditional flexibility. As Grenadier [86] pointed out the traditional real option paradigm is limited by not enough consideration of strategic environment in which many decision makers interact with each other. Integrating the knowledge of flexibility and game theory provide more profound understanding about the behavior of real world decision makers.

There are two approaches to harmonize flexibility and game theory; option exercise games and stochastic differential games. Option exercise games are based on the real option approach extend to game theoretic modification. On the other hand, stochastic differential games are grounded on game theory, and extend to include stochastic factors which are expressed with stochastic differential equations. Researchers in economics, business, and engineering fields mainly take option exercise games approach, and applied mathematicians have contributed to stochastic differential games area.

2.4.1 Option Exercise Games

Huisman [87], Smit and Trigeorgis [88] and Chevalier-Roignant and Trigeorgis [89] provide a text book of option exercise games. Grenadier [90] edited selected papers of option exercise games. Ferreira et al. [91] emphasized option exercise games is an appropriate approach to analyze the competitive advantage in strategic stochastic environment. This section will review option exercise games studies by subjects

1. Real Estate Investment: Real estate investment has important properties that make strategic environments significant in decision making. Williams [92] asserted that real asset development has finite elasticity of demand, limited number of and capacities of developers and limited supply of investment opportunities. Therefore, real estate development is one of the popular application areas of option exercise games. He derived a sub-game perfect Nash equilibrium focusing on the limited supply of undeveloped real estate and finite elasticity of

demand due to the limited number of developer. Grenadier [93] found optimal investment time for real estate development and strategic equilibrium.

2. Option to defer under competition: Smit and Ankun [94] considered the option to defer investment in manufacturing under competition. They found that the postponement of an investment decision may lose first mover's advantage.

3. Strategic Growth: The exercise of options is an appropriate model to explain strategic growth such as R&D, advertising campaigns, and logistical planning, under competition. Since these activities can yield long term competitive advantages and growth opportunities, those are called strategic growth options. Loury [95] is the seminal paper in strategic growth subject, R&D investment with competition. Kulatilaka and Perotti [96] studied the optimal decision of strategic growth under imperfect competition. Joaquin and Butler [97, Chapter 16] developed a strategic investment model when one firm has a competitive advantage thanks to an asymmetric cost structure. They derived an optimal exercising strategy of output level and timing and found a unique sub-game perfect equilibrium under the assumption of duopoly market. Weeds [98] studied strategic growth option under two stochastic factors; the value of patent and the probability of success of the project. Her results showed that there are various optimal strategies according to the parameter values.

4. Incomplete Information and Preemption: When the competitors have incomplete information, the value of preemption, the first mover's advantage, is not the same to the case of complete information. Grenadier [99] studied the problem that each player behaves based on asymmetric private information, and found an equilibrium framework. Interesting results include that if a player chooses exercising policy based on the observed behavior of a competitor, a "follow the leader" type of policy is optimal. Lambrecht and Perraudin [100] investigated optimal strategies when there is a threat of competitor's entry. The information about a competitor's entry is known with a probability distribution.

5. Duopoly or Oligopoly Market Competition: Dias and Teixeira [101] reviewed Smets [102]. He studied a stochastic symmetric duopoly problem. In his model, an impact on market price follows geometric Brownian motion, and each firm has an identical cost structure to its competitor. He found symmetric equilibrium of the market using dynamic programming approach with simplified assumption about production level. Joaquin and Buttler [97, Chapter 16] expand Smets's work into asymmetric duopoly market and derived mixed strategy theorem. Grenadier [86] derived the Nash equilibrium for a Cournot competition where both of the players have symmetric payoff structures and information. The results show that the payoff of each player is determined by the underlying stochastic process and the strategies of the players.

2.4.2 Stochastic Differential Games

The researchers in the stochastic differential games field focus on the structure of the problems and problem characteristics, such as pursuit evasion, zero sum games, cooperative and non-cooperative games, rather than the application. Recently, Ramachandran and Tsokos [103] provided a book about stochastic differential games focusing on pursuit-evasion games, concept of solutions and solving techniques. Friesz [104] and Dockner et al. [105] focused on the application of stochastic differential games in economics and management context. Bardi et al. [106] and Cardaliaguet and Cressman [107] dealt with technical methods, especially numerical method, for solving stochastic differential games. Contrary to other books which focus on non-cooperative games, Yeung and Petrosyan [108] provided a text book treat about cooperative stochastic differential games. This section contains a brief review about stochastic differential games with traditional research topics and solving technique point of views.

Roxin and Tsokos [109] define stochastic differential games. According to the definition, games are stochastic if there is noise in the players' observations of the state of the system or the transition equation. Bardi and Gaghavan [106] reviewed many aspects of differential games such as pursuit evasion games, zero-sum games, cooperative and non-cooperative games and other types of dynamic games. Pursuit Evasion games believed the first application area of stochastic differential games. The seminal work of Von Neumann and Morgenstern [110] applied to pursuit evasion problem [111] since 1954. Basar and Haurie [112], a problem of pursuit-evasion is considered where the pursuer has perfect knowledge whereas the evader can only make noisy measurements of the state of the game.

This dissertation is interested in the solution techniques which are developed in stochastic differential games. In the early development of solving techniques of stochastic differential games, it is believed that a control process where each player choose the optimal control variable to accomplish his or her objective. However, subsequent research showed that the optimal control approach is inappropriate to be applied directly to solve stochastic differential games [111, 113].

Ho [114] solved a stochastic differential game problem with variational techniques. Stimulated by [114], martingale approach and variational inequality approach were popular in the 1970s. The existence and uniqueness of a solution was investigated by many researchers, for example, Elliott [115], Bensoussan and Friedman [116].

A type of dynamic programming approach, Hamiltonian-Jacobi-Issacs (HJI) equation, was another widespread method to attack stochastic differential games. The early works on differential games are based on the dynamic programming method now called as Hamiltonian-Jacobi-Issacs (HJI) [117]. However, is an HJI equation may not have smooth solutions, and existing non-smooth solutions may not be unique. Therefore, in 1980s viscosity solution, a generalized solutions for Hamilton-Jacobi equation, emerged. If a HJI equation satisfies a class of boundary conditions, viscosity solution represents the unique solution of the HJI equation. The notion of viscosity solution is also useful to show a convergence property of algorithms based on dynamic programming [118, 119].

3. FLEXIBLE SYSTEM WITH EXERCISE DELAY

3.1 Introduction

This research considers an irreversible decision under a stochastic environment. This chapter postulates that a system designer builds a flexible system that enables the operator of the system to postpone an irreversible change of system capability until the underlying uncertainty is resolved enough to cope with a stochastic environment. There exists "exercise delay" between the time the change is initiated and the time that the change is completed. Since the operational decision is a premise of the design problem, it is required to derive a comprehensive optimal solution of the operational level decision. This research employs an optimal control theory that is based on variational inequality approach to solve the operational level problem, and derives a comprehensive closed form solution of operational level problem. Based on the closed form solution, this research effort analyzes the effect of exercise delay on the operational policy and identifies the interdependency of the operational task and design task. Given the optimal operational decision, the system designer decides how much flexibility, i.e., how long the exercise delay, is embedded in the flexible system.

The system design framework is developed upon the findings from operational level decisions. Given the optimal operational decision, the system designer decides how much flexibility, i.e., how long the exercise delay, is embedded in the flexible system. The framework suggests that the design problem is decomposed into sub-problems according to the effect of decided design variables on the operational level decision. The developed framework is so concrete that the design level problem is solvable with usual non-linear optimization methods with once differentiability cost function.

The rest of this chapter is outlined as follows. Section 3.2 provides a brief review of delayed flexible systems and design of flexible system design. Section 3.3 models a delayed flexible system that has only one flexible alternative with respect to the capability change. The operational task is to determine the optimal time to start the change the system's capability, and the design task is choosing the optimal exercise delay. The operational problem and design problem are stated in Section 3.3.1. The comprehensive description of the optimal operational policy is reported in Section 3.3.2. Section 3.3.3 provides the decomposition framework of design level problem and an illustrative example to show the usefulness of the framework. Section 3.4 considers the case that the system designer decides both the length of exercise delay and the level of capability change. The results in this section are reported with the similar structure of previous section. This chapter ends with the summary in Section 3.5.

3.2 Literature Review

3.2.1 Flexible System with Exercise Delay

Review papers and conceptual studies about flexibility agree with the importance of exercise delay. Buzacott and Mandelbaum [3] highlight exercise delay as a measure of flexibility. Slack emphasizes system response, which is defined as "the ease with which it moves from one state to another, in terms of cost, time or organizational disruption" as a measure of flexibility [29–31].

This research leverages the relatively well-developed literature devoted to the optimal operation or control of a flexible system. Bar-Ilan and Strange [120] delivered a seminal research article in this area. They consider the case when the recourse decision can be abandoned during the delay, and find that the option of abandonment can make option exercise time early. Other extensions dealing with delayed options or delayed stopping problems include Alvarez and Keppo [121], who consider the case where delivery lags and revenues are correlated; Bayraktar and Egami [122], who provide a constructive solution and model the magnitude of the option decision; and Sødal [123], who considers extensions such as multi-agent systems. Applications of these and similar models include mergers and acquisitions [124], power generation [125, 126], and the decision to implement advanced manufacturing equipment [127]. Relative to the existing results in the literature, primary contributions of this research are the measurement of flexibility in terms of the exercise delay, and the modeling and solution of the system design problem.

Optimal stopping problems are a classic formulation from the operations research and real options communities, where dynamic programming [1] and variational inequality approaches [128] are two common solution approaches. The operation of a flexible system with a single irreversible decision can be modeled as an optimal stopping problem, where the typical solution to the control problem is a threshold policy which delineates between exercise and continuation regions. The most general approach for a flexible system with exercise delay is by modeling the optimal operation or control problem as a delayed optimal stopping time problem. Øksendal [129] provides the method to convert a delayed optimal stopping time problem to a normal optimal stopping time problem without delay, under the assumption of strong Markov property. Moreover, he delivers a rigorous method to solve a optimal stopping time problem, and this paper leverages these results to solve the system design problem [128, Chapter 10].

3.2.2 Design of Flexible Systems

A number of studies devoted to study the optimal design of a flexible system. Fine and Freund [130] studies the optimal investment in flexibility of a firm that can distribute its capacity between two products. The uncertainty that the firm faced was the quantity of demands described with discrete probability distributions. This research addresses the optimal design of a prior flexibility. Van Mieghem [131] models the optimal flexibility investment problem as a news-vendor problem with arbitrary multivariate demand distribution. The research finds the optimal investment policies according to costs, prices and demand uncertainty. Bish and Wang [132] develop van Mieghem's work to the case that there are correlations among the uncertain amount of demands. Research about partial flexibility provides significant insights into the investment of flexibility.

Partial flexibility means that facilities can only produce a certain range of products. Jordan and Graves [33] provide important concepts, smart limited flexibility and chaining. Smart limited flexibility means the appropriate limited flexibility provides most of the benefits of full flexibility. Moreover, chaining strategy, combination of limited flexible facilities, enhances the performance of limited flexible systems. Jordan and Graves' work [33] has been extended in many application areas; Graves and Tomlin [133] extend the result in multistage supply chains context; Gurumurthi and Benjaafar [134] apply the result to queuing systems; Hopp et al. [135] apply the results for scheduling flexible workforce. The flexibility in a network context is also an interesting research area in the systems flexibility point of view. Iravani et al. [136] suggested structural flexibility that is the flexibility concept for serial, parallel, open and closed networks. Based on network flow model, Akşin and Karaesmen [137] showed that the throughput of a network is concave with respect to the level of flexibility. Using the property, they studied the relationship between flexibility and capacity. Chou et al. [138] studied the worst-case performance of a symmetric system, and provided design guidelines. Although numerous studies are devoted to design of flexible system, the author has not found the research considering the exercise delay as a design variable, yet.

3.3 One Alternative Model

3.3.1 Model

This research considers a system with an infinite lifetime $[0, \infty)$. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space equipped with a filtration $(\mathcal{F}_t)_{t \geq 0}$ satisfying the usual conditions, and $B(t)$ be a standard Brownian motion. The state variable, $X(t)$, represents the underlying uncertainty in the system environment, and is assumed to follow a geometric Brownian motion with constant coefficients, i.e.,

$$dX(t) = \mu X(t)dt + \sigma X(t)dB(t), \qquad X(0) = x \in [0, \infty), t \in [0, \infty) \tag{3.1}$$

where $\mu \in \mathbb{R}$ and $\sigma \in [0, \infty)$ are the drift and volatility parameters. Geometric Brownian motion is widely accepted to model underlying uncertainties of flexible systems, such as a price process in a perfectly competitive market. Notice that the stochastic differential equation (3.1) has a unique solution, and satisfies strong Markov Property.

Suppose that the system operator has one opportunity to change from the current system to an alternative system, for example, by upgrading or downgrading equipment. The current and alternative systems yield discounted linear profit rate of $e^{-\rho t}(a_1 X(t) + b_1)$ and $e^{-\rho t}(a_2 X(t) + b_2)$, respectively. The parameters, a_1, a_2, b_1, b_2, and ρ, are given constants. The discount factor ρ is assumed to be positive and $\rho > \mu$ to ensure existence of expected payoff. This setting can describe various system performances which is linear to underlying uncertainty. For example, in the context of production, a_i stands for the production levels of system i, and $b_i < 0$ represents manufacturing cost of system i, for $i = 1, 2$. In terms of real options, this model includes entry option with $a_1 = b_1 = 0$, exit option with $a_2 = b_2 = 0$, expansion option with $a_1 < a_2$, and downsizing option with $a_1 > a_2$. For the sake of convenience, this research refers to the case when the alternate system provides less variable yield than the initial system $(a_1 > a_2)$ as a downgrade, and the opposite case $(a_1 < a_2)$ as a upgrade. To avoid trivial solutions, this research assumes that $a_1 \neq a_2$. For notational simplicity, let $\theta = \{\mu, \sigma, \rho, a_1, a_2, b_1, b_2\}$ represent the full set of parameters.

When the system operator decides to change the system at time τ, there is a delay δ until the alternative system takes effect at time $\tau + \delta$. During the exercise delay the system is assumed to continue the initial yield mode. Moreover, the author assumes that there exists only one opportunity to change the system. According to the results of [129], the operational level problem with multiple opportunities can be solved by iterative application of the method that this research employs. Therefore, this assumption does not undermine the value of this research but assist to focus on the design level problem. Making the decision to change to the alternative system incurs the cost ($\tilde{c}(\delta) < 0$) or revenue ($\tilde{c}(\delta) > 0$); this study models the one-time cash flow as occurring at the time when the transformation is completed, $\tau + \delta$. Upgrading the system would typically incur a cost, whereas downgrading the system may produce revenue through the sale of equipment. At this point, the system operators control problem can be formulated. The operational performance function, $j(x, \tau|\delta)$, and the operational value function, $v(x, \delta)$ can be written as

$$j(x, \tau|\delta) = \mathbb{E}\left[\begin{array}{c} \int_0^{\tau+\delta} e^{-\rho t}(a_1 X(t) + b_1)dt + e^{-\rho(\tau+\delta)}\tilde{c}(\delta) \\ + \int_{\tau+\delta}^{\infty} e^{-\rho t}(a_2 X(t) + b_2)dt \end{array}\right] \tag{3.2}$$

$$v(x, \delta) = \sup_{\tau \in [0, \infty)} j(x, \tau|\delta) \tag{3.3}$$

where the expectation is taken with respect to the probability law of $X(t)$ starting at $X(0) = x$. The operational level problem is for the system operator to determine the optimal time, τ^* to exercise the option.

At the design level, the system designer builds the system with optimal delay, δ^*, considering the costs of designing, acquiring, and constructing a system, given that system will be operated optimally. The possible choice set of δ is $[\delta_{min}, \delta_{max}]$ where $0 \leq \delta_{min} \leq \delta_{max} < \infty$. The initial cost of designing and building the flexible system is $C(\delta)$. Then the value of optimally designed flexible system under optimal operational control becomes

$$V(x) = \sup_{\delta \in [\delta_{min}, \delta_{max}]} [v(x, \delta) - C(\delta)] \tag{3.4}$$

One additional assumption is that the system designer and operator have the same objective functions, and therefore any principal-agent scenarios requiring incentives is not a subject of this research. This research now turns to analyzing this model in order to derive optimal control and design policies.

3.3.2 Operational Level Decision

Optimal Operational Policy

The operational control problem of this flexible system is a delayed optimal stopping time problem. The results of [129] show how to transform this problem into an optimal stopping time problem without delay based on the strong Markov condition. In general, the optimal control policy is a threshold policy where the continuation region is either below the threshold when the alternative system is an upgrade or above the threshold when the alternative system is a downgrade [1, pp.128-130]. However, based on system parameters, the decision to exercise the option could be made immediately, when the state variable crosses an optimal threshold, or never. The combinations of parameters which determine the boundaries of the optimal policies are described in the following theorem.

Theorem 3.3.1 *The solution of operational problem (3.2) and (3.3) is summarized in Table 3.1.*

Proof in Appendix A.1 ■

The first three rows of Table 3.1 correspond to the upgrade case. Here the solution yields two cases, when it is desirable to exercise the option immediately, and when it is optimal to stay in the current system until the state variable rises above a threshold. The second three rows of Table 3.1 correspond to the downgrade case. Here the solution yields three cases, where the option is never desirable, immediately desirable, and desirable once the state variable falls below an optimal threshold. The reason there is no set of parameters within the upgrade case which produce a solution to never exercise the option is that the state variable $X(t)$ is not bounded above, and therefore the greater variable yield from the alternate system can always become great enough to compensate for the exercise costs as well as possibly greater system fixed operation costs. It can easily be verified that when the state variable is exactly

Table 3.1
Optimal Operational Policy and Operational Value

x, θ			Optimal Operational Policy	$v(x, \delta)$
$a_1 < a_2$	$\tilde{c}(\delta) < \frac{b_1 - b_2}{\rho}$	$0 \leq x < x_1^*(\delta)$	Continue in current mode until $\tau_1^* = \inf\{t \geq 0 \| x \geq x_1^*(\delta)\}$	$v_{c,1}(x, \delta)$
		$x_1^*(\delta) \leq x$	Exercise the option immediately	$v_0(x, \delta)$
	$\tilde{c}(\delta) \geq \frac{b_1 - b_2}{\rho}$		Exercise the option immediately	$v_0(x, \delta)$
$a_1 > a_2$	$\tilde{c}(\delta) \leq \frac{b_1 - b_2}{\rho}$		Never exercise the option	$v_\infty(x)$
	$\tilde{c}(\delta) > \frac{b_1 - b_2}{\rho}$	$0 \leq x \leq x_2^*(\delta)$	Exercise the option immediately	$v_0(x, \delta)$
		$x_2^*(\delta) < x$	Continue in current mode until $\tau_2^* = \inf\{t \geq 0 \| x \leq x_2^*(\delta)\}$	$v_{c,2}(x, \delta)$

$$x_i^*(\delta) = \frac{r_i(\mu - \rho)}{(a_1 - a_2)(r_i - 1)} \left[\frac{b_1 - b_2}{\rho} - \tilde{c}(\delta) \right] e^{-\mu\delta}, \quad \text{for } i = 1, 2 \tag{3.5}$$

$$r_1 = \frac{(\sigma^2 - 2\mu) + \sqrt{(2\mu - \sigma^2)^2 + 8\rho\sigma^2}}{2\sigma^2} \tag{3.6}$$

$$r_2 = \frac{(\sigma^2 - 2\mu) - \sqrt{(2\mu - \sigma^2)^2 + 8\rho\sigma^2}}{2\sigma^2} \tag{3.7}$$

$$v_0(x, \delta) = \frac{\{(a_1 - a_2)e^{(\mu - \rho)\delta} - a_1\}x}{\mu - \rho} - \frac{b_1 - b_2}{\rho} e^{-\rho\delta} + \frac{b_1}{\rho} + e^{-\rho\delta}\tilde{c}(\delta) \tag{3.8}$$

$$v_\infty(x) = \frac{a_1 x}{\rho - \mu} + \frac{b_1}{\rho} \tag{3.9}$$

$$v_{c,i}(x, \delta) = \frac{e^{-\rho\delta}}{r_i - 1} \left[\frac{b_1 - b_2}{\rho} - \tilde{c}(\delta) \right] \left(\frac{x}{x_i^*(\delta)} \right)^{r_i} - \frac{a_1 x}{\mu - \rho} + \frac{b_1}{\rho}, \quad \text{for } i = 1, 2 \tag{3.10}$$

equal to the threshold level, the system operator is indifferent between exercising and continuing.

Effect of Delay on Optimal Control

In analyzing the effect of exercise delay on the optimal operational policy, the key insight is that lengthening the exercise delay of the system increases the risk exposure an operator faces when controlling the system flexibility. It turns out this increased exposure has varying effects on the expected exercise time depending both on the trend of the stochastic process and the effect of the option itself.

To concentrate on the effect of additional risk exposure, consider a constant exercise cost. If the exercise cost is a constant, $\frac{d}{d\delta}x_i^*(\delta) = -\mu x_i^*(\delta)$. When the stochastic process has positive drift, $\mu > 0$, the future state is likely to be favorable to an upgraded system. This expectation hastens execution of a upgrade option given a realization of underlying uncertainty, since $\frac{d}{d\delta}x_1^*(\delta) < 0$, and may eventually make the option immediately desirable. On the other hand, the trend in the state variable means those longer delays make downgrade options increasingly dour. A longer exercise delay lowers the threshold value for downgrading, because $\frac{d}{d\delta}x_2^*(\delta) < 0$, and may eventually bring the option into the never desirable case. It implies that the longer delay defers downgrading the system. In the case that $\mu < 0$, the exercise delay has opposite effects on the optimal operational policies, since $\frac{d}{d\delta}x_1^*(\delta) > 0$ and $\frac{d}{d\delta}x_2^*(\delta) > 0$.

In the interesting solution cases where the option is not currently desirable but will be exercised if the state variable crosses some threshold this research provides the following interpretation. If the trend of underlying uncertainty is favorable to the alternative system, the system operator is essentially trading off the optimality of the current system configuration in the current state against the future optimality of the alternative system configuration in the uncertain future states. With highly flexible systems (short exercising delays), the operator is able to delay execution when the trend of the stochastic process is favorable to the alternate system to squeeze ever last drop out of the current system's optimality. With less flexibility (longer exercise delays) the system operator's hand is forced to more quickly act upon the expected optimality of the alternative system in future states.

Even when the trend of the underlying uncertainty is unfavorable to the alternative system, flexibility may be exercised. It is the case that the realized uncertainty is so favorable (to the alternative system) that the alternative system can harvest enough benefits from the temporary advantageous status. In this case, a longer delay requires more advantageous status to exercise the flexible option, and it results postponements of execution time. In the view point of exposure delay, a longer delay increases the risk that the realized advantageous status becomes unfavorable against the alternative system. Therefore, the system operator exercises the flexible option with more favorable realization of underlying uncertainty to compensate the increased risk exposure due to the exercise delay.

The above analysis can be extended into the case that the exercise cost is a function of exercise delay to consider both risk exposure effect and cost effect of exercise delay upon optimal operational policy. Suppose that the exercise cost is once differentiable with respect to exercise delay δ. The first derivative of the threshold value with respect to exercise delay is expressed as

$$\frac{d}{d\delta}x_i^*(\delta) = -\left[\frac{r_i(\mu-\rho)}{(a_1-a_2)(r_i-1)}\right]\left[\mu\frac{b_2-b_1}{\rho} - \mu\tilde{c}(\delta) + \frac{d}{d\delta}\tilde{c}(\delta)\right]e^{-\mu\delta} \tag{3.11}$$

As shown in (3.11), even strong assumptions such as convexity or monotonicity of the exercise cost function do not guarantee straightforward relationships between exercise delay and exercise time. If $\frac{d}{d\delta}\tilde{c}(\delta) > \mu\left[\tilde{c}(\delta) - \frac{b_2-b_1}{\rho}\right]$ then $x_i^*(\delta)$ is increasing, otherwise $x_i^*(\delta)$ is decreasing. Given that $\frac{d}{d\delta}\tilde{c}(\delta) > \mu\left[\tilde{c}(\delta) - \frac{b_2-b_1}{\rho}\right], \forall\delta \in [\delta_{min}, \delta_{max}]$, the optimal threshold value of upgrade is a monotone decreasing function and that of downgrade is a monotone increasing function in exercise delay. Hence the expected exercise time is monotone decreasing with respect to exercise delay in both of the cases. On the other hand, if $\frac{d}{d\delta}\tilde{c}(\delta) < \mu\left[\tilde{c}(\delta) - \frac{b_2-b_1}{\rho}\right], \forall\delta \in [\delta_{min}, \delta_{max}]$, the relationships are reversed. The integrated effect of exercise delay is summarized in Table 3.2.

Table 3.2
The Effect of Exercise Delay upon Optimal Operational Policy

	$\frac{d}{d\delta}\tilde{c}(\delta) > \mu\left[\tilde{c}(\delta) - \frac{b_2-b_1}{\rho}\right]$	$\frac{d}{d\delta}\tilde{c}(\delta) < \mu\left[\tilde{c}(\delta) - \frac{b_2-b_1}{\rho}\right]$
Upgrade ($a_1 < a_2$)	Longer delay postpones the expected exercise time.	Longer delay hastens the expected exercise time.
Downgrade ($a_1 > a_2$)	Longer delay hastens the expected exercise time.	Longer delay postpones the expected exercise time.

3.3.3 Optimal Design of Flexible System

This section investigates the problem faced by a system designer; how much exercise delay (flexibility) to build into a system. At the design stage, a system designer chooses the optimal length of exercise delay assuming the system is operated optimally as summarized in Theorem 3.1. The first insight this dissertation discovers is that the design problem is decomposed into two sub-problems based on the optimal operation. Moreover, this research effort discovers that when the exercise cost and system design cost are once differentiable with respect to the length of exercise delay, the design problem is well posed and solvable with usual non-linear optimization method such as KarushKuhnTucker (KKT) conditions.

Decomposition of Design Problem

To illustrate the structure of optimal design problem, consider an upgrade case. Here, this research postulates that the exercise cost is a continuous function with respect to exercise delay to reflect more situations. Figure 3.1 demonstrates an example of the structure of an operational value of a flexible system under optimal operation.

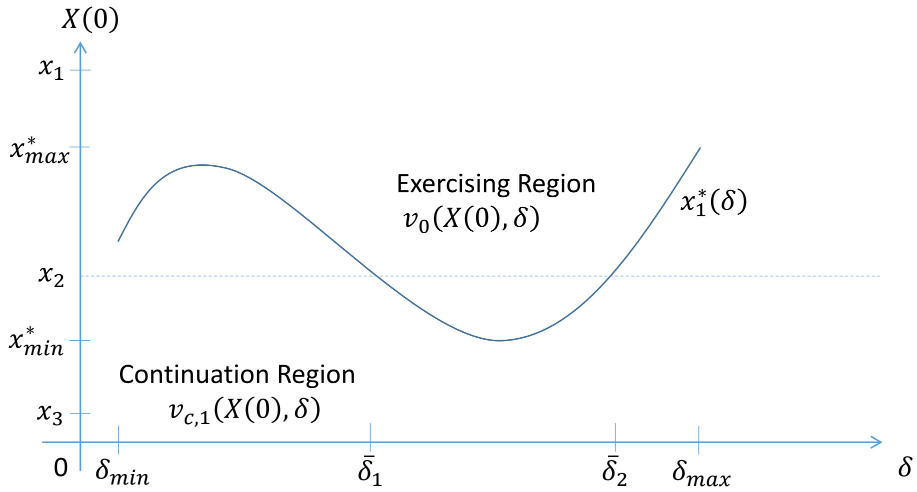


Figure 3.1. Structure of Flexible System Design Problem with Delay

The horizontal and vertical axis stands for the length of exercise delay, δ, and the current value of state variable, $X(0)$, respectively. In the assumed case there exists a threshold value $x^*_1(\delta)$ that determines the optimal operational policy. If the current state variable $X(t)$ is smaller than $x^*_1(\delta)$, staying current mode is optimal. In opposition, if $X(t) \geq x^*_1(\delta)$, exercising the flexible option is optimal. Therefore the region above $x^*_1(\delta)$ is called exercising region and that below $x^*_1(\delta)$ is continuation region. In the exercising region, the operational value of flexible system is $v_0(X(0), \delta)$ and that in the continuation region is $v_{c,1}(X(0), \delta)$.

Let x^*_{min} and x^*_{max} be the minimum and maximum value of $x^*_1(\delta)$. If the current state is higher than x^*_{max} or lower than x^*_{min}, the the design problem is simple. If $X(0)x^*_{max}$, such as $X(0) = x_1$, the operational value function is $v(X(0), \delta) = v_0(X(0), \delta), \forall\delta \in [\delta_{min}, \delta_{max}]$, and $v_0(X(0), \delta)$ is given in Theorem 3.3.1 explicitly. Therefore the design problem (3.3) becomes $V(x) = \sup_{\delta\in[\delta_{min},\delta_{max}]}[v_0(X(0), \delta) - C(\delta)]$. Similarly, when $X(0)x^*_{min}$, for instance $X(0) = x_3$, the operational value function is $v_{c,1}(X(0), \delta)$, for all $\delta \in [\delta_{min}, \delta_{max}]$. Therefore the design problem is $V(x) = \sup_{\delta\in[\delta_{min},\delta_{max}]}[v_{c,1}(X(0), \delta) - C(\delta)]$. Given the once differentiability assumptions of cost functions, it is a nice behave non-linear optimization problem that is solvable with usual techniques.

When $X(0) \in (x^*_{min}, x^*_{max})$, the design problem becomes complicated. Suppose that the system designer sets $\delta \in [0, \bar{\delta}_1) \cup (\bar{\delta}_2, \delta_{max}]$. Then the operator waits until the underlying uncertainty $X(t)$ hits the threshold value $x^*_1(\delta)$ to exercise the flexible option. Hence, the operational value function $v(X(0), \delta) = v_{c,1}(X(0), \delta)$. Otherwise, the operator exercise the option immediately and the operational value function is $v(X(0), \delta) = v_0(X(0), \delta)$. Notice that the objective function of design problem is the sum of operational value function and the system design cost. Since the length of delay changes the objective function of design problem, the system design problem is decomposed into two sub problems. Following theorem provides the characteristics of design level problem in complicated cases, which is helpful for computation.

Theorem 3.3.2 *When the exercise cost and system design cost are continuous with respect to δ, the design problem (3.3) is decomposable into two nonlinear optimization problems. For upgrade cases, the optimal solution of design problem is obtained by comparing the two solutions of sub-problems:*

$$P_1 = \begin{cases} \max_\delta & v_{c,1}(X(0), \delta) - C(\delta) \\ s.t. & \delta_{min} \le \delta \le \delta_{max} \\ & X(0) \le x^*_1(\delta) \end{cases} \quad and \quad P_2 = \begin{cases} \max_\delta & v_0(X(0), \delta) - C(\delta) \\ s.t. & \delta_{min} \ge \delta \le \delta_{max} \\ & X(0) \ge x^*_1(\delta) \end{cases} \tag{3.12}$$

For downgrade cases, the design level problem is solvable by decomposing the original design problem into two sub-problems:

$$P_1 = \begin{cases} \max_\delta & v_{c,2}(X(0), \delta) - C(\delta) \\ s.t. & \delta_{min} \le \delta \le \delta_{max} \\ & X(0) \ge x^*_2(\delta) \end{cases} \quad and \quad P_2 = \begin{cases} \max_\delta & v_0(X(0), \delta) - C(\delta) \\ s.t. & \delta_{min} \ge \delta \le \delta_{max} \\ & X(0) \le x^*_2(\delta) \end{cases} \tag{3.13}$$

Proof in Appendix A.2 ■

The sub-problem P_1 can be interpreted as finding the optimal configuration with the constraints forcing the system operator to wait until the execution criterion is satisfied. On the other hand, P_2 means that with constraints compelling the system operator exercise the flexible option immediately. Since each sub-problem attains its maximum in the feasible region, the objectives of decomposed problems are maximization instead of finding supremum in (3.4). Moreover, when the cost functions are smooth, the objective functions are smooth as well. It implies that usual non-linear optimization techniques (such as KKT conditions) are able to be employed to solve the design problem.

Illustrative Examples

This subsection provides an illustrative example in the context of renewable energy source expansion flexibility [89, pp.164-165]. Suppose that an electric utility considers investing in an expandable power plant. The power plant starts with a limited number of turbines, and if the electricity price goes up the utility can double up the capacity. The expansion takes time and the utility can expedite the expansion by paying additional amount of money. At the design phase, the utility system designer decides how fast the power plant is expanded. The system operators task is to increase the capacity at the right time observing the electricity price.

The electricity price follows a geometric Brownian motion described in (3.1) with $\mu = 0.02$ and $\sigma = 0.1$. It means that the price is expected to be increased with 2% per year continuous compound growth rate, and the price process has 10% volatility. Before the expansion, the power plant generates a unit of electricity with a unit cost, i.e., $a_1 = 1$ and $b_1 = -1$. When the generation capacity is doubled up, the generating cost increases proportionally ($a_2 = 2$ and $b_2 = -2$). The discount rate is assumed to be 5%, i.e., $\rho = 0.05$. The system building cost and the exercise cost are assumed to be

$$\tilde{c}(\delta) = -e^{-\delta}, \qquad C(\delta) = \frac{1}{\delta}$$

The system building cost is inversely proportional to the exercise delay and is motivated by [139, 140]. Both of the papers asserted that the value of a supply chain is inverse proportional to response time. The exponential decreasing exercise cost is intuitively acceptable. Moreover, the exercise cost yields a monotone decreasing $x_1^*(\delta)$, and it is helpful to clarify the structure of decomposition. This research sets the range of exercise delay as $\delta_{min} = 0.1 \leq \delta \leq \delta_{max} = 10$, where δ is the exercise delay in years.

This section focuses on exemplifying the process of solving the design problem using Theorem 3.3.2 rather than providing the specific results. For the readers who are interested in replicating the results, the numerical solution is given in Appendix B.3. Since this example is an upgrade case, $x_1^*(\delta)$ divides the design problem into two

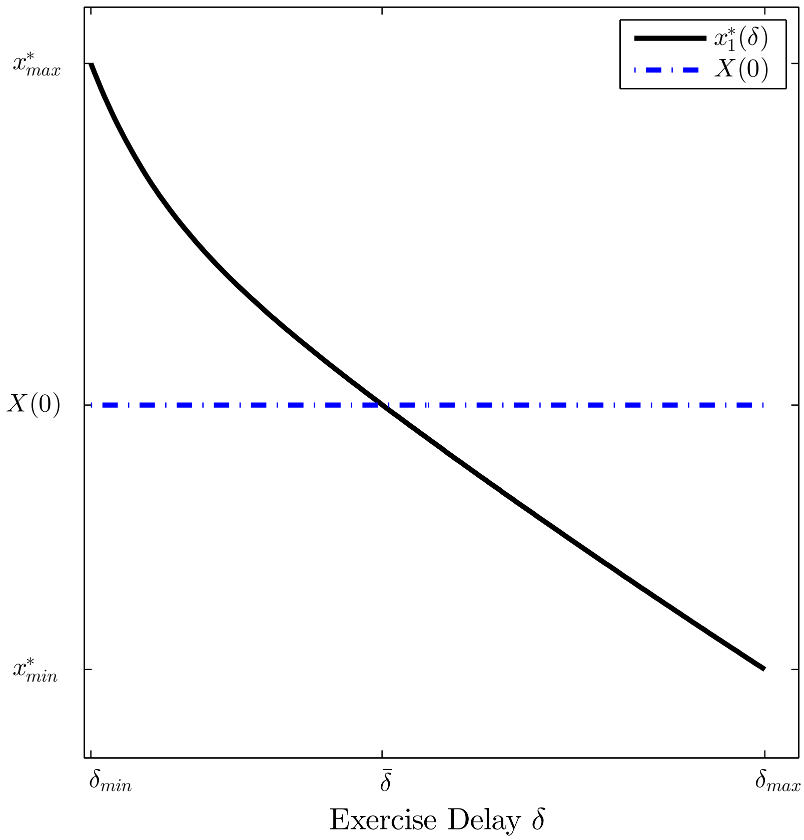


Figure 3.2. Exercising Threshold Value and Decomposition

sub-problems. With the postulated parameters and exercise cost, $x_1^*(\delta)$ decreases as the exercise delay increases. The maximum of $x_1^*(\delta)$ is attained when the exercised delay is set to be δ_{min}, and the maximum is attained at δ_{max}. The horizontal axis represents the length of delay (δ), and the vertical axis represents the price of electricity. The solid line of Figure 3.2 shows the threshold value $x_1^*(\delta)$, and the dotted line represents the current electricity price $X(0)$. Notice that the current price and exercising threshold value are identical at $\delta = \bar{\delta}$, and $x_1^*(\delta)$ is higher than $X(0)$ for $\delta < \bar{\delta}$. This implies that if the system designer sets the length of delay less than $\bar{\delta}$, the system operator waits until the price goes up $x_1^*(\delta)$ to expand the capacity. The optimal configuration of the expandable power plant for $\delta < \bar{\delta}$ is obtained by solving P_1. On the other hand, the optimal operational decision is expanding the capacity as soon as possible when the power plant is designed with exercise delay δ that is longer than $\bar{\delta}$. By solving P_2, the system designer can find the optimal design.

Figure 3.3 represents the objective function value of design problem $v(X(0), \delta, \theta) - C(\delta)$ with respect to exercise delay, given the current electricity price. The solid line represents the objective function value of P_1 and the dashed line stands for that of P_2. The optimal solution of each sub-problems are denoted by δ_1^* and δ_2^*. Since the optimal solution of P_1 yields higher value V^* than that of P_2, the optimal solution of design problem is δ_1^*. Therefore, the expandable power plant is designed to increase its capacity in δ_1^*, and the system operator expand the capacity when the electricity price goes up to $x_1^*(\delta_1^*)$, given the current price X(0). In analyzing the optimal flexibility design with respect to the current price, this research focuses on the area, $X(0) \in [x_{min}^*, x_{max}^*]$ in which the design problem need to be decomposed into two sub-problem.

Figure 3.4 shows the optimal objective function values of sub-problems with respect to current electricity price $X(0)$. The solid line is the optimal value of P_1 and the dotted line is that of P_2. In Figure 3.5, the solid line (δ_1^*) and the dotted line (δ_2^*) represents the optimal solution of P_1 and that of P_2, respectively. In Figure 3.4, the optimal value of P_1 is greater than that of P_2 when the current price is lower than $\bar{x}$. Among the exercise delays those force the system operator stay in the current

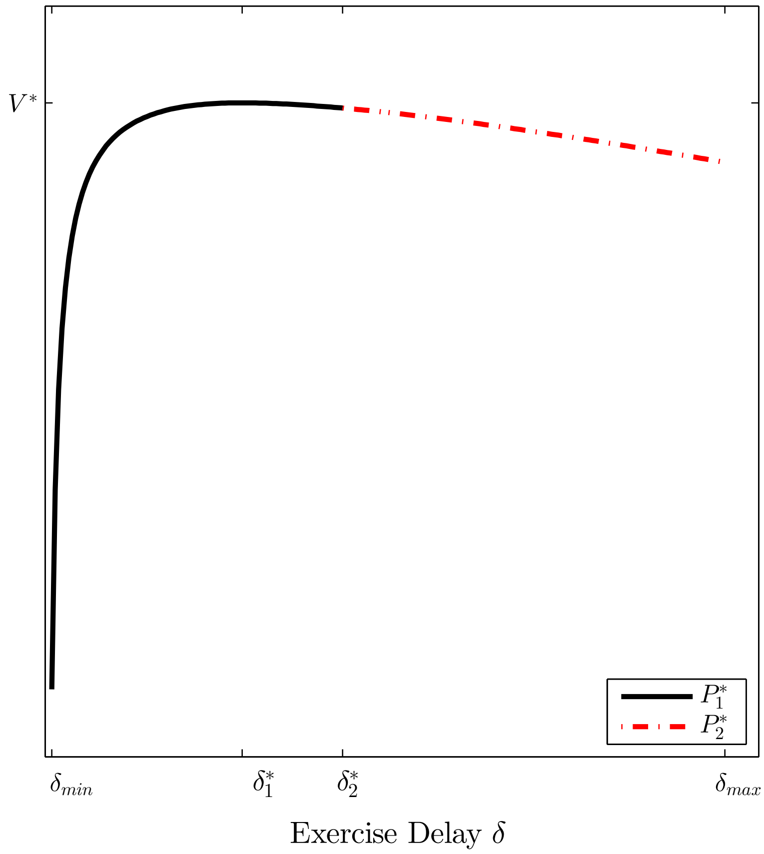


Figure 3.3. Design Problem Objective Function Value

capacity until the price goes up high enough, δ_1^* is the best choice. On the other hand, δ_2^* yields the highest value among the exercise delays those compel the system operator to expand the capacity immediately. Since P_1^* is higher than P_2^* when the current price is lower than $\bar{x}$, it is optimal for the system designer to set the exercise delay as δ_1^*, and for the system operator to wait until the electricity price grows up to $x_1^*(\delta_1^*)$. On the other hand, when the current price is higher than $\bar{x}$, the value of expandable power plant is maximized by starting the expansion immediately within δ_2^*. Therefore, the shaded δ^* in Figure 3.4 is the optimal solution of overall design problem based on the current price of electricity.

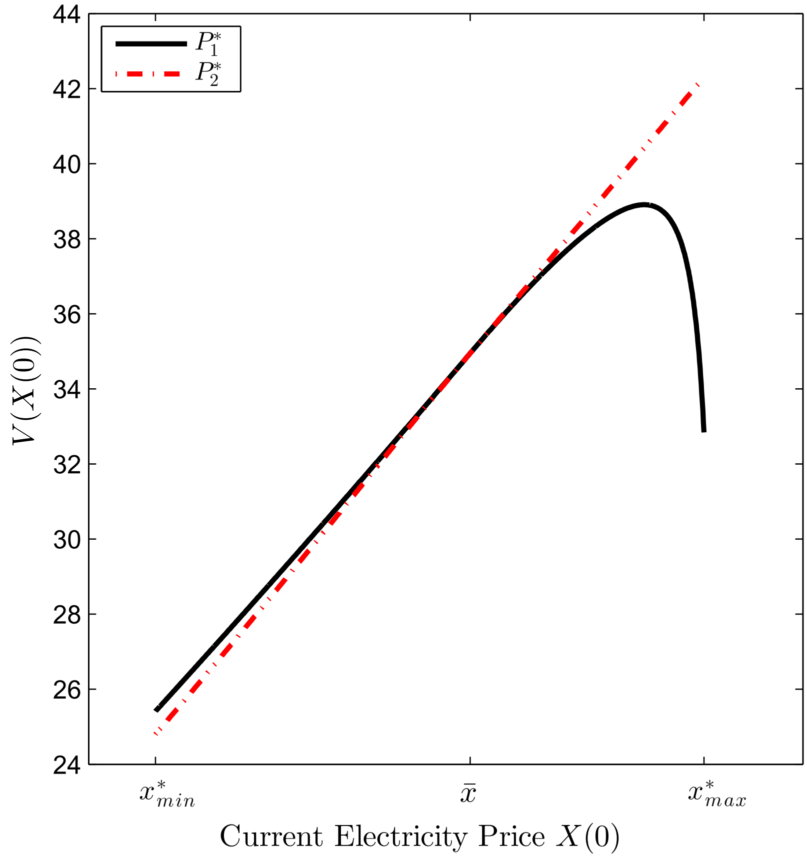


Figure 3.4. Optimal Objective Function Values of Sub-problems

With the given assumptions about the cost functions, an exercise delay that enforces the system operator to hold the expansion option is lower than that of immediate expansion. Therefore, δ_1^* is always lower than δ_2^*. Since a higher electricity price is more desirable for expanding the capacity, higher price provides an incentive to shorten the exercise delay at the expense of higher costs. The downward slopes of δ_1^* and δ_2^* reflect this incentive. When the current price is low, immediate upgrade of the system is not attractive. Therefore to save on costs, the system designer chooses a longer delay among the delays forcing an immediate upgrade. However, if the system designer selects a relatively short delay among the delays which make the system operator hold off on the expansion option until the electricity price becomes favor-

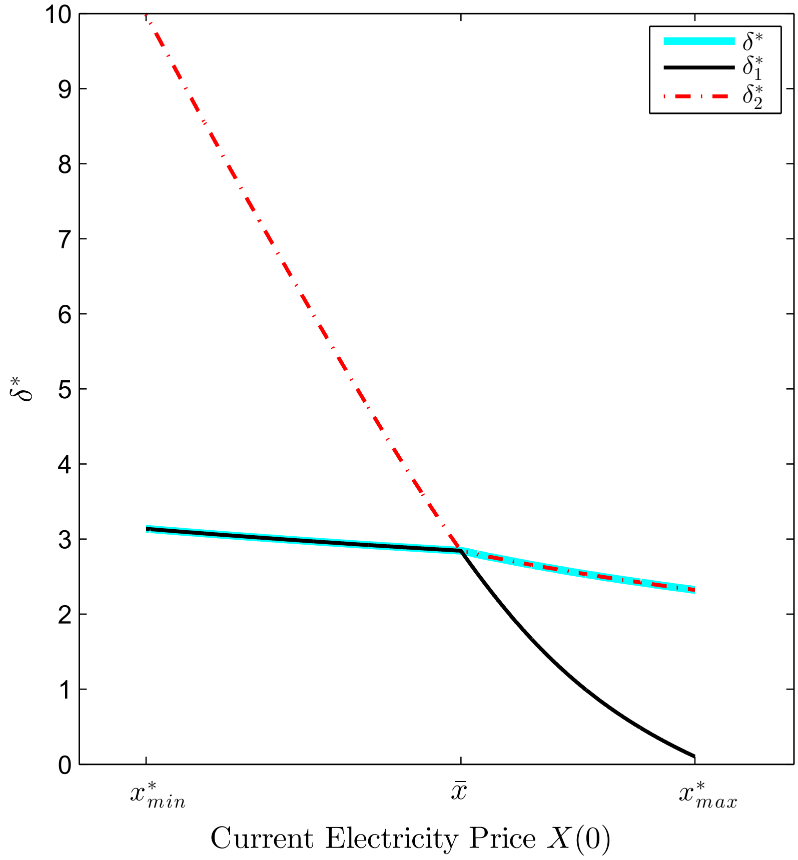


Figure 3.5. Optimal Solutions of Sub-problems

able enough for the capacity expansion. Between the optimal choices, δ_1^* yields higher value than δ_2^* when the current price is lower than $\bar{x}$ as shown in Figure 3.4. Therefore the optimal solution of overall design problem is δ_1^* where $X(0) < \bar{x}$. Notice that the slope of δ_1^* in this region is determined by the additional costs to shorten the exercise delay and incremental value of $v_(c, 1)$ (operational value function given that holding the option is optional). Once the price reaches $\bar{x}$, immediate expansion becomes more beneficial than holing the expansion option for better chances. Hence, the optimal solution of the overall design problem is δ_2^* where $X(0) < \bar{x}$. In this region the slope changes because the additional costs and incremental value of v_0 determine the slope.

3.4 Multiple Alternatives Model

This section extends the model of Section 3.3 including the level of capacity change as a design variable. In Section 3.3, the flexible system has only one alternative, and the assumption helps to elucidate how exercise delay affects optimal decisions. However, the extent of capacity change has been the main focus of flexible system design. By considering both exercise delay and level of capacity change, this research contributes to the research area of flexibility. Similar to the previous section, the problem is postulated as a two levels decision problem. The system designer decides the length of exercise delay and the level of capacity change imposing that the system operator behaves optimally. At the operational level, a system operator chooses the time to execute the flexible option.

3.4.1 Model

At the operational level, system operator decides the time to exercise the flexible option, given the exercise delay δ and the level of capacity change ζ those are determined in design level. The operational performance function $j(X(0), \tau|\delta, \zeta)$ and value function $v(X(0), \delta, \zeta)$ are defined as

$$j(X(0), \tau|\delta, \zeta) = \mathbb{E}\left[\begin{array}{c} \int_0^{\tau+\delta} e^{-\rho t}\{aX(t) + b\}dt + e^{-\rho(\tau+\delta)}\tilde{c}(\delta, \zeta) \\ + \int_{\tau+\delta}^{\infty} e^{-\rho t}\zeta\{aX(t) + b\}dt \end{array}\right] \tag{3.14}$$

$$v(X(0), \delta, \zeta) = \sup_{\tau} j(X(0), \tau|\delta, \zeta) \tag{3.15}$$

Consider a flexible energy utility, whose initial configuration generates a units of electricity with the operating cost $-b$. By paying the exercise cost or harvesting the salvage value $\tilde{c}(\delta, \zeta)$, the capacity can be increased or decreased to ζ units. It takes time δ to adjust the capacity of the flexible power plant. The generating cost is changed proportionally to the capacity change. The unit price of electricity follows the geometric Brownian motion defined in (3.1). The system designer's task is determining the length of exercise delay and the extent of capacity change assuming that the system operator optimally exercises the designed flexible option. The design problem is expressed with

$$V(X(0)) = \sup_{(\delta,\zeta)\in\mathbb{A}} \mathbb{E}\left[v(x(0),\delta,\zeta) - C(\delta,\zeta)\right] \tag{3.16}$$

The system designer can choose one of the flexible system configuration, which belongs to $\mathbb{A} = [\delta_{min}, \delta_{max}] \times [\zeta_{min}, \zeta_{max}]$. Since negative δ and ζ do not have practical meanings, this research assumes that $0 \leq \delta_{min} \leq \delta_{max}$ and $0 \leq \zeta_{min} \leq \zeta_{max}$. If $\zeta > 1$, the capacity is increased when the expansion is completed. On the other hand, $\zeta < 1$ means decrease the capacity.

This research considers continuous and once differentiable cost functions on $\mathbb{A}$ and imposes following additional assumptions on cost functions.

$$\tilde{c}(\delta, 1) = 0, \quad \forall\delta \in [\delta_{min}, \delta_{max}] \tag{3.17}$$

$$C(\delta, 1) = 0, \quad \forall\delta \in [\delta_{min}, \delta_{max}] \tag{3.18}$$

Suppose that $\zeta = 1$. Then the exercising flexible option does not change the capacity of the system. The cases of $\tilde{c}(\delta, 1) \neq 0$ and $C(\delta, 1) \neq 0$ yield obvious solutions of operational and design problems. To avoid these obvious solutions, (3.17) and (3.18) are imposed. For example, if $\tilde{c}(\delta, 1) > 0$ and $\zeta = 1$, the system operator exercise the option as soon as possible to acquire the free cash inflow.

3.4.2 Optimal Operational Policy

Given the exercise delay δ and the extent of change ζ, the optimal operational policy is derived using Theorem 3.3.1 by setting $a_1 = a, a_2 = \zeta a, b_1 = b$ and $b_2 = \zeta b$. Theorem 3.4.1 summarizes the optimal policy.

Theorem 3.4.1 *The optimal operational policy, the solution of (3.15), is summarized in Table 3.3, where r_1 and r_2 are identical to those in Theorem 3.3.1.*

Notice that the threshold value of changing the system feature is expressed with $x_i^*(\delta, \zeta)$ to emphasize on the effect of design variables on the operational policy. Theorem 3.4.1 is a straight forward application of Theorem 3.3.1 except for the case that

Table 3.3
Optimal Operational Policy Given the System Configuration

$X(0), \theta$		Optimal Operational Decision	$v(X(0), \delta, \theta)$
$\zeta > 1$	$0 \le X(0) < x_1^*(\delta, \zeta)$	Continue in the current mode until $\tau_1^*\{t \ge 0 \vert X(t) \ge x_1^*(\delta, \zeta)\}$	$v_{c,1}(X(0), \delta, \zeta)$
	$X(0) \ge x_1^*(\delta, \zeta)$	Exercise the option immediately	$v_0(X(0), \delta, \zeta)$
$\zeta = 1$		Indifferent	$\frac{aX(0)}{\rho-\mu} + \frac{b}{\rho}$
$\zeta < 1$	$0 \le X(0) < x_2^*(\delta, \zeta)$	Exercise the option immediately	$v_0(X(0), \delta, \zeta)$
	$x_2^*(\delta, \zeta) < X(0)$	Continue in the current mode until $\tau_2^* = \{t \ge 0 \vert X(t) \le x_2^*(\delta, \zeta)\}$	$v_{c,2}(X(0), \delta, \zeta)$

where for $i = 1, 2$ (3.19)

$$x_i^*(\delta, \zeta) = \left[\frac{r_i(\mu-\rho)}{a(1-\zeta)(r_i-1)}\right]\left[\frac{b(1-\zeta)}{\rho} - \tilde{c}(\delta, \zeta)\right] e^{-\mu\delta} \tag{3.20}$$

$$v_0(X(0), \delta, \zeta) = \left[\frac{a(1-\zeta)}{\mu-\rho} e^{(\mu-\rho)\delta} - \frac{a}{\mu-\rho}\right] X(0) - \frac{b(1-\zeta)}{\rho} e^{-\rho\delta} + \frac{b}{\rho} + e^{-\rho\delta}\tilde{c}(\delta, \zeta) \tag{3.21}$$

$$v_{c,i}(X(0), \delta, \zeta) = \frac{e^{-\rho\delta}}{r_i - 1}\left[\frac{b(1-\zeta)}{\rho} - \tilde{c}(\delta, \zeta)\right]\left[\frac{X(0)}{x_i^*(\delta, \zeta)}\right]^{r_i} - \frac{a}{\mu-\rho} X(0) + \frac{b}{\rho} \tag{3.22}$$

$\zeta = 1$. The result for $\zeta = 1$ is obtained by plugging $\zeta = 1$ and $\tilde{c}(\delta, 1) = 0$ into (3.14) and (3.15). Notice that the system would not be changed by exercising the flexible option when $\zeta = 1$. Therefore, the system operator is indifferent to whether to exercise the flexible option or not.

3.4.3 Optimal Design

The design problem is also decomposable with sub-problems, given the current value of the underlying uncertainty $X(0)$. If all the available alternatives are upgrade options, $1 < \zeta_{min}$, the decomposition is similar to (3.12). In the case that $1 > \zeta_{max}$, all the alternatives are downgrade options. Therefore, the the decomposition is similar to (3.13). However, when the set of alternatives includes upgrade, downgrade and staying with the current system mode, i.e., $1 \in [\zeta_{min}, \zeta_{max}]$, the design problem is not well-posed, since $x_i^*(\delta, \zeta)$ is not defined at $\zeta = 1$. This research finds that the design problem can be decomposed into well behaved sub-problems by imposing a fictional threshold if it is necessary. For notational simplicity, define $\underline{\mathbb{A}} = [\delta_{min}, \delta_{max}] \times [\zeta_{min}, 1]$ and $\bar{\mathbb{A}} = [\delta_{min}, \delta_{max}] \times [1, \zeta_{max}]$ for the case that $\zeta_{min} \leq 1 \leq \zeta_{max}$.

Theorem 3.4.2 *Assume that $\tilde{c}(\delta,\zeta)$ and $C(\delta,\zeta)$ are once differentiable on $\mathbb{A}$. The design problem (3.15) is decomposed into four sub-problems.*

$$P_1 = \begin{cases} \max_{\delta,\zeta} & v_0(X(0),\delta,\zeta) - C(\delta,\zeta) \\ s.t. & (\delta,\zeta) \in \underline{\mathbb{A}} \\ & X(0) \leq x_2^*(\delta,\zeta) \end{cases} \tag{3.23}$$

$$P_2 = \begin{cases} \max_{\delta,\zeta} & v_{c,2}(X(0),\delta,\zeta) - C(\delta,\zeta) \\ s.t. & (\delta,\zeta) \in \underline{\mathbb{A}} \\ & X(0) \geq x_2^*(\delta,\zeta) \end{cases} \tag{3.24}$$

$$P_3 = \begin{cases} \max_{\delta,\zeta} & v_{c,1}(X(0),\delta,\zeta) - C(\delta,\zeta) \\ s.t. & (\delta,\zeta) \in \bar{\mathbb{A}} \\ & X(0) \leq x_1^*(\delta,\zeta) \end{cases} \tag{3.25}$$

$$P_4 = \begin{cases} \max_{\delta,\zeta} & v_0(X(0),\delta,\zeta) - C(\delta,\zeta) \\ s.t. & (\delta,\zeta) \in \bar{\mathbb{A}} \\ & X(0) \geq x_1^*(\delta,\zeta) \end{cases} \tag{3.26}$$

If $\frac{b}{\rho} + \left.\frac{\partial \tilde{c}(\delta,\zeta)}{\partial \zeta}\right|_{\zeta=1} < 0$, the threshold value is defined as

$$x_i^*(\delta,1) = \frac{r_i(\mu-\rho)}{a(r_i-1)} \left[\frac{b}{\rho} + \left.\frac{\partial \tilde{c}(\delta,\zeta)}{\partial \zeta}\right|_{\zeta=1} \right] e^{-\mu\delta}$$

.

Proof in Appendix A.3 ■

It is worthwhile to review the economic meanings of decomposed problems. Postulate that the constraints of (3.23) are satisfied in the design phases. Then the designed features of the flexible system is downgrading the capacity with a exercise delay. Moreover the designed features force the system operator to start the reducing capacity immediately. In the feasible region of (3.24), the system capacity is decreased by exercising the flexible option, and the capacity reduction starts when the underlying uncertainty hits $x_2^*(\delta,\zeta)$ that is lower than the current status. If the constraints of (3.25) are satisfied, the flexible system is designed to expand its capacity, and the expansion starts when the underlying uncertainty reaches the threshold value $x_1^*(\delta, \zeta)$ that is higher than current state. The region that the constraints of (3.26) are satisfied, the flexible system has expansion option, and the expansion starts as soon as possible, because the exercising threshold value is lower than the current state.

Illustrative Example

This subsection provides another illustrative example. An electric utility considers investing in a flexible power plant whose capacity change takes time δ. The generation capacity is changed from 1 to ζ by exercising the flexible option. The system operator's task is finding the optimal time to start altering the capacity, and the system designer's is determining optimal δ and ζ considering the related costs and benefits. The parameters representing the market conditions are the same to the previous example, i.e., $\mu = 0.02, \sigma = 0.1$ and $\rho = 0.05$. The range of possible choices for system designer is assumed to be $\delta_{min} = 0.1 \leq \delta \leq \delta_{max} = 10$ and $\zeta_{min} = 0 \leq \zeta \leq \zeta_{max} = 3$. The economic implication of the range of possible capacity change is that the utility can increase its capacity by three times of current capacity at most, and can reduce the capacity as much as the company wants. The reduction of capacity even includes exiting from the industry by choosing $\zeta = 0$.

The the exercise cost $\tilde{c}(\delta, \zeta)$ and system building cost $C(\delta, \zeta)$ are assumed to be

$$\tilde{c}(\delta, \zeta) = (1 - \zeta)e^{-\delta}, \qquad C(\delta, \zeta) = \frac{(\zeta - 1)^2}{\delta}$$

These assumptions concurs with the assumptions in (3.17) and (3.18). Moreover, these costs represent that quick response system costs more in both exercise and design phases, because $\frac{\partial}{\partial \delta}\tilde{c}(\delta, \zeta) < 0$ and $\frac{\partial}{\partial \delta}C(\delta, \zeta) < 0$. When the capacity is increased, the system operator pays change cost, i.e., $\tilde{c}(\delta, \zeta) < 0$ for $\zeta > 1$. On the other hand, a capacity scale down causes a cash inflow from salvage values of existing facilities, i.e., $\tilde{c}(\delta, \zeta) > 0$ for $\zeta < 1$. However, the more the system can change its capacity, the more flexible the system is no matter what the direction of the change is. Therefore, the system designing cost is a convex function in the extend of capacity change and is minimized at $\zeta = 1$. The system building cost reflects this aspect, since $\frac{\partial}{\partial \zeta} C(\delta, \zeta)\Big|_{\zeta=1} = 0$ and $\frac{\partial^2}{\partial \zeta^2} C(\delta, \zeta) > 0$.

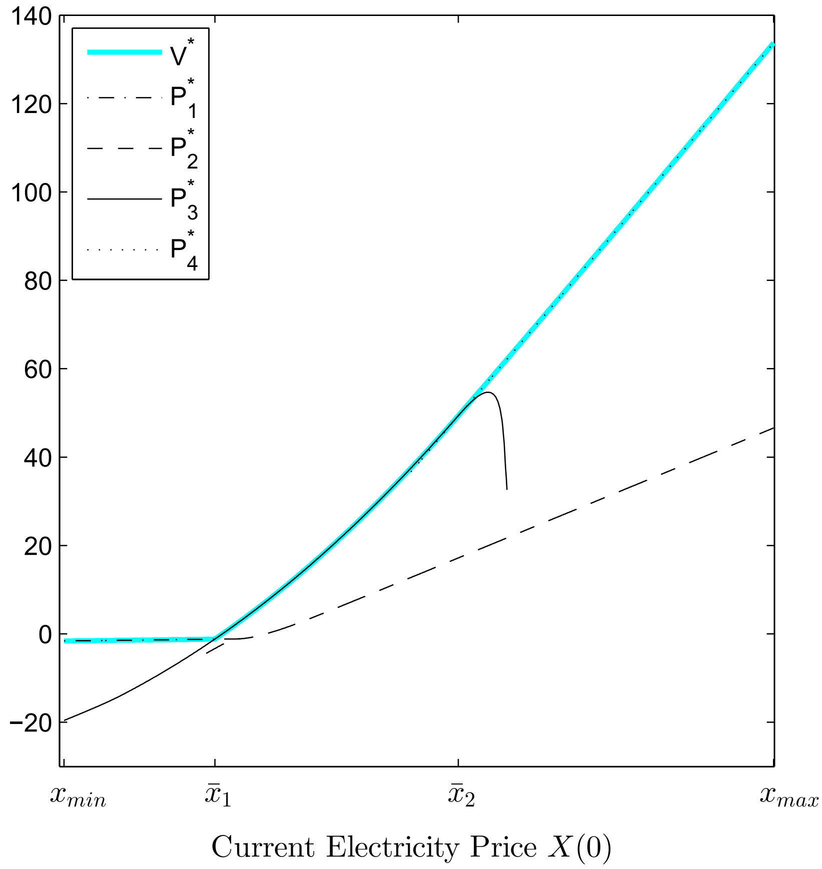
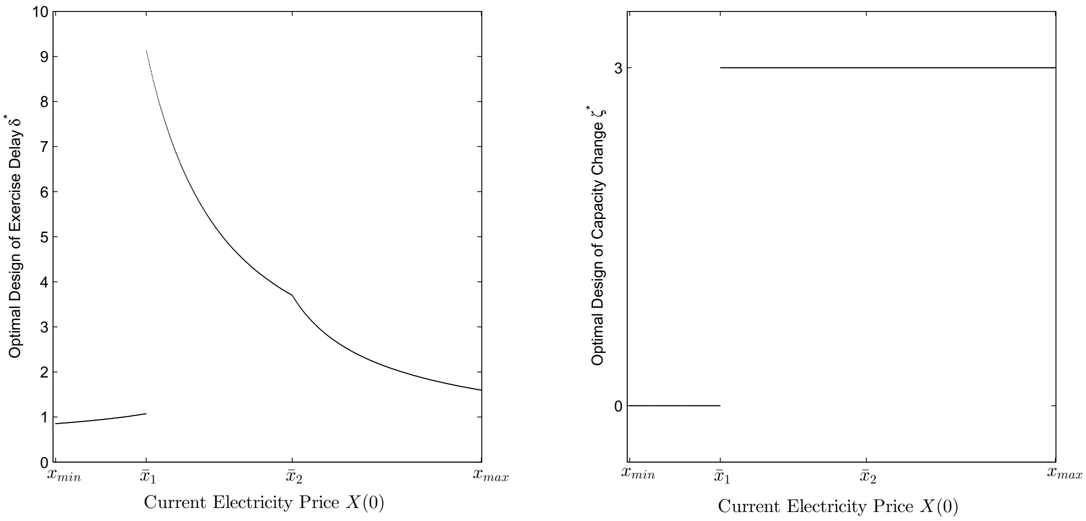


Figure 3.6. Value Functions of Sub-Problems

The lines P_1^*, P_2^*, P_3^* and P_4^* in Figure 3.6 represent the optimal objective function values of the sub-problems P_1, P_2, P_3 and P_4, respectively. The shaded line V^* represents the optimal function value of overall design problem. Notice that the sub-problems are not defined for the entire domain of $[x_{min}, x_{max}]$. For example, P_3 is not defined for high $X(0)$ close to x_{max}. It means that no configuration of the expandable power plant satisfies the constraints of the sub-problem P_3 for the high enough initial electricity price.

Figure 3.6 provides more information than the value of the optimally designed power plant. In the region of $[0, \bar{x}_1]$, P_1 yields the highest value. Considering the economic meaning of P_1, this result implies that the flexible option of the power plant is designed to be reduction of the capacity, and the system operator exercise the option immediately. If the current price is higher than $\bar{x}_1$ and lower than $\bar{x}_2$, the flexible power plant must be designed to expand its capacity by exercising the flexible option, and the system operator waits until the price rises to $x_1^*(\delta^*, \zeta^*)$. When the current price is higher than $\bar{x}_2$,



(a) Optimal Exercise Delay

(b) Optimal Capacity Change

Figure 3.7. Optimal Flexible Wind Farm Configurations

Figure 3.7 shows the optimal design of the flexible power plant. Figure 3.7(a) and Figure 3.7(b) illustrate the optimal exercise delay and the optimal capacity change with respect to the current electricity price, respectively. In the region of $[x_{min}, \bar{x}_1]$, the utility exits from the energy market in a relatively short time, and the exit starts immediately. When the current price belongs to the intermediate region $[\bar{x}_1, \bar{x}_2]$, the flexible power plant is designed with an expansion option that expands the capacity of the power plant three times in a relatively long expansion period. The system operator wait until the electricity price reaches to a threshold value $x_1^*(\delta^*, \zeta^*)$ that is higher than the current price. If the current price is higher than $\bar{x}_2$, the utility starts the capacity expansion immediately. The capacity is increased up to the maximum capacity and it takes shorter time than the time of the intermediate region and longer time than the exit case. For the readers who are interested in replicating this example, the author provides the critical values, $\bar{x}_1 \approx 0.4349$ and $\bar{x}_2 \approx 1.1154$.

3.5 Delayed Flexible System Summary

Systems engineering and design are called on to develop increasingly complex and costly systems. These systems must have appropriate levels of flexibility in order to maintain relevance and capitalize on opportunities. Section 3.3 modeled the control of a flexible system as a delayed optimal stopping problem assuming the available capability change is fixed. Section 3.4 extended the model of Section 3.3 by including the level of capability change into the design variables. The measure of flexibility considered at the design phase was the delay between the decision to exercise flexible alternatives and the implementation of such decisions. First solving for optimal control policies, the author constructed the parameter settings and thresholds to guide the system operator to exercise the option, continue, or never exercise. This research finds a non-trivial effect of exercise delay upon operational decision. Turning to the system design problem, when the cost (or revenue) resulting from exercising flexibility is once differentiable, the author provides concrete optimization problems for the optimal system design that is solvable with usual non-linear optimization methods.

Although this research employs a relatively simple model of flexible systems in order to preserve tractability, the author believes that these results can aid system designers in choosing how flexible to make systems. Highlighting the delay between decision time and implementation time will be especially important for systems in which there is considerable value of flexibility, where system effectiveness is fundamentally tied to uncertainties which can be capitalized on.

4. DUOPOLY MARKET SHARE COMPETITION WITH ASYMMETRIC EXERCISE DELAY

4.1 Introduction

The research of flexibility has expanded to option exercise games which consider the other decision makers' behaviors since the 1990s. As investigated in Chapter 3, exercise delay has significant impacts on the optimal operational policy and the value of flexible systems. What if the flexible system with delay is exposed to other decision maker's action? To answer this question, this chapter extends the model in Chapter 3 including the interactions between decision makers by assuming one decision maker operates a flexible system without exercise delay and the other manages another flexible system with exercise delay.

An option exercise game is an appropriate model to evaluate flexible options when the options interact with each other, since it is an integrated approach of game and real option theories. One of the applicable areas of this research is evaluating an exclusive patent with potential entry of alternative technology. When a firm acquires a patent, the firm may not implement the patent protected technology to produce a new product right away. The firm can wait until the new product market becomes profitable enough to compensate the implementation costs. However, if the firm confronts the threat from the other firm's R&D opportunity, the threat should be considered. When the growth of the market is stochastic, the option exercise games approach provides valuable insight to evaluate the patent [89]. This research interprets an exclusive patent as flexibility without exercise delay, and R&D opportunity as that with exercise delay. If a firm has an exclusive patent, the firm can introduce a new product in a short amount of time, by paying a relatively small implementation fee. On the other hand, the firm without patent should do R&D to introduce a comparable product, and it takes considerable time and resources. Each firm decides when it launches the new product or it starts the R&D project, reacting to uncertain market profitability and the other competitor's action. Therefore, the implementation of patent protected technology is execution of a flexible option without exercise delay and initiating a R&D project is that with exercising delay. This dissertation studies a patent and R&D competition game in the prospect of option exercise games.

This research postulates two players are competing market share in a duopoly market where the total market volume is stochastic. Both of the players are risk-neutral, i.e., they are only interested in the expected profit, rather than including the accompanying risks in their decisions. The players have complete information about the competition. It means every player knows the payoffs and possible actions of other players. About the information structure, this research considers both the open loop structure and the close loop structure. In the open loop model, players cannot observe the actions of other players after the beginning of the game. In the context that this chapter considers, the players cannot detect when the other player introduces the new product into the market and when the R&D project is initiated. On the other hand, the closed loop model assumes that players have perfect information about the past. So, as soon as one player starts to produce a new product or initiates the R&D project, the other player knows about it.

This chapter is organized as follows. Section 4.2 provides brief literature review about preemption game, option exercise games with exercise delay and asymmetric option exercise games. Section 4.3 provides the model of patent and R&D competition under deterministic R&D duration and cost assumption. This section contains the open loop and closed loop equilibrium of the option exercise game. Section 4.4 extends the deterministic period and cost assumption to stochastic situation. Conclusions are in section 4.5.

4.2 Literature Review

This research models the problem as a stochastic preemption game in continuous time. The continuous time preemption game between two identical firms under a deterministic environment studied in the early 1980s. Reinganum [141, 142] studied the equilibrium of the game. She assumed that the roles of players, the first mover and the second mover, is predetermined. Therefore, the resulting equilibrium is an open loop equilibrium. Moreover, she assumed that the market profitability is not matured so that no player invests at the outset of the game. Under the assumption, she found that either of the player exercises the option when the market is matured, but simultaneous investment is not a equilibrium. Extending Reinganum's work, Fudenberg and Tirole [6] constructed a mixed strategy space and derived mixed strategy perfect equilibria. Since the role of each player is not determined at the beginning of the game, the equilibrium is a closed loop equilibrium.

Following [141, 142], Smets [102] extended the deterministic setting to stochastic environment, and applied it to international investment context. He considered two identical firms as well and investigated pure strategy equilibrium. Huisman and Kort extended deterministic mixed strategy equilibrium framework to stochastic games in their series of studies [87, 143]. Spencer and Brander [144] studied duopoly with quantity competition and derived closed form solution considering a random demand which is determined by both of the participant's production. Demand function was modeled as a linear function that follows a random distribution on a closed interval. Williams [92] provided the rigorous derivation of a Nash equilibrium in a real options framework. He found the equilibrium in a strategic setting without delay, and the fact that increasing competition leads to earlier exercise of options. Baldursson [145] found Nash equilibrium and derived stochastic processes adapted optimal strategies considering the exogenous process influencing demand. This is an open-loop strategy, in the sense that there is no feedback from the investment of any firm to the investment of any other firm.

Grenadier [86] asserted that he derived the closed-loop solution of stochastic differential Cournot games. Based on dynamic programming approach, he derived a differential equation which solves the differential game with boundary conditions; continuity, smooth-pasting and super-contact conditions. He provided a closed-loop strategy with the assumption of symmetric structure. Novy-Marx [146] extended Grenadier's work. He considered the case that firms are heterogeneous and found the optimal investment decision under the assumptions of geometric Brownian motion and a constant elasticity demand function. Back and Paulsen [147] contradicted Grenadier's results. They proved the trigger strategies of [86] are not the best responses and derived the best response function under the assumption of geometric Brownian motion and linear inverse demand curve. Back and Paulsen pointed out the preemption opportunity is the incentive to deviate from the symmetric closed-loop strategy provided in Grenadier [86]. Thijssen et al. [148] focused on the value of preemption. When a player acquires the advantage of first mover, the player need to take additional risk. Comparing the benefit of preemption and additional risk, they provided the insight of optimal decision under a discontinuous stochastic environment. Thijssen et al. [149] studied an extended definition of strategy spaces under jump diffusion stochastic environments. Steg [150] derived explicit solutions under the assumptions that the exogenous uncertainty follows a Lévy process and the inverse demand curve has a positive constant elasticity. Chevalier-Roignant et al. [151] delivered a well-organized overview research about competitive investment including quantity competition in oligopoly market and provided the mixed strategy equilibrium framework that this research employs.

Asymmetric option exercising games, in which players have uneven profit structures or information, are an intensive research area. Pawlina and Kort [152] investigated an asymmetric investment costs case. They found that small cost differences cause coordination problem which makes open loop equilibrium and close loop equilibrium different. However when the cost difference is significant, the two equilibria are identical. Miltersen and Schwartz [153] considered asymmetry in the development and commercialization of a new product, in the context of patent protected R&D investments. Murto [154] studied the production scale difference.

Several studies devoted themselves to the effect of exercising delay on option exercise games. Grenadier [93] and Weeds [98] considered exercising delay as a component of duopoly option exercise games. Grenadier [93] studied deterministic exercising delay and Weeds [98] included stochastic exercising delay, which follows an exponential distribution. However, to the best of author's knowledge, asymmetric exercise delay has not been studied in the context of option exercise games.

4.3 Fixed R&D Duration and Cost

4.3.1 Model

Two risk neutral players compete in a duopoly market. Firm P procured an exclusive patent, and is waiting for the optimal time to implement the technology into the market. As soon as firm P decides to implement the the technology, it can increase its market share by paying an implementation fee I_P. On the other hand, firm D, the competitor of firm P, does not have the patent. So it takes time, δ , for firm D to invent a new technology that enables firm D to produce a comparable product and does not infringe on the patent. The cost of R&D project is denoted by I_D and assumed to be paid at the beginning of the project. The initial market share of firm D is π_0 and the incremental market share is K_D. Those of firm P are denoted by $1-\pi_0$ and K_P, respectively. Since the negative market share does not make sense, the market shares satisfy $0 \leq \pi_0 \leq 1$, $0 \leq K_D \leq 1-\pi_0$ and $0 \leq K_P \leq \pi_0$.

A geometric Brownian motion with constant coefficients describes the total market profit.

$$dX(t) = \mu X(t)dt + \sigma X(t)dB(t), \quad X(0) = x_0 \tag{4.1}$$

where $B(t)$ is the standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

Firm P launches the new product using patent protected technology at time τ_P, and firm D starts its R&D project at τ_D. The R&D project will be completed at $\tau_D+\delta$ and from this time firm D will earn higher profit. Each firm decides the optimal τ_P and τ_D. The performance functions are described as the followings

$$
\begin{aligned}
&\tilde{J}_P(\tau_P|\tau_D, x_0) = \int_0^{\tau_m} e^{-\rho t}(1-\pi_0)X(t)dt + \int_{\tau_m}^{\tau_P} e^{-\rho t}(1-\pi_0-K_D)X(t)dt - e^{-\rho\tau_P}I_P \\
&+ \int_{\tau_P}^{\tau_M} e^{-\rho t}(1-\pi_0+K_P)X(t)dt + \int_{\tau_M}^{\infty} e^{-\rho t}(1-\pi_0+K_P-K_D)X(t)dt \\
&\tilde{J}_D(\tau_D|\tau_P, x_0) = \int_0^{\tau_m} e^{-\rho t}\pi_0 X(t)dt + \int_{\tau_m}^{\tau_D+\delta} e^{-\rho t}(\pi_0-K_P)X(t)dt - e^{-\rho\tau_D}I_D \\
&+ \int_{\tau_D+\delta}^{\tau_M} e^{-\rho t}(\pi_0+K_D)X(t)dt + \int_{\tau_M}^{\infty} e^{-\rho t}(\pi_0+K_D-K_P)X(t)dt
\end{aligned}
\tag{4.2}
$$

where $\tau_m = \min(\tau_P, \tau_D+\delta)$ and $\tau_M = \max(\tau_P, \tau_D+\delta)$. The discount factor $\rho > 0$ satisfies $\mu - \rho < 0$ to ensure the existence of expected value.

4.3.2 Open Loop Equilibrium

In an open loop game, players decide their strategies at the beginning of the game and would not change them, because the players do not acquire any further information about other player's action after the beginning of the game. Because the firms are risk neutral, each player decides the optimal time to invest to maximize the expected value of performance function. The relatively simple structure of this game yields dominant strategy equilibrium. Each player has the dominant strategy which is not affected by the other player's strategy, and the equilibrium is unique. Theorem 4.3.1 summarizes the open loop equilibrium.

Theorem 4.3.1 *In the open loop game, it is optimal for firm D to start it's R&D project at* $\tau_D^* = \inf\{t \geq 0|X(t) \geq x_D^*\}$, *where* $x_D^* = -\frac{r_1(\mu-\rho)I_D e^{-(\mu-\rho)\delta}}{(r_1-1)K_D}$. *For firm P the optimal policy is* $\tau_P^* = \inf\{t \geq 0|X(t) \geq x_P^*\}$, *where* $x_P^* = -\frac{r_1(\mu-\rho)I_P}{(r_1-1)K_P}$. *The expected values of the systems are summarized in Table 4.1 for each case.*

Proof See Appendix A.4. ■

Table 4.1
The Value of Flexible Systems in the Open Loop Equilibrium

Optimal Trigger Points	Initial Market Profit	Player P	Player D
$x_P^* < x_D^*$	$x_0 \geq x_D^*$	$V_1^P(x_0, x_0)$	$V_1^D(x_0, x_0)$
	$x_P^* \leq x_0 < x_D^*$	$V_1^P(x_D^*, x_0)$	$V_2^D(x_D^*, x_0, x_0)$
	$x < x_P^*$	$V_2^P(x_P^*, x_D^*, x)$	$V_2^D(x_D^*, x_P^*, x_0)$
$x_P^* = x_D^*$	$x_0 \geq x_D^* = x_P^*$	$V_1^P(x_0, x_0)$	$V_1^D(x_0, x_0)$
	$x_0 < x_D^* = x_P^*$	$V_2^P(x_D^*, x_P^*, x_0)$	$V_2^D(x_P^*, x_D^*, x_0)$
$x_P^* > x_D^*$	$x_0 \geq x_P^*$	$V_1^P(x_0, x_0)$	$V_2^D(x_0, x_0)$
	$x_D^* \leq x_0 < x_P^*$	$V_2^P(x_P^*, x_D^*, x_0)$	$V_1^D(x_P^*, x_0)$
	$x_0 < x_D^*$	$V_2^P(x_P^*, x_D^*, x_0)$	$V_2^D(x_D^*, x_P^*, x_0)$

where

$V_1^P(x_D^*, x_0) = \frac{K_D e^{(\mu-\rho)\delta}(x_D^*)^{1-r_1}}{\mu-\rho} x_0^{r_1} - \frac{K_P}{\mu-\rho} x_0 - I_P - \frac{1-\pi_0}{\mu-\rho} x_0$

$V_2^P(x_P^*, x_D^*, x_0) = \left[\frac{I_P(x_P^*)^{-r_1}}{r_1-1} + \frac{K_D e^{(\mu-\rho)\delta}(x_D^*)^{1-r_1}}{\mu-\rho}\right] x_0^{r_1} - \frac{1-\pi_0}{\mu-\rho} x_0$

$V_1^D(x_P^*, x_0) = \frac{K_P(x_P^*)^{1-r_1}}{\mu-\rho} x_0^{r_1} - \frac{K_D e^{(\mu-\rho)\delta}}{\mu-\rho} x_0 - I_D - \frac{\pi_0}{\mu-\rho} x_0$

$V_2^D(x_D^*, x_P^*, x_0) = \left[\frac{I_D(x_D^*)^{-r_1}}{r_1-1} + \frac{K_P(x_P^*)^{1-r_1}}{\mu-\rho}\right] x_0^{r_1} - \frac{\pi_0}{\mu-\rho} x_0$

In the open loop equilibrium, the role of each player is determined by the profitability of technologies. When $x_P^* < x_D^*$, $\tau_P^* < \tau_D^*$. Therefore, Player P will be the first mover in this case. On the other hand, if $x_D^* < x_P^*$, the role of each player is reversed. The case that $x_P^* < x_D^*$ is equivalent to $\frac{K_D e^{(\mu-\rho)\delta}}{I_D} < \frac{K_P}{I_P}$, and $x_P^* > x_D^*$ implies $\frac{K_D e^{(\mu-\rho)\delta}}{I_D} > \frac{K_P}{I_P}$. Notice that K_P and K_D represent the benefit from implementing or developing new technologies, and I_D and I_P denote the costs. Therefore $\frac{K_D e^{(\mu-\rho)\delta}}{I_D}$ and $\frac{K_P}{I_P}$ stand for the profitability of each technology. The open loop equilibrium suggests that the player who has more profitable technology moves first. Since Player P already procured the patent, the implementation fee I_P of the patent protected technology is probably lower than the R&D cost of Player D. When the patent protected technology and the researched and developed technology are comparable, K_P and K_D tend to be close. In this case, Player P is inclined to be the first mover.

Notice that the dominant strategy holds for the oligopoly model as well. Suppose that there are N players in the oligopoly market. Let I_n, δ_n and K_n denote player n's exercise cost, exercise delay and incremental market share, respectively for $n = 1, 2, \cdots, N$. Then each player's dominant strategy is expressed with $\tau_n^* = \inf\{t \geq 0 | X(t) \geq x_n^*\}$ where $x_n^* = -\frac{r_1(\mu-\rho) I_n e^{-(\mu-\rho)\delta_n}}{(r_1-1)K_n}$.

4.3.3 Closed Loop Equilibrium

In a closed loop game, every player observes the realized value of the stochastic factor and competitors' behavior as time goes by. Based on the observed information, each player updates his or her strategy. This section investigates the mixed strategy closed loop equilibria of the patent and R&D competition.

As soon as one player exercises the flexible option, the remainder faces a one decision maker's decision problem. Because there is only one chance of investment and the investment is irreversible, the first mover cannot react to the second mover's action. Therefore, there is no reason the follower deviates from the optimal decision as a follower. It implies that the game ends when at least one player exercises his or her option.

Henceforth, this research assumes $x_D^* > x_P^*$ by considering the usual real world situation that is stated in the open loop equilibrium. Since the analysis structure for the case of $x_D^* < x_P^*$ is similar to the assumed case, this assumption does not deteriorate the value of this research. This research investigated the closed loop equilibrium where the initial state of the sub-game is lower than any player's open loop exercising trigger point, i.e., $x_0 < x_P^*$ where x_0 is the initial market volume of the sub-game starting at t_0. Moreover, the author does not include collaboration of the players in the analysis.

Strategy and Equilibrium

It is worthwhile to review the development of strategy space in a preemption game. Consider a duopoly market share competition game starting at time t_0 with initial total market volume $X_(t_0)$. Suppose that no player exercises its flexible option until time t which is later than t_0, i.e., $t \geq t_0$, and the total market volume at that time is X_t. Then the decision making structure from time t is exactly same to that from the time t_0. Therefore the decision from time $t \geq t_0$ can be considered as another game with initial state X_t. This game is called as a sub-game. A sub-game of an original game should satisfy independence conditions [155, pp. 274]. The imposed assumptions of this research guarantee to satisfy the assumptions.

In the deterministic continuous preemption game Fudenberg and Tirole [6] defined strategy space with two real value function $G_i^t(s)$ and $q_i(s)$, for $s \geq t$, where t is the starting time of a sub-game. The payoffs of players are deterministic and expressed explicitly with respect to time. The first element of the strategy, $G_i^t(s)$, stands for the cumulative distribution function of the probability that Player i has exercised the player's option before or at time $s \geq t$, given that the other player has not invested yet. On the other hand, $q_i(s)$ represents Player i's intensity of exercising option at time s.

The intensity function $q_i(s)$ is employed to compensate the loss of information from the naïve extension of a discrete-time mixed strategy into a continuous-time game. When both players have incentives to exercise option, at least one player will invest. In this case the coordination problem occurs. The coordination means that which player moves first or simultaneous move, when both players have incentive to be the leader. The coordination of the players becomes a measure zero event if the strategy is defined only with the cumulative distribution function. To address this coordination problem, an atoms function in the sense of optimal control theory is needed. An intuitive interpretation of the intensity function is a tie breaker. When both of the players have incentive to invest, at least one of the players exercises the option. In this case, the coordination is determined by the intensity function. In sum when $G_i^t(s) > 0$ for all i, who moves first or simultaneous investment is decided by the intensity of the players. This strategy space is expended to stochastic contexts [149, 151, 152], and this research follows Chevalier-Roignant and Trigeorgis's framework [151, chapter 12].

A simple strategy consists of the two real value functions, $G_i^t(s)$ and $q_i(s)$. A pair of simple strategies $(G_P^t(s), q_P(s))$ and $(G_D^t(s), q_D(s))$ is a Nash equilibrium of the sub-game starting at time t with neither player having exercised, if each player's strategy maximizes his payoff given the other player's strategy fixed. Moreover, a pair of closed-loop strategies $\{(G_P^t(s), q_P^t(s))\}_{t\geq t_0}$ and $\{(G_D^t(s), q_D^t(s))\}_{t\geq t_0}$ is a perfect equilibrium of a game beginning at time t_0, if the simple strategies are Nash equilibria $\forall t \geq t_0$.

For stochastic option exercise games, the simple strategies are $\mathcal{F}_t$ adapted rather than explicit functions of time. Thijssen et al. [149] focused on this characteristics, but they did not clarified the arguments of simple strategy. This research elucidates the arguments of the simple strategies at the equilibrium relying on the Markov property and existence of threshold types exercise trigger. Let $M^t(s)$ and $m^t(s)$ be the running maximum and minimum of a stochastic process $X(s)$ of a sub-game starting at t, i.e., $M^t(s) = \max_{t\leq u\leq s} X(u)$ and $m^t(s) = \min_{t\leq u\leq s} X(u)$. The cumulative distribution function $G_i^t(s)$ is explicitly expressed with the arguments as $G_i^t(M^t(s), m^t(s))$. The intensity function is defined as $q_i(s) = q_i(X(s))$.

Terminal Payoff

There are three possible scenarios terminating the patent and R&D competition. The first case is that both of the players exercise options, i.e., Player P starts to produce a new product and Player D launches the R&D project at the same time. When the total market volume at the time of exercising the option is given as $X(t) = x$, $P_M(x)$ and $D_M(x)$ represent the payoff of Player P and that of Player D, respectively.

Suppose that Player P preempts Player D by exercising his option when $X(t) = x$. Then Player D optimally chooses the time to start R&D project as a follower. In this case $P_L(x)$ and $D_F(x)$ denote the payoff of Player P as a leader and that of Player D as a follower, respectively. On the other hand, when Player D starts the R&D project first when $X(t) = x$, the payoff of Player P as a follower is denoted by $P_F(x)$ and $D_L(x)$ represents that of Player D as a leader. The terminal payoffs are calculated as followings and the detail calculation procedure is in Appendix B.1.

Player P's terminal payoffs

$$P_M(x) = \left[\frac{K_D e^{(\mu-\rho)\delta} - K_P}{\mu - \rho}\right] x - I_P - \frac{1-\pi_0}{\mu-\rho} x \tag{4.3}$$

$$P_L(x) = \begin{cases} P_M(x) & x \geq x_D^* \\ -\frac{I_D r_1}{r_1 - 1}\left(\frac{x}{x_D^*}\right)^{r_1} - \frac{K_P}{\mu-\rho} x - I_P - \frac{1-\pi_0}{\mu-\rho} x & x < x_D^* \end{cases} \tag{4.4}$$

$$P_F(x) = \begin{cases} P_M(x) & x \geq x_P^* \\ \frac{I_P}{r_1 - 1}\left(\frac{x}{x_P^*}\right)^{r_1} + \frac{K_D e^{(\mu-\rho)\delta}}{\mu-\rho} x - \frac{1-\pi_0}{\mu-\rho} x & x < x_P^* \end{cases} \tag{4.5}$$

Player D's terminal payoffs are

$$D_M(x) = \left[\frac{-K_D e^{(\mu-\rho)\delta} + K_P}{\mu-\rho}\right] x - I_D - \frac{\pi_0}{\mu-\rho} x \tag{4.6}$$

$$D_L(x) = \begin{cases} D_M(x) & x \geq x_P^* \\ -\frac{I_P r_1}{r_1 - 1}\left(\frac{x}{x_P^*}\right)^{r_1} - \frac{K_D e^{(\mu-\rho)\delta}}{\mu-\rho} x - I_D - \frac{\pi_0}{\mu-\rho} x & x < x_P^* \end{cases} \tag{4.7}$$

$$D_F(x) = \begin{cases} D_M(x) & x \geq x_D^* \\ \frac{I_D}{r_1 - 1}\left(\frac{x}{x_D^*}\right)^{r_1} + \frac{K_P}{\mu-\rho} x - \frac{\pi_0}{\mu-\rho} x & x < x_D^* \end{cases} \tag{4.8}$$

where

$$x_D^* = \frac{I_D e^{-(\mu-\rho)\delta}(\mu-\rho) r_1}{-K_D(r_1 - 1)} \tag{4.9}$$

$$x_P^* = \frac{I_P(\mu-\rho) r_1}{-K_P(r_1 - 1)} \tag{4.10}$$

Notice that $P_M(x)$ and $D_M(x)$ are straight lines with positive slope. Moreover, $P_F(x)$ and $D_F(x)$ are positive, increasing and convex $\forall x > 0$, and $P_L(x), x \in (0, x_D^*)$, and $D_L(x), x \in (0, x_P^*)$ are concave. With the assumption $x_P^* < x_D^*$, firm P's payoff structure is unique, but there are two possible structures of firm D's payoff according to the parameter values. Figure 4.1 shows the structure of Player P's payoffs. The blue solid, the red dash-dot and the black dash lines represent $P_M(x), P_L(x)$ and $P_F(x)$, respectively. The terminal payoffs inform fundamental incentive of Player P's behavior. The intersection of $P_L(x)$ and $P_F(x)$ in $(0, x_P^*)$ is unique and denoted by $\underline{x}_P$. In the region of $[0, \underline{x}_P)$, Player P does not have incentive to be a leader, since being follower provides higher payoff than being leader. When the current market volume belongs to $(\underline{x}_P, x_P^*)$, Player P wants to be a leader, but there is a risk having undesirable payoff $P_M(x)$ if Player D starts R&D project at the same time. Figures 4.3.3 and 4.3.3 illustrate two cases of Player D's payoff. When Player D's payoff as a leader, $D_L(x)$, never exceeds that as a follower, $D_F(x)$, Figure 4.3.3 illustrates the payoffs. In this case, being a follower is preferable to moving first when $x \in [0, x_D^*)$ and indifferent when $x = [x_D^*, \infty)$ for Player D. It implies that Player D has no incentive to start an R&D project earlier than Player P's introduction of new product because of the assumption $x_P^* < x_D^*$. Because the quick response of Player P enables Player

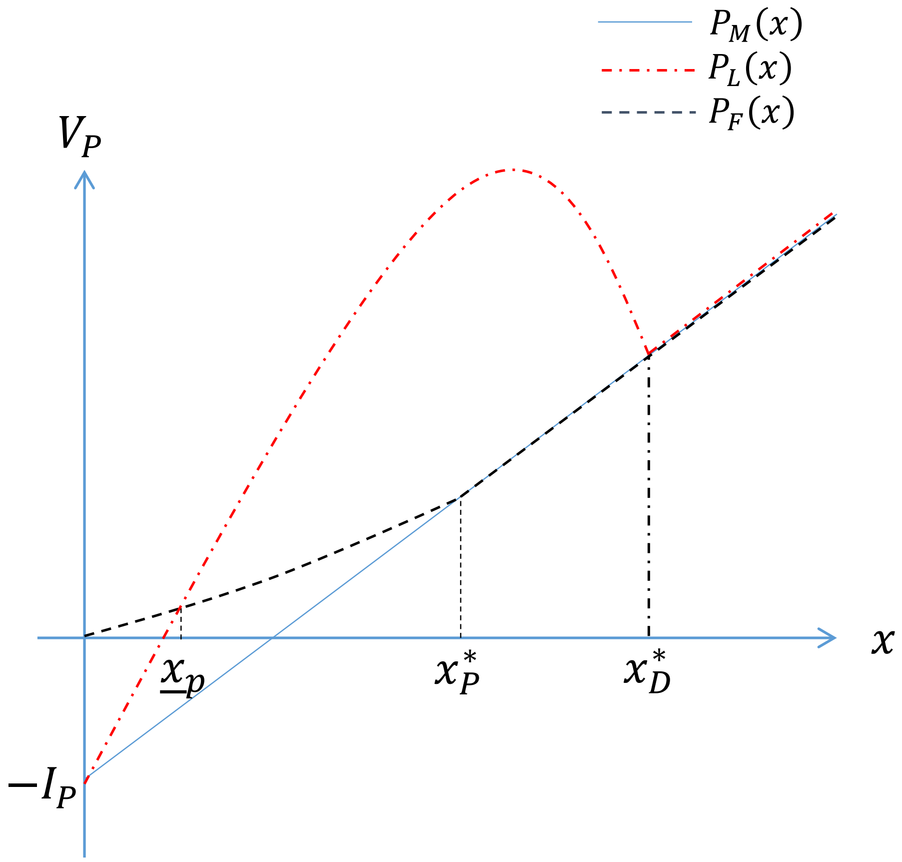


Figure 4.1. Player P's Payoff Structure

P not to be afraid of being preempted, this case is referred to a 'Large Asymmetry' case.

The other case, which is represented in Figure 4.3.3, is that $D_L(x)$ is higher than $D_F(x)$ for $x \in (\underline{x}_D, \bar{x}_D)$. Notice that $\bar{x}_D$, the bigger intersection of $D_L(x)$ and $D_F(x)$, is smaller than x_P^*. This means when the total market volume belongs to an open interval $(\underline{x}_D, \bar{x}_D)$, Player D has incentive to preempt Player P, even though there is a risk of simultaneous movement. This case is referred to 'Small Asymmetry' case

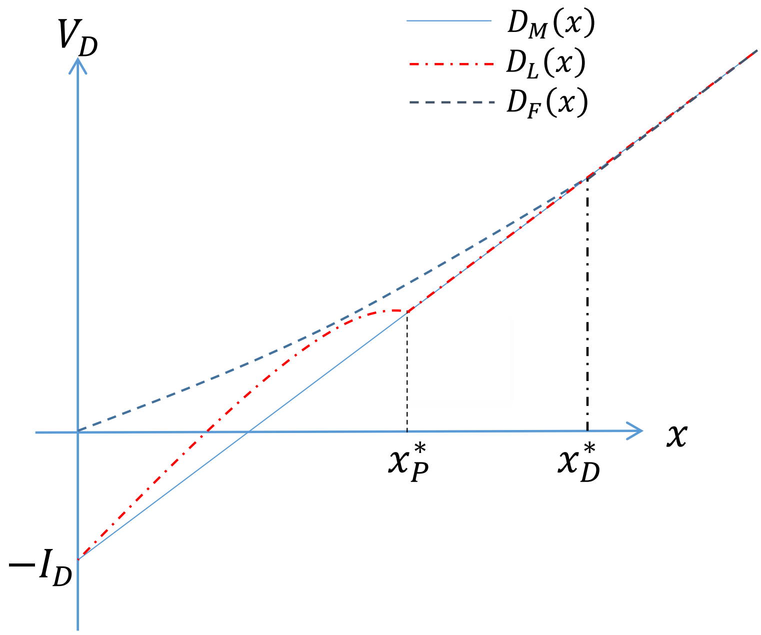


Figure 4.2. Player D's Payoff with Large Asymmetry

Closed Loop Equilibrium with Large Asymmetry

When the patent provides large advantage to Player P, Player D does not have an incentive to be the leader. With a large advantage of procuring a patent, the closed loop equilibrium has a relatively simple strategy profile.

Theorem 4.3.2 *If the set of parameters satisfies (4.11), $D_L(x) \leq D_F(x)$ for all $x \geq 0$.*

$$I_D \left\{ \frac{I_D}{(x_D^*)^{r_1}} + \frac{I_P r_1}{(x_P^*)^{r_1}} \right\}^{\frac{1}{r_1-1}} > \left[\frac{r_1 - 1}{r_1} \cdot \frac{K_P + K_D e^{(\mu-\rho)\delta}}{\rho - \mu} \right]^{\frac{r_1}{r_1-1}} \tag{4.11}$$

In this case, the following strategies are a perfect equilibrium for a game starting at t_0 with $X(t_0) < x_p^*$, where $t_0 \leq t \leq s$.

$$G_P^t(s) = \begin{cases} 0 & M^t(s) < x_P^* \\ 1 & M^t(s) \geq x_P^* \end{cases}, q_P(s) = \begin{cases} 0 & X(s) < x_P^* \\ 1 & X(s) \geq x_P^* \end{cases} \tag{4.12}$$

$$G_D^t(s) = \begin{cases} 0 & M^t(s) < x_D^* \\ 1 & M^t(s) \geq x_D^* \end{cases} \quad q_D(s) = \begin{cases} 0 & X(s) < x_D^* \\ 1 & X(s) \geq x_D^* \end{cases} \tag{4.13}$$

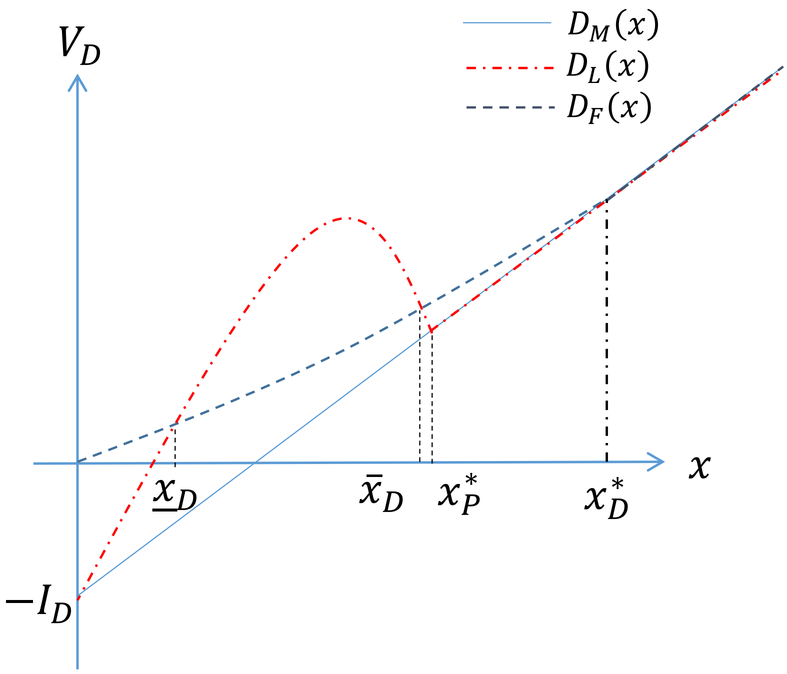


Figure 4.3. Player D's Payoff with Small Asymmetry

Proof See Appendix A.5 ■

All games starting with $X(t_0) = x_0 < x_P^*$ end when Player P moves first at the time $\tau_P^* = \inf\{s \geq t_0 | X(s) \geq x_P^*\}$. Player D exercises its option when $\tau_D^* = \inf\{s \geq t_0 | X(s) \geq x_D^*\}$. The author remarks that this equilibrium is identical to the open loop equilibrium in Theorem 4.3.1. This result implies that the coordination problem does not occur with large asymmetry, and coincides with the previous research [151, pp. 393-394] [152].

Closed Loop Equilibrium with Small Advantage

If the set of parameters does not satisfy the inequality (4.11), Player D has incentive to preempt Player P in the region $(\underline{x}_D, \bar{x}_D)$. Given $x_0 < x_P^*$, Player P cannot wait until the market volume grows up to x_P^* due to the preemption threat from firm D. This research seeks epsilon-equilibrium as the closed loop equilibrium. Intuitively speaking, in the context of a preemption game, a player can exercise his or her option right before the time that the other player intends to. For more formal discussion, refer to [156]. Here, this research assumes that $\underline{x}_P < \underline{x}_D$, and Section 4.4.3 discuss the case that $\underline{x}_P > \underline{x}_D$

Theorem 4.3.3 *Suppose that Player P has small advantage by procuring the patent, i.e., the inequality (4.11) does not hold, and $\underline{x}_P < \underline{x}_D$. Then, the following simple strategies consist of perfect equilibrium for a game starting at t_0 with $X(t_0) = x_0 < x_P^*$ and arbitrary small $\varepsilon > 0$, where $t_0 \le t \le s$.*

$$G_P^t(s) = \begin{cases} 0 & M^t(s) < \underline{x}_D - \varepsilon \\ 0 & \bar{x}_D < m^t(s) + \varepsilon \quad \textit{and} \quad M^t(s) < x_P^* \\ 1 & \textit{Otherwise} \end{cases} \tag{4.14}$$

$$q_P(s) = \begin{cases} 0 & X(s) < \underline{x}_D \\ \frac{D_L(X(s)) - D_F(X(s))}{D_L(X(s)) - D_M(X(s))} & \underline{x}_D \le X(s) \le \bar{x}_D \\ 0 & \bar{x}_D < X(s) < x_P^* \\ 1 & x_P^* \le X(s) \end{cases} \tag{4.15}$$

$$G_D^t(s) = \begin{cases} 0 & M^t(s) < \underline{x}_D \\ 0 & \bar{x}_D < m^t(s) \quad \textit{and} \quad M^t(s) < x_D^* \\ 1 & \textit{Otherwise} \end{cases} \tag{4.16}$$

$$q_D(s) = \begin{cases} 0 & X(s) < \underline{x}_D \\ \frac{P_L(X(s)) - P_F(X(s))}{P_L(X(s)) - P_M(X(s))} & \underline{x}_D \le X(s) \le \bar{x}_D \\ 0 & \bar{x}_D < X(s) < x_D^* \\ 1 & x_D^* \le X(s) \end{cases} \tag{4.17}$$

Proof See Appendix A.6 ■

The profile of perfect equilibrium strategies implies the players' behaviors according to the initial state of the game x_0 as follows.

1. $x_0 < \underline{x}_D$

   Player P preempts Player D by implementing the patent protecting technology at the moment right before $X(s)$ hits $\underline{x}_D$. In other words, Player P moves first when $\tau_P^* = \inf\{s \geq t_0 | X(s) \geq \underline{x}_D - \varepsilon\}$. Therefore, the simultaneous investment, which is called 'coordination failure,' would not happen.

2. $\underline{x}_D \leq x_0 \leq \bar{x}_D$

   In this case, both players have incentive to exercise their option at the beginning of the game. Therefore, the game ends immediately, since at least one player exercises his or her option. The coordination is determined by the intensity of the players, $q_P(x_0)$ and $q_D(x_0)$. Let $\mathbb{P}_{(\text{Leader}=P)}(x_0) = \mathbb{P}_{(\text{Follower}=D)}(x_0)$ denote the probability of Player P is the leader and Player D is the follower. Similarly, $\mathbb{P}_{(\text{Leader}=D)} = \mathbb{P}_{(\text{Follower}=P)}(x_0)$ represents the probability that the roles of players are reversed. The probability of simultaneous execution is connoted by $\mathbb{P}_{(\text{Simultaneous Investment})}(x_0)$. At the equilibrium, the probabilities are calculated as

$$\mathbb{P}_{(\text{Leader}=P)}(x_0) = \frac{q_P(x_0)(1 - q_D(x_0))}{q_P(x_0) + q_D(x_0) - q_P(x_0)q_D(x_0)} \tag{4.18}$$

$$\mathbb{P}_{(\text{Leader}=D)}(x_0) = \frac{q_D(x_0)(1 - q_P(x_0))}{q_P(x_0) + q_D(x_0) - q_P(x_0)q_D(x_0)} \tag{4.19}$$

$$\mathbb{P}_{(\text{Simultaneous Investment})}(x_0) = \frac{q_D(x_0)q_P(x_0)}{q_P(x_0) + q_D(x_0) - q_P(x_0)q_D(x_0)} \tag{4.20}$$

3. $\bar{x}_D < x_0 < x_P^*$

   Player P moves first at the time either the market volume grows up to x_P^* or shrinks to $\bar{x}_D - \varepsilon$. Therefore Player P is the leader, and simultaneous investment does not happen.

Comparing to the open loop equilibrium stated in Theorem 4.3.1, the closed loop information structure hastens the players' investment time when the asymmetry of exercise delay is small. Moreover, the expected payoff of the player who retains a competitive advantage from a short exercise delay with closed loop information is less than or equal to that with open loop information.

4.4 Stochastic R&D Duration and Cost

In this section, the assumptions about fixed R&D project period and cost in Section 4.3 are relaxed to stochastic variables. In the real world, required time and cost of a R&D project is not known in advance. To consider this aspect, the delay of implementing decision is assumed to follow an exponential distribution with mean $1/\gamma$, i.e. $\delta(\gamma) \sim \exp(\gamma)$ and $\gamma > 0$. This assumption is accepted by previous research such as [98]. Since the structure of this game is almost identical to the structure of the game in Section 4.3, this section focuses on the difference from the previous section.

4.4.1 Model

The unit cost per time of the R&D project is fixed as $c > 0$. Although the cost per unit time is fixed, the total cost of the R&D project is random due to the random duration of the project. The other settings are identical to section 4.3. Player P is endowed with the initial market share $(1 - \pi_0)$, and possesses the patent protected technology that enables introducing a new product without exercise delay. By investing I_P at the time τ_P, Player P increases his or her market share by $0 \leq K_P \leq \pi_0$. Player D starts the R&D project at time τ_D and the project is completed at $\tau_D + \delta(\gamma)$. Initially, Player D's market share is π_0, and it can be increased by K_D from $\tau_D + \delta(\gamma)$. The increase of a player's market share causes the other player's loss of market share. The other conditions for parameter values hold as well. Reflecting the stochastic duration of R&D project, the performance functions for players are expressed as the following.

$$\begin{aligned} J_P(\tau_P|\tau_D, x_0) \quad &= \int_0^{\tau_m} e^{-\rho t}(1-\pi_0)X(t)dt + \int_{\tau_m}^{\tau_P} e^{-\rho t}(1-\pi_0-K_D)X(t)dt - e^{-\rho\tau_P}I_P \\ &+ \int_{\tau_P}^{\tau_M} e^{-\rho t}(1-\pi_0+K_P)X(t)dt + \int_{\tau_M}^{\infty} e^{-\rho t}(1-\pi_0+K_P-K_D)X(t)dt \\ J_D(\tau_D|\tau_P, x_0) \quad &= \int_0^{\tau_m} e^{-\rho t}\pi_0 X(t)dt + \int_{\tau_m}^{\tau_D+\delta(\gamma)} e^{-\rho t}(\pi_0-K_P)X(t)dt \\ &+ \int_{\tau_D+\delta(\gamma)}^{\tau_M} e^{-\rho t}(\pi_0+K_D)X(t)dt - \int_{\tau_D}^{\tau_D+\gamma} e^{-\rho t}cdt \\ &+ \int_{\tau_M}^{\infty} e^{-\rho t}(\pi_0+K_D-K_P)X(t)dt \end{aligned} \tag{4.21}$$

4.4.2 Open Loop Equilibrium

This game also yields the following dominant strategy equilibrium.

Theorem 4.4.1 *In the open loop setting, it is optimal for Player D to start its R&D project at $\tau_D^* = \inf\{t \geq 0|X(t) \geq x_D^*\}$. For Player P, it is optimal to implement the new technology at $\tau_P^* = \inf\{t \geq 0|X(t) \geq x_P^*\}$. The optimal trigger points are*

$$x_P^* = -\frac{r_1 I_P(\mu-\rho)}{K_P(r_1-1)} \tag{4.22}$$

$$x_D^* = \frac{cr_1(\mu-\rho)(\mu-\rho-\gamma)}{\gamma K_D(r_1-1)(\rho+\gamma)} \tag{4.23}$$

Proof See Appendix A.7. ■

Similar to the fixed R&D duration and cost case, the open loop equilibrium implies that the profit-abilities of patent and R&D determine the coordination of players. If $\frac{I_P}{K_P} < -\frac{c(\mu-\rho-\gamma)}{\gamma K_D(\rho+\gamma)}$, Player P moves first, because $x_P^* < x_D^*$. On the other hand, if $\frac{I_P}{K_P} > -\frac{c(\mu-\rho-\gamma)}{\gamma K_D(\rho+\gamma)}$, Player D becomes the leader.

4.4.3 Closed Loop Equilibrium

To investigate the closed loop equilibrium, this research assumes $x_P^* < x_D^*$ same as the previous section.

Terminal Payoff

As shown in Section 4.3.3, the terminal payoffs are the cornerstones of deriving the closed loop equilibrium. The author summarizes the terminal payoffs in (4.24) - (4.29) and provides the detail procedure of calculation in Appendix B.2. If both players exercise the options at the same time when $X(t) = x$, Player P's payoff is $P_M(x)$, and that of Player D is $D_M(x)$. When Player P preempts Player D by implementing its patented technology at $X(t) = x$, Player P earns $P_L(x)$, and Player D receives $D_F(x)$. Suppose that Player D starts the R&D project when $X(t) = x$ before Player P introduces a new product using the patent. Then Player D and Player P gross $D_L(x)$ and $P_F(x)$, respectively.

$$P_M(x) = -\left[\frac{\gamma K_D}{(\mu-\rho-\gamma)(\mu-\rho)} + \frac{K_P}{\mu-\rho}\right]x - I_P - \frac{1-\pi_0}{\mu-\rho}x \tag{4.24}$$

$$P_L(x) = \begin{cases} P_M(x) & x \geq x_D^* \\ -\frac{cr_1}{(r_1-1)(\rho+\gamma)}\left(\frac{x}{x_D^*}\right)^{r_1} - \frac{\gamma K_D}{(\mu-\rho)(\mu-\rho-\gamma)}x - \frac{1-\pi_0}{\mu-\rho}x & x < x_D^* \end{cases} \tag{4.25}$$

$$P_F(x) = \begin{cases} P_M(x) & x \geq x_P^* \\ \frac{I_P}{r_1-1}\left(\frac{x}{x_P^*}\right)^{r_1} - \frac{\gamma K_D}{(\mu-\rho)(\mu-\rho-\gamma)}x - \frac{1-\pi_0}{\mu-\rho}x & x < x_P^* \end{cases} \tag{4.26}$$

$$D_M(x) = \left[\frac{\gamma K_D}{(\mu-\rho-\gamma)(\mu-\rho)} + \frac{K_P}{\mu-\rho}\right]x - \frac{c}{\rho+\gamma} - \frac{\pi_0}{\mu-\rho}x \tag{4.27}$$

$$D_L(x) = \begin{cases} D_M(x) & x \geq x_P^* \\ -\frac{I_P r_1}{(r_1-1)}\left(\frac{x}{x_P^*}\right)^{r_1} + \frac{\gamma K_D}{(\mu-\rho)(\mu-\rho-\gamma)}x - \frac{c}{\rho+\gamma} - \frac{\pi_0}{\mu-\rho}x & x < x_P^* \end{cases} \tag{4.28}$$

$$D_F(x) = \begin{cases} D_M(x) & x \geq x_D^* \\ \frac{c}{(r_1-1)(\rho+\gamma)}\left(\frac{x}{x_D^*}\right)^{r_1} + \frac{K_P}{\mu-\rho}x - \frac{\pi_0}{\mu-\rho}x & x < x_D^* \end{cases} \tag{4.29}$$

Notice that the terminal payoffs have the identical structures to the deterministic R&D duration and cost case. The payoffs, $P_M(x)$ and $D_M(x)$, are straight lines with positive slopes, and $P_F(x)$ and $D_F(x)$ are increasing convex. Moreover, $P_L(x)$ for $x \in [0, x_D^*]$ and $D_L(x)$ for $x \in [0, x_P^*]$ are concave.

Closed Loop Equilibrium

Since the terminal payoff structures are identical to those with fixed R&D duration and costs, the equilibrium is the same to the case of fixed R&D duration and costs except for the value of focal points such as $\underline{x}_P, \underline{x}_D$ and $\bar{x}_D$. When the patent provides a small advantage, i.e., $D_L(x) \leq D_F(x), \forall x \geq 0$, the closed loop equilibrium is identical to the equilibrium stated in (4.12) and (4.13) with the threshold values in (4.22) and (4.23). If the patent provides a large advantage, i.e., $D_L(x) \geq D_F(x)$ for $\underline{x}_D \leq x \leq \bar{x}_D$, the equilibrium is derived with the same procedure in section 4.3.3. Since the equilibrium stated in Theorem 4.3.3 assumed $\underline{x}_P < \underline{x}_D$, the author states the closed equilibrium when $\underline{x}_P > \underline{x}_D$. The proof in Appendix A.6 covers this theorem as well.

Theorem 4.4.2 *Suppose that Player P has small advantage by procuring the patent, i.e.,* $D_L(x) \geq D_F(x)$ *for* $\underline{x}_D \leq x \leq \bar{x}_D$*, and* $\underline{x}_P > \underline{x}_D$*. Then, the following simple* *strategies consist of perfect equilibrium for a game starting at* t_0 *with* $X(t_0) = x_0 < x_P^*$ *and arbitrary small* $\varepsilon > 0$*, where* $t_0 \le t \le s$*.*

$$G_P^t(s) = \begin{cases} 0 & M^t(s) < \underline{x}_P \\ 0 & \bar{x}_D < m^t(s) + \varepsilon \quad and \quad M^t(s) < x_P^* \\ 1 & Otherwise \end{cases} \tag{4.30}$$

$$q_P(s) = \begin{cases} 0 & X(s) < \underline{x}_P \\ \frac{D_L(X(s)) - D_F(X(s))}{D_L(X(s)) - D_M(X(s))} & \underline{x}_P \le X(s) \le \bar{x}_D \\ 0 & \bar{x}_D < X(s) < x_P^* \\ 1 & x_P^* \le X(s) \end{cases} \tag{4.31}$$

$$G_D^t(s) = \begin{cases} 0 & M^t(s) < \underline{x}_P - \varepsilon \\ 0 & \bar{x}_P < m^t(s) \quad and \quad M^t(s) < x_D^* \\ 1 & Otherwise \end{cases} \tag{4.32}$$

$$q_D(s) = \begin{cases} 0 & X(s) < \underline{x}_P \\ \frac{P_L(X(s)) - P_F(X(s))}{P_L(X(s)) - P_M(X(s))} & \underline{x}_P \le X(s) \le \bar{x}_D \\ 0 & \bar{x}_D < X(s) < x_D^* \\ 1 & x_D^* \le X(s) \end{cases} \tag{4.33}$$

Proof See Appendix A.6 ■

The players' behaviors on the perfect equilibrium are summarized as the following.

1. $x_0 < \underline{x}_P$: Player D starts R&D project before Player P introduce a new product when the market volume grows to $X(s) \ge \underline{x}_P$.

2. $\underline{x}_D \le x_0 \le \bar{x}_D$: At least one player exercises the flexible option, and the game ends at the outset of the game. The probability of the coordination is the same as (4.18)-(4.20).

3. $\bar{x}_D < x_0 < x_P^*$: Player P preempts Player D.

4.5 Strategic Flexibility Summary

Under strategic environment, an operator of a flexible system must consider and react properly to both the stochastic circumstance and other decision makers behavior. This chapter extended the environments considered in Chapter 3 to strategic environments including the interactions between decision makers. Duopoly market share competition was employed in this chapter to clarify the context of option exercise games. This research interpreted an exclusive patent as flexibility without exercise delay, and R&D opportunity as that with exercise delay. Section 4.3 modeled the asymmetric option exercise game when the exercise delay is fixed, and Section 4.4 considered a stochastic exercise delay.

In the open loop model, the game yields unique dominant strategy equilibrium, and the profitability of each technology decides who exercises the option first. For the closed loop model, this research suggested a strategy space with respect to the running maximum, the running minimum and the current state of the stochastic factor. Based on the construction of strategy space, this dissertation successfully assessed the closed loop equilibria of the option exercise game. When the asymmetry between the players is large, the closed loop equilibrium is identical to the open loop equilibrium. However, if the short exercise delay does not provide a sufficient competitive advantage, both players exercise their option earlier than the case of large asymmetry. Moreover, in this case the expected payoffs of the players are lower than the large asymmetry case.

5. CONCLUSIONS

The primary objectives of this dissertation were developing a concrete framework for designing a flexible system by considering the exercise delay as a measure of flexibility and investigating the rational behaviors of decision makers who operate flexible systems under strategic environments. The general approach employed to develop the theoretical models for this dissertation included the optimal control theory, non-linear optimization, stochastic differential equation and game theory. The impact of exercise delay as a measure of flexibility was investigated with respect to operational level and design level under a stochastic environment. By deriving a comprehensive profile of optimal operational policies of a flexible system with exercise delay and identifying the interdependency of design and operational level decisions, a well-organized concrete framework to solve the design level problem was developed. Moreover, the impact of interactions between decision makers who manage a flexible system was studied under both the open loop and the closed loop information structures. The derived equilibria provided insights into a market share competition under stochastic duopoly market.

This chapter summarizes the research conducted in this dissertation, highlights the contributions to the current literature and proposes potential extensions for future research.

5.1 Summary

This study considered two distinct environments, stochastic and strategic environments, of a flexible system. The models for each environments focused on exercise delay as the essential component of flexibility.

5.1.1 Designing a Flexible System under Stochastic Environment

The first part of this research was presented in Chapter 3 and studied the optimal decisions on a flexible system with exercise delay within stochastic environments. The model postulates two level decisions, operational level and design level decisions.

The first model of this chapter assumed that the operational level problem is deciding the optimal time to exercise the designed option, and the design problem is choosing the optimal level of flexibility, i.e., the length of exercise delay. The operational level problem was modeled as a delayed optimal stopping time problem, and this research provided a comprehensive profile of the optimal operational policies. The profile provides a guideline for optimal operational policies according to the parameters representing the market conditions and characteristics of the alternative and designed features of the flexible system. Leveraging a general approach developed in optimal control theory, the first goal of this research, successful derivation of the operational level solution, was accomplished.

In addition, the profile elucidates the interdependence between the operational level decision and the design level decision separating the entire domain of the design problem into sub-regions. This finding contributes to the area of flexible systems engineering. By analyzing the characteristics of the design problem in each sub-region, this research effort found that the design problem is decomposable with well-behaved non-linear optimization problems. With an illustrative example, the usefulness of the developed framework was shown.

The second model of Chapter 3 expanded the previous model by including the extent of capability change in the design level decision variables. The possible choices of the capability change included expanding the capability, reducing the capability, terminating the operation of flexible system and staying the initial mode forever. The inclusive optimal operational policy developed with the previous model was utilized to assess the operational level solution. Following the similar process of the previous model, the interdependence between the two level problems was also confirmed for this model. With a careful examination of the possible singularity, this research showed that the design problem of the extended model is also decomposable, and another illustrative example is provided to clarify the usefulness of the suggested flexible system design framework.

5.1.2 Management of Flexible System in Duopoly Market

Strategic environments force a decision maker to cope with both exogenous uncertainty and endogenous interactions among decision makers. Investigating decision makers' rational behaviors in equilibrium was another main goal of this dissertation. As the strategic environment, a duopoly market share competition was postulated where the total market profit was regarded as the underlying uncertainty. The player retaining an exclusive patent was regarded as a player competing in the market with a flexible system that does not have exercise delay. The other competitor was interpreted as a player operating a flexible system with exercise delay.

Because two types of exercise delay and the two types of information structure were considered in this dissertation, four equilibria were derived. This research effort stared with a fixed exercise delay model and evolved into the model with a stochastic exercise delay. The open loop and closed loop information structures were considered for each model. The results showed that the open loop equilibria are unique dominant strategy equilibria in both of the models with respect to exercise delay. An interesting implication of the open loop equilibria was the profitability of flexible option decides the role of its owner in the duopoly market competition.

Although construction of an appropriate strategy space is a premise of deriving closed loop equilibrium, a satisfactory strategy space has not yet reported for a stochastic preemption games. Bearing in mind the strategy space for deterministic preemption games, the author suggested strategy space that is adapted to the set of cumulated information up to the present time of the game. With the suggested strategy space, the closed loop equilibria were described properly. This is a contribution of this research effort to the area of option exercise games.

This research found that the closed loop equilibrium has two distinctive forms. When the asymmetry of exercise delay is small, the closed loop equilibrium is identical to the open loop equilibrium. On the other hand, if the asymmetry provides a large enough advantage to the player who has a flexible option with no exercise delay, the rational behaviors of the players are complicated in the closed loop equilibrium. Comparing the closed loop equilibrium with the open loop equilibrium, this research discovered the following interesting insights.

1. The closed loop information structure hastens the execution of flexible options, and it results in lower payoffs to both of the players.

2. The role of each player is determined not only by the characteristics of the flexible options but also by the value of stochastic factor.

3. Even the player with a competitive disadvantage from the asymmetry has a positive chance to be the leader of the market.

5.2 Future Work

The author summarizes the possible extensions to this research effort as follows:

1. This research mainly focused on theoretical development of flexible systems management under stochastic and strategic environments. However, the developed models are applicable in many practical contexts with minor modifications. The illustrative examples in Chapter 3 and the context employed in Chapter 4 showed the application potentials of this research. Moreover, the constructive frameworks of this research are useful to find insights from the real world problems. For example, this dissertation reported closed form solutions for the operational level problem of flexible system management under stochastic envi-

ronments. If this result is applied to a real world problem, it can make sensitive analysis easily

2. The stochastic process $X(t)$ was assumed to follow a geometric Brownian motion and employed to represent the underlying uncertainty of the flexible systems. However, this assumption may not be appropriate to model all underlying uncertainties. Especially, the assumed stochastic process satisfies the strong Markov property. If the underlying uncertainty does not satisfy the property, the theorem, which this research heavily relied on for deriving the optimal operational policy of delayed flexible system, does not hold. Furthermore, when the underlying uncertainty is not Markov, empowerment to the system operator can be an interesting research topic. For example, it can be more efficient for system designer to set the range of exercise delay in which the system operator decides the specific exercise delay when he or she exercises the flexible option, instead of setting a fixed length of delay.

3. The flexible systems considered in this dissertation allowed only one change of to the system configuration over the life time of the flexible system. If a flexible system allows multiple changes, an appropriate numerical method may be required to solve the problem. For example, this extension would explain the automation flexibility. Once an automated system is established, the change within the embedded alternatives can occur frequently. Unfortunately, the results in this dissertation have limits to assess the frequently changing flexible system, especially computation-wise. Investigation of the practical methods for the frequently changing flexible system would be an interesting extension of this research.

4. The flexible options were implicitly assumed to have infinite life time, i.e., as long as the irreversible change of the system has not been made, it is possible to change the system whenever the system operator wants. However, in reality many of the opportunities disappear within a certain amount of time. When

the flexible option is perishable, the operational problem does not yield a useful closed form solution. Investigating a flexible system whose alternatives are available for limited time is an interesting research topic.

5. The closed loop equilibrium derived in this dissertation was unique with the assumption that players exercise their options at the outset of a game if the game starts with a high initial state. However, other closed loop equilibria are reported for both stochastic and deterministic preemption games when both of the players payoff structures and available strategies are identical. The studies showed that it is equilibrium that both of the players wait until the state variable increased to very high state. Yet, for asymmetric option exercise games, the existence of other closed loop equilibrium is still matter of study.

6. There could be the first movers advantage or the second movers advantage. In terms of flexible option, the first movers advantage implies that the gain from exercising the flexible option ahead of the competitor is greater than the gain from following the competitor. If there is the first movers advantage, no useful closed form solution of option exercise policy has been reported for asymmetric exercise delay case. Even an appropriate numerical method for this problem is still under research. Obstacle problem approach and front tracking method are the current candidates for the numerical method. Since the first movers advantage exists in many of the real world situations, this could be a valuable research topic.

7. This dissertation does not consider design problems under strategic environments. It is mainly for two reasons. The first reason is the possible existence of multiple equilibria. The other reason is difficulties of constructing design level games. When there are multiple equilibria, it may be challenging to assess which equilibrium will be attained. Moreover there must be interactions in the design phase. The players can acquire a flexible system without exercise delay through an auction, or through a negotiation with the system seller. The mechanism to

acquire the flexible system is also open to research. An integrated approach for both the operational level option exercise games and design level game would contribute to the research areas of option exercise games and flexible decision making.

These contributions will impact decision making with flexible systems.

APPENDICES

APPENDIX A
PROOFS OF THEOREMS

A.1 Proof of Theorem 3.3.1

Proof Let $f_i(t, X(t)) = e^{-\rho t}(a_i X(t) + b_i)$ for $i = 1, 2$ and

$$G(x) = \mathbb{E}^x \left[\int_0^\infty f_2(t, X(t))dt \right] = -\frac{a_2}{\mu - \rho}x + \frac{b_2}{\rho}$$

Then

$$\begin{aligned} v(x,\delta) &= \sup_\tau \mathbb{E}^x \left[\int_0^{\tau+\delta} f_1(t, X(t))dt + e^{-\rho(\tau+\delta)}\tilde{c}(\delta) + \int_{\tau+\delta}^\infty f_2(t, X(t))dt \right] \\ &= \sup_\tau \mathbb{E}^x \left[\int_0^{\tau+\delta} f_1(t, X(t)) - f_2(t, X(t))dt + e^{-\rho(\tau+\delta)}\tilde{c}(\delta) \right] + G(x) \end{aligned}$$

Let $f_1(t, X(t)) - f_2(t, X(t)) = e^{-\rho t}(aX(t) + b)$, where $a = a_1 - a_2, b = b_1 - b_2$,

$$\begin{aligned} &\mathbb{E}^x \left[\int_0^{\tau+\delta} f_1(t, X(t)) - f_2(t, X(t))dt + e^{-\rho(\tau+\delta)}\tilde{c}(\delta) \right] \\ &= \mathbb{E}^x \left[\int_0^{\tau+\delta} e^{-\rho t}(aX(t) + b)dt + e^{-\rho(\tau+\delta)}\tilde{c}(\delta) \right] \end{aligned}$$

By [129, Theorem 2.1], The delayed optimal stopping time problem can be transformed to a classic optimal stopping time problem without delay based on Markov property as follows:

$$\begin{aligned} &\sup_\tau \mathbb{E}^x \left[\int_0^{\tau+\delta} e^{-\rho t}(aX(t) + b)dt + e^{-\rho(\tau+\delta)}\tilde{c}(\delta) \right] \\ &= \sup_\tau \mathbb{E}^x \left[\int_0^{\tau} e^{-\rho t}(aX(t) + b)dt + \mathbb{E}^x \left[\int_0^{\delta} e^{-\rho t}(aX(t) + b)dt + e^{-\rho\delta}\tilde{c}(\delta) \middle| x = X(\tau) \right] \right] \\ &= \sup_\tau \mathbb{E}^x \left[\int_0^{\tau} e^{-\rho t}(aX(t) + b)dt + e^{\rho\tau}(F_1 X(\tau) + F_2) \right] \end{aligned}$$

where $F_1 = a(e^{(\mu-\rho)\delta} - 1)/(\mu - \rho)$ and $F_2 = (b/\rho)(1 - e^{-\rho\delta}) + e^{-\rho\delta}\tilde{c}(\delta)$. Therefore,

$$v(x,\delta) = \tilde{v}(x,\delta) + G(x) \tag{A.1}$$

where

$$\tilde{v}(x,\delta) = \sup_\tau \mathbb{E}^x \left[\int_0^\tau e^{-\rho t}(aX(t) + b)dt + e^{\rho\tau}(F_1 X(\tau) + F_2) \right] \tag{A.2}$$

The solution of equation (A.2) can be found using the approach of [128, Chapter 10]. Let $S = \{(x,\tau) \in (0,\infty) \times (0,\infty)\}$ be the solvency region. Define an operator for a function of s and x, $A := \frac{\partial}{\partial s} + \mu x \frac{\partial}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2}{\partial x^2}$, based on Ito's formula. Let $h_1(s,x) := e^{-\rho s}(ax+b)$ and $h_2(s,x) := e^{-\rho s}(F_1 x + F_2)$. Consider the set $U = \{(x,\tau) \in (0,\infty) \times (0,\infty) | A(h_2) + h_1 > 0\}$

$$\begin{aligned} A(h_2) + h_1 &= -\rho e^{-\rho s}(F_1 x + F_2) + \mu x e^{-\rho s} F_1 + e^{-\rho s}(ax+b) \\ &= e^{-\rho(s+\delta)} \left\{ a e^{\mu\delta} x - \rho \tilde{c}(\delta) + b \right\} \end{aligned}$$

Therefore $U = \{(x,\tau) \in (0,\infty) \times (0,\infty) | A(h_2) + h_1 > 0\}$ $= \left\{(x,\tau) \in (0,\infty) \times (0,\infty) \middle| ax > e^{-\mu\delta}(\rho\tilde{c}(\delta) - b)\right\}$. We can consider the following cases.

1. $a > 0$ and $\tilde{c}(\delta) \le b/\rho$

$$ax > e^{-\mu\delta}(\rho\tilde{c}(\delta) - b) \Leftrightarrow x > \frac{e^{-\mu\delta}}{a}(\rho\tilde{c}(\delta) - b)$$

This means $U = S$, because above inequality is satisfied $\forall x \in (0,\infty)$. Therefore, the flexible option should not be exercised [157, Proposition 2.3 and Proposition 2.4].

2. $a < 0$ and $\tilde{c}(\delta) \ge b/\rho$

$$ax > e^{-\mu\delta}(\rho\tilde{c}(\delta) - b) \Leftrightarrow x < \frac{e^{-\mu\delta}}{a}(\rho\tilde{c}(\delta) - b)$$

In this case $U = \emptyset$. Then exercising option immediately is optimal.

3. $a > 0$ and $\tilde{c}(\delta) > b/\rho$

$$ax > e^{-\mu\delta}(\rho\tilde{c}(\delta) - b) \Leftrightarrow x > \frac{e^{-\mu\delta}}{a}(\rho\tilde{c}(\delta) - b)$$

It means $U = \left\{(s,x) \middle| x > \frac{e^{-\mu\delta}}{a}(\rho\tilde{c}(\delta) - b) > 0\right\}$. The continuation region D has the form

$$D = \{(s,x) | x^* < x\}$$

for some x^* that satisfies $0 < x^* \le \frac{e^{-\mu\delta}}{a}(\rho\tilde{c}(\delta) - b)$.

4. $a < 0$ and $\tilde{c}(\delta) \leq b/\rho$

$$ax > e^{-\mu\delta}(\rho\tilde{c}(\delta) - b) \Leftrightarrow x < \frac{e^{-\mu\delta}}{a}(\rho\tilde{c}(\delta) - b)$$

In this case, $U = \left\{(s,x)\middle|0 < x < \frac{e^{-\mu\delta}}{a}(\rho\tilde{c}(\delta) - b)\right\}$. The continuation region D has the form

$$D = \{(s,x)|0 < x < x^*\}$$

for some x^* that satisfies $0 < \frac{e^{-\mu\delta}}{a}(\rho\tilde{c}(\delta) - b) \leq x^*$

Let $v_\infty(x)$ denote the value of system when never exercising the flexible option is optimal given $X(0) = x$ and δ.

$$\begin{aligned} v_\infty(x) &= \mathbb{E}^x\left\{\int_0^\infty aX(t) + bdt + G(x)\right\} \\ &= \int_0^\infty a_1\mathbb{E}^x[X(t)] + b_1 dt \\ &= -\tfrac{a_1}{\mu-\rho}x + \tfrac{b_1}{\rho} \end{aligned} \tag{A.3}$$

Let $v_0(x,\delta)$ represent the value of the flexible system when immediate exercise of the flexible option is optimal given $X(0) = x$ and δ.

$$\begin{aligned} v_0(x,\delta) &= \mathbb{E}^x\left\{\int_0^\delta aX(t) + bdt + e^{-\rho\delta}\tilde{c}(\delta) + G(x)\right\} \\ &= \int_0^\delta a\mathbb{E}^x[X(t)] + bdt + e^{-\rho\delta}\tilde{c}(\delta) + G(x) \\ &= \tfrac{ae^{(\mu-\rho)\delta} - a_1}{\mu-\rho}x + \tfrac{b_1 - be^{-\rho\delta}}{\rho} + e^{-\rho\delta}\tilde{c}(\delta) \end{aligned} \tag{A.4}$$

In the case that $a < 0$ and $\tilde{c}(\delta) \leq b/\rho$, the solution of (A.2) satisfies [128, Theorem 10.4.1], and $\phi(s,x) = e^{-\rho s}\psi(x)$ is a well-known candidate for the solution of (A.2). By the condition (vii) of [128, Theorem 10.4.1],

$$A\phi + h_1 = -\rho e^{-\rho s}\psi(x) + \mu x e^{-\rho s}\psi'(x) + \frac{1}{2}\sigma^2 x^2 e^{-\rho s}\psi''(x) + e^{-\rho s}(ax+b) = 0, \quad \forall (s,x) \in D$$

Equivalently,

$$-\rho\psi(x) + \mu x\psi'(x) + \frac{1}{2}\sigma^2 x^2\psi''(x) + ax + b = 0, \quad \forall (s,x) \in D \tag{A.5}$$

Let $\psi_0(x)$ be the solution of homogeneous ordinary differential equation

$$-\rho\psi_0(x) + \mu x\psi_0'(x) + \frac{1}{2}\sigma^2 x^2\psi_0''(x) = 0$$

The general solution of above differential equation, Cauchy-Euler equation, is $\psi_0(x) = \Lambda_1 x^{r_1} + \Lambda_2 x^{r_2}$, where r_1 and r_2 are the solutions of the auxiliary equation, $u(r) = -\rho + \mu r + \frac{1}{2}\sigma^2 r(r-1) = 0$. Notice that the auxiliary equation has two solutions, r_1 and r_2, which satisfy $r_2 < 0 < 1 < r_1$, because $u(0) = -\rho < 0$, $u(1) = \mu - \rho < 0$ and $\lim_{r\to\infty} u(r) > 0$. A function $\psi_1(x) = \lambda_1 x + \lambda_2$ is a candidate of the solution of non-homogeneous equation, $-\rho\psi_1(x) + \mu x\psi_1'(x) + \frac{1}{2}\sigma^2 x^2 \psi_1''(x) + ax + b = 0$, with unknown constants λ_1 and λ_2.

$$\begin{aligned} -\rho\psi_1(x) + \mu x\psi_1'(x) + \tfrac{1}{2}\sigma^2 x^2 \psi_1''(x) + ax + b &= 0 \\ \{(\mu - \rho)\lambda_1 + a\}x - \rho\lambda_2 + b &= 0 \end{aligned}$$

Therefore, $\lambda_1 = -\frac{a}{\mu-\rho}$ and $\lambda_2 = \frac{b}{\rho}$. Because we are considering $a < 0$ and $\tilde{c}(\delta) \le b/\rho$, and the continuation region $D = \{(s,x)|0 < x < x^*\}$. Since $\psi(x)$ is bounded near $x = 0$, the solution of (A.5) has the following form.

$$\psi(x) = \begin{cases} \Lambda_1 x^{r_1} - \frac{a}{\mu-\rho}x + \frac{b}{\rho} & 0 < x < x^* \\ F_1 x + F_2 & x^* \le x \end{cases} \tag{A.6}$$

By the condition (i) of [128, Theorem 10.4.1], the continuity and smooth pasting conditions,

$$\Lambda_1 (x^*)^{r_1} - \frac{a}{\mu - \rho}x^* + \frac{b}{\rho} = F_1 x^* + F_2 \tag{A.7}$$

$$\Lambda_1 r_1 (x^*)^{r_1 - 1} - \frac{a}{\mu - \rho} = F_1 \tag{A.8}$$

By solving above equations,

$$x_1^*(\delta) = \frac{r_1(\mu-\rho)}{(a_1 - a_2)(r_1 - 1)}\left[\frac{b_1 - b_2}{\rho} - \tilde{c}(\delta)\right] e^{-\mu\rho}, \quad \Lambda_1 = \frac{ae^{(\mu-\rho)\delta}}{r_1(\mu-\rho)}(x_1^*)^{1-r_1} \tag{A.9}$$

Notice that s is a time shift parameter [128, Section 10.4]. Since the current value of the system is calculated by setting $s = 0$. Then $\phi(0,x) = \psi(x)$. When $a < 0$ and $\tilde{c}(\delta) \le b/\rho$, the solution of (A.1) is expressed as the follows by plugging (A.9) into (A.6).

$$v_{c,1}(x,\delta) = \frac{e^{-\rho\delta}}{r_1 - 1}\left[\frac{b_1 - b_2}{\rho} - \tilde{c}(\delta)\right]\left(\frac{x}{x_1^*(\delta)}\right)^{r_1} - \frac{a_1 x}{\mu - \rho} + \frac{b_1}{\rho}, \qquad x_1^* > x \tag{A.10}$$

$$v_0(x,\delta) = \frac{\{(a_1 - a_2)e^{(\mu-\rho)\delta} - a_1\}x}{\mu - \rho} - \frac{b_1 - b_2}{\rho}e^{-\rho\delta} + \frac{b_1}{\rho} + e^{-\rho\delta}\tilde{c}(\delta), \quad x_1^* \le x \tag{A.11}$$

In the case that $a > 0$ and $\tilde{c}(\delta) > b/\rho$, the continuation region has the form $D = \{(s,x)|x^* < x\}$ for some x^* which satisfies $x^* \leq \frac{e^{-\mu\delta}}{a}(\rho\tilde{c}(\delta) - b)$. With the similar procedure of above, the solution of (A.2) is

$$\psi(x) = \begin{cases} F_1 x + F_2 & 0 < x < x^* \\ \Lambda_2 x^{r_2} - \frac{a}{\mu-\rho}x + \frac{b}{\rho} & x^* \leq x \end{cases} \tag{A.12}$$

where

$$x_2^* = \left[\frac{r_2(\mu-\rho)}{a(r_2-1)}\right]\left[\tilde{c}(\delta) + \frac{b}{\rho}\right]e^{-\mu\delta}, \quad \Lambda_2 = \frac{ae^{(\mu-\rho)\delta}}{r_2(\mu-\rho)}(x^*)^{1-r_2} \tag{A.13}$$

Therefore, the solution of (A.1) is expressed as the follows.

$$v_0(x,\delta) = \frac{\{(a_1-a_2)e^{(\mu-\rho)\delta} - a_1\}x}{\mu-\rho} - \frac{b_1-b_2}{\rho}e^{-\rho\delta} + \frac{b_1}{\rho} + e^{-\rho\delta}\tilde{c}(\delta), \quad x \leq x_2^* \tag{A.14}$$

$$v_{c,2}(x,\delta) = \frac{e^{-\rho\delta}}{r_2-1}\left[\frac{b_1-b_2}{\rho} - \tilde{c}(\delta)\right]\left(\frac{x}{x_2^*(\delta)}\right)^{r_2} - \frac{a_1 x}{\mu-\rho} + \frac{b_1}{\rho}, \quad x > x_2^* \tag{A.15}$$

■

A.2 Proof of Theorem 3.3.2

Proof Suppose that $a_1 < a_2$ and $\tilde{c}(\delta) < (b_1 - b_2)/\rho$. If $X(0) \geq x_1^*(\delta)$, the operational value function is $v_0(X(0),\delta)$ according to Theorem 3.3.1, and the objective function of design problem is $v_0(X(0),\delta) - C(\delta)$ and continuous. Because $\tilde{c}(\delta)$ is once differentiable, $x_1^*(\delta)$ is continuous. Therefore, the level set $\{\delta \in [\delta_{min}, \delta_{max}]|X(0) \geq x_1^*(\delta)\}$ is compact, and P_2 has maximizer. If $X(0) < x_1^*(\delta)$, the object function of design problem is $v_{c,1}(X(0),\delta) - C(\delta)$. Because of value matching condition, $v_0(X(0),\delta) = v_{c,1}(X(0),\delta)$, where $X(0) = x_1^*(\delta)$. Therefore, $v_{c,1}(X(0),\delta) - C(\delta)$ is the objective function at $\delta = \bar{\delta}$ as well. Since the level set $\{\delta \in [\delta_{min}, \delta_{max}]|X(0) \leq x_1^*(\delta)\}$ is compact, P_1 has maximizer.

Let $\Delta = \{\delta \in [\delta_{min}, \delta_{max}]|x_i^*(\delta) = X(0)\}$. For $\bar{\delta} \in \Delta$, $v_{c,i}(X(0),\bar{\delta}) = v_0(X(0),\delta)$, because of the value matching condition. With the similar procedure, the design problem is decomposed into P_1 and P_2 in (3.13), in the case that $a_1 > a_2$ and $\tilde{c}(\delta) > (b_1 - b_2)/\rho$. ■

A.3 Proof of Theorem 3.4.2

Proof To proof the optimization problem (3.15) is decomposable into four maximization problems (3.23)-(3.26), it is necessary to show that all the feasible areas are compact, and all objective functions are continuous.

Notice that $x_i^*(\delta,\zeta)$ is continuous on $\mathbb{A}\setminus\{(\delta,\zeta)|\zeta=1\}$. Using the L'Hôpital's rule, by setting

$$x_i^*(\delta,1)=\frac{r_i(\mu-\rho)}{a(r_i-1)}\left[\frac{b}{\rho}+\left.\frac{\partial\tilde{c}(\delta,\zeta)}{\partial\zeta}\right|_{\zeta=1}\right]e^{-\mu\delta} \tag{A.16}$$

, the threshold values, $x_i^*(\delta,\zeta)$ is continuous on $\mathbb{A}$. Since the threshold value has meanings when it is positive, this setting is required only when $\frac{b}{\rho}+\left.\frac{\partial\tilde{c}(\delta,\zeta)}{\partial\zeta}\right|_{\zeta=1}<0$. With this setting, the feasible areas of all the sub-problems are compact, because a level set of a continuous function on a compact domain is also compact.

To investigate possible singularities, let $\Delta=\{(\delta,\zeta)\in\mathbb{A}|x_i^*(\delta,\zeta)=X(0)\}$, where $\mathbb{A}=[\delta_{min},\delta_{max}]\times[\zeta_{min},\zeta_{max}]$. If $\zeta\in[\zeta_{min},1)$ and $X(0)\le x_2^*(\delta,\zeta)$, the objective function of design problem is $v_0(X(0),\delta,\zeta)-C(\delta,\zeta)$. Notice that $\lim_{\zeta\uparrow 1}v_0(X(0),\delta,\zeta)=\frac{a}{\rho-\mu}X(0)+\frac{b}{\rho}$. Therefore the objective function of (3.23) is continuous on $\underline{\mathbb{A}}$. With the same procedure, the objective function of (3.26) is also continuous on its feasible area.

For (3.24), if $\zeta\in[\zeta_{min},1)$ and $X(0)>x_2^*(\delta,\zeta)$, the objective function of design problem is $v_{c,2}(X(0),\delta,\zeta)-C(\delta,\zeta)$ and continuous on this open set. Because of the value matching condition, $v_{c,2}(X(0),\delta,\zeta)$ is continuous at $(\delta,\zeta)\in\Delta$. If $\frac{b}{\rho}+\left.\frac{\partial\tilde{c}(\delta,\zeta)}{\partial\zeta}\right|_{\zeta=1}\ge 0$, $\underline{\mathbb{A}}$ does not contain $\zeta=1$. Therefore, $v_{c,2}(X(0),\delta,\zeta)$ is continuous on the feasible area. On the other hand, if $\frac{b}{\rho}+\left.\frac{\partial\tilde{c}(\delta,\zeta)}{\partial\zeta}\right|_{\zeta=1}<0$, with the fictitious threshold value defined at (A.16) $\lim_{\zeta\uparrow 1}v_{c,2}(X(0),\delta,\zeta)=\frac{a}{\rho-\mu}X(0)+\frac{b}{\rho}$. Therefore $v_{c,2}(X(0),\delta,\zeta)$ is continuous. The proof for (3.25) is almost identical to this proof.

■

A.4 Proof of Theorem 4.3.1

Proof Player P's decision problem at the outset of the market share competition is

$$\begin{aligned}
&\sup_{\tau_P} \mathbb{E}\left[\tilde{J}_P(\tau_P|\tau_D, x_0)\right] \\
&= \sup_{\tau_P} \mathbb{E}^{x_0}\left[\int_{\tau_P}^{\infty} e^{-\rho t}K_P X(t)dt - e^{-\rho\tau_P}I_P\right] \\
&+ \mathbb{E}^{x_0}\left[\int_0^{\infty} e^{-\rho t}(1-\pi_0-K_D)X(t)dt + \int_0^{\tau_D+\delta} e^{-\rho t}K_D X(t)dt\right]
\end{aligned}$$

Following calculations of expected discount factors are useful to prove this theorems. Let τ be the first hitting time of the stochastic process $X(t)$ given in (4.1) to a trigger point X^*, i.e., $\tau = \inf\{t \geq 0 | X(t) = X^*\}$.

$$\mathbb{E}^{x_0}\left[e^{-\rho\tau}\right] = \left(\frac{x_0}{X^*}\right)^{r_1} \qquad X^* > x_0 \tag{A.17}$$

$$\mathbb{E}^{x_0}\left[e^{-\rho\tau}\right] = \left(\frac{x_0}{X^*}\right)^{r_2} \qquad X^* < x_0 \tag{A.18}$$

$$\mathbb{E}^{x_0}\left[\int_0^{\tau} e^{-\rho t}X(t)dt\right] = \frac{(X^*)^{1-r_1}}{\mu-\rho}x_0^{r_1} - \frac{x_0}{\mu-\rho} \qquad X^* > x_0 \tag{A.19}$$

$$\mathbb{E}^{x_0}\left[\int_0^{\tau+\delta} e^{-\rho t}X(t)dt\right] = \frac{e^{(\mu-\rho)\delta}(X^*)^{1-r_1}}{\mu-\rho}x_0^{r_1} - \frac{x_0}{\mu-\rho} \qquad X^* > x_0 \tag{A.20}$$

The proofs of (A.17) and (A.19) are in [1, Page 315-316]. Because the proof of (A.18) and (A.20) are similar to the cited proof, the sketch of proof is enough. Let $f(x) := \mathbb{E}\left[e^{-\rho\tau}\right]$. Then $f(x)$ satisfies

$$-\rho f(x) + \mu x f'(x) + \frac{1}{2}\sigma^2 x^2 f''(x) = 0, \quad f(X^*) = 1, \quad \lim_{x\to\infty} f(x) = 0$$

Let $f(x) := \mathbb{E}\left[\int_0^{\tau+\delta} e^{-\rho t}X(t)dt\right]$, then

$$\begin{aligned}
&-\rho f(x) + \mu x f'(x) + \frac{1}{2}\sigma^2 x^2 f''(x) + f(x) = 0 \\
&f(0) = 0, \quad f(X^*) = \mathbb{E}\left[\int_0^{\delta} e^{-\rho t}X(t)dt\right] = \frac{e^{(\mu-\rho)\delta}-1}{\mu-\rho}X^*
\end{aligned}$$

By solving above ordinary differential equations, (A.18) and (A.20) are obtained.

Notice that $\mathbb{E}^{x_0}\left[\int_0^{\infty} e^{-\rho t}(1-\pi_0-K_D)X(t)dt\right] = -\frac{(1-\pi_0-K_D)x_0}{\mu-\rho}$. Using (A.20),

$$\mathbb{E}^{x_0}\left[\int_0^{\tau_D+\delta} e^{-\rho t}K_D X(t)dt\right] = K_D\left(\frac{e^{(\mu-\rho)\delta}{x_D}^{1-r_1}}{\mu-\rho}x_0^{r_1} - \frac{x_0}{\mu-\rho}\right)$$

where x_D is the trigger value of Player D to start R&D project. The decision problem,

$$\sup_{\tau_P} \mathbb{E}^{x_0}\left[\int_{\tau_P}^{\infty} e^{-\rho t} K_P X(t)dt - e^{-\rho\tau_P} I_P\right]$$

is a special case of Theorem 3.3.1 with $a_1 = 0, a_2 = K_P \geq 0, b_1 = b_2 = 0, \tilde{c}(\delta) = -I_P$ and $\delta = 0$. Therefore,

$$\sup_{\tau_P} \mathbb{E}^{x_0}\left[\int_{\tau_P}^{\infty} e^{-\rho t} K_P X(t)dt - e^{-\rho\tau_P} I_P\right] = \begin{cases} -I_P - \frac{K_P}{\mu-\rho}x_0 & x_0 \geq x_P^* \\ \frac{I_P}{r_1-1}\left(\frac{x_0}{x_P^*}\right)^{r_1} & x_0 < x_P^* \end{cases} \tag{A.21}$$

where $x_P^* = \frac{r_1(\mu-\rho)I_P}{-K_P(r_1-1)}$. By summing up all terms and rearranging them,

$$\sup_{\tau_P} \mathbb{E}\left[\tilde{J}_P(\tau_P|\tau_D, x_0)\right] = \begin{cases} \frac{K_D e^{(\mu-\rho)\delta} x_D^{1-r_1}}{\mu-\rho} x_0^{r_1} - \frac{K_P}{\mu-\rho}x_0 - I_P - \frac{1-\pi_0}{\mu-\rho}x_0 & x_0 \geq x_P^* \\ \left[\frac{I_P(x_P^*)^{-r_1}}{r_1-1} + \frac{K_D e^{(\mu-\rho)\delta} x_D^{1-r_1}}{\mu-\rho}\right] x_0^{r_1} - \frac{1-\pi_0}{\mu-\rho}x_0 & x_0 < x_P^* \end{cases} \tag{A.22}$$

Player D's decision problem at the beginning of the game is

$$\begin{aligned} &\sup_{\tau_D} \mathbb{E}\left[\tilde{J}_D(\tau_D|\tau_P, x_0)\right] \\ &= \sup_{\tau_D} \mathbb{E}^{x_0}\left[\int_{\tau_D+\delta}^{\infty} e^{-\rho t} K_D X(t)dt - e^{-\rho\tau_D} I_D\right] \\ &+ \mathbb{E}^{x_0}\left[\int_0^{\infty} e^{-\rho t}(\pi_0 - K_P)X(t)dt + \int_0^{\tau_P} e^{-\rho t} K_P X(t)dt\right] \end{aligned}$$

Notice that $\mathbb{E}^{x_0}\left[\int_0^\infty e^{-\rho t}(\pi_0 - K_P)X(t)dt\right] = -\frac{(\pi_0 - K_P)x_0}{\mu-\rho}$. Using (A.19),

$$\mathbb{E}^{x_0}\left[\int_0^{\tau_P} e^{-\rho t} K_P X(t)dt\right] = K_P\left(\frac{{x_P}^{1-r_1}}{\mu-\rho}x_0^{r_1} - \frac{x_0}{\mu-\rho}\right)$$

where x_P is the threshold value of Player P to implement the patent protected technology. Since

$$\sup_{\tau_D} \mathbb{E}^{x_0}\left[\int_{\tau_D+\delta}^{\infty} e^{-\rho t} K_D X(t)dt - e^{-\rho\tau_D} I_D\right]$$

is a special case of Theorem 3.3.1 with $a_1 = 0, a_2 = K_D \geq 0, b_1 = b_2 = 0$ and $\tilde{c}(\delta) = -I_D e^{\rho\delta}$,

$$\sup_{\tau_D} \mathbb{E}^{x_0}\left[\int_{\tau_D+\delta}^{\infty} e^{-\rho t} K_D X(t)dt - e^{-\rho\tau_D} I_D\right] = \begin{cases} -I_D - \frac{K_D e^{(\mu-\rho)\delta}}{\mu-\rho}x_0 & x_0 \geq x_D^* \\ \frac{I_D}{r_1-1}\left(\frac{x_0}{x_D^*}\right)^{r_1} & x_0 < x_D^* \end{cases} \tag{A.23}$$

where $x_D^* = \frac{I_D e^{-(\mu-\rho)\delta}(\mu-\rho)r_1}{-K_D(r_1-1)}$. By summing up all terms and rearranging them,

$$\sup_{\tau_D} \mathbb{E}\left[\tilde{J}_D(\tau_D|\tau_P, x_0)\right] = \begin{cases} \frac{K_P(x_P)^{1-r_1}}{\mu-\rho} x_0^{r_1} - \frac{K_D e^{(\mu-\rho)\delta}}{\mu-\rho} x_0 - I_D - \frac{\pi_0}{\mu-\rho} x_0 & x_0 \geq x_D^* \\ \left[\frac{I_D(x_D^*)^{-r_1}}{r_1-1} + \frac{K_P(x_P)^{1-r_1}}{\mu-\rho}\right] x_0^{r_1} - \frac{\pi_0}{\mu-\rho} x_0 & x_0 < x_D^* \end{cases} \tag{A.24}$$

Because the optimal threshold values, x_P^* and x_D^*, are independent from the other player's behavior, each player has dominant strategy. If $x_0 \geq x_D^*$, then $x_D = x_0$, otherwise $x_D = x_D^*$ in (A.22). Similarly, if $x_0 \geq x_P^*$, then $x_P = x_0$, otherwise $x_P = x_P^*$ in (A.24). ■

A.5 Proof of Theorem 4.3.2

Proof Let $f(x) = D_F(x) - D_L(x)$ for $x \in [0, x_P^*]$.

$$f(x) = \left[\frac{I_D}{(r_1-1)(x_D^*)^{r_1}} + \frac{I_P r_1}{(r_1-1)(x_P^*)^{r_1}}\right] x^{r_1} + \left[\frac{K_P + K_D e^{(\mu-\rho)\delta}}{\mu-\rho}\right] x + I_D$$

The first order condition, $f'(x) = 0$, is satisfied at

$$(x^*)^{r_1-1} = -\left[\frac{K_P + K_D e^{(\mu-\rho)\delta}}{\mu-\rho}\right] \cdot \frac{r_1-1}{r_1} \left[\frac{I_D}{(x_D^*)^{r_1}} + \frac{I_P r_1}{(x_P^*)^{r_1}}\right]$$

Because $f''(x) > 0$ at $x = x^*$, if $f(x^*) > 0$, then $D_F(x) > D_L(x)$. This condition is rewritten as (4.11). Figure A.5 represents the structure of terminal payoffs of the players when the patent provides a large advantage to Player P.

Let x_t denote the initial state of the sub-game starting at time t. Unless $x_t < x_P^*$, either of the players will exercise the option at the outset of the sub-game according to the open loop equilibrium strategy. Hence, the interest of this proof is the case that $x_t < x_P^*$.

Since $D_L(x) < D_F(x)$, $\forall x < x_P^* < x_D^*$, Player D has no incentive to exercise the option as the leader when $X(s) < x_D^*$. As proved in Theorem 4.3.1, it is optimal to exercise the delayed option at the time $X(t) = x_D^*$. Therefore, $G_D^t(s) = 0$ when $M^t(s) < x_D^*$, and $G_D^t(s) = 1$ when $M^t(s) \geq x_D^*$. Because the optimality does not depend on Player P's action, $q_D(s) = 0$ when $X(s) < x_D^*$, and $q_D(s) = 1$ when $X(s) \geq x_D^*$.

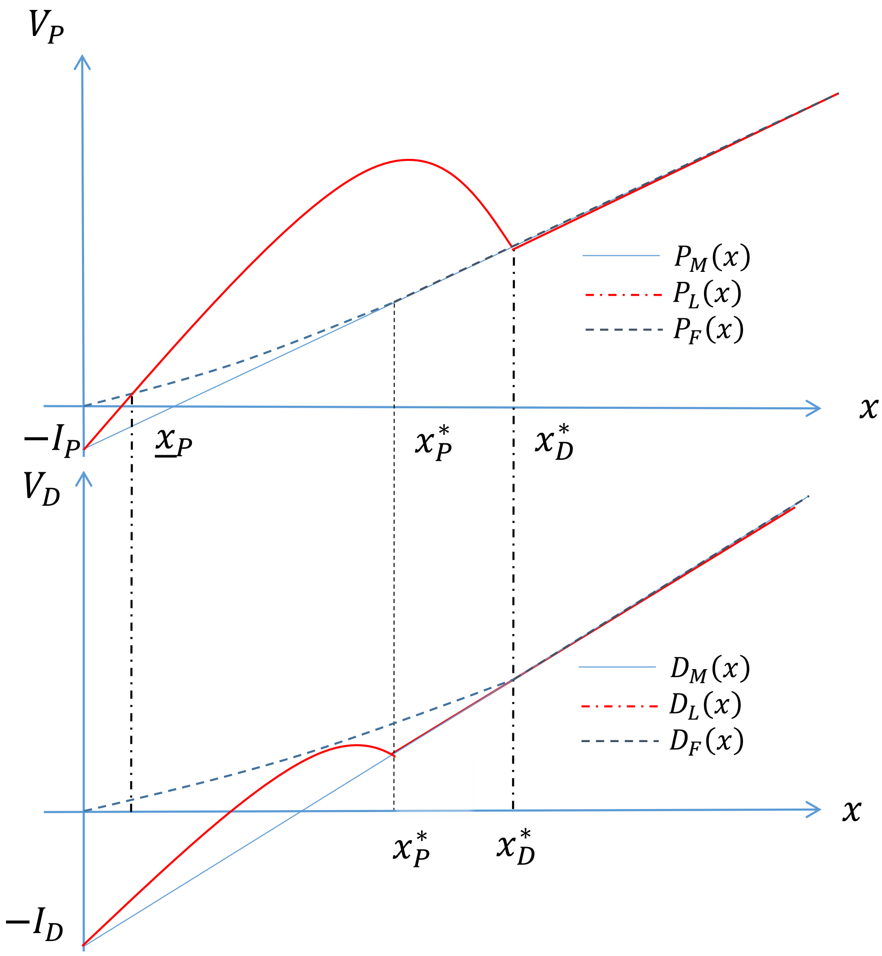


Figure A.1. Payoffs Comparison with Large Asymmetry

The Player D's strategy implies that Player P does not fear being preempted in the region of $[0, x_D^*)$. It is obvious that Player P never exercise his or her option for $X(s) < \underline{x}_P$, since $P_L(X(s)) < P_F(X(s))$. Define $P_L^{x_t}(x_s)$ be the player P's payoff as the leader of the sub-game that starts with initial state $x_t \in [\underline{\mathrm{x}}_P, x_P^*)$ and ends when $X(t) = x_s$. Suppose that $x_s > x_t$. Then,

$$
\begin{aligned}
P_L^{x_t}(x_s) &= \mathbb{E}^{x_t}\left[\begin{array}{c} \int_t^\infty e^{-\rho u}(1-\pi_0)X(u)du + \int_{\tau_P}^\infty e^{-\rho u}K_P X(u)du \\ -\int_{\tau_D^*+\delta}^\infty e^{-\rho u}K_D X(u)du \end{array}\right] \\
&= \left(\frac{x_t}{x_s}\right)^{r_1}\left[-\frac{I_D r_1}{r_1-1}\left(\frac{x_s}{x_D^*}\right)^{r_1} - \frac{K_P}{\mu-\rho}x_s - I_P\right] - \frac{1-\pi_0}{\mu-\rho}x_t
\end{aligned}
$$

where $\tau_P = \inf\{s \geq t | X(u) \geq x_s\}$ and $\tau_D^* = \inf\{s \geq t | X(u) \geq x_D^*\}$. Using the first and the second order conditions, $P_L^{x_t}(x_s)$ is maximized at

$$x_s^* = \frac{r_1 I_P(\mu - \rho)}{K_P(1 - r_1)} = x_P^* \tag{A.25}$$

If $\underline{x}_P < x_s < x_t$,

$$\begin{aligned} P_L^{x_t}(x_s) &= \left(\frac{x_t}{x_s}\right)^{r_2} \left[-\frac{I_D r_1}{r_1 - 1} \left(\frac{x_s}{x_D^*}\right)^{r_1} - \frac{K_P}{\mu - \rho} x_s - I_P \right] - \frac{1 - \pi_0}{\mu - \rho} x_t \\ &\leq P_L^{x_t}(x_t) \end{aligned}$$

Therefore, without preemption threat from Player D, it is optimal for Player P to exercise the option when $X(s)$ hits x_P^*. This results are summarized as Theorem 4.3.2 with respect to the simple strategy,. ■

A.6 Proof of Theorem 4.3.3

Proof Figure A.6 illustrates the terminal payoff structure when Player P retains a small advantage from procuring the patent. Let x_t denote the initial state of the sub-game starting at time t, which satisfies $x_t < x_P^*$. Suppose that $x_t \in [0, \underline{x}_D)$ and $M^t(s) \leq \underline{x}_D$ for $s \geq t$. It is obvious that $G_D^t(s) = 0$ and $q_D(s) = 0$, because being the follower yields higher payoff than being the leader for Player D. With the same reason, if $x_t \in (\bar{x}_D, x_D^*), m^t(s) > \bar{x}_D$ and $M^t(s) \leq x_P^*$, $G_D^t(s) = 0$ and $q_D(s) = 0$. For Player P, if $x_t \in [0, \underline{x}_P)$ and $M^t(s) < \underline{x}_P$, $G_P^t(s) = 0$ and $q_P(s) = 0$.

For $\max(\underline{x}_P, \underline{x}_D) \leq x_t \leq \bar{x}_D$, it is optimal for both of the players to exercise the option immediately with positive intensities [151, Chapter 12]. Thus $G_P^t(s) = 1$ and $G_D^{x_t}(s) = 1$. Since both players have positive $G_i^t(s)$, they play the game with intensity functions. Given $q_P(x_t)$ and $q_D(x_t)$, the probability that Player P becomes the leader is given as

$$\mathbb{P}_{(\text{Leader}=P)}(x_t) = \frac{q_P(x_t)(1 - q_D(x_t))}{q_P(x_t) + q_D(x_t) - q_P(x_t)q_D(x_t)}$$

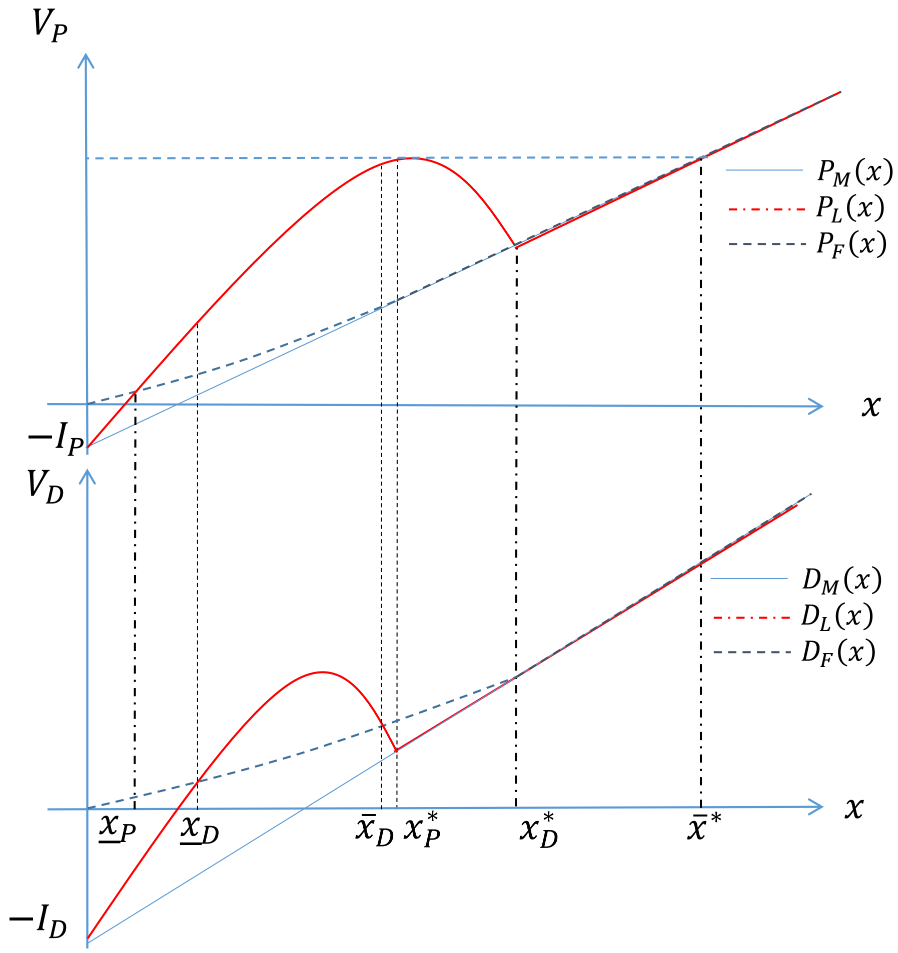


Figure A.2. Payoffs Comparison with Small Asymmetry

Similarly,

$$\mathbb{P}_{(\text{Leader}=D)}(x_t) = \frac{q_D(x_t)(1-q_P(x_t))}{q_P(x_t)+q_D(x_t)-q_P(x_t)q_D(x_t)}$$

$$\mathbb{P}_{(\text{Simultaneous Investment})}(x_t) = \frac{q_D(x_t)q_P(x_t)}{q_P(x_t)+q_D(x_t)-q_P(x_t)q_D(x_t)}$$

The value of each player is expressed as

$$V_P(x_t,q_P,q_D) = \frac{\{q_P(1-q_D)\}P_L(x_t)+\{q_D(1-q_P)\}P_F(x_t)+\{q_Dq_P\}P_M(x_t)}{q_P+q_D-q_Pq_D}$$

$$V_D(x_t,q_P,q_D) = \frac{\{q_D(1-q_P)\}D_L(x_t)+\{q_P(1-q_D)\}D_F(x_t)+\{q_Dq_P\}D_M(x_t)}{q_P+q_D-q_Pq_D}$$

By taking the first derivative,

$$\begin{aligned}
\frac{\partial}{\partial q_P}V_P(x_t,q_P,q_D) &> 0, \quad \text{if } q_D(x_t) < \frac{P_L(x_t)-P_F(x_t)}{P_L(x_t)-P_M(x_t)} \\
\frac{\partial}{\partial q_P}V_P(x_t,q_P,q_D) &< 0, \quad \text{if } q_D(x_t) > \frac{P_L(x_t)-P_F(x_t)}{P_L(x_t)-P_M(x_t)} \\
\frac{\partial}{\partial q_D}V_D(x_t,q_P,q_D) &> 0, \quad \text{if } q_P(x_t) < \frac{D_L(x_t)-D_F(x_t)}{D_L(x_t)-D_M(x_t)} \\
\frac{\partial}{\partial q_D}V_D(x_t,q_P,q_D) &< 0, \quad \text{if } q_P(x_t) > \frac{D_L(x_t)-D_F(x_t)}{D_L(x_t)-D_M(x_t)}
\end{aligned}$$

Figure A.3 shows the best response functions q_P^* and q_D^*. Therefore the equilibrium

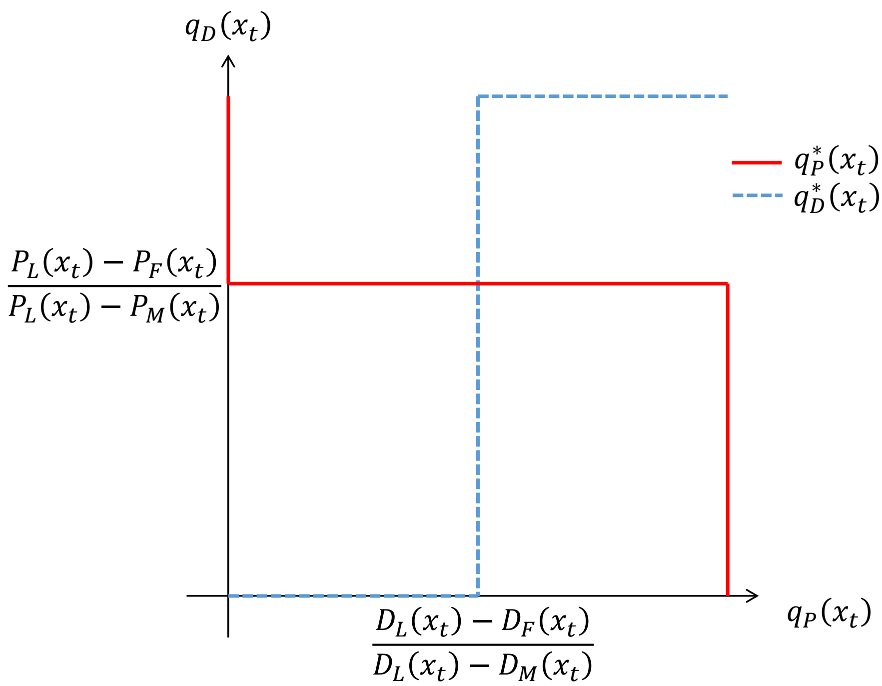


Figure A.3. Best Response Intensity Functions

is obtained at

$$\begin{aligned}
q_P(x_t) &= \frac{D_L(x_t)-D_F(x_t)}{D_L(x_t)-D_M(x_t)} \\
q_D(x_t) &= \frac{P_L(x_t)-D_F(x_t)}{P_L(x_t)-P_M(x_t)}
\end{aligned}$$

Consider the case that $\underline{x}_P < x_t < \underline{x}_D$. Player P does not fear to be preempted by Player D as long as $M(s) \leq \underline{x}_D$. As shown in A.5, it is optimal for Player P to wait until $X(s)$ hits x_P^* given no preemption threat from Player D. However, Player D has incentive to exercise its option when $X(s) \geq \underline{x}_D$ because $q_D(X(s)) > 0$. Therefore, Player P is able to postpone the implementation until $\underline{x}_D - \varepsilon$ for arbitrary small $\varepsilon > 0$.

Suppose that $\underline{x}_D < x_t < \underline{x}_P$. As long as $M(s) \leq \underline{x}_P$, Player D is not afraid of being preempted by Player P. Let $D_L^{x_t}(x_s)$ represent Player D's payoff as the leader of the sub-game that starts with initial state x_t and is terminated when $X(t) = x_s \geq x_t$.

$$\begin{aligned} D_L^{x_t}(x_s) &= \int_0^\infty e^{-\rho t}\pi_0 X(t)dt - \int_{\tau_P}^\infty e^{-\rho t}K_P X(t)dt + \int_{\tau_D+\delta}^\infty e^{-\rho t}K_D X(t)dt \\ &= \begin{cases} \left(\frac{x_t}{x_s}\right)^{r_1}\left[-\frac{I_P r_1}{r_1-1}\left(\frac{x_s}{x_P^*}\right)^{r_1} - \frac{K_D e^{(\mu-\rho)\delta}}{\mu-\rho}x_s - I_D\right] - \frac{\pi_0}{\mu-\rho}x_t & x_s \geq x_t \\ \left(\frac{x_t}{x_s}\right)^{r_2}\left[-\frac{I_P r_1}{r_1-1}\left(\frac{x_s}{x_P^*}\right)^{r_1} - \frac{K_D e^{(\mu-\rho)\delta}}{\mu-\rho}x_s - I_D\right] - \frac{\pi_0}{\mu-\rho}x_t & x_s \leq x_t \end{cases} \end{aligned}$$

Remark that if $\underline{x}_D \leq x_s \leq x_t$, $D_L^{x_t}(x_t) \geq D_L^{x_t}(x_s)$, and $d/dx_s D_L^{x_t}(x_s) > 0, \forall x_s \in (x_t, x_D^*)$. Therefore, it is optimal for Player D to exercise the delayed option at $\underline{x}_P - \varepsilon$, due to Player P's preemption threat in $X(s) \geq \underline{x}_P$.

When $\bar{x}_D < x_0 < x_P^*$, Player P is not afraid of being preempted, because Player D has no incentive to be the leader in this region. Therefore Player P can maximize his or her payoff by waiting until $X(t)$ hits the optimal threshold value x_P^*. However, if Player P does not exercise the option until $X(t)$ becomes less or equal to $\bar{x}_D$, Player D has incentive to move first, and it is not desirable to Player P. Hence, Player P exercises his or her option right before $X(t) \leq \bar{x}_D$.

■

A.7 Proof of Theorem 4.4.1

Proof Based on the performance function (4.21), Player P's decision problem is

$$\begin{aligned}
&\sup_{\tau_P} \mathbb{E}\left[J_P(\tau_P|\tau_D, x_0)\right] \\
&= \sup_{\tau_P} \mathbb{E}\left[\int_{\tau_P}^{\infty} e^{-\rho t} K_P X(t)dt - e^{-\rho\tau_P} I_P\right] \\
&+ \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}(1-\pi_0-K_D)X(t)dt + \int_0^{\tau_D+\delta(\gamma)} e^{-\rho t} K_D X(t)dt\right]
\end{aligned}$$

As shown in A.21,

$$\sup_{\tau_P} \mathbb{E}\left[\int_{\tau_P}^{\infty} e^{-\rho t} K_P X(t)dt - e^{-\rho\tau_P} I_P\right] = \begin{cases} -I_P - \frac{K_P}{\mu-\rho}x_0 & x_0 \geq x_P^* \\ \frac{I_P}{r_1-1}\left(\frac{x_0}{x_P^*}\right)^{r_1} & x_0 < x_P^* \end{cases}$$

where $x_P^* = -\frac{r_1(\mu-\rho)I_P}{(r_1-1)K_P}$.

Player D's decision problem is

$$\begin{aligned}
&\sup_{\tau_D} \mathbb{E}\left[J_D(\tau_D|\tau_P, x_0)\right] \\
&= \sup_{\tau_D} \mathbb{E}\left[\int_{\tau_D+\delta(\gamma)}^{\infty} e^{-\rho t} K_P X(t)dt - \int_{\tau_D}^{\tau_D+\delta(\gamma)} e^{-\rho t} c dt\right] \\
&+ \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}(1-\pi_0-K_P)X(t)dt + \int_0^{\tau_P} e^{-\rho t} K_P X(t)dt\right]
\end{aligned}$$

Focusing on the relevant part,

$$\begin{aligned}
&\sup_{\tau_D} \mathbb{E}\left[\int_{\tau_D+\delta(\gamma)}^{\infty} e^{-\rho t} K_P X(t)dt - \int_{\tau_D}^{\tau_D+\delta(\gamma)} e^{-\rho t} c dt\right] \\
&= \mathbb{E}\left[e^{-\rho\tau_D}\mathbb{E}\left\{\int_0^{\infty} e^{-\rho t} K_D X(t) - e^{-(\rho+\gamma)t} K_D X(t) - e^{-(\rho+\gamma)t} c dt \middle| X(\tau_D)\right\}\right] \\
&= \left(\frac{x_0}{X(\tau_D)}\right)^{r_1}\left[\frac{\gamma K_D}{(\mu-\rho)(\mu-\rho-\gamma)}X(\tau_D) - \frac{c}{\rho+\gamma}\right]
\end{aligned}$$

Using the first and second order conditions,

$$x_D^* = \frac{c r_1(\mu-\rho)(\mu-\rho-\gamma)}{\gamma K_D(r_1-1)(\rho+\gamma)} \tag{A.26}$$

■

APPENDIX B
SUPPLEMENTARY CALCULATIONS AND NUMERICAL RESULTS

B.1 Calculation of Terminal Payoffs with Fixed R&D Period and Cost

When both players exercise their options simultaneously,

$$\begin{aligned}
P_M(x) &= \mathbb{E}^x\left[\int_0^\infty e^{-\rho t}(1-\pi_0+K_P)X(t)dt - \int_\delta^\infty e^{-\rho t}K_D X(t)dt - I_P\right]\\
&= -\frac{1-\pi_0+K_P}{\mu-\rho}x + \frac{K_D}{\mu-\rho}xe^{(\mu-\rho)\delta} - I_P\\
&= \left[\frac{K_D e^{(\mu-\rho)\delta}-K_P}{\mu-\rho}\right]x - I_P - \frac{1-\pi_0}{\mu-\rho}\\
D_M(x) &= \mathbb{E}^x\left[\int_0^\infty e^{-\rho t}(\pi_0-K_P)X(t)dt + \int_\delta^\infty e^{-\rho t}K_D X(t)dt - I_D\right]\\
&= -\frac{\pi_0-K_P}{\mu-\rho}x - \frac{K_D}{\mu-\rho}xe^{(\mu-\rho)\delta} - I_D\\
&= \left[\frac{K_P - K_D e^{(\mu-\rho)\delta}}{\mu-\rho}\right]x - I_D - \frac{\pi_0}{\mu-\rho}x
\end{aligned}$$

The first derivatives of $P_M(x)$ and $D_M(x)$ are positive constants because of the assumptions $0 \le \pi_0 \le 1$, $0 \le K_D \le 1-\pi_0$ and $0 \le K_P \le \pi_0$. Therefore $P_M(x)$ and $D_M(x)$ are upward straight lines with respect to x.

Suppose that Player P introduces a new product, when $X(t) = x$, before Player D launches the R&D project. Then the Player D's decision problem becomes

$$\begin{aligned}
D_F(x) &= \sup_{\tau_D}\mathbb{E}^x\left[\int_0^\infty e^{-\rho t}(\pi_0-K_P)X(t) - e^{-\rho\tau_D}I_D + \int_{\tau_D+\delta}^\infty e^{-\rho t}K_D X(t)dt\right]\\
&= \sup_{\tau_D}\mathbb{E}^x\left[\int_{\tau_D+\delta}^\infty e^{-\rho t}K_D X(t)dt - e^{-\rho\tau_D}I_D\right] - \frac{\pi_0-K_P}{\mu-\rho}x\\
&= \begin{cases} -I_D - \frac{K_D e^{(\mu-\rho)\delta}}{\mu-\rho}x - \frac{\pi_0-K_P}{\mu-\rho}x & x \ge x_D^* \\ \frac{I_D}{r_1-1}\left(\frac{x}{x_D^*}\right)^{r_1} - \frac{\pi_0-K_P}{\mu-\rho}x & x < x_D^* \end{cases}
\end{aligned}$$

where $x_D^* = \frac{I_D e^{-(\mu-\rho)\delta}(\mu-\rho)r_1}{-K_D(r_1-1)}$. The last equality holds by (A.23), and $D_F(x) = D_M(x)$ when $x \geq x_D^*$. Consider the first and the second derivatives of $D_F(x)$.

$$\begin{aligned}\frac{d}{dx}D_F(x) &= \frac{I_D r_1}{r_1-1}(x_D^*)^{-r_1}x^{r_1-1} - \frac{\pi_0 - K_P}{\mu-\rho}\\ \frac{d^2}{dx^2}D_F(x) &= \frac{I_D r_1(r_1-1)}{r_1-1}(x_D^*)^{-r_1}x^{r_1-1}\end{aligned}$$

These derivatives are positive for $x \in [0, x_D^*)$, because $r_1 > 1$, $\mu - \rho < 0$,$I_D > 0$ and $0 \leq K_P \leq \pi_0$. Therefore $D_F(x)$ is an increasing and convex function with respect to x.

If Player P exercises his or her option when $X(t) = x \geq x_D^*$, Player D will starts R&D project immediately. Therefore, $P_L(x) = P_M(x)$ for $x \geq x_D^*$. Given $x < x_D^*$, $P_L(x)$ is

$$\begin{aligned}P_L(x) &= \mathbb{E}^x\left[\int_0^\infty e^{-\rho t}(1-\pi_0+K_P)X(t)dt - I_P - \int_{\tau_D+\delta}^\infty e^{-\rho t}K_D X(t)dt\right]\\ &= \mathbb{E}^x\left[\int_0^\infty e^{-\rho t}(1-\pi_0+K_P-K_D)X(t)dt - I_P + \int_0^{\tau_D+\delta} e^{-\rho t}K_D X(t)dt\right]\\ &= -\frac{1-\pi_0+K_P-K_D}{\mu-\rho} - I_P + \frac{K_D e^{(\mu-\rho)\delta}}{\mu-\rho}x_D^*\left(\frac{x}{x_D^*}\right)^{r_1} - \frac{K_D}{\mu-\rho}x\\ &= -\frac{I_D r_1}{r_1-1}\left(\frac{x}{x_D^*}\right)^{r_1} - \frac{K_P}{\mu-\rho}x - I_P - \frac{1-\pi_0}{\mu-\rho}x\end{aligned}$$

The second last equality is obtained by applying (A.20), and plugging x_D^* into the second last line yields the last line. Notice that $\frac{d^2}{dx^2}P_L(x) = -I_D(r_1)^2(x_D^*)^{-r_1}x^{r_1-2}$ for $0 \leq x < x_D^*$ because $x_P^* > 0$ and $I_D > 0$. Hence, $P_L(x)$ is concave for $x < x_D^*$.

The other terminal values are expressed as the followings.

$$\begin{aligned}P_F(x) &= \sup_{\tau_P}\mathbb{E}^x\begin{bmatrix}\int_0^\infty e^{-\rho t}(1-\pi_0)X(t)dt - \int_\delta^\infty e^{-\rho t}K_D X(t)dt\\ -I_P e^{-\rho\tau_P} + \int_{\tau_P}^\infty e^{-\rho t}K_P X(t)dt\end{bmatrix}\\ D_L(x) &= \mathbb{E}^x\left[\int_0^\infty e^{-\rho t}\pi_0 X(t)dt + \int_\delta^\infty e^{-\rho t}K_D X(t)dt - I_D - \int_{\tau_P}^\infty e^{-\rho t}K_P X(t)dt\right]\end{aligned}$$

With similar procedure, (4.4), (4.7) and their properties are obtained.

B.2 Calculation of Terminal Payoffs with Stochastic R&D Period and Cost

$$
\begin{aligned}
P_M(x) &= \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}(1-\pi_0+K_P-K_D)X(t)dt - I_P + \int_0^{\delta(\gamma)} e^{-\rho t}K_D X(t)dt\right] \\
&= -\frac{1-\pi_0+K_P}{\mu-\rho}x_0 - I_P + \frac{K_D}{\mu-\rho}x_0 + \mathbb{E}\left[\int_0^{\infty} e^{-(\rho+\gamma)t}K_D X(t)dt\right] \\
&= \left[-\frac{1-\pi_0+K_P}{\mu-\rho} - \frac{\gamma K_D}{(\mu-\rho-\gamma)(\mu-\rho)}\right]x_0 - I_P \\
\frac{dP_M(x)}{dx} &= -\frac{1-\pi_0+K_P}{\mu-\rho} - \frac{\gamma K_D}{(\mu-\rho-\gamma)(\mu-\rho)} \\
&= -\frac{-(\mu-\rho)(1-\pi_0+K_P)+\gamma(1-\pi_0+K_P-K_D)}{(\mu-\rho)(\mu-\rho-\gamma)} > 0
\end{aligned}
$$

Therefore $P_M(x)$ is a straight line with positive slope. For $P_L(x)$, if $\tau_D = 0$, $P_L(x) = P_M(x)$. Otherwise,

$$
\begin{aligned}
P_L(x) &= \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}(1-\pi_0+K_P)X(t)dt - I_P - \int_{\tau_D^*+\delta(\gamma)}^{\infty} e^{-\rho t}K_D X(t)dt\right] \\
&= -\frac{1-\pi_0+K_P-K_D}{\mu-\rho}x - I_P + \frac{K_D x_D^*}{\mu-\rho}\left(\frac{x}{x_D^*}\right)^{r_1} - \frac{K_D}{\mu-\rho}x \\
&+ e^{-\rho\tau_D^*}\mathbb{E}\left[\int_0^{\delta(\gamma)} e^{-\rho t}K_D X(t)dt \middle| x_D^*\right] \\
&= -\frac{cr_1}{(r_1-1)(\rho+\gamma)}\left(\frac{x}{x_D^*}\right)^{r_1} - \frac{1-\pi_0+K_P}{\mu-\rho}x - I_P
\end{aligned}
$$

The derivatives of $P_L(x)$ for $x \in [0, x_D^*]$ are

$$
\begin{aligned}
\frac{dP_L(x)}{dx} &= -\frac{cr_1}{(r_1)(\rho+\gamma)}\left(\frac{x_0}{x_D^*}\right)^{r_1} - \frac{1-\pi_0+K_P}{\mu-\rho}x_0 - I_P \\
\frac{d^2P_L(x)}{dx^2} &= -\frac{cr_1}{\rho+\gamma}r_1(x_D^*)^{r_1}x^{r_1-2} < 0
\end{aligned}
$$

Therefore, $P_L(x)$ is concave $\forall x \in [0, x_D^*]$.

$$
\begin{aligned}
P_F(x) = \sup_{\tau_P}\mathbb{E}&\left[\int_{\tau_P}^{\infty} K_P X(t)dt - e^{-\rho\tau_P}I_P\right] \\
&+ \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}(1-\pi_0)X(t)dt - \int_{\delta(\gamma)}^{\infty} e^{-\rho t}K_D X(t)dt\right]
\end{aligned}
$$

Using Theorem 3.3.1, with $a_1 = b_1 = b_2 = 0, a_2 = K_P$ and $\delta = 0$, the supremum is calculated as

$$\sup_{\tau_P} \mathbb{E}\left[\int_{\tau_P}^{\infty} K_P X(t)dt - e^{-\rho\tau_P} I_P\right] = \begin{cases} \frac{-K_P}{\mu-\rho}x_0 - I_P & x_P^* \leq x_0 \\ \frac{I_P}{r_1-1}\left(\frac{x_0}{x_P^*}\right)^{r_1} & x_0 < x_P^* \end{cases}$$

where $x_P^* = -\frac{r_1(\mu-\rho)I_P}{K_P(r_1-1)}$. By taking expectation and summing up the rest of the terms,

$$P_F(x) = \begin{cases} P_M(x) & x \geq x_P^* \\ \frac{I_P}{r_1-1}\left(\frac{x_0}{x_P^*}\right)^{r_1} - \frac{\gamma K_D}{(\mu-\rho)(\mu-\rho-\gamma)}x - \frac{1-\pi_0}{\mu-\rho}x & x < x_P^* \end{cases}$$

The derivatives of $P_F(x)$ for $x \in [0, x_P^*]$ are

$$\begin{aligned} \frac{dP_F(x)}{dx} &= \frac{r_1 I_P}{(r_1-1)x}\left(\frac{x}{x_P^*}\right)^{r_1} - \frac{\gamma K_D}{(\mu-\rho)(\mu-\rho-\gamma)} - \frac{1-\pi_0}{\mu-\rho} > 0 \\ \frac{d^2P_F(x)}{dx^2} &= \frac{r_1 I_P}{x^2}\left(\frac{x}{x_P^*}\right)^{r_1} > 0 \end{aligned}$$

Therefore, $P_F(x)$ is increasing and convex $\forall x \in [0, x_P^*]$. Player D's payoffs and their properties can be found with similar procedures from the following equations.

$$\begin{aligned} D_M(x) &= \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}(\pi_0 - K_P)X(t)dt + \int_{\delta(\gamma)}^{\infty} e^{-\rho t}K_D X(t)dt - \int_0^{\delta(\gamma)} e^{-\rho t}cdt\right] \\ D_L(x) &= \mathbb{E}\left[\begin{array}{c} \int_0^{\infty} e^{-\rho t}\pi_0 X(t)dt + \int_{\delta(\gamma)}^{\infty} e^{-\rho t}K_D X(t)dt \\ -\int_0^{\delta(\gamma)} e^{-\rho t}cdt - \int_{\tau_P^*}^{\infty} e^{-\rho t}K_P X(t) \end{array}\right] \\ D_F(x) &= \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}(\pi_0 - K_P)X(t)dt + \int_{\tau_D^*+\delta(\gamma)}^{\infty} e^{-\rho t}K_D X(t)dt - \int_{\tau_D^*}^{\tau_D^*+\delta(\gamma)} e^{-\rho t}cdt\right] \end{aligned}$$

B.3 Numerical Results

Followings are the critical number for replicating the illustrative example in Section 3.3.3.

Table B.1
Numerical Results of the Illustrative Example

x^*_{max}	1.125
x^*_{min}	0.983
$\bar{\delta}$	4.375
δ^*_1	2.906
δ^*_2	4.375
V^*	32.602
$\bar{x}$	1.137

LIST OF REFERENCES

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[1] R.K. Dixit and R.S. Pindyck. *Investment under uncertainty*. Princeton university press, 2008.

[2] Yash P Gupta and Toni M Somers. Business strategy, manufacturing flexibility, and organizational performance relationships: a path analysis approach. *Production and Operations Management*, 5(3):204–233, 1996.

[3] J.A. Buzacott and M. Mandelbaum. Flexibility in manufacturing and services: achievements, insights and challenges. *Flexible services and manufacturing journal*, 20(1):13–58, 2008.

[4] J.H. Saleh, D.E. Hastings, and D.J. Newman. Flexibility in system design and implications for aerospace systems. *Acta Astronautica*, 53(12):927–944, 2003.

[5] Peter Dapiran. Benetton–global logistics in action. *International Journal of Physical Distribution & Logistics Management*, 22(6):7–11, 1992.

[6] Drew Fudenberg and Jean Tirole. Preemption and rent equalization in the adoption of new technology. *The Review of Economic Studies*, 52(3):383–401, 1985.

[7] Drew Fudenberg and David K Levine. Open-loop and closed-loop equilibria in dynamic games with many players. *Journal of Economic Theory*, 44(1):1–18, 1988.

[8] George Stigler. Production and distribution in the short run. *The Journal of Political Economy*, 47(3):305–327, 1939.

[9] Thomas Marschak and Richard Nelson. Flexibility, uncertainty, and economic theory. *Metroeconomica*, 14(1-2-3):42–58, 1962.

[10] Albert Gailord Hart. *Anticipations, uncertainty, and dynamic planning*. University of Chicago Press., 1940.

[11] James Feibleman and Julius W Friend. The structure and function of organization. *The Philosophical Review*, 54(1):19–44, 1945.

[12] D. Upton. The management of manufacturing flexibility. *California management review*, 36(2):72–89, 1994.

[13] Jim Browne, Didier Dubois, Keith Rathmill, Suresh P Sethi, and Kathryn E Stecke. Classification of flexible manufacturing systems. *The FMS magazine*, 2(2):114–117, 1984.

[14] A.K. Sethi and S.P. Sethi. Flexibility in manufacturing: a survey. *International journal of flexible manufacturing systems*, 2(4):289–328, 1990.

[15] Robert J Vokurka and Scott W O'Leary-Kelly. A review of empirical research on manufacturing flexibility. *Journal of Operations Management*, 18(4):485–501, 2000.

[16] Yash P Gupta and Sameer Goyal. Flexibility of manufacturing systems: concepts and measurements. *European journal of operational research*, 43(2):119–135, 1989.

[17] Bhaba R Sarker, Sembian Krishnamurthy, and Srinivasa G Kuthethur. A survey and critical review of flexibility measures in manufacturing systems. *Production Planning & Control*, 5(6):512–523, 1994.

[18] Alberto De Toni and Stefano Tonchia. Manufacturing flexibility: a literature review. *International journal of production research*, 36(6):1587–1617, 1998.

[19] John P Shewchuk and Colin L Moodie. Definition and classification of manufacturing flexibility types and measures. *International Journal of Flexible Manufacturing Systems*, 10(4):325–349, 1998.

[20] Rodney P Parker and Andrew Wirth. Manufacturing flexibility: measures and relationships. *European journal of operational research*, 118(3):429–449, 1999.

[21] Roger Beach, AP Muhlemann, DHR Price, Andrew Paterson, and JA Sharp. A review of manufacturing flexibility. *European Journal of Operational Research*, 122(1):41–57, 2000.

[22] Jens Bengtsson. Manufacturing flexibility and real options: A review. *International Journal of Production Economics*, 74(1):213–224, 2001.

[23] Lori L Koste, Manoj K Malhotra, and Subhash Sharma. Measuring dimensions of manufacturing flexibility. *Journal of Operations Management*, 22(2):171–196, 2004.

[24] SH Lim. Flexible manufacturing systems and manufacturing flexibility in the united kingdom. *International Journal of Operations & Production Management*, 7(6):44–54, 1987.

[25] Ramchandran Jaikumar. *Flexible manufacturing systems: a managerial perspective*. Division of Research, Graduate School of Business Administration, Harvard University, 1984.

[26] Sten-Olof Gustavsson. Flexibility and productivity in complex production processes. *THE INTERNATIONAL JOURNAL OF PRODUCTION RESEARCH*, 22(5):801–808, 1984.

[27] Raghavan Parthasarthy and S Prakash Sethi. Relating strategy and structure to flexible automation: a test of fit and performance implications. *Strategic Management Journal*, 14(7):529–549, 1993.

[28] Judith Gebauer and Arno Scharl. Between flexibility and automation: an evaluation of web technology from a business process perspective. *Journal of Computer-Mediated Communication*, 5(2):0–0, 1999.

[29] Nigel Slack. Flexibility as a manufacturing objective. *International Journal of Operations & Production Management*, 3(3):4–13, 1983.

[30] Nigel Slack. The flexibility of manufacturing systems. *International Journal of Operations & Production Management*, 7(4):35–45, 1987.

[31] Nigel Slack. Manufacturing systems flexibility-an assessment procedure. *Computer integrated manufacturing systems*, 1(1):25–31, 1988.

[32] Donald Gerwin. Manufacturing flexibility in the cam era. *Business Horizons*, 32(1):78–84, 1989.

[33] W.C. Jordan and S.C. Graves. Principles on the benefits of manufacturing process flexibility. *Management Science*, 41(4):577–594, 1995.

[34] Helen Schonenberg, Ronny Mans, Nick Russell, Nataliya Mulyar, and Wil van der Aalst. Process flexibility: A survey of contemporary approaches. In *Advances in Enterprise Engineering I*, pages 16–30. Springer, 2008.

[35] LUDWIG H AVONTS and LUK N VAN WASSENHOVE. The part mix and routing mix problem in fms: a coupling between an lp model and a closed queueing network. *The International Journal Of Production Research*, 26(12):1891–1902, 1988.

[36] Donald Gerwin. Do's and don'ts of computer integrated manufacturing. *Harvard Business Review*, 60:107–116, 1982.

[37] Brian Tomlin and Yimin Wang. On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing & Service Operations Management*, 7(1):37–57, 2005.

[38] Andrea Rossi and Gino Dini. Flexible job-shop scheduling with routing flexibility and separable setup times using ant colony optimisation method. *Robotics and Computer-Integrated Manufacturing*, 23(5):503–516, 2007.

[39] Rahul Caprihan and Subhash Wadhwa. Impact of routing flexibility on the performance of an fmsa simulation study. *International Journal of Flexible Manufacturing Systems*, 9(3):273–298, 1997.

[40] Donald Gerwin and Jean-Claude Tarondeau. International comparisons of manufacturing flexibility. *Managing International Manufacturing*, pages 169–85, 1989.

[41] PK Palani Rajan, Michael Van Wie, Matthew I Campbell, Kristin L Wood, and Kevin N Otto. An empirical foundation for product flexibility. *Design Studies*, 26(4):405–438, 2005.

[42] J Robb Dixon. Measuring manufacturing flexibility: an empirical investigation. *European Journal of Operational Research*, 60(2):131–143, 1992.

[43] Panagiotis Kouvelis, Abbas A Kurawarwala, and Genaro J Gutierrez. Algorithms for robust single and multiple period layout planning for manufacturing systems. *European Journal of Operational Research*, 63(2):287–303, 1992.

[44] Panagiotis Kouvelis. Design and planning problems in flexible manufacturing systems: a critical review. *Journal of Intelligent Manufacturing*, 3(2):75–99, 1992.

[45] Ehap H Sabri and Benita M Beamon. A multi-objective approach to simultaneous strategic and operational planning in supply chain design. *Omega*, 28(5):581–598, 2000.

[46] Eric P Jack and Amitabh Raturi. Sources of volume flexibility and their impact on performance. *Journal of Operations Management*, 20(5):519–548, 2002.

[47] E Ertugrul Karsak and C Okan Özogul. An options approach to valuing expansion flexibility in flexible manufacturing system investments. *The Engineering Economist*, 47(2):169–193, 2002.

[48] E Ertugrul Karsak and C Okan Özogul. Valuation of expansion flexibility in flexible manufacturing system investments using sequential exchange options. *International Journal of Systems Science*, 36(5):243–253, 2005.

[49] William Golden and Philip Powell. Towards a definition of flexibility: in search of the holy grail? *Omega*, 28(4):373–384, 2000.

[50] J.H. Saleh, G. Mark, and N.C. Jordan. Flexibility: a multi-disciplinary literature review and a research agenda for designing flexible engineering systems. *Journal of Engineering Design*, 20(3):307–323, 2009.

[51] Donald Gerwin. An agenda for research on the flexibility of manufacturing processes. *International Journal of Operations & Production Management*, 7(1):38–49, 1987.

[52] Rajan Suri and Cynthia K Whitney. Decision support requirements in flexible manufacturing. *Journal of Manufacturing Systems*, 3(1):61–69, 1984.

[53] D Gupta. On measurement and valuation of manufacturing flexibility. *THE INTERNATIONAL JOURNAL OF PRODUCTION RESEARCH*, 31(12):2947–2958, 1993.

[54] Andrew Mair. Hondas global flexifactory network. *International Journal of Operations & Production Management*, 14(3):6–23, 1994.

[55] Mark Stevenson and Martin Spring. Flexibility from a supply chain perspective: definition and review. *International Journal of Operations & Production Management*, 27(7):685–713, 2007.

[56] B Carlsson. Management of flexible manufacturing: An international comparison. *Omega*, 20(1):11–22, 1992.

[57] Stanley B. Gershwin, R.R. Hildebrant, Rajan Suri, and S.K. Mitter. A control perspective on recent trends in manufacturing systems. *Control Systems Magazine, IEEE*, 6(2):3–15, 1986.

[58] Diwakar Gupta and John A Buzacott. A framework for understanding flexibility of manufacturing systems. *Journal of manufacturing systems*, 8(2):89–97, 1989.

[59] Xavier De Groote. The flexibility of production processes: a general framework. *Management Science*, 40(7):933–945, 1994.

[60] Marvin Mandelbaum and John Buzacott. Flexibility and decision making. *European Journal of Operational Research*, 44(1):17–27, 1990.

[61] Shiv K Gupta and Jonathan Rosenhead. Robustness in sequential investment decisions. *Management Science*, 15(2):B–18, 1968.

[62] Vinod Kumar. Entropic measures of manufacturing flexibility. *International Journal of Production Research*, 25(7):957–966, 1987.

[63] Eyas Shuiabi, Vince Thomson, and Nadia Bhuiyan. Entropy as a measure of operational flexibility. *European Journal of Operational Research*, 165(3):696–707, 2005.

[64] F. Black and M. Scholes. The pricing of options and corporate liabilities. *The journal of political economy*, 13:637–654, 1973.

[65] Robert C Merton. Theory of rational option pricing. *The Bell Journal of Economics and Management Science*, pages 141–183, 1973.

[66] Stewart C Myers. Determinants of corporate borrowing. *Journal of financial economics*, 5(2):147–175, 1977.

[67] Lenos Trigeorgis. *Real options: Managerial flexibility and strategy in resource allocation*. MIT press, 1996.

[68] Sheridan Titman. Urban land prices under uncertainty. *The American Economic Review*, 75(3):505–514, 1985.

[69] Jonathan E Ingersoll Jr and Stephen A Ross. Waiting to invest: investment and uncertainty. *Journal of Business*, pages 1–29, 1992.

[70] Robert McDonald and Daniel Siegel. The value of waiting to invest. *The Quarterly Journal of Economics*, 101(4):707–727, 1986.

[71] Saman Majd and Robert S Pindyck. Time to build, option value, and investment decisions. *Journal of financial Economics*, 18(1):7–27, 1987.

[72] Peter Carr. The valuation of sequential exchange opportunities. *The Journal of Finance*, 43(5):1235–1256, 1988.

[73] Michael J Brennan and Eduardo S Schwartz. Evaluating natural resource investments. *Journal of business*, pages 135–157, 1985.

[74] Lenos Trigeorgis and Scott P Mason. Valuing managerial flexibility. *Midland Corporate Finance Journal*, 5(1):14–21, 1987.

[75] Robert S Pindyck. Irreversible investment, capacity choice, and the value of the firm, 1986.

[76] Robert L McDonald and Daniel R Siegel. Investment and the valuation of firms when there is an option to shut down. *International Economic Review*, 26(2):331–349, 1985.

[77] Stewart C Myers and Saman Maid. Abandonment value and project life. *Real Options and Investment under Uncertainty: Classical Readings and Recent Contributions*, pages 295–313, 2001.

[78] Nalin Kulatilaka. Valuing the flexibility of flexible manufacturing systems. *Engineering Management, IEEE Transactions on*, 35(4):250–257, 1988.

[79] Nalin Kulatilaka and Lenos Trigeorgis. The general flexibility to switch: Real options revisited. *Real options and investment under uncertainty: classical readings and recent contributions, 1st edn. MIT Press, Massachusetts*, pages 179–198, 2004.

[80] Adam M Ross, Donna H Rhodes, and Daniel E Hastings. Defining changeability: Reconciling flexibility, adaptability, scalability, modifiability, and robustness for maintaining system lifecycle value. *Systems Engineering*, 11(3):246–262, 2008.

[81] Robert Neches and Azad M Madni. Towards affordably adaptable and effective systems. *Systems Engineering*, 2012.

[82] James Edwin Neely. *Improving the valuation of research and development: a composite of real options, decision analysis and benefit valuation frameworks.* PhD thesis, Massachusetts Institute of Technology, 1998.

[83] Roshanak Nilchiani and Daniel E Hastings. Measuring the value of space systems flexibility: a comprehensive six-element framework. In *PhD thesis in Aeronautics and Astronautics. Massachusetts Institute of Technology*, 2005.

[84] Roshanak Nilchiani and Daniel E. Hastings. Measuring the value of flexibility in space systems: A six-element framework. *Systems Engineering*, 10(1):26–44, 2007.

[85] Phelim P Boyle, Jeremy Evnine, and Stephen Gibbs. Numerical evaluation of multivariate contingent claims. *Review of Financial Studies*, 2(2):241–250, 1989.

[86] Steven R Grenadier. Option exercise games: An application to the equilibrium investment strategies of firms. *Review of financial studies*, 15(3):691–721, 2002.

[87] Kuno JM Huisman. *Technology Investment: a game theoretic real options approach*, volume 28. Kluwer Academic Pub, 2001.

[88] Han TJ Smit and Lenos Trigeorgis. *Strategic investment: Real options and games.* Princeton University Press, 2008.

[89] Benoît Chevalier-Roignant and Lenos Trigeorgis. *Competitive Strategy: Options and Games.* Mit Pr, 2011.

[90] Steven R Grenadier. *Game choices: The intersection of real options and game theory.* Risk Books London, 2000.

[91] Nelson Ferreira, Jayanti Kar, and Lends TRIGEORGIS. Option games: The key to competing in capital-intensive industries. *Harvard business review*, 87(3), 2009.

[92] Joseph T Williams. Equilibrium and options on real assets. *Review of Financial Studies*, 6(4):825–850, 1993.

[93] Steven R Grenadier. The strategic exercise of options: Development cascades and overbuilding in real estate markets. *The Journal of Finance*, 51(5):1653–1679, 1996.

[94] Han TJ Smit and LA Ankum. A real options and game-theoretic approach to corporate investment strategy under competition. *Financial Management*, pages 241–250, 1993.

[95] Glenn C Loury. Market structure and innovation. *The Quarterly Journal of Economics*, pages 395–410, 1979.

[96] Nalin Kulatilaka and Enrico C Perotti. Strategic growth options. *Management Science*, 44(8):1021–1031, 1998.

[97] M.J.A. BRENNAN and L.A. TRIGEORGIS. *Project Flexibility, Agency, and Competition: New Developments in the Theory and Application of Real Options Analysis*. Oxford University Press, 2000.

[98] Helen Weeds. Strategic delay in a real options model of r&d competition. *The Review of Economic Studies*, 69(3):729–747, 2002.

[99] Steven R Grenadier. Information revelation through option exercise. *Review of financial studies*, 12(1):95–129, 1999.

[100] Bart Lambrecht and William Perraudin. Real options and preemption under incomplete information. *Journal of Economic Dynamics and Control*, 27(4):619–643, 2003.

[101] Marco Antonio Guimarães Dias and José Paulo Teixeira. Continuous-time option games: review of models and extensions. *Multinational Finance Journal*, 14(3/4):219–254, 2010.

[102] F. R. Smets. *Essays on foreign direct investment*. PhD thesis, Yale University, 1993.

[103] K.M. Ramachandran and C.P. Tsokos. *Stochastic Differential Games: Theory and Applications*. Atlantis Studies in Probability and Statistics. Atlantis Press (Zeger Karssen), 2012.

[104] Terry L Friesz. *Dynamic optimization and differential games*, volume 135. Springer, 2010.

[105] Engelbert J Dockner, Steffen Jorgensen, Ngo Van Long, and Gerhard Sorger. *Differential games in economics and management science*. Cambridge University Press, 2000.

[106] M. Bardi, T.E.S. Raghavan, and T. Parthasarathy. *Stochastic and Differential Games: Theory and Numerical Methods*. Annals of the International Society of Dynamic Games, V. 4. Birkhäuser, 1999.

[107] P. Cardaliaguet and R. Cressman. *Advances in Dynamic Games: Theory, Applications, and Numerical Methods for Differential and Stochastic Games*. Annals of the International Society of Dynamic Games. Birkhäuser, 2012.

[108] D.W.K. Yeung and L.A. Petrosyan. *Cooperative Stochastic Differential Games*. Springer Series in Operations Research and Financial Engineering. Springer, 2010.

[109] Emilio Roxin and Chris P Tsokos. On the definition of a stochastic differential game. *Theory of Computing Systems*, 4(1):60–64, 1970.

[110] John Von Neumann and Oskar Morgenstern. *Theory of Games and Economic Behavior (60th Anniversary Commemorative Edition).* Princeton university press, 2007.

[111] Rufus Isaacs. *Differential games: a mathematical theory with applications to warfare and pursuit, control and optimization.* Courier Dover Publications, 1999.

[112] Alain Haurie. Feedback equilibria in differential games with structural and modal uncertainties. In *in Advances in Large Scale Systems.* Citeseer, 1984.

[113] Nikolai Nikolaevich Krasovskii, Andrei Izmailovich Subbotin, and Samuel Kotz. *Game-theoretical control problems.* Springer-Verlag New York, Inc., 1987.

[114] Yu-Chi Ho. Optimal terminal maneuver and evasion strategy. *SIAM Journal on Control*, 4(3):421–428, 1966.

[115] Robert Elliott. The existence of value in stochastic differential games. *SIAM Journal on Control and Optimization*, 14(1):85–94, 1976.

[116] Alain Bensoussan and Avner Friedman. Nonzero-sum stochastic differential games with stopping times and free boundary problems. *Transactions of the American Mathematical Society*, 231(2):275–327, 1977.

[117] Wendell H Fleming. The convergence problem for differential games ii. In *Advances in game theory*, pages 195–210. Princeton Univ. Press Princeton, NJ, 1964.

[118] Michael G Crandall and Pierre-Louis Lions. Viscosity solutions of hamilton-jacobi equations. *Transactions of the American Mathematical Society*, 277(1):1–42, 1983.

[119] Wendell Helms Fleming and H Mete Soner. *Controlled Markov processes and viscosity solutions*, volume 25. Springer New York, 2006.

[120] A. Bar-Ilan and W.C. Strange. Investment lags. *The American Economic Review*, 86(3):610–622, 1996.

[121] L.H.R. Alvarez and J. Keppo. The impact of delivery lags on irreversible investment under uncertainty. *European Journal of Operational Research*, 136(1):173–180, 2002.

[122] E. Bayraktar and M. Egami. The effects of implementation delay on decision-making under uncertainty. *Stochastic processes and their applications*, 117(3):333–358, 2007.

[123] S. Sødal. Entry and exit decisions based on a discount factor approach. *Journal of Economic Dynamics and Control*, 30(11):1963–1986, 2006.

[124] J.J.J. Thijssen. Optimal and strategic timing of mergers and acquisitions motivated by synergies and risk diversification. *Journal of Economic Dynamics and Control*, 32(5):1701–1720, 2008.

[125] R. Madlener, G. Kumbaroğlu, and V.Ş. Ediger. Modeling technology adoption as an irreversible investment under uncertainty: the case of the turkish electricity supply industry. *Energy Economics*, 27(1):139–163, 2005.

[126] G. Kumbaroğlu, R. Madlener, and M. Demirel. A real options evaluation model for the diffusion prospects of new renewable power generation technologies. *Energy Economics*, 30(4):1882–1908, 2008.

[127] S.L. MacDougall and R.H. Pike. Consider your options: changes to strategic value during implementation of advanced manufacturing technology. *Omega*, 31(1):1–15, 2003.

[128] B. Øksendal. *Stochastic differential equations: an introduction with applications*. Springer, 2010.

[129] B. Øksendal. Optimal stopping with delayed information. *Stochastics and Dynamics*, 5(02):271–280, 2005.

[130] Charles H Fine and Robert M Freund. Optimal investment in product-flexible manufacturing capacity. *Management Science*, 36(4):449–466, 1990.

[131] Jan A Van Mieghem. Investment strategies for flexible resources. *Management Science*, 44(8):1071–1078, 1998.

[132] Ebru K Bish and Qiong Wang. Optimal investment strategies for flexible resources, considering pricing and correlated demands. *Operations Research*, 52(6):954–964, 2004.

[133] Stephen C Graves and Brian T Tomlin. Process flexibility in supply chains. *Management Science*, 49(7):907–919, 2003.

[134] Suri Gurumurthi and Saif Benjaafar. Modeling and analysis of flexible queueing systems. *Naval Research Logistics (NRL)*, 51(5):755–782, 2004.

[135] Wallace J Hopp, Eylem Tekin, and Mark P Van Oyen. Benefits of skill chaining in serial production lines with cross-trained workers. *Management Science*, 50(1):83–98, 2004.

[136] Seyed M Iravani, Mark P Van Oyen, and Katharine T Sims. Structural flexibility: A new perspective on the design of manufacturing and service operations. *Management Science*, 51(2):151–166, 2005.

[137] O Zeynep Akşin and Fikri Karaesmen. Characterizing the performance of process flexibility structures. *Operations Research Letters*, 35(4):477–484, 2007.

[138] Mabel C Chou, Geoffrey A Chua, Chung-Piaw Teo, and Huan Zheng. Process flexibility revisited: the graph expander and its applications. *Operations research*, 59(5):1090–1105, 2011.

[139] Herbert Kotzab. Improving supply chain performance by efficient consumer response? a critical comparison of existing ecr approaches. *Journal of Business & Industrial Marketing*, 14(5/6):364–377, 1999.

[140] Martin Christopher and Denis R Towill. Supply chain migration from lean and functional to agile and customised. *Supply Chain Management: An International Journal*, 5(4):206–213, 2000.

[141] Jennifer F Reinganum. On the diffusion of new technology: a game theoretic approach. *The Review of Economic Studies*, pages 395–405, 1981.

[142] Jennifer F Reinganum. Market structure and the diffusion of new technology. *Bell Journal of Economics*, 12(2):618–624, 1981.

[143] Kuno JM Huisman and Peter M Kort. Strategic technology adoption taking into account future technological improvements: A real options approach. *European Journal of Operational Research*, 159(3):705–728, 2004.

[144] Barbara J Spencer and James A Brander. Pre-commitment and flexibility: Applications to oligopoly theory. *European Economic Review*, 36(8):1601–1626, 1992.

[145] Fridrik M Baldursson. Irreversible investment under uncertainty in oligopoly. *Journal of Economic Dynamics and Control*, 22(4):627–644, 1998.

[146] Robert Novy-Marx. An equilibrium model of investment under uncertainty. *Review of Financial Studies*, 20(5):1461–1502, 2007.

[147] Kerry Back and Dirk Paulsen. Open-loop equilibria and perfect competition in option exercise games. *Review of Financial Studies*, 22(11):4531–4552, 2009.

[148] Jacco JJ Thijssen. Preemption in a real option game with a first mover advantage and player-specific uncertainty. *Journal of Economic Theory*, 145(6):2448–2462, 2010.

[149] Jacco JJ Thijssen, Kuno JM Huisman, and Peter M Kort. Symmetric equilibrium strategies in game theoretic real option models. *Journal of Mathematical Economics*, 48(4):219–225, 2012.

[150] Jan-Henrik Steg. Irreversible investment in oligopoly. *Finance and Stochastics*, 16(2):207–224, 2012.

[151] Benoît Chevalier-Roignant, Christoph M Flath, Arnd Huchzermeier, and Lenos Trigeorgis. Strategic investment under uncertainty: a synthesis. *European Journal of Operational Research*, 215(3):639–650, 2011.

[152] Grzegorz Pawlina and Peter M Kort. Real options in an asymmetric duopoly: Who benefits from your competitive disadvantage? *Journal of Economics & Management Strategy*, 15(1):1–35, 2006.

[153] Kristian R Miltersen and Eduardo S Schwart. R&d investments with competitive interactions. *Review of Finance*, 8(3):355–401, 2004.

[154] Pauli Murto. Exit in duopoly under uncertainty. *RAND Journal of Economics*, pages 111–127, 2004.

[155] Andreu Mas-Colell, Michael Dennis Whinston, Jerry R Green, et al. *Microeconomic theory*, volume 1. Oxford university press New York, 1995.

[156] Roy Radner. Collusive behavior in noncooperative epsilon-equilibria of oligopolies with long but finite lives. *Journal of economic theory*, 22(2):136–154, 1980.

[157] B.K. Øksendal and A. Sulem. *Applied stochastic control of jump diffusions.* Springer Verlag, 2005.

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This dissertation was typed entirely by the author.