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Vahid Raissi Dehkordi

CanmetENERGY - Natural Resources Canada, Canada, vahid.raissidehkordi@canada.ca

José Agustín Candanedo

CanmetENERGY - Natural Resources Canada, Canada, jose.candanedoibarra@canada.ca

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State-Space Modeling of Thermal Spaces in a Multi-Zone Building

Vahid R. DEHKORDI^{1*}, Jose A. CANDANEDO²

¹CanmetENERGY, Natural Resources Canada,
Varenes, Quebec, Canada
vahid.raissidehkordi@canada.ca

²CanmetENERGY, Natural Resources Canada,
Varenes, Quebec, Canada
jose.candanedoibarra@canada.ca

* Corresponding Author

ABSTRACT

In this paper, a study on system identification and modeling of thermal spaces in a large institutional building is presented. Thermal models are essential in predictive control since they are required to predict the thermal load of a single building zone, a collection of various thermal spaces, or a whole building. The main goal of this paper is to show how the optimum model order associated which each thermal zone depends on factors such as the location of the zone within the building, its orientation and its exposure to outdoor space. The results of this study will serve as a guideline for choosing the appropriate order of linear models in similar buildings.

1. INTRODUCTION

Control-oriented modeling of heating, ventilation, and air conditioning (HVAC) systems has a significant potential to improve the operation of HVAC systems, as it is an essential step in the optimization of energy consumption and indoor environment quality. Thermal space models, specifically, can be used to calculate building energy needs over a prediction horizon of up to several days, when a reasonably accurate weather forecast and occupancy data are available. This is a valuable asset for the formulation and optimization of a model predictive control (MPC) setup (Prívará *et al.*, 2012; Candanedo *et al.*, 2015).

HVAC systems are modeled using various approaches, which result in quite diverse types of models (Janczura *et al.*, 2013). The modeling approaches most commonly used can be categorized roughly in three types (Afram and Janabi-Sharifi, 2014). The data-driven “black box” approach consists of collecting system performance data under different conditions and then deriving a mathematical relationship between the input and output variables. Data-driven models can be obtained in numerous ways, for example, by using statistical regression or artificial neural network (ANN) techniques. Conversely, the “white box” approach involves deriving the model based on the knowledge of physical dynamics governing the plant to be modeled. Finally, an intermediate “grey box” approach is also available which uses physical characteristics of the system to form the basic structure of the model. Subsequently, measured data of the system are processed using parameter estimation algorithms to tune the model parameters. Data-driven RC models used for characterization of thermal spaces are a typical example of grey box models, as the RC circuit structure is determined first via physical considerations; the value of the RC parameters can then be determined by using recorded weather data and indoor temperature (Saber-Derakhtenjani *et al.*, 2015; Candanedo *et al.*, 2013). This classification in “black box”, “grey box” and “white box” is of course only a very coarse approximation of a continuum of model types, as all models are really some form of grey-box model.

White box models require detailed knowledge of the system and its processes. On the other hand, data driven gray box and black box models can be obtained with little or no knowledge of the system. White box models can be

generalized easily since they follow the ‘true’ physical notions and relations between different parts of a system, but at the same time, on account of their reliance on abundant *a priori* information, they are prone to function poorly when evaluated numerically against actual processes. On the other hand, data driven models function well under situations closely mimicking a defined set of training data, but their performance varies considerably when different datasets are applied as input data. Grey box models benefit from the advantages of the other two types, providing good generalization capabilities in comparison with data driven models, as well as better accuracy in comparison with the physics-based models.

The above mentioned approaches to the modeling of building dynamics and HVAC systems result not only in diverse types of models, but also in different levels of complexity. For example, the building simulation software EnergyPlus creates and uses a highly detailed, white box RC network model; this model is a valuable tool for design, but it is arguably too complex for the formulation and later optimization of a MPC setup. Conversely, a low-order model can provide a satisfactory description of large-scale dynamic phenomena in the building (Kim and Braun, 2012; Cole *et al.*, 2014). For instance, a low-order thermal space model of the whole building with very few inputs/outputs to a coarse estimation of thermal load of a commercial building during the next 48 hours. Other examples are low-order thermal networks which are used to estimate the thermal response of thermal zones from office spaces to whole communities. The simplicity of low-order models makes them more manageable and flexible. Moreover, they provide clarity and insight about the effect of different inputs and can facilitate the management of uncertainty. However, the simplicity of low-order models also means that they fail to represent the smaller-scale details of their subject. Representing the whole building with a single model provides a good example of such limitation; while the building model predicts well the overall load, it does not contain enough information to show how much cooling or heating is required by each individual zone.

This paper explores system identification and modeling of thermal spaces in a large institutional building as nearly “black box” models with very few input signals. These data-driven models are in the form of state-space representations which do not take the physical characteristics of the thermal zones into account. The main goal is to find how the optimum state-space model order associated with each thermal zone depends on factors such as the location of the zone within the building, its orientation and its exposure to outdoor space, *without* explicitly incorporating the physical structure of each thermal space into the model.

2. PRELIMINARIES

This section provides information regarding the simulation building considered for this study, as well as some fundamental definitions regarding state-space models.

2.1 Case Study Building

The case study building is a model of a two-story school building, with a floor area of 24,000m² (258,000ft²) with 46 thermal zones. A detailed thermal model of the building was created in EnergyPlus. The advantages of having such a model will be explained in Section 3.

The zones considered include a large diversity of spaces: small offices, classrooms, long hallways and two gymnasias. Because of this diversity, some zones span both the main and the upper floor. Also, as it can be seen in Figure 1, the western side of the building is divided into three wings. All these factors cause the number of zones adjacent to each zone varies greatly.

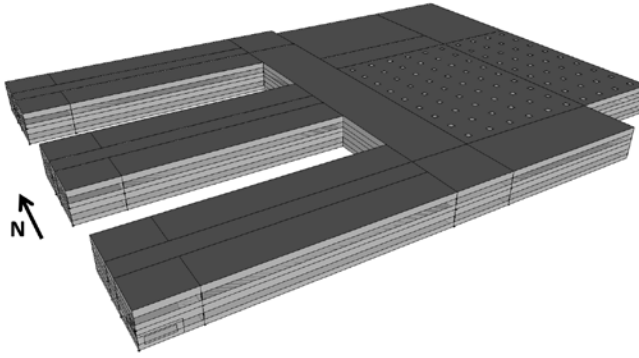


Figure 1: The sketch of the school building

2.2 State-space Representation

A state-space representation is a mathematical model of any physical system in which inputs, outputs and state variables are related using first-order differential equations. Consider the LTI discrete-time multi-input multi-output (MIMO) model of a system, expressed in state-space form:

	$\mathbf{x}[i+1] = \mathbf{A}_{n \times n} \mathbf{x}[i] + \mathbf{B}_{n \times p} \mathbf{u}[i]$ $\mathbf{y}[i] = \mathbf{C}_{q \times n} \mathbf{x}[i] + \mathbf{D}_{q \times p} \mathbf{u}[i]$	(1)
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where matrices **A**, **B**, **C** and **D** relate the input and output signals; i indicates the discrete time sample and vectors $\mathbf{u} \in \mathbb{R}^p$, $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^q$ represent input signal, state vector and output signal, respectively. In case of a multi-input single-output (MISO) system, **C** and **D** will become row vectors.

2.3 State-space models: determination through RC networks vs. direct derivation from data

According to the definition of state-space models, the RC models of thermal spaces can also be represented in state-space format. In this case, the order of the state-space model or n , as in equation (1), will be equal to the number of capacitors included in the RC model, which is the same as the number of differential equations required to express the dynamics of the RC circuit. A standardized method for the transformation from RC network to state-space for thermal networks was proposed by (Candanedo et al., 2013).

Since the heat transfer phenomena in the building are mainly described by linear differential equations, it is reasonable to assume that, as a whole, the thermal space behaves a linear dynamical system (Putta *et al.*, 2014). Given this important assumption, it is also possible to obtain a state-space model *directly from recorded data*. Directly identifying a state space model from measured data provides a significantly higher number of degrees of freedom as compared to a derivation from a calibrated RC model. This is because every individual coefficient in the matrices **A**, **B**, **C** and **D** can be considered as a degree of freedom, whereas in the RC model the degree of freedom is equal to the total number of resistors and capacitors. The main drawback of obtaining such “free-form” state-space model is that the state vector \mathbf{x} cannot be readily associated with physical quantities or features of the thermal space. In contrast, in a state-space representation derived from an RC thermal network model, \mathbf{x} (the vector of states) corresponds to temperatures in the building. Figure 2 summarizes the above discussion.

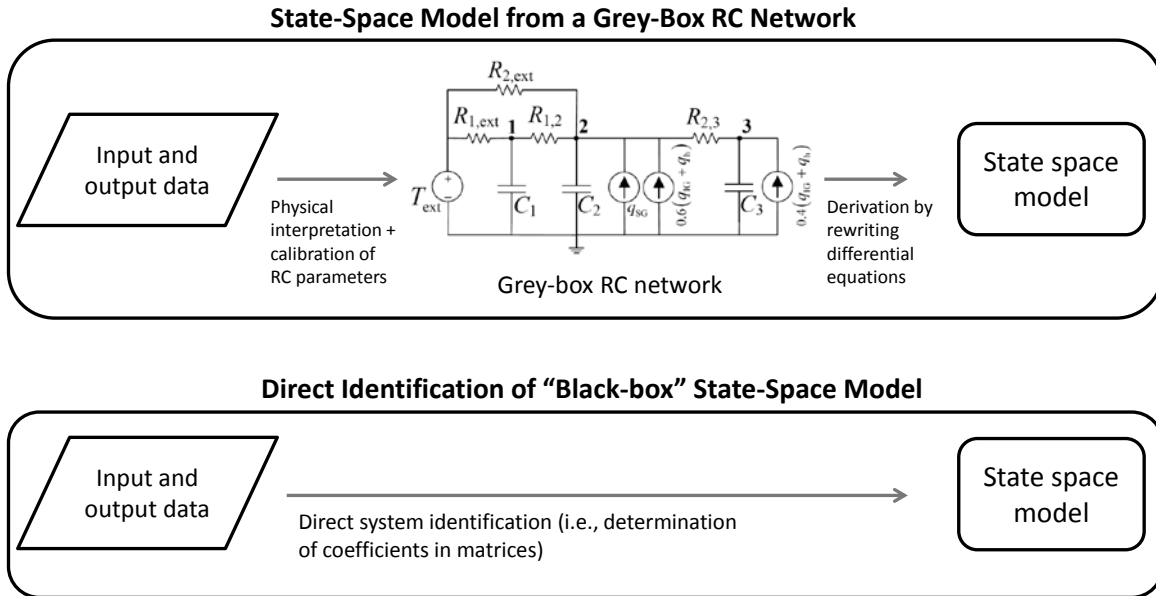


Figure 2: Comparison of state-space models obtained through RC circuits and directly from data.

3. MODELING

As mentioned in Section 2.1, a detailed model of the school building was developed on the EnergyPlus platform, which allows running long-term simulation scenarios or “virtual experiments” and recording practically any required performance index. The weather data file used for simulations contains various weather indices, such as solar radiation, which determines the solar gains (q_{SG}) into the building, and outdoor temperature (T_{ext}). Internal heat gains (q_{IG}), corresponding to the heat released by lights, equipment and occupants, are assumed to follow semi-rectangular, uniform pulses according to the typical operation of a school building (Dehkordi and Candanedo, 2014).

Each thermal zone is individually modeled as a MISO LTI discrete-time state-space system. The output of each model is the indoor temperature (T_{in}) of its corresponding thermal zone (EnergyPlus output), while T_{ext} , q_{SG} and q_{IG} are considered to be the input signals. The heating/cooling power (q) injected to each thermal zone is also treated as an input.

The structure of the EnergyPlus thermal zones is assumed not to be adiabatic, meaning that there will be an exchange of heat between different thermal spaces. In other words, the thermal dynamics of each thermal zone will affect the thermal dynamics of neighboring zones. Knowing this, four modeling scenarios are considered:

- 1) The mutual effects between zones are ignored. In other words, the thermal response of each zone is considered to be solely affected by the input factors directly affecting it, i.e., T_{ext} , q_{SG} , q_{IG} and q .
- 2) The heating/cooling power delivered to a certain thermal zone is considered to have an effect on thermal dynamics of its adjacent thermal zones. It means that beside T_{ext} , q_{SG} , q_{IG} and q , a fifth input is considered which is the sum of the q delivered to adjacent zones (q_{adj}).
- 3) The indoor temperature of a certain thermal zone is considered to have an effect on thermal dynamics of its adjacent thermal zones. Similar to the previous case, beside T_{ext} , q_{SG} , q_{IG} and q , the indoor temperature of the adjacent zones, T_{adj} are also individually considered as inputs. In other words, the number of inputs in this case will vary depending on the number of adjacent zones.
- 4) Similar to 3, the indoor temperature is considered to have an effect on thermal dynamics of its adjacent thermal zones. In this case, however, the *average* indoor temperature of the adjacent zones, \bar{T}_{adj} , is considered as the fifth input.

Note that using models generated following scenarios 2 to 4 in a building control algorithm requires for each zonal model to have access to real-time data from its adjacent zones. Figure 3 shows the model structures considered:

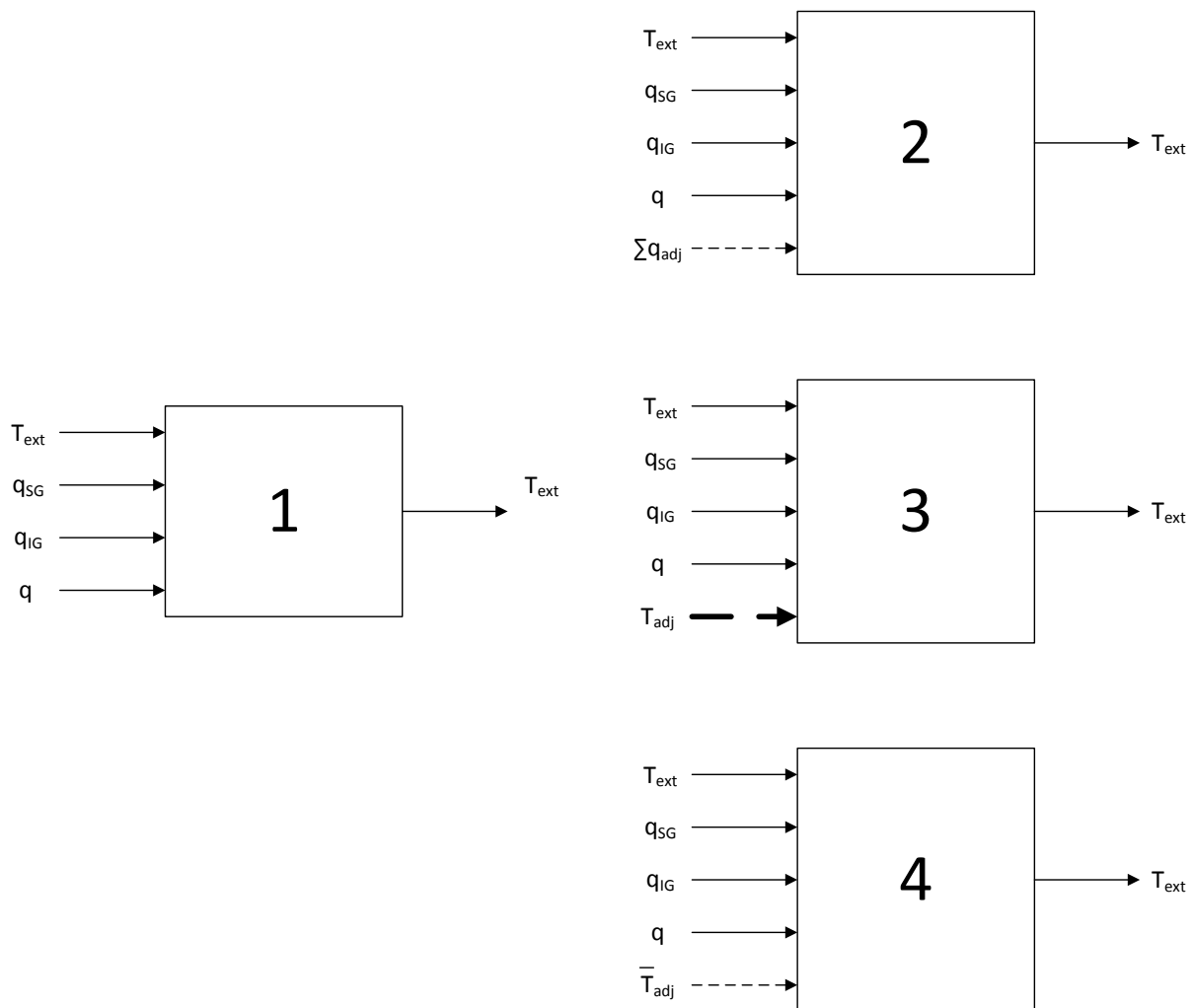


Figure 3: The four modeling scenarios

The EnergyPlus simulation data is reported at 15-minute intervals for each thermal zone. It provides the portion of total solar gains received by the building applicable to each thermal zone, which in practice can be measured on-site. Also, the internal gains follow a typical school profile and the outdoor temperature is the same for all thermal zones. The q for each thermal zone is provided for a typical temperature setpoint of 23°C.

For every zone, the adjacent zones are defined as the ones who share a wall or floor, i.e., corners are not considered as connection between any two zones. Figure 4 shows the floor map as well as the number of adjacent zones for each zone. Note that the empty spaces in the second floor are mostly the zones which extend to both floors (e.g., gymnasium with a high ceiling). Also, the color map is chosen in such a way that the darker color, the smaller the value (here the number of adjacent zones) assigned value is.

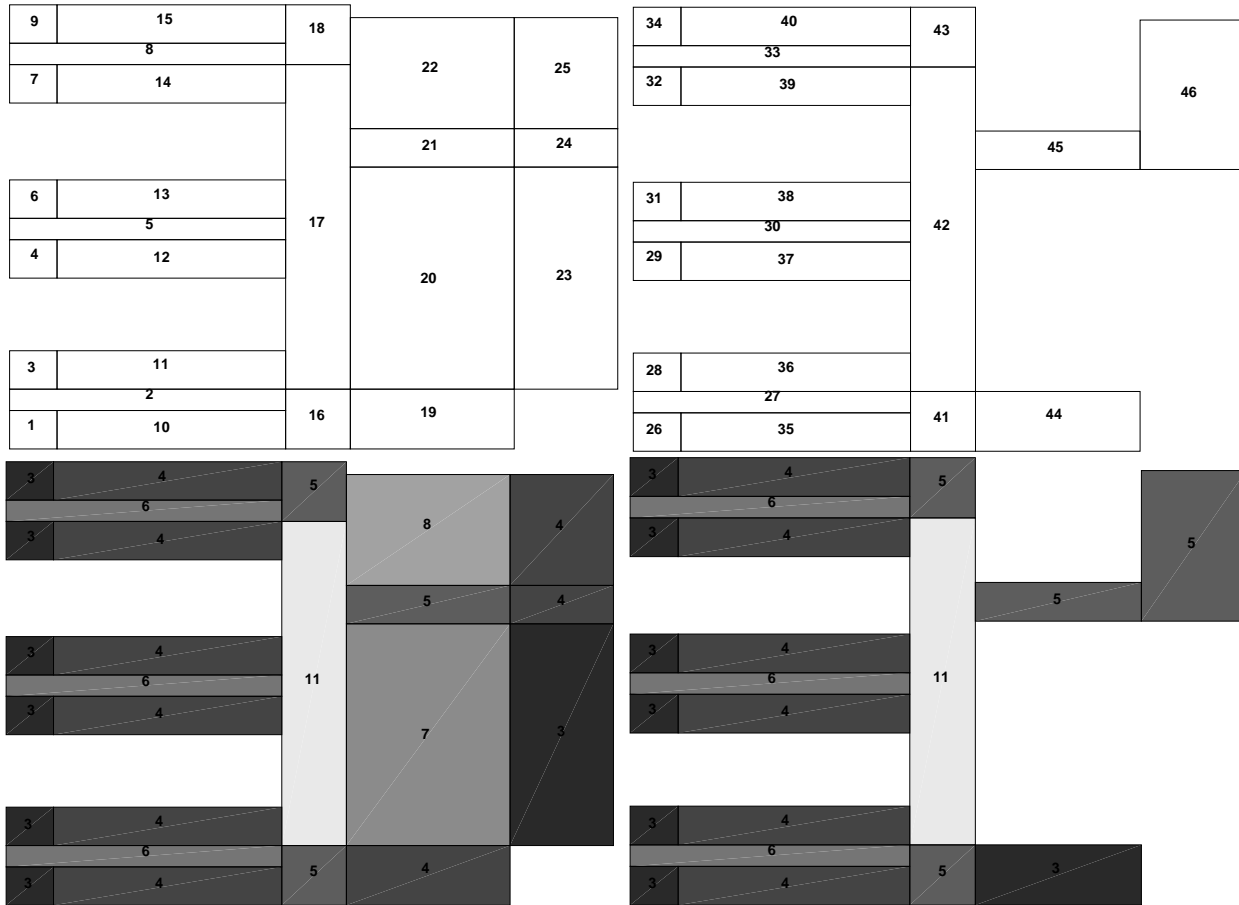


Figure 4: The floor map of first (left) and second (right) floors with zone indices (top) and the color-coded number of adjacent zones (bottom)

4. SIMULATION RESULTS

As discussed in Section 3, state-space models are developed for each thermal zone, following all four scenarios depicted in Figure 3. The order of the models is set to be varying between 1 and 10, therefore putting a hard cap on the maximum order of the state-space models. The system identification is performed using *ssest* command in MATLAB software environment, which uses a state-space subspace system identification (4SID) approach. Following Equation (1), no noise model is taken into account. Figure 5 shows the best modeling RMSE achieved for each zone, for all four scenarios.

As depicted in Figure 5, the performance of scenario 2 is not that much different from the base scenario 1 since the least RMSE achieved for individual thermal zones are generally very close. Also, the number of cases indicating otherwise is far less than being meaningful. The performance of scenario 3, however, shows a substantial improvement of the performance as the average RMSE value drops by a factor of 3.

Figure 6 shows the order of the model resulting in the least RMSE for each thermal zone following scenarios 1, 3 and 4. As it can be clearly seen, the location and the orientation of the zones have an important impact on the modeling performance when adiabatic conditions are assumed (scenario 1). For example, at the first floor, the zones facing southwest and north west – zones 1, 3, 4, 6, 7 and 9 (see Figure 4) - have higher orders compared to south facing zones (10, 12 and 14). Observing the numerical results in Figure 5 reveals that using the same modeling setup, the error level is more or less in the same range for zones 35 to 40. Zones 36, 38 and 40 require only a second-order model to reach the lowest error level. This becomes more interesting since at the same time, the south-

facing zones 35, 37 and 39 require have optimal modeling orders of 6 and 7. Therefore, the orientation of thermal zones definitely plays a role in their thermal response and subsequently, the performance of modeling.

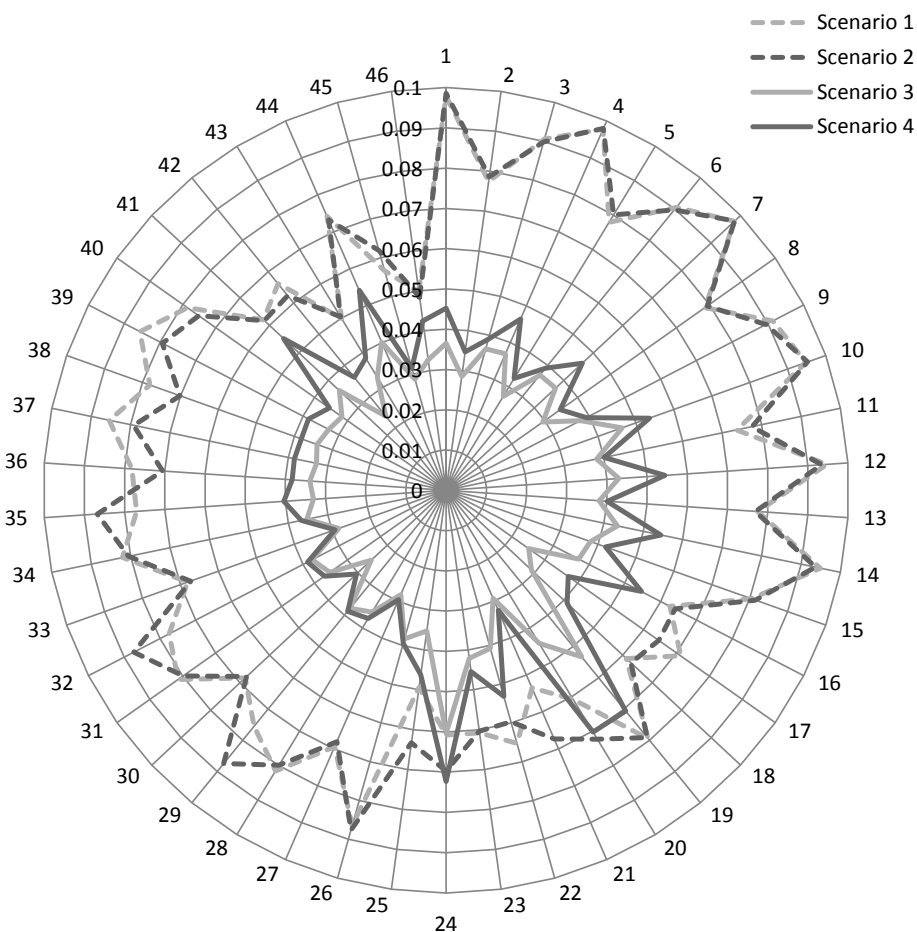


Figure 5: The modeling RMSE achieved for all 46 zones using all modeling scenarios

Focusing now on the results of scenario 3, the optimum model orders for both floors seem to be quite high for all thermal zones. However, it should be noted that upon observing all RMSE values obtained from all ten models developed for each zone (not shown in this paper), the variation in modeling performance for each individual thermal zone in this scenario is relatively low, often negligible – sometimes less than 5% of the best value. Therefore, one can safely reduce the order to much lower values, even 2 to 4, and still benefit from a low energy of modeling error. In other words, relatively high order of the models depicted in figure 6 (right) is purely because of numerical optimization and does not have anything to do with the geometrical characteristics of the thermal zones.

Scenario 4 plays the role of an alternative between scenarios 2 and 3: similar to the model structure in scenario 2, it has only one additional input compared to that of scenario 1, but its performance is more or less similar to that of scenario 3, as it can be seen in Figure 5. By observing Figure 6, it is even more interesting to see that (unlike scenario 2) the model orders can be somewhat associated with certain orientations and indoor locations. For example, the north facing zones have relatively smaller orders while the corner zones facing west, somehow similar to scenario 1, have higher orders. Interior thermal zones have relatively low orders; the ones with high orders exhibit the same numerical situation of scenario 3: the variation in modeling performance is often negligible which means that, for example, zones 2, 27 and 33 can safely be modeled with orders as low as 4 or 5, similar to the zones 5, 8 and 30.

Back to Figure 5, it can be seen that in all modeling scenarios, in general, the least RMSE associated with a certain thermal zone on the second floor is lower than that of its equivalent on the first floor. Also, given the fact that the optimum model order for some zones in scenario 1, e.g. zone 1 or 4, is low while their corresponding RMSE is

relatively high, one can conclude that there is a very little chance that increasing the order of model beyond the maximum 10 would do any better. At the same time, if the modeling performance is crucial, i.e. very low error level/RMSE is needed, scenarios 3 and 4 are the most prone to leading to better results if the model order increased, although by a small extent.

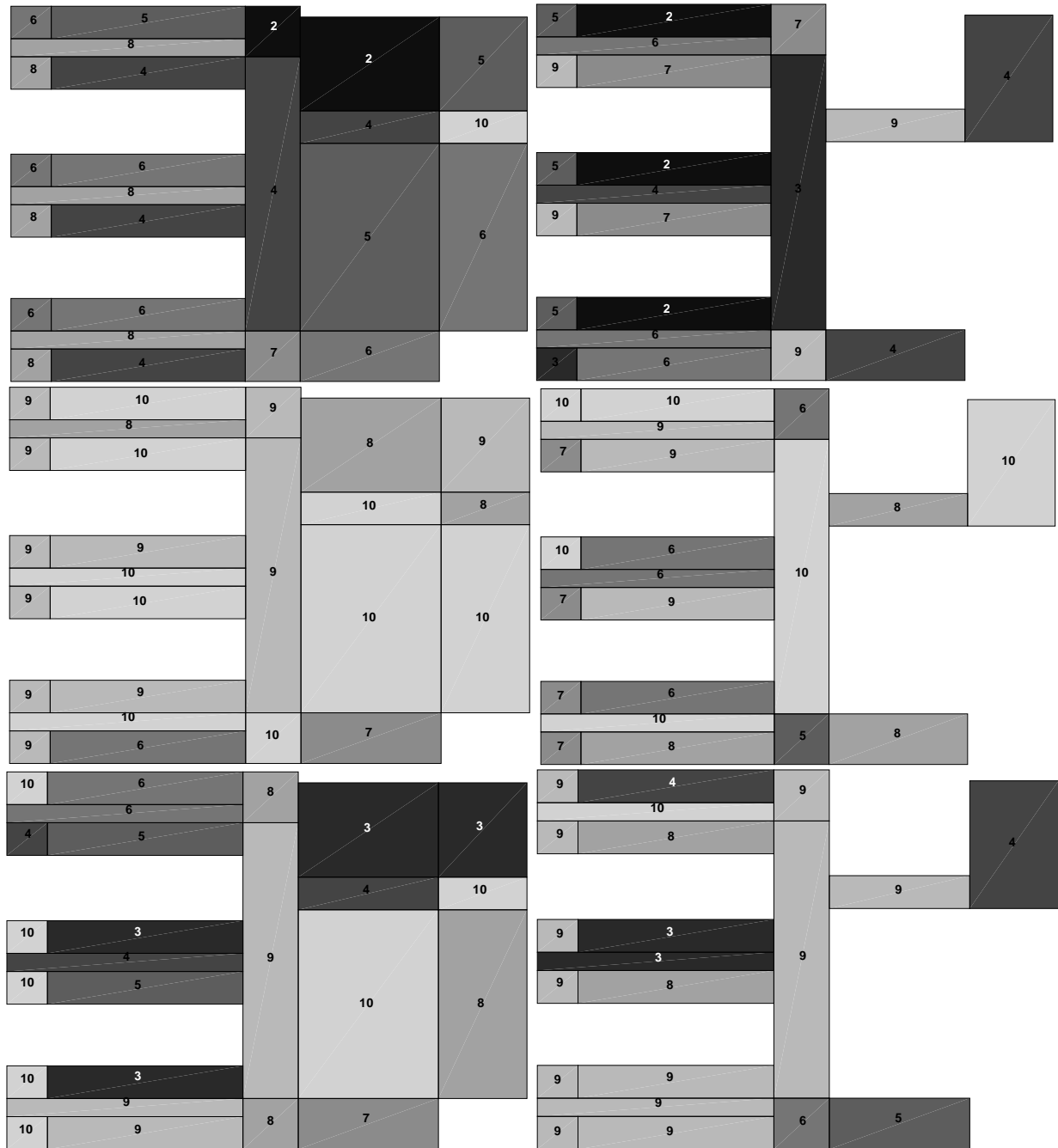


Figure 6: Optimum model orders in 1st (top), 3rd (middle) and 4th (bottom) scenarios

5. CONCLUSIONS

Four modeling scenarios have been presented in this paper to assess the performance of linear state-space models for the representation of thermal response of the thermal spaces in a model school building. Simulation data provided by EnergyPlus (the building simulation environment) has been used directly to create the state-space models (black-box modeling). When adiabatic conditions are assumed, the optimum order of the models can be associated with the geographical location and orientation of the thermal zones. Depending on which parameters are considered as input variables, taking the indoor temperature of the adjacent zones into account proves to be useful in reducing the error levels while still maintaining a ≤ 10 model order. Also, taking the average temperature of the adjacent zones proves to be useful in reducing the error while leading to even lower model orders and keeping the number of input variables low. Future works will include the comparison of the performance of different types of RC models with that of state-space models with the same order.

REFERENCES

- Afram, A., Janabi-Sharifi, F., 2014. Review of modeling methods for HVAC systems. *Applied Thermal Engineering* In Press.
- Candanedo, J. A., Dehkordi, V. R., Lopez, P., A control-oriented simplified modelling strategy. 13th International Building Simulation Conference (IBPSA 2013), August 26-28 2013 Chambéry, France.
- Candanedo, J. A., Dehkordi, V. R., Tamasauskas, J., Impact of heat pumps and predictive control on residential energy use, load and grid interaction: a Canadian perspective. 14th International Building Simulation Conference (IBPSA 2015), December 7th-9th 2015 Hyderabad, India.
- Cole, W. J., Powell, K. M., Hale, E. T., Edgar, T. F., 2014. Reduced-order residential home modeling for model predictive control. *Energy and Buildings* 74, pp. 69-77.
- Dehkordi, V. R., Candanedo, J., Multi-level model predictive control of a building. ASME 2014 Dynamic Systems and Control Conference (DSCC 2014), October 22-24 2014 San Antonio, Texas.
- Janczura, J., Maciejewska, M., Szczurek, A., Wyłomanska, A., 2013. Stochastic modeling of indoor air temperature. *Journal of Statistical Physics*.
- Kim, D., Braun, J. E., Reduced-order building modeling for application to Model-Based Predictive Control. SimBuild 2012 August 1-3 2012 Madison, Wisconsin.
- Prívará, S., Cigler, J., Vana, Z., Oldewurtel, F., Sagerschnig, C., Zacekova, E., 2012. Building modeling as a crucial part for building predictive control. *Energy and Buildings* In Press.
- Putta, V., Kim, D., Cai, J., Hu, J., Braun, J. E., Distributed model predictive control for building HVAC systems: a case study. Third High Performance Buildings Conference, July 14-17 2014 Purdue University, Indiana.
- Saberi-Derakhtenjani, A., Candanedo, J. A., Chen, Y., Dehkordi, V. R., Athienitis, A. K., 2015. Modeling approaches for the characterization of building thermal dynamics and model-based control: A case study. *Science and Technology for the Built Environment* 21, pp. 824-836.