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For my parents

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NOMENCLATURE

f	index of flight
i	index of flight on path 1
j	index of flight on path 2
K	set of path
L	required fly time from arrival fix to airport
N_1	number of flights scheduled for path 1
N_2	number of flights scheduled for path 2
$P\{q\}$	probability of scenario q
q	index of scenario
Q	total number of scenarios
S_{ij}^{q}	reversal decision variables for flight i on path 1 and flight j on path 2 under scenario q
t	time step
T	planning time horizon of the problem
$T_{k,f}$	time window for each flight
$\underline{T_{k,f}}$	The first time period in the set $T_{k,f}$
$Y_{k,f,t}^{plan}$	scheduled arrival time at arrival fix
$\mathcal{H}$	set of safety time separation profile scenarios
ξ_q	safety time separation profile scenario q
τ_q	the time where scenarios tree diverges to produce a new branch
Δ	set of safety time separation
$\Delta(t)^q$	specific time separation under scenario q

$\underline{\Delta}$	small separation
$\bar{\Delta}$	big separation
Γ	set of time periods
λ	cost ratio between one unit of airborne holding and ground holding
$\mathcal{F}$	set of flight
$\upsilon_{k,f,t}^{q}$	Lagrange multiplier for flight f on path k at time t under scenario q
υ_q	Lagrange multiplier for each scenario

ABSTRACT

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Weather accounts for the majority of congestion in the National Airspace System which highlights the importance of addressing weather uncertainty to mitigate delays, and this paper presents an effort in this direction.

Firstly, a new dynamic stochastic 0-1 Integer Programming (IP) model is proposed, which models the Single Airport Ground Holding Problem (SAGHP) with respect to uncertainty in the separation between flights instead of Airport Acceptance Rate (AAR) or landing capacity. Uncertainty in separation according to different weather conditions is represented through the scenario tree by using stochastic linear programming. Considering time separation constraints instead of AAR constraints, our model is able to schedule a more accurate plan for the individual flight in minutes.

Secondly, a converging inbound air traffic model is formulated based on our dynamic stochastic IP model. We address a problem involving two paths inbound air traffic merging into a single airport in which uncertainty in separation from Minute-In-Trail restrictions is considered. Although "First Come, First Serve" policy is still obeyed by flights on the same path, the experimentation has shown that, allowing flights on different paths to switch arrival orders can help reduce the total delays.

Finally, in order to tackle the running time problem faced by the disaggregate integer model we built, we introduce dual decomposition method into the model to improve the computing efficiency. The original problem is decomposed scenario by scenario into several sub-problems based on the dual decomposition method; then a

parallel computing algorithm is developed to handle these sub-problems. Such combination increases the model's computational efficiency.

CHAPTER 1. INTRODUCTION

1.1 Background

Motivated by the continuing growth of air transportation demand, the Next Generation Air Transportation System (NextGen) has been proposed to address the challenge rising from constant growing air traffic [1]. With more congested airspace in the future, the automation of the Air Traffic Control (ATC) system is needed to help reduce the workload of air traffic controllers. The primary purpose of ATC is to prevent collisions between aircraft by enforcing traffic separation rules, which ensure aircraft maintaining minimum amount of safety space at all times. Besides that, the automation of the ATC system will also benefit the airlines and passages by reducing the delays and improving the safety.

Miles-in-Trail (MIT) is often used by air traffic controllers in metering operations for arrival assignment, which manages aircraft to achieve a schedule time of arrival. MIT describes the minimum allowable number of miles required between successive aircraft departing/arriving an airport, over a fix, through a sector, or on a specific route. MIT is used to apportion traffic into a manageable flow, as well as to provide space for additional traffic (merging or departing) to enter the flow of traffic. For example, standard separation between aircraft in the en route environment is five nautical miles. During a weather event, this separation may increase significantly. Many delays are directly attributable to MIT in an adverse weather event. A variation on MIT is Minutes-in-Trail (MINIT), which describes the minimum allowable minutes needed between successive aircraft. MINIT can be easily derived from MIT with a consideration of aircraft speed.

Ground Delay Program (GDP) is the most common action used to alleviate congestion costs and ensure safe and efficient air traffic. A GDP is often issued to control air traffic volume to airports where the projected traffic demand is expected to exceed the airport's Airport Acceptance Rate (AAR) for a length period of time (usually 15 minutes or more) [6]. Lengthy period of demand exceeding AAR are normally a result of the AAR being reduced for some reason and the most common reason is adverse weather. In a GDP, some flights are assigned a later time slot of arrival to avoid airborne delay, because it is cheaper and safer to delay flights on the ground than hold them when they are airborne.

Weather accounts for the majority of congestion in the National Airspace System (NAS). Adverse weather such as fog, snow, wind and reduced visibility may require greater separation between flights. Approximately 60% of total delay in the NAS is caused by adverse weather across 12 months of 2009 [8]. The imperfect weather forecast brings uncertainty into the air traffic management problem. Decisions made under uncertainty can cause airborne delays when the separation between flights is greater than the original forecast. On the other hand, if the forecast is too conservative, unnecessary ground delays will happen. This highlights the importance of addressing weather uncertainty to mitigate delays.

1.2 Literature Review

In past two decades, the Ground Holding Problem (GHP) has been studied by many researchers to support GDP action at airports. The objective of this class of problem is to minimize the sum of airborne and ground delay costs. Most of the GHPs are modeled in response to AAR (landing capacity) reductions caused by adverse weather. Efforts to tackle GHP problems dates back to 1987 when, Odoni was among the first to systematically describe this [2]. Following this, Richetta and Odoni(1993) formulated a static stochastic Integer Programming (IP) model for the single airport ground holding problem(SAGHP), in which ground holding strategies are decided "once and for all" at the beginning of planning time horizon and cannot be revised [3]. Later Richetta and Odoni (1994) formulated a dynamic multistage stochastic IP model for the SAGHP to overcome this limitation [4]. In this dynamic model, the ground holding decisions are made at the scheduled departure time of the flights instead of “once for all” at the beginning. However the ground holding decision still cannot be revised after it has been made. Mukherjee and Hansen (2007) improved this dynamic model by allowing for ground holding revisions contingent on scenario realizations [5]. In all above models, uncertainty in airport arrival capacities is represented through a finite number of scenarios arranged in a probabilistic decision tree. As time progresses, branches of the tree are realized, resulting in better information about future capacities.

On the other side, a 0-1 IP model is proposed by Bersimas and Stock-Patterson, known as the Bersimas Stock-Paterson (BSP) model [7]. This model is formulated to address the air Traffic Flow Management (TFM) problem, but it can also handle the GHP as a special case. This model is a Lagrangian model, which is based on trajectories of each individual aircraft. A limitation of Lagrangian models is that the dimension of this model is related to the number of aircraft involved in the planning time horizon. And Bertsimas proved that the 0-1 IP problem is NP-hard by deriving the equivalent job-shop scheduling problem. Another limitation is that it only addressed the deterministic problem. Gupta and Bertsimas (2011) improved this model to address the capacity uncertainty [8]. However this method is not really addressing the stochastic problem because it considers the uncertainty from the robust optimization aspect and only solves the “worst case” in the same fashion as the deterministic one.

In summary, many models have been applied to solve GHP, but almost all of the GHP models are formulated accounting for the landing capacity (AAR) constraints. The limitation is that they cannot schedule the individual flight very accurately because the basic period length used for the AAR is normally 15 min or more. To overcome this limitation, this thesis will handle GHP accounting for the MIT/MINIT constraints by using stochastic linear programming method.

1.3 This Thesis's Contributions

In this thesis, a dynamic stochastic optimization model is formulated by using linear stochastic programming, which can utilize dynamic updates of information about the minute-in-trail separation in a single airport. This thesis's contributions are as following:

First, we present a dynamic stochastic model that accounts for uncertainty in separation of MIT/MINIT restriction in a single airport. According to our best knowledge, our study is the first attempt to model GHP with respect to uncertainty in the separation between flights instead of AARs or landing capacity. Uncertainty in separation according to different weather conditions is represented through a scenario tree. This model is able to handle the time varying separation of MIT and the uncertainty rising from the imperfect forecast of weather conditions.

Second, we address a problem involving two paths inbound air traffic merging into a single airport in which uncertainty in separation from MIT restriction is considered. Allowing flights on different paths to switch arrival order will help reduce total delays.

Finally, we present a decomposition method for the stochastic problem modeled by the scenario tree method, in which the stochastic problem can be decomposed scenario by scenario to improve the computational efficiency.

1.4 Organization of This Thesis

The rest of this thesis is organized as follows. Chapter 2 introduces the formulation of a stochastic dynamic model for converging two paths inbound flights into a single airport. It can handle the uncertainty in MINIT and is adaptive to updated information as time progresses. After the model, a small size problem is used to demonstrate how our model works. Chapter 3 describes the dual decomposition method used to solve the large-scale stochastic optimization problem based on the scenario tree method. In Chapter 4 the numerical application results are presented and a discussion of the results follows. Finally we summarize conclusions and recommendations for future work in Chapter 5.

CHAPTER 2. A DYNAMIC STOCHASTIC MODEL FOR CONVERGING INBOUND AIR TRAFFIC

In this Chapter, we present the development of the dynamic stochastic model for converging inbound air traffic. We consider two sets of flights are scheduled to fly to a single airport from two paths; each set of flights arrives at the airport via an arrival fix. Flights on same path obey "First Come First Serve" policy, but they can change order with flight on the other path. For each flight there is a time window (slot) for arriving at the arrival fix. Flights are planned to reach their arrival fixes at their schedule time or later (but still in time window), but it depends on the weather conditions of the airport at that time. If the weather condition is not good, the time separation between successive landing flights will be greater than the normal one. Therefore flights that arrived at their arrival fixes at their schedule time will be airborne held. Another way to handle reduced weather condition is to impose ground holding to delay flights before their departure to avoid airborne holding because airborne holding costs more than ground holding and it has higher safety risk.

2.1 Scenario Tree

Following Richeta and Odoni (1994), we use a scenario tree to represent the evolution of weather condition at airport [4] [9]. Each node of the tree represents a status. As time progresses, each scenario realizes along each branch of the tree. Let $\mathcal{H}$ denote the set of safety time separation profile scenarios and a scenario $\xi_q \in \mathcal{H}$ will occur with a probability $\mathrm{P}\{q\}$. We assume that in the beginning of the time horizon (t=0), there are Q alternative scenarios, each scenario providing a possible time-varying safety time separation profile forecast for the entire time interval [0,T]. So each node of the tree represents the time separation at that time. Let τ_q denote the time where scenarios tree diverges to produce a new branch. Figure 2.1 shows our notation using the scenario tree representation.

2.2 Uncertainty Weather Model

We consider that weather condition only affects safety time separation in the airport, i.e. adverse weather will make the safety time separation to be larger. Let Δ denote the set of safety time separation. Safety time separation is time-varying and different from scenario to scenario, that is the key fact why our model is dynamic and stochastic, we use $\Delta(t)^q$ denote the specific time separation. For simple, we assume that weather can only change once, from bad weather (big separation $\bar{\Delta}$) to good weather (small separation $\underline{\Delta}$). But the exact timing of weather changing is uncertain.

For example: Suppose we have a time-horizon of 7 periods, i.e. T=7. Moreover there are 3 scenarios in the beginning, Q=3. We assume weather may change at $\tau_1 = 2, \tau_2 = 3$ and $\tau_3 = 4$. Let $\bar{\Delta} = 3$, $\underline{\Delta} = 1$. Then we have:

$$\Delta^1 = (3,3,1,1,1,1,1), \Delta^2 = (3,3,3,1,1,1,1) \text{ and } \Delta^3 = (3,3,3,3,1,1,1)$$

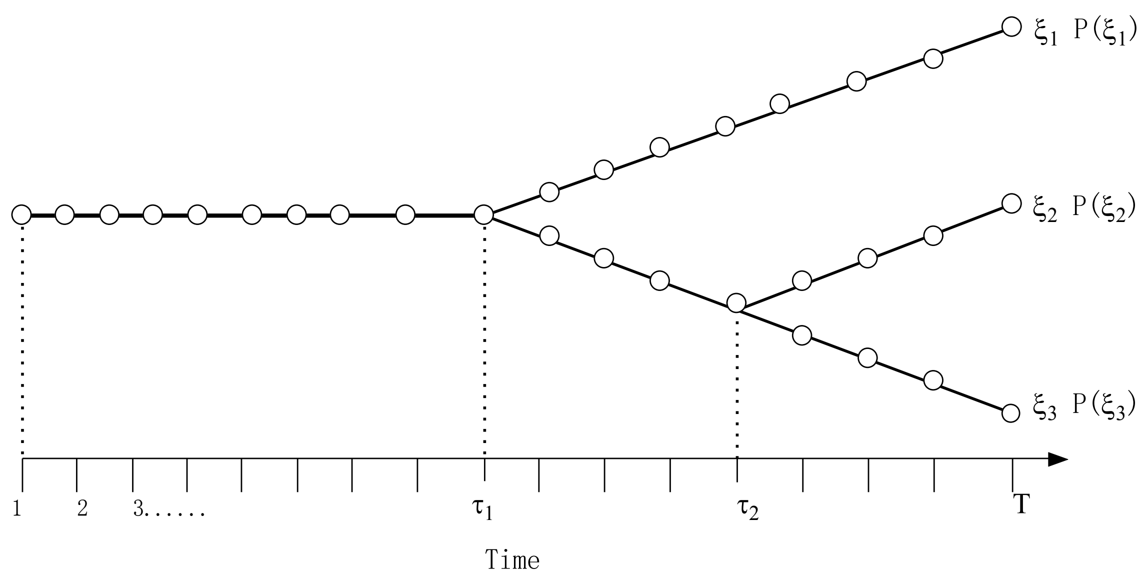

Figure 2.1 Scenario Tree represents the evolution of safety time separation at airport

2.3 Problem Formulation

Notation

Let N_1 and N_2 be the number of flights scheduled for each path, and the path is denoted by the set $K = \{1,2\}$. Let $T_{k,f}$ denote the time window for each flight. The time is a set of T time periods of equal duration, and is denoted by the set $\Gamma = \{1,2 \ldots, T\}$. Any flight from each path must pass the according arrival fix before landing at the airport, the required fly time from arrival fix to airport is denoted by L. Here we do not consider the difference of required fly time from different fixes to airport; they are the same in this model. Let λ denote the cost ratio between one unit of airborne holding and ground holding. And we consider $\lambda > 1$because airborne holding is more expensive and we assume it is the same for all flights.

Decision Variables

The decision variables in the model are binary variables defined as follows:

$$W_{k,f,t}^{q} = \begin{cases} 1 & \text{if flight } f \text{ on path } k \text{ is landed by time } t \text{ under scenario } \xi_q \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{k,f,t}^{q} = \begin{cases} 1 & \text{if flight } f \text{ on path } k \text{ arrives at arrival fix by time } t \text{ under scenario } \xi_q \\ 0 & \text{otherwise} \end{cases}$$

Note that our decision variables are similar to the Bertsimas-Stock model (1998), this definition using “by” instead of “at” is important to understand this model. Once flight f arrives at fix or lands at airport at time t, then both variable of time t and subsequent will be set to 1. We can record the status changing time as arrival time or landing time.

The Objective Function

The objective of the model is to minimize the expected combination cost of airborne holding delay and ground holding delay for all flight.

$$\text{Min } \sum_{\xi_q \in \mathcal{H}} P\{q\} \left\{ \sum_{k\in K} \sum_{f\in\mathcal{F}} \left(\sum_{t\in \mathrm{T}_{k,f}} \left(Y_{k,f,t}^{plan} - Y_{k,f,t}^{q} \right) \right) + \lambda \left(\sum_{k\in K} \sum_{f\in\mathcal{F}} \left(\sum_{t\in \mathrm{T}_{k,f}} \left(Y_{k,f,t}^{q} - W_{k,f,t+\mathrm{L}}^{q} \right) \right) \right) \right\} \tag{2.1}$$

Where

$$Y_{k,f,t}^{plan} = \begin{cases} 1 & \text{if flight f on path k is planned to arrive at arrival fix by time t} \\ 0 & \text{otherwise} \end{cases}$$

Let $Y_{k,f,t}^{plan}$ denote the corresponding binary values of the scheduled arrival plan at arrival fix, it is prior information for this model and it is deterministic and same for all scenarios. Here the first component is the difference between schedule arrival time and actual arrival time at arrival fix which expresses the ground holding delay. Note if a flight is planned to arrive later than its scheduled time, we assumed that the delay occurs at its original airport. We only consider the delay as ground holding delay and ignore the delay in en route, because airborne delay is more expensive than ground holding. Second component is the difference between planned landing time and actual landing time at the airport, which is the airborne holding delay. The airborne holding delay multiplies the delay cost ratio λ for the difference in ground and airborne delay costs per unit.

The Constraints

$$W_{k,f,t}^{q} \le W_{k,f,t+1}^{q} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma \tag{2.2}$$

$$Y_{k,f,t}^{q} \le Y_{k,f,t+1}^{q} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, t \in \Gamma \tag{2.3}$$

$$W_{k,f+1,t+\Delta(t)^q}^{q} - W_{k,f,t}^{q} \le \sigma^q(t) \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma, \mathrm{t} \le T - inf\Delta(t)^q \tag{2.4}$$

Where:

$$\sigma^q(t) = \begin{cases} W_{k,f,t+(\overline{\Delta}-\underline{\Delta})}^{q} & t \in \left[\tau_q, \tau_q - \left(\overline{\Delta} - \underline{\Delta}\right) + 1\right] \\ W_{k,f,t-(\overline{\Delta}-\underline{\Delta})}^{q} & t \in \left[\tau_q + 1, \tau_q + \left(\overline{\Delta} - \underline{\Delta}\right)\right] \\ 0 & otherwise \end{cases}$$

$$W_{k,f,t+L}^{q} \le Y_{k,f,t}^{q} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \mathrm{T}_{k,f}, \mathrm{t} \le T - L \tag{2.5}$$

$$Y_{k,f,t}^{q} \le Y_{k,f,t}^{plan} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, t \in \mathrm{T}_{k,f} \tag{2.6}$$

$$W_{2,j,t+\Delta(t)^q}^{q} - W_{1,i,t}^{q} - S_{ij}^{q} \le \sigma^q(t) \quad \forall\, i,j \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma, \mathrm{t} \le T - inf\Delta(t)^q \tag{2.7}$$

$$W_{1,i,t+\Delta(t)^q}^{q} - W_{2,j,t}^{q} + S_{ij}^{q} \le \sigma^q(t) + 1 \quad \forall\, i,j \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma, \mathrm{t} \le T - inf\Delta(t)^q \tag{2.8}$$

$$Y_{k,f,t}^{\xi_q} = Y_{k,f,t}^{\xi_{q+1}} \quad \forall\, k \in K, f \in \mathcal{F}\,, t \in \mathrm{T}_{k,f}, q \in \{1, \dots Q-1\}, \xi_q \in \mathcal{H}, 1 \le t \le \tau_q \tag{2.9}$$

$$W_{k,f,t}^{q}, Y_{k,f,t}^{q} \in \{0,1\} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma \tag{2.10}$$

Constraints (2.2) and (2.3) represent connectivity in time, which means if a flight has arrived (landed) by time t, then $Y_{k,f,t}^{q}(W_{k,f,t}^{q})$ will be set to 1 for all subsequent time periods.

Constraints (2.4) represent connectivity between flights in same path. This constraint separates flights in the same path by required safety separation depending on weather condition. If one flight lands at airport at time t, then the next flight from the same path must lands after time $t + \Delta(t)^q$. Here the required safety separation $\Delta(t)^q$is time-varying and different from scenario to scenario.

The term $\sigma^q(t)$ on the right side works as a switch key at the weather changing time. It ensures that either the constraints before the weather changing time work or the ones after the weather changing time work. For example, we assume T=5 and weather may change at $\tau = 2$. Let $\overline{\Delta} = 3$, $\underline{\Delta} = 1$. Then we have $\Delta = (3,3,1,1,1)$ and if we do not have $\sigma^q(t)$, our constraints look like followings:

$$W_{k,f+1,4}^{q} - W_{k,f,1}^{q} \le 0 \tag{2.11}$$

$$W_{k,f+1,5}^{q} - W_{k,f,2}^{q} \le 0 \tag{2.12}$$

$$W_{k,f+1,4}^{q} - W_{k,f,3}^{q} \le 0 \tag{2.13}$$

$$W_{k,f+1,5}^{q} - W_{k,f,4}^{q} \le 0 \tag{2.14}$$

We can find constraints (2.11),(2.13) or (2.12),(2.14) cannot be satisfied at the same time. Constraint (2.11),(2.12) will make constraint (2.13),(2.14) redundant because of constraint (2.2). In other words, if flight $f$doesn't land on time period 1, flight $f+1$ cannot land on time period 4, even if the weather has become good and the separation is small on time period 4. So if we want either of them to work, we just add the two pair constraints together, which makes it:

$$W_{k,f+1,4}^{q} - W_{k,f,1}^{q} - W_{k,f,3}^{q} \le 0 \tag{2.15}$$

$$W_{k,f+1,5}^{q} - W_{k,f,2}^{q} - W_{k,f,4}^{q} \le 0 \tag{2.16}$$

And the term $\sigma^q(t)$ has the same function above.

Constraints (2.5) represent connectivity between arrival fix and airport. If a flight lands at airport by time $t + L$, it must have arrived arrival fix by time t. In other words, flight cannot land at airport until it has spent L time units flying from arrival fix to airport.

Constraints (2.6) ensure that flight will not arrive at arrival fix before the scheduled time.

Constraints (2.7) and (2.8) represent connectivity between two paths. Flights on one path obey "First Come First Serve" rule, but they can change order with flight on the other path. In other words, any pair of flights$(f_i f_j)$ can reverse, f_i is any flight on path 1 and f_j is any flight on path 2. If flight f_i lands before flight f_j, then we set $S_{ij} = 0$. So constraints (2.8) become redundant and constraints (2.7) ensure the safety time separation between these two flights. Similarly, if flight f_j is landing before flight f_i, i.e. $S_{ij} = 1$. Then constraints (2.7) become redundant and constraints (2.8) ensure the safety time separation between these two flights.

Constraints (2.9) are a set of coupling constraints (Richetta and Odoni 1994) on the decision variables of arriving time at arrival fix $Y^q_{k,f,t}$. These constraints equate the specific planned arrival decisions under different scenarios, which force ground holding decisions to be the same for all scenarios passing through the same node at that time. For example in Figure 2.1 scenario ξ_1 and ξ_2 pass through the same nodes before scenarios tree diverges, which starts from time 1 to time τ_1. So all the decision variables of both scenario ξ_1 and ξ_2 must be the same, which means $Y^1_{k,f,t} = Y^2_{k,f,t}\ \forall\ k \in K, f \in \mathcal{F}, 1 \le t \le \tau_1$. And similarly for scenario ξ_2 and ξ_3, we also have $Y^2_{k,f,t} = Y^3_{k,f,t}\ \forall\ k \in K, f \in \mathcal{F}, 1 \le t \le \tau_2$. Note here scenario ξ_1 and ξ_3 also pass through the same nodes before $t = \tau_1$, but the two previous constraints already include this relationship, there is no need to add more constraints here.

2.4 Example with Small Size Problem

To illustrate the properties of our model presented in the last section, we apply it to a small problem. We assume there are 4 aircrafts in total, each two of them are on each path (N_1=2, N_1=2), the total time period is 8, T=8. Let the cost ratio between airborne holding and ground holding be 2, $\lambda = 2$. The time window for each flight is set as followings: $\mathrm{T}_{1,1} = \{1,2\}, \mathrm{T}_{1,2} = \{3,4,5,6,\}, \mathrm{T}_{2,1} = \{1,2,3\}, \mathrm{T}_{2,2} = \{4,5,6\}$ and the first time period $\underline{\mathrm{T}_{k,f}}$ is set as the scheduled arrival time at arrival fix for each aircraft. We set the required flight time from arrival fix to airport to be 2, L=2. There are two separation scenarios: $\mathcal{H} = \{\xi_1, \xi_2\}$. This example with small number of flights and scenarios will help illustrate how our model works clearly.

First we define our scenarios for our example. As shown in Figure 2.2, both of the scenarios begin with greater separation $\bar{\Delta}$= 2 which might account for the fog in the morning. And they will change to a small separation $\underline{\Delta}$= 1 later, which means the fog disappears. The only difference between these two scenarios is the timing of separation changing. For scenario 1, the changing time is at t=4. And it is two units time later for scenario 2 (t=6). The detail for scenario tree of our example is shown in Figure 2.3.

Case I: The Difference between Uncontrolled Mode and GDP Mode

Table 2.1 Difference between Uncontrolled Mode and GDP Mode

Flight	Uncontrolled				GDP			
	Fix	Airport	GH	AH	Fix	Airport	GH	AH
1.1	1	3	0	0	1	3	0	0
1.2	3	6	0	1	4	6	1	0
2.1	1	5	0	2	3	5	2	0
2.2	4	7	0	1	5	7	1	0

We use scenario 1 as the basic deterministic scenario in this case to compare the uncontrolled result and GDP result. The solutions are shown in Table 1, in which GH and AH stand for ground holding and airborne holding. We can find implementing GDP with perfect weather forecast (deterministic model), all airborne holding can be replaced by ground holding. Due to the high cost of airborne holding, it is much cheaper to implement ground delay.

Case II: The Difference between Two Paths and One Single Path

We use scenario 1 as a deterministic model to demonstrate the difference between two paths and one single path. For one single path, it obeys "First Come, First Serve" rule. A.M. Bayen et al. attempt to solve a similar problem by transfering it into a schedule problem and prove the fixed arrival order is not the optimal solution [13][14]. Here we only consider that aircrafts on different paths can switch arrival order, but the fixed arrival order is still fixed on each path.

Table 2.2 Difference between Two Paths and One Single Path

Time window	Flight	One Single Path				Two Paths			
		Fix	Airport	GH	AH	Fix	Airport	GH	AH
{1,2}	1.1	2	5	1	1	1	3	0	0
{3,4,5,6}	1.2	4	6	1	0	4	6	1	0
{1,2,3}	2.1	1	3	0	0	3	5	2	0
{4,5,6}	2.2	5	7	1	0	5	7	1	0

We assume for one single path, the fixed arrival order is: $f_{2.1}, f_{1.1}, f_{1.2}, f_{2.2}$. We apply GDP on both situation and the result is shown in Table 2. We can find that for two paths problem, the order of $f_{2.1}, f_{1.1}$ is switched to reduce the total cost. Instead of assigning one unit ground delay and one unit airborne delay to flight $f_{1.1}$ which costs 1+2=3, two paths problem let flight $f_{1.1}$ arrive first and assign two units ground delay to flight $f_{2.1}$ which costs only 2. So the advantage of two paths problem is that it allows aircrafts on different paths to change arrival order to mitigate total delay cost; at the same time, it still apply "First Come, First Serve" policy on each path to make it easy for implementation in reality.

Case III: Dynamic Stochastic Model with Different Probability Mass Function of Scenarios

We need to specify scenario probabilities for each scenario first before we apply our dynamic stochastic model. Frist, let's set $\mathrm{P}\{\xi_1\} = 0.9$ and $\mathrm{P}\{\xi_2\} = 0.1$, which means the first scenario has a very high probability to realize. In other words, the weather condition will become good early (t=4) with high probability. So we will prefer to schedule flight arrival time earlier to reduce the unnecessary ground delay. Even though this decision could risk into airborne delay, the probability of airborne delay to happen is very low. The result shown in Table 3 proved our above assumption. As we can see, flight $f_{1.2}$ and $f_{2.2}$'s actual landing time is different in each scenario. For flight $f_{1.2}$, the decision is made before the scenario tree diverges, so their decision is the same (it will arrive at arrival fix at t=4). Although it could face 1 unit airborne delay after it arrives at arrival fix if scenario 2 happens, the expected cost is low. For flight $f_{2.2}$, the decision is made after the diverge time, so they can choose the best strategy to reduce total delay respectively. Similarly, if we set $\mathrm{P}\{\xi_1\} = 0.1$ and $\mathrm{P}\{\xi_2\} = 0.9$ which means scenario 2 has a high chance to realize, the weather will probably become good late (t=6). As the result shown in Table 4, one more unit ground delay is assigned to flight $f_{1.2}$. It could be unnecessary ground delay if scenario 1 happens in reality, but the expected cost is low. As a result of the conservative decision on flight $f_{1.2}$, one more ground delay is also assigned to flight $f_{2.2}$ to ensure the separation between flights.

So our dynamic stochastic model can adjust the schedule based on different probability mass function to make the best strategy for the weather forecast at that time.

Table 2.3 Stochastic Model with Probability Mass Function($\mathrm{P}\{\xi_1\} = 0.9$,$\mathrm{P}\{\xi_2\} = 0.1$)

Time window	Flight	Scenario 1($\mathrm{P}\{\xi_1\} = 0.9$)				Scenario 2($\mathrm{P}\{\xi_2\} = 0.1$)			
		Fix	Airport	GH	AH	Fix	Airport	GH	AH
{1,2}	1.1	1	3	0	0	1	3	0	0
{3,4,5,6}	1.2	4	6	1	0	4	7	1	1
{1,2,3}	2.1	3	5	2	0	3	5	2	0
{4,5,6}	2.2	5	7	1	0	6	8	2	0

Table 2.4 Stochastic Model with Probability Mass Function($P\{\xi_1\} = 0.1$,$P\{\xi_2\} = 0.9$)

Time window	Flight	Scenario 1($P\{\xi_1\} = 0.1$)				Scenario 2($P\{\xi_2\} = 0.9$)			
		Fix	Airport	GH	AH	Fix	Airport	GH	AH
{1,2}	1.1	1	3	0	0	1	3	0	0
{3,4,5,6}	1.2	5	7	2	0	5	7	2	0
{1,2,3}	2.1	3	5	2	0	3	5	2	0
{4,5,6}	2.2	6	8	2	0	6	8	2	0

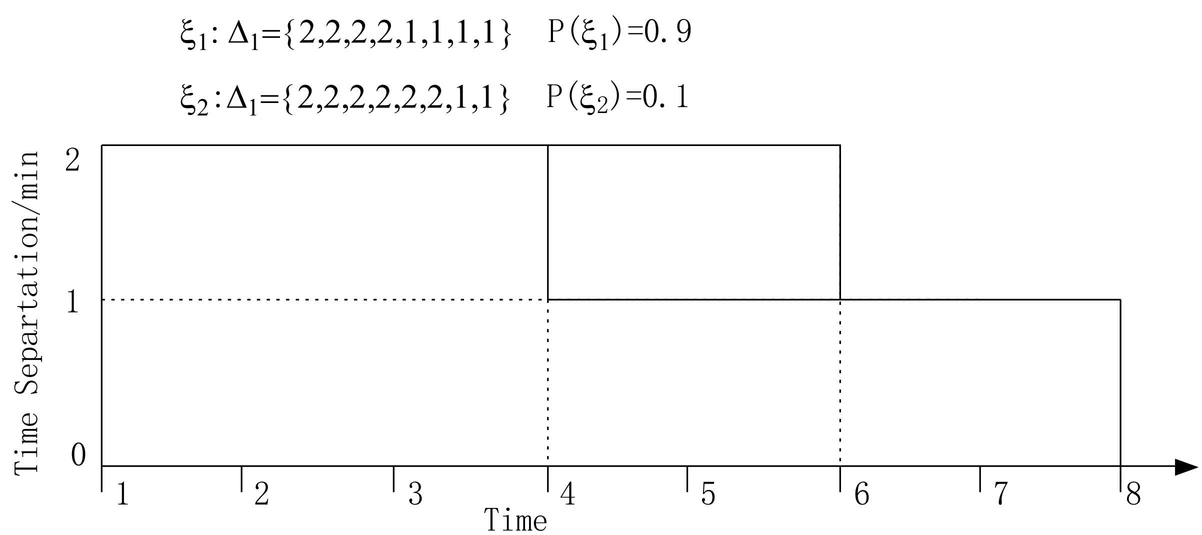

Figure 2.2 Separations Corresponding to Different Scenarios

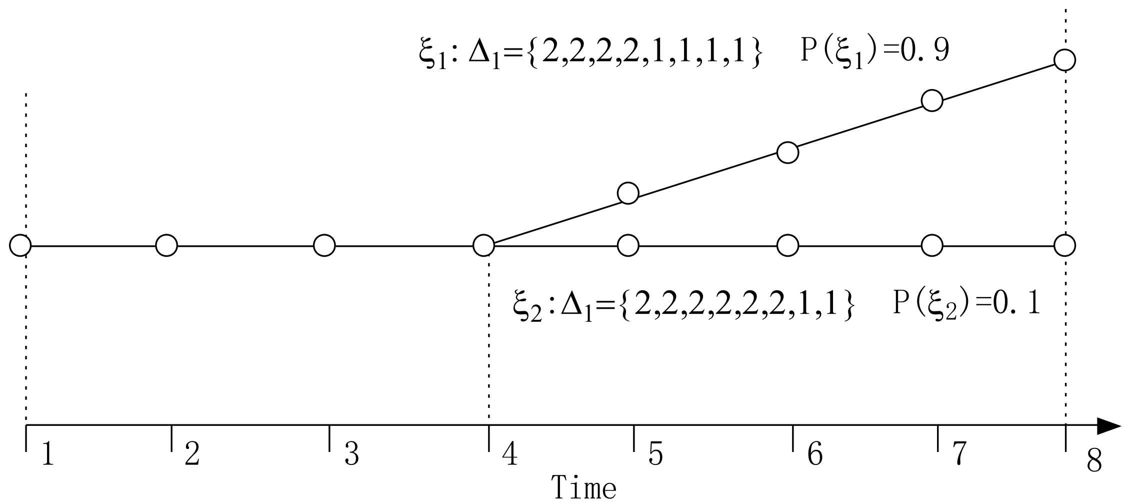

Figure 2.3 Scenarios tree for small example

CHAPTER 3. DUAL DECOMPOSITION METHOD

3.1 Complexity of The Problem

Above model is a disaggregate model, the decision variables are related to each individual flight. The number of variables is determined by number of flights N, time period T and scenario numbers Q. The variables could be up to hundreds of thoughts for a busy hub airport like Hartsfield–Jackson Atlanta International Airport (ATL). For example a 2-hour problem involves approximately 100 flights and 4 scenarios. Then there are $120 \times 100 \times 4 = 48{,}000$ landing variables $W_{k,f,t}^{q}$ and the same for variable $Y_{k,f,t}^{q}$. So the number of decision variable is up to 96,000. Moreover the reversal decision variables S_{ij}^{q} should be considered. So the total number of variable could be more than 100,000. For such large size Integer Program (IP), even the most up to date optimization solver cannot solve it in a reasonable time [12]. However, all constraints are separate for each scenario except for the coupling constraints (2.9). In large scale optimization, dual decomposition method is often used to separate the problem into several small problems.

Dual decomposition method was first proposed by Danzig et al. (1960) to solve large scale problems [10]. More recently, Sun et al. used dual decomposition method to tackle the arrival scheduling problem which is known to be NP hard (2011). By using dual decomposition method, each scenario becomes a smaller sub-problem, which can be solved separately or even in parallel. Note even though our scenario number is not very large, the solving time is not linear to the problem size. Solving each scenario separately is much faster than solving them as a whole. For a large scale problem, the difference could even be whether this problem can be solved or not.

3.2 Dual Problem Formulation

Step 1 Decompose the terms scenario by scenario, the objective function is a summation of the total delay of each scenario.

We define:

$$f^q(Y^q, W^q) = P\{q\}\left\{\sum_{k\in K}\sum_{f\in\mathcal{F}}\left(\sum_{t\in \mathrm{T}_{k,f}}\left(Y^{plan}_{k,f,t} - Y^q_{k,f,t}\right)\right)\right.$$
$$\left. + \lambda\left(\sum_{k\in K}\sum_{f\in\mathcal{F}}\left(\sum_{t\in \mathrm{T}_{k,f}}\left(Y^q_{k,f,t} - W^q_{k,f,t+L}\right)\right)\right)\right\}$$

(3.1)

So the objective function can be rewritten as following:

$$\text{Min } \textstyle\sum_{\xi_q\in\mathcal{H}} f^q(Y^q, W^q) \tag{3.2}$$

Step 2 By forming the partial Lagrangian for the last constraints (coupling constraints), we can obtain the dual problem:

$$\mathrm{g}(\upsilon) := \text{Max}_{\upsilon\geq 0} \text{ Min} \sum_{\xi_q\in\mathcal{H}} f^q(Y^q, W^q) + \sum_{q=1}^{Q-1}\sum_{k\in K}\sum_{f\in\mathcal{F}}\sum_{t=1}^{\tau_q}\left(\upsilon^q_{k,f,t}\left(Y^{\xi_q}_{k,f,t} - Y^{\xi_{q+1}}_{k,f,t}\right)\right)$$

(3.3)

s.t.

$$W^q_{k,f,t} \leq W^q_{k,f,t+1} \quad \forall\, k\in K, f\in\mathcal{F}\ \xi_q\in\mathcal{H}, \mathrm{t}\in\Gamma \tag{3.4}$$

$$Y^q_{k,f,t} \leq Y^q_{k,f,t+1} \quad \forall\, k\in K, f\in\mathcal{F}\ \xi_q\in\mathcal{H}, t\in\Gamma \tag{3.5}$$

$$W^q_{k,f+1,t+\Delta(t)^q} - W^q_{k,f,t} \leq \sigma^q(t) \quad \forall\, k\in K, f\in\mathcal{F}\ \xi_q\in\ \mathcal{H}, \mathrm{t}\in\Gamma, \mathrm{t}\leq T - inf\Delta(t)^q \tag{3.6}$$

Where:

$$\sigma^q(t) = \begin{cases} W^q_{k,f,t+(\overline{\Delta}-\underline{\Delta})} & t\in\left[\tau_q, \tau_q - \left(\overline{\Delta}-\underline{\Delta}\right)+1\right] \\ W^q_{k,f,t-(\overline{\Delta}-\underline{\Delta})} & t\in\left[\tau_q+1, \tau_q + \left(\overline{\Delta}-\underline{\Delta}\right)\right] \\ 0 & otherwise \end{cases}$$

$$W^q_{k,f,t+L} \leq Y^q_{k,f,t} \quad \forall\, k\in K, f\in\mathcal{F}\ \xi_q\in\mathcal{H}, \mathrm{t}\in \mathrm{T}_{k,f}, \mathrm{t}\leq T-L \tag{3.7}$$

$$Y_{k,f,t}^{q} \leq Y_{k,f,t}^{plan} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, t \in \mathrm{T}_{k,f} \tag{3.8}$$

$$W_{2,j,t+\Delta(t)^q}^{q} - W_{1,i,t}^{q} - S_{ij}^{q} \leq \sigma^q(t) \quad \forall\, i,j \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma, \mathrm{t} \leq T - inf\Delta(t)^q \tag{3.9}$$

$$W_{1,i,t+\Delta(t)^q}^{q} - W_{2,j,t}^{q} + S_{ij}^{q} \leq \sigma^q(t) + 1 \quad \forall\, i,j \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma, \mathrm{t} \leq T - inf\Delta(t)^q \tag{3.10}$$

$$W_{k,f,t}^{q}, Y_{k,f,t}^{q} \in \{0,1\} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma \tag{3.11}$$

Step 3 Combine the coupling constraints which belong to the same scenario. Use array v_q to express Lagrangian multiplier for each scenario and use array Y^q to express the corresponding decision variables. Then re-arrange the terms in objective function of dual problem to group the terms scenario by scenario, we can obtain the master problem:

$$\mathrm{g}^* = \mathrm{Max}_{v \geq 0} \sum_{q=1}^{Q} g^q(v_1, v_2, \ldots, v_{Q-1}) \tag{3.12}$$

Where

$$g^q(v_1, v_2, \ldots, v_{Q-1}) = \min\left(f^q(Y^q, W^q) + \sum_{j=1}^{Q-1} v_j{}^T Y^q\right) \tag{3.13}$$

which is the sub-problem for each scenario q.

Step 4 Iterations

Sub-problem: $g^q(v_1, v_2, \ldots, v_{Q-1}) = \min\left(f^q + \sum_{j=1}^{Q-1} v_j{}^T Y^q\right), q \in \{1, \ldots, Q\}$ (3.14)

s.t.

$$W_{k,f,t}^{q} \leq W_{k,f,t+1}^{q} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma \tag{3.15}$$

$$Y_{k,f,t}^{q} \leq Y_{k,f,t+1}^{q} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, t \in \Gamma \tag{3.16}$$

$$W_{k,f+1,t+\Delta(t)^q}^{q} - W_{k,f,t}^{q} \leq \sigma^q(t) \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma, \mathrm{t} \leq T - inf\Delta(t)^q \tag{3.17}$$

Where:

$$\sigma^q(t) = \begin{cases} W_{k,f,t+(\overline{\Delta}-\underline{\Delta})}^{q} & t \in \left[\tau_q, \tau_q - \left(\overline{\Delta} - \underline{\Delta}\right) + 1\right] \\ W_{k,f,t-(\overline{\Delta}-\underline{\Delta})}^{q} & t \in \left[\tau_q + 1, \tau_q + \left(\overline{\Delta} - \underline{\Delta}\right)\right] \\ 0 & otherwise \end{cases}$$

$$W_{k,f,t+L}^{q} \leq Y_{k,f,t}^{q} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \mathrm{T}_{k,f}, \mathrm{t} \leq T - L \tag{3.18}$$

$$Y_{k,f,t}^{q} \leq Y_{k,f,t}^{plan} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, t \in \mathrm{T}_{k,f} \tag{3.19}$$

$$W_{2,j,t+\Delta(t)^q}^{q} - W_{1,i,t}^{q} - S_{ij}^{q} \leq \sigma^q(t) \quad \forall\, i,j \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma, \mathrm{t} \leq T - inf\Delta(t)^q \tag{3.20}$$

$$W_{1,i,t+\Delta(t)^q}^{q} - W_{2,j,t}^{q} + S_{ij}^{q} \leq \sigma^q(t) + 1 \quad \forall\, i,j \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma, \mathrm{t} \leq T - inf\Delta(t)^q \tag{3.21}$$

$$W_{k,f,t}^{q}, Y_{k,f,t}^{q} \in \{0,1\} \quad \forall\, k \in K, f \in \mathcal{F}\ \xi_q \in \mathcal{H}, \mathrm{t} \in \Gamma \tag{3.22}$$

The dual decomposition algorithm flowchart is shown in Figure 3.1. The whole problem is decomposed scenario by scenario. Each sub-problem is an independent optimization problem which is easier to solve. To solve the dual problem, we need to compute the sub-gradient of the dual function and update Lagrange multiplier and step size each loop, the detail is shown in Table 3.1. It is easy to find that the sub-problem can be solved in parallel to improve the computing efficiency.

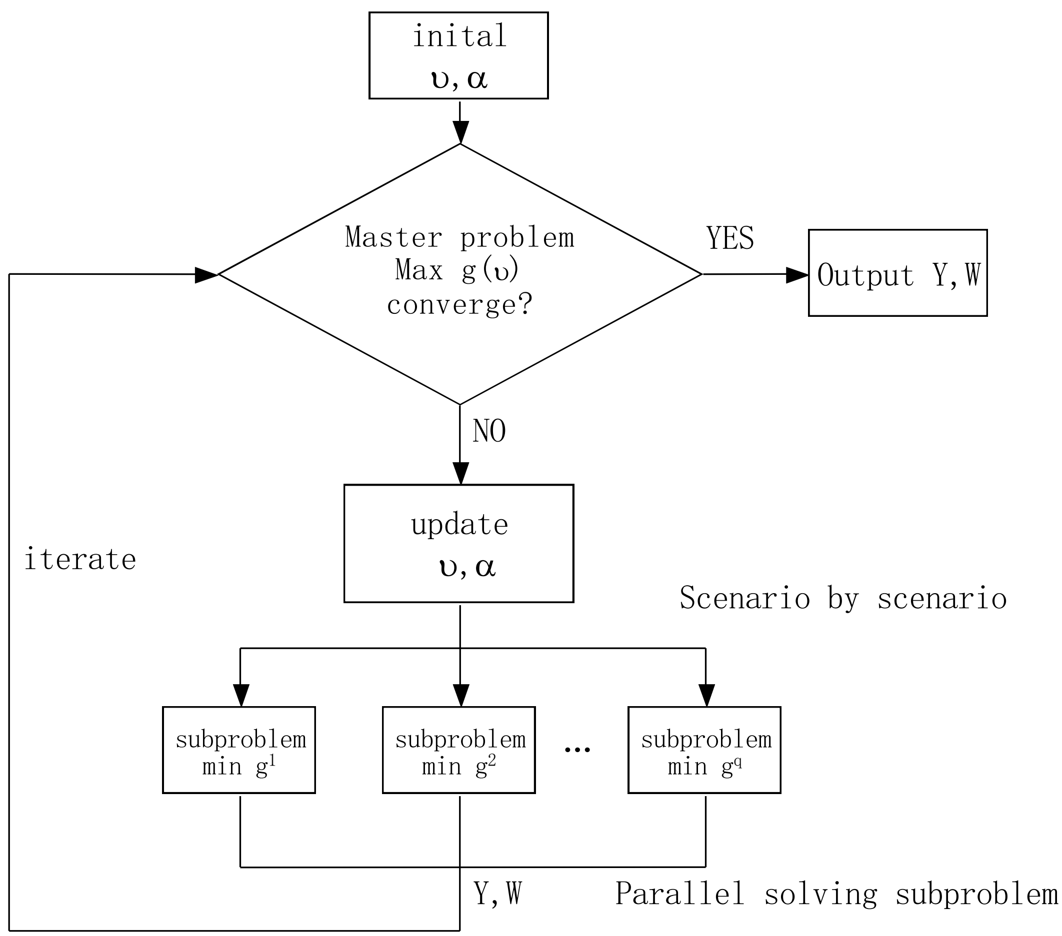

Figure 3.1 Flowchart for dual decomposition algorithm

Table 3.1Dual Decomposition Algorithm

Inputs:
Planning time horizon T
Schedule flights plan N_1, N_2, $T_{k,f}$.
Scenario Tree information τ_q, $\Delta(t)^q$, $P\{q\}$.
Initial Lagrange multiplier v_q.
Initial step size α_0.
Step1: Solve sub-problems one by one (for each scenario)
Step2: update master problem
If master problem g^* converge or i=max iteration
Output Y and W, stop
else update:
master algorithm sub-gradients
$d(t) = Y_{k,f,t}^{\xi_q} - Y_{k,f,t}^{\xi_{q+1}} 1 \leq t \leq \tau_q$
$v_q = (v_q + \alpha_i d(t))_+$
$i = i + 1$
Go to Step 1.
Where
$\alpha_i = \frac{1}{\sqrt{i+1}}$ is the step size and i is the index of iteration.

3.3 Computing Improvement

To demonstrate the computing improvement by using the dual decomposition method, a half hour case is studied which has 10 flights on each path and three scenarios in total. Based on our inputs, the experiment problem has 25370 constraints and 3988 decision variables. The solving time is sensitive to the parameters of input. The model was solved ten times and the average solving time was 893s. However, after the problem is decomposed scenario by scenario, each scenario is a sub-problem, whose solving time is much shorter. Figure 3.2 shows the solving time for each scenario in each step. The average solving time for three scenarios was around 0.2011s, 0.1926s and 1.2742s. On average, the objective value converges in 14 steps. This means the total computing time by dual decomposition method is 38 times faster. Moreover if we consider solving the sub-problems in parallel, the computing time could be up to 50 times faster.

Since the computing time is not linearly related to the problem size, the dual decomposition method with parallel computing will improve the computing efficiency. More importantly, the unsolvable, large-size problem can be converted into several solvable sub-problems, and be solved step by step. This is the key advantage of the dual decomposition method.

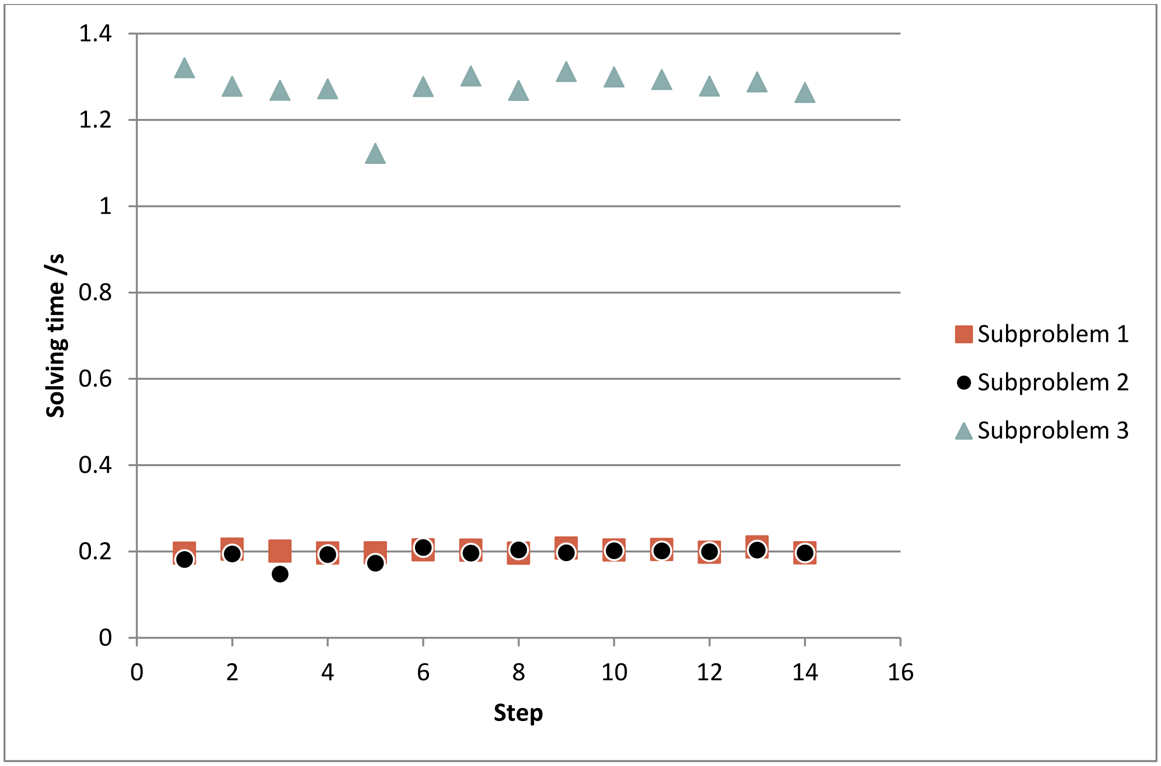

Figure 3.2 Computing time for each scenario's sub-problem

CHAPTER 4. EXPERIMENTS WITH A LARGE SCALE PROBLEM

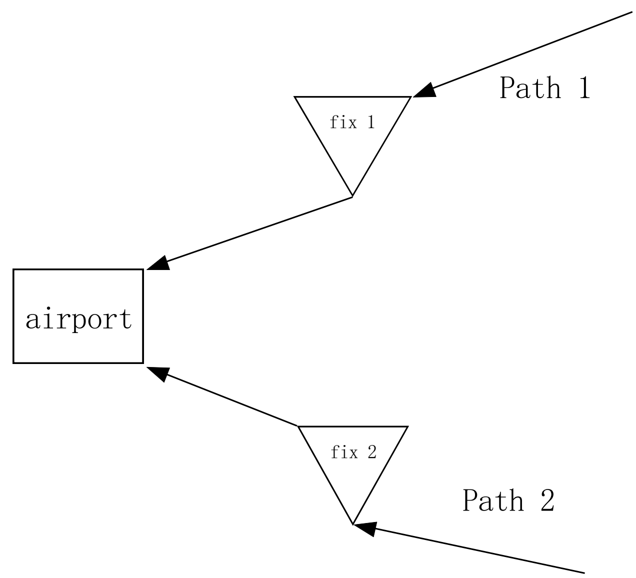

Figure 4.1 The conceptual airspace used in our model

4.1 Experimental Setup

Now we consider a large-scale problem with many more flights and longer planning time. The arrival schedule between 11:00 and 12:00 AM on October 13, 2013 at Hartsfield–Jackson Atlanta International Airport (ATL) is used in our experiment, shown in Table 4.1. The Bureau of Transportation Statistics (BTS) database is the source of data on scheduled arrival times of individual flights [15].

The model was programmed with C++ as a single thread program on a 2.8GHz INTEL i7 CPU, 16G RAM DELL workstation running LINUX. The mathematical programming solver Gurobi5.6.3 was used, which is capable of solving IP problem [16].

Table 4.1 Arrival Schedule between 11-12 am on Oct 13, 2013 at ATL

Path1 Total 22 flights	11:05	11:07	11:09	11:17	11:23	11:27	11:28
	11:30	11:35	11:36	11:37	11:38	11:40	11:43
	11:46	11:50	11:51	11:52	11:54	11:57	11:58
	11:59						
Path2 Total 18 flights	11:02	11:04	11:07	11:11	11:12	11:13	11:14
	11:20	11:25	11:26	11:28	11:32	11:37	11:38
	11:39	11:51	11:57	11:58			

We will study the sensitivity of our results and do model validation in this chapter. First we build a baseline case and a set of alternative cases in which particular model inputs are varied. Through comparing the results of baseline case with result of each alternative case, we can get interesting insight from our model.

Case I: The baseline case

We consider a total of 40 flights scheduled to arrive at Hartsfield–Jackson Atlanta International Airport (ATL) between 11:00 and 12:00 AM. As shown in Table 5, there are 22 scheduled flights on path 1 and 18 scheduled flights on path 2. The window slot duration is random assigned between 20-30 min. We will set T=90 min and we extend half hour in case some flights may face longer delay. Note here, the problem size is related to the planning time periods, so it is critical to choose a proper value for T. A large T value will make the problem size too large to solve but too small T value may face the situation that not all flights have landed. Let the cost ratio between airborne holding and ground holding be 2 ($\lambda = 2$) for the baseline case. We assume the required fly time from arrival fixes to airport is 10 min, which means L=10.

We will consider three time-separation scenarios: $\mathcal{H} = \{\xi_1, \xi_2, \xi_3\}$. In each of them, we assume weather only change once, from bad weather (big separation $\bar{\Delta}$= 5min) to good weather (small separation $\underline{\Delta}$= 1min). But the exact timing of weather changing depends on the scenario. As shown in Figure 4.2, for the first scenario the weather will become good starting 11:10am, inducing a small separation. The changes in second scenario and third scenario will happen at 11:20 am and 11:30 am respectively.

Figure 4.3 shows the scenario tree of our case I. We can see that at 11:10 am and 11:20 am, two new branches come out corresponding to different scenarios. Each scenario occurs with a probability. Here we set the probability Mass Function for case 1 as following:

$$\mathrm{P}\{\xi_1\} = 0.8; \mathrm{P}\{\xi_2\} = 0.1; \mathrm{P}\{\xi_3\} = 0.1$$

Which means that the first scenario will happen with a high chance; we expect to observe early arrival decisions and less ground hold.

Then based on the case I, we will modify one parameter each time to define a new case in order to study the impact of that parameter for this model. We defined 3 alternative cases in total. Now we will describe the detail of the 3 alternative cases.

Case II: Change the Probability Mass Function of Scenarios

In this case we change the probability mass function of scenario; the worst case scenario will have the highest probability, which means the weather will probably keep bad for a long time. So the Probability Mass Function is set as following:

$$\mathrm{P}\{\xi_1\} = 0.1; \mathrm{P}\{\xi_2\} = 0.1; \mathrm{P}\{\xi_3\} = 0.8$$

The other parameters are set as the same with the baseline case.

Case III: Change the delay cost Ratio between airborne delay and ground delay

In this case we change the cost Ratio between airborne delay and ground delay. We will increase it from $\lambda = 2$ up to $\lambda = 20$. So the airborne delay is much more expensive than the ground delay.

Case IV: Fix the arrival order of flights

In this case we demonstrate our model's ability to reduce delay by allowing flights to switch arrival order with other flights on the other path. We fixed the arrival order based on the original schedule shown in Table 4.1. So it is equal to a single path problem with fix arrival order, all flights obey the "First Come, First Serve" rule. The detail of the fixed arrival order schedule is shown in Table 4.2.

Table 4.2 Arrival Schedule with Fixed Arrival Order

	Time	#	Time	#	Time	#	Time	#	Time	#	Time	#
Path1:Total 22 flights	11:05	3	11:07	4	11:09	6	11:17	8	11:23	13	11:27	16
	11:28	17	11:30	19	11:35	21	11:36	22	11:37	23	11:38	25
	11:40	28	11:43	29	11:46	30	11:50	31	11:51	33	11:52	34
	11:54	35	11:57	36	11:58	39	11:59	40				
Path2:Total 18 flights	11:02	1	11:04	2	11:07	5	11:11	7	11:12	9	11:13	10
	11:14	11	11:20	12	11:25	14	11:26	15	11:28	18	11:32	20
	11:37	24	11:38	26	11:39	27	11:51	32	11:57	37	11:58	38

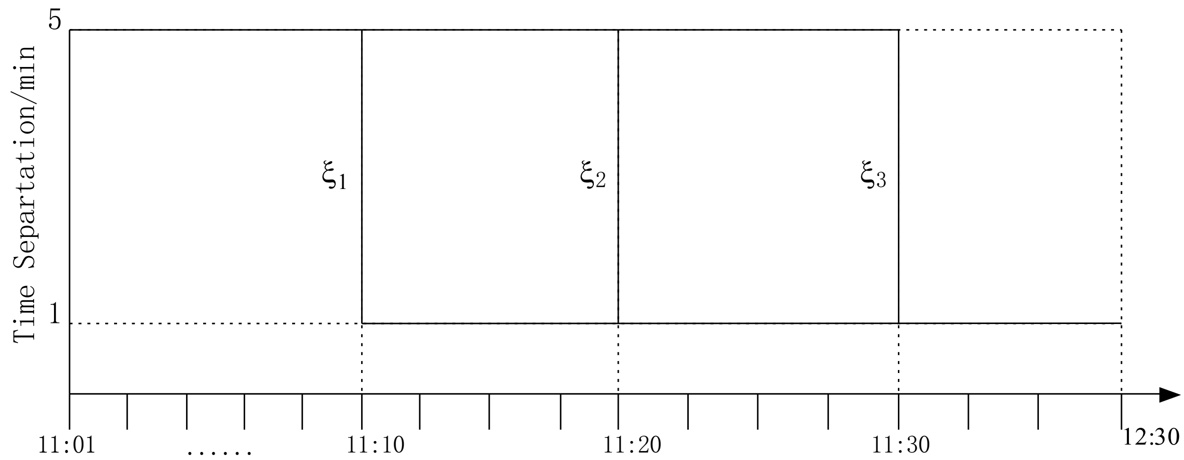

Figure 4.2 Separations Corresponding to Different Scenarios in Experimental

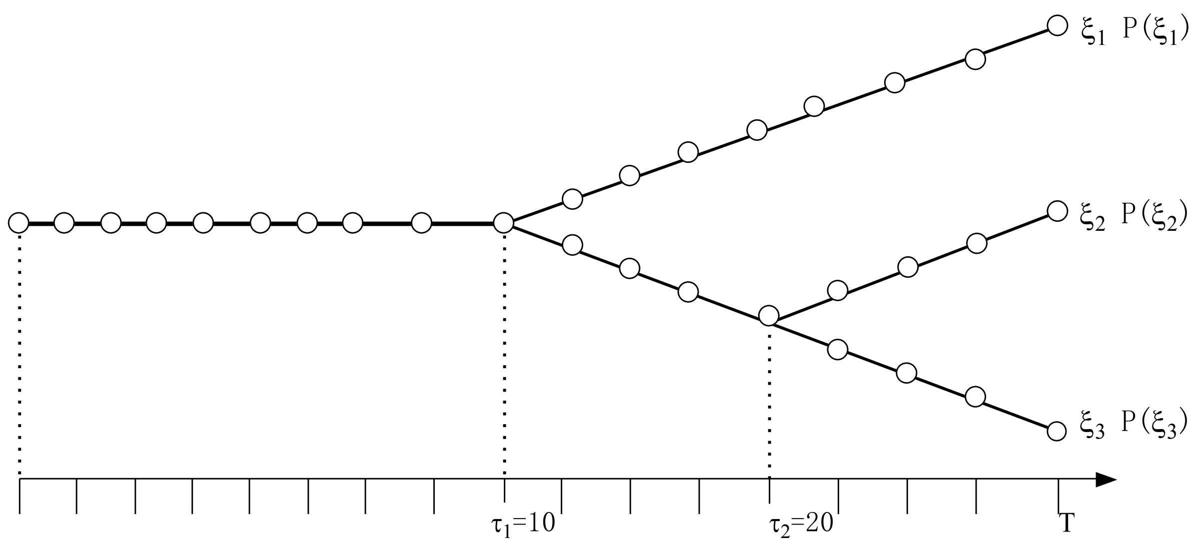

Figure 4.3 Scenario tree for case 1

4.2 Results

We applied our model to all of the four cases described above. The four cases result of expected delay costs is summarized in Table 4.3. Besides the four cases, a perfect information case is calculated to work as an ideal case. The perfect information case is actually the deterministic case, in which we calculated the schedule for each scenario separately accounting for its specific deterministic separation profile. Then they are multiplied with their associated scenario probabilities to get the ideal delay cost. And we compare the result of the four stochastic cases with the deterministic case to measure the total delay in percentage.

In Case 1, we compare our stochastic model with the deterministic model. In our stochastic model, about 25% more delays are assigned, especially some airborne delays are among them. For deterministic case, information is perfect for each scenario, which means we can assign ground delays to replace the airborne delays. For example, if we know a flight will face airborne delay for 4 units of time after it arrives at the arrival fix, we can assign 4 more units time of ground delays to make sure this flight will not wait when it approaches the airport. But for stochastic case, the information about future weather condition is not perfect, each scenario has chance to occur, which may cause airborne delays. For example, the first scenario will happen with a high chance in case 1. Most flights will be assigned less ground delays to arrive at the fix as the scheduled plan due to the high probability for good weather to occur at 11:10 am. The detail of each flight's schedule is shown in Figure 4.4 and 4.5. We can find that most decisions coincide with the scheduled plan before the scenario trees diverging point (t=10). Flights will face airborne delays if the second scenario and the third scenario happen, but their probabilities are very low relative to the first scenario. So the expected total delay cost is optimal even there are airborne delays here. On the other hand, our delay moderate cost ratio ($\lambda = 2$) also contributes to this result.

Another difference between the deterministic case and stochastic case is that all three scenarios are calculated separately as three small problems in the deterministic case; but for stochastic case, all three scenarios are solved together as a whole big problem. As a result of the coupling constraint, all decisions are the same before the scenario trees diverging point because decision can only be made on information available at that time. In Figure 4.4 and Figure 4.5, we can see that all three scenarios' arrival decisions at fix are the same before the first scenario trees diverging point (t=10). And the second scenario's arrival decisions at fix are also the same with the third scenario before they diverge (t=20). This feature makes stochastic case have more delays because it cannot adjust decisions separately for each scenario while the deterministic case with the perfect information can.

In Case 2, the Probability Mass Function of scenarios is changed. The worst weather condition scenario is going to happen with a high probability, which means more ground delays will be assigned in the early decision stage to avoid airborne delays. But this strategy will product unnecessary ground delays to the first two scenarios at the same time. This is confirmed by the result shown in Table 4.3. There is no airborne delay for any scenario in Case 2. The expected delay cost is much higher than that in Case 1 because more unnecessary ground delays are assigned with this conservative weather forecast ; on the other hand, the probability associated with the worst weather condition scenario increase a lot from 0.1 to 0.8. These two reasons contribute to the expected cost increasing.

In Case 3: we increase the delay cost ratio between airborne delay and ground delay from $\lambda = 2$ up to $\lambda = 20$, which means airborne delays unit cost is much higher than that of ground delay. So we can expect that more ground delays will be assigned in all three scenarios to avoid the high cost airborne delays. The expected cost result shown in Table 7 confirmed this. Also no airborne delay is assigned by any scenario in this case. The high delay cost ratio force the model to make the decision mainly based on the worst weather condition scenario (the third scenario). The flight schedule result is the same with that in Case 2, the lower expected total cost is because the probability associated with the third scenario is much less than the one in Case 2.

Finally, the result from Case 4 demonstrates our model's ability to reduce the expected delay cost comparing with the fixed arrival order schedule. The fixed arrival order case is equal to a single path problem in which all flights obey the "First Come, First Serve" rule. Add one more path is like giving a little more freedom to the optimization problem to find a more optimal solution. The performance improvement is not much by our two paths model in this experiment; it only reduces the expected delay cost from 158% to 125%. This is due to our loose schedule plan. With a tighter schedule with many overlaps in the scheduled arrival time period, our model will reduce much more delay cost. We can also try to give more freedom to the problem, such as allowing all of the flights to switch orders with each other. Under this assumption, we can expect a more optimal schedule with much less delay cost. But the complexity of the problem will also increase significantly, which cannot be handled.

In addition, the solving time of the original problem is long according to our experimentations. Based on our inputs, the experiment problem size is large with 254,570 constraints and 22,788 decision variables. The solving time can be up to 200,000s and it is sensitive to the parameters of input. In most cases, the problem was not solved even after 200,000s. After the problem is decomposed scenario by scenario, each scenario is a sub-problem, whose solving time is much shorter. The average solving times for the three scenarios are 238s, 975s and 8742s. On average, the objective value will converge in 17 steps. Moreover if we solve the sub-problem in parallel, the computing time could be around $8742\text{s} \times 17 = 148{,}614\text{s}$. The solving time is still long, but we converted an unsolvable large size problem into a solvable one. This is the critical improvement by the dual decomposition method.

Table 4.3 Expected delay costs for all cases

	Ground Delay Cost	Airborne Delay Cost	Total Delay Cost	Delay in percentage
Deterministic case	42.8	0.0	42.8	100%
Case 1	43.7	9.8	53.5	125%
Case 2	186.4	0.0	186.4	436%
Case 3	87.8	0.0	87.8	205%
Case 4	53.3	14.2	67.5	158%

Note: Airborne delay cost is already multiplied by the cost ratio in this table.

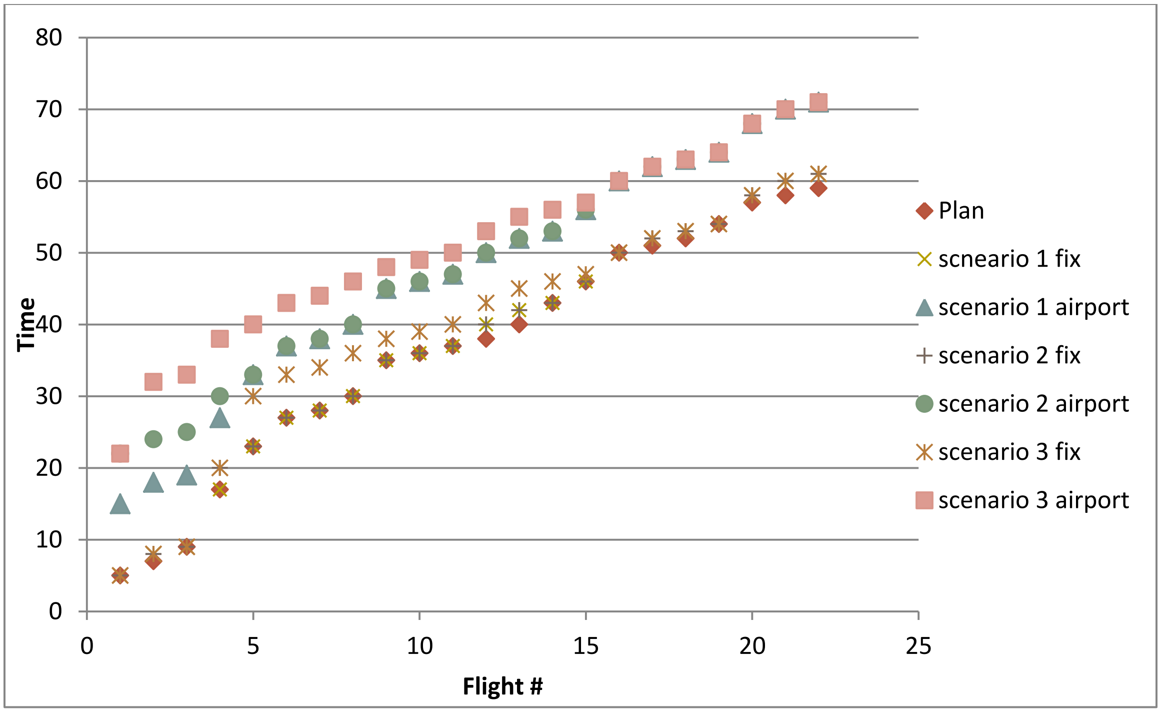

Figure 4.4 Schedule Result for flights on path 1 in Case 1

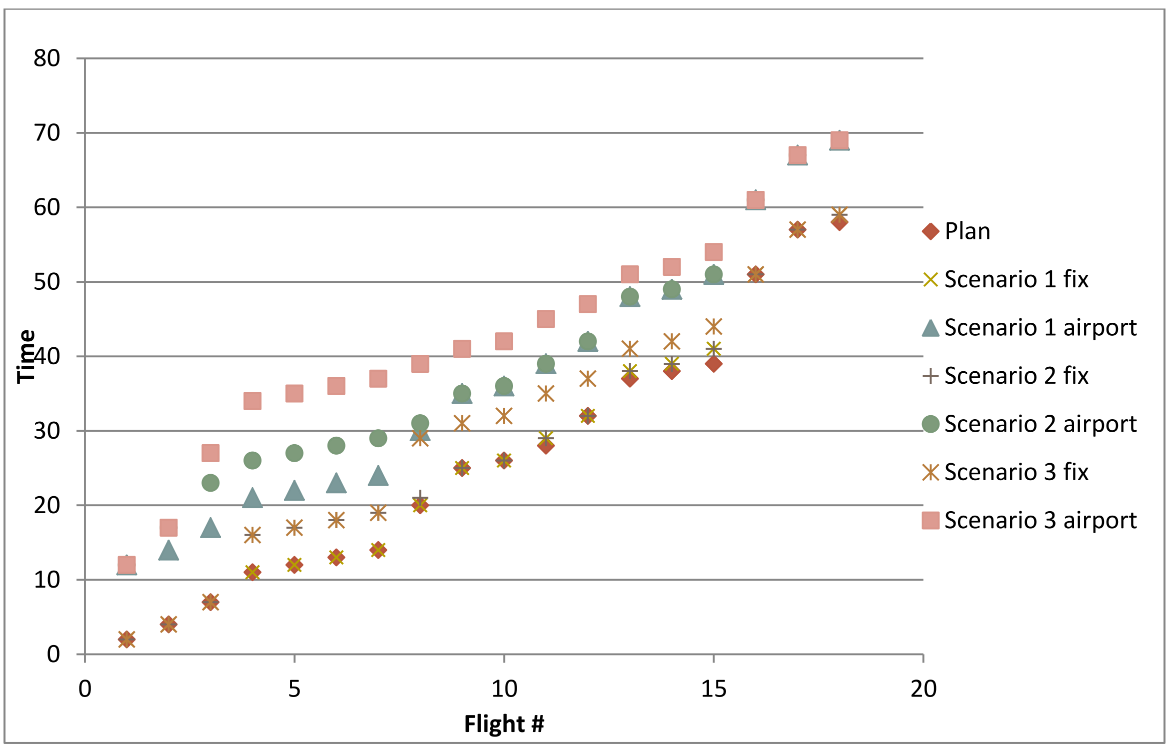

Figure 4.5 Schedule Result for flights on path 2 in Case 1

CHAPTER 5. CONCLUSION AND FUTURE WORK

5.1 Conclusion

This thesis presents a dynamic stochastic 0-1 IP model for converging inbound air traffic. This model addresses the single airport ground holding problem (SAGHP) with respect to uncertainty in the separation between flights instead of Airport Acceptance Rates or landing capacity. This model can overcome the limitation that the individual flight cannot be scheduled very accurately in previous models because the basic length period of time of AAR is normally 15 min or more. Uncertainty in separation between flights according to different weather conditions in airport is represented through the scenario tree method. This model is able to handle the time varying separation of minute-in-trail and the uncertainty rising from the imperfect forecast of weather condition.

Based on our dynamic stochastic IP model, We address a problem involving two paths inbound air traffic merging into a single airport in which uncertainty of separation from minute-in-trail restriction is considered. Although “First Come, First Serve” policy is still obeyed by flights on the same path, the simulation experiment has shown that, allowing flights on different paths to switch arrival orders will help reduce the total delays. Ideally, this model can be extended to perform with more freedom on the arrival order, such as all flights can switch arrival order with each other. But it will increase the complexity of the problem very quickly as more freedom is given to the flight’s arrival order. According to our experiment, it will become untraceable very quickly when the problem size increases.

In order to tackle the running time problem faced by the disaggregate model we built, we introduce dual decomposition method into the model to improve the computing efficiency. The original problem is decomposed scenario by scenario into several sub-problems based on the dual decomposition method; then a parallel computing algorithm is developed to handle these sub-problems. Such combination of dual decomposition method and parallel greatly increases computational efficiency. In our experiment, even though the computing time is still long for a large size problem after decomposition, this method can convert an unsolvable large size problem into a solvable one. This is the critical improvement by the dual decomposition method with our model.

5.2 Future research recommendation

There are primarily two approaches to address decision-making under uncertainty, Stochastic Programming and Robust optimization. Most of the models are built with Stochastic Programming to deal with the uncertainty in GHPs, in which scenarios are generated with associate probability to represent the uncertainty. However, in practice it is difficult to know the exact distribution of the uncertainty to help generate the corresponding scenarios. Moreover, as the scenario number increases, the complexity of the problem increases quickly and the problem becomes intractable even using decomposition method. With the development of Robust Optimization recently (Bertsimas et al. and the recent book by Ben Tal et al), it could presents a tractable framework to model optimization problems under uncertainty [17][18]. But few work has been done to deal with uncertainty in GHP by using Robust Optimization, there is still room for further improvement in this area.

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