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Buckling analysis of laminated composite box beams

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ABSTRACT

Paper presents buckling analysis of thin-walled laminated composite box beam type structures. The nonlinear displacement field of thin-walled cross-section is adopted in order to insure the geometric potential of semitangential type for both the internal torsion and bending moments. The cross-section mid-line contour is assumed to remain not deformed in its own plane and the shear strains of middle surface are neglected. The laminates are modeled on the basis of classical lamination theory. Analysis is performed in an eigenvalue manner and it attempts to determine the critical loads as well as corresponding buckling modes in a direct manner without calculating the deformations. The model is validated on a few test examples comparing the results with those reported in the literature.

Keywords: composite, box beams, buckling

1. INTRODUCTION

Thin-walled composite beam structures are widespread in lot of engineering areas spatially because of their high strength-to-weight ratio. However due to their slenderness and specific mechanical behaviour such structures are very susceptive to instability and buckling failure.

Thinn-walled beam theory was firstly developed by (Vlasov, 1961)[and then further accomplished by (Gjelsvik, 1981) while the theoretical background for closed section laminated profiles are given by (Song & Librescu, 1993; Kollar & Pluzsik, 2002; Cortinez & Piovan, 2006; and Vo & Lee, 2007). This paper is partially based on the some of these previouslly established theories.

Numerical model adopted in this paper is based on assumptions of large displacements and small strains, the Euler-Bernoulli-Navier beam bending theory and the Vlasov torsion theory. The members of thin-walled beam are considered as prismatic and straight. The model further assume the static and conservative external loads. The Clasical lamination theory (CLT) is implemented in the model. The stability problem is approached in an eigenvalue manner.

2. BASICS

2.1. Displacement field

In this paper, two sets of coordinate systems, which are mutually interrelated, are used. The first coordinate system is Cartesian coordinate system (z, x, y), for which z-axis coincides with the beam axis passing through the centroid O of each cross-section, while the x- and y-axes are the principal inertial axes of the cross-section taken along the width and height of the beam. The second coordinate system is contour coordinate (z, n, s) as shown in Fig. 1, wherein coordinate z coincident with beam z-axis, the coordinate s is measured along the tangent of the middle surface in a counter-clockwise direction, while n is the coordinate perpendicular to s. Incremental displacement measures of a cross-section are defined as:

$$\left.\begin{aligned} & w_o = w_o(z), u_o = u_o(z), v_o = v_o(z), \\ & \varphi_z = \varphi_z(z), \quad \varphi_x = -v_o' = \varphi_x(z), \\ & \varphi_y = u_o' = \varphi_z(z), \theta = -\varphi_z' = \theta(z) \end{aligned}\right\} \qquad (1)$$

where w0, u0 and v0 are rigid body displacements in z, x and y directions; φ_z, φ_x and φ_y are the rigid-body rotations about the z-, x- and y-axis while θ is the warping parameter. The displanement field is defined as:

$$\boldsymbol{U}_{\text{uk}}^{\text{T}} = \{W \quad U \quad V\} = \{w+\tilde{w} \quad u+\tilde{u} \quad v+\tilde{v}\} \tag{2}$$

Where w, u and v are standard linear displacement field components while $\tilde{w}, \tilde{u}$ and $\tilde{v}$ are second order components due to the large rotations (Turkalj et.al., 2011; and Lanc at. al., 2014). I contour coordinate system (*z*, *n*, *s*), Figure 1., the mid-line contour are $\bar{w}, \bar{u}, \bar{v}$, while out of mid-line displacements are defined as:

$$w(z,s,n) = \bar{w} - n\frac{\partial \bar{u}}{\partial z}; \quad v(z,s,n) = \bar{v} - n\frac{\partial \bar{u}}{\partial s}; \quad u(z,s,n) = \bar{u} \tag{3}$$

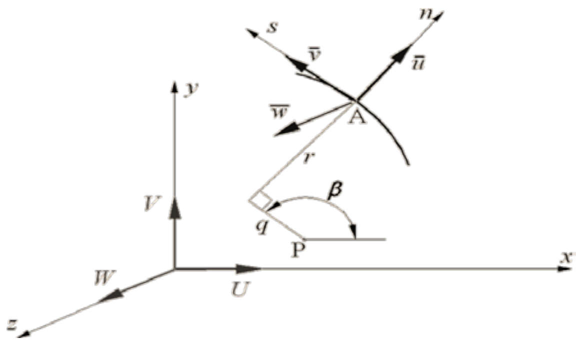

Figure 1 *Contour coordinate system*

Abeam to contour relation is defined as:

$$\bar{w} = W(z,s,n); \quad \bar{v} = U(z,s,n)\cos\beta + V(z,s,n)\sin\beta; \quad \bar{u} = U(z,s,n)\sin\beta - V(z,s,n)\cos\beta \tag{4}$$

2.2. Strains

The strain tensor consist of three parts:

$$\varepsilon_{ij} = e_{ij} + \eta_{ij} + \tilde{e}_{ij}; \quad e_{ij} = 0.5\left(u_{i,j} + u_{j,i}\right), \quad \eta_{ij} = 0.5\, u_{k,i} u_{k,j}, \quad \tilde{e}_{ij} = 0.5\left(\tilde{u}_{i,j} + \tilde{u}_{j,i}\right) \tag{5}$$

with the non-zero components:

$$e_{zz} = \frac{\partial \bar{w}}{\partial z} - n\frac{\partial^2 \bar{u}}{\partial z^2}; \quad e_{zs} = \bar{\gamma}_{zs} - 2n\frac{\partial^2 \bar{u}}{\partial s \partial z}; \tag{6}$$

$$\eta_{zz} = \frac{1}{2}\left[\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]; \quad \eta_{zs} = \frac{\partial w}{\partial z}\frac{\partial w}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial u}{\partial s} + \frac{\partial v}{\partial z}\frac{\partial v}{\partial s}; \tag{7}$$

$$\tilde{e}_{zz} = \frac{\partial \tilde{w}}{\partial z}; \quad \tilde{e}_{zs} = \frac{\partial \tilde{w}}{\partial s} + \frac{\partial \tilde{v}}{\partial z} \tag{8}$$

where $\bar{\gamma}_{zs}$ is defined as:

$$\bar{\gamma}_{zs} = \frac{\partial \bar{w}}{\partial s} + \frac{\partial \bar{v}}{\partial z} = \frac{F_s}{t} \cdot \frac{d\varphi_z}{dz} \tag{9}$$

In equation above *t* is tehe wall thickness while F_s is St.Venant shear flow, for constant wall thickness defined as:

$$F_s = \frac{bht}{h+b} \tag{10}$$

2.3. Internal forces

The constitutive equation for one lamina is:

$$\begin{pmatrix} \sigma_z \\ \tau_{zs} \end{pmatrix} = \begin{pmatrix} \bar{Q}_{11}^{\;*} & \bar{Q}_{16}^{\;*} \\ \bar{Q}_{16}^{\;*} & \bar{Q}_{66}^{\;*} \end{pmatrix} \cdot \begin{pmatrix} \varepsilon_z \\ \gamma_{zs} \end{pmatrix} \tag{11}$$

where $\bar{Q}_{ii}^{\;*}$ are so called reduced stiffnesses according to (Vo & Lee, 2007). Integrating over the cross-sectional area, the internal beam forces follow as:

$$F_z = \int_A \sigma_z \, dn\, ds; \quad M_z = \int_A \tau_{zs}\left(n + \frac{F_s}{2t}\right) dnds; \quad M_\omega = \int_A \sigma_z (\omega - nq) dnds; \; M_x = \int_A \sigma_z (y - n\cos\beta) dnds; \quad M_y = \int_A \sigma_z (x + n\sin\beta) dnds; \tag{12}$$

2.4. Finite element

Figure 2. presents the two-noded spatial finite element with 7 DOF per node. The nodal displacement and the nodal force vectors are:

$$\left(\mathbf{u}_i^e\right)^{\text{T}} = \left\{w_i, u_i, v_i, \varphi_{zi}, \varphi_{xi}, \varphi_{yi}, \theta_i\right\} \tag{13}$$

$$\left(\mathbf{f}_i^e\right)^{\text{T}} = \left\{F_{zi}, F_{xi}, F_{yi}, M_{zi}, M_{xi}, M_{yi}, M_{\omega i}\right\} \tag{14}$$

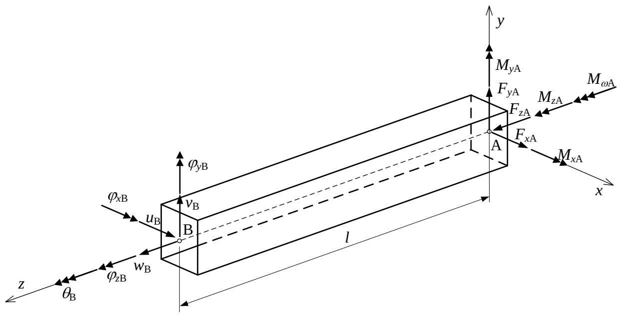

Figure 2 *Box-beam finite element*

Applying the virthual work principle on beam finite element folows:

$$\delta\mathcal{U}_{\mathrm{E}} + \delta\mathcal{U}_{\mathrm{G}} = \delta\mathcal{W}\,, \tag{15}$$

where $\delta\mathcal{U}_{\mathrm{E}}$ and $\delta\mathcal{U}_{\mathrm{G}}$ are the potential energy and geometric potential:

$$\delta\mathcal{U}_{\mathrm{E}} = \int_V S_{\mathrm{ij}}\,\delta e_{\mathrm{ij}}\,\mathrm{d}V = \left(\delta\mathbf{u}^{\mathrm{e}}\right)^{\mathrm{T}}\mathbf{k}_{\mathrm{E}}^{\mathrm{e}}\ \mathbf{u}^{\mathrm{e}} \tag{16}$$

$$\delta\mathcal{U}_{\mathrm{G}} = \int_V S_{\mathrm{ij}}\left(\delta\eta_{\mathrm{ij}} + \delta\tilde{e}_{\mathrm{ij}}\right)\mathrm{d}V - \int_{A_\sigma} t_{\mathrm{i}}\,\delta\tilde{u}_{\mathrm{i}}\,\mathrm{d}A = \left(\delta\mathbf{u}^{\mathrm{e}}\right)^{\mathrm{T}}\mathbf{k}_{\mathrm{G}}^{\mathrm{e}}\ \mathbf{u}^{\mathrm{e}} \tag{17}$$

while $\delta\mathcal{W}$ is the virthual work of external forces:

$$\delta\mathcal{W} = \int_V S_{\mathrm{ij}}\,\delta e_{\mathrm{ij}}\,\mathrm{d}V = \left(\delta\mathbf{u}^{\mathrm{e}}\right)^{\mathrm{T}}\mathbf{f}^{\mathrm{e}}. \tag{18}$$

In expressions above $\mathbf{k}_{\mathrm{E}}^{\mathrm{e}}$ and $\mathbf{k}_{\mathrm{G}}^{\mathrm{e}}$ are elastic and geometric stiffnes matrices while $\mathbf{f}^{\mathrm{e}}$ is the nodal force vector.

3. EXAMPLE

Simply supported beam is axially loaded at cross section centroid by force *F*. The length oft he beam is *L* = 8 m and the box cross-section has dimensions *b* = 200 mm, *h* = 100 mm, *t* = 10 mm, Figure 3.

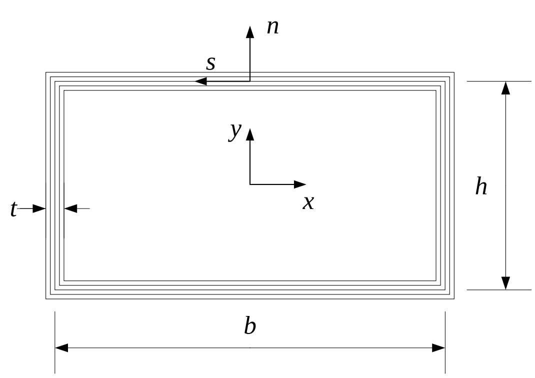

Figure 3 *Box beam cross-section*

All beam flanges are four layer laminates with stacking sequence $[\pm\varphi]_s$. Material parameters are:: E_1= 250.0 GPa, E_2= 10.0 GPa, G_{12}= 6.0 GPa and ν_{12}=0.25. Non-dimensional critical buckling load, $\overline{F} = FL^2/b^3 t E_2$, for flexural buckling modes in x and y directions with respect to fibre orientation angle φ are ploted on Figure 4. together with the results of Vo and Lee [12] for comparison.

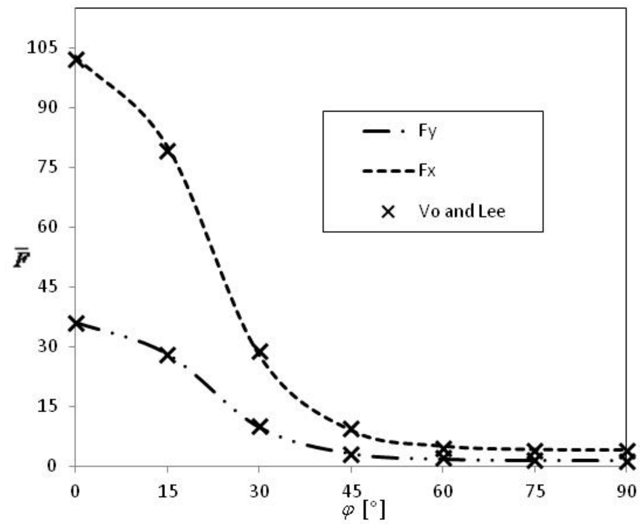

Figure 4 *Non-dimensional buckling loads vs fiber orientation angle*

4. CONCLUSION

Paper presents finite element buckling analysis of thin walled box section laminated beams. Developed computer code has been verified comparing with the results available in literature. Very good result coincidence is achieved. A further extension of algorithm is planned to include open section beam profiles.

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