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A Distributed Model Predictive Control Approach for Optimal Coordination of Multiple Thermal Zones in a Large Open Space

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ABSTRACT

Model Predictive Control (MPC) based approaches have recently seen a significant increase in applications to the supervisory control of building heating, ventilation and air-conditioning (HVAC) systems, thanks to their ability to incorporate weather, occupancy, and utility price information in the optimization of heating/cooling strategy while satisfying the physical constraints of HVAC equipment. Many of the proposed MPC solution approaches are centralized ones that often suffer from high computational cost caused by the large number of decision variables and the overhead in information gathering and distribution. This paper investigates a distributed MPC approach based on a variant of the Alternating Direction Method of Multipliers (ADMM). The proposed method is highly scalable, and facilitates a device level plug-and-play implementation. A case study is carried out on one of the Purdue Living Labs, which is an open office space of multiple thermal zones with individual thermostat controls. In view of significant thermal couplings due to direct air exchange and noticeable load gradient between zones, a multiple thermal zones coordination problem is formulated with the objective of optimally scheduling the different thermostat setpoints for energy minimization and comfort delivery while satisfying actuation constraints. Simulation results demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

The building sector consumes about 40% of the energy used and is responsible for about 40% of the green house gas emissions in the United States (McQuade, 2009). Hence, reducing the energy consumption of buildings is of huge economical and environmental importance. Many commercial buildings consist of multiple thermal zones. Each zone may represent an individual room, a partially enclosed space, or even a part of a room in an open space. The thermal dynamics of different zones in a building are typically coupled, either through shared HVAC equipments, or via heat transfer across walls, door openings or direct air exchange in the case of open space.

The problem of controlling multi-zone buildings with coupled zone thermal dynamics has received increased attention in the building control community. Various intelligent control techniques have been proposed for reducing the overall energy consumption of such buildings while maintaining and improving the occupant comfort. To name a few, genetic algorithms are used in (Mossolly, Ghali, & Ghaddar, 2009) to obtain an optimal control strategy for a multi-zone air conditioning system. Particle Swarm Optimization (PSO) is utilized to optimize the building energy management in (Yang & Wang, 2013).

In particular, a control technique that has been especially popular in the recent years is Model Predictive Control (MPC). Due to its ability of incorporating weather forecasts and occupancy profiles in real time decision making, MPC provides a powerful and practical solution to mamy building control problems (Ma et al., 2012), (Privara, Široký, Ferkl, & Cigler, 2011), (Oldewurtel et al., 2012). In the MPC framework, A receding horizon optimal control problem is formulated and solved at each time step, but only the first control action will be applied to the building system while the procedure is repeated at the next time step with updated information (weather, solar radiation, occupancy gains, etc).

However, centralized MPC approaches often have difficulty in dealing with large-scale building clusters or even a single building consisting of multiple thermal zones due to the large number of decision variables that are distributed across different parts of the system. In addition, repetive engineering costs of designing controllers for different building systems with similar multiple thermal zone structure is formidable. Therefore, a distributed control strategy that utilizes the special structure of multi-zone buildings, while at the same time have good scalability is highly desired.

This paper aims at designing a general Distributed MPC (DMPC) strategy that is capable of dealing with not only coupling in the thermal dynamics, but also coupling in HVAC equipments. The DMPC method takes advantage of a decomposition technique called Proximal Jacobian Alternating Direction Method of Multipliers (PJ-ADMM) and is highly scalable with respect to the size of the building system.

The proposed method is applied to a case study on one of the Purdue Living Labs. A optimal coordination problem of multi-zones with individual thermostat controls is formulated and solved with the proposed controller. Simulation results demonstrates the effectiveness of the proposed distributed controller, and the electricity saving potential of inter-zonal coordination.

The rest of this paper is organized as follows. In Section 2, the case study building and HVAC system models are discussed. The centralized optimal multiple thermal zone coordination problem is formulated in Section 3. The proposed distributed MPC approach is introduced and explained in Section 4. Section 5 presents the simulation results and discusses their implications. Finally, some concluding remarks and future directions are given in Section 6.

2. CASE STUDY MODEL

In this section, a detailed description of the case study model is given. The case study model consists of two parts, namely the room envelope model and the rooftop units (RTUs) equipment model. The room envelope model captures the thermal dynamics of the room space under the influence of sensible cooling/heating provided by HVAC systems. The RTU equipment subsection explains the input-output relation of the RTU model used as well as some realistic simplifications and assumptions.

2.1 Envelope Model

The Purdue Living Lab is a large open space located at the Center for High Performance Buildings, West Lafayette, IN. It has a south-facing double façade. The room serves not only as a test-bed for building thermal, HVAC system research, but also as the office for 16 graduate students. A multi-zone thermal network state-space model of the room is obtained with estimated model parameters from building construction information. The structure of the room is given in Figure 1.

As can be observed from Figure 1, the room space is divided into three zones based on their relative distance to the south facing double façade. The double façade system is equivalent to a "thermal buffer" between room and ambient environment from the heat transfer point of view. In addition, solar radiation, occupancy schedule and even local thermal comfort preference may be significantly different from one zone to another. All these factors could cause a load imbalance between different zones. Indeed, preliminary experiment results show that there is a noticeable 2°C zone temperature gradient across the office space under normal operation conditions.

A general linear discrete-time thermal dynamics of one individual zone in a multiple zone building is given by

$$\begin{aligned} x_i(k+1) &= A_{ii}x_i(k) + \sum_{j\in\mathcal{N}_i} A_{ij}x_j(k) + B_{ii}Q_i(k) + \sum_{j\in\mathcal{M}_i} B_{ij}Q_j(k) + F_i w_i(k), \\ T_i(k) &= C_i x_i(k), \end{aligned} \tag{1}$$

where $k \in \mathbb{N}$, $x_i(k) \in \mathbb{R}^{n_i}$ is the local state variable of zone i, including average zone temperature and a lumped wall/floor temperature. $Q_i(k) \in \mathbb{R}^{m_i}$ is the sensible cooling or heating directly injected into zone i by the HVAC, which is the controllable input. $w_i(k) \in \mathbb{R}^{p_i}$ are the exogenous inputs including solar radiation, ambient temperature, occupancy gains, etc., which are uncontrollable but assumed to be predictable. $T_i(k)$ is the average temperature of zone i, and C_i capture its dependency on $x_i(k)$. $A_{ij} \in \mathbb{R}^{n_i \times n_j}$ are constant matrices denoting the thermal coupling between different zones. In our case, this coupling is caused by the direct air exchange between adjacent zones; in other cases, the heat

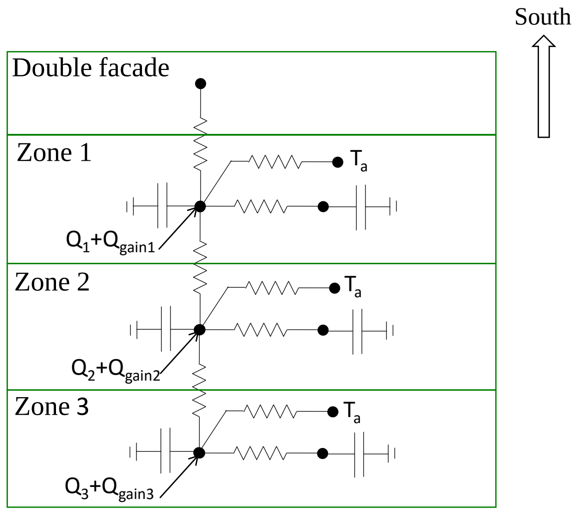

Figure 1: Purdue Living Lab multi-zone thermal network structure

exchange channel may be an open door, a shared corridor or windows between two rooms. $B_{ij} \in \mathbb{R}^{n_i \times m_i}$ are constant matrices that representing the coupling in the HVAC system (zero matrices in this case study). $\mathcal{N}_i$ and $\mathcal{M}_i$ represent the set of neighbouring zones that have influence on x_i through x_j and Q_j, respectively.

The lumped envelope model of the whole thermal system can be obtained by combining the dynamics of individual zones

$$\begin{aligned} x(k+1) &= Ax(k) + BQ(k) + Fw(k), \\ T(k) &= Cx(k), \end{aligned} \tag{2}$$

where $x(k)$, $Q(k)$, $w(k)$ and $T(k)$ are the concatenation of corresponding zone-level local variables. Because of the thermal couplings between zones, A is not block diagonal. The thermal interactions between zones give opportunity for coordination between zones in order to minimize the energy consumption of the HVAC system, more details will be discussed in later sections.

2.2 RTU Equipment Model

In the case study, we assume that each zone in the Purdue Living Lab is served by a dedicated RTU with individual thermostat control. RTUs are packaged direct-expansion type cooling systems that regulate the temperature and circulate air as part of a HVAC system. Such air-conditioning systems have straightforward configurations and are easy to install and maintain. Thus, they are widely used in small to medium commercial buildings. In the US, RTUs are responsible for approximately 60% of the installed cooling capacity in the commercial building sector. The proposed DMPC methodology provides a generic means to coordinate the different RTUs serving a large open space.

In this study, a correlation-based RTU model is developed from manufacturer catalogue data and the capacity of RTUs have been scaled down to meet the requirements of this specific study. However, the characteristics of the model remain unchanged. The input and output relation of the RTU model used is given in

$$[Power, PLR] = RTU(Q_{sen}, T_{amb}, T_{wb}, T_{db}), \tag{3}$$

where inputs of the RTU are: (1) Q_{sen}, sensible cooling load of the building; (2) T_{amb}, ambient temperature; (3) T_{wb}/T_{db}, nominal wet/dry bulb temperatures and are assumed to take constant values of 15.3/28.9^{o}C. The outputs are the instant power consumption *Power* of unit and its part-load ratio *PLR*. The coefficient of performance of the unit, namely, *COP* varies with the ambient temperature T_{amb}, and in general lower ambient temperature provides higher unit efficiency.

Note that at part load conditions, RTUs primarily rely on cycling to match the building loads and the RTU models use degradation curves (coefficients of degradation assumed to equal 0.2) to capture the efficiency decrease caused by

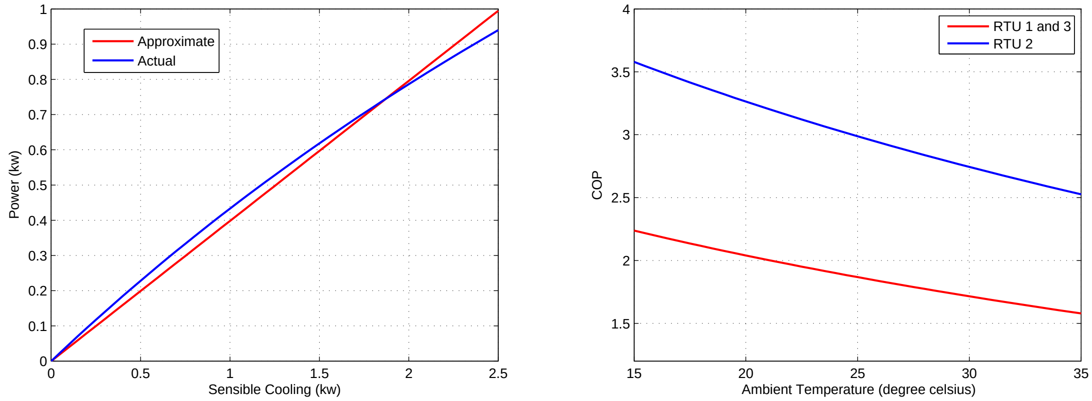

Figure 2: RTU characteristics left: *Power* **vs.** Q_{sen} **(**T_{amb} = 17.9°**C) right:** *COP* **vs.** T_{amb}

RTU cycling. However, such degradation effect creates a slightly concave mapping from Q_{sen} to the RTU power. To convexify the problem, linear approximations of the power consumption function with respect to Q_{sen} were obtained under various ambient temperatures T_{amb}. Note that this approximation captures the averaged unit efficiency within a range of cooling load and is different than a zero coefficient of degradation. This approximation will not cause noticeable inaccuracy in practice, but can significantly help guarantee the theoretical performance of the controller to be designed. The power consumption vs. sensible cooling relation under $T_{amb} = 17.9$ °C is plotted in Figure 2 (left) as an example.

Among the three RTUs, RTU 2 (serving zone 2) has the largest capacity (around 3kw), while RTU 1 and 3 are relatively smaller in size (around 2.5kw). In addition, under the same T_{amb}, RTU 2 is the most efficient unit, that is, it has a larger *COP* compared to RTU 1 and 3 under the same ambient temperature. The plot of *COP* vs. T_{amb} is given in Figure 2 (right) to demonstrate the efficiency difference between RTUs under various ambient temperatures.

3. OPTIMAL MULTI-ZONE COORDINATION

As mentioned before, RTUs are prevalent in many open-spaced commercial buildings (restaurant, retail stores, etc). Most of the current RTU operation strategies are based on simple feedback control logic in response to individual thermostat setpoints for the zone temperature. The lack of inter-zonal coordination in the traditional control strategy usually results in a higher total energy consumption of the building.

It is particularly beneficial to seek coordination between different units in a large open space, especially when unit efficient differences are significant. A multiple thermal zone coordination problem is a supervisory control problem, with individual thermostat setpoint temperatures as well as sensible coolings provided by RTUs being the decision variables. It is assumed that the RTU equipment level local (PID) controller is fast enough to respond to the optimal Q_{sen} and thermostat setpoint temperatures. In other words, we are trying to find the optimal thermostat setpoint temperature *trajectories* and the corresponding RTUs operations.

The optimal multiple thermal zone coordination problem will be formulated as a finite horizon optimal control problem with the objective of minimizing the total electricity consumptions in a receding horizon fashion, i.e. via model predictive control (MPC). Besides incorporating exogenous inputs information (weather, occupancy, etc.) into the optimization, another major reason for using MPC is that it is able to take advantage of the thermal storage capability of building itself so that load shifting from peak hours (possibly higher utility rates) to off-peak hours becomes a possibility.

3.1 Centralized MPC Formulation

For notational simplicity, constant variables T_{wb}, T_{db} will be neglected from the RTU model. Also, index of the current time step will be omitted, and we will use $x_i(k)$ to represent the predicted state variables of zone i that are k steps ahead of the current step. If we denote the approximate power consumption function for RTU i as $P_i(Q_i(k), T_{amb}(k))$, the centralized optimal multi-zone coordination problem can be formulated under the MPC framework as

$$\underset{T(k),\,Q(k)}{\text{minimize}} \quad J = \sum_{k=0}^{N-1}\sum_{i=1}^{3} \mu(k)\cdot P_i(Q_i(k),\, T_{amb}(k)) \tag{4}$$

$$\text{subject to} \quad x(k+1) = Ax(k) + BQ(k) + Fw(k), \quad T(k) = Cx(k), \tag{5}$$

$$T_{min,i}(k) \le T_i(k) \le T_{max,i}(k), \tag{6}$$

$$0 \le PLR_i(Q_i(k),\, T_{amb}(k)) \le 1, \tag{7}$$

$$i = 1,2,3, \quad k = 0,1,\ldots,N-1,$$

where N is the look-ahead horizon (prediction horizon), $\mu(k)$ is the utility price (possibly time-varying). $T_{min,i}$ and $T_{max,i}$ are the lower and upper bounds of the temperature of zone i, respectively, which together specify the "comfort interval" for the occupants sitting in zone i, and can be locally customized. PLR_i is the part-load ratio of RTU i, and it should be less or equal to 1 at all times.

In this study, we assume that the ambient temperature information is predictable through weather forecast. Therefore, at each time instant, a new linear approximation of P_i can be obtained for the current $T_{amb}(k)$. Similarly, unit capacities $Q_{max,i}(k)$ at each time instant could be obtained according to $T_{amb}(k)$. With this simplification, the cost function (4) can be re-written as a single linear function in $Q_i(k)$, and RTU operation constraint (7) can be replaced by

$$0 \le Q_i(k) \le Q_{max,i}(k),$$

which consists of two linear constraints on part of the decision variables. Notice that the other two constraints (5) and (6) are also linear constraints on the decision variables $T(k)$ and $Q(k)$. Therefore, the centralized MPC problem (4) for optimal multi-zone coordination under the current formulation is a linear program.

Optimization problem (4) can be more compactly written as

$$\underset{\mathbf{x},\,\mathbf{Q}}{\text{minimize}} \quad J = \sum_{i=1}^{3} \tilde{f}_i^{\top}\mathbf{Q}_i \tag{8}$$

$$\text{subject to} \quad \mathbf{x} = \mathbf{\Omega}x(0) + \mathbf{\Phi}\mathbf{Q} + \mathbf{\Psi}\mathbf{w}, \tag{9}$$

$$\mathbf{x}_{min,i} \le \mathbf{x}_i \le \mathbf{x}_{max,i}, \quad i = 1,\ldots,3, \tag{10}$$

$$\mathbf{Q}_{min,i} \le \mathbf{Q}_i \le \mathbf{Q}_{max,i}, \quad i = 1,\ldots,3, \tag{11}$$

where $\tilde{f}_i$ is the linear coefficient for the power consumption approximation function of RTU i, and

$$\mathbf{x} = [x^{\top}(1)\cdots x^{\top}(N)]^{\top}, \quad \mathbf{Q} = [Q^{\top}(0)\cdots Q_i^{\top}(N-1)]^{\top}, \quad \mathbf{x_i} = [x_i^{\top}(1)\cdots x_i^{\top}(N)]^{\top}, \quad \mathbf{Q}_i = [Q_i^{\top}(0)\cdots Q_i^{\top}(N-1)]^{\top},$$

$$\mathbf{x}_{min,i} = [x_{min,i}^{\top}(1)\cdots x_{min,i}^{\top}(N)]^{\top}, \quad \mathbf{Q}_{min,i} = [Q_{min,i}^{\top}(0)\cdots Q_{min,i}^{\top}(N-1)]^{\top},$$

$$\mathbf{x}_{max,i} = [x_{max,i}^{\top}(1)\cdots x_{max,i}^{\top}(N)]^{\top}, \quad \mathbf{Q}_{max,i} = [Q_{max,i}^{\top}(0)\cdots Q_{max,i}^{\top}(N-1)]^{\top},$$

$$\mathbf{\Omega} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}, \quad \mathbf{\Phi} = \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix}, \quad \mathbf{\Psi} = \begin{bmatrix} F & 0 & \cdots & 0 \\ AF & F & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}F & A^{N-2}F & \cdots & F \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} w(0) \\ w(1) \\ \vdots \\ w(N-1) \end{bmatrix}.$$

Observing optimization problem (8), it should be noted that the cost function (8), as well as the constraints (10) and (11) are all readily separable with respect to each zone and their RTUs. However, the thermal dynamics equality constraint (9) is a coupling constraint between all three zones.

Linear programs in general can be solved very efficiently. However, in practice, as the number of zones and its corresponding variables grow, the communication costs in collecting and diffusing variables between local zones and central computing facility through Building Management System (BMS) could become prohibitive. In particular, if we are dealing with building clusters, in which case a zone may be a single building or a group of buildings (optimal coordination problem becomes coordinating demands between buildings to reduce total demand charge), this problem becomes more challenging.

In addition, we are seeking local control strategies that can be potentially embedded into HVAC devices (RTUs in this case), such that instead of transmitting all local information to a central computer for computation, the optimization problem could be solved distributedly and cooperatively by local agents through simple information exchange with each other. This type of distributed control strategies will have much better scalability compared to centralized ones when dealing with large-scale building systems. One of the possible ways to design such distributed control strategies is through distributed MPC (DMPC), and a novel DMPC algorithm for solving problems in the form of (8) is proposed in Section 4.

4. DISTRIBUTED MPC VIA PROXIMAL JACOBIAN ADMM

In a general DMPC formulation, the centralized problem are solved cooperatively by a group agents through information exchanging and broadcasting. Each agent takes responsibility for a small part of the decision variables (corresponding to one zone in this case) and may not be aware of the global system model. Many existing works on DMPC are based on agent negotiation (Maestre, De La Pena, Camacho, & Alamo, 2011) or distributed optimization. Dual decomposition method was applied to DMPC in (Wakasa, Arakawa, Tanaka, & Akashi, 2008) and (Farokhi, Shames, & Johansson, 2014), while (Farokhi et al., 2014) also used Gauss-Seidel alternating direction method of multipliers (ADMM). However, dual decompostion needs very strong conditions for convergence, and the convergence speed is often slow in practice due to improper tunings of stepsizes for gradient update of the dual problem. As for standard Gauss-Seidel ADMM (Boyd, Parikh, Chu, Peleato, & Eckstein, 2011), it uses a sequential update scheme between agents instead a parallel one, which can be more efficiently implemented. We refer interested readers to (Scattolini, 2009) and (Christofides, Scattolini, de la Peña, & Liu, 2013) for more recent work on DMPC.

In this study, a new DMPC method using Proximal Jacobian ADMM (PJ-ADMM), a variant of the traditional ADMM is proposed. PJ-ADMM is based on primal-dual optimization scheme, and it incorporates the parallel update scheme from dual decomposition while at the same time shares robust convergence behavior as Gauss-Seidel ADMM.

4.1 Distributed Primal-Dual Optimization Scheme

First, we concatenate decision variables $\mathbf{x}$ and $\mathbf{Q}$ into a single variable $\mathbf{z}$, and rewrite optimization problem (8) as

$$\underset{\mathbf{z}}{\text{minimize}} \quad J = \sum_{i=1}^{3} f_i^\top \mathbf{z}_i \tag{12}$$

$$\text{subject to} \quad \mathbf{A}\mathbf{z} = \mathbf{b}, \tag{13}$$

$$\mathbf{z}_{min,i} \leq \mathbf{z}_i \leq \mathbf{z}_{max,i}, \quad i = 1, \ldots, 3, \tag{14}$$

where $\mathbf{z} = \left[\mathbf{x}^\top,\, \mathbf{Q}^\top\right]^\top$, and

$$f_i = \begin{bmatrix} 0^\top & \tilde{f}_i^\top \end{bmatrix}^\top, \quad \mathbf{A} = \begin{bmatrix} \mathbf{I} & -\mathbf{\Phi} \end{bmatrix}, \quad \mathbf{b} = \mathbf{\Omega} x(0) + \mathbf{\Psi}\mathbf{w}$$

Notice that, the elements in $\mathbf{z}$ can be re-ordered as $\mathbf{z} = [\mathbf{z}_1^\top, \mathbf{z}_2^\top, \mathbf{z}_3^\top]^\top$, where $\mathbf{z}_i = [\mathbf{x}_i^\top,\, \mathbf{Q}_i^\top]^\top$ is the decision variables related to zone i. In the following development, we want to design a parallel update protocol of $\mathbf{z}_i$, implemented by the computing agent of RTU i. In this regard, constraint (14) can be thought of as the local constraint for zone i and RTU i, while constraint (13) is a shared constraint across zones.

For optimization problem (12), an augmented Lagrangian function is formulated as,

$$\begin{aligned}
\mathcal{L}_\rho(\mathbf{z}, \lambda) &= \sum_{i=1}^{3} f_i^\top \mathbf{z}_i + \lambda^\top(\mathbf{A}\mathbf{z} - \mathbf{b}) + \frac{\rho}{2}\|\mathbf{A}\mathbf{z} - \mathbf{b}\|^2 \\
&= \sum_{i=1}^{3} f_i^\top \mathbf{z}_i + \lambda^\top\Big(\sum_{i=1}^{3} \mathbf{A}_i\mathbf{z}_i - \mathbf{b}\Big) + \frac{\rho}{2}\Big\|\sum_{i=1}^{3} \mathbf{A}_i\mathbf{z}_i - \mathbf{b}\Big\|^2 \\
&= \sum_{i=1}^{3} \mathcal{L}_i(\mathbf{z}_i, \lambda) - \lambda^\top\mathbf{b} + \varphi(\mathbf{z})
\end{aligned}$$

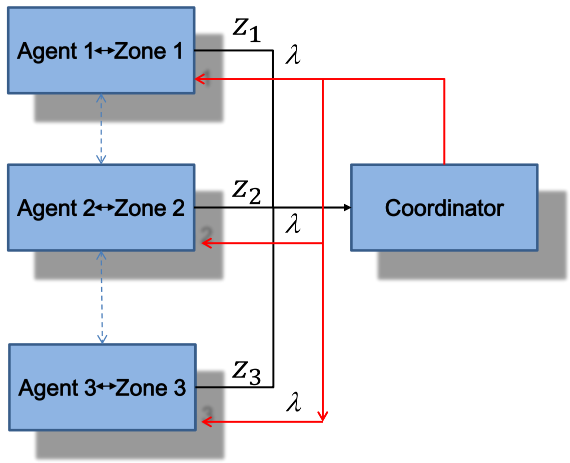

Figure 3: Distributed primal-dual optimization scheme: information structure

where λ is the lagrange multiplier, or dual variable, $\rho > 0$ is a penalty parameter, $\mathcal{L}_i(\mathbf{z}_i, \lambda) = f_i^\top \mathbf{z}_i + \lambda^\top \mathbf{A}_i \mathbf{z}_i$ and $\varphi(\mathbf{z}) = \frac{\rho}{2} \| \sum_{i=1}^3 \mathbf{A}_i \mathbf{z}_i - \mathbf{b} \|^2$. Notice that $\mathbf{A}_i$ is the columns of $\mathbf{A}$ that correspond to the elements of $\mathbf{z}_i$. The dual function is obtained by minimizing the Lagrangian function with respect to the primal variable $\mathbf{z}$,

$$\begin{aligned} d(\lambda) &= \inf_{\mathbf{z}} \mathcal{L}_\rho(\mathbf{z}, \lambda) \\ &= \sum_{i=1}^{3} \left(\inf_{\mathbf{z}_i \in \mathcal{Z}_{Ni}} \mathcal{L}_i(\mathbf{z}_i, \lambda) \right) - \lambda^\top \mathbf{b} + \varphi(\mathbf{z}), \end{aligned}$$

and the corresponding dual problem is

$$\underset{\lambda}{\text{maximize}} \ d(\lambda)$$

4.2 Algorithm

The proposed PJ-ADMM based DMPC algorithm is an iterative algorithm that contains two steps in each iteration: primal optimization and dual update. The primal optimization step will be carried out by local (zone-level) agent in parallel, and each agent will do a best-response coordinate descent for the Lagrangian function with respect to only part of the decision variables, namely $\mathbf{z}_i$. Dual update will be carried out by a "coordinator", and the objective of dual update is to maximize the dual function $d(\lambda)$. Interestingly, one can also think of the dual variable as an "enforcer" that facilitates the satisfaction of the shared constraint (13).

A diagram illustrating the information structure is given in Figure 3. The overall structure of the proposed algorithm is given in Algorithm 1.

Notice that in Algorithm 1, a proximal term $\frac{\phi_i}{2}\|\mathbf{z}_i - \mathbf{z}_i^\nu\|^2$ is added to regularize each agent's subproblem, where $\mathbf{z}_i^\nu$ is the value of $\mathbf{z}_i$ from the previous iteration. Intuitively, the larger ϕ_i is, the slower $\mathbf{z}_i$ should be moving away from its previous value (or equivalently, the less selfishly agent i responds to other agents' actions). With this extra term, the global convergence of the algorithm could be established.

Theorem 1. *(Global Convergence Theorem) Suppose the parameters in Algorithm 1 satisfy that* $\rho > 0$*,* $0 < \gamma < 2$*, and*

$$\phi_i > \frac{1+\gamma}{2-\gamma} \cdot \rho \|\mathbf{A}_i\|^2, \quad i = 1, 2, 3,$$

then the sequence $\{\mathcal{S}^\nu = (\mathbf{z}_1^\nu, \mathbf{z}_2^\nu, \mathbf{z}_3^\nu, \lambda^\nu)\}$ *generated by Algorithm 1 converges to a solution* $\mathcal{S}^*$ *of problem* (12).

Proof. See (Hou, Xiao, Cai, Hu, & Braun, 2016). Algorithm 1 is a special case of Theorem 1 therein. □

Theorem 1 provides us the theoretical guarantee of algorithmic convergence. However, in practice, one often should expect some fine tuning of parameters for a better convergence speed.

Algorithm 1 Distributed MPC via Proximal Jacobian ADMM

1: Initialize $(\mathbf{z}^0, \lambda^0, \mu^0)$, set $v = 0$;
2: **repeat**
3: Update $\mathbf{z}_i$ (in parallel) according to

$$\begin{cases} \mathbf{z}_1^{v+1} = \arg\min_{\mathbf{z}_{min,1} \le \mathbf{z}_1 \le \mathbf{z}_{max,1}} \left(\mathcal{L}_1(\mathbf{z}_1, \lambda^v) + \frac{\phi_1}{2} \|\mathbf{z}_1 - \mathbf{z}_1^v\|^2 + \varphi(\mathbf{z}_1, \mathbf{z}_2^v, \mathbf{z}_3^v) \right); \\ \mathbf{z}_2^{v+1} = \arg\min_{\mathbf{z}_{min,2} \le \mathbf{z}_2 \le \mathbf{z}_{max,2}} \left(\mathcal{L}_2(\mathbf{z}_2, \lambda^v) + \frac{\phi_2}{2} \|\mathbf{z}_2 - \mathbf{z}_2^v\|^2 + \varphi(\mathbf{z}_1^v, \mathbf{z}_2, \mathbf{z}_3^v) \right); \\ \mathbf{z}_3^{v+1} = \arg\min_{\mathbf{z}_{min,3} \le \mathbf{z}_3 \le \mathbf{z}_{max,3}} \left(\mathcal{L}_3(\mathbf{z}_3, \lambda^v) + \frac{\phi_3}{2} \|\mathbf{z}_3 - \mathbf{z}_3^v\|^2 + \varphi(\mathbf{z}_1^v, \mathbf{z}_2^v, \mathbf{z}_3) \right) \end{cases} \tag{15}$$

4: Update λ according to $\lambda^{v+1} = \lambda^v + \gamma\rho(\mathbf{A}\mathbf{z}^{v+1} - \mathbf{b})$;
5: $v \leftarrow v + 1$;
6: **until** some stopping criterion is satisfied.

4.3 Extention to a More General Setting

This subsection discusses some possible extensions of our problem formulation in (4). All of these extensions could still be handled by the DMPC algorithm proposed in this paper, and may lead to more applications other than the multiple thermal zone coordination problem.

The first possible extension is on the objective function. Although in this case study, the objective function is approximated by a linear function, the distributed control strategies is applicable to general convex HVAC cost functions. Namely, the only requirement on function $P_i(\cdot)$ in (4) is convexity (we do not require strong convexity).

In addition to the shared equality constraint in the form of $\sum_{i=1}^{L} \mathbf{A}_i \mathbf{z}_i = \mathbf{b}$, where L is the number of subproblems, we can also deal with shared inequality constraint in the form of $\sum_{i=1}^{L} \mathbf{D}_i \mathbf{z}_i \le \mathbf{e}$. Many "shared resource" kind of constraints can be modelled by this form. For instance, in a building where one central air handling unit (AHU) serves multiple zones through VAVs, the total amount of sensible cooling provided to all zones should be smaller or equal to the capacity of the AHU. Another example is demand response management. Assuming $\mathbf{z}_i$ is the load of a single building in a building cluster, we can use this type of shared inequality constraint to enforce an upperbound for the demand of the whole cluster.

In the situation where one also has the shared inequality constraint in the form of $\sum_{i=1}^{L} \mathbf{D}_i \mathbf{z}_i \le \mathbf{e}$, the Lagrangian function should be modified by adding a corresponding penalty term (with an extra dual variable μ) for the violation of the shared inequality constraint. Details for this formulation can be found in (Hou et al., 2016).

5. SIMULATION RESULTS

In the following simulations, the proposed DMPC with 24-hour prediction horizon is implemented and the sampling time is 30 minutes. The thermal comfort interval for all zones are set to be [21.5, 23.5]°C during occupied hours (9am to 17pm) and [20.5, 24.5]°C during unoccupied hours (17pm to 9am). A seven day warm up period is implemented to obtain the intitail states. Actual weather measurements from May 2015 are used in the simulations and perfect weather prediction is assumed in the DMPC optimization. Electricity price is assumed to be \$0.1/kw. Optimization problems are numerically solved using CVX (Grant & Boyd, 2014) and the SDPT3 solver (Toh, Todd, & Tütüncü, 1999).

To evaluate the energy saving potential of our proposed optimal coordination formulation, a baseline control strategy is implemented for comparison. The baseline control is based on simple feedbacks on the zone temperatures. Whenever the temperature in zone i is about to go above the pre-specified comfort interval, RTU i turns on to maintain it at its upperbound. This greedy control strategy is not expected to explore inter-zonal coordination.

Table 1: Electricity costs comparison

Control Strategy	Baseline	Optimal Coordination with DMPC	**Saving**
Electricity Cost ($)	55.95	48.49	13.3%

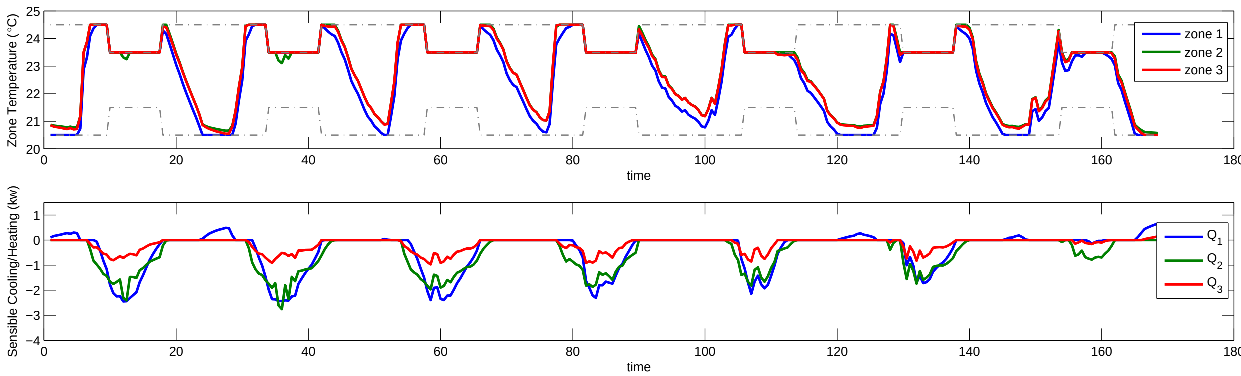

Figure 4: Baseline control

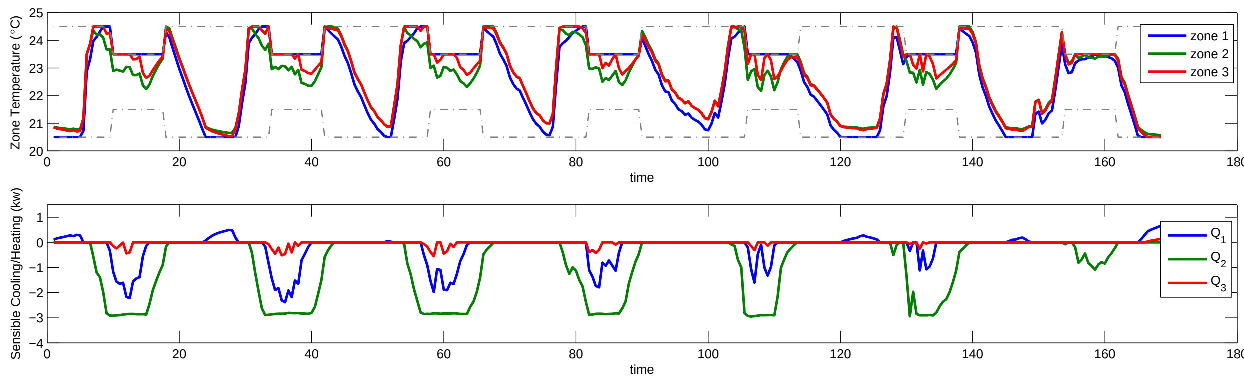

Figure 5: Optimal coordination with proposed DMPC

Simulation results with the proposed DMPC are plotted in Figure 5 and the baseline results are plotted in Figure 4. The total electricity costs of the two controllers are given in Table 1. Comparing these results, we can make the following observations and comments.

- As expected, the double façade adjacent zone 1 has the highest load during most of the days simulated. All three zone temperatures are well maintained in the pre-specified comfort interval.
- In the baseline control, each RTU only cares about the corresponding zone temperature it is "looking after", but does not take the thermal interactions between zones into consideration.
- In the optimal coordination formulation with DMPC control, the most efficient unit, RTU 2, is more heavily utilized during the peak load hours. Through direct air exchange between zones, RTU 2 over cools zone 2 to help out RTU 1 and 3, whose loads are reduced compared to the baseline case. This can also be verified from the fact that the setpoint temperature for zone 2 is noticeably lower than the comfort upperbound.
- It is confirmed that the simulation results with the proposed DMPC is almost identical to the results obtained with a centralized controller.

Some other issues that could be explored but omitted due to space limitation include,

- Time-of-use price. Some preliminary results have already been obtained considering time-of-use price. The proposed coordination formulation with DMPC control is capable of responding to the higher peak hour electricity price by pre-cooling the room at off-peak hours.
- Local comfort delivery. Instead of using a uniform zone temperature constraints for all zones, customized comfort intervals based on local occupants' preferences could easily be included in our formulation.
- Demand response. (Cai, Braun, Kim, & Hu, 2016) proposed a novel multi-agent based demand reduction strategy. A similar method could also be incorporated in our formulation.

6. CONCLUSIONS

This paper investigated a case study for the optimal coordination of multiple thermal zones in a large open space. A general distributed MPC method based on Proxiaml Jacobian ADMM is proposed to solve the optimal control problem. The proposed DMPC algorithm can not only deal with the current case study, but also a family of other building control problems. One possible future direction is a more sophisticated case study that includes a more realistic envelope model and at the same time investigates demand charge reduction.

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