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PURDUE UNIVERSITY GRADUATE SCHOOL Thesis/Dissertation Acceptance

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By Ali Elghariani

Entitled

Application of Integer Quadratic Programming to Detection of High-Dimensional Wireless Systems

For the degree of _____ Doctor of Philosophy

Is approved by the final examining committee:

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Head of the Department Graduate Program

Date

APPLICATION OF INTEGER QUADRATIC PROGRAMMING IN DETECTION OF HIGH-DIMENSIONAL WIRELESS SYSTEMS

A Dissertation

Submitted to the Faculty

of

Purdue University

by

Ali Ahmed Elghariani

In Partial Fulfillment of the

Requirements for the Degree

of

Doctor of Philosophy

December 2014

Purdue University

West Lafayette, Indiana

To my wife and children.

ACKNOWLEDGMENTS

First of all, I would like to express my deep appreciation to my wife Sondos and my kids Shada, Shahed, Mohamed, and Jana, for their enormous love and support throughout my PhD years.

I would like to give a special thanks to my advisor, Professor Michael Zoltowski, for his encouragement and continued guidance throughout my years in graduate school. It has been a pleasure and honor to work with such an excellent advisor, and I just cannot thank him enough. I would also like to thank the other members of my advisory committee for their comments and advice: Professor Mark Bell, Professor James Lehnert, and Professor David Love. Special thanks to Professor David Love for his support and also for the courses that he offered duringmy time at grad school. I really enjoyed his classes in Estimation Theory and Multiple Antenna Communication Systems. They were a great source of help in my research.

I would like to thank my fellow ECE graduate students, especially Abdurahman Maud and Song Noh for their friendship and valuable research discussions.

Finally, I am very grateful to my parents, to family and friends from Libya, and to the faculty members of the Electrical Engineering department of Tripoli University for their support and encouragement.

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SYMBOLS

- $(\mathbf{A})^T$ Transpose of A $(\mathbf{A})^H$ Conjugate Transpose of A $\tilde{\mathbf{x}}$ Complex quantity Real quantity \mathbf{x} $\mathbf{z}^{(j)}$ Continuous solution of an optimization problem at node j of BB tree. $f^{(j)}$ Objective function value of node j of BB tree. $\parallel \mathbf{a} \parallel$ two-norm of a vector ${\bf a}$ $\Re(\tilde{\mathbf{x}})$ Real part of $\tilde{\mathbf{x}}$ $\Im(\tilde{\mathbf{x}})$ Imaginary part of $\tilde{\mathbf{x}}$ \mathbf{I}_N $N \times N$ Identity Matrix $\mathcal{Q}[.]$ Quantization function $\|\mathbf{a}\|_{\infty}$ Infinity norm of a vector **a** $\left\lceil . \right
 floor$ Rounding operation to the nearest integer O(.)Complexity order <u>0</u> Column vector of all zeros
- $\underline{1}$ Column vector of all ones

ABBREVIATIONS

2QPTwo-stage Quadratic Programming BBBranch and Bound Integer Quadratic Programming IQP IDD Iterative Detection and Decoding IP Interior Point LAS Local Ascent Search LML Local Maximum Likelihood ML Maximum Likelihood MMSE Minimum Mean Square Error MIQP Mixed Integer Quadratic Programming OFDM Orthogonal Frequency Division Multiplexing \mathbf{QP} Quadratic Programming RTS Reactive Tabu Search SIC Successive Interference Cancellation SDSphere Decoding SOFDM Spread OFDM

ABSTRACT

Elghariani, Ali Ph.D., Purdue University, December 2014. Application of Integer Quadratic Programming in Detection of High-Dimensional Wireless Systems . Major Professor: Michael Zoltowski.

High-dimensional wireless systems have recently generated a great deal of interest due to their ability to accommodate increasing demands for high transmission data rates with high communication reliability. Examples of such large-scale systems include single-input, single-output symbol spread OFDM system, large-scale single-user multi-input multi-output (MIMO) OFDM systems, and large-scale multiuser MIMO systems. In these systems, the number of symbols required to be jointly detected at the receiver is relatively large. The challenge with the practical realization of these systems is to design a detection scheme that provides high communication reliability with reasonable computational complexity, even as the number of simultaneously transmitted independent communication signals becomes very large.

Most of the optimal or near-optimal detection techniques that have been proposed in the literature of relatively low-dimensional wireless systems, such as MIMO systems in which number of antennas is less than 10, become problematic for high-dimensional detection problems. That is, their performance degrades or the computational complexity becomes prohibitive, especially when higher-order QAM constellations are employed.

In the first part of this thesis, we propose a near-optimal detection technique which offers a flexible trade-off between complexity and performance. The proposed technique formulates the detection problem in terms of Integer Quadratic Programming (IQP), which is then solved through a controlled Branch and Bound (BB) search tree algorithm. In addition to providing good performance, an important feature of this approach is that its computational complexity remains roughly the same even as we increase the constellation order from 4-QAM to 256-QAM. The performance of the proposed algorithm is investigated for both symbol spread OFDM systems and large-scale MIMO systems with both frequency selective and flat fading channels.

The second part of this work focuses on a reduced complexity version of IQP referred to as relaxed quadratic programming (QP). In particular, QP is used to reformulate two widely used detection schemes for MIMO OFDM: (1) Successive Interference Cancellation (SIC) and (2) Iterative Detecting and Decoding (IDD). First, SIC-based algorithms are derived via a QP formulation in contrast to using a linear MMSE detector at each stage. The resulting QP-SIC algorithms offer lower computational complexity than the SIC schemes that employ linear MMSE at each stage, especially when the dimension of the received signal vector is high. Three versions of QP-SIC are proposed based on various trade-offs between complexity and receiver performance; each of the three QP-SIC algorithms outperforms existing SIC techniques. Second, IDD-based algorithms are developed using a QP detector. We show how the soft information, in terms of the Log Likelihood Ratio (LLR), can be extracted from the QP detector. Further, the procedure for incorporating the a-priori information that is passed from the channel decoder to the QP detector is developed. Simulation results are presented demonstrating that the use of QP in IDD offers improved performance at the cost of a reasonable increase in complexity compared to linear detectors.

1. INTRODUCTION

1.1 Integer Quadratic Programming for Signal Detection

The field of mixed-integer nonlinear programming (MINLP) optimization has applications in many areas of engineering, applied mathematics, applied science, and operations research [1]. MINLP problems basically involve general constraints and nonlinear objective functions with both continuous and integer variables. The applications of MINLP are extensively surveyed in [2] and the references therein. The Integer quadratic programming (IQP) problem is a subclass of MINLP problems, where the objective function is quadratic and all variables are restricted to be integers.

There has been an interest in applying IQP in the area of communication and signal processing, such as filter design [3], cognitive radios [4], and network routing and scheduling [5]. Binary IQP problems, in which all variables belong to $\{0, 1\}$ set, have received a considerable amount of attention in research because of their simplicity in dealing with 0 and 1 variables and the potential of lower computational complexity [6], [7]. In wireless detection problems, this can fit several communication systems in which the mapped signal is BPSK or QPSK.

A known algorithm that is used to solve integer programming problems is the Branch and Bound (BB) algorithm, which was first proposed by Land and Doig [8]. BB algorithm is based on relaxing the integer constraints of the IQP problem, then through a systematic recursive search, variables are successively forced to take integer values using branching rules [9]. This is done in a tree-structured manner, the details of which can be seen in section 2.4. Each node of this tree represents a subproblem which can be split into two subproblems by adding two mutually exclusive constraints. BB algorithm does not search the complete tree. It prunes some

sub-trees by a bounding strategy [10] so that the searching space is reduced.

The concept of a search tree is known in the detection problems of wireless communications, such as in Maximum Likelihood (ML), Sphere Decoding (SD) [11], and QR Decomposition combined with M algorithm (QRD-M) [12], where the search is based on enumerating all possible solutions. In this thesis, the BB search tree is introduced in conjunction with the IQP problem which provides interesting features. These features could contribute to complexity savings and allow easy modifications to the algorithm. These features are [13]: 1) Every node in the search tree generates only two sub-nodes regardless of the modulation level. 2) At each node of the tree, the continuous solution of all variables is available, which can be utilized for obtaining a quick suboptimal solution. 3) No tracing back at the end of the search tree to get the final solution, which consumes fewer memory spaces. 4) No need to reach the end leaf of the tree to find the solution. Several nodes could be pruned based on the rules of BB algorithm (see section 2.4) and as such, the solution could be found before the last layer of the tree.

There has been little research presented in relevant literature that uses IQP based on BB algorithm in wireless communication detection problems. The work in [14] uses BB algorithm for a conventional small MIMO, such as: 4×4 , and shows promising results that motivated us to continue exploring this technique. It shows that the complexity of using BB algorithm is more computationally efficient than the sphere decoder algorithm at a low SNR regime and at high QAM modulation orders. Another work in [15], which uses binary IQP formulation with BB for QPSK 4×4 MIMO in a spatial multiplexing setting. It shows an interesting complexity reduction when preprocessing techniques are employed.

1.2 Spread OFDM System

Orthogonal Frequency Division Multiplexing (OFDM) is being considered as a promising transmission technique to combat frequency selectivity of the wireless channel. It has already been accepted for several wireless communication standards, such as digital audio/video broadcasting (DAB/DVB), wireless local area networks (WLAN) [16], and Advanced Long Term Evolution (LTE) [17]. OFDM transforms a frequency selective fading channel into a large number of flat fading subchannels which leads to easy equalization and symbol decoding. However, one of the disadvantages of OFDM is that it does not fully exploit the frequency diversity inherent in the frequency selective channel. This is due to the fact that each symbol is transmitted over a single subcarrier independently, and as a result, if this subcarrier experiences a deep fade, the modulated symbol through this subcarrier is most likely to be unreliably detected at the receiver.

There has been a fair amount of research that works towards alleviating this problem. The use of error-correcting codes (ECC) across OFDM subcarriers is a known technique for achieving frequency diversity in OFDM [16], but at the expense of some data rate reduction. It is known as Coded-OFDM (COFDM), and is used in DVBT and WLAN standards [16]. The other technique that increases the frequency diversity of OFDM is the use of symbol spreading over all or part of OFDM subcarriers. This is called Spread OFDM (SOFDM) [18], [19], [20], [21]. The idea is to spread the generated data symbols across all subcarriers so that each subcarrier modulates a linear combination of all the data symbols. The Spreading process can be done through a unitary matrix, such as Hadamard or Vandermonde spreading matrices [22].

Unlike the conventional OFDM system, the effective channel matrix of the spread OFDM system, in frequency domain, is no longer diagonal due to the spreading transformation. This in turn complicates the equalization process of the spread OFDM signal. Therefore, the detection problems of the SOFDM system is considered as a high-dimensional problem in which the number of jointly detected symbols at the receiver is approximately the same as the number of OFDM subcarriers used in the system, which is practically in the order of 256, 512, and 1024. Most of the work presented in the literature regarding signal detection of SOFDM was based on linear detection techniques, such as in [18], [23], and [20], which improves the frequency

diversity of the SOFDM system compared to the non spread system; however, the frequency diversity is still not efficiently exploited. To fully exploit the diversity potential of the SOFDM, an optimal detection technique is required, such as Maximum Likelihood detector (ML) or Sphere Decoding (SD) [11], but these techniques would entail prohibitive complexity which makes them impractical for real systems, especially when the number of OFDM subcarriers is large. Different techniques have been proposed in the literature for achieving near-ML performance with reduced complexity, such as reduced complexity SD and QRD-M [12] techniques. Most of this work focuses on MIMO systems and shows efficacy for small dimensions, where the number of transmit antennas is less than 10, and also where the order of modulation is small, such as QPSK and 16QAM. Work in [22] shows that a Local ML (LML) detector can provide a lower bit error rate than MMSE when QPSK symbol mapping is used.

In this work, we formulate the detection problem in terms of IQP, which is then solved through BB search tree algorithm. Then we introduce simplified techniques that approximate integer solutions with reduced complexity and also yield sub-optimal performance that is better than the MMSE and LML (see chapters 2 & 4).

1.3 Large-Scale MIMO Systems

Large-scale MIMO is an emerging technology that uses tens to hundreds of antennas compared to a small MIMO that uses less than 10 antennas. The more antennas the transmitter/receiver is equipped with, the better the performance in terms of data rate, link reliability, and spectral efficiency [24]. When the number of antennas at the transmitter and receiver are n_t and n_r , respectively, the point-to-point system diversity scales up to the multiplication of both n_t and n_r , and the achievable rate could increase by a factor min (n_t, n_r) [24], [25]. In addition, large-scale MIMO has the potential to reduce the operational power consumption at the base station in the multiuser scenario [26], [27]. As shown in [24], the large-scale MIMO systems are attracting a lot of attention in the research community. There are some theoretical concepts that highly motivated the research in this field, such as the asymptotic of random matrix theory, and the applicability of the law of large numbers [28]. These concepts could provide more insight into the analysis of these systems. For example, when n_t and n_r are large, the distribution of the singular values of the channel matrix **H** approaches a deterministic and $\frac{1}{n_t} \mathbf{H} \mathbf{H}^H \approx \mathbf{I}$ [24], [29].

Similar to the small MIMO systems, large-scale MIMO can always be combined with OFDM technology (MIMO-OFDM) to achieve higher data rates and spectral efficiency without increasing bandwidth, which makes it more attractive for high data rate wireless applications.

Given the aforementioned benefits, a large-scale MIMO system poses challenges in several design aspects of the MIMO system, such as channel estimation, hardware implementation, and detection complexity [24]. In MIMO systems, generally the detector is often the bottleneck for the overall performance and complexity, and it is obviously exacerbated in the large-scale case. There have been many linear detectors and near Maximum Likelihood (ML) detectors proposed in the literature of conventional (small) MIMO systems; however, they become noncompetitive when used to serve large-scale systems. One reason is because their computational complexity becomes exponential, such as in the case of Sphere Decoding (SD) and its variants [30], [31], [32]. Another reason is because the performance worsens as the number of antennas increases, as in the cases of minimum mean square error (MMSE), MMSE with ordered successive interference cancellation (MMSE-OSIC) [33], Chase type detector [34], QR Decomposition combined with M-algorithm (QRDM) [35], and Fixed Sphere Decoder (FSD) [36], [37]. Various algorithms have been presented in the literature that exhibit a large-system behavior where the BER performance improves and becomes increasingly closer to ML performance as the number of antennas increases. A family of low complexity detectors termed Likelihood Ascent Search (LAS) detectors have been proposed in [38], [39], [40], [30] for large-scale

MIMO systems. They are based on successively searching the local neighborhood of some good initial vectors, such as MMSE vector. LAS detectors show near single antenna AWGN performance when hundreds of antennas are used with an average per-received vector complexity of $O(n_r^2)$, where $n_r = n_t$ and n_t and n_r denote the number of transmit and receive antennas, respectively. LAS detectors have also been generalized for higher order modulations and showed increased performance as the number of antennas increases. However, they still suffer from performance deterioration as the modulation order increases. They also require hundreds of antennas to achieve near-optimum performance. This number increases as the modulation level increases [39]. Another neighborhood search algorithm that is based on reactive tabu search (RTS) [41] has also been proposed for large-scale MIMO systems with various QAMs in [42], [36], [43]. This algorithm is a heuristic-based combinatorial optimization technique which achieves near-ML performance with much lower complexity compared to ML and SD. However, its computational complexity scales up significantly with increasing QAM modulation levels accompanied with performance deterioration. There are a few other detection algorithms that have been presented for large-scale MIMO systems and shown to achieve near-optimal performance, but only for $\{-1, +1, \}$ alphabet, such as the work in [32], and in [44].

In this work, we further the idea of utilizing IQP formulation of the ML problem to formulate a large-scale MIMO detection problem in both frequency selective and flat fading channels. Although the concept of using IQP was also used in a previous work [14], using BB algorithm in a small MIMO detection, our work extends the idea by proposing a controlled size search tree algorithm using BB technique to solve the IQP problem so that it can easily accommodate a large-scale MIMO system with various QAM levels. In addition, we propose other versions of the QP algorithm, which can provide more flexible trade-offs between complexity and performance. The advantages of our proposed algorithms are: (*i*) complexity is flexible as it depends on two main parameters: the selected depth and width of the search tree, and (*ii*) complexity is nearly independent of the selected modulation order. Simulation results demonstrate that our proposed algorithms outperform most of the existing large-scale detection algorithms.

1.3.1 Successive Interference Cancellation Detectors

Successive interference cancellations (SIC) detectors [33], [45], [25] are known detectors in MIMO systems. They are classified as non-linear detectors with a proven performance superior to linear detectors [33]. However, they require more computational complexity than their linear counterparts to achieve this performance. The basic idea of SIC is to cancel out the effect of the already-detected symbol(s) by subtracting it(them) out from the received signal vector, which leads to a modified received vector in which fewer interferers are present [33]. The general drawback of all SIC is error propagation, because some of the estimated symbols are not reliable, which in turn impair the subsequent symbols estimates [46]. To mitigate this problem, ranking symbols based on certain reliability measure were introduced in the literature of SIC [33], [47]. One such technique is called V-BLAST, which improves the performance of the SIC detector using channel power ordering [33]; it detects data symbols with the greatest channel power first, and then cancels out its effect on the remaining data streams. It can be seen as an ordered SIC with either zero forcing (ZF) or minimum mean-square-error (MMSE) criterion for interference nulling.

A fair amount of research has been presented utilizing the concept of SIC. Most of this work focuses on MMSE with ordered SIC, because MMSE is more accurate than ZF. The work in [47] shows that the original V-BLAST ordered SIC can be further improved if Log-Likelihood Ratio (LLR) reliability ordering is used instead of channel power ordering. Furthermore, it depicts that the complexity can be reduced if symbol grouping is introduced using a multi-stage MMSE detector, however, the bit error rate (BER) performance is penalized as the number of symbols per group increases, leading to a higher BER. For instance, in this reference, if the number of stages is one, the technique boils down to the conventional MMSE detector, and if the number of stages equal to the number of symbols, the technique becomes the conventional MMSE V-BLAST. In [48], [46], another SIC approach was proposed, in multi-user MIMO, to reduce the effect of error propagation issue in SIC. It is based on the concept of multi-feedback, which is similar to the conventional SIC technique bu puts extra constraints on the MMSE estimated symbol. That is, if the MMSE estimated point lies within the shadow area of the constellation lines, then before canceling out its effect, the algorithm searches several constellation points from the constellation set and chooses the most appropriate point that minimizes the Euclidean distance rule. It shows that as the width of the shadow area increased, the performance improved, but as a result, more computational complexity was incurred. Most of the SIC techniques proposed are designed for small MIMO systems and rely on ZF or MMSE detectors because of their low complexity.

In massive MIMO systems where a large number of transmit and receive antennas is required, more interferers are present and subsequently, SIC process requires more efficient detectors that can combat the error propagation issue. Moreover, complexity is an important issue when it comes to large MIMO systems, as most of the proposed techniques require a large number of pseudo-inverse computations each time a new symbol is detected, though there has been some work in this regard to reduce the number of matrix inversions [49], [50], [51]. This is a huge burden on the receiver when number of antennas grows large.

In this thesis, we exploit the formulation of a QP, which can be obtained by relaxing the constraints of the IQP problem, to propose a new SIC algorithm that suits a large-scale MIMO system and can provide better performance together with low computational complexity compared to the existing techniques. We extend the applicability of this technique to other high-dimensional systems, such as SOFDM system since the detection problem is similar to the large-scale MIMO case.

1.3.2 Iterative Detection and Decoding

During the last decade, inspired by the development in turbo codes [52], there were a number of important advances made using the concept of joint equalization and decoding in which traditional equalization methods and decoding methods exchange soft information in an iterative fashion until convergence is achieved [53], [54]. This technique actually contrasts the conventional way that implements the equalizer and channel decoder separately, that is, a channel equalizer produces hard decision symbols which are then passed to a channel decoder that uses a certain error correcting strategy to improve final data quality.

Previous work shows that addressing joint equalization and decoding based on an optimal criteria, such as MAP provides the best performance, but it is usually impractically complex [55]. It has been presented for small systems with a small constellation size, such as in [56] and the references therein. Various approaches and techniques have been considered to implement this joint detection and decoding process using low complexity equalizers, in particular linear equalizers, such as ZF, MMSE, and decision feedback (DFE) [56], [57], [58], [55].

In this thesis, we propose an alternative equalization technique that could replace the linear MMSE equalizer in the joint detection and decoding receiver that can provide better performance with a penalty of some complexity increase. We propose to use QP detector in the turbo equalization type receiver. The challenges in using QP in the turbo equalization setting are: first, how to incorporate prior information in the form of LLR provided by the channel decoder into the QP optimization problem, and second, how we can make QP detector provide soft information in the form of LLR with reasonable complexity so that it can be passed to the channel decoder.

1.4 Outline

The rest of the thesis proceeds as follows: chapters 2 and 3 show the application of IQP using BB algorithm in the detection problems of SOFDM and large-scale MIMO OFDM systems. Chapter 4 demonstrates the application of various QP based algorithms in a large-scale MIMO system with a flat fading channel. Chapter 5 proposes a new successive interference cancellation technique using QP formulation. Chapter 6 focuses on implementing iterative detection and decoding type of receiver for SOFDM using an MMSE equalizer, while chapter 7 implements iterative detection and decoding for the MIMO OFDM system using a QP detector. Finally, concluding remarks and a summary of this work are provided in chapter 8.

2. INTEGER QUADRATIC PROGRAMMING APPROACH FOR SPREAD OFDM SYSTEMS

Spread OFDM (SOFDM) is introduced in [59], [60], and [21] to increase frequency diversity in OFDM systems. The idea is to spread the data symbols across all subcarriers so that each subcarrier contains a linear combination of all the data symbols. In order to fully exploit the diversity potential of SOFDM, an optimal detection technique is required, such as ML. Unfortunately, the computational complexity of the ML makes it impractical for real systems, especially when a large number of subcarriers is adopted. Various other techniques have been proposed for making ML detection less complex and more practical, such as [61], [62]. Most of that work focused on small-dimensional detection problems. In this work, we consider SOFDM with a large number of subcarriers (e.g. 128, 256).

In this thesis, we are inspired by a few research papers that used an IQP formulation in combination with the BB algorithm for MIMO signal detection, such as [14], [15], and [63]. The main observation in these research papers is that using IQP with BB can make the complexity independent of the type of QAM constellation adopted. In this thesis, we use their work and explore several other important features in the formulation of the IQP problems as well as the structure of the BB search tree, which we then utilize for further complexity reduction and to provide flexible trade-off between performance and complexity.

In this chapter, an IQP detection approach based on the BB search tree algorithm is introduced to a SOFDM system. The formulation of the detection problem is presented and the bit error rate (BER) performance and complexity are studied using the standard BB algorithm. Furthermore, a preprocessing procedure and some modifications to the standard BB algorithm are proposed.

2.1 Spread OFDM System Model

The idea behind the SOFDM is to spread the data symbols across all OFDM carriers prior to modulation such that each carrier contains a linear combination of all of the data symbols. Thus, if several carriers are lost due to spectral null, it may still be possible to retrieve all of the transmitted symbols [19]. This is in contrast to the conventional OFDM where the symbol is lost if the carrier modulating it was nulled by the channel. Figure 2.1 shows the block diagram representation of the SOFDM.

The mapped symbols $\tilde{\mathbf{x}} = [\tilde{x}_0, \dots, \tilde{x}_{N_b-1}]^T \in \mathbb{C}^{N_b \times 1}$, are modulated from the informa-



Fig. 2.1. Block diagram representation of the SOFDM

tion data bits after binary phase shift keying (BPSK), quadrature phase shift keying (QPSK), or any other quadrature amplitude modulation (QAM). Each \tilde{x}_i is taken from a finite constellation $\tilde{\chi}$ ($\tilde{x}_i \in \tilde{\chi}$) with normalized energy, $E[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^H] = \mathbf{I}_{N_b}$. N_b is the number of data symbols, E[.] is the expectation operator, $(.)^H$ is the conjugate transpose operation and \mathbf{I}_N is the identity matrix of size N. These N_b symbols will be spread across all OFDM subcarriers, N, using the spreading matrix, $\tilde{\mathbf{D}}$, of size $N \times N_b$, which could be real (e.g Hadamard Matrix [64]) or complex (e.g. Vandermonde [22]). Thus, the resulting spread symbols vector, X_{spr} , will be as follows:

$$X_{spr} = \mathbf{D}\tilde{\mathbf{x}} \tag{2.1}$$

where the vector $X_{spr} = [X_{spr_0}, ..., X_{spr_{N-1}}]^T$. There are two cases of interest in the SOFDM. The first one is when $N_b = N$, which is known as the full spread OFDM [22], and the other case is when $N_b < N$, which is known as the partial spread OFDM (i.e. partially loaded OFDM system) [22]. The resulting spread symbols are then

modulated onto N OFDM carriers through inverse FFT (IFFT) process to yield the time domain symbols

$$\mathbf{x}_{spr} = \mathrm{IFFT}(\mathbf{X}_{spr}) = \mathbf{F}^H X_{spr} = \mathbf{F}^H \tilde{\mathbf{D}} \tilde{\mathbf{x}}$$
(2.2)

where \mathbf{F} is the unitary Fourier Matrix and its elements can be generated from

$$\mathbf{F}_{i,k} = \frac{1}{\sqrt{N}} \exp^{\frac{-j2\pi ik}{N}}, \quad i,k = 0, 1, 2, ..., N - 1$$
(2.3)

After parallel to serial conversion, the \mathbf{x}_{spr} sequence will be appended with cyclic prefix (CP) symbols as a guard interval in order to avoid inter-symbol-interference (ISI) and inter-carrier-interference (ICI). The time length of this CP should be greater than the expected delay spread of the channel. The complex baseband signal is then up converted and sent over a multipath wireless channel with channel impulse response length L_{ch} , that is, the number of channel taps. We are assuming that the channel is a frequency selective fading channel and it is a constant during one OFDM block (block-fading channel [65]). In this thesis, we refer to one spread OFDM block as a block that contains N + CP symbols. The radio channel is assumed to exhibit Rayleigh fading with channel impulse response $\mathbf{h} = [h_0, h_1, h_{L_{ch}-1}]$. Each component of \mathbf{h} is identically independent distributed (i.i.d) complex Gaussian with zero mean and unit variance ($E[|h_i|^2] = \sigma_h^2 = 1$). The assumptions are also that \mathbf{h} remains constant during one OFDM block and that it is perfectly known at the receiver. The additive white Gaussian noise $\mathbf{n} = [n_0, n_1, n_{N-1}]$, which is a zero mean complex normal random vector with variance σ_n^2 , will be added at the receiver.

The discrete time convolution of \mathbf{x}_{spr} with the channel impulse response \mathbf{h} can be represented as a circular convolution due to the presence of CP. Thus, the received symbols vector (\mathbf{r}), before FFT is applied, can be written in a matrix form as

$$\mathbf{r} = \mathbf{H}_c \mathbf{x}_{spr} + \mathbf{n} \tag{2.4}$$

where \mathbf{H}_c is a circulant matrix over N samples of interest and each column of \mathbf{H}_c is a circular shift by one relative to the previous column, with the first column formed from the zero padded impulse response of the channel **h**. Now when applying FFT operation to **r** and using (2.2) and (2.4), the received vector in frequency domain can be derived as follows:

$$\mathbf{Fr} = \mathbf{FH}_c \mathbf{x}_{spr} + \mathbf{Fn}$$

$$\mathbf{Fr} = \mathbf{FH}_c \mathbf{F}^H \tilde{\mathbf{D}} \tilde{\mathbf{x}} + \mathbf{Fn}$$

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}} \tilde{\mathbf{D}} \tilde{\mathbf{x}} + \tilde{\mathbf{v}}$$

(2.5)

where $\tilde{\mathbf{H}}$ is a diagonal matrix of eigenvalues of \mathbf{H}_c [66], $\tilde{\mathbf{y}} = \mathbf{Fr}$, and $\tilde{\mathbf{v}} = \mathbf{Fn}$. The unitary Fourier Matrix \mathbf{F} does not alter the statistics of \mathbf{h} or \mathbf{n} . $\tilde{\mathbf{y}}$ represents the received symbols vector in the frequency domain after the FFT operation, where $\tilde{\mathbf{y}} = [y_0, \dots, y_{N-1}]^T \in \mathbb{C}^{N \times 1}$. $\tilde{\mathbf{H}}$ is a complex diagonal matrix whose diagonal entries are N points DFT of frequency selective channel vector \mathbf{h} . In this chapter, we consider the case of full spread OFDM, where $N_b = N$.

2.2 Linear MMSE Detection for Spread OFDM

MMSE is a linear suboptimal detection technique that is based on minimizing the mean square error between the estimated and the actual data symbols. It is characterized by its low complexity compared to the ML technique and it is unlike zero forcing detectors because it uses a filter matrix that takes noise variance into consideration. To find the matrix, \mathbf{W} , that minimizes the mean square error, the following optimization problem needs to be solved:

$$\underset{\mathbf{W}}{\operatorname{argmin}} \quad \mathbf{E}[(\tilde{\mathbf{x}} - \mathbf{W}^{H} \tilde{\mathbf{y}})^{2}] \tag{2.6}$$

The solution to (2.6) is given as follows [67]:

$$\mathbf{W}^{H} = (\mathbf{G}^{H}\mathbf{G} + \frac{\mathbf{I}_{N_{b}}}{\mathrm{SNR}})^{-1}\mathbf{G}^{H}$$
(2.7)

where $\mathbf{G} = \tilde{\mathbf{H}}\tilde{\mathbf{D}}$, with dimension of $N \times N_b$. In the above equations, SNR refers to the received signal-to-noise ratio, which equals to $1/\sigma_v^2$, because the energy per symbol is normalized to 1. $\tilde{\mathbf{D}}$ is a unitary spreading matrix and the covariance of $\tilde{\mathbf{H}}$ is \mathbf{I}_N .

Using **W** from (2.7) and substituting the values of **G** and SNR, the resulted MMSE estimate is as follows:

$$\tilde{\mathbf{x}}_{mmse} = \mathbf{W}^H \tilde{\mathbf{y}} \tag{2.8}$$

$$\tilde{\mathbf{x}}_{mmse} = (\tilde{\mathbf{D}}^H \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \tilde{\mathbf{D}} + \sigma_v^2 \mathbf{I}_N) \tilde{\mathbf{D}}^H \tilde{\mathbf{H}}^H \tilde{\mathbf{y}}$$
(2.9)

$$\hat{\tilde{\mathbf{x}}}_{mmse} = \mathcal{Q}[\tilde{\mathbf{x}}_{mmse}] \tag{2.10}$$

where Q[.] denotes the quantization (slicing) function to the appropriate constellation. The performance of SOFDM using the MMSE detector has been studied extensively with various spreading matrices in [22]. In this chapter, the MMSE performance will be compared to the performance of our proposed technique. In addition, the linear MMSE detector will also be utilized as a preprocessing technique for the proposed modifications to the standard Branch and Bound algorithm as shown in Section 2.5.1.

2.3 Formulating IQP Detection Problem

Given the set of all possible transmitted symbol vectors as $\tilde{\chi}^N$, the optimal detector rule chooses one of these possible symbol vectors that maximizes the a posteriori probability given the observation vector $\tilde{\mathbf{y}}$ and the channel matrix $\tilde{\mathbf{H}}$ [68]. More explicitly :

$$\hat{\tilde{\mathbf{x}}} = \underset{\tilde{\mathbf{x}} \in \tilde{\chi}^N}{\arg \max} \quad p(\tilde{\mathbf{x}} | \tilde{\mathbf{y}}, \tilde{\mathbf{H}})$$
(2.11)

Equation (2.11) is known as the Maximum A posteriori Probability (MAP) detection rule. Applying the standard assumption, that all possible symbols vectors are equiprobable, the MAP problem becomes an ML problem. Further, by assuming the noise to be white and Gaussian, the ML problem can be easily shown to become equivalent to a squared Euclidean distance minimization problem. That is,

$$\hat{\tilde{\mathbf{x}}}_k = \underset{\tilde{\mathbf{x}} \in \tilde{\chi}^N}{\operatorname{argmin}} \| \tilde{\mathbf{y}} - \tilde{\mathbf{H}} \tilde{\mathbf{D}} \tilde{\mathbf{x}} \|_2^2$$
(2.12)

For the detection problem to be solved efficiently, it is usually advantageous to convert the complex system in (2.5) into a real representation model. It can be easily shown that (2.5) can be rewritten into an equivalent real system as

$$\mathbf{y} = \mathbf{H}\mathbf{D}\mathbf{x} + \mathbf{v} \tag{2.13}$$

where

$$\mathbf{y} = \begin{bmatrix} \Re\{\tilde{\mathbf{y}}\}\\\Im\{\tilde{\mathbf{y}}\} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} \Re\{\tilde{\mathbf{x}}\}\\\Im\{\tilde{\mathbf{x}}\} \end{bmatrix}, \mathbf{v} = \begin{bmatrix} \Re\{\tilde{\mathbf{v}}\}\\\Im\{\tilde{\mathbf{v}}\} \end{bmatrix}$$
(2.14)

$$\mathbf{H} = \begin{bmatrix} \Re\{\tilde{\mathbf{H}}\} & -\Im\{\tilde{\mathbf{H}}\}\\ \Im\{\tilde{\mathbf{H}}\} & \Re\{\tilde{\mathbf{H}}\} \end{bmatrix}, \mathbf{D} = \begin{bmatrix} \Re\{\tilde{\mathbf{D}}\} & -\Im\{\tilde{\mathbf{D}}\}\\ \Im\{\tilde{\mathbf{D}}\} & \Re\{\tilde{\mathbf{D}}\} \end{bmatrix}$$
(2.15)

Then the equivalent ML detection problem for the real model can be written as

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \chi^{2N}}{\arg \min} \| \mathbf{y} - \mathbf{H} \mathbf{D} \mathbf{x} \|^2$$
(2.16)

where set $\chi = \{-\sqrt{C} + 1, ..., -3, -1, 1, 3, ..., \sqrt{C} - 1\}$, *C* is the QAM constellation size. Each element of this real set can be transformed into a positive integer via the following linear transformation:

$$\mathbf{z} = \frac{\mathbf{x} + (\sqrt{C} - 1)}{2} \tag{2.17}$$

The norm 2 term in 2.16 can be simplified using the above linear transformation as follows:

$$\| \mathbf{y} - \mathbf{H}\mathbf{D}\mathbf{x} \|^{2} = (\mathbf{y} - \mathbf{H}\mathbf{D}(2\mathbf{z} - (\sqrt{C} - 1)))^{T}(\mathbf{y} - \mathbf{H}\mathbf{D}(2\mathbf{z} - (\sqrt{M} - 1)))$$
$$= \mathbf{y}^{T}\mathbf{y} + (2\mathbf{z} - (\sqrt{C} - 1))^{T}\mathbf{D}^{T}\mathbf{H}^{T}\mathbf{H}\mathbf{D}(2\mathbf{z} - (\sqrt{C} - 1)) - 2\mathbf{y}^{T}\mathbf{H}\mathbf{D}(2\mathbf{z} - (\sqrt{C} - 1))$$

when substituting the simplified norm 2 term back into (2.16) and removing the terms that do not depend on \mathbf{z} , the ML problem is reformulated to the following optimization problem:

$$\hat{\mathbf{z}} = \underset{\mathbf{z}\in\Omega^{2N}}{\arg\min}f(\mathbf{z}) = \frac{1}{2}\mathbf{z}^{T}\mathbf{Q}\,\mathbf{z} + \mathbf{b}^{T}\mathbf{z}$$
(2.18)

where set $\Omega = \{0, 1, 2, 3, .., \sqrt{C} - 1\}$, $\mathbf{Q} = \mathbf{D}^T \mathbf{H}^T \mathbf{H} \mathbf{D}$, $\mathbf{b} = -\mathbf{D}^T \mathbf{H}^T (\mathbf{y} + (\sqrt{C} - 1)\mathbf{H}\mathbf{D}\mathbf{1})/2$, and $\mathbf{1}$ is a column vector of ones and length 2N. Table 2.1 shows the elements of the two sets (χ and Ω) for various QAM constellation levels. Therefore, the exhaustive search of all possible transmitted vectors in (2.16) becomes an IQP problem, as in (2.18).

Modulation level	\sqrt{C}	X	Ω
4-QAM	2	$\{-1,1\}$	$\{0,1\}$
16-QAM	4	$\{-3, -1, 1, 3\}$	$\{0, 1, 2, 3\}$
64-QAM	8	$\{-7, -5, -3, -1, 1, 3, 5, 7\}$	$\{0, 1, 2, 3, 4, 5, 6, 7\}$
256-QAM	16	$\{-15, -13,, 13, 15\}$	$\{0, 1, 2, 3,, 14, 15\}$

Table 2.1 Elements of χ and Ω Sets

2.4 Standard Branch and Bound Algorithm for Solving IQP

Branch and Bound Algorithm is a general search tree-based algorithm [9] that can be used to find the exact solution of combinatorial optimization problems or any NPhard problems, such as ML [10]. It is based on solving a relaxed constraint problem in a recursive way until either its integer solution is found, or it is proven that the problem has no optimal integer solution. The BB algorithm consists of three major rules [10]: the branching rule, the bounding rule, and the search strategy rule. The branching rule divides the solution set into several non-overlapping subsets, which helps to narrow the search and prune several nodes in the tree. The bounding rule determines whether to continue branching certain nodes in the search tree or not. This means that, in the minimization problem, such as (2.18), whenever a node has a cost function that is greater than any known upper bound, this node is pruned from any further expansion because it cannot provide any better solution. Note also that in the minimization problems, BB algorithm has non-decreasing node cost along any path from the root node to the leaf node; that is, any parent node should have a cost function that is no greater than its child node [10]. The search strategy rule determines the sequence in which the nodes of the tree should be explored. Though there have been several strategies presented in relevant literature, we focus mainly on the Breadth First (BF) strategy [69], which is a search strategy that visits the nodes of the tree in a level-by-level manner, meaning that it does not explore any nodes from level L of the tree until it finishes exploring nodes from level L - 1. We focus on this strategy because it suits our proposed algorithms in this thesis, specifically in this chapter and the next one.

The detailed procedures of using BB to solve problem (2.18) are as follows: the BB algorithm starts with solving a relaxed version of problem (2.18), removing the integer constraints on the variables and allowing them to be included in the real set. This process makes the problem much easier to solve as it becomes convex, provided that \mathbf{Q} is non-negative definite. Thus problem (2.18), after relaxing the constraints, can be expressed as:

$$\min_{\mathbf{z}} \{\frac{1}{2}\mathbf{z}^{\mathbf{T}}\mathbf{Q}\,\mathbf{z} + \mathbf{b}^{\mathbf{T}}\mathbf{z}\}$$
subject to $\underline{\mathbf{0}} \leq \mathbf{z} \leq (\sqrt{C} - 1)\underline{\mathbf{1}}$

$$(2.19)$$

where $\underline{\mathbf{0}}$ represents $2N \times 1$ vector of all zeros and the constraints $\underline{\mathbf{0}} \leq \mathbf{z} \leq (\sqrt{C} - 1)\underline{\mathbf{1}}$ represents the box constrains of all elements of \mathbf{z} , that is, each element(symbol) of \mathbf{z} is lower bounded by 0 and upper bounded by $\sqrt{C} - 1$. This form of convex optimization problem is well known and can be solved iteratively by various algorithms, such as the Interior Point (IP) method [70], [71], or the Active Set (AS) method [70], though it was shown in [71] that the IP method is more suitable for such problems, especially when the size of the problem is large. Solving (2.19) provides a 2N dimensional solution vector $\mathbf{z}^{(0)} = [z_1^{(0)}, \ldots, z_{2N}^{(0)}]^T \in \mathbb{R}^{2N}$ and a scalar cost function value $f^{(0)}$, where superscript (0) refers to node 0 in the BB search tree as depicted in Fig. 2.2. If all elements of $\mathbf{z}^{(0)}$ satisfy the integer constraints, then $\mathbf{z}^{(0)}$ is the optimum solution for problems (2.19) and consequently an optimum integer solution for (2.18) and the BB search tree is terminated. Otherwise, $f^{(0)}$ is a lower bound to the optimal

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cost of problem (2.18) and the branching is needed on the fractional variable which splits the problem (2.19) into two subproblems by adding two mutually exclusive and exhaustive constraints, as in (2.20) and (2.21). The new subproblems are called children node problems and the original problem is called the parent node problem. The new constraints divide the feasible set of the parent node into two disjoint subsets:

$$\begin{array}{ll} \underset{\mathbf{z}}{\operatorname{minimize}} & \{\frac{1}{2}\mathbf{z}^{T}\mathbf{Q}\,\mathbf{z} + \mathbf{b}^{T}\mathbf{z}\}\\ \text{subject to} & \mathbf{lb} \leq \mathbf{z} \leq \mathbf{ub} \\ & z_{i} \leq \left\lfloor z_{i}^{(0)} \right\rfloor \\ \\ \underset{\mathbf{z}}{\operatorname{minimize}} & \{\frac{1}{2}\mathbf{z}^{T}\mathbf{Q}\,\mathbf{z} + \mathbf{b}^{T}\mathbf{z}\}\\ \text{subject to} & \mathbf{lb} \leq \mathbf{z} \leq \mathbf{ub} \\ & z_{i} \geq \left\lfloor z_{i}^{(0)} \right\rfloor \end{array}$$
(2.21)

where where z_i is called the branching variable at index i, where $i \in \{1, 2, ..., 2N\}$, and $\lfloor z_i^{(0)} \rfloor$ ($\lceil z_i^{(0)} \rceil$) denotes the largest (smallest) integer smaller (greater) than or equal to $z_i^{(0)}$. Solving these new subproblems return ($\mathbf{z}^{(1)}$, $f^{(1)}$) and ($\mathbf{z}^{(2)}$, $f^{(2)}$) for nodes 1 and 2 respectively. Subproblems (2.20) and (2.21) are also solved in the same way as problem (2.19) using interior point algorithm. If the solution to these subproblems does not satisfy the integer constraint, each of them will be branched into two more subproblems and the process of branching will continue in the order shown in Fig. 2.2 until the optimal integer solution is found.

The advantage of the BB search algorithm is that it does not search the complete tree, as some subtrees are pruned according to these rules, 1) the node is pruned (and hence the subtree below it) whenever its solution satisfies the integer constraints, and 2)the node is pruned (and hence the subtree below it) whenever its cost value is greater than a known upper bound, $f^{(up)}$. Every time an integer solution is found, a corresponding upper bound is calculated and the minimum bound is kept as the so-far best upper bound". When the upper bound is properly chosen, many subtrees will be pruned and, hence, the computational complexity will be significantly reduced. A summary of the BB algorithm for solving IQP problem is presented in Table 3.1


Fig. 2.2. Representation of Breath First Branch and Bound search tree

2.4.1 Interior Point Algorithm for Node Problem

The Interior Point (IP) method is an algorithm that finds a point where the Karush-Kuhn-Tucker (KKT) conditions hold. It does this through successive descent iterations, where each iteration is a Newton-like step. In the problem at hand, (2.19), IP solves a simple bounded convex optimization problem of size N, where the objective function is quadratic and the constraints are only subjected to lower and upper bounds. In our formulation, each variable in the optimization problem is lower bounded by zero and upper bounded by $\sqrt{C} - 1$. A summary of the IP algorithm that solves (2.19) can be found in [71], [72].

In practice, the interior-point method converges in a number of iterations which is almost a constant, independent of the problem dimension [73].

- 1 Initialize a LIST with the root node problem
- 2 Initialize upper bound $f^{(up)} = \infty$
- 3 WHILE (LIST is not empty) DO
- 4 Pop a sub-problem from the LIST and solve it to get $\mathbf{z}^{(j)}$ and $f^{(j)}$, where j is the node number, j = 0, 1, 2, ...
- 5 **if** $f^{(j)} > f^{(up)}$ or $\mathbf{z}^{(j)}$ is infeasible,

delete the current problem from the LIST

elseif $\mathbf{z}^{(j)}$ is satisfying all integer constraints,

update the "best so far" integer solution as $\mathbf{z}^{(j)}$ and the

"best so far" upper bound as $f^{(j)}$ and then

delete the current problem from the LIST

else branch the current problem into two new

sub-problems and push them on the LIST, and then

delete the current problem from the LIST

6 END WHILE

2.4.2 Complexity Analysis

Each node of the BB tree requires solving a quadratic programming problem of the form of (2.19) with size 2N, and solving a quadratic programming problem requires an iterative algorithm, such as the IP algorithm mentioned in section 2.4.1. From [71] and [70], each iteration of the IP method boils down to solving a system of linear equations, where it is required to perform a matrix inversion of the same size in every iteration. Therefore, the complexity of one iteration is in the order of $O(N)^3$). We focus here on the number of multiplication operations as a measure for the computational complexity. Thus, with n iterations, the complexity of one node rises to the order of $O(nN^3)$. In practice, the interior point algorithm converges in a number of iterations which is almost a constant, independent of the problem dimension [73]. It is a complexity advantage, especially when N is large. In considering the total number of visited nodes in the BB search tree as N_v , the total expected complexity for detecting one received vector becomes $O(N_v nN^3)$. It is important to note that the only variable that can change in the complexity formula is the number of visited nodes, N_v , which is estimated through simulation experiments. In the SOFDM system, N is usually large, in the order of 32, 64, 256, 1024, ..., which means that the complexity could be very large, though it is not exponential like ML or SD [32]. In the next section and the next chapter, we propose techniques to further reduce the computational complexity per each received vector.

2.4.3 Simulation Results

The simulation experiments in this section were conducted to investigate the performance of SOFDM when BB is applied. It is assumed that the QPSK and QAM data symbols are transmitted on 32 OFDM carriers across complex Gaussian multipath channels of length $L_{ch} = 8$ with perfect knowledge of the channel at the receiver. To prove the optimality of the BB algorithm on solving an IQP problem, the BER performance of SOFDM using BB has been compared to ML and SD algorithms. We have investigated the cases of N = 4 and N = 8 only, as it is computationally prohibitive to simulate ML and SD algorithms with a larger N, such as 16, 32, especially with higher QAM levels. It can be observed from Figs 2.3 and 2.4 that the performance of the SOFDM system using the BB algorithm agrees with the performance of the conventional optimum algorithms, SD and ML.

Fig. 2.5 shows the computational complexity of BB in terms of multiplication operations at low SNR (SNR = 10dB). It shows that the complexity of BB algorithm at various QAM levels grows slowly compared to the exponential growth of SD and

ML algorithms. Although it is not simulated here, it is expected that the complexity of standard BB grows exponentially when N becomes large.

Fig. 2.6 shows the performance of QPSK SOFDM, at N = 32 OFDM carriers, using the BB algorithm compared to the performance of the linear MMSE detector. It can be discerned from the figure that there is a huge improvement in diversity and BER performance when compared to the non-spread and MMSE spread cases.



Fig. 2.3. BER performance of 16QAM SOFDM using BB, SD, and ML : (a) N = 4, (b) N = 8



Fig. 2.4. BER performance of SOFDM (N = 4) using BB, SD, and ML : (a) 64 QAM, (b) 256 QAM



Fig. 2.5. Computational complexity comparison between ML, SD and BB when used for SOFDM detection at SNR = 10dB



Fig. 2.6. BER performance of SOFDM using standard BB Algorithm (N=32)

2.5 Reduced Complexity Search Algorithm

The two major factors that contribute to the complexity of the BB algorithm are the number of visited nodes by the search tree and the complexity pertaining to each node of the tree, which is basically the iterative solution of the constrained optimization problem using the IP algorithm (section 2.4.1). Our proposed techniques in this chapter aim at reducing the computational complexity of the BB algorithm by reducing the number of visited nodes. To achieve this goal, we applied two strategies. One strategy is based on applying preprocessing procedures prior to the BB algorithm. The other strategy is to work directly inside of the BB search tree and perform some approximations to simplify the computations.

2.5.1 Complexity Reduction Using Preprocessing

One of the important parameters that speeds up the BB search tree is the good choice of the starting upper bound, $f^{(up)}$. To get a good estimate of $f^{(up)}$, it is required to have a good initial integer solution. In the problem at hand, the linear MMSE solution is a good choice for estimating the initial solution of the symbols vector, \mathbf{z} , because it has a linear complexity and gives a good estimate, especially at a high SNR. Therefore, in this strategy, we propose to use the linear MMSE as a suboptimal detector prior to applying the standard BB algorithm as shown in Fig. 2.7. The point of this proposal is to have an initial integer solution to all variables so that we can help the BB search algorithm avoid this step, which may cost several node computations. In fact, finding the first integer solution costs the standard BB algorithm a large number of nodes to be solved on average. Thus, the MMSE detected symbols from (2.10) are used to estimate the starting upper bound of the BB search tree as follows:

$$f^{(up)} = \| \mathbf{y} - \tilde{\mathbf{H}} \tilde{\mathbf{D}} \hat{\tilde{\mathbf{x}}}_{mmse} \|^2$$
(2.22)

This initial f^{up} gets updated inside the search tree as soon as a better integer solution is found (see section (2.4)). The advantages of using this initial upper bound are: 1) it is a one-time calculation needed for the whole tree, and 2) it works as a tight bound, especially at a high SNR regime, which results in the pruning of a large number of nodes. For instance, in one simulation experiment of QPSK SOFDM with N = 32 at



Fig. 2.7. BB with MMSE as a preprocessing step

SNR = 20 dB, the number of visited nodes declined from around 90000 to 80 nodes.

This preprocessing step does not seem to prune the search tree nodes as efficiently at low SNR as it does at high SNR. This is due to the fact that at low SNR, symbols are erroneously estimated, and as a consequence, the MMSE bound becomes inaccurate. It can also be said that, on the one hand, this preprocessing step provides a noticeable complexity reduction at high SNR, but on the other hand, it makes the detection complexity an SNR dependent.

2.5.2 Complexity Reduction by Forcing BB to Stop at an Intermediate Level of the Search Tree

The motivation for this proposal comes from several points that are related to the features of the standard BB algorithm. As the BB search proceeds down the tree, some nodes are pruned and some nodes are further explored but diverge from the optimum integer solution. The rest of the nodes are also explored but converge gradually to the optimum integer solution. With this and the availability of the continuous solutions in every node in mind, it can naturally be suggested that a suboptimal integer solution can be obtained at each level of the search tree. This can be achieved by applying a hard decision decoding (i.e. quantization to the nearest integer set Ω) to the most converged solution in that level of the tree.

This process of quantization can occur at any level of the tree between the root

node (level L = 0) and the end level of the tree. When the quantization is performed at level L, all the steps prior to this level follow the rules of the standard BB (see section 2.4). The steps are as follows:

- 1) Predefine the level L at which BB search tree should stop.
- 2) Follow the standard BB algorithm until level L.
- 3) At level L, find $\mathbf{z}^{(j)}$ and $f^{(j)}$ for all j live nodes.
- 4) Choose $\mathbf{z}^{(j)}$ that corresponds to the minimum cost function $f^{(j)}$.

5) Quantize the chosen $\mathbf{z}^{(j)}$ to the nearest integer constellation set Ω (see Table 2.1). Let the number of the live nodes in the stopping level equal m. The solution and the objective function values of these nodes in this level are $\mathbf{z}^{(j)}$ and $f^{(j)}$, respectively, where j = 1, ..., m. Thus, the suboptimal integer solution at the level L is

$$\mathbf{z} = \mathcal{Q}(\mathbf{z}^{(i)})$$

$$i = \underset{j}{\operatorname{argmin}}(f^{(j)}) \quad j = 1, \dots, m$$
(2.23)

A special case of the above proposed technique occurs when the quantization is performed at level 0, that is, at the root node problem (2.19). In this case, the detection problem is reduced to a Quadratic Programming detector (QP) [74], [75], [76]. Then the suboptimal integer solution is thus expressed as

$$\mathbf{z} = \mathcal{Q}(\mathbf{z}^{(0)}) \tag{2.24}$$

2.5.3 Simulation Results

In this section, we evaluate bit error rate (BER) performance of the SOFDM using the reduced complexity BB technique with different QAM modulations and OFDM subcarriers. The SOFDM symbols (N) are transmitted across a complex Gaussian multipath channel of length $L_{ch} = 8$. It is also assumed that the channel is perfectly known at the receiver and the algebraically structured Vandermonde spreading matrix [22] is used throughout the simulation unless otherwise mentioned. The solutions to the node problems of the BB algorithm are obtained through the use of the MAT-LAB function "quadprog" by specifying the interior point method in the function's options.

The performance of the SOFDM using BB with MMSE preprocessing is shown in Fig. 2.8 when 32 subcarriers are used in the QPSK SOFDM system. This figure shows that using BB with MMSE preprocessing keeps the optimum performance and the frequency diversity as the standard BB (which was shown to provide the ML performance in section 2.4.3). The advantage of using MMSE preprocessing becomes apparent with regards to computational complexity. Table 2.3 shows the number of visited nodes (N_v) required by both the standard BB and the BB with the preprocessing step (at N = 32 carriers), where it clearly manifests the complexity saved at moderate to high SNR. The drawback, however, is that the complexity becomes a SNR dependent.

Though the MMSE as a preprocessing step with BB shows better complexity performance compared to the standard BB, it can still become computationally expensive when N grows large, such as when N = 128 and 256. Therefore, the second proposed technique, based on performing quantization at certain levels of the BB tree, can provide more flexibility as well as a trade-off between BER performance and complexity. Fig. 2.9 shows the BER performance of QPSK SOFDM for various values of L when N = 32 subcarriers. This figure demonstrates the fact that the performance increases as the quantization is implemented at a deeper level of the BB tree. For instance, BER performance of BB at L = 16 is better than that at L = 5, and both are superior than the case of L = 0. Note also that the performance at the root node level (L = 0) is better than the MMSE performance. Fig. 2.10 demonstrates the same performance improvement as the level of BB increases, but for 16QAM symbol mapping.

Due to the fact that as the level L gets deeper in the tree, more nodes in the BB tree are explored and consequently, more complexity is experienced. Thus, choosing L = 0 is the best choice for the SOFDM when a large N is adopted. Performing quantization at L = 0 attains better performance than the MMSE and LML [22]. Fig. 2.11 shows the improvement of BB at L = 0 compared to the MMSE and LML techniques for the 16QAM modulation SOFDM symbols. Fig. 2.12 illustrates further the BER performance improvements due to quantization at the root node level for various QAM modulations at 256 OFDM subcarriers. Fig. 2.13 shows the corresponding complexity comparison.

Fig. 2.14 reveals that in the case of full spread OFDM, the choice of spreading matrices does affect BER performance, especially at a high SNR regime, which is unlike the case of MMSE [22].



Fig. 2.8. BER of QPSK SOFDM with BB and MMSE as a preprocessing step (N=32 OFDM carriers)

Table 2.3 Complexity in terms of Number of Visited Nodes for QPSK SOFDM with ${\cal N}=32$

method	5dB	15dB	20dB
Standard BB	150000	100000	90000
MMSE-BB	40000	950	80
BB with $L = 0$	1	1	1
BB with $L = 5$	60	20	15



Fig. 2.9. BER of QPSK SOFDM at various levels of the BB tree (N=32)



Fig. 2.10. BER of 16QAM SOFDM at various levels of the BB tree (N=32)



Fig. 2.11. BER performance of 16QAM SOFDM using MMSE, LML, and BB at L = 0, (N = 256)



Fig. 2.12. BER performance of various QAM SOFDM using BB at L = 0, (N = 256)



Fig. 2.13. Complexity comparison of BB at L = 0 for various QAM SOFDM (N = 256)



Fig. 2.14. BER performance comparison of 16QAM SOFDM with BB at L = 0 and MMSE with various spreading matrices (Hm: Hadamard spreading, VD: Vandermonde spreading, and DFT: DFT spreading)

3. A CONTROLLED SIZE SEARCH TREE ALGORITHM FOR MIMO OFDM SYSTEMS

Most of the optimal or near-optimal detection techniques that have been proposed in the literature for small MIMO systems [77], [31], [78] become problematic when extended to serve large-scale MIMO systems. This is partly because the incurred complexity is high, especially when a higher QAM modulation level is used. In this chapter, we continue working with the IQP and BB algorithms for signal detection (see section (2.5)). However, the focus here is on their application to MIMO OFDM signal detection, specifically, in MIMO OFDM systems with a large number of antennas at both the transmitter and receiver (the so-called *Large-Scale* MIMO systems). We exploit the similarity between the large MIMO system model and the SOFDM model in (2.5), where the detection problem in both systems can be considered as a large dimensional problem.

In chapter 2, in order to accommodate a large number of jointly detected symbols with reduced complexity, we proposed a MMSE preprocessing step and an early termination of the BB search tree at level L. Furthermore, in this chapter, we adopt the concept of the M-algorithm (widely used in QRD-M [35] MIMO detection problems) in the BB search tree for further complexity reduction. The goal is to make the size of the BB search tree controlled in both depth, through parameter L, and in width, through parameter M. This accelerates the computations and provides flexible tradeoffs between complexity and performance. The advantages of this proposal are: (*i*) complexity is flexible, as it depends on two main parameters, the selected depth and width of the search tree, (*ii*) complexity is almost independent of the selected modulation order, and *iii*) this detector can be efficiently extended to provide the soft information represented in Log Likelihood Ratio (LLR). In this chapter, we focus on systems employing a large MIMO in conjunction with OFDM modulation in a spatial multiplexing setup and frequency selective fading channel.

3.1 System Model

Consider a MMO OFDM system with n_t transmit antennas and n_r receive antennas as shown in Fig. 3.1, with a single OFDM modulator per antenna. At the transmitter side, the information is generated in the source and mapped into symbols of a different alphabet (QPSK, 16QAM, 64QAM, etc). The mapped complex symbols are then demultiplexed into n_t separate data streams. Each stream is subjected to OFDM modulation, after serial-to-parallel conversion, using N points IFFT. The generated time domain OFDM symbol from the i^{th} transmit branch is subjected to a parallel-to-serial conversion and cyclic prefix (CP) insertion with a length greater than or equal to the length of the channel impulse response. This is done to avoid inter-symbol-interference (ISI) impairments. The wireless channel is assumed in this work as a quasi-static frequency selective fading channel [65], meaning the channel remains constant during the transmission of one OFDM symbol (or block). The channel impulse response between the i^{th} transmit antenna and the j^{th} receive antenna is a frequency selective with L_{ch} channel paths, that is, $\mathbf{h}_{j,i} = [h_{j,i,0}, \ldots, h_{j,i,L_{ch}-1}]$. Each channel path is modeled as an independent complex Gaussian random variable with zero mean and unity variance. Let the frequency domain transmitted signal sequence from the i^{th} transmit antenna represented by $\mathbf{x}_i = [x_{i,1}, \ldots, x_{i,k}, \ldots, x_{i,N}],$ where k = 1, 2, ..., N represents the k^{th} OFDM subcarrier. The frequency domain received data symbols at the j^{th} receive antenna can be expressed as

$$\mathbf{y}_j = \sum_{i=1}^{n_t} H_{i,j} \mathbf{x}_i + \mathbf{v}_j \tag{3.1}$$

where $H_{i,j}$ is an $N \times N$ diagonal matrix composed of the DFT values of the channel between the i^{th} transmit antenna and the j^{th} receive antenna and \mathbf{v}_j represents the AWGN vector of $N \times 1$ dimension at the j^{th} receive antenna with variance σ^2 per each element. Eq. (3.1) shows that the received vector \mathbf{y}_j at the j^{th} receive antenna is a superposition of the transmit signals from all of the n_t transmit antennas.

To obtain the per carrier MIMO OFDM received signal vector model from all transmit antennas, let the signal transmitted on the k^{th} subcarrier from all antennas be represented as an n_t dimensional vector $\tilde{\mathbf{x}}_k = [x_{1,k}, x_{2,k}, \ldots, x_{n_t,k}]^T \in \mathbb{C}^{n_t \times 1}$, where, for instance, $x_{1,k}$ is the transmitted symbol from antenna 1 at the k^{th} OFDM subcarrier and $(.)^T$ is the transpose operation. Similarly, after the CP removal and FFT operation, the received signal at all receive antennas can also be expressed as an n_r dimensional vector $\tilde{\mathbf{y}}_k = [\mathbf{y}_{1,k}, \ldots, \mathbf{y}_{n_r,k}]^T \in \mathbb{C}^{n_r \times 1}$. Thus, given the knowledge of the Channel State Information at the receiver (CSI), the MIMO OFDM signal model is

$$\tilde{\mathbf{y}}_{k} = \mathbf{H}_{k}\tilde{\mathbf{x}}_{k} + \tilde{\mathbf{v}}_{k}$$

$$\tilde{\mathbf{H}}_{k} = \begin{bmatrix} H_{1,1,k} & \cdots & H_{1,n_{t},k} \\ \vdots & \ddots & \vdots \\ H_{n_{r},1,k} & \cdots & H_{n_{r},n_{t},k} \end{bmatrix}, \tilde{\mathbf{v}}_{k} = \begin{bmatrix} v_{1,k} \\ \vdots \\ v_{n_{r},k} \end{bmatrix}$$
(3.2)

where $H_{j,i,k}$ is the k^{th} DFT value of the channel $\mathbf{h}_{i,j}$, $\tilde{\mathbf{x}}_k = [x_{1,k}, \dots, x_{n_t,k}]^T$ is $n_t \times 1$ transmitted vector from n_t antennas, $\mathbf{y}_k = [y_{1,k}, \dots, y_{n_r,k}]^T$ is $n_r \times 1$ received vector at all n_r antennas and $\tilde{\mathbf{v}}_k$ is the n_r dimensional vector represents zero mean complex AWGN with covariance matrix $E[\tilde{\mathbf{v}}_k \tilde{\mathbf{v}}_k^H] = \sigma^2 I_{n_r}$, where $(.)^H$ is the hermitian operation. Note that the channel state information is available at the receiver, but not at the transmitter. Consequently, the transmit power is equally allocated among all transmit antennas. This chapter treats spatial multiplexing MIMO, where independent data streams are mapped to distinct OFDM symbols and are transmitted simultaneously from transmit antennas.



Fig. 3.1. MIMO OFDM system block diagram

3.2 Controlled Size BB Search Algorithm

3.2.1 Formulation of the Problem

The system model in (3.2) is similar to the model of the SOFDM system in (2.5), because both of them are interference channel models. Thus the ML problem of (3.2)can be expressed as

$$\hat{\tilde{\mathbf{x}}}_k = \operatorname*{argmin}_{\tilde{x}_k \in \tilde{\chi}^{n_t}} \| \tilde{\mathbf{y}}_k - \tilde{\mathbf{H}}_k \tilde{\mathbf{x}}_k \|_2^2$$
(3.3)

where $\tilde{\chi}^{n_t}$ is the set of all possible n_t -dimensional complex candidate symbol vectors of the n_t transmitted vector at the k^{th} OFDM tone, $\tilde{\boldsymbol{X}}_k$. And similar to (2.13), (3.2) can be rewritten into an equivalent real system as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \tag{3.4}$$

where

$$\mathbf{y}_{k} = \begin{bmatrix} \Re\{\tilde{\mathbf{y}}_{k}\}\\ \Im\{\tilde{\mathbf{y}}_{k}\} \end{bmatrix}, \quad \mathbf{x}_{k} = \begin{bmatrix} \Re\{\tilde{\mathbf{x}}_{k}\}\\ \Im\{\tilde{\mathbf{x}}_{k}\} \end{bmatrix}, \quad \mathbf{v}_{k} = \begin{bmatrix} \Re\{\tilde{\mathbf{v}}_{k}\}\\ \Im\{\tilde{\mathbf{v}}_{k}\} \end{bmatrix}$$
(3.5)

$$\mathbf{H}_{k} = \begin{bmatrix} \Re\{\tilde{\mathbf{H}}_{k}\} & -\Im\{\tilde{\mathbf{H}}_{k}\}\\ \Im\{\tilde{\mathbf{H}}_{k}\} & \Re\{\tilde{\mathbf{H}}_{k}\} \end{bmatrix}$$
(3.6)

Thus, the equivalent ML detection problem for the real model can also be written as:

$$\hat{\mathbf{x}}_{k} = \underset{\mathbf{x}_{k} \in \chi^{2n_{t}}}{\arg\min} \| \mathbf{y}_{k} - \mathbf{H}_{k} \mathbf{x}_{k} \|^{2}$$
(3.7)

where set $\chi = \{-\sqrt{C}+1, ..., -3, -1, 1, 3, ..., \sqrt{C}-1\}$, *C* is the QAM constellation size. Following the same steps in section (2.3), The MIMO OFDM detection problem can be formulated as

$$\hat{\mathbf{z}}_{k} = \underset{\mathbf{z}_{k} \in \Omega^{2n_{t}}}{\arg\min} f(\mathbf{z}_{k}) = \{\frac{1}{2}\mathbf{z}_{k}^{T}\mathbf{Q} z_{k} + \mathbf{b}^{T}\mathbf{z}_{k}\}$$
(3.8)

where set $\Omega = \{0, 1, 2, .., \sqrt{C} - 1\}$, and

$$\mathbf{Q}_{k} = \mathbf{H}_{k}^{T} \mathbf{H}_{k}$$

$$\mathbf{b} = -\mathbf{H}_{k}^{T} (\mathbf{y}_{k} + (\sqrt{C} - 1)\mathbf{H}_{k} \mathbf{1})/2 \qquad (3.9)$$

$$\mathbf{1} = [1, 1, \dots, 1]^{T}, \text{ colomn vector of dimension } (2n_{t} \times 1)$$

As shown in Section 2.4, problem (3.8) can be solved using the BB algorithm (see chapter 2), which starts by relaxing (3.8) into the following problem:

$$\hat{\mathbf{z}}_{k} = \underset{\mathbf{z}_{k}}{\operatorname{argmin}} \{ \frac{1}{2} \mathbf{z}_{k}^{T} \mathbf{Q} \, \mathbf{z}_{k} + \mathbf{b}^{T} \mathbf{z}_{k} \}$$

$$\underline{\mathbf{0}} \le \mathbf{z}_{k} \le (\sqrt{C} - 1) \underline{\mathbf{1}}$$
(3.10)

In addition to the techniques proposed in chapter 2, we propose further complexity reduction processing that suit MIMO OFDM systems. This is discussed in the following section. For the sake of simplicity, from now on, we omit the subscript k that indicates the k^{th} OFDM subcarrier in (3.4), (3.8), and (3.10).

3.2.2 Proposed Technique

Consider a general node (n) in the BB search tree (see Fig. 2.2), with the assumption that it is not pruned. Exploring this node will result in two more nodes; we denote them as (n + 1) and (n + 2). Based on the BB rules of minimization [9], [79], (i) $f^{(n)} \leq \min(f^{(n+1)}, f^{(n+2)})$, and (ii) $S^{(n+1)} \cap S^{(n+2)} = \emptyset$ and $S^{(n+1)} \bigcup S^{(n+2)} \subseteq S^{(n)}$, where $S^{(n)}$ represents the feasible set of node (n), $(S^{(n+1)}, \text{ and } S^{(n+2)}$ are similarly defined). This means that if the optimal integer solution lies in $S^{(n+2)}$, for instance, it will not be in $S^{(n+1)}$, therefore, $f^{(n+2)} < f^{(n+1)}$. Now if we assume that node (n + 2)is not pruned, then exploring it produces two more nodes; we call them (n + 3) and (n + 4). If for instance the feasible solution lies in $S^{(n+3)}$ and not in $S^{(n+4)}$, then $f^{(n+3)} < f^{(n+4)}$. The idea here is that as we proceed down the tree and perform splitting (branching), the BB algorithm successively pushes variables to become integers in the path that leads to the optimum integer solution. This means that the values of the variables in these nodes steadily approach integers. Also, the objective function values of the nodes in this path converge steadily, while that of the other paths may diverge. Therefore, as we come closer to the node of the optimum integer solution, the absolute value of the difference between the estimated value of each variable z_i and its quantized (rounded) version becomes smaller and smaller. It follows that

$$\| \mathbf{z}^{(n+3)} - \lfloor \mathbf{z}^{(n+3)} \rfloor \|_{\infty}$$

$$\leq \| \mathbf{z}^{(n+2)} - \lfloor \mathbf{z}^{(n+2)} \rfloor \|_{\infty}$$

$$\leq \| \mathbf{z}^{(n)} - \lfloor \mathbf{z}^{(n)} \rfloor \|_{\infty}$$
(3.11)

where $\| \boldsymbol{x} - \boldsymbol{y} \|_{\infty} := max(|x_1 - y_1|, \dots |x_n - y_n|)$ and $\lfloor x \rceil$ is the rounding operation of x to the nearest integer.

Based on the above analysis, and by adopting the BF strategy in exploring the nodes of the BB tree, we propose three different ways to reduce the size of the search tree:

1) Search tree depth Reduction: which we put forth in section 2.5.2 for SOFDM system. In this proposal, we force the BB search tree to stop at a predefined level, L $(0 \le L < 2N_t)$, even if the optimum integer solution has not yet been reached. Denote the number of nodes in level L to be m. Thus, the solution and the corresponding cost function values of the nodes in this level are $\mathbf{z}_L^{(j)}$ and $f_L^{(j)}$, respectively, where j = 1, ..., m. Therefore, the near-optimum integer solution becomes the quantized (rounded) version of the solution that corresponds to the minimum cost function value in level L:

$$\mathbf{z} = \mathcal{Q}[\mathbf{z}_L^{(j)}]$$

$$j = \underset{j}{\operatorname{argmin}}(f_L^{(j)}) \quad j = 1, ..., m$$
(3.12)

2) **Search tree width Reduction**: In this proposal, we adopt the concept of the *M*-Algorithm [80], which is a breadth-first algorithm widely used in QRD-M MIMO

detection [81]. The idea is to keep the most M probable nodes that may lead to the optimum solution for further exploration while the remaining nodes are discarded (pruned). The selection criteria is based on the objective function values of these nodes as a metric.

3) Search tree depth and width Reduction: in which the depth and width reduction strategies mentioned above are now combined. As the BB search tree proceeds downwards, only M nodes per level are retained and the last level of the tree becomes L, and the near-optimal solution is found as follows:

$$\mathbf{z} = \mathcal{Q}[\mathbf{z}_L^{(j)}]$$

$$j = \underset{j}{\operatorname{argmin}} (f_L^{(j)}) \quad j = 1, \dots, M \quad , M < m$$
(3.13)

For example, when using this proposed combined reduction strategy with L = 3 and M = 2, the search tree in Fig. 2.2 is reduced to the one shown in Fig. 3.2. Note that in the sequel, we refer to our proposed algorithm as BB(L,M), where L is the stopping level of the search tree and M is the number of nodes maintained in each level.



Fig. 3.2. Reduced search BB tree

3.2.3 Interior Point Algorithm for Node Problem

As mentioned in section 2.4.1, interior point algorithm can solve a quadratic optimization problem in successive descent iterations where each iteration is a Newton-like step. This algorithm suits large problems well because the number of iterations it takes is approximately independent of the problem size [73]. The ultimate goal in using IP algorithm is to solve the relaxed QP, such as (3.10), and then quantization operation is applied to this optimum real solution to get the integer solution that is appropriate for the selected constellation. Therefore, we suggest in this section to perform early termination to the IP algorithm for the sake of speeding up computations. Normally, the last couple of iterations of IP is performed to reach a level of convergence within 10^{-4} or smaller, thus, it is logical to perform early terminations to avoid unnecessary iterations. Then the quantization operation is performed. To implement this modification, we use one of the following to intervene the standard IP algorithm:

1) Force IP iteration to terminate at a predefined number of iterations.

2) Relax the tolerance criteria so that the algorithm can terminate quickly.

Simulation results illustrate that this proposed simplification does not significantly penalize BER performance.

3.2.4 Complexity Analysis

Similar to our analysis in section 2.4.2, the complexity of one IP iteration is in the order of $O(n_t^3)$, and it is escalated to $nO(n_t^3)$ for n iterations. This in turn leads to a total complexity of $O(N_v n n_t^3)$ per each received vector, where N_v is the number of visited nodes in the BB search tree. n is usually $<< n_t$ and N_v is a function of Land M values of the BB tree. Our simulation shows that, approximately, $N_v \leq LM$. Moreover, using the early termination idea for an IP algorithm, n could be reduced to $\frac{n}{2}$ without a major loss in BER performance, as will be seen in the simulation results. The complexity of MMSE-OSIC is also in the order of more than $O(n_t^3)$ because it performs matrix inversion and orders the channel matrix columns for each symbol of the received vector. In large-scale MIMO systems, the QRD-M algorithm has two complexity challenges, one is that its complexity approaches exponential, especially when 64 and 256 QAM is used. The other complexity issue is that QRD-M requires sorting procedures in each level of the search tree for MC elements, which is very large compared to our proposed algorithm, which requires sorting only 2M elements in each level.

3.2.5 Simulation Results

In this section, we show simulation results for an uncoded MIMO OFDM system in a frequency selective block fading channel with N = 128 OFDM subcarriers, $L_{ch} = 8$ taps, which is assumed to be perfectly known at the receiver, and $n_t = n_r = 20$ for various QAM levels. We refer to our algorithm as BB(L,M).

Fig. 3.3 and Fig. 3.4 demonstrate how BB(L,M) works for 20 × 20 MIMO OFDM systems. Fig. 3.3 shows that, as expected, the deeper the stopping level, L, the better the BER performance. The smallest value of L in BB is the root node level, where L = 0. The BER performance of BB(0,1) can provide a roughly 5 dB gain over MMSE detector at 10^{-2} BER. When the stopping level is fixed, say, at 16 and the number of explored nodes per level, M, varies, we observe that as M increases from 2 to 4, a clear improvement in BER can be seen, especially at high SNR. While, on the other hand, if M increases further from 4 to 6 and then to 8, hardly any noticeable improvement in the performance can be seen, as shown in Fig. 3.4. This is the benefit of incorporating an M-Algorithm [80] in the BB search tree. It avoids exploring unnecessary nodes in the search tree while preventing major performance loss.

Fig. 3.5 shows the performance comparison between BB(L,M) and other existing techniques, such as, MMSE-OSIC [33], MMSE-chase [82], MMSE-LAS [38], and QRD-M. BB(0,1) performs better than MMSE-chase and MMSE-LAS, but worse than V-BLAST. Increasing L to 4 and restricting M to 4 makes BB(4,4) clearly outperform V-BLAST. It can also be observed that BB(8,4) can achieve lower BER than QRD-M with M = 16. Interestingly, this figure shows that QRD-M with M = 16could not achieve ML performance as is usually claimed in small MIMO systems. In other words, the minimum value of M (which makes QRD-M achieves ML performance) is equal the number of constellation points [83], [84]. To see this contrast, the results in this figure can be compared to the results of the conventional MIMO case in Fig. 3.7.

Fig 3.6 represents the plot of uncoded BER performance as a function of the number of antennas $(n_t = n_r)$ at 20 dB average SNR. It can be observed that, like MMSE-LAS [38], BB(L,M) can successfully pick up some of the diversity of the system as the number of antennas increases, though with better performance. Detectors that use MMSE, such as linear MMSE and V-BLAST, could not handle the increase in the number of antennas and therefore, this results in no increase in the BER performance as n_t increases (similar to the BPSK modulated large-scale MIMO results provided in [38]).

Predicting the ML/SD performance is prohibitively complex for large-scale MIMO systems. Therefore, to assess how well our algorithm performs w.r.t. to the ML, we compare its performance to ML/SD and QRD-M with M =16 in a small 16QAM (4×4) MIMO OFDM setup. As shown in in Fig. 3.7, we see clearly that full BB yields the ML/SD performance, while BB(8,2), which explores only 16 nodes, can get near-ML performance.

Fig. 3.8 demonstrates our suggestion that when performing early termination of the IP algorithm, no major loss in performance is experienced. For instance, the performance degradation in achieving an uncoded BER of 10^{-2} is 0 dB for 16QAM 20×20 MIMO OFDM for both cases of BB(0,1) and BB(4,4). At a BER of 10^{-3} , a loss of about 0.5 dB is incurred for BB(0,1) as a penalty for reducing the IP iterations from 9 to 4, while it is a little less than 0.5 dB for BB(4,4) when the IP iterations are reduced from 10 to 5. Note that the minimum number of IP iterations required so that no major performance loss is encountered could change depending on the constellation size.

Fig. 3.9 demonstrates the application of the BB(L,M) algorithm for different QAM modulation orders and various values of L and M when a 20 × 20 MIMO OFDM system is used. As was analyzed in section 3.2.3, the computational complexity depends mainly on the number of visited nodes (N_v) and the number of iterations of the IP algorithm per node (n). Thus, Fig. 3.10 shows the computational complexity of 20×20 MIMO OFDM when the size of the BB search tree is BB(0,1) and BB(4,4). It can be observed that for BB(0,1) the complexity of all the QAM levels considered is nearly the same.



Fig. 3.3. BER performance of 16QAM 20×20 MIMO OFDM



Fig. 3.4. BER performance for 16QAM 20×20 MIMO OFDM



Fig. 3.5. BER performance of various QAM 20×20 MIMO OFDM



Fig. 3.6. BER performance vs. number of antennas $n_t = n_r$ for 16QAM MIMO OFDM



Fig. 3.7. BER performance of 16QAM 4×4 MIMO OFDM



Fig. 3.8. The effect of early termination of the interior point algorithm on the uncoded BER performance of 16QAM 20 \times 20 MIMO OFDM



Fig. 3.9. BER performance of various QAM 20×20 MIMO OFDM



Fig. 3.10. BER performance of various QAM 20×20 MIMO OFDM

3.3 Application of the Proposed Technique for the Detection of Overloaded MIMO Systems

A MIMO system is called overloaded when the number of spatially multiplexed signals (or the number transmit antennas) is greater than the number of receive antennas, i.e., $n_t > n_r$ [85]. the MIMO channel matrix in this system becomes a fat matrix and the system becomes underdetermined. Linear detectors, such as ZF and MMSE cannot do well in an overloaded MIMO, even at high SNR, ecause they fail to exploit the available MIMO diversity [85], [86]. In general, the difficulty caused by the overloaded MIMO detection problem stems from the underdetermined system, in which the number of interference signals exceeds the number of receive antennas [87]. An overloaded system is a practical system, one that is used, for example, in a terrestrial mobile system where there is a minimal number of antennas needed at the receiver [87]. Previous work has been done to improve the detection performance of the overloaded MIMO system using Genetic Algorithm (GA) optimization, such as in [85], [87], [86]. In this section we utilize the proposed controlled search tree algorithm BB(L,M) to provide exact and near-ML performance for a small overloaded MIMO system.

We consider an overloaded MIMO system with n_t transmit and n_r receive antennas where $n_t > n_r$. The transmitted signals are assumed to be an independent multistream of data (Spatial Multiplexing) using QPSK modulation. The received signal vector $\mathbf{r} \in \mathbb{C}^{n_r \times 1}$ is

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{3.14}$$

where $\tilde{\mathbf{H}}$ is an $(n_r \times n_t)$ flat fading channel matrix whose entries are independent and identically distributed (i.i.d) complex Gaussian distributed variables. We assume perfect channel estimation is available at the receiver. \mathbf{s} denotes a transmitted signal vector of length n_t at a particular time instant. The entries of the vector $\mathbf{n} \in \mathbb{C}^{n_r \times 1}$ are i.i.d complex Gaussian random variables with zero mean and a variance of σ^2 . The same formulation procedures presented in section 3.2.1 will be applied for model (3.14).

Simulation experiments for a system of 6 transmit antennas and 3 to 4 receive antennas is used to evaluate the symbol error rate (SER) performance of this overloaded MIMO system. Fig. 3.11 shows the degraded performance resulting from MMSE detector, which is just a high error floor. On the other hand, using BB(8,4) can improve system performance and exploit some of the MIMO diversity. BB(12,M), where the stopping level is L = 12 and M is kept unrestricted, can achieve the ML performance. Fig. 3.11 also shows the SER performance for an overloaded MIMO system but for a 6 × 4 setting. Full BB algorithm achieves the exact ML performance. Similar to the previous results in section 3.2.5, the SER performance improves as the value of Lincreases from 4 to 6 and then to 8; in all cases, they outperform MMSE performance.



Fig. 3.11. QPSK SER performance of 6×3 MIMO



Fig. 3.12. QPSK SER performance of 6×4 MIMO
4. APPLICATION OF QP ALGORITHMS IN LARGE-SCALE MIMO SYSTEMS

The use of a large number of antennas at the transmitter and receiver becomes of interest in MIMO systems due to the possibility of gaining high spectral efficiencies without the need for increasing bandwidth [88], [89], [24].

In this chapter, three potential algorithms are proposed for a large-scale MIMO detection problem. They can provide near single antenna AWGN performance with only tens of antennas and with nearly constant average complexity over all modulation orders. The first algorithm is simply the conventional quadratic programming detector that was already studied in the previous chapters. We show in this chapter that it provides better performance than the LAS detector with no major increase in average complexity. We also show that its complexity does not grow significantly from a low order to a high order modulation. QP detectors are also studied in conventional MIMO systems [76], [90], [78]; however, we note a lack of performance studies in relevant literature for this type of detector in large-scale MIMO systems. Therefore, we present the QP detector's performance and complexity analysis, and point out that it is one of the detectors that exhibits large-system behavior. Thus, any other algorithm based on QP can have the same behavior in a large-scale system.

The second proposed algorithm improves the performance of the first algorithm with a minor complexity increase. The improvement is based on the use of a two-stage quadratic programming detector with a successive interference cancellation strategy that utilizes a shadow area constraint [48] to measure symbols reliability. Finally, the third algorithm uses the already proposed technique, namely a controlled size search tree algorithm, where a few nodes are explored in the BB tree based on two criteria: one reduces the depth of the BB tree and the other reduces the width of the BB tree. Although the complexity of this algorithm is still high when n_t is large at all SNRs, we were able to reduced it dramatically (although, only at high SNR regime) by applying a new pruning rule based on the duality gap between the integer problem and its relaxed problem. The contribution of this chapter can be summarized in the following points:

- We present the application of QP-based detection algorithms in a large-scale MIMO system with uncoded BER performance results that have not been reported so far. These results are reported for QPSK, 16QAM, 64QAM, and 256QAM modulations.
- In addition to the conventional QP detector, two new algorithms are proposed to enhance the performance with various complexity trade-offs. The idea of our new proposed algorithms has not been used before in conjunction with a QP detector; therefore, we point out their advantages in boosting the performance of large-scale MIMO systems.
- We show that the complexity of the proposed algorithms does not change significantly across various QAM modulation orders. In addition, there is no modification or extension needed to implement these algorithms in various QAMs.
- The performance of the three algorithms exceeds the performance of LAS and RTS algorithms at higher QAM orders with even lower complexity, especially with the first and the second algorithms.
- The computational complexity of the standard QP detector can be reduced with no major performance loss using a few interior-point iterations (e.g. two iterations when QPSK is used).
- The complexity of the proposed algorithms is analyzed and compared with other large-scale MIMO detectors.

4.1 System Model

Consider a MIMO system with a flat fading channel with n_t transmit antennas and n_r receive antennas employing spatial multiplexing (V-BLAST) transmission. At the transmitter side, the information is generated in the source and mapped to symbols of different alphabets. The mapped complex symbols are demultiplexed into n_t separate independent data streams with a transmitted signal vector $\tilde{\mathbf{x}} = [\tilde{x}_1, \ldots, \tilde{x}_{n_t}]^T \in \mathbb{C}^{n_t \times 1}$. The general MIMO channel model is

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{n}} \tag{4.1}$$

where $\tilde{\mathbf{y}} = [\tilde{y}_1, \dots, \tilde{y}_{n_r}]^T \in \mathbb{C}^{n_r \times 1}$ is the received signal vector at all n_r antennas, $\tilde{\mathbf{H}} \in \mathbb{C}^{n_r \times n_t}$ denotes the flat fading channel gain matrix whose entries are modeled as $\mathbb{C}\mathcal{N}(0, 1)$, and $\tilde{\mathbf{n}}$ represents the receiver AWGN noise vector whose entries are modeled as i.i.d $\mathbb{C}\mathcal{N}(0, \sigma^2)$. The tilde symbol in (4.1) is made to distinguish the complex model from the real model that will be shown in the next section. We assume ideal channel estimation and synchronization at the receiver end.

4.2 Formulation of the Problem

Formulation procedures similar to those done in chapter 3 are repeated except that the channel is characterized by a flat fading. Thus, the ML problem of model (4.1), which is equivalent to Euclidean distance minimization, can be expressed as

$$\hat{\tilde{\mathbf{x}}} = \underset{\tilde{\mathbf{x}} \in \tilde{\chi}^{n_t}}{\operatorname{argmin}} \| \tilde{\mathbf{y}} - \tilde{\mathbf{H}} \tilde{\mathbf{x}} \|_2^2$$
(4.2)

where $\tilde{\chi}^{n_t}$ is the set of all possible n_t -dimensional complex candidate vectors of the transmitted vector $\tilde{\mathbf{x}}$. The equivalent real system model of (4.1) is:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{4.3}$$

where

$$\mathbf{y} = \begin{bmatrix} \Re\{\tilde{\mathbf{y}}\}\\\Im\{\tilde{\mathbf{y}}\} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} \Re\{\tilde{\mathbf{x}}\}\\\Im\{\tilde{\mathbf{x}}\} \end{bmatrix}, \mathbf{n} = \begin{bmatrix} \Re\{\tilde{\mathbf{n}}\}\\\Im\{\tilde{\mathbf{n}}\} \end{bmatrix}$$
(4.4)

$$\mathbf{H} = \begin{bmatrix} \Re\{\tilde{\mathbf{H}}\} & -\Im\{\tilde{\mathbf{H}}\}\\ \Im\{\tilde{\mathbf{H}}\} & \Re\{\tilde{\mathbf{H}}\} \end{bmatrix}$$
(4.5)

In this real-valued system model, the real part of the complex data symbols is mapped to $[x_1, \ldots, x_{Nt}]$ and the imaginary part of these symbols is mapped to $[x_{Nt+1}, \ldots, x_{2Nt}]$. Now the equivalent ML detection problem of the real model is

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \chi^{2n_t}}{\operatorname{argmin}} \| \mathbf{y} - \mathbf{H} \mathbf{x} \|_2^2$$
(4.6)

where, set $\chi = \{-\sqrt{C}+1, ..., -1, 1, ..., \sqrt{C}-1\}$, *C* is the QAM constellation size. Each element of this real set can be transformed to a positive integer using the following linear transformation: $\mathbf{z} = \frac{\mathbf{x} + (\sqrt{C}-1)}{2}$. Norm 2 term in (4.6) can be simplified and the ML problem can be reformed as

$$\hat{\mathbf{z}} = \arg\min_{\mathbf{z}\in\Omega^{2n_t}} \{\frac{1}{2}\mathbf{z}^T \mathbf{Q}\,\mathbf{z} + \mathbf{b}^T \mathbf{z}\}$$
(4.7)

where

$$\Omega = \{0, 1, 2, .., \sqrt{C-1}\}$$

$$\mathbf{Q} = \mathbf{H}^T \mathbf{H}, \text{ is a symmetric positive semidefinite matrix}$$

$$\mathbf{b} = -\mathbf{H}^T (\mathbf{y} + (\sqrt{C} - 1)\mathbf{H}\underline{1})/2$$

$$\underline{\mathbf{1}} = [1, 1, ..., 1]^T, \text{ colomn vector of dimension}(2n_t \times 1)$$

$$(4.8)$$

4.3 Proposed Algorithms

4.3.1 Algorithm I: Quadratic Programming Detector

One way to solve (4.7) is to use standard QP solvers that rely on relaxing the integer constraints. Thus, problem (4.7) becomes:

argmin
_z
$$\frac{1}{2} \mathbf{z}^{\mathbf{T}} \mathbf{Q} \mathbf{z} + \mathbf{b}^{\mathbf{T}} \mathbf{z}$$
 (4.9)
subject to $\underline{\mathbf{0}} \le \mathbf{z} \le (\sqrt{C} - 1) \underline{\mathbf{1}}$

where $\underline{\mathbf{0}}$ represents $2n_t \times 1$ vector of all zeros and the constraints $\underline{\mathbf{0}} \leq \mathbf{z} \leq (\sqrt{C} - 1)\underline{\mathbf{1}}$ represents the box constraints of all elements of \mathbf{z} , i.e. each element (symbol) of \mathbf{z} is lower bounded by 0 and upper bounded by $\sqrt{C} - 1$. This form of an optimization problem is a convex QP minimization problem. A unique global continuous solution \mathbf{z}^* can be obtained using efficient interior-point solvers with reduced computational complexity [71]. The importance of using an interior-point solver is that in practice, the interior-point algorithm converges in a number of iterations that is constant, independent of the problem dimension [73]. This becomes attractive from a complexity point of view, especially when the number of antennas increases. Solving (4.9) provides a $2n_t$ dimensional solution vector $\mathbf{z}^* = [z_1^{(*)}, \ldots, z_{2N}^{(*)}]^T \in \mathbb{R}^{2n_t}$ and a scalar cost function value $f(\mathbf{z}^*)$. If all elements of $\mathbf{z}^{(*)}$ satisfy the integer constraints, then $\mathbf{z}^{(*)}$ is the optimum solution for problems (4.7) and (4.9). In general, the integer solution of (4.9) is provided by quantizing $\mathbf{z}^{(*)}$ to the nearest constellation set Ω , that is :

$$z_i = \mathcal{Q}[z_i^*], \quad i = 1, 2, \dots, 2n_t$$
 (4.10)

where, $\mathcal{Q}[.]$ is a quantization function to the appropriate constellation levels of the set Ω . In the next subsections, we propose to improve the performance of the QP detector in a large MIMO system through performing further analysis of the problem (4.9) using first, two-stage QP detection with interference cancellation, and second, the concept of the Branch and Bound search tree algorithm [9], [91], [92].

4.3.2 Algorithm II: Two-Stage Quadratic Programming Detector

The idea of this algorithm is to implement two stages of QP detection with interference cancellation to further improve the detection of the unreliable symbols (non-integer variables of \mathbf{z}^* found in (4.9)). One drawback of algorithm I is that all symbols are quantized simultaneously, irrespective of their reliabilities. Therefore, in this algorithm, a shadow area between positive integers of the constellation set, Ω , is proposed before performing quantization in (4.10). Any z_i^* that falls in this shadow area will be considered unreliable. In other words, the unreliable symbols are selected based on the following: from the solution of (4.9), the variables with fractions that are far from their nearest integers by a value greater than or equal to δ are considered noisy and, therefore, need another stage of QP detection. We denote the positions of these unreliable symbols by the set of indices \mathcal{J} . On the other hand, the variables with small fractions ($< \delta$), or those with purely integers, can be quantized and considered the optimum integer for both (4.9) and (4.7). Thus, their effects need to be canceled out so that the solution of the noisy variables can be improved. The set of indices represents the positions of these integer variables is denoted as \mathcal{I} , which can be estimated using the following criteria:

$$\mathcal{I} = \{i \in \{1, 2, \dots, 2n_t\} \mid || \mathbf{z}^* - \lfloor \mathbf{z}^* \rceil ||_{\infty} \le \delta\}$$

$$(4.11)$$

where, $\| \boldsymbol{x} - \boldsymbol{y} \|_{\infty} := max(|x_1 - y_1|, \dots |x_n - y_n|), |x|$ is the rounding operation of x to the nearest integer, and $0 < \delta < 0.5$ is a measure of how close each element from (4.9) to its nearest integer. The maximum value of δ is 0.5 because the integer feasible set Ω in (4.8) is made up of consecutive positive numbers.

Consider, for example, that the i^{th} value of \mathbf{z}^* from (4.9) is an integer or satisfies the condition in (4.11), that is, $|z_i - \lfloor z_i^* \rceil| \leq \delta$. In order to perform an interference cancellation for this symbol in the second QP stage formulation, the new modified received vector becomes $\bar{\mathbf{y}} = \mathbf{y} - x_i \mathbf{g}_i$, where $x_i = 2z_i - (\sqrt{C} - 1)$, and \mathbf{g}_i represents the i^{th} column of \mathbf{H} . Now with one symbol z_i , which is assumed known, the new reduced ML problem can be formulated with a new norm 2 term as $\| \bar{\mathbf{y}} - \bar{\mathbf{H}}\bar{\mathbf{x}} \|_2^2$, where $\bar{\mathbf{H}} = \mathbf{H}_{[i]}$ is obtained by omitting i^{th} column of matrix \mathbf{H} . Similarly, $\bar{\mathbf{x}}$ is obtained by omitting i^{th} element of \mathbf{x} . This can be generalized if more than one symbol are integers or satisfying the condition in (4.11). That is, simply replace the index i by \mathcal{I} . To obtain the reduced size QP after the interference cancellation step, the same formulation procedures that are shown to get (4.7), (4.8) can be repeated, but for $\| \bar{\mathbf{y}} - \bar{\mathbf{H}}\bar{\mathbf{x}} \|_2^2$. This yields the following QP problem:

argmin

$$\bar{z}$$
 $\frac{1}{2} \bar{\mathbf{z}}^T \bar{\mathbf{Q}} \bar{\mathbf{z}} + \bar{\mathbf{b}}^T \bar{\mathbf{z}}$
(4.12)
subject to $\underline{\bar{\mathbf{0}}} \leq \bar{\mathbf{z}} \leq (\sqrt{C} - 1) \underline{\bar{\mathbf{1}}}$

where,

$$\bar{\mathbf{Q}} = \bar{\mathbf{H}}^T \bar{\mathbf{H}}, \text{ is a symmetric positive semidefinite matrix}$$
$$\bar{\mathbf{b}} = -\bar{\mathbf{H}}^T (\bar{\mathbf{y}} + (\sqrt{C} - 1)\bar{\mathbf{H}}\underline{\mathbf{1}})/2 \qquad (4.13)$$
$$\underline{\mathbf{1}} = [1, 1, \dots, 1]^T, \text{ colomn vector of length}(2n_t - |\mathcal{I}|)$$

and $|\mathcal{I}|$ is the cardinality of the set \mathcal{I} . In order to avoid recomputing $\bar{\mathbf{Q}}$ and $\bar{\mathbf{b}}$ in (4.13), we further simplify them to be evaluated in terms of the \mathbf{Q} and \mathbf{b} as follows:

$$\bar{\mathbf{Q}} = \mathbf{Q}(\mathcal{J}, \mathcal{J})$$

$$\bar{\mathbf{b}} = \mathbf{Q}(\mathcal{I}, \mathcal{J})^T \mathbf{z}(\mathcal{I}) + \mathbf{b}(\mathcal{J})$$
(4.14)

where $\mathbf{Q}(\mathcal{I}, \mathcal{J})$ denotes the submatrix composed of rows \mathcal{I} and columns \mathcal{J} of \mathbf{Q} for sets \mathcal{I} and \mathcal{J} . Also, $\mathbf{b}(\mathcal{J})$ denotes a subvector consisting of elements of \mathbf{b} corresponding to the indices of set \mathcal{J} .

Note that unlike the conventional successive interference cancellation techniques [33], [47], this algorithm provides symbol ordering that is based on the non-integral measure, δ , that indicates how many reliable or unreliable received symbols there are. The parameter δ in (4.11) is a design parameter and needs to be optimized. When δ is chosen to be very small (e.g. $\delta < 0.1$), a large number of variables fall into the second QP stage because they don't pass the condition in (4.11). In this case, the interference cancellation cannot do much in improving detection performance of the first QP, especially at low SNR. When δ is large (e.g. $\delta > 0.4$), most of the symbols will pass the integer condition, even though they might be far from their nearest integers. With this δ , interference cancellation may improve the detection of some symbols, especially at a high SNR regime. In this algorithm, δ is optimized based on both minimum BER and complexity across various SNR using simulation experiments, since the analytical optimization seems cumbersome. We found that the optimum δ is around 0.2 to 0.3 for various QAM levels. A summary of the Algorithm II steps is shown in Table 4.1.

Table 4.1 Two-stage QP Algorithm

Input: Q, b 1 $\mathbf{z}^* = \text{quadprog}(\mathbf{Q}, \mathbf{b}) \text{ from } (4.9)$ 2Find \mathcal{I} that satisfies $\parallel \mathbf{z}^* - \lfloor \mathbf{z}^* \rceil \parallel_{\infty} \leq \delta$ 3 $\mathbf{z}(\mathcal{I}) = \mathcal{Q}[\mathbf{z}^*(\mathcal{I})]$ 4 Find set of indices \mathcal{J} 5Find $\bar{\mathbf{Q}} = \mathbf{Q}(\mathcal{J}, \mathcal{J})$, and 6 $ar{\mathbf{b}} = \mathbf{Q}(\mathcal{I}, \mathcal{J})^T \mathbf{z}(\mathcal{I}) + \mathbf{b}(\mathcal{J})$ 7 $\mathbf{z}^*(\mathcal{J}) = \text{quadprog}(\bar{\mathbf{Q}}, \bar{\mathbf{b}}) \text{ from } (4.12)$ 8 $\mathbf{z}(\mathcal{J}) = \mathcal{Q}[\mathbf{z}^*(\mathcal{J})]$ 9

4.3.3 Algorithm III: Controlled Size Branch and Bound Algorithm

The proposed algorithm in chapter 3, BB(L,M) is used here for the sake of comparison with other algorithms that are widely used for the detection of large-scale MIMO systems. In addition, for faster simulation time and a reduced number of visited nodes (hence lower computations), we further propose another approximation in conjunction with the BB(L,M) search tree. This approximation depends on the duality gap between the relaxed problem and the integer problem of any node in the tree. The duality gap is defined as the difference between the objective function values of these two problems. The key idea is that whenever this gap is small (based on some criteria), we can approximate the relaxed continuous solution to be the integer solution using the basic rounding. This adds one more pruning rule to the BB algorithm because more integer solutions will be available. Hence, it reduces the number of visited nodes significantly, especially at high SNR. Following the same notation in this section, we denote the optimum continuous solution of the relaxed problem of a node k by $\mathbf{z}^{(k)}$ and its objective function value as $f(\mathbf{z}^{(k)})$, where $k = 0, 1, 2, ..., N_v$ and N_v is the number of visited nodes in the search tree. Similarly, we denote the optimum integer solution of the same node, k, by $\mathcal{Q}[\mathbf{z}^{(k)}]$, and its objective function value as $f(\mathcal{Q}[\mathbf{z}^{(k)}])$. Thus, the approximation can be represented as

$$\mathbf{z}^{(k)} = \begin{cases} \mathcal{Q}[\mathbf{z}^{(k)}] & \text{if } |f(\mathbf{z}^{(k)}) - f(\mathcal{Q}[\mathbf{z}^{(k)}])| \le |\alpha f(\mathbf{z}^{(k)})| \\ \mathbf{z}^{(k)} & \text{otherwise} \end{cases}$$
(4.15)

where |(.)| represents the absolute value operation, and α is a small number $\in (0, 1)$. The larger the α , the lower the performance and the complexity is reduced. Note that this approximation is different from the one presented in [14] which prunes the node only if its objective value is close to the best available upper bound so far.

4.4 Complexity Analysis

The main ingredient of the computations of the QP detector is the interior-point algorithm, which finds a point where the Karush-Kuhn-Tucker (KKT) conditions hold for the optimization problem (4.9) in an iterative manner. As shown in [71] and [70], each iteration of the interior-point algorithm boils down to solving a system of linear equations where it is required to perform a matrix inversion of the same size in every iteration. Therefore, the complexity of one interior-point iteration is in the order of $O(n_t^3)$, and becomes $nO(n_t^3)$ for n iterations. In practice, the interior point converges in a number of iterations which is almost always a constant, independent of the problem dimension [73]. This is one of the reasons why the interior-point is selected for high dimensional optimization problems. From our simulation experiments, we found that when using the standard interior point algorithm, the average number of iterations required for various number of antennas is 6,7, 8, and 9, when the symbol mapping is QPSK, 16QAM, 64QAM, and 256QAM, respectively. In this work, we further reduce the number of iterations to 2, 4, 5, and 6 without major performance loss. The idea here is that the algorithm makes, in every iteration, a huge step towards convergence to the optimum continuous solution. Since the aim of the QP detector is to find the integer solution, an early termination to the algorithm, achieved by relaxing the tolerance constraints of the convergence before applying quantization step in (4.10), can speed up the convergence to the integer solution without major loss in the performance.

The second algorithm requires more computations than the first algorithm, due to the presence of the second round of QP. Fortunately, the problem size of the second QP is much smaller than the first, especially for medium to high SNR and when the δ parameter is optimized. This makes the computational complexity of algorithm I and II is nearly the same when the number of antennas becomes large. The interior-point algorithm in the second QP requires complexity in the order of $O(n(|\mathcal{J}|)^3)$. Therefore, the total complexity of algorithm II is in the order of $O(nn_t^3 + n(|\mathcal{J}|)^3)$. Based on algorithms I and II, the difference in computations for various modulation orders arises from the different number of iterations of the interior-point algorithm, which, for example, can be at most 3 times between 256QAM and QPSK cases.

Finally, as shown in chapter 3, the controlled size BB algorithm needs more computations compared to the first two algorithms because of the computations needed in every node of the small tree. Thus, the total complexity can be of the order of $O(N_v n n_t^3)$ per received vector, where N_v is the number of visited nodes in the proposed BB search tree. In large-scale MIMO systems, $n \ll n_t$ and N_v is a function of both L and M values of the tree (approximately, from simulations, $N_v \approx \text{LM}$ at low SNR, whereas $N_v \ll \text{LM}$ at high SNR).

For various QAM modulations, the complexity of the proposed algorithms does not change significantly. In fact, the small variations in complexity are due to the difference in the number of interior-point iterations required for each modulation case, which is at most 3 times between the cases of QPSK and 256QAM. This is an important advantage for the QP-based detectors compared to other algorithms in the literature of large-scale MIMO such as, RTS and R3TS [43], and Fixed Complexity SD [37], which require a large variation in complexity when the modulation order changes from low to high (e.g. it is in the order of 100 times between QPSK and 64QAM for R3TS [43], and more for FSD). As shown in [38], the complexity per received vector of MMSE-LAS is in the order of $O(n_t^3) + O(n_t^3)$; one $O(n_t^3)$ due to the MMSE initial vector, and one $O(n_t^3)$ due to the LAS procedures. Therefore, clearly the extra complexity needed by QP over MMSE-LAS mainly arises from the number of interior-point iterations, n, of QP detector. Moreover, BB(L,M) requires approximately nN_v times the complexity of MMSE-LAS.

4.5 Simulation results

In this section, we show simulation results for an uncoded large-scale MIMO system in a block flat fading channel with $n_t = n_r$ for various QAM modulation levels, assuming perfect knowledge of channel state information at the receiver. We refer to our proposed algorithms as QP for algorithm I, 2QP for the two-stage QP detector (algorithm II), and BB(L,M) for the third algorithm that uses the controlled size BB search tree. We compare our proposed algorithms with other detectors including MMSE, MMSE-OSIC, MMSE-LAS, and MIV-LAS. MIV-LAS is a LAS algorithm that uses three initial input vectors, such as matched filter (MF), zero forcing (ZF), and MMSE. Since the performance gain from using multiple symbol update LAS algorithm [30] over MMSE-LAS is small, we limit our comparison to MIV-LAS and MMSE-LAS only. In the following BER performance figures, the x-axis represents the average received SNR per received antenna in dB, and the y-axis represents the average BER resulting from more than 1000 channel realizations of Monti Carlo simulations per each SNR. For fair comparison between various detection techniques, all implementation is done using real system model shown in (4.3)

4.5.1 Optimizing δ in Algorithm II and the Number of Iterations of the Interior Point Algorithm

Figs. 4.1 and 4.2 demonstrate, in the case of QPSK modulation, that the choice of parameter δ can significantly improve the performance of a 2QP detector over the conventional QP detector. In this particular example of 32×32 , it can be said that the vale of δ between 0.25 and 0.3 provides the best performance over other values. For instance, when $\delta = 0.25$ or 0.3, 2QP has a 2 dB improvement over QP at 10^{-3} BER. Given that the size of the first QP problem is $2n_t$, the computational complexity of the second stage of QP is far below that of the first stage, especially at medium to high SNR. The size of the second QP problem decreases as the value of δ increases (see Fig. 4.2). This makes the computational complexity of the 2QP detector very close to the QP detector. For example, in the QPSK case with $n_t = 32$, at 10^{-3} BER for $\delta = 0.25$, the average size of the second stage of QP is 5 variables compared to 64 variables in the first stage. For various QAM modulations, various SNRs, and different n_t , Fig. 4.3 demonstrates that the value of $\delta = 0.25$ can be a good optimized value, and thus, it is used in the 2QP detector over the rest of the simulation results.

As we mentioned in Section 4.2, the main computational burden in a QP detector comes from the iterative interior point solver. We proposed to reduce its computations by forcing the algorithm to perform early termination, thus reducing the number of iterations. We performed simulation experiments using both QP and 2QP detectors for QPSK and 16 QAM modulations with various interior point iterations. Figs. 4.4-a and 4.4-b show that 2 and 4 iterations for QPSK and 16QAM modulations, respectively, are the minimum numbers that guarantee no major loss in BER performance. The same reduction procedures were done for 64QAM and 256QAM where the minimum number of iterations was 5 for 64QAM and 6 for 256QAM. These numbers are used in the rest of the simulation experiments.



Fig. 4.1. QPSK BER performance of a Two-stage QP detector using various values of δ



Fig. 4.2. Size of the 2^{nd} stage of a QP problem in the Two-stage QP detector at various values of δ

4.5.2 Uncoded BER Performance vs. SNR

We choose a relatively large number of antennas, such as $n_t = 32$, to demonstrate the performance of our proposed techniques. In Figs. 4.5 and 4.6, we present the



Fig. 4.3. BER performance vs. δ of the Two-stage QP detector at various QAM modulations

average uncoded BER performance for 32×32 MIMO with QPSK, 16QAM, 64QAM, and 256QAM modulations. In a 2QP detector, $\delta = 0.25$, and in the BB(L,M) detector, $\alpha = 0.01$ for QPSK, $\alpha = 0.001$ for 16QAM, and $\alpha = 0.0001$ for both 64QAM and 256 QAM. From Figs. 4.5 and 4.6, the following observations can be made:

Figs. 4.5 and 4.6 clearly show that both 2QP and BB(L,M) algorithms improve the performance of the QP detector at all displayed SNRs and at all QAM modulation orders. Interestingly, when comparing 2QP with BB(L,M), say BB(16,2), in Fig. 4.5-a, the 2QP algorithm performs better than BB(16,2) in QPSK (+ 0.5 dB at 10⁻³ BER), but then it steadily becomes worse than BB(16,2) as the modulation order increases, see Figs. 4.6 c and d. In 256QAM, for instance, 2QP is worse than BB(16,2) by 5 dB at 10⁻³ BER. As we will see in the rest of simulation results later, this may change at a larger number of antennas, such as > 40 for 16QAM and > 100 for 256QAM.

- A more detailed illustration of the BB(L,M) algorithm is shown in Fig. 4.7 for 16QAM, as an example only. It shows that as the depth of the BB search tree, L, increases, the performance increases, along with diversity. For instance, BB(16,2) outperforms BB(2,4) with 2 dB at 10⁻³ BER, with a cost of about a 4-times increase in complexity. On the other hand, increasing the width of the BB search tree cannot always provide improved performance. The same figure shows that BB(16,4) has the same BER as BB(16,6); however, the benefit is that BB(16,4) has a lower number of visited nodes.
- Fig. 4.5 a shows that LAS detectors perform better than the QP detector when QPSK is used, especially at SNR < 15 dB. It also shows that the 2QP algorithm outperforms MMSE-LAS and MIV-LAS at all displayed SNRs. Performance changes as the modulation order moved to high levels, at which point all three proposed algorithms outperform LAS algorithms with at least 4 dB at BER = 10^{-3} . Moreover, the QP algorithm, which provides an upper bound BER performance to the other two proposed techniques, when used for 256 QAM, can provide a 5 dB improvement over LAS algorithms at BER = 10^{-2} .
- Although the RTS algorithm performs better than the proposed algorithms in QPSK, with about 1 dB closer to the single antenna AWGN bound (see Fig. 4.5 a), its performance tends to deteriorate at higher QAM modulations, such as 16QAM, 64QAM, and 256QAM, especially at high SNR, as shown in Fig. 4.5 b and Figs. 4.6 c & d. The performance of RTS was improved using a hybrid of RTS and Belief Propagation (BP) (RTS-BP) in [93], but this only achieved a 1.6 dB improvement at 10⁻³ BER (see Fig. 3 in [93]), while our 2QP algorithm provides a 2 dB improvement and BB(32,4) provides a 4 dB improvement over the RTS.
- It is worth mentioning that the performance of our proposed algorithms can be further improved by combining any one of them with any of the LAS algorithms, meaning that the starting initial vector of any LAS detector can be the

vector results from QP, 2QP, or BB(L,M) detectors. The simulation results for this claim are not extensively shown here, but two examples for 2QP with LAS using QPSK, and BB(32,4) with LAS using 16QAM are depicted in Figs. 4.5-b and 4.7, respectively. The improvement of the combination over conventional 2QP and BB(32,4) are clearly shown.

Table 4.2 presents a sample of a complexity computation at relatively low SNR that is required to achieve 10^{-2} BER. The important observations are as follows: (i) there is no significant increase in the computational complexity of QP and 2QP detectors over MMSE-LAS detector; however, performance is substantially improved, especially at higher QAM modulations. For example QP has a 5 dB improvement at 256QAM, see Fig. 4.6 d. (ii) As the modulation order increases, the performance increase of 3MIV-LAS over MMSE-LAS does not pay off complexity added. And (iii) At fixed $n_t = n_r$, complexity of QP, 2QP, and BB(L,M) does not change significantly from QPSK to 256 QAM. The complexity of BB(4,4) and BB(16,2) in Table II, which is measured at a relatively low SNR (10^{-2} BER), is reduced significantly at a high SNR. For example, at an SNR that achieves 10^{-4} BER for 256QAM 64×64 , the complexity of BB(4,4) drops from 203×10^6 to 25×10^6 , and that of BB(16,2) drops from 404×10^6 to 46×10^6 . This huge reduction in the complexity is due to the efficacy of the duality gap pruning rule at high SNR.

4.5.3 Uncoded BER Performance vs. n_t

In Figs. 4.8, 4.9, 4.10, we plot an uncoded BER performance as a function of $n_t = n_r$, for various detectors at an average received SNR of 15 dB, 26 dB, and 39 dB for QPSK, 16QAM and 256QAM, respectively. We compare the proposed algorithms against MMSE-LAS, RTS, MMSE-OSIC, and QRDM. Other LAS algorithms are not simulated here because their improvement over MMSE-LAS is small, especially at higher QAM modulations. MF and MMSE are also plotted for reference. The values

	QPSK		16QAM		256QAM	
Algorithm	32×32	64×64	32×32	64×64	32×32	64×64
MMSE-LAS	0.524	4.194	0.528	4.210	0.786	6.291
3MIV-LAS	1.25	10.06	1.27	10.10	1.88	15.09
RTS	0.68	1.36	6.5	12.56	80.23	118.34
QP	0.786	6.291	1.04	8.385	1.575	12.58
2QP	0.798	6.371	1.11	9.25	1.70	14.027
BB(4,4)	8.79	69.37	12.66	103.28	25.52	203.20
BB(16,2)	17.59	136.74	25.32	203.57	50.51	404.40
FSD [36], [43]	8.59	138.74	4599.53	*	*	*

Table 4.2 Avg. Complexity in terms of # of real operations $\times 10^6$ at 10^{-2} BER.

of δ and α are the same as mentioned above. The main observations from the simulation results of this section are as follows:

- Figs. 4.8, 4.9, 4.10 demonstrate that at higher QAM orders (16QAM and 256QAM), our proposed algorithms outperform the MMSE-LAS algorithm at all displayed n_t . However, in the case of QPSK, MMSE-LAS provides better performance than QP and BB(4,4) only at $n_t \ge 30$ and $n_t \ge 40$, respectively. Note that in this particular case, as more levels are considered in the BB(L,M) algorithm, the crossover point between MMSE-LAS and BB(L,M) moves to the right of the figure (see Fig. 4.8).
- Since the inherent search strategy of RTS is more effective than LAS [36], we can see clearly from Figs. 4.8, 4.9, and 4.10 that RTS outperforms LAS at all displayed dimensions. It also outperforms our proposed algorithms when QPSK modulation is used. However, when RTS is compared to the QP-based algo-

rithms at higher QAM orders, it only outperforms our proposed techniques at a certain range of n_t s, this range gets smaller as we move to a higher modulation.

- An interesting result regarding the 2QP algorithm, across various QAM modulations, is that although it requires lower complexity than BB(L,M), it has superior performance. For example, in the QPSK modulation, it outperforms BB(4,4) and BB(16,2) when $n_t > 10$ and $n_t > 28$, respectively. At higher QAM modulations, the value of n_t at which 2QP starts to outperform BB(L,M) is increased (see Figs. 4.9, and 4.10). For example, at $n_t > 40$ and $n_t > 100$, 2QP outperforms BB(16,2) for 16QAM and 256QAM, respectively.
- We also observe a flooring behavior with respect to BB(L,M) performance. This is due to the fact that while we increase n_t = n_r, we keep the same depth of the BB tree, which is not enough to further reduce errors. This effect can be reduced if the depth of the BB is made to adaptively increase with n_t, For instance, Fig. 4.8 shows that when BB(16,2) is replaced by BB(2n_t,2), the flooring effect is reduced.
- V-BLAST successive interference cancellation with ordering, MMSE-OSIC, performs well only at smaller n_t ; using QPSK, it performs better than QP at $n_t \leq 12$; using 16QAM, it performs better than QP and 2QP at $n_t \leq 16$; using 256QAM, interestingly, it performs better than QP, 2QP, and BB(4,4) at $n_t \leq 45$; however it requires more computations. In general, MMSE-OSIC starts to exhibit a high error floor as n_t increases. This can be explained due to the larger interference generated by larger n_t . This falls line with the results showed in [38] for BPSK.
- The performance of reduced complexity search tree algorithms that are extensively studied in conventional MIMO detection problems, such as Fixed SD (FSD) [37], K-best SD, and QRDM, demonstrate poor performance in largescale MIMO systems [94]. We present here, as an example, the performance

of the QRDM algorithm, as we were able to simulate this algorithm for n_t up to 60 for both QPSK (with M=4) and 16QAM (with M=16) cases (but not for 256QAM). We show that QRDM (with M = QAM size) can provide the best performance at Nt < 10, which is the ML performance; however, as n_t gets higher, the BER performance deteriorates due to the fact that the reduced search space becomes very small compared to the ML space.

An important conclusion from the comparison of the three proposed algorithms is that at medium to high SNR with a large number of antennas (e.g. at $n_t \ge 40$) and with low modulation order, such as QPSK and 16QAM, the 2QP algorithm seems to be the winner algorithm in terms of complexity and performance, while at a higher modulation order, such as 256QAM, BB(L,M) with L \ge 16 is the best choice. Also, at medium to high SNR, but at small $n_t = n_r$, BB(L,M) is always the best choice, especially in terms of performance.

4.5.4 Turbo Coded BER Performance

In this subsection, we evaluate the turbo coded BER performance of the QP-based detectors compared to MMSE, MMSE-LAS, and RTS detectors. In this simulation, a 32×32 MIMO system is examined with 16QAM and a 1/3 rate turbo decoder of 10 iterations. ± 1 output valued vector from all detectors is fed as an input to the turbo decoder. In Fig. 4.11, the 2QP detector perform close to the RTS detector with less than 0.5 dB difference, even though the RTS performance is superior in the uncoded case. It can also be seen that turbo coded QP exceeds the coded BER performance of MMSE-LAS and MMSE with 1 dB and 2 dB, respectively. The $n_t = n_r = 32$ with 16QAM and rate-1/3 turbo coded corresponds to $32 \times 1/3 \times 4 = 42.67$ bit/sec/Hz spectral efficiency. The minimum SNR required to achieve this capacity is shown in Fig. 4.11, which is 3.25 dB. This is obtained from plotting the value of ergodic capacity expression, from [95], against average received SNR. Therefore, at coded BER 10⁻³, the 2QP and RTS detectors are about 14 dB away from the capacity.

The advantages of turbo coded QP and 2QP detectors becomes clearer at high QAM modulations, such as 256 QAM, as shown in Fig. 4.12



(a)



Fig. 4.4. Effect of reducing the interior point method iters. on the BER performance of 32×32 MIMO system (a) QP Detector (b) Two-stage QP detector







(b)

Fig. 4.5. BER performance of 32×32 MIMO (a) QPSK (b)16QAM



Fig. 4.6. BER performance of 32×32 MIMO (a) 64QAM (b)256QAM



Fig. 4.7. 16QAM BER performance using BB(L,M)



Fig. 4.8. QPSK BER performance vs. number of antennas at SNR =15 dB



Fig. 4.9. 16 QAM BER performance vs. number of antennas at SNR =26 dB



Fig. 4.10. 256 QAM BER performance vs. number of antennas at SNR =39 dB



Fig. 4.11. BER performance of 16QAM 1/3 rate Turbo coded 32×32 MIMO system



Fig. 4.12. BER performance of 256QAM 1/3 rate Turbo coded 32×32 MIMO system

5. QP-SUCCESSIVE INTERFERENCE CANCELLATION APPROACH FOR LARGE-SCALE MIMO OFDM SYSTEMS

In this chapter, we focus on a reduced complexity version of IQP, referred to as relaxed quadratic programming (QP). We exploit the formulation of this problem to propose a new low complexity successive interference cancellation (SIC) technique that suits a large-scale MIMO OFDM system in a spatial multiplexing setup. In a large-scale MIMO system, a large number of transmit and receive antennas is required, and more interferers are present; therefore, the SIC process requires more efficient detectors that can combat the error propagation issue. Moreover, computational complexity is an important issue to consider when it comes to implementing a detection technique for this large MIMO system, as most of the proposed SIC techniques require a large number of pseudo inverse computations each time a new symbol is detected, putting a huge burden on the receiver complexity. The proposed technique in this chapter formulates SIC procedures using QP formulation (QP-SIC) in a symbol-by-symbol or group-by-group manner.

Three versions of the QP-SIC technique are introduced in this chapter with various performance and complexity trade-offs. Simulation results show that the proposed schemes significantly outperform the existing SIC schemes. They also show that these proposed algorithms improve the system diversity as the number of antennas increases and it gradually approaches the AWGN single antenna performance. We refer to the three versions of QP-SIC as Algorithm I, Algorithm II, and Algorithm III. Algorithm I performs symbol-by-symbol SIC and within each iteration of symbol detection and nulling, only a linear operation is required and no matrix inversion operation is performed. Algorithm II improves the performance gained by Algorithm I via performing SIC in a grouping manner and using QP detector for each group. Finally, Algorithm III is similar to Algorithm II, though instead it performs SIC procedures only for selected groups, which in turn saves more computational complexity. In all of the aforementioned algorithms, an initial MMSE estimation is required to help in both the symbol ordering and interference cancellation processes. The reliability ordering is implemented based on approximated LLR evaluation [47].

Proposed schemes will be compared against some of the known SIC schemes in both performance and complexity. We extend the applicability of this technique to the Spread OFDM (SOFDM) system since the detection problem is similar to the massive MIMO case.

5.1 System Description

5.1.1 System Model

We consider a MMO-OFDM system model similar to the one presented in chapter 3, with n_t transmit antennas and n_r receive antennas, and a single OFDM modulator per each antenna. At the transmit side, the information is generated in the source and mapped to symbols of a different alphabet (we focus here on QPSK mapping). The mapped complex symbols are then demultiplexed into n_t separate data streams. Each stream is subjected to OFDM modulation, after serial to parallel conversion, using the *N*-points IFFT module. At the receiver side, the CP is removed and the received signal is subjected to FFT operation. With perfect knowledge of the Channel State Information (CSI) at the receiver, the per tone MIMO OFDM signal model can be expressed as

$$\tilde{\mathbf{y}}_k = \mathbf{H}_k \tilde{\mathbf{x}}_k + \tilde{\mathbf{v}}_k \tag{5.1}$$

5.1.2 Linear MMSE Detector

MMSE detection is a linear process that assumes a priori knowledge of noise variance and channel covariance. The idea is to design an $n_r \times n_t$ matrix, **W**, based on the MMSE criterion as follows:

$$\underset{W}{\operatorname{argmin}} E\left[\left|\tilde{\mathbf{x}} - \mathbf{W}^{H}\tilde{\mathbf{y}}\right|^{2}\right]$$
(5.2)

where the $\mathbf{W}^{H}\tilde{\mathbf{y}}$ term represents the MMSE estimation of $\tilde{\mathbf{x}}$. Thus, \mathbf{W} in MMSE sense can be expressed as [22]

$$\mathbf{W} = \mathbf{R}_{\tilde{y}}^{-1} \mathbf{H} \tag{5.3}$$

where $\mathbf{R}_{\tilde{y}}$ denotes the autocorrelation matrix of $\tilde{\mathbf{y}}$

$$\mathbf{R}_{\tilde{y}} = (\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{H} + \sigma_{v}^{2}\mathbf{I}_{n_{t}})^{-1}$$
(5.4)

Therefore, the estimated MMSE vector can be written as

$$\hat{\tilde{\mathbf{x}}} = \tilde{\mathbf{H}}^{H} (\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{H} + \sigma_{v}^{2}\mathbf{I}_{n_{t}})^{-1}\tilde{\mathbf{y}}$$
(5.5)

where $\hat{\tilde{\mathbf{x}}} \in \mathbb{C}^{n_t \times 1}$. The corresponding real-valued representation of $\hat{\tilde{\mathbf{x}}}$ can be expressed as

$$\hat{\mathbf{x}} = \left[\Re\{\hat{\tilde{\mathbf{x}}}\} \ \Im\{\hat{\tilde{\mathbf{x}}}\} \right]^T \tag{5.6}$$

where, $\hat{\mathbf{x}} = [\hat{x}_1, \dots, \hat{x}_{2n_t}]^T \in \mathbb{R}^{(2n_t \times 1)}$. The initial MMSE solution, **b**, is obtained by quantizing $\hat{\mathbf{x}}$ to the set $\{-1, 1\}$ as follows;

$$\mathbf{b} = \mathcal{Q}[\hat{\mathbf{x}}] \tag{5.7}$$

where $\mathcal{Q}[.]$ denotes the quantization (slicing) function. Vector **b** can be transformed to a binary representation, which will be used later in the proposed SIC algorithms, as follows:

$$\mathbf{a} = (\mathbf{b} + 1)/2, \quad \mathbf{a} \in \{0, 1\}^{2n_t}$$
 (5.8)

5.1.3 Quadratic Programming Detector

To formulate the detection problem as a quadratic programming, let's use the corresponding real-valued model of (5.1) as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{5.9}$$

where

$$\mathbf{y} = \begin{bmatrix} \Re\{\tilde{\mathbf{y}}\}\\\Im\{\tilde{\mathbf{y}}\}\end{bmatrix}, \mathbf{x} = \begin{bmatrix} \Re\{\tilde{\mathbf{x}}\}\\\Im\{\tilde{\mathbf{x}}\}\end{bmatrix}, \mathbf{v} = \begin{bmatrix} \Re\{\tilde{\mathbf{v}}\}\\\Im\{\tilde{\mathbf{v}}\}\end{bmatrix}$$
(5.10)

$$\mathbf{H} = \begin{bmatrix} \Re\{\tilde{\mathbf{H}}\} & -\Im\{\tilde{\mathbf{H}}\} \\ \Im\{\tilde{\mathbf{H}}\} & \Re\{\tilde{\mathbf{H}}\} \end{bmatrix}$$
(5.11)

The maximum likelihood (ML) detection problem of \mathbf{x} can be expressed as:

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \chi^{2n_t}}{\operatorname{argmin}} \| \mathbf{y} - \mathbf{H} \mathbf{x} \|^2$$
(5.12)

where, the set $\chi = \{-1, 1\}$ for the case of QPSK constellation. Using the same procedures and transformation shown in section 3.2.1, we get the following IQP optimization problem

$$\underset{\mathbf{z}}{\operatorname{argmin}} \quad \frac{1}{2} \mathbf{z}^{T} \mathbf{Q} \, \mathbf{z} + \mathbf{c}^{T} \mathbf{z}$$

$$\mathbf{z} \in \{0, 1\}^{2n_{t}} \tag{5.13}$$

where

$$\mathbf{Q} = \mathbf{H}^{T} \mathbf{H}$$

$$\mathbf{c} = -\mathbf{H}^{T} (\mathbf{y} + \mathbf{H} \underline{\mathbf{1}})/2$$

$$\mathbf{z} = \frac{(\mathbf{x} + 1)}{2}$$
(5.14)

 $\underline{\mathbf{1}} = [1, 1, \dots, 1]^T$, colomn vector of dimension $(2n_t \times 1)$

When relaxing the constraints of \mathbf{z} , equation (5.13) becomes

$$\underset{\mathbf{z}}{\operatorname{argmin}} \quad \frac{1}{2} \mathbf{z}^{T} \mathbf{Q} \, \mathbf{z} + \mathbf{c}^{T} \mathbf{z}$$

$$\underline{\mathbf{0}} \le \mathbf{z} \le \underline{\mathbf{1}}$$
(5.15)

Problem (5.15) seeks to minimize a quadratic objective function over a convex feasible region. Therefore, it is a convex QP minimization problem. A unique global solution \mathbf{z}^* can be obtained using efficient interior-point (IP) solvers with reduced computational complexity [71]. The solution of (5.15) is then quantized to the nearest element in the binary set $\{0, 1\}$, that is :

$$z_i = \mathcal{Q}[z_i^*], \quad i = 1, 2, \dots, 2n_t$$
 (5.16)

5.2 Proposed SIC Algorithms

Each of the proposed SIC algorithms is comprised of two main steps. The first one is to order the reliability of the received symbols based on their LLR measure using the initial MMSE estimate vector given by (5.6). The second step performs SIC for the ordered symbols using QP formulation.

5.2.1 Reliability Ordering using LLR

The a posteriori log-likelihood ratios (LLR) of bit b_i based on the MMSE information in (5.6) can be evaluated as follows:

LLR
$$(b_i) = \ln \frac{\mathbf{p}(b_i = +1 | \hat{\mathbf{x}})}{\mathbf{p}(b_i = -1 | \hat{\mathbf{x}})}, \quad \forall i = 1, \dots, 2n_t$$
 (5.17)

Using Bayes' rule and considering equal priors (i.e. $p(b_i = -1) = p(b_i = +1)$), Eq. (5.17) can be equivalently written as:

LLR
$$(b_i) = \ln \frac{\mathbf{p}(\hat{\mathbf{x}}|b_i = +1)}{\mathbf{p}(\hat{\mathbf{x}}|b_i = -1)}, \quad \forall i = 1, \dots, 2n_t$$
 (5.18)

The optimum LLR for each bit can be evaluated using the maximum a posteriori (MAP) algorithm, but the complexity could be very high, especially when n_t is large. This is due to the fact that $\hat{\mathbf{x}}$ depends on the entire block of bits, which can be explained from the LLR(b_i) expression [47], [96] :

$$LLR(b_i) = \ln \frac{\sum_{\mathbf{b}_i^+} p(\hat{\mathbf{x}} | \mathbf{b}_i^+) p(\mathbf{b}_i^+)}{\sum_{\mathbf{b}_i^-} p(\hat{\mathbf{x}} | \mathbf{b}_i^-) p(\mathbf{b}_i^-)}, \quad \forall i = 1, \dots, 2n_t,$$
(5.19)

where \mathbf{b}_i^+ and \mathbf{b}_i^- are $2n_t$ -length binary vectors with the i^{th} entries being +1 and 1, respectively, whereas $\mathbf{p}(\mathbf{b}_i^+)$ and $\mathbf{p}(\mathbf{b}_i^-)$ are their corresponding probabilities. In this work, we follow the same assumption in [47], [46], where real elements of $\hat{\mathbf{x}}$ in (5.6) are assumed to be independently identically distributed (iid) random variables that can be modeled as Gaussian distributions with mean μ and variance σ^2 as follows:

$$p(\hat{x}_i|b_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \left[e^{(\hat{x}_i - \mu)^2} / 2\sigma^2 \right]$$
(5.20)

Thus, $LLR(b_i)$ can be simplified to :

LLR
$$(b_i) = \ln \frac{p(\hat{x}_i | b_i = +1)}{p(\hat{x}_i | b_i = -1)}, \quad \forall i = 1, \dots, 2n_t,$$
 (5.21)

and by substituting (5.20) into (5.21) we get:

$$LLR(b_i) = \left[\frac{\mu_{+1} - \mu_{-1}}{\sigma^2}\right] \hat{x}_i = \gamma_i \hat{x}_i$$
(5.22)

where μ_{+1} and μ_{-1} are the means when $b_i = +1$ and -1, respectively. γ_i represents the instantaneous signal-to-interference-plus-noise ratio, which we are not considering it in computing LLR, as it may require extra computations for each LLR(b_i), see [47] for more details on the difference in performance between either considering γ_i and \hat{x}_i or \hat{x}_i only. In sum, it does not contribute much to the LLR value, especially when SNR is high and when the number of antennas gets large. Thus, we resort only to the absolute value of \hat{x}_i to measure the reliability of bit b_i . The resulting LLR reliability measurement for all bits can be represented as :

$$LLR(b_i) = \hat{x}_i, \quad \forall i = 1, \dots, 2n_t, \tag{5.23}$$

5.2.2 QP-SIC Algorithm I

The proposed algorithm starts by computing the reliability measure for each bit using (5.23), considering that the initial MMSE estimate is available from (5.6). Then all bits are ordered and indexed according to the value of |LLR|. Let the order index set $J = \{k_1, k_2, \ldots, k_{2n_t}\}$ be a permutation of the integers $1, 2, \ldots, 2n_t$ specifying the order in which components of |LLR| vector are ordered from the most reliable bit to the least reliable bit. Without loss of generality, the lower index k_1 is assigned to the highest value of |LLR|, and the highest index k_{2n_t} is assigned to the lowest value of |LLR|, see Fig. 5.1 for illustration.

Unlike most of the SIC techniques, this proposal tackles the least reliable bit first and then the second least reliable bit and so on until it reaches the last bit, which is the highest reliable bit based on (5.23). In the first iteration $(m = 2n_t)$, it formulates the QP problem (5.24) (shown below) for the least reliable bit (with the index k_{2n_t}) through canceling out the effects of all other bits, assuming that they are reliable and their values are obtained from the MMSE solution in (5.8). The second iteration $(m = 2n_t-1)$ formulates another QP problem for the second least reliable bit (with the index k_{2n_t-1}) by canceling out the effects of bits that have indices $(k_1, k_2, \ldots, k_{2n_t-2})$ using their MMSE values from (5.8).In addition, it also cancel out the effect of the bit positioned at the index k_{2n_t} by using its QP value from the previous iteration. The same process continues for the remaining bits until the last bit, which is indexed by k_1 and the iteration number (m = 1), where, in this case, the QP is formulated by canceling out the effects of all previous bits that were evaluated via (5.24). Thus, based on the general formulation of QP detection in (5.15), the formulation of QP for each bit is expressed as follows:

where $z^{(m)}$ is a variable representing the bit that has index k_m . The process of interference cancellation is included in forming quantities $q^{(m)}$ and $c^{(m)}$ as follows:

$$q^{(m)} = \mathbf{Q}(k_m, k_m)$$

$$c^{(m)} = \mathbf{Q}(\bar{J}^{(m)}, k_m)^T \mathbf{a}(\bar{J}^{(m)}) + \mathbf{c}(k_m)$$

$$\bar{J}^{(m)} = \{J \setminus k_m\}$$

$$m = 2n_t, 2n_t - 1, \dots, 2, 1$$
(5.25)

where $\mathbf{Q}(k_m, k_m)$ is the diagonal element of \mathbf{Q} matrix in (5.14) that corresponds to the bit of index k_m and $\mathbf{c}(k_m)$ is the element of vector \mathbf{c} that corresponds to the bit of index k_m from (5.14). $\overline{J}^{(m)}$ denotes the complementary set of indices other than index k_m , and $\mathbf{a}(\overline{J}^{(m)})$ is the vector of already known bits for which their effects need to be canceled out. This vector is updated in each iteration of m with the new detected value, $z^{(m)}$, from (5.24) until all bits from the initial MMSE solution are replaced by their QP values. Note that (5.25) can be obtained using (5.15) and the initial MMSE solution in (5.8). Now problem (5.24) is a quadratic constraint problem in one variable and can be easily solved through exploiting the binary property of $z = z^2$ when $z \in \{0, 1\}$. Thus, (5.24) can be reduced to

The sign of the term $(\frac{1}{2}q^{(m)} + c^{(m)})$ can be used to force the variable $z^{(m)}$ to be 0 or 1 according to the following criterion:

$$z^{(m)} = \begin{cases} 0 & \left(\frac{1}{2}q^{(m)} + c^{(m)}\right) \ge 0\\ 1 & \left(\frac{1}{2}q^{(m)} + c^{(m)}\right) < 0 \end{cases}$$
(5.27)

This simplification avoids the need to perform any operation of matrix inversion or matrix multiplication, and therefore, it becomes more attractive from a complexity implementation point of view. A summary of Algorithm I procedures is shown in Table 5.1.



Fig. 5.1. Example of ordering and grouping in the proposed algorithms ($n_t = 8$ Antennas and, $2n_t = 16$ QPSK bits)



- 1 Find the initial MMSE vector in binary form, **a**.
- 2 Estimate LLR
- 3 Order |LLR| vector and identify the indices of ordered bit positions set, J
- 4 **for** $m = 2n_t : -1 : 1$

Force $z^{(m)}$ according to criteria (5.27)

update the initial estimated vector ${\bf a}$ with ${\bf a}(J^{(m)})=z^{(m)}$

\mathbf{end}

6 Reorder bit positions to their original sequence

5.2.3 QP-SIC Algorithm II

In this algorithm, we propose to perform SIC in a grouping manner instead of a bit-by-bit manner, which was shown in Algorithm I. Thus, after performing ordering and indexing steps, the ordered bits are divided into a number of groups, M ($1 < M < 2n_t$), where the number of bits per group, N_g , is equal to $2n_t/M$ bits. These generated groups are ordered in accordance with the order index set J. That is, group number 1 contains the most reliable bits (highest |LLR| values), and group number M contains the least reliable bits and so on, as shown in illustration example in Fig (5.1). The set of indices for each group is referred to as $J^{(m)}$, where m = 1, 2, ..., M and $\bigcup_{m=1}^{M} J^{(m)} = J$. Also, we refer to the complement set of ordered grouped indices as $\bar{J}^{(m)}$, where $\bar{J}^{(m)} = J \setminus J^{(m)}$ and $\bar{J}^{(m)} \bigcup J^{(m)} = J$. Note that when $M = 2n_t, N_g = 1$ bit and Algorithm II simplifies to Algorithm I, whereas M = 1 group corresponds to no SIC and hence all symbols are detected jointly using a conventional QP detector in (5.15).

The QP problem is formulated for each group starting with the least reliable group (m = M) and then with the second least reliable group m = M - 1, and so on till we reach the first group (m = 1), as follows:

$$\operatorname{argmin}_{\mathbf{z}^{(m)}} \frac{1}{2} \mathbf{z}^{(m)T} \mathbf{Q}^{(m)} \mathbf{z}^{(m)} + \mathbf{c}^{(m)T} \mathbf{z}^{(m)}$$

$$\underline{\mathbf{0}} \leq \mathbf{z}^{(m)} \leq \underline{\mathbf{1}}$$

$$m = M, M - 1, \dots, 1$$
(5.28)

where vectors $\underline{\mathbf{0}}$ and $\underline{\mathbf{1}}$ are of length N_g . Note that in Algorithm I, m refers to a single index, while here in Algorithm II, it refers to a group of indices. The interference cancellation process in each iteration, m, is included in the construction of $\mathbf{Q}^{(m)}$ and $\mathbf{c}^{(m)}$ as follows:

$$\mathbf{Q}^{(m)} = \mathbf{Q}(J^{(m)}, J^{(m)})$$

$$\mathbf{c}^{(m)} = \mathbf{Q}(\bar{J}^{(m)}, J^{(m)})^T \mathbf{a}(\bar{J}^{(m)}) + \mathbf{c}(J^{(m)})$$

$$\bar{J}^{(m)} = \{J \setminus J^{(m)}\}$$

$$m = M, M - 1, \dots, 1$$

(5.29)


Fig. 5.2. Algorithm II illustration with 4 groups ($n_t = 8$ Antennas, and, $2n_t = 16$ QPSK bits)

where $\mathbf{Q}(J^{(m)}, J^{(m)})$ represents the \mathbf{Q} matrix in (5.14) with columns and rows corresponding to indices of the set $J^{(m)}$, and $\mathbf{c}(J^{(m)})$ represents the elements of \mathbf{c} that correspond to the indices of the set $J^{(m)}$. The vector $\mathbf{a}(\bar{J}^{(m)})$ denotes the vector of detected bits that correspond to the indices of the set $\bar{J}^{(m)}$. This vector is updated in each iteration of m with $\mathbf{z}^{(m)}$ in the corresponding indices until all initial MMSE is updated with values from grouped QP in (5.28), see Fig. (5.2) for an illustration of the algorithm. That is, at the first iteration, where m = M, $\mathbf{a}(\bar{J}^{(m)})$ consists of the initial MMSE solution that corresponds to indices of the set $\bar{J}^{(m)}$, while the second iteration, where m = M - 1, $\mathbf{a}(\bar{J}^{(m)})$ consists partly of the previously detected bits using QP (at m = M) and partly of the initial MMSE solution. This process continues until the last iteration (m = 1), where $\mathbf{a}(\bar{J}^{(m)})$ consists of all the previous detected bits from QP in (5.28) in groups $m = M, M - 1, \ldots, 2$. A summary of Algorithm II is presented in Table 5.2.

Detecting each group of bits in this proposed SIC requires solving an optimiza-

- 1 Find the initial MMSE vector in binary form, **a**.
- 2 Estimate LLR
- 3 Order |LLR| vector and identify the indices of ordered bit positions set, J
- 4 Select number of groups, M, and accordingly $N_g = \frac{2n_t}{M}$
- 5 **for** m = M : -1 : 1

construct $\mathbf{Q}^{(m)}, \mathbf{c}^{(m)}$

 $\mathbf{z}^{(m)} = QP(\mathbf{Q}^{(m)}, \mathbf{c}^{(m)})$

update the initial estimated vector **a** with $\mathbf{a}(J^{(m)}) = \mathbf{z}^{(m)}$

end

6 Reorder bit positions to their original sequence

tion problem of the form (5.28), which cannot be solved in a closed form solution as in the case of (5.24). Hence, we will use the same iterative algorithm that was used in the previous chapters: interior-point (IP), which solves the problem efficiently and with low complexity. We once again attempt to reduce the number of iterations that the IP algorithm requires to solve the QP problem (as stated in section 3.2.3) by performing early termination to the algorithm. In the case of QPSK mapping, where the quantized set is $\in \{0, 1\}$, the early termination to the IP algorithm with two iterations provides no major loss in BER performance. This may be due to the fact that in QPSK mapping, the quantization operation is confined to a small set of $\{0, 1\}$, which reduces the approximation error.

5.2.4 QP-SIC Algorithm III

To reduce the complexity of computations in Algorithm II, we propose to modify it so that the successive group cancellation is performed only for a few groups, say K groups, instead of M, where 1 < K < M, as shown in Fig. 5.1. The bits in the rest of the groups (from 1 to M - K) are deemed reliable and correctly received based on their initial MMSE solution evaluated from (5.8). This is due to the fact that bits in groups 1 to M - K are considered highly reliable compared to bits in groups M - K + 1 to M based on their LLR measure. An illustration is shown in Fig. (5.1), where M = 4 groups, and therefore, if we let K = 2, then groups 3 and 4 are detected using Algorithm II, while groups 1 and 2 keep their initial MMSE solution as the final detected values. This algorithm speeds up the computations with negligible performance loss compared to Algorithm II, as we will see in the simulation results. This proposed algorithm suits high-dimensional systems well, such as the large MIMO and SOFDM systems. Note that when K = 1, the idea in this algorithm becomes similar to a previous work done in [97] for small MIMO with soft detection. However, that work uses ML criteria instead of QP, which is too complex to be implemented in our problem setup.

5.3 Complexity Analysis

Following the same analysis done in chapters 2 and 3, QP detection complexity can be evaluated using the complexity of the Interior Point algorithm. Each iteration of IP boils down to solving a system of linear equations, where it is required to perform matrix inversion of the same size in every iteration. The complexity of one iteration is in the order of $O(N_g^3)$ where $N_g < n_t$. Thus, with *n* iterations, the complexity rises to the order of $O(nN_g^3)$. In practice, the IP algorithm converges in a number of iterations that is almost a constant, independent of the problem dimension [73]. As will be shown in the simulation results, the number of iterations, *n*, can be reduced to 2 iterations in the case of QPSK mapping without major loss in BER performance. Regarding Algorithm II, Equation (5.28) is required to be solved M times, which leads to a total complexity of $O(MnN_g^3)$. Moreover, the initial MMSE estimation in (5.5) adds complexity in the order of $O(n_t^3)$, which is only required one time per each received vector. Thus, the total complexity per each received vector becomes $O(n_t^3) + O(MnN_g^3)$. This complexity can be further reduced to $O(n_t^3) + O(2MN_g^3)$, as pointed out in section 5.2.3. In the case of Algorithm III, the complexity becomes $O(n_t^3) + O(KnN_g^3)$, while in the case of Algorithm I, where $N_g = 1$, the total complexity is approximately $O(n_t^3)$.

On the other hand, the complexity of MMSE-VBLAST is in the order of more than $O(n_t^3)$; more specifically, it is $O(n_t^3) + O((n_t - 1)^3) + O((n_t - 2)^3) + ...$ because it performs matrix inversion and ordering of the channel matrix columns each time one symbol is detected and its effect is canceled out. In a large MIMO system, this complexity becomes more burdensome because $n_t, n_t - 1, n_t - 2, n_t - 3, ...$, and even $n_t/2$ are considerably large. The complexity of MF-SIC [48] is even greater than the complexity of MMSE V-BLAST [33] because, in addition to performing MMSE-SIC, MS-SIC is required to perform a partial ML search each time the MMSE estimated symbols lie within the shadow area of the constellation. This extra computation is higher, especially at low SNR [46].

5.4 Simulation Results

In this section, we provide results for BER performance and complexity analysis of the proposed algorithm compared to MMSE V-BLAST [33] and MF-SIC [48] in a large-scale MIMO OFDM system. We consider a spatial multiplexing MIMO-OFDM system with QPSK modulation in a frequency selective fading channel of length $L_{ch} = 8$ that is assumed perfectly known at the receiver (for the sake of simplicity, we assume uniform tap spacing). The number of OFDM subcarriers, N, used in this simulation is 128. In the following figures, the three variants of QP-SIC are referred to as Alg. I, Alg. II, and Alg. III. The AWGN single antenna BER performance describes the performance with an interference-free scenario.

Fig. 5.3 compares the uncoded BER performance of our proposed algorithm (QP-SIC) with MMSE V-BLAST and MF-SIC in a 100×100 MIMO-OFDM system. In this simulation experiment, both MF-SIC and MMSE V-BLAST techniques use LLR ratio for reliability ordering. We also compare against the original MMSE V-BLAST, which uses channel power for symbol ordering. It can be seen that QP-SIC Alg. I outperforms both techniques, especially at an SNR that is greater than 10 dB. This performance improvement of Alg. I is accompanied by lower computational complexity compared to MF-SIC and MMSE V-BLAST, as it can be depicted from Fig. 5.4. For instance, when $n_t = 50$, Alg. I requires about 10 times fewer computations compared to other techniques, while at $n_t = 100$, it requires about 50 times fewer computations. This is due to the fact that QP-SIC Alg. I requires a one-time computation of matrix inversion, while both MF-SIC and MMSE V-BLAST algorithms require multiple psuedoinverse computations per each received vector (i.e. in every step of SIC, one matrix inversion is needed). Moreover, Alg. I attains this performance improvement with only a one-time reliability ordering, unlike MF-SIC and MMSE V-BLAST, where the reliability ordering is done dynamically; that is, ordering is done every time a symbol is detected and its effect is canceled out. Of course, this adds extra complexity to those techniques.

Fig. 5.3 also shows the performance improvement of Alg. II and III over Alg. I when grouping is implemented in QP-SIC procedures. Alg. II with M = 20 and Alg. III with M = 20 and K = 10 (i.e only the 10 least reliable groups perform QP-SIC using Alg. II and the rest use their initial MMSE solution) perform almost the same; however, Alg. III saves some computations, as can be seen in Fig. 5.4.

Fig. 5.5 and Fig. 5.6 show that QP-SIC can improve the diversity of the system as the number of antennas increases, which is similar to the results provided in [38], [44] for a large MIMO system. This illustrates the large system behavior of the proposed algorithms, where the performance moves towards the single antenna AWGN performance. Fig. 5.5 shows that when using 300×300 , the performance of Alg. I can be of 2-3 dB away from the single antenna AWGN bound. This gap is reduced when Alg. II is used, as depicted in Fig. 5.6. Similarly, Fig. 5.7 shows that when $\frac{1}{2}$ rate convolutional channel coding is applied, algorithms I and II can provide performance that is ≤ 1 dB away from the coded single antenna AWGN bound at 10^{-6} BER with even fewer antennas, such as to 50×50 and 100×100 .

In all of tree proposed algorithms, the SIC idea was based on starting with the least reliable bit or group of bits, as shown in equations (5.24) and (5.28), where $m = 2n_t : -1 : 1$ in Alg. I and m = M : -1 : 1 in Alg. II. Fig. 5.8 demonstrates that this claim is better than performing QP-SIC procedures starting from the highest reliable to bit or group of bits to the least, that is, letting the order be $m = 1 : 1 : 2n_t$ in Alg.I and m = 1 : 1 : M in Alg. II.

Fig. 5.9 shows that when performing the standard interior point algorithm to solve the QP problem in (5.28), the required number of iterations is nearly constant for various SNR and various problem sizes. Therefore, towards reducing the computational complexity of (5.28) of QP-SIC Alg. II, we perform early termination to the IP algorithm, as was suggested in Chapter 3. Fig. 5.10 and Fig. 5.11 show that in QPSK mapping with various group sizes in MIMO OFDM setting, two iterations of IP algorithm result in no loss in BER performance at various SNR values.



Fig. 5.3. Uncoded BER performance of QPSK (100×100) MIMO OFDM



Fig. 5.4. Computational complexity comparison $(n_t = n_r)$



Fig. 5.5. Uncoded BER performance of QPSK MIMO OFDM $\left(n_t=n_r\right)$



Fig. 5.6. Uncoded BER performance of QPSK MIMO OFDM $(n_t = n_r)$



Fig. 5.7. Coded BER performance of QPSK MIMO OFDM $(n_t = n_r)$



Fig. 5.8. Effect of reversing order of groups in QP-SIC on uncoded BER performance



Fig. 5.9. Average number of iterations of the standard IP algorithm



Fig. 5.10. Effect of reducing number of IP iterations on uncoded BER performance



Fig. 5.11. Effect of reducing number of interior point (IP) iterations on uncoded BER performance

5.5 Application of QP-SIC to Spread OFDM Systems

Consider a single antenna Spread OFDM (SOFDM) system as described in Chapter 1, with N complex data symbols generated based on QPSK constellation mapping. The frequency domain SOFDM signal model can be represented as:

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{D}}\tilde{\mathbf{x}} + \tilde{\mathbf{v}} \tag{5.30}$$

where $\tilde{\mathbf{H}} \in \mathbb{C}^{N \times N}$ is a complex diagonal matrix whose diagonal entries are N points FFT of the time domain frequency selective channel vector $\mathbf{h} = [h_1, \ldots, h_{L_{ch}-1}]$ and $\tilde{\mathbf{v}}$ is the frequency domain AWGN with mean zero and covariance matrix $\sigma_v^2 \mathbf{I}_N$. Applying a linear MMSE detector in (5.5) to a Spread OFDM system in (5.30), by replacing n_t with N, and $\tilde{\mathbf{H}}$ with the effective channel matrix $\tilde{\mathbf{H}}\tilde{\mathbf{D}}$, we get :

$$\hat{\tilde{\mathbf{x}}} = \tilde{\mathbf{D}}^{H} \tilde{\mathbf{H}}^{H} (\tilde{\mathbf{H}} \tilde{\mathbf{D}} \tilde{\mathbf{D}}^{H} \tilde{\mathbf{H}}^{H} + \sigma_{v}^{2} \mathbf{I}_{N})^{-1} \tilde{\mathbf{y}}$$
(5.31)

Equation (5.31) can be further simplified using the properties of unitary matrix (i.e. $\tilde{\mathbf{D}}\tilde{\mathbf{D}}^{H} = \mathbf{I}_{N}$) as follows:

$$\hat{\tilde{\mathbf{x}}} = \tilde{\mathbf{D}}^{H} \tilde{\mathbf{H}}^{H} (\tilde{\mathbf{H}} \tilde{\mathbf{H}}^{H} + \sigma_{v}^{2} \mathbf{I}_{N})^{-1} \tilde{\mathbf{y}}$$
(5.32)

this provides interesting complexity reduction because the $\tilde{\mathbf{H}}$ matrix is diagonal and so is $\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{H}$. This makes (5.31) easy to compute.

Conversely, applying ordered SIC using MMSE V-BLAST to the Spread OFDM in (5.30) can be computationally expensive. Each iteration of SIC reduces the size of both the $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{D}}$ matrices by one column, which makes $\tilde{\mathbf{D}}\tilde{\mathbf{D}}^H$ no longer an identity matrix, and therefore, equation (5.32) cannot be used in the subsequent iterations of MMSE V-BLAST. Instead, (5.31) is used, but it requires matrix inversion in every step of nulling and interference cancellation, hence requiring more computations. In fact, applying QP-SIC to this system provides interesting complexity saving because the initial MMSE, which is required for the QP-SIC algorithm, can be estimated using a diagonal matrix inversion (the term $(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H + \sigma_v^2\mathbf{I}_N)^{-1}$ in (5.32)). The rest of the procedures of QP-SIC require either linear complexity, as in Algorithm I, or QP complexity of a smaller size, as in Algorithms II and III.

Using the same manner and definition in section 5.2, the same procedures are applied to the SOFDM system, except taking into account the following changes in formulating the quadratic programming problem; $\mathbf{Q} = \mathbf{D}^T \mathbf{H}^T \mathbf{H} \mathbf{D}$; $\mathbf{c} = -\mathbf{D}^T \mathbf{H}^T (\mathbf{y} + \mathbf{H} \mathbf{D} \mathbf{1})/2$; and $\mathbf{z} = [z_1, \ldots, z_{2N}]^T \in \mathbb{R}^{(2N \times 1)}$. Also, the initial MMSE estimation vector becomes $\hat{\mathbf{x}} = [\hat{x}_1, \ldots, \hat{x}_{2N}]^T \in \mathbb{R}^{(2N \times 1)}$.

Fig. 5.12 shows the BER performance for QPSK SOFDM using QP-SIC Algorithm I and II compared to MMSE V-BLAST. At various group settings, Alg. II outperforms MMSE V-BLAST, while Alg. I has a slightly higher BER than V-BLAST starting from SNR around 13 dB. However, the complexity saving of Alg. I and II is better than V-BLAST, as depicted in Fig. 5.13. Fig. 5.14 shows that Alg. III provides the best trade-off for the SOFDM system in terms of both performance and complexity. It shows that for an SOFDM system with N =256, Alg. III can save about 87 % of the computational complexity of Alg. II by performing SIC of one group only (K=1) out of 8 groups. This saving in computation is penalized by only less than 0.5 dB SNR at a higher SNR regime.



Fig. 5.12. BER performance of SOFDM using the proposed algorithms (N = 256)



Fig. 5.13. An Example of complexity comparison of SOFDM using Algorithm II and V-BLAST



Fig. 5.14. BER performance of SOFDM using Algorithms II and III (N = 256 and M = 8)

6. MMSE-ITERATIVE DETECTION AND DECODING FOR SPREAD OFDM SYSTEMS

In this chapter, we introduce the iterative detection and decoding (turbo equalization) technique for the SOFDM system using an MMSE equalizer and a channel decoder, where the MMSE equalizer and the channel decoder exchange soft information in an iterative fashion to improve detection performance. BER performance is investigated with both full and partial spread OFDM scenarios and also with and without channel decoding. Simulation results show improved performance, especially at low SNR regime. Also, more improvement can be obtained using the partial SOFDM scenario.

6.1 Spread OFDM System Model

Fig. 6.1 shows the overall system that consists of a channel encoder, an interleaver (\prod) and a SOFDM modulator at the transmitter side, and an iterative channel equalization and decoding at the receiver side. The interleaved encoded bits are mapped to N_b QPSK symbols, according to Table 6.1. Each symbol is spread over N OFDM subcarriers using a spreading matrix, D. This means that after the IFFT process, each subcarrier modulates a linear combination of all symbols. Note that $N_b \leq N$, and they are only equal when the full spread scenario is considered. Thus, the received signal vector in the frequency domain, after CP removal and inverse FFT, can be expressed as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{D}}\tilde{\mathbf{x}} + \tilde{\mathbf{v}} \tag{6.1}$$

where $\tilde{\mathbf{x}} = [x_1, \dots, x_N]^T \in \mathbb{C}^{N \times 1}, x_i \in \tilde{\chi}$, is the transmitted symbol taken from a finite constellation alphabet, $\tilde{\chi} = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$, as in Table 6.1, with $E[\tilde{\mathbf{x}}^H \tilde{\mathbf{x}}] = \mathbf{I}_N$, and $\tilde{\mathbf{y}} = [y_1, \dots, y_N]^T \in \mathbb{C}^N$ is the received vector. $\tilde{\mathbf{H}}$ is a complex diagonal matrix whose diagonal entries are N points FFT of the frequency selective channel, which

is the same as modeled in chapter 2. $\tilde{\mathbf{D}}$ is the spreading matrix, which could be real (e.g. Hadamard matrix) or complex (e.g. Vandermonde) with size $N \times N$ and \boldsymbol{v} is the zero mean AWGN with covariance $\sigma^2 \mathbf{I}_N$. The MMSE estimator that minimizes



Fig. 6.1. Block diagram representation of an SOFDM system with turbo equalization

k	1	2	3	4
$(\alpha_{k,1}, \alpha_{k,2})$	(0, 0)	(1, 0)	(0, 1)	(1, 1)
α_k	$\left(\frac{1+i}{\sqrt{2}}\right)$	$\left(\frac{-1+i}{\sqrt{2}}\right)$	$\left(\frac{1-i}{\sqrt{2}}\right)$	$\left(\frac{-1-i}{\sqrt{2}}\right)$

Table 6.1 QPSK Alphabet and Mapping

 $E\left[\left|\tilde{\mathbf{x}}-\hat{\tilde{\mathbf{x}}}\right|^{2}\right]$ is derived in [98] based on the available *a priori* information, and the resulting MMSE estimate is as follows:

$$\hat{\tilde{\mathbf{x}}} = E(\tilde{\mathbf{x}}) + \mathbf{f}^H(\tilde{\mathbf{y}} - E(\tilde{\mathbf{y}}))$$
(6.2)

where

$$\mathbf{f} = cov(\tilde{\mathbf{y}}, \tilde{\mathbf{y}})^{-1}cov(\tilde{\mathbf{y}}, \tilde{\mathbf{x}})$$

From the received signal model in (6.1)

$$cov(\tilde{\mathbf{y}}, \tilde{\mathbf{y}}) = \tilde{\mathbf{H}}\tilde{\mathbf{D}}cov(\tilde{\mathbf{x}}, \tilde{\mathbf{x}})(\tilde{\mathbf{H}}\tilde{\mathbf{D}})^{H} + \sigma^{2}I_{N}$$
$$cov(\tilde{\mathbf{y}}, \tilde{\mathbf{x}}) = cov(\tilde{\mathbf{x}}, \tilde{\mathbf{x}})(\tilde{\mathbf{H}}\tilde{\mathbf{D}})^{H}$$
$$E(\tilde{\mathbf{y}}) = \tilde{\mathbf{H}}\tilde{\mathbf{D}}E(\tilde{\mathbf{x}})$$

Here, we assume that symbols constituting the vector $\tilde{\mathbf{x}}$ are independent. This assumption is supported by the fact that QPSK symbols are generated from the interleaved bits. Therefore, the covariance matrix, $\mathbf{V} = cov(\tilde{\mathbf{x}}, \tilde{\mathbf{x}})$, is non-zero only in the diagonal (i.e. $V = diag[v_1, \ldots, v_N]$ where $v_i = cov(x_i, x_i)$). $E(\tilde{\mathbf{x}}) = \mathbf{m} =$ $[m_1, m_2, \ldots, m_{N-1}]$ represents the vector of all means of symbols such that $m_i =$ $E(x_i)$. The estimated MMSE vector in (6.2) becomes

$$\hat{\mathbf{x}} = \mathbf{m} + \mathbf{V}(\tilde{\mathbf{H}}\tilde{\mathbf{D}})^{H}[(\tilde{\mathbf{H}}\tilde{\mathbf{D}})\mathbf{V}(\tilde{\mathbf{H}}\tilde{\mathbf{D}})^{H} + \sigma^{2}\mathbf{I}_{N}]^{-1}(\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\tilde{\mathbf{D}}\mathbf{m})$$
(6.3)

Equation (6.3) shows that the prior mean and variance of each symbol are required to estimate the symbol. If, however, no prior information is available, then $E(\tilde{\mathbf{x}}) = \mathbf{0}$ and $V = I_N$ and the MMSE estimator becomes the standard form, as in [60]. This is always the case in the first iteration of the turbo equalization.

The importance of turbo equalization lies not only in the iterative process, but also in the process of exchanging soft information between both the equalizer and the decoder. This soft information provides new information about a certain symbol gathered from the rest of the symbols except the symbol itself, which is called extrinsic information [54]. In other words, the extrinsic information exchanged between the equalizer and the decoder about a particular symbol is independent from *a priori* information of that symbol. Therefore, MMSE estimation in (6.3) needs to be implemented in a symbol-by-symbol manner, as follows:

$$\hat{x}_i = E(x_i) + \mathbf{f}_i^H(\tilde{\mathbf{y}} - E(\tilde{\mathbf{y}}))$$
(6.4)

where

$$\mathbf{f}_i = cov(\tilde{\mathbf{y}}, \tilde{\mathbf{y}})^{-1} cov(\tilde{\mathbf{y}}, x_i) = ((\tilde{\mathbf{H}}\tilde{\mathbf{D}}) \mathbf{V}(\tilde{\mathbf{H}}\tilde{\mathbf{D}})^H + \sigma^2 \mathbf{I}_N)^{-1} v_i(\tilde{\mathbf{H}}\tilde{\mathbf{D}})_i$$

 $(\tilde{\mathbf{H}}\tilde{\mathbf{D}})_i$ represents the i^{th} column of the resulting matrix $\tilde{\mathbf{H}}\tilde{\mathbf{D}}$.

6.2 Soft-In Soft-Out MMSE Equalizer

The soft information is generally represented as a Log-likelihood Ratio (LLR) per bit. Thus, for a QPSK symbol, x_i , that consists of 2 bits (let's refer to them as $(c_{i,1}, c_{i,2})$), the soft information per bit is expressed as follows:

$$\mathcal{L}_a(c_{i,j}) = \ln \frac{\mathbf{p}(c_{i,j} = 0)}{\mathbf{p}(c_{i,j} = 1)} \quad , \quad j = 1, 2$$
(6.5)

This is called *a priori* information about bit $c_{i,j}$. At the output of the MMSE equalizer, the estimated value of each \hat{x}_i is mapped to an extrinsic information $(L_E(c_{i,j}))$, which can be expressed in terms of *a posterior* and *a priori* information as follows:

$$L_{E}(c_{i,j}) = \ln \frac{p(c_{i,j} = 0 | \hat{x}_{i})}{p(c_{i,j} = 1 | \hat{x}_{i})} - L(c_{i,j}) = \ln \frac{p(\hat{x}_{i} | c_{i,j} = 0)}{p(\hat{x}_{i} | c_{i,j} = 1)}$$

$$= \ln \frac{\sum_{\forall \alpha_{k}:\alpha_{k,j} = 0} p(\hat{x}_{i} | x_{i} = \alpha_{k}) p(c_{i,l} = \alpha_{k,l})}{\sum_{\forall \alpha_{k}:\alpha_{k,j} = 1} p(\hat{x}_{i} | x_{i} = \alpha_{k}) p(c_{i,l} = \alpha_{k,l})}$$
(6.6)

where $j, l = \{1, 2\}$ and $j \neq l$. It is apparent that the *a posteriori* LLR of the bit $(c_{i,j})$ in (6.6) is conditioned only on the symbol \hat{x}_i rather than on the entire estimated $\hat{\mathbf{x}}$ vector because we use the same simplification as was used in [54] and [57], which allows for easier calculation of the extrinsic LLR. Equation 6.6 also shows that calculating the extrinsic information per bit requires the knowledge of the distribution of $p(\hat{x}_i | x_i = \alpha_k)$. This conditional probability is modeled as a complex Gaussian density function with mean and variance, $\mu_{i,k}$ and $\sigma_{i,k}$, respectively, as follows:

$$p(\hat{x}_i | x_i = \alpha_k) = \frac{1}{\pi \sigma_{i,k}^2} e^{\frac{\left|x_i - \mu_{i,k}\right|^2}{\sigma_{i,k}^2}}$$
(6.7)

Now, by substituting (6.7) in (6.6), the extrinsic LLR of each bit of the i^{th} symbol becomes:

$$L_E(c_{i,1}) = \ln \frac{p(\hat{x}_i | x_i = \alpha_1) p(c_{i,2} = 0) + p(\hat{x}_i | x_i = \alpha_3) p(c_{i,2} = 1)}{p(\hat{x}_i | x_i = \alpha_2) p(c_{i,2} = 0) + p(\hat{x}_i | x_i = \alpha_4) p(c_{i,2} = 0)}$$
(6.8)

where α_1 and α_3 represent the case of $c_{i,1} = 0$ (i.e. the first bit in the symbol x_i is 0) and α_2 and α_4 represent the case of $c_{i,1} = 1$. The same thing is done for the extrinsic LLR of the second bit of the QPSK symbol:

$$L_E(c_{i,2}) = \ln \frac{p(\hat{x}_i | x_i = \alpha_1) p(c_{i,2} = 0) + p(\hat{x}_i | x_i = \alpha_2) p(c_{i,2} = 1)}{p(\hat{x}_i | x_i = \alpha_3) p(c_{i,2} = 0) + p(\hat{x}_i | x_i = \alpha_4) p(c_{i,2} = 0)}$$
(6.9)

Each extrinsic LLR becomes a function of both $\mu_{i,k}$ and $\sigma_{i,k}$, which can be evaluated as follows [57] using (6.4) and (6.7):

$$\mu_{i,k} = E[\hat{x}_i | x_i = \alpha_k] = m_i + \mathbf{f}_i^H (\tilde{\mathbf{H}} \tilde{\mathbf{D}})_i (\alpha_k - m_i)$$
(6.10)

$$\sigma_{i,k} = cov(\hat{x}_i, \hat{x}_i | x_i = \alpha_k) = v_i^2 t_i (1 - v_i t_i)$$
(6.11)

where

$$t_i = (\tilde{\mathbf{H}}\tilde{\mathbf{D}})_i^H [(\tilde{\mathbf{H}}\tilde{\mathbf{D}})\mathbf{V}(\tilde{\mathbf{H}}\tilde{\mathbf{D}})^H + \sigma^2 \mathbf{I}_N]^{-1} (\tilde{\mathbf{H}}\tilde{\mathbf{D}})_i$$

Finally, the extrinsic information L_E can be expressed as:

$$\mathcal{L}_{E}(c_{i,1}) = \frac{\sqrt{8}\Re(\hat{x}_{i})}{v_{i}(1 - v_{i}t_{i})}$$
(6.12)

$$\mathcal{L}_{E}(c_{i,2}) = \frac{\sqrt{8\Im(\hat{x}_{i})}}{v_{i}(1 - v_{i}t_{i})}$$
(6.13)

This extrinsic soft information from the MMSE equalizer will be transformed as *a* priori information to the channel decoder using a de-interleaver (\prod^{-1}) module (see Fig. 6.1). The soft input soft output decoder uses these *a priori* LLR to generate *a posteriori* information for each bit $(b_{i,j})$, and it is referred to as $L_{post}(b_{i,j})$. The corresponding extrinsic information output from the decoder is

$$\mathcal{L}_E(b_{i,j}) = \mathcal{L}post(b_{i,j}) - \mathcal{L}_a(b_{i,j})$$
(6.14)

The interleaved version of the $L_E(b_{i,j})$ produces $L_a(c_{i,j})$, which is considered as *a* priori information to the equalizer. $L_a(c_{i,j})$ is then used to calculate the prior mean and variance of the new estimated symbol in the new iteration as follows [54]:

$$m_i = \frac{\tanh(\mathcal{L}_a(c_{i,1})/2) + i \tanh(\mathcal{L}_a(c_{i,2})/2)}{\sqrt{2}}$$
(6.15)

$$v_i = 1 - |m_i|^2 \tag{6.16}$$

As it was pointed out in the previous section and also in several turbo equalization papers, such as in [54] and in [99], the extrinsic information of a certain symbol x_i should be independent of the *a priori* information of that particular symbol. One way to comply with this assumption is to put $m_i = 0$ and $v_i = 1$ in (6.4) for this particular symbol and keep the rest. This means that \hat{x}_i uses *a priori* information of x_i that comes from all other symbols except the i^{th} symbol. Therefore, equation (6.4) can be reformulated to reflect this manipulation as follows [55] :

$$\hat{x}_{i} = \mathbf{f}_{i}^{H} [\tilde{\mathbf{y}} - \mathbf{H}\mathbf{D}\mathbf{m} + m_{i}(\mathbf{H}\mathbf{D})_{i}]$$

$$\mathbf{f}_{i}^{H} = (\tilde{\mathbf{H}}\tilde{\mathbf{D}})_{i}^{H} [(\tilde{\mathbf{H}}\tilde{\mathbf{D}})\mathbf{V}(\tilde{\mathbf{H}}\tilde{\mathbf{D}})^{H} + \sigma^{2}\mathbf{I}_{N} + (1 - v_{i})(\tilde{\mathbf{H}}\tilde{\mathbf{D}})_{i}(\tilde{\mathbf{H}}\tilde{\mathbf{D}})_{i}^{H}]^{-1}$$
(6.17)

It is important to note that with this manipulation, the values of m_i and v_i in (6.17) are the computed values from (6.15) and (6.16) and not $m_i = 0$ and $v_i = 1$.

6.3 Low Complexity Formulation Using Matrix Inversion Lemma

The drawback of estimating MMSE symbols in turbo equalization using (6.17) is that, for symbol-by-symbol estimation, it is required to perform matrix inversion for the term $[(\tilde{H}\tilde{D})V(\tilde{H}\tilde{D})^{H} + \sigma^{2}I_{N} + (1 - v_{i})(\tilde{H}\tilde{D})_{i}(\tilde{H}\tilde{D})^{H}]$ for all symbols in every iteration. This requires large computational complexity, especially for large N, such as 256, 512, etc. In this work, we utilize the matrix inversion lemma and the manipulation in [57] to simplify the formula so that the matrix inversion can be performed once for all symbols in every iteration:

$$\hat{x}_i = \frac{1}{(1 + t_i(1 - v_i))} [t_i m_i + (\tilde{\mathbf{H}}\tilde{\mathbf{D}})_i^H [(\tilde{\mathbf{H}}\tilde{\mathbf{D}}) \mathbf{V}(\tilde{\mathbf{H}}\tilde{\mathbf{D}})^H + \sigma^2 \mathbf{I}_N]^{-1} (\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\tilde{\mathbf{D}}\mathbf{m})] \quad (6.18)$$

Another possible simplification, though this may result in a loss of BER performance, is to replace the diagonal matrix of all variance, V, with $\bar{v}I_N$ where \bar{v} is a constant, which could be the average value of all variances, i.e $\bar{v} = \frac{1}{N} \sum_{i=1}^{N} v_i$, or the maximum value of all variance. Provided that $\tilde{\mathbf{D}}$ is a unitary matrix, and with this manipulation of matrix V, (6.18) can be further reduced to the following:

$$\hat{x}_i = \frac{1}{(1+t_i(1-v_i))} [t_i m_i + \frac{1}{\bar{v}} (\tilde{\mathbf{H}} \tilde{\mathbf{D}})_i^H [\tilde{\mathbf{H}} \tilde{\mathbf{H}}^H + \frac{\sigma^2}{\bar{v}} \mathbf{I}_N]^{-1} (\mathbf{y} - \tilde{\mathbf{H}} \tilde{\mathbf{D}} \mathbf{m})]$$
(6.19)

where the term $[\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{H} + \frac{\sigma^{2}}{\bar{v}}\mathbf{I}_{N}]$ is a diagonal matrix and can be easily inverted.

6.4 Simulation Results

In this section, we consider the SOFDM system with N = 128 subcarriers. The generated bits of information at the source are encoded using $\frac{1}{2}$ rate convolutional coding. The resulted bits are interleaved and mapped to QPSK symbols. Before transmission, these symbols are spread across N OFDM subcarriers using the vandermonde spreading matrix and OFDM modulation. The frequency selective fading channel is simulated using 16 channel taps. Perfect channel knowledge at the receiver is assumed, and therefore, no pilots are used in the simulation. Also, in this simulation, both full spread OFDM scenario with $N_b = N = 128$, and partial spread OFDM scenario with $N_b = \frac{N}{2} = 64$ are examined.

Fig. 6.2 shows simulation results of BER for the SOFDM system when 3 iterations of turbo equalization is implemented at the receiver. It can be said that turbo equalization works well with a SOFDM system and clearly outperforms coded SOFDM, which obviously suffers from high error rate at low SNR. At BER of 10^{-2} , for instance, the first iteration of turbo equalization saves more than 2.5 dB relative to the coded case, and even more with the second and third iterations. The performance of turbo case with 3 iteration is still worse than the uncoded case at around 0 dB SNR. Furthermore, implementing turbo equalization with the partial spread case of $N_b = 64$ greatly improves the performance at low SNR, but the penalty is the decrease in data rate. This is due to the fact that using $\frac{1}{2}$ rate spreading and $\frac{1}{2}$ rate convolutional decoding results in an overall rate of $\frac{1}{4}$.

Fig. 6.3 shows the performance comparison between model (6.18), which is more accurate in terms of prior variance calculation, and model (6.19), which is an approximated version of (6.18). The advantage of using model (6.19) is that it avoids matrix inversion computations; it does only a matrix inversion for a diagonal matrix, which is a low computational operation. On the other hand, however, using model (6.19) does not show improvement of performance from iteration to iteration at low SNR, only at high SNR.

We also investigated the performance of the MMSE equalizer in the setup of turbo equalization when there is no channel decoding is used. That is, the extrinsic information generated from the MMSE equalizer, in the current iteration, is fed back again to the equalizer and then used to calculate prior means and variances of the next iteration. The BER performance of this setup is shown in Fig. 6.4. It shows that although there is an improvement from iteration to iteration, the apparent improvement occurs only at the moderate to high SNR region (SNR > 8 dB).

Next, we investigate various scenarios for implementing a channel equalizer and a channel decoder in the SOFDM system, see Fig. 6.5. Scenario I represents the case of no turbo equalization, that is, independent equalization and decoding. Scenario II represents turbo equalization but with separate channel decoding, and scenario III represents the joint equalization and decoding (iterative detection and decoding). BER performance comparison for these three scenarios is presented in Fig. 6.6. It can be observed that scenario III, which is the joint detection and decoding technique, provides the best performance. However, scenario II, which iterates around the equalizer only and uses the channel decoder only once, could achieve the performance of scenario III at high SNR.



Fig. 6.2. BER of QPSK of SOFDM system with turbo equalization



Fig. 6.3. BER comparison between the model (6.18) and its approximation (6.19) for a QPSK SOFDM system with turbo equalization



Fig. 6.4. BER comparison with and without a channel decoding for a QPSK SOFDM system



Fig. 6.5. Different scenarios for channel equalization and decoding



Fig. 6.6. BER comparison of different scenarios for channel equalization and decoding

7. QP-ITERATIVE DETECTION AND DECODING IN A LARGE-SCALE MIMO SYSTEM

In Chapter 5, iterative detection and decoding (IDD) was implemented using a linear MMSE detector for the SOFDM system. In this chapter, the aim is to develop an IDD-type technique using the QP detector. The QP-based detectors were studied in the previous chapters in large dimensional systems using BB(L,M), two-stage QP, and QP-SIC algorithms. When the MMSE was used to derive the IDD, prior information that passed from the channel decoder was incorporated by updating the mean and variance of each symbol. These updates were then used to provide a better MMSE estimate and better LLR information. However, the following challenges are presented when a QP detector is used in an IDD setting: 1) How to incorporate prior information, in the form of LLR, provided by a channel decoder into the QP optimization problem, and 2) How to make the QP detector provide soft information, in the form of LLR, to be used by the channel decoder. Addressing these challenges with implementation and performance study is presented in this chapter using a MIMO system in a spatial multiplexing setup.

We use the same technique in [100] to incorporate *a priori* information into the QP optimization problem, although with reduced number of optimization problems needed to compute the LLR of all bits through the use of local neighborhood solutions of the QP solution. A receiver block diagram with turbo equalization is shown in Fig. 7.1.

The focus of this chapter is on a flat fading spatial multiplexing MIMO system. The model that will be used for this system is the equivalent real model, as in (5.9), except that the channel is flat fading:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{7.1}$$



Fig. 7.1. Receiver side of a MIMO system with IDD using a QP detector

The information is generated in the source and randomly interleaved and convolutionally encoded. Then it is mapped to symbols of different alphabets (we focus in this chapter on the QPSK mapping).

7.1 Incorporating Prior Information into a QP Detector

Consider QPSK symbols, mapped from coded and interleaved bits, to be transmitted over a MIMO flat fading channel. At the receiver side the complex channel model is transformed to a real equivalent one, as shown in (5.10) and (5.11). The real part of the complex data symbols is mapped to $[x_1, \ldots, x_{n_t}]$, and the imaginary part of these symbols is mapped to $[x_{n_t+1}, \ldots, x_{2n_t}]$, where bit $x_i \in \{-1, +1\}, \forall i = 1, \ldots, 2n_t$. Therefore, the *a posteriori* LLR for bit x_i is

$$\mathcal{L}_{post}(\mathbf{x}_i) = \ln \frac{\mathbf{p}(\mathbf{x}_i = +1 | \mathbf{y}, \mathbf{H})}{\mathbf{p}(\mathbf{x}_i = -1 | \mathbf{y}, \mathbf{H})}, \quad \forall i = 1, \dots, 2n_t$$
(7.2)

Using Bayes' theorem, Eq. (7.2) can be equivalently written as

$$L_{post}(\mathbf{x}_i) = \ln \sum_{\mathbf{x} \in \chi_{\mathbf{x}_i^{+1}}} p(\mathbf{y} | \mathbf{x}, \mathbf{H}) P(\mathbf{x}) - \ln \sum_{\mathbf{x} \in \chi_{\mathbf{x}_i^{-1}}} p(\mathbf{y} | \mathbf{x}, \mathbf{H}) P(\mathbf{x})$$
(7.3)

where $\chi_{\mathbf{x}_{i}^{\pm 1}}$ is the set of all possible vectors of \mathbf{x} satisfying $\mathbf{x}_{i} = \pm 1$. P(\mathbf{x}) is the vector of *a priori* probabilities, which in the case of turbo equalization, is delivered by the outer channel decoder in the form of an *a priori* LLR ratio as follows:

$$L_a(\mathbf{x}_i) = \ln \frac{\mathbf{p}(\mathbf{x}_i = +1)}{\mathbf{p}(\mathbf{x}_i = -1)}, \quad \forall i = 1, \dots, 2n_t$$
(7.4)

If the noise in the system is considered white Gaussian, the probability density function $p(\mathbf{y}|\mathbf{x}, \mathbf{H})$ can be represented by $\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-||\mathbf{y} - \mathbf{H}\mathbf{x}||^2/2\sigma^2\right)$. This can be used in (7.3), and with the aid of max-log approximation from [101], $\ln(\sum_i \exp(\phi_i)) \approx \max_i \{\phi_i\}$, Eq. (7.3) can be simplified to the following:

$$L_{post}(\mathbf{x}_{i}) \approx \min_{\mathbf{x} \in \chi_{\mathbf{x}_{i}^{-1}}} \left\{ \frac{1}{2\sigma^{2}} ||\mathbf{y} - \mathbf{H}\mathbf{x}||^{2} - \ln[\mathbf{P}(\mathbf{x})] \right\} - \min_{\mathbf{x} \in \chi_{\mathbf{x}_{i}^{+1}}} \left\{ \frac{1}{2\sigma^{2}} ||\mathbf{y} - \mathbf{H}\mathbf{x}||^{2} - \ln[\mathbf{P}(\mathbf{x})] \right\}$$
(7.5)

In order to find the relation between the vector of *a priori* probability $\mathbf{P}(\mathbf{x})$ and the *a priori* LLR \mathbf{L}_a , we use the following:

1) the fact that $p(x_i = +1) = \frac{\exp(L_a(x_i))}{(1 + \exp(L_a(x_i)))}$ and $p(x_i = -1) = \frac{1}{(1 + \exp(L_a(x_i)))}$. They can can be derived based on Eq. (7.4)).

2) adopt the same assumption in [101] and [54], where symbols \mathbf{x}_i , $i = 1, ..., 2n_t$, are assumed statistically independent across spatial streams due to the interleaver in the transmitter, so that we can model $P(\mathbf{x})$ using $P(\mathbf{x}) = \prod_{i=1}^{2n_t} P(\mathbf{x}_i)$.

3) by some manipulations, we can express the relation between $P(\mathbf{x})$ and L_a , as in [102]:

$$\ln(\mathbf{P}(\mathbf{x})) = A + \sum_{i=1}^{2Nt} \mathbf{x}_i \frac{\mathbf{L}_a(\mathbf{x}_i)}{2}$$

where A is a constant that does not depend on x_i . Substituting this relation back into (7.5) and ignoring constant terms that does not depend on bit x_i , Eq. (7.5) can be written as

$$L_{post}(\mathbf{x}_{i}) \approx \min_{\mathbf{x} \in \chi_{\mathbf{x}_{i}}^{-1}} \left\{ \frac{1}{2\sigma^{2}} ||\mathbf{y} - \mathbf{H}\mathbf{x}||^{2} - \frac{1}{2} \mathbf{x}^{T} \mathbf{L}_{a} \right\} - \\ \min_{\mathbf{x} \in \chi_{\mathbf{x}_{i}}^{+1}} \left\{ \frac{1}{2\sigma^{2}} ||\mathbf{y} - \mathbf{H}\mathbf{x}||^{2} - \frac{1}{2} \mathbf{x}^{T} \mathbf{L}_{a} \right\}$$
(7.6)

where $\mathbf{x} = [x_1, \dots, x_{2n_t}]^T$ is the vector of all interleaved bits, and $\mathbf{L}_a = [L_{a(1)}, \dots, L_{a(2n_t)}]^T$ is the vector of LLR ratios of all interleaved bits.

Consider one term from (7.6) and perform the same quadratic programming formulation procedures done in section 3.2.1:

$$\min_{\mathbf{x}\in\chi_{c_{i,j}}^{-1}} \left\{ \frac{1}{2\sigma^2} ||\mathbf{y} - \mathbf{H}\mathbf{x}||^2 - \frac{1}{2} \mathbf{x}^T \mathbf{L}_a \right\}$$

$$= \min_{\mathbf{x}\in\chi_{c_{i,j}}^{-1}} \left\{ ||\mathbf{y} - \mathbf{H}\mathbf{x}||^2 - \sigma^2 \mathbf{x}^T \mathbf{L}_a \right\}$$

$$= \min_{\mathbf{z}\in\Omega_{z_i}^{0}} \left\{ \frac{1}{2} \mathbf{z}^T \mathbf{Q} \, \mathbf{z} + \mathbf{b}^T \mathbf{z} \right\}$$
(7.7)

where, $\Omega = \{0, 1\}^{2n_t}$,

$$\mathbf{Q} = \mathbf{H}^T \mathbf{H},$$

$$\mathbf{z} = \frac{\mathbf{x} + 1}{2}$$

$$\mathbf{b} = -\frac{1}{2} \mathbf{H}^T (\mathbf{y} + \mathbf{H}\underline{1}) - \frac{\sigma^2}{4} \mathbf{L}_a$$

The result of (7.7) can be applied to (7.6), in addition to relaxing integer constraints. Therefore, (7.6) can be expressed in the following form:

$$L_{post}(\mathbf{x}_{i}) \approx \min_{\underline{\mathbf{0}} \leq \mathbf{z} \leq \underline{\mathbf{1}}, \mathbf{z}_{i}=0} \{ \frac{1}{2} \mathbf{z}^{T} \mathbf{Q} \, \mathbf{z} + \mathbf{b}^{T} \mathbf{z} \} - \\ \min_{\underline{\mathbf{0}} \leq \mathbf{z} \leq \underline{\mathbf{1}}, \mathbf{z}_{i}=1} \{ \frac{1}{2} \mathbf{z}^{T} \mathbf{Q} \, \mathbf{z} + \mathbf{b}^{T} \mathbf{z} \}$$
(7.8)

Eq (7.8) shows that to evaluate LLR per one bit, it is required to solve two QP problems of length $2n_t - 1$ each. Although the computation of each QP problem can be reduced based on reducing interior-point iterations to as low as 2 iterations with QPSK, the LLR computations for all $2n_t$ bits require a total number of $4n_t$ QP problems to solve, which are large computations. Thus, we follow the same idea in [100], where we solve $2n_t + 1$ QP problems instead of $4n_t$. The idea is to solve the following problem without any bit constraints

$$\breve{\mathbf{z}} = \operatorname*{argmin}_{\underline{\mathbf{0}} \leqslant \mathbf{z} \leqslant \underline{\mathbf{1}}} \frac{1}{2} \mathbf{z}^T \mathbf{Q} \, \mathbf{z} + \mathbf{b}^T \mathbf{z}$$
(7.9)

then the same problem is solved again $2n_t$ times, but with bit constraints based on the results from $\mathbf{\breve{z}}$ as follows:

$$\min_{\mathbf{z}} \quad \frac{1}{2} \mathbf{z}^T \mathbf{Q} \, \mathbf{z} + \mathbf{b}^T \mathbf{z}$$
st
$$\underbrace{\mathbf{0}}_{\mathbf{z}_i} \leq \mathbf{z} \leq \underline{\mathbf{1}}, \qquad (7.10)$$

$$z_i = \operatorname{xor}(\breve{z}_i, 1) \qquad i = 1, \dots, 2n_t$$

The cost function of the minimization problems in (7.9) and (7.10) are used to find $L_{post}(x_i)$ in (7.8). As shown in [54], the exchange of extrinsic information between the channel detector and channel decoder is more effective in improving performance of the turbo equalization receiver. Thus, the required extrinsic information can be calculated as follows:

$$\mathcal{L}_e(\mathbf{x}_i) = \mathcal{L}_{post}(\mathbf{x}_i) - \mathcal{L}_a(\mathbf{x}_i)$$
(7.11)

7.2 Reduced Complexity Algorithm

Although the above technique may suit the conventional small MIMO systems because the size of the QP is small, it is not computationally efficient for a large-scale MIMO system. For instance, if $n_t = 64$ with a QPSK modulation, 129 QP optimization problems are needed to be solved to evaluate the LLR of 128 bits (i.e. using (7.9) and (7.10)). Therefore, in this section, we reduce the complexity of this technique by exploiting the neighborhood set of solutions of the vector \breve{z} (from (7.9)), as follows:

- Solve QP problem (7.9) one time to find **ž**
- Find the closest neighborhood solutions to ž
- Construct a list of solutions using both \breve{z} and its neighbors
- Use (7.6) to compute $L_{post}(x)$
- Use (7.11) to compute $L_e(x)$

The construction of neighborhood solutions can be done according to the following way. Let the alphabet set in the case of QPSK modulation be $\Omega = \{0, 1\}$, Thus, the symbol neighborhood to a symbol $\{0\}$ (i.e. $\mathcal{N}(0)$) is $\{1\}$, and $\mathcal{N}(1)$ is $\{0\}$. The vector neighborhood to $\mathbf{\breve{z}}$ is the vector that differs from $\mathbf{\breve{z}}$ in just one coordinate, hence there will be $2n_t$ neighbor vectors to $\mathbf{\breve{z}}$. Let the neighbor vectors be $\mathbf{z}_{neighbors} = [\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(j)}, \dots, \mathbf{z}^{(2n_t)}]$, where $\mathbf{z}^{(j)} = [z_1^{(j)}, \dots, z_i^{(j)}, \dots, z_{2n_t}^{(j)}]^T$, $i, j = 1, \dots, 2n_t$, and $z_i^{(j)} = \begin{cases} \breve{z}_i & \text{for } i \neq j \\ \mathcal{N}(\breve{z}_i) & \text{for } i = j \end{cases}$ (7.12)

7.3 Simulation Results

The simulation of this section is implemented using a soft-in soft-out 1/2 rate convolution channel decoder that is based on the BCJR algorithm. Note that in the transmit side a convolutional encoder (rate R = 1/2, generator polynomials [133 171], and constraint length 7) is used with a random interleaver and a QPSK large-scale MIMO system with $n_t = n_r = 16$ and 64. The number of iterations represents the number of times the soft-input soft-output MIMO detector and the soft-input softoutput channel decoder are used.

Fig. 7.2 demonstrates the BER performance of three iterations of IDD when a soft-in soft-out QP detector is used. It can be seen that as the number of iterations increases, a lower BER is obtained for both cases of $n_t = 16$ and $n_t = 64$, though the difference in performance between $n_t = 16$ and $n_t = 64$ can be seen clearly at higher iteration numbers, such as iteration 3. The uncoded and convolutionally coded MIMO cases are also plotted in the same figure to point out the advantages of IDD, especially at a low SNR regime. The convolutional coded performance of 16×16 and 64×64 MIMO in Fig. 7.2 represents the case where a hard decision QP detector is followed by a hard decision Viterbi decoder. As expected, the performance difference between the hard decision and the soft decision (represented by iteration number 1 of IDD) is about 2 dB. Note that in this figure, the large system behavior between 16×16 and 64×64 can be observed in both uncoded and coded cases; however, when IDD is used, the large system behavior can be observed only in higher iteration numbers.

The performance of our proposed technique for reducing LLR computations is

shown in Fig. 7.3, with $n_t = 16$ and $n_t = 64$. In this figure, QP refers to the technique that uses (7.9) and (7.10) to compute LLR, while the proposed technique refers to using (7.9) with the set of neighborhood solutions. When n_t is relatively small, such as 16, the performance of the two techniques become very close as the number of iterations increases, such as the case of iteration 3 in Fig. 7.3 a. While on the other hand, for a relatively large n_t , such as 64, the performance of the proposed technique is quite similar to the QP technique. It becomes even slightly better at the third iteration, as shown in Fig. 7.3 b. This may be due to the large system effect that appears more clearly in our proposed technique at $n_t = 64$ because it combines a QP technique with some sort of neighborhood search technique [38] in computing LLR.



Fig. 7.2. BER performance of turbo equalization using a QP detector in a MIMO system



Fig. 7.3. BER performance of the proposed LLR computation technique compared to the multiple QP optimization (a) 16×16 (b) 64×64

8. SUMMARY

This dissertation presents the use of integer quadratic programming (IQP) in detection problems of high-dimensional wireless systems, such as the SOFDM system and the large-scale MIMO system. An ML detection problem for these systems is reformulated in terms of an integer quadratic optimization problem, which is solved recursively using the BB search tree algorithm. This technique was able to gain the ML performance and diversity with a reasonable complexity when the number of antennas is small in a MIMO system, and also when the number of OFDM subcarriers is relatively small in the SOFDM system.

Computational complexity of IQP with BB in an SOFDM system increases as the number of OFDM subcarriers increases. However, at a fixed number of subcarriers, the increase in complexity is not significant when SOFDM system varies its modulation order from low QAM, such as QPSK, to high QAM, such as 256 QAM. This is in contrast to the ML and SD techniques. Furthermore, when an MMSE detector is used as a preprocessing step for BB, complexity saving occurs at a high SNR regime due to the fact that several nodes have been pruned in the BB tree. Also, when the BB search tree is forced to stop at various levels and a quantization operation is performed on the most probable node, a flexible trade-off between performance and complexity is gained.

The complexity of joint symbol detection in SOFDM using IQP is considered very high, especially for a large number of OFDM subcarriers. Therefore, forcing BB to stop at the root node level (L = 0) is more suitable from a complexity point of view. In addition, the BER performance shows that using BB with L = 0 is enough to provide better performance and diversity compared to the MMSE and LML techniques at large N, such as N = 256.

To speed up the search of the BB tree, a controlled size BB search tree algorithm
is proposed with flexible trade-offs between complexity and performance. This proposed technique is referred to as BB(L,M), where L is the layer at which the search tree stops and M is the number of nodes retained per each layer. At different values of L and M, BB(L,M) outperforms existing techniques such as MMSE-OSIC, MMSE chase, MMSE-LAS, and QRD-M in large-scale MIMO OFDM systems. It also shows its efficacy in providing near-ML diversity and performance in the overloaded MIMO systems when QPSK modulation is used.

The performance of the BB(L,M) detector and the two-stage QP detectors are investigated in a large-scale MIMO system with a flat fading channel. Their performance was superior than the existing LAS and RTS algorithms as the number of antennas increases, especially when the modulation level is 64QAM or greater. The BB(L,M) algorithm performs better than the two-stage QP algorithm when a relatively small number of antennas is considered or when a large value of L is chosen (e.g. > 16). on the other hand, however, the two-stage QP algorithm requires much less computational complexity compared to BB(L,M), especially at large $n_t = n_r$.

The QP detector shows interesting results when used to reformulate SIC detection schemes in a large-scale MIMO OFDM system. It offers both lower computational complexity and better BER performance compared to the existing widely used SIC techniques. In fact, three versions of this technique are proposed with flexible tradeoffs between complexity and BER performance, as each of the three QP-SIC algorithms outperforms the existing SIC techniques. QP-SIC is able to demonstrate the large system behavior at a large number of antennas. It shows its ability to continuously extract more diversity from the system as the number of antennas increases. It shows near single antenna AWGN performance within 2 dB at BER $10^{(-3)}$ when $n_t = 300$.

QP is also used to derive a turbo equalization based receiver algorithm. More specifically, a QP detector is developed so that it can provide soft information suitable for use by the channel decoder, and also can accept soft information that is passed from the channel decoder. Simulation results demonstrate that the use of QP in turbo equalization improves performance at the cost of a reasonable increase in complexity when compared to linear detectors.

Since the focus of the last part of this thesis was on the application of using QP techniques to formulate SIC and IDD techniques with QPSK symbol mapping, future work is aimed at developing these proposed techniques for higher QAM levels. Straightforward extension may not be applicable because unlike the case of QPSK mapping, in higher QAM modulation, the real and imaginary parts of the complex symbol represent more than one bit.

Throughout this thesis, the channel is assumed perfectly known at the receiver and also time invariant within each block of transmission. However, in a fast-fading channel, Doppler spread caused by user mobility destroys the orthogonality among subcarriers, which in turn leads to intercarrier interference (ICI) and degrades system performance. Thus, a BER performance study using this type of channel with the techniques proposed in this thesis would be required.

Finally, it is important to examine the performance of the proposed algorithms in this thesis in other high-dimensional wireless systems, such as full rate non-orthogonal space time block codes (non-orthogonal STBC) and large-scale multi user detection systems. LIST OF REFERENCES

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VITA

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