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Modeling and Optimization of Care Transitions

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献给我的父母。

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NOTATION	DESCRIPTION
A_i	The state that the patient is admitted by the inpatient care setting for the i^{th} episode.
A_{ij}	The state that the patient has spent j consecutive days in the inpatient care setting during his i^{th} episode.
B_i	The state that the patient enters for the i^{th} time and is staying at the home- and community-based care setting.
R	The state that the patient is cured and leaves the system.
D	The state that the patient is forcedly transited out of the system after N inpatient episodes.
p_i^A	The probability of transiting from $A_{ij}(j \in [1, m-1])$ to B_i.
p_i^B	The probability of transiting from B_i to $A_{(i+1)1}$.
q_i^A	The probability of transiting from $A_{ij}(j \in [1, m])$ to R.
q_i^B	The probability of transiting from B_i to R.
N	The maximum number of episodes covered by the insurance company.
m_i	The maximum coverage LOS in the inpatient care setting during the i^{th} episode.
m_i^*	The optimal coverage LOS during the i^{th} episode.
$c_S^{A_i}, c_S^{B_i}$	The daily cost of staying at A_i or B_i.
$c_T^{A_i}$	The cost of forced transiting into B_i.
$c_T^{B_i}$	The cost of readmission into A_i.
C	The expected total cost.

C_S	The expected cumulative cost of staying in the system.
C_S^A, C_S^B	The expected cumulative cost of staying in the inpatient care setting or the home- and community-based care setting.
C_T	The expected cumulative cost of transition in the system.
C_T^A	The expected cumulative cost of (forcedly) transiting from the inpatient care setting to the home- and community-based care setting.
C_T^B	The expected cumulative cost of transiting from the home- and community-based care setting to the inpatient care setting.
$P_E^{A_i}, P_E^{B_i}$	The probability that the patient will enter $A_i \; or \; B_i$.
$P_j^{A_i}, P_j^{B_i}$	The probability that the patient will stay in $A_i \; or \; B_i$ for j days, given the condition that the patient enters $A_i \; or \; B_i$.
$P_S^{A_i}, P_S^{B_i}$	The probability that the patient not be cured during the i^{th} inpatient episode or the i^{th} time in the home- and community-based care setting, given the condition that the patient enters $A_i \; or \; B_i$.
$P_F^{A_i}$	The probability that the patient is forcedly transited after the i^{th} inpatient episode, given the condition that the patient enters A_i.
$E[T_i^A], E[T_i^B]$	The expected number of days spend in $A_i \; or \; B_i$, given the condition that the patient enters $A_i \; or \; B_i$.

ABSTRACT

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More than 145 million people live with at least one chronic condition, and almost half of them have multiple conditions. As a result, many managed care and integrated delivery systems have taken a great interest in alleviating the many deficiencies in managing the current care system that spans across various care delivery settings. In addition, many Americans have to rely on some social health insurance plan to cover her care expenses. As a result, these patients often may not been sufficiently cured but have to be transitioned to less expensive but less medically intensive facilities, due to the increasing pressure on the social health insurance programs to save their total spending. This in turns increases the risk of being readmitted to more expensive facilities sooner.

In this thesis, we systematically study stochastic transitions within a system of care delivery. We investigate how to modify insurable length of stay to reduce the total care spending and improve the quality of care to individual patients.

We first develop a chronic care cycle model to optimize the transitions between two types of settings: the inpatient care setting and the home- and community-based care setting. By optimizing the number of covered episodes, and the coverage LOS for each episode, this model is intended to balance the tradeoff between the cost of staying and the cost of (forced) transition, as well as the tradeoff between current cost and future (opportunity) cost. The results indicate that as a public insurer, the best strategy will be only focusing on the early episodes and covering them unlimitedly.

We also develop a three-layer rehabilitation service process model and use discrete event simulation to study the transitions among three levels of rehabilitations: primary rehab, secondary rehab, and tertiary rehab. We test different values for on the coverage LOS for primary rehab and secondary rehab to balance the tradeoff between the current cost and future cost. We assume some relationship between the quantity of care and care transition probability, and observe their joint effect on cost and rehospitalization incidences in the given length of period. The results indicate that as a public insurer, the best strategy will be remaining current coverage on primary rehab but limiting coverage on secondary rehab.

CHAPTER 1. INTRODUCTION

1.1 Background and Motivation

More than 145 million people, or almost half of all Americans, live with at least one chronic condition. That number is projected to increase by more than one percent per year by 2030, resulting in an estimated chronically ill population of 171 million (Wu & Green, 2000). Almost half of all people with chronic illness have multiple conditions (Anderson, 2004). As a result, many managed and integrated care delivery systems have taken a great interest in alleviating the many deficiencies in current management of diseases such as diabetes (Kimura, DaSilva, & Marshall, 2008), heart disease, depression, asthma and others. As a result, cyclic chronic care management models (Carson, Cramp, Morgan, & Roudsari, 1998) and coordinated care system (Battersby, 2005) have been proposed to improve the operations in care coordination and transition. In a coordinated care delivery system, patient will receive care services in different levels of settings. Based on the quantity of service provided, the settings can be described as upstream settings and downstream settings. Upstream settings are expensive but provide higher quantity of care, while downstream settings are more affordable but the quantity of care is lower. For the upstream settings, the process of admission, staying and receiving care, and leaving for downstream settings is referred as one episode. However, it remains unclear how the practice guideline should be developed at the individual level for either proposed integrated care delivery system, especially when it is under pressure of reducing readmissions while cutting down inpatient spending.

Many Americans have to rely on some social health insurance plan to cover her care expenses, which typically specifies the number of episodes to cover, and the number of days (referred as LOS) to cover for each episode. For these people, after staying in the inpatient care setting for a certain number of days, they may have not been sufficiently cured but have to leave for the home- and community-based care setting, then they will be forced to transition out of the inpatient care setting if without the continued coverage of the insurance plan. As a result, the patients may be more likely to be readmitted back to the inpatient care setting sooner.

It is common for patients in the United States to be readmitted to inpatient care hospitals after a short period of time post hospital discharge. Hospitals readmissions incur unnecessary cost. It is estimated that preventable readmissions for Medicare patients alone cost $17 billion annually (Jencks, Williams, & Coleman, 2009) which is equivalent to more than 10% of Medicare benefit payment for hospital inpatient services (Centers for Medicare & Medicaid Services, 2013). Hence, readmission reduction is critical to the U.S. public funding agencies, such as Medicare and Medicaid, whose spending increases rapidly in recent years with the population aging and increased prevalence of chronic conditions.

1.2 Problem Description

In this thesis, we develop two models for optimizing a coordinated care delivery system. In Chapter 3, we develop a stochastic process model to optimize the cyclic care delivery process between the inpatient care setting and the home- and community-based care setting. In Chapter 4, we developed a discrete event simulation model to study transitions among three levels of rehabilitations services: primary rehab, secondary rehab and tertiary rehab.

1.2.1 Chronic Care Cycle

In a chronic care cycle model, we consider a cyclic care delivery process optimization problem involving care transitions between two distinct care delivery settings: the inpatient care setting and the home- and community-based care setting (referred as HCBC). We take the viewpoint of minimizing care spending for a pubic insurer. We consider two types of costs: the cost of staying in health care settings and the cost of transition between settings. One important part of transition cost is the forced transition cost, which occurs when the inpatient care setting fails to sufficiently cure the patient within the insurable LOS in a specific episode. Sometimes other forms of cost can also be counted as forced transition cost. For instance, in order to ensure that patients will receive adequate support from the insurance plans, when the insurer fails to cover a patient for a specific episode, a forced transition cost will be incurred, which can be regarded as a penalty to the insurer.

With a longer coverage LOS, the patient will spend more time in the inpatient care setting, incurring higher current cost of staying, however also increasing her chance of being cured and leaving the system for good. While the cost of staying will decrease with a shorter coverage LOS, and the insurance company will be more likely to be charged for forced transition cost. On the other hand, with shortened coverage LOS, the patient will be less likely to be cured, which incurs more transitions and higher opportunity cost. Hence an optimal insured LOS will need to balance between the cost of staying and the cost of (forced) transition, as well as the tradeoff between current cost and future (opportunity) cost.

To make this tradeoff optimally, we propose a stochastic process model for the insurance duration decision. Given a maximum number of covered episodes, the probabilities of being cured and transitioned between settings, as well as the unit cost of staying and transition, this model minimizes the expected total cost of staying and transition over the whole inpatient process, by deciding the optimal number of covered episodes, and the coverage LOS for each episode.

1.2.2 Three-Layer Rehabilitation Service Process

In the three-layer rehabilitation service process model, we consider three layers along the rehabilitation service process. The three layers are primary rehab, secondary rehab and tertiary rehab. The first two types are provided in an inpatient setting whereas the last one is given in a home- and community-based setting. Usually a patient will receive initial care in the primary rehab, while she will be serviced in the secondary rehab when rehospitalization occurs. We use discrete-event simulation to describe the transition process among them. We consider two measures: total care spending and rehospitalization incidences. We test different coverage LOS for primary rehab and secondary rehab, to observe their effect on care spending and rehospitalization frequencies.

With a longer coverage LOS, the patient will spend more time in primary and secondary rehab, incurring higher current cost, however the patient will have a lower risk of rehospitalization with higher future cost. While with a shorter coverage LOS, though the current cost can be reduced, the patient will be more likely to incur rehospitalization because of the reduced quantity of care, and the insurer will have to pay for higher cost in the future. Hence an optimal insured LOS will need to balance between the current cost and future cost.

We assume some relationship between the quantity of care and rehospitalization probability, and observe their joint effect on cost and rehospitalization incidences for the given length of period. With these observations, we can obtain a more systematic view of the system and offer insights into policy making.

1.3 Contribution

In the chronic care cycle model, we propose an innovative way of defining states in stochastic process. Instead of only using states to present the health conditions of the patient, this model also incorporates the inpatient LOS into the states, which enables us to incorporate the effect of forced transition. We also use the idea of control limit to develop algorithms to solve the optimization problem.

In the three-layer rehabilitation service process model, we introduce discrete event simulation method to study the transition system. It provides a flexible and intuitive way to test the effect of different policies on the coordinated rehab system.

1.4 Outline of Thesis

The remainder the thesis is organized as follows. In chapter 2, literatures on modeling health care transitions are presented. In chapter 3, the chronic care cycle model is developed and the optimization problem is analyzed with a case study. In chapter 4, the three-layer rehabilitation service process is simulated and the effect of shortened LOS on total cost and rehospitalization incidence is presented. Finally, in chapter 5, main conclusions are summarized, and future research is discussed.

CHAPTER 2. LITERATURE REVIEW

Several articles that propose the problem and optimization method of health care transition problem have be have been cited in this chapter. These papers motivate the problem, analyze it on multiple aspects such as rehabilitation length of stay, pattern recognition and care setting, and optimize the system by various method such as discrete event simulation, system dynamics and decision analysis.

In Thomas, Guire, & Horvat (1997), the relationship between hospital length-of-stay (LOS) and quality of care was investigated. It indicated that LOS was widely used as an indicator of hospital performance and sometimes was also assumed to be related to quality. Under some assumption, longer than expected LOSs was viewed as indicative of poor quality care. It also showed that in 1/13 of the clinical conditions examined, cases that received poor quality care had significantly longer risk-adjusted LOSs than cases with acceptable quality.

In Brailsford & Hilton (2001), two different simulation approaches, which were widely used in health care domain, were discussed: discrete event simulation and system dynamics. The aim of this paper is to discuss the root cause of the choice of methodology, and whether one approach is superior to the other. Also, it provided guidelines for modelers to choose from these approaches.

In M. Murtaugh & Litke (2002), a 2-year longitudinal record was conducted on the use of short-stay hospitals and post-acute and long-term care settings. This analysis provided new information on post-acute and long-term care use patterns and showed which types of transitions were most likely to be followed by potential problems. It also proposed three broad strategies which can improve the outcome of transitions through post-acute and long-term care settings.

In Coleman, Min, Chomiak, & Kramer (2004), The Medicare Current Beneficiary Survey was used to identify and describe patterns of post-hospital care transitions, which was characterized as uncomplicated (a sequence of transfers from higher-to lower-intensity care environments without recidivism) or complicated (the opposite sequence of events). Also, this paper developed indices to identify patients at risk for complicated transitions. It concluded that post-hospital care transitions were common among Medicare beneficiaries and patterns of care varied greatly.

In Naylor, Kurtzman, & Pauly (2009), a case was made which enhanced health care quality and outcomes among these elders by reducing preventable hospitalizations and improving transitions to and from hospitals. It recommended immediate actions targeting diffusion of evidence-based care to decrease avoidable re-hospitalizations and save cost. It was also suggested that policy changes were needed to address barriers to high-quality transitional care, including deficits in health professionals' and caregivers' knowledge and resources, regulatory obstacles, and inadequate financial incentives and clinical information systems.

In Arango-Lasprilla et al.(2010), a prediction rule was developed to acutely identify patients at risk for extended rehabilitation length of stay (LOS) after traumatic brain injury (TBI) by using demographic and injury characteristics. This prediction rule considered a series of predictors such as FIM motor and cognitive scores at admission, pre-injury level of education, cause of injury, punctate/petechial hemorrhage, acute-care LOS, and primary payer source. It was concluded that this model might allow for enhanced rehabilitation team planning, improved patient and family education, and better use of health care resources.

CHAPTER 3. A STOCHASTIC MODEL FOR CHRONIC CARE CYCLE

3.1 System Description

In this section, a stochastic process model is developed to analyze the health care transitions for a single patient between the inpatient care setting and the home- and community-based care setting. With a series of homogeneity assumptions on costs and transition probabilities, the expected total cost for a patient during her whole health care transition process will be computed, based on which the optimal coverage LOS for each episode will be derived.

Without loss of generality, we assume the health transition process contains *N* episodes in our model. There are four sets of states used to indicate the patient's condition: inpatient care setting (A_i), home- and community-based care setting (B_i), recovery (R) and death (D). The transitions between these states are illustrated in the following two diagrams.

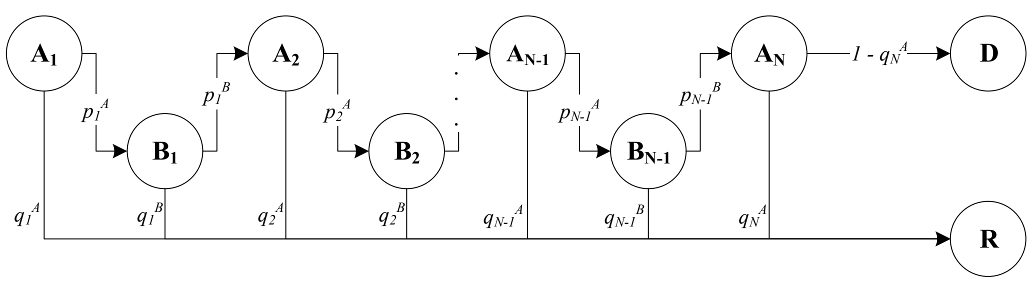

Figure 3.1 Care Transition Diagram between Settings

Figure 3.1 shows how the patient is transitioned between the inpatient care setting and the home- and community-based care setting. Initially, the patient is admitted by the inpatient care setting for the initial episode (A_1). Then a series of transitions will occur between the inpatient care setting (A_i) and the home- and community-based care setting (B_i), until one of two following conditions occurs: in one case, whenever the patient is staying in A_i or B_i, it is possible for him to be cured and enter state *R*; in the other case, after *N* episodes, the insurer will no longer provide insurance coverage, thus the patient will eventually transition to state *D*.

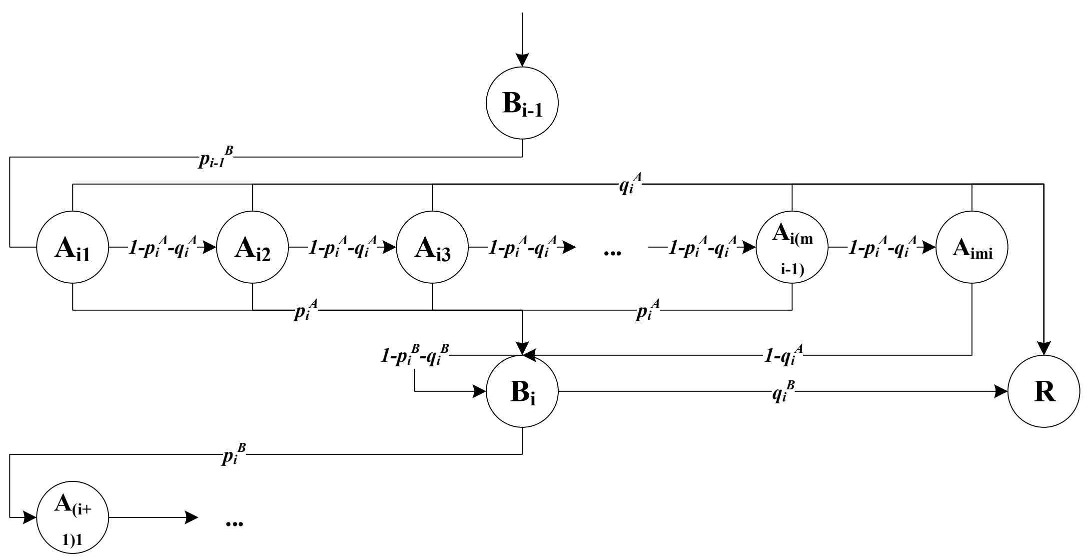

Figure 3.2 Care Transition Diagram within Settings

During each inpatient episode, for instance, the i^{th} episode $A_i (i \in [1, N])$, the state A_i is further decomposed to a series of states $A_{ij} (1 \le j \le m_i)$, which indicates that the patient is on the j^{th} day in his i^{th} inpatient episode. During this episode, the patient will be covered for a maximum m_i days. For the first $(m_i - 1)$ days, after each day staying in the inpatient care setting, she will encounter three conditions: 1) being transitioned to the home- and community-based care setting B_{i+1} with a constant probability p_i^A; 2) being cured and entering state *R* with a constant probability q_i^A; or 3) continue to stay at the inpatient care setting for another day with probability $(1 - p_i^A - q_i^A)$. After the m_i^{th} day, if the patient is still not cured, she will be forcedly transitioned to B_{i+1} with probability $\left(1 - q_i^A\right)$. The subscript $(i + 1)$ indicates that the patient is transitioned from the i^{th} episode in the inpatient care setting.

After the patient enters the home- and community-based care setting for a particular time, for instance, the i^{th} time being in the home- and community-based care setting $B_i (i \in [1, N - 1])$, similarly, after each day staying in this center, she will also encounter three conditions: 1) being readmitted by the inpatient care setting A_{i+1} with a condition probability p_i^B; 2) being cured and enters state *R* with a condition probability q_i^B; or 3) remaining at the home- and community-based care setting for another day with probability $(1 - p_i^B - q_i^B)$. Note that there will be no limit on the insured LOS at the home- and community-based care setting. Finally, after being transitioned to state *R* or *D*, the patient will never enter the inpatient care setting or the home- and community-based care setting again. These two states are absorbing states, while states $A_{ij}\ and\ B_i$ are transient states.

3.2 Steady State Analysis: Expected Total Cost over N Episodes

In order to optimize the coverage LOS, denoted by m_i, during each episode *i*, we analyze the steady state of the system and derive the expected total cost over *N* episodes. The insurance plan will cover the cost of staying and transitions for the patient. Therefore, the total cost *C* consists of four parts: the total cost of staying in the inpatient care setting

(C_S^A) and in the home- and community-based care setting (C_S^B), as well as the total cost of transition from the inpatient care setting to the home- and community-based care setting (C_T^A), and from the home- and community-based care setting to the inpatient care setting (C_T^B). For the i^{th} episode, the daily cost of staying in the inpatient care setting is $c_S^{A_i}$, and the one-time cost of forced transition is $c_T^{A_i}$. While when the patient stays in B_i, the daily cost of staying in the home- and community-based care setting is $c_S^{B_i}$, and the one-time cost of transiting to the inpatient care setting for readmission is $c_T^{B_i}$. The daily cost of staying in the inpatient care setting $c_S^{A_i}$ is higher than staying in the home- and community-based care setting $c_S^{B_i}$.

With the introduced notation above, the objective function of the model, which is a function of the coverage LOS for each episode $m_1, m_2, \cdots, m_N$, is presented as:

$$C(m_1, m_2, \cdots, m_N) = (C_S^A + C_S^B) + (C_T^A + C_T^B)$$

$$= \sum_{i=1}^{N}\left[c_S^{A_i} P_E^{A_i} E\left(T_i^A\right)\right] + \sum_{i=1}^{N-1}\left[c_S^{B_i} P_E^{B_i} E(T_i^B)\right] + \sum_{i=1}^{N}\left(c_T^{A_i} P_E^{A_i} P_F^{A_i}\right)$$

$$+ \sum_{i=1}^{N-1}\left(c_T^{B_i} P_E^{A_{i+1}}\right)$$

To derive the expected total cost over *N* episodes, three groups of intermediate variables need to be calculated: the probability of entering each setting during a care cycle, i.e. $P_E^{A_i}, P_E^{B_i}$, the expected LOS in each setting during each care cycle, i.e. $E[T_i^A], E[T_i^B]$, and the probability of forced transition after each inpatient care episode, i.e. $P_F^{A_i}$.

Whether the patient is staying in the inpatient care setting or the home- and community-based care setting, after each day, she will probably be cured and leave the system. It is possible that she will not go through all *N* episodes, or equivalently all $(2N-2)$ transitions. Therefore, when trying to derive the expected lengths of stay in settings as well as the transition costs, these dwelling durations and transitions are conditioned on whether the transition will occur and whether the patient will enter a setting for a particular episode.

3.2.1 Probability of Entering a Setting

Let $P_E^{A_i}, P_E^{B_i}$ denote the probabilities that the patient will enter the inpatient care setting or the home- and community-based care setting for the i^{th} episode. It is related to the probability that the patient will not be cured during her i^{th} time staying in the inpatient care setting or the home- and community-based care setting, which are denoted as $P_S^{A_i}, P_S^{B_i}$. Thus, the following relations exist.

$$P_E^{A_i} = \prod_{k=1}^{i-1}\left(P_S^{A_k} P_S^{B_k}\right), P_E^{B_i} = \prod_{k=1}^{i-1}\left(P_S^{A_k} P_S^{B_k}\right) P_S^{A_i}$$

For state A_i, the probability of not being cured will be conditioned on the probability that the patient will stay for *j* days, which is

$$P_S^{A_i} = \sum_{j=1}^{m_i}\left(P_S^{A_i} \middle| LOS = j\right) P_j^{A_i} = \frac{p_i^A + q_i^A\left(1 - p_i^A - q_i^A\right)^{m_i}}{p_i^A + q_i^A}$$

Similarly, for state B_i, the probability of not being cured will also be conditioned the probability that the patient will stay for *j* days, which is

$$P_S^{B_i} = \sum_{j=1}^{\infty}\left(P_S^{B_i} \middle| LOS = j\right) P_j^{B_i} = \frac{p_i^B}{p_i^B + q_i^B}$$

The proof can be referred in Appendix A.

3.2.2 Expected LOS in Each Setting for Each Time

For states A_i, based on the probability that the patient will stay for j days $P_j^{A_i}$, given the condition that she enters A_i, the expected LOS is

$$\mathrm{E}[T_i^A] = \sum_{j=1}^{m_i} jP_j^{A_i} = m_i\left(1 - p_i^A - q_i^A\right)^{j-1} + \sum_{j=1}^{m_i-1} j\left(1 - p_i^A - q_i^A\right)^{j-1}\left(p_i^A + q_i^A\right)$$

$$= \frac{1 - \left(1 - p_i^A - q_i^A\right)^{m_i}}{p_i^A + q_i^A}$$

Similarly, for states B_i, based on the probability that the patient will stay for j days $P_j^{B_i}$, given the condition that she enters B_i, the expected LOS is

$$E(T_i^B) = \sum_{j=1}^{\infty} jP_j^{B_i} = \sum_{j=1}^{\infty} j(1 - p_i^B - q_i^B)^{j-1}(p_i^B + q_i^B) = \frac{1}{p_i^B + q_i^B}$$

3.2.3 Probability of Forced Transition

Forced transitions will occur when the patient is not cured, instead being transited after m_i days during the i^{th} episode. Therefore, given the condition that the patient enters A_i, the probability that the patient will be forcedly transited is

$$P_F^{A_i} = \left(1 - p_i^A - q_i^A\right)^{m_i-1}\left(1 - q_i^A\right)$$

Additionally, it is obvious that before the initial inpatient episode, the patient is not cured for sure. Therefore, for the simplicity of calculation,

$$P_S^{A_0} = 1, \prod_{k=1}^{i} P_S^{A_k} = \prod_{k=0}^{i} P_S^{A_k}$$

3.3 Optimization of the Covered Episodes and LOS

Given all the transition probabilities and unit costs, the expected total cost *C* will be a function of coverage LOSs for each episode $m_1, m_2, \dots, m_N$, as follows.

$$C(m_1, m_2, \dots, m_N)$$

$$= \sum_{i=1}^{N} \left[c_S^{A_i} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) \frac{1 - \left(1 - p_i^A - q_i^A\right)^{m_i}}{p_i^A + q_i^A} \right]$$

$$+ \sum_{i=1}^{N-1} \left[c_S^{B_i} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) P_S^{A_i} \frac{1}{p_i^B + q_i^B} \right] + \sum_{i=1}^{N} \left[c_T^{A_i} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) P_F^{A_i} \right]$$

$$+ \sum_{i=1}^{N-1} \left[c_T^{B_i} \left(\prod_{k=0}^{i} P_S^{A_k} P_S^{B_k} \right) \right]$$

The optimal $m_1^*, m_2^*, \dots, m_N^*$ can be decided by the properties of the partial derivatives of *C* over m_i. And under certain assumptions, it is proved that the best decision will be cover the first n episodes without limited LOS, and not cover the rest episodes at all, i.e.,

$$m_1^* = m_2^* = \cdots = m_n^* = \infty, m_{n+1}^* = m_{n+2}^* = \cdots = m_N^* = 0$$

It is proved by two theorems. The first theorem proves that for each episode A_i, the total cost is always monotonic over m_i. The second theorem proves that under certain assumptions, if the best decision is to cover n episodes, then the optimum will be cover the first n episodes, and not cover the subsequent episodes.

3.3.1 Bang-bang Control

Given the total number of considered episodes *N*, for any arbitrary episode A_i, the total cost *C* is always monotonic over the coverage LOS is always monotonic over m_i.

Whether the total cost will monotonically increase or decrease is decided by both the unit cost of staying and transition, and the transition probabilities.

It can be prove by mathematical induction. First, it is proved that given the total number of covered episodes *N*, for the last episode A_N, the expected total cost *C* is always monotonic over m_N. Next it is proved that given arbitrary episode A_i, if the expected total cost is monotonic over m_i, then if the total number of considered episodes is increased by 1, the new expected total cost will also be monotonic over m_i. With these two steps, the monotonicity over arbitrary m_i is established.

Proposition 1: Given the total number of considered episode *N*, the total cost *C* is monotonic over the coverage LOS of the last episode m_N. Specifically when condition

$$\left[-c_S^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)\frac{1-p_N^A-q_N^A}{p_N^A+q_N^A}+c_T^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)\left(1-q_N^A\right)\right]\ln\left(1-p_N^A-q_N^A\right) < 0$$

is satisfied, the partial derivative of *C* over m_N will always be less or equal to 0, regardless of the value of m_N. While when condition

$$\left[-c_S^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)\frac{1-p_i^A-q_i^A}{p_i^A+q_i^A}+c_T^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)\left(1-q_i^A\right)\right]\ln\left(1-p_i^A-q_i^A\right) > 0$$

is satisfied, the partial derivative of *C* over m_N will always be greater or equal to 0, regardless of the value of m_N.

The proof can be referred in Appendix B.

Proposition 2: Given the maximum number of considered episode $\boldsymbol{N}$ and an arbitrarily given episode $\boldsymbol{A_i}$, If the total cost *C* is monotonic over $\boldsymbol{m_i}$, when the maximum number of covered episode is increased by 1, the incremental cost will also be monotonic over $\boldsymbol{m_i}$.

This proposition gives the situation when the insurer decide to consider covering one more episode for m_{N+1} days. When the coverage LOSs for previous episodes remain unchanged, the increased expected cost is defined as the incremental cost $\Delta C(m_{N+1})$.

$$\Delta C(m_{N+1}) = C(m_1, m_2, \dots, m_N, m_{N+1}) - C(m_1, m_2, \dots, m_N)$$

The partial derivative of incremental cost over m_i will be in the form of

$$\partial \Delta C(m_{N+1})/\partial m_i = Y_{N+1}\left(1 - p_i^A - q_i^A\right)^{m_i}$$

in which Y_{N+1} is a function independent of m_i.

The proof can be referred in Appendix C.

Theorem 1: Given the maximum considered episode *N*, for any episode $\boldsymbol{A_i}$, the expected total cost *C* will always be monotonic over $\boldsymbol{m_i}$. Specifically when condition

$$\frac{\partial C(m_1, m_2, \dots, m_i)}{\partial m_i} + \frac{\partial \Delta C(m_{i+1})}{\partial m_i} + \dots + \frac{\partial \Delta C(m_N)}{\partial m_i}$$

$$= \left[X_N + \sum_{k=i+1}^{N} Y_k\left(1 - p_i^A - q_i^A\right)\right]\left(1 - p_i^A - q_i^A\right)^{m_i - 1} < 0$$

is satisfied, the partial derivative of *C* over $\boldsymbol{m_i}$ will always be less or equal to 0, regardless of the value of $\boldsymbol{m_i}$. While when condition

$$\left[X_N + \sum_{k=i+1}^{N} Y_k\left(1 - p_i^A - q_i^A\right)\right]\left(1 - p_i^A - q_i^A\right)^{m_i - 1} > 0$$

is satisfied, the partial derivative of C over m_i will always be greater or equal to 0, regardless of the value of m_i.

The proof can be referred in Appendix D.

3.3.2 Control Limit

Under certain assumptions, it can also be proved that given N, the maximum number of episodes, there will be certain control limit n so that if $N > n$, the insurer should covered only the first n episode without limited LOS. On the other hand, if $N \leq n$, the insurer should cover all N episodes and each of the covered episodes should not be imposed by any limitation on the coverage LOS.

Specifically, two assumptions are made. In terms of the transition probabilities, it is assumed that the probability of being transferred to the home- and community-based care setting will decrease after every episode, i.e. $p_{i+1} < p_i$, and the probability of being cured will significantly decrease after every episode, i.e. $q_{i+1} \ll q_i$. While in terms of the cost, the daily cost of staying will significantly increase after every episode, i.e. $c_S^{A_{i+1}} \gg c_S^{A_i}$, but the cost of forced transition after every episode will remain the same, i.e. $c_T^{A_{i+1}} = c_T^{A_i}$.

Proposition 3: Given two consecutive episodes, if only one of them will be covered, then the expected total cost for covering the earlier episode will always be greater than the expected total cost for covering the latter episode.

The proof can be referred in Appendix E.

Theorem 2: Given the maximum number of considered episodes N, if the decision is to cover N^* of the N episodes, then it will be best to only cover the first N^* episodes without limited LOS, and not cover the subsequent episodes.

Theorem 2 can be proved by contradiction. Assume that the optimal decision is to cover N^* episodes, but not the first N^* episode. Therefore, there must be one covered episode i, with the $(i-1)^{th}$ episode not covered. However, according to Proposition 3, it will be better to cover the $(i-1)^{th}$ episode, and not cover the i^{th} episode. Therefore, the given decision cannot be optimal. Q.E.D.

3.3.3 Relationship between Forced Transition Cost and Optimal Number of Covered Episodes

Since in the health care transition system, the cost of forced transition is one of the most important factors in the decision making of the insurer. Therefore, this part also proposes a theorem on the relationship between the cost of forced transition and the optimal number of covered episodes.

Theorem 3: Given the maximum number of considered episodes N, if the previous decision is to cover the first n of the N episodes, then when condition

$$\boldsymbol{c_T^{A_{n+1}} + \left(1 - P_S^{A_{n+1}}\right) \sum_{i=n+2}^{N} \left(\prod_{j=n+1}^{i} \frac{p_j^B}{p_j^B + q_j^B} \right) c_T^{A_i}}$$

$$\boldsymbol{> c_S^{A_{n+1}} \frac{1}{p_{n+1}^A + q_{n+1}^A}}$$

$$\boldsymbol{- \left(1 - P_S^{A_{n+1}}\right) \left[c_S^{B_{n+1}} \frac{1}{p_{n+1}^B + q_{n+1}^B} + c_T^{B_{i+1}} \frac{p_{n+1}^B}{p_{n+1}^B + q_{n+1}^B} \right.}$$

$$\boldsymbol{\left. + \sum_{i=n+2}^{N} \left(\prod_{j=n+1}^{i} \frac{p_j^B}{p_j^B + q_j^B} \right) \left(c_S^{B_j} \frac{1}{p_i^B + q_i^B} + c_T^{B_j} \frac{p_i^B}{p_i^B + q_i^B} \right) \right]}$$

is satisfied, it is better to cover one more episode.

The proof can be referred in Appendix F.

3.4 Numerical Study

In this section, we apply our model to a case study, and draw some conclusions. On one hand, we analyze the relationship between cost and covered episodes, on the other hand, by conducting sensitivity analysis, we study the relationship between maximum number of considered episodes and optimal number of covered episodes, as well as the relationship between penalty/transition cost and optimal number of covered episodes.

3.4.1 Case Description

In our case study, we assume that the patient with some disease, will normally go through no more than 10 episodes, and the cost will be covered by an insurance plan approved in advance.

For each episode, if the insurer decides to cover it, then it will cover the patient until she is being cured or good enough to be naturally transited a community center, and the cost of staying in the episode is covered. While if the insurer decides not to cover it, since it would be harmful for the patient, the insurer will face a large amount of penalty. The more episodes the insurer decides not to cover, the higher penalty it will face. When the patient is in the community center, the cost of staying is covered. Also, if the patient gets deteriorated and has to be readmitted by the inpatient care setting, the insurer also needs to cover the cost of transition.

Based on the cost and survival rates for each episode, the insurer needs to make a decision on how many episodes, and which episodes should be covered by the insurance plan, so that the expected total cost for each patient will be minimized.

3.4.2 Parameter Estimation

According to the statistical analysis based on a data set of Medicare beneficiaries who had traumatic brain injury, the expected LOS within the first episode will be 12 days, therefore the probability of being transferred out will be $p_1^A + q_1^A = 0.08$. Also, according to the data, 73% of them will stay in the community for the 2 years, which is considered being cured, therefore $p_1^A = 0.02, q_1^A = 0.06$. As for the remaining episodes a patient may enter due to readmission, we assume that after each episode, the probability of being cured will decrease by 40%, and the probability of being transited to the community will decrease by 20%. For a patient in the community, the expected LOS is about 30 days, without significant difference among different times of being admitted. Therefore, the probability of being readmitted and being cured is approximately identical through different times of being in the community, which is $p_i^B = 0.027, q_i^B = 0.003$. With the same data set, we estimate the daily cost in the first episode to be 316.5, and for the subsequent episodes, the average incremental daily cost is about 20%. Therefore, $c_S^{A_i} = 316.5 * 1.2^{i-1}$. While for the community center, the daily cost of staying is approximately stable as $c_S^{B_i} = 10.768$. Also, it is observed that for the community center, there is a fixed cost, which can be considered as the transition cost from the community center to the inpatient care setting, which is $c_T^{B_i} = 130.15$. In addition, the insurer can choose to cover an episode (without limitation on the insured LOS) or not cover the episode at all. However, if the insurer chooses not to cover an episode, since it would be harmful for the patient, there will be a large amount of penalty $c_T^{A_i} = 10000$.

3.4.3 Relationship between Cost and Covered Episodes

According to theorem 2, the best decision for the insurer will always be cover the first n episode, and not cover the last $(10 - n)$ episode. Thus, the problem will be determining the best n.

According to the calculation, the relationship between the expected total cost per capita and covered episodes n is illustrated as follows. The optimal decision will be cover the first 4 episode, with which the expected cost for each patient will be 7489.

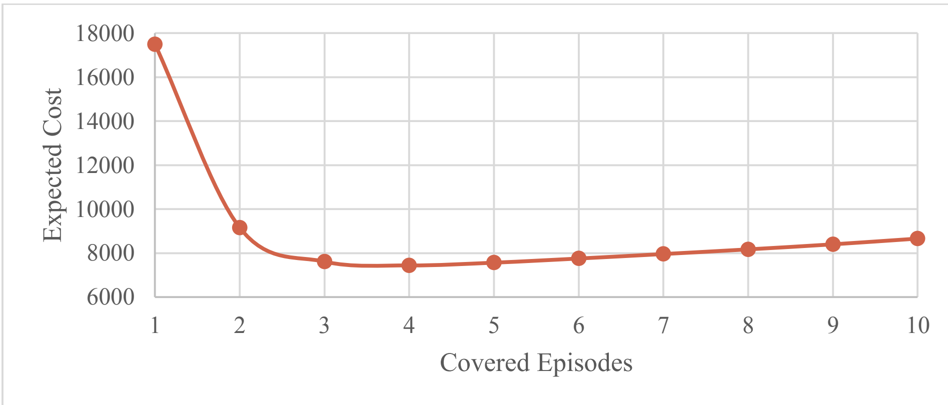

Figure 3.3 Relationship between Cost and Covered Episodes

When $n \leq 4$, as more episodes covered, the insurer will less likely to encounter the penalty of not covering the patient before he is cured, and thus the cost will decrease along with n. When $n \geq 4$, on one hand, since it is less likely that the patient will enter the following episodes, the expected penalty will significantly go down. On the other hand, since the cost of staying is increasing, and the probability of being transferred out of the inpatient care setting is decreasing, the expected cost of staying will go up. Therefore it is noticed that the expected cost will increase mildly. After trading off the cost of staying and penalty, as well as the current and future cost, it is concluded that the best decision will be cover the first four episodes.

3.4.4 Sensitivity Analysis

3.4.4.1 Relationship between *N* and *n**

In general, when *N*, the maximum number of considered episodes increases, the opportunity cost for a specific insurance plan will also increase, which encourages the insurer to cover more episodes to lower down the risk of high penalty. According to the numerical study, when *N* increases from 1 to 50, the optimal number of covered episodes *n** also increases from 1 to 5.

However, since the patient is likely to be cured in earlier episodes, the probability of entering future episodes is decreasing, which implies that if one more episode is considered, the expected total cost will increase, but the cost incremental will decrease. Convergence is shown for the optimal number of covered episodes. Specifically, when 16 or more episodes are considered, the optimum will be 5 episodes, and will not continue to increase as *N* increases.

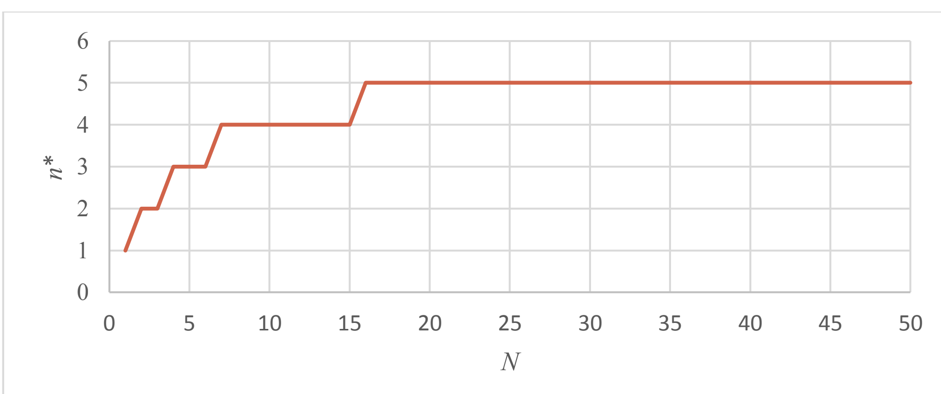

Figure 3.4 Relationship between *N* and *n**

For the expected cost of the optimum, convergence is also noticed. It is shown that if the insurer decides to cover 5 episodes, as the number of considered episodes increases, the expected cost will also increase, but the cost incremental is decreasing. When considering 50 episodes, the expected cost for covering 5 episodes is 7825, and it can be observed that if *N* continues to increase, the expected cost will converge to about 7830.

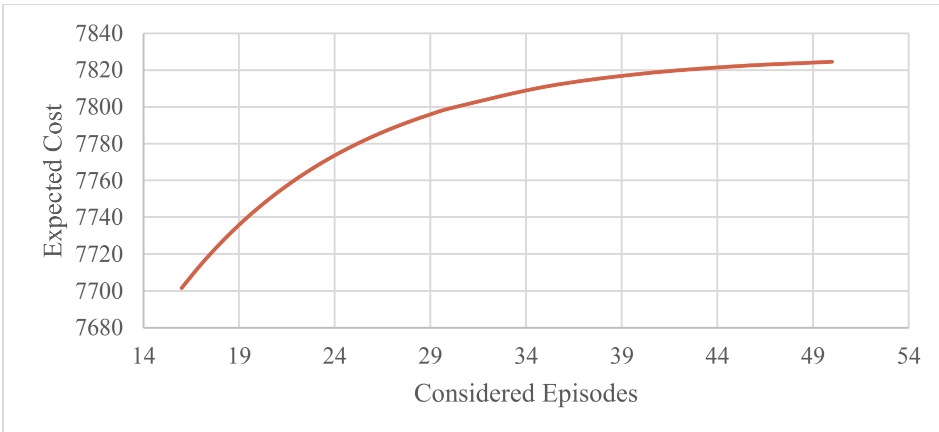

Figure 3.5 Relationship between *N* and Expected Cost

3.4.4.2 Relationship between Penalty and n*

In this numerical study, penalty is the most influential factor for the insurer to decide how many episodes to cover. A heavier penalty will encourage the insurer to cover more in order to avoid high future cost, whereas a lighter penalty will lead him to cover less in order to lower the current inpatient cost. In the baseline case, the insurer will choose to cover 4 episodes under the penalty of $10,000 for every uncovered episode. However, this decision does not incorporate the level of cure for the patient. If the majority of patients need more episodes, then the government may need to increase the penalty to drive the insurer to cover more.

In the baseline case, by varying parameter value of penalty based on Theorem 3, it is shown that when the penalty is greater than $14,400, the insurer will choose to cover 5 episodes. If the penalty is less than $6,600, the insurer will choose to cover only 3 episodes. If penalty continues to increase (decrease), then the insurer will cover more (less) episodes. The following line chart shows the relationship between penalty and n*. When the penalty increases from 0 to 600,000, the optimal *n** will switch from 0 all along to 10.

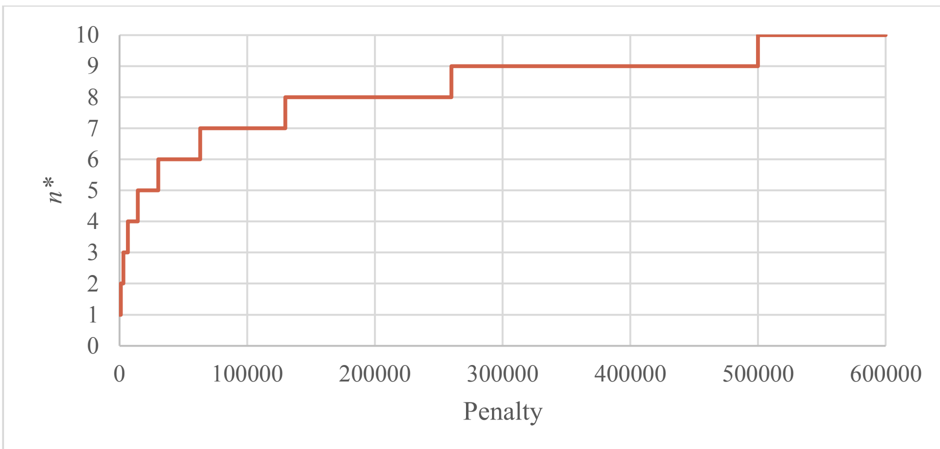

Figure 3.6 Relationship between Penalty and *n**

3.5 Conclusions

This paper develops a stochastic process model to study the health transitions between the inpatient care setting and the home- and community-based care setting. Specifically, instead of only using states to present the health conditions of the patient, this model also incorporates the inpatient LOS into the state, which enables us to incorporate the effect of forced transition. Then, an unconstrained optimization problem is developed to minimize the expected total cost by setting the coverage LOS within each episode.

According to the properties of the partial derivatives, it is proved that for each episode, there exists a bang-bang control policy. That is, the optimal solution for each coverage LOS is either 0 or infinity. Further, under certain assumptions on the monotonicity of cost and transition probabilities, it is also proved that there exists a control limit on which episodes to cover, i.e., the best option is always to cover the first several episodes entirely, and then stop covering at all in the remaining episodes.

In the sensitivity analysis of our numerical study, two important parameters are discussed: the total number of considered episodes and the penalty (forced transition cost). Increasing the total number of considered episodes will encourage the insurer to cover more episodes. However, as the number of considered episodes increases, both the expected total cost and the optimal number of covered episodes will coverage. Increasing the penalty will also encourage the insurer to cover more episodes. As the penalty is increased to some large number, the optimal decision will switch from not covering any episodes to covering all episodes.

CHAPTER 4. A SIMULATION MODEL FOR THREE-LAYER REHABILITATION SERVICE PROCESS

4.1 The Use of Simulation

The analytical stochastic model presents a good way to describe the health care transitions between an inpatient care setting and home- and community-based care setting. However, since homogeneity assumptions are used on cost and transition probabilities, it may be inadequate when dealing with the transitions among different levels of services, for which non-linear cost and transition probability functions are more appropriate.

In order to better evaluate the impact of coverage LOS on various performances in a care transition system, we developed a simulation model for three-layer rehabilitation service processes. This simulation model is based on VBA, which is compatible with Microsoft Excel. By developing a Microsoft Excel Macro, both the logical programs in VBA and statistical functions in Excel can be fully integrated, which provides a very effective way to simulate the system.

4.2 Simulation Modeling Development

4.2.1 Conceptual Design

This model simulates the transitions of a single patient between three categories of rehabs: primary rehab (A_1), secondary rehab (A_2) and tertiary rehab (B). Generally, patients will enter the system by being admitted by primary rehab with highest utility and cost. After staying for some days, she will be transitioned to tertiary rehab with lowest utility and cost, stay here until she gets deteriorated and has to be transitioned to secondary rehab with moderate utility and cost. For some patients, there may be multiple transitions between secondary and tertiary rehabs. By simulating with different coverage LOS setting for primary and secondary rehab, this simulation model will decide the optimal coverage LOS at each rehab layer, and minimize the total cost.

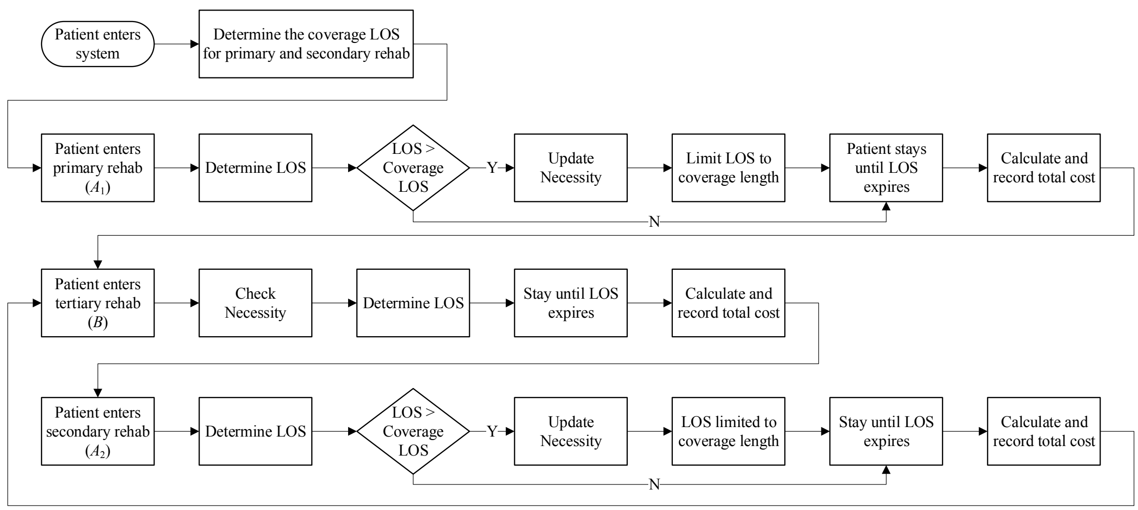

Figure 4.1 Simulation Flow Chart

At the beginning of the simulation, a patient is created. When entering the simulation model, a series of parameters on costs and transition probabilities are assigned to the model.

During the simulation, the patient will be transitioned among three states: $A_1, B\ or\ A_2$. When the patient enters one state, a random number will be generated. Then this number will be looked up in the Kaplan–Meier survival analysis table and compared with the given survival probabilities, with which the LOS in this state will be decided. Then, the

LOS will be plugged in the regressed cost functions and decide the cost in this rehab. After recording the LOS and total cost, the patient will be transitioned to the next state. At the end of the simulation duration, two outcomes will be evaluated for this individual patient: the total cost and the total number of transitions from tertiary rehab to secondary rehab within 720 days. On one hand, every time the patient leaves a state, the cost within this states will be calculated. On the other hand, every time the patient enters state A_2, there will be a counter which records and updates the number of transitions.

Additionally, after simulating the baseline case, we will then include the decision variables for the optimization simulation model: the coverage LOSs for the primary and secondary rehabs. Every time when the LOS in states $A_1 \, or \, A_2$ is generated, it will be compared with the coverage LOS. When the generated LOS is longer than the coverage LOS, then the patient will leave the state after the coverage LOS expires. In this case, the uncovered LOS will be recorded and updated as necessity information, which will affect the survival probability and LOS in state B.

The necessity variable at the beginning of simulation is 1. Every time when a patient leaves the primary or secondary rehab, if the generated LOS is larger than the coverage LOS, then the necessity variable will be updated to the ratio of coverage LOS to generated LOS. For example, when the coverage LOS in secondary rehab is 30 days, and generated LOS is 40 days, then the necessity will decrease to $30/40 = 0.75$. Therefore, the LOS will be decreased, and the risk of being transitioned back to secondary rehab will be increased.

4.2.2 Input Modeling

We use cost and utilization data collected from Indiana Medicaid beneficiaries who suffered from initial modest-to-severe traumatic brain injury and received rehabilitation services. This simulation model intends to describe the care transition system for patients with homogeneity on service requirement at each rehab facility. These patients are with ICD-9 ranging from 801 to 803 (described as "Fracture of base of skull, fracture of face bones, other and unqualified skull fractures") and a short acute-hospitalization LOS (ranging from 3 to 21 days). The input modeling contains two parts: input cost modeling and input LOS modeling.

According to the data, 64.3% of the patient will only transition once from the primary rehab to the tertiary. Since these patients do not require more transitions, we define them as low risk patients. While the remaining 35.7% will go through several transitions between the secondary and tertiary rehab, we define them as high risk patients. Since high risk patients will incur higher cost and more transitions, this simulation model will focus on these patients.

4.2.2.1 Input LOS Modeling

First, the LOSs in all three levels of rehabs are estimated by Kaplan-Meier survival analysis.

The likelihood of a patient being discharged from her current location was represented using a survival function of the current LOS via Kaplan-Meier analysis (Kaplan & Meier, 1958). The Kaplan-Meier estimator is used to estimate the survival function of the patients from lifetime data, which, in this case would be the LOS at a particular facility obtained from the claims data. This survival function captures the probability that a patient will survive, which, in this case represents the probability that a patient will continue to stay in her current rehab an additional day. Using Kaplan-Meier analysis allows estimation of survival over time, even when patients drop out or are studied for different lengths of time.

For each day, the survival probability is estimated as the number of patients surviving divided by the number of patients at risk. Patients who have dropped out or not reached the current day of study due to some reason are censored and thus not accounted for in the number of patients at risk. The lifetime of a patient in any rehab facility which represents the LOS is estimated from the cumulative probability of surviving each day. This LOS has an upper bound that is bounded by the maximum LOS at each facility. Therefore, at any time $t<T$, where T is the upper bound, the set of transition probabilities between one rehab and another can be obtained from the survival model. The below figure represents the survival curve for patients receiving care at the primary rehab.

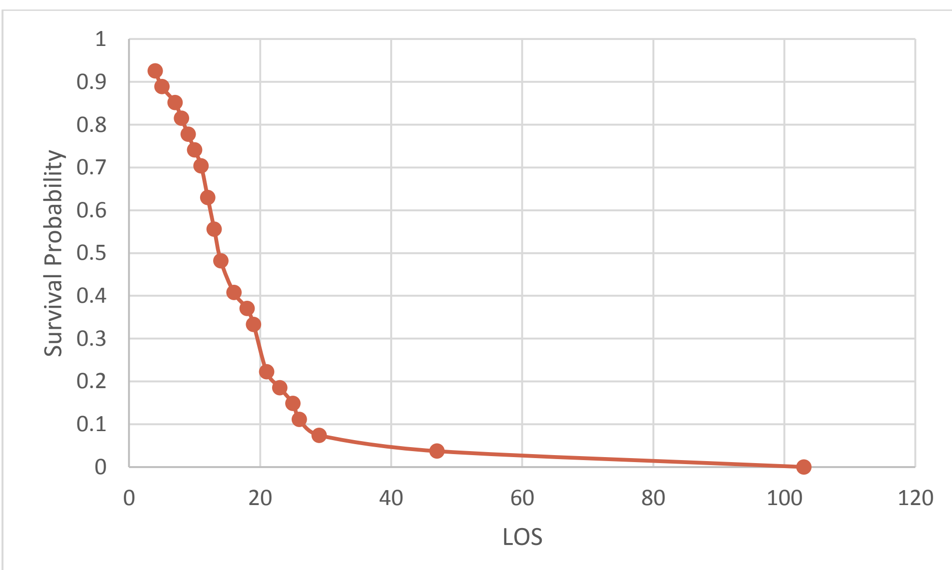

Figure 4.2 Survival Curve

4.2.2.2 Input Cost Modeling

Once the LOS in each rehab is inputted, then we can decide the related cost within each rehab. The costs in the model are estimated with polynomial regression models. With the data in LOS and its related total cost, we can use polynomial functions to represent the relationship. According to the polynomial functions, since the y intercept is not equal to 0, we consider it to be the fixed cost, or transition cost. Due to the imperial meaning of the cost function, both the y intercept and slop should be positive. Thus, any data points in this regression model that drove either the y intercept or slope in the negative direction are identified as outliers and eliminated. The functions are regressed with linear, quadratic or cubic models, and we use R-square values to verify the goodness of fit for the regression model, and choose the one with good fitness and lower order. The figure below represents the regression model for the cost in the secondary rehab as a function of the LOS.

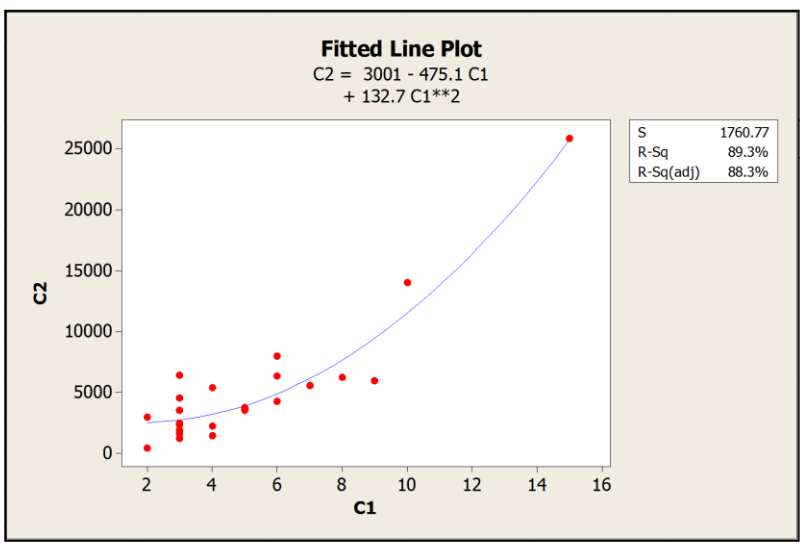

Figure 4.3 Regressed Cost Function

4.2.3 Verification and Validation

After inputting the system and parameters into the simulation model, we run the baseline case for verification and validation.

First, we run the model for one replication and one single patients, and record all the LOSs and costs during the whole process. According to the below table, after leaving A_1, she has been transitioned between B and A_2, being admitted by B for five times and by A_2 for four times. The total LOSs add up to 730 days, which is two years. The total cost is $25158.42. This run perfectly verifies the model, showing the right pathway of the patient in the system, and also generated right LOSs and costs.

Table 4.1 Verification

State	LOS (days)	Cost ($)
A_1	26	8092.862
B	388	3133.537
A_2	4	1192.806
B	64	821.0522
A_2	5	1458.91
B	37	628.3451
A_2	9	3505.306
B	124	1249.29
A_2	10	4262.4
B	63	813.9149
Sum	730	25158.42

After verification, we increase the number of replications, and validate the model. Here, in order to simplify the gathering and analysis of statistics, we uses batch mode and have 50 runs in each replication. Then, in order to decide the replication of the simulation, we choose the patient's total cost as the indicator. We collected the simulated cost data, and average them over each replication to observe their trend. The following line graph shows the relationship between the number of replications and the average total cost. It indicates that after about 100 replications, the average total cost over replications has converged, indicating 100 replications is large enough and the results is statistically significant. Therefore, we run 100 replications and 50 runs in each replication.

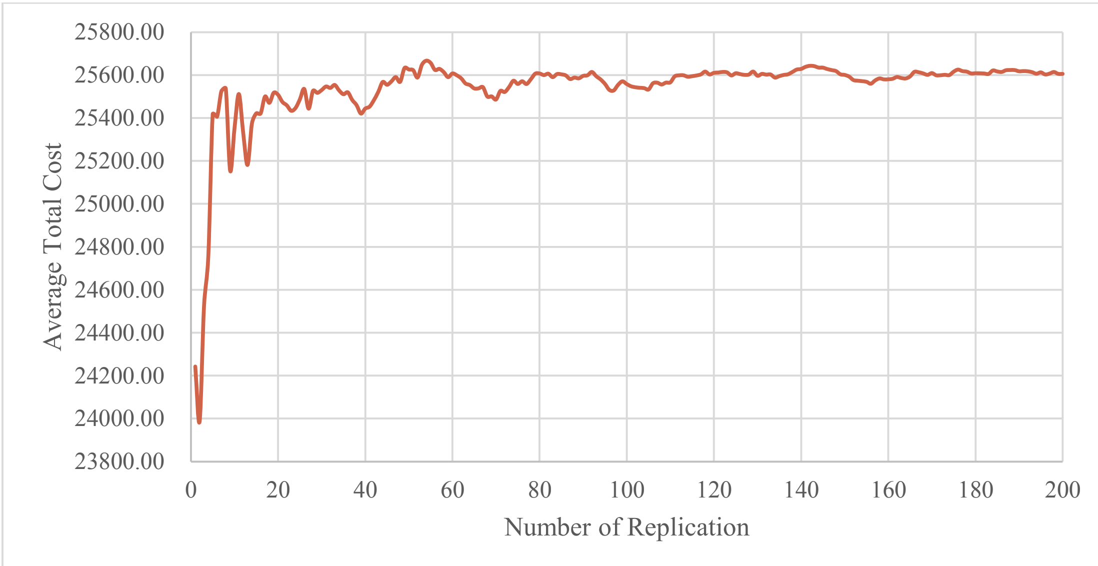

Figure 4.4 Average Total Cost over Number of Replication

With a proper number of replication, we validate the simulated total cost with two sorts of descriptive statistics: LOS in each rehab facilities and average historic cost per capita for high risk patient. According to the comparison, all the chosen statistics falls into or are very close to the 95% CI of the simulated cost, which indicates the model is well validated.

Table 4.2 Validation

System Parameters Measured	**Historic Mean**	**Simulated Mean**	**95% CI for Simulated Stats**
Primary rehab LOS for low risk patients (days)	14.84	14.59	[14.23, 14.96]
Tertiary rehab LOS for low risk patients (days)	715.16	715.26	[714.92, 715.6]
Primary rehab LOS for high risk patients (days)	13.13	12.69	[12.18, 13.21]

Secondary rehab LOS for high risk patients (days)	111.50	89.92	[89.71, 90.12]
Tertiary rehab LOS for high risk patients (days)	5.54	4.86	[4.76, 4.96]
Total Cost for high risk patients ($)	25,829	25,979	[25544.14, 26413.03]

4.3 Effect of Shortened LOS on Total Cost and Rehospitalization Number

In the baseline case, the survival analysis indicates that the coverage LOS for the primary rehabs is roughly 30 days, the one for secondary rehab is roughly 20 days, and there is no limited LOS for tertiary rehab. According to the simulation data, for the high risk patient, the average total cost per capita is $25,829, and the average rehospitalization number (the incidence of transitions to secondary rehab) per capita is 7.58. In this section, we use the simulation model to assess the effect on the trend of total cost and number of transitions to secondary rehab, when we shorten the coverage LOS in primary or secondary rehab. First, we set the coverage LOS for secondary rehab to the baseline case, 20 days, while shortening the coverage LOS for primary rehab. According to the simulation, when the coverage LOS for primary rehab decreases from 30 to 1, both the total cost and the rehospitalization number will monotonically increase. It indicates that when patients are not cured enough before leaving the primary rehab, she will need more future services in secondary rehab, and higher total cost will incur. Therefore, for the primary rehab, it is not beneficial to shorten the current coverage LOS.

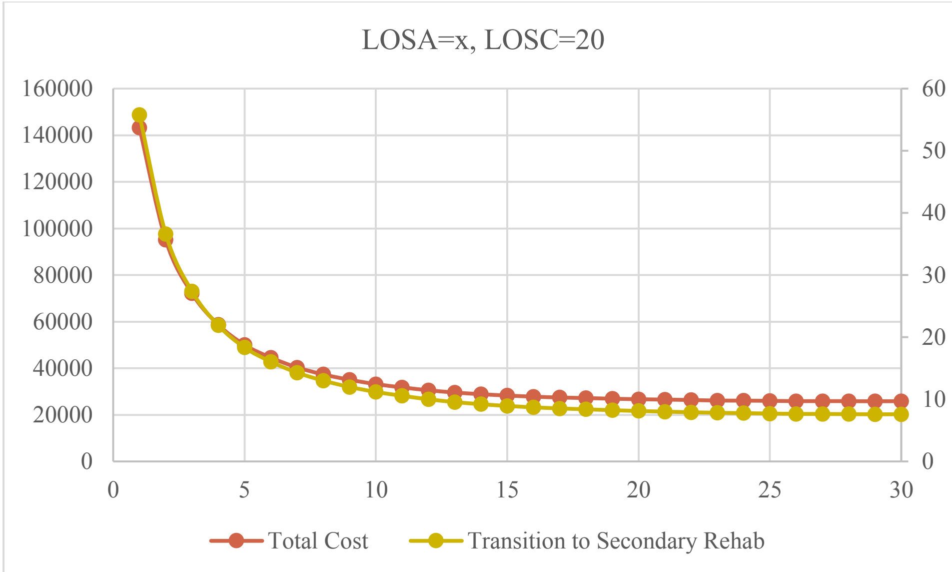

Figure 4.5 Effect of Shortened LOS for Primary Rehab

Second, we set the coverage LOS for primary rehab to the baseline case, 30 days, while shortening the coverage LOS for secondary rehab. According to the simulation, when the coverage LOS for the secondary rehab decreases from 20 to 1, the number of rehospitalization number will monotonically increase, while the total cost will decrease first, and start to increase dramatically when less than 5. It indicates that when patients are not cured enough before leaving secondary rehab, she will have an increased likelihood of needing more future services from secondary rehab in the future. But since there exists a tradeoff between current cost and future cost, the total cost will present unimodality. Therefore, for the secondary rehab, when only considering cost, the best policy will be covering five days.

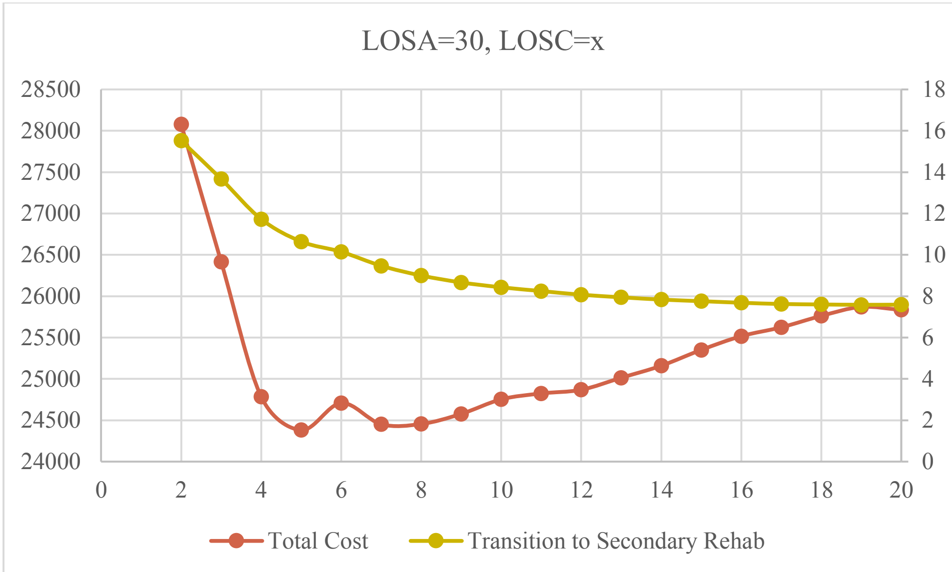

Figure 4.6 Effect of Shortened LOS for Secondary Rehab

4.4 Conclusions

For this work, we make three major conclusions.

1) The patient in the studied cohort can be divided into two categories: low risk patients who keep staying in the tertiary rehab after leaving primary rehab, and high risk patients who will need rehospitalization and multiple transitions between the secondary and tertiary rehabs.
2) For primary rehab, decreasing LOS will increase both total cost and rehospitalization number. Therefore, the optimal LOS for primary rehab will be 30 days (current setting).
3) For the secondary rehab, decreasing LOS will on one hand increase the rehospitalization number, and on the other hand decrease the total cost first and then dramatically increase it after a turning point. Therefore, when only considering cost, the optimal LOS for secondary rehab will be 5 days.

CHAPTER 5. CONCLUSIONS

In this thesis, we developed two models to describe and optimize the care transition system. Both models took a public health insurer's viewpoint, and minimized the total cost for a single patient within the transition system by deciding the best care setting: covered number of episodes and coverage LOS for each episodes.

First, we studied the problem of optimizing the number of episodes to cover and how many days to cover for a cyclic care delivery process. We modeled the randomness of the underlying transitions with a Markov process. We analyzed the steady-state condition and formulated a steady-state process optimization problem. In addition to the health condition of a patient, our Markov model also incorporates the inpatient LOS into the state description. This enables us to incorporate the effect of forced transitions. With this model, we drew three main conclusions.

1) For each episode, there exists a bang-bang control policy. That is, the optimal solution for each covered LOS is either 0 or infinity. Further, under mild monotonicity assumptions on the costs and transition probabilities, it is also proved that there exists a control limit on which episodes to cover, i.e., the best option is always to cover the first several episodes entirely, and then stop covering the remaining episodes.
2) Increasing the total number of considered episodes will encourage the insurer to cover more episodes. However, as the number of considered episodes

increases, both the expected total cost and the optimal number of covered episodes will coverage.

3) Increasing the penalty will also encourage the insurer to cover more episodes. As the penalty increases to some large number, the optimal decision will switch from not covering any episodes to covering all episodes.

Second, a simulation model was developed to assess the impact of LOS on the transitions between three levels of rehabilitations: primary rehab, secondary rehab and tertiary rehab. We modeled the relationships between LOS with cost and rehospitalization based on data collected from Indiana Medicaid beneficiaries who suffered from initial modest-to-severe traumatic brain injury and received rehabilitation services. We conducted polynomial regression and Kaplan–Meier analysis to estimate parameters, developed a discrete event simulation model to capture the stochastic transitions, and assessed the relationship between the coverage LOS and survival probability, as well as their joint effect on average total cost per capita and incidences of transition from tertiary to secondary rehab. In this model, we drew four major conclusions.

In this model, we made three major conclusions.

1) The patient in the studied cohort can be divided into two categories: low risk patients who keep staying in the tertiary rehab after leaving primary rehab, and high risk patients who need multiple transitions between the secondary and tertiary rehabs.

2) For the primary rehab, decreasing LOS from baseline will increase both total cost and number of readmission. In this case, the optimal LOS will be 30 days (current setting).
3) For the secondary rehab, decreasing LOS from baseline will on one hand increase the number of readmission, and on the other hand decrease the total cost first and then dramatically increase it after a turning point. In this case, the optimal LOS will be 5 days.

In our future research, for the stochastic process model, we will relax the homogeneity assumptions on the transition probabilities. In this paper, the transition probabilities are identical and independent within each episode, which may not fully capture the nature of the real world care transitions. For example, it makes more sense to assume that patient is more likely to leave in the first few days than after a longer LOS. In addition, we will model the cases where the transition probabilities are dependent on the coverage decisions. For example, with a longer coverage LOS during each episode, the patient will be less likely to be readmitted to the hospital. For the simulation model, first, we will further identity different transition patterns for the patients and classify them into more detailed categories to increase the accuracy of parameter estimation and system description. Second, since this thesis only considers a subset of the patients, we can extend the conceptual model to study multiple clinical pathways and their joint impact on the care delivery system. Also, with the support of larger sample size on LOS, we will use regression instead of Kaplan-Meier to model the transition probabilities, so that we can study the effect of increasing LOS from baseline on total cost and number of readmissions.

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APPENDICES

Appendix A Expected LOS in Each Setting for Each Time

For states A_i, the patient will stay in this episode no more that m_i days, and the probability of not being cured will be conditioned on his LOS of this episode. Given the condition that the patient enters A_i, the probability that the patient will stay for *j* days is

$$P_i^j = \begin{cases} \left(1 - p_i^A - q_i^A\right)^{j-1}\left(p_i^A + q_i^A\right), for\ j \in [1, m_i - 1] \\ \left(1 - p_i^A - q_i^A\right)^{j-1}, for\ j = m_i \end{cases}$$

Therefore, the probability that the patient will not be cured during this episode is

$$P_S^{A_i} = \sum_{j=1}^{m_i} \left(P_S^{A_i} \middle| LOS = j\right) P_j^{A_i}$$

$$= \sum_{j=1}^{m_i - 1} \frac{p_i^A}{p_i^A + q_i^A} \left(1 - p_i^A - q_i^A\right)^{j-1}\left(p_i^A + q_i^A\right)$$

$$+ \left(1 - q_i^A\right)\left(1 - p_i^A - q_i^A\right)^{m_i - 1} = \frac{p_i^A + q_i^A\left(1 - p_i^A - q_i^A\right)^{m_i}}{p_i^A + q_i^A}$$

For states B_i, the probability of not being cured will also be conditioned on his LOS in the home- and community-based care setting. Given the condition that the patient enters B_i, the probability that the patient will stay for *j* days is

$$P_j^{B_i} = (1 - p_i^B - q_i^B)^{j-1}(p_i^B + q_i^B), for\ j \in [1, \infty]$$

Therefore, the probability that the patient will not be cured during this episode is

$$P_S^{B_i} = \sum_{j=1}^{\infty} \left(P_S^{B_i} \middle| LOS = j\right) P_j^{B_i} = \sum_{j=1}^{m_i - 1} \frac{p_i^B}{p_i^B + q_i^B} (1 - p_i^B - q_i^B)^{j-1}(p_i^B + q_i^B) = \frac{p_i^B}{p_i^B + q_i^B}$$

Appendix B Proof of Proposition 1

The partial derivative on total cost C over the coverage LOS for the last episode m_N is

$$\frac{\partial C(m_1, m_2, \dots, m_N)}{\partial m_N}$$

$$= \left[c_S^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)\frac{-(1-p_N^A-q_N^A)^{m_N}}{p_N^A+q_N^A}\right.$$

$$\left. + c_T^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)(1-q_N^A)(1-p_N^A-q_N^A)^{m_N-1}\right]'$$

$$= \left\{\left[-c_S^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)\frac{1-p_N^A-q_N^A}{p_N^A+q_N^A}\right.\right.$$

$$\left.\left. + c_T^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)(1-q_N^A)\right](1-p_N^A-q_N^A)^{m_N-1}\right\}'$$

$$= \left[-c_S^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)\frac{1-p_N^A-q_N^A}{p_N^A+q_N^A}\right.$$

$$\left. + c_T^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)(1-q_N^A)\right](1-p_N^A-q_N^A)^{m_N-1}\ln(1-p_N^A-q_N^A)$$

Let

$$X_N = \left[-c_S^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)\frac{1-p_N^A-q_N^A}{p_N^A+q_N^A} + c_T^{A_N}\left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k}\right)(1-q_N^A)\right]$$

Then,

$$\frac{\partial C(m_1, m_2, \dots, m_N)}{\partial m_N} = X_N(1-p_N^A-q_N^A)^{m_N-1}$$

In which X_N is a function of a series of parameters without m_i.

When condition (I)

$$X_N = \left[-c_S^{A_N} \left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k} \right) \frac{1 - p_N^A - q_N^A}{p_N^A + q_N^A} + c_T^{A_N} \left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k} \right) (1 - q_N^A) \right] \ln(1 - p_N^A - q_N^A) < 0 \quad \text{(I)}$$

is satisfied, then no matter what the value of m_N is, the partial derivative will always be negative. Thus the cost will be decreasing monotonically along with the coverage LOS. Therefore, the insurer should cover this episode, and there should be no limited LOS.

When condition (II)

$$X_N = \left[-c_S^{A_N} \left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k} \right) \frac{1 - p_i^A - q_i^A}{p_i^A + q_i^A} + c_T^{A_N} \left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k} \right) (1 - q_i^A) \right] \ln(1 - p_i^A - q_i^A) > 0 \quad \text{(II)}$$

is satisfied, then no matter what the value of m_N is, the partial derivative will always be positive. Thus the cost will be increasing monotonically along with the coverage LOS. Therefore, the insurer should not cover this episode.

Appendix C Proof of Proposition 2

When the maximum number of covered episode is increase from N to $N+1$, the increased cost will be

$$C(m_1, m_2, \dots, m_N, m_{N+1}) - C(m_1, m_2, \dots, m_N)$$

$$= c_S^{A_{N+1}} \left(\prod_{k=0}^{N} P_S^{A_k} P_S^{B_k} \right) \frac{1 - (1 - p_{N+1}^A - q_{N+1}^A)^{m_{N+1}}}{p_{N+1}^A + q_{N+1}^A}$$

$$+ c_S^{B_N} \left(\prod_{k=0}^{N-1} P_S^{A_k} P_S^{B_k} \right) P_S^{A_N} \frac{1}{p_N^B + q_N^B}$$

$$+ c_T^{A_{N+1}} \left(\prod_{k=0}^{N} P_S^{A_k} P_S^{B_k} \right) (1 - q_{N+1}^A)(1 - p_{N+1}^A - q_{N+1}^A)^{m_{N+1}-1}$$

$$+ c_T^{B_N} \left(\prod_{k=0}^{N} P_S^{A_k} P_S^{B_k} \right)$$

The partial derivative of the incremental cost over the coverage LOS for the i^{th} episode m_i is

$$\frac{\partial[C(m_1,m_2,\ldots,m_N,m_{N+1})-C(m_1,m_2,\ldots,m_N)]}{\partial m_i}$$

$$=\left[c_S^{A_{N+1}}\left(\prod_{k=0}^{N}P_S^{A_k}P_S^{B_k}\right)\frac{1-(1-p_{N+1}^A-q_{N+1}^A)^{m_{N+1}}}{p_{N+1}^A+q_{N+1}^A}\right.$$

$$+c_S^{B_N}\left(\prod_{k=0}^{N-1}P_S^{A_k}P_S^{B_k}\right)P_S^{A_N}\frac{1}{p_N^B+q_N^B}$$

$$\left.+c_T^{A_{N+1}}\left(\prod_{k=0}^{N}P_S^{A_k}P_S^{B_k}\right)(1-q_{N+1}^A)(1-p_{N+1}^A-q_{N+1}^A)^{m_{N+1}-1}+c_T^{B_N}\left(\prod_{k=0}^{N}P_S^{A_k}P_S^{B_k}\right)\right]'$$

$$=\left\{\left[c_S^{A_{N+1}}\left(\prod_{k=0}^{i-1}P_S^{A_k}P_S^{B_k}\right)P_S^{B_i}\left(\prod_{k=i+1}^{N}P_S^{A_k}P_S^{B_k}\right)\frac{1-(1-p_{N+1}^A-q_{N+1}^A)^{m_{N+1}}}{p_{N+1}^A+q_{N+1}^A}\right.\right.$$

$$+c_S^{B_N}\left(\prod_{k=0}^{i-1}P_S^{A_k}P_S^{B_k}\right)P_S^{B_i}\left(\prod_{k=i+1}^{N-1}P_S^{A_k}P_S^{B_k}\right)P_S^{A_N}\frac{1}{p_N^B+q_N^B}$$

$$+c_T^{A_{N+1}}\left(\prod_{k=0}^{i-1}P_S^{A_k}P_S^{B_k}\right)P_S^{B_i}\left(\prod_{k=i+1}^{N}P_S^{A_k}P_S^{B_k}\right)(1-q_{N+1}^A)(1-p_{N+1}^A-q_{N+1}^A)^{m_{N+1}-1}$$

$$\left.\left.+c_T^{B_N}\left(\prod_{k=0}^{i-1}P_S^{A_k}P_S^{B_k}\right)P_S^{B_i}\left(\prod_{k=i+1}^{N}P_S^{A_k}P_S^{B_k}\right)\right]P_S^{A_i}\right\}'$$

$$= \Bigg[c_S^{A_{N+1}} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) P_S^{B_i} \left(\prod_{k=i+1}^{N} P_S^{A_k} P_S^{B_k} \right) \frac{1 - (1 - p_{N+1}^A - q_{N+1}^A)^{m_{N+1}}}{p_{N+1}^A + q_{N+1}^A}$$

$$+ c_S^{B_N} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) P_S^{B_i} \left(\prod_{k=i+1}^{N-1} P_S^{A_k} P_S^{B_k} \right) P_S^{A_N} \frac{1}{p_N^B + q_N^B}$$

$$+ c_T^{A_{N+1}} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) P_S^{B_i} \left(\prod_{k=i+1}^{N} P_S^{A_k} P_S^{B_k} \right) (1$$

$$- q_{N+1}^A)(1 - p_{N+1}^A - q_{N+1}^A)^{m_{N+1}-1}$$

$$+ c_T^{B_N} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) P_S^{B_i} \left(\prod_{k=i+1}^{N} P_S^{A_k} P_S^{B_k} \right) \Bigg] \frac{q_i^A \left(1 - p_i^A - q_i^A\right)^{m_i}}{p_i^A + q_i^A} \ln\big(1$$

$$- p_i^A - q_i^A\big)$$

Let

$$Y_{N+1} = \Bigg[c_S^{A_{N+1}} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) P_S^{B_i} \left(\prod_{k=i+1}^{N} P_S^{A_k} P_S^{B_k} \right) \frac{1 - (1 - p_{N+1}^A - q_{N+1}^A)^{m_{N+1}}}{p_{N+1}^A + q_{N+1}^A}$$

$$+ c_S^{B_N} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) P_S^{B_i} \left(\prod_{k=i+1}^{N-1} P_S^{A_k} P_S^{B_k} \right) P_S^{A_N} \frac{1}{p_N^B + q_N^B}$$

$$+ c_T^{A_{N+1}} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) P_S^{B_i} \left(\prod_{k=i+1}^{N} P_S^{A_k} P_S^{B_k} \right) (1$$

$$- q_{N+1}^A)(1 - p_{N+1}^A - q_{N+1}^A)^{m_{N+1}-1}$$

$$+ c_T^{B_N} \left(\prod_{k=0}^{i-1} P_S^{A_k} P_S^{B_k} \right) P_S^{B_i} \left(\prod_{k=i+1}^{N} P_S^{A_k} P_S^{B_k} \right) \Bigg] \frac{q_i^A}{p_i^A + q_i^A} \ln\big(1 - p_i^A - q_i^A\big)$$

Then,

$$\frac{\partial[C(m_1, m_2, \ldots, m_N, m_{N+1}) - C(m_1, m_2, \ldots, m_N)]}{\partial m_i} = Y_{N+1}\left(1 - p_i^A - q_i^A\right)^{m_i}$$

in which Y_{N+1} is a function of a series of parameters without m_i.

Appendix D Proof of Theorem 1

Denote $\Delta C(m_N) = C(m_1, m_2, \dots, m_{N-1}, m_N) - C(m_1, m_2, \dots, m_{N-1})$, then

$$C(m_1, m_2, \dots, m_N) = C(m_1, m_2, \dots, m_i) + \Delta C(m_{i+1}) + \cdots + \Delta C(m_N)$$

$$\therefore \frac{\partial C(m_1, m_2, \dots, m_N)}{\partial m_i} = \frac{\partial [C(m_1, m_2, \dots, m_i) + \Delta C(m_{i+1}) + \cdots + \Delta C(m_N)]}{\partial m_i}$$

$$= \frac{\partial C(m_1, m_2, \dots, m_i)}{\partial m_i} + \frac{\partial \Delta C(m_{i+1})}{\partial m_i} + \cdots + \frac{\partial \Delta C(m_N)}{\partial m_i}$$

According to Proposition 1,

$$\frac{\partial C(m_1, m_2, \dots, m_i)}{\partial m_i} = X_N \left(1 - p_i^A - q_i^A\right)^{m_i - 1}$$

According to Proposition 2,

$$\frac{\partial \Delta C(m_{i+1})}{\partial m_i} = Y_{i+1} \left(1 - p_i^A - q_i^A\right)^{m_i}$$

$$\frac{\partial \Delta C(m_{i+2})}{\partial m_i} = Y_{i+2} \left(1 - p_i^A - q_i^A\right)^{m_i}$$

$$\vdots$$

$$\frac{\partial \Delta C(m_N)}{\partial m_i} = Y_N \left(1 - p_i^A - q_i^A\right)^{m_i}$$

$$\therefore \frac{\partial C(m_1, m_2, \dots, m_N)}{\partial m_i} = \frac{\partial C(m_1, m_2, \dots, m_i)}{\partial m_i} + \frac{\partial \Delta C(m_{i+1})}{\partial m_i} + \cdots + \frac{\partial \Delta C(m_N)}{\partial m_i}$$

$$= X_N \left(1 - p_i^A - q_i^A\right)^{m_i - 1} + \sum_{k=i+1}^{N} Y_k \left(1 - p_i^A - q_i^A\right)^{m_i}$$

$$= \left[X_N + \sum_{k=i+1}^{N} Y_k \left(1 - p_i^A - q_i^A\right)\right] \left(1 - p_i^A - q_i^A\right)^{m_i - 1}$$

In which X_N, Y_k are functions of a series of parameters without m_i.

When condition

$$X_N + \sum_{k=i+1}^{N} Y_k\left(1 - p_i^A - q_i^A\right) < 0$$

is satisfied, then no matter what the value of m_i is, the partial derivative will always be negative. Thus the cost will be decreasing monotonically along with the coverage LOS. Therefore, the insurer should cover this episode, and there should be no limited LOS.

When condition

$$X_N + \sum_{k=i+1}^{N} Y_k\left(1 - p_i^A - q_i^A\right) > 0$$

is satisfied, then no matter what the value of m_i is, the partial derivative will always be positive. Thus the cost will be increasing monotonically along with the coverage LOS. Therefore, the insurer should not cover this episode.

Appendix E Proof of Proposition 3

This proposition can be proved by comparing two cases with the same total number of considered episodes. In the first case, N^* of the episodes are covered without limited LOS, including the i^{th} episode, but not the $(i+1)^{th}$ episode. In the second case, the same N^* episodes are covered without limited LOS, except that the i^{th} episode is not covered, while the $(i+1)^{th}$ episode is covered.

Assumption 1: The probability of being cured will significantly decrease after every episode, i.e. $q_{i+1} \ll q_i$.

Assumption 2: The probability of being transferred to the home- and community-based care setting will decrease after every episode, i.e. $p_{i+1} < p_i$.

Assumption 3: The daily cost of staying will significantly increase after every episode, i.e. $c_S^{A_{i+1}} \gg c_S^{A_i}$.

Assumption 4: The cost of forced transition after every episode will remain the same, i.e. $c_T^{A_{i+1}} = c_T^{A_i}$.

Consider a total number of N episodes, and two cases. In the first case, N^* of the episodes are covered without limited LOS, including the i^{th} episode, but not the $(i+1)^{th}$ episode. In the second case, the same N^* episode are covered without limited LOS, except that the i^{th} episode is not covered, while the $(i+1)^{th}$ episode is covered. For the two cases, the cost is divided into three parts: the cost within the first $(i-1)$ episodes, the cost of the i^{th} and $(i+1)^{th}$ episode, and the cost within the last $(N-i-1)$ episodes.

For the first $(i-1)$ episodes, the cost for the two cases will be the same, since same episodes are covered.

For the last $(N-i-1)$ episodes, the cost will be dependent on the probability of entering the $(i+2)^{th}$ episode. Also, since same episodes are covered, the independent cost will be the same, which is assumed to be $C(N-i-1)$.

Then the cost for case 1 will be

$$P_E^{A_{i+2}}C(N-i-1) = P_E^{A_i}P_S^{A_i}P_S^{B_i}P_S^{B_{i+1}}C(N-i-1)$$

While the cost of case 2 will be

$$P_E^{A_{i+2}}C(N-i-1) = P_E^{A_i}P_S^{A_{i+1}}P_S^{B_i}P_S^{B_{i+1}}C(N-i-1)$$

According to Assumption 1 and 2,

$$P_S^{A_{i+1}} = \frac{p_{i+1}^A}{p_{i+1}^A + q_{i+1}^A} = \frac{1}{1+\frac{q_{i+1}^A}{p_{i+1}^A}} > \frac{1}{1+\frac{q_i^A}{p_i^A}} = P_S^{A_i}$$

Therefore, for the last $(N-i-1)$ episodes, the cost for case 1 will be lower than case 2.

For the i^{th} and $(i+1)^{th}$ episode, the cost for case 1 will be

$$C(Case\ 1) = P_E^{A_i}\left(c_S^{A_i}\frac{1}{p_i^A+q_i^A} + c_S^{B_i}P_S^{A_i}\frac{1}{p_i^B+q_i^B} + c_T^{A_{i+1}}\frac{1-q_{i+1}^A}{1-p_{i+1}^A-q_{i+1}^A}\frac{p_i^B}{p_i^B+q_i^B} + c_T^{B_i}P_S^{A_i}\frac{p_i^B}{p_i^B+q_i^B}\right)$$

While the cost for case 2 will be

$$C(Case\ 2) = P_E^{A_i}\left(c_S^{A_{i+1}}\frac{1}{p_{i+1}^A+q_{i+1}^A}\frac{p_i^B}{p_i^B+q_i^B} + c_S^{B_i}\frac{1}{p_i^B+q_i^B} + c_T^{A_i}\frac{1-q_i^A}{1-p_i^A-q_i^A} + c_T^{B_i}\frac{p_i^B}{p_i^B+q_i^B}\right)$$

According to Assumption 1, 2 and 3,

$$c_S^{A_i}\frac{1}{p_i^A+q_i^A} < c_S^{A_i}\frac{1}{p_{i+1}^A+q_{i+1}^A} \ll c_S^{A_i+1}\frac{1}{p_{i+1}^A+q_{i+1}^A}\frac{p_i^B}{p_i^B+q_i^B}$$

According to Assumption 1, 2 and 4,

$$c_T^{A_{i+1}}\frac{1-q_{i+1}^A}{1-p_{i+1}^A-q_{i+1}^A}\frac{p_i^B}{p_i^B+q_i^B} < c_T^{A_{i+1}}\frac{1-q_i^A}{1-p_i^A-q_i^A} = c_T^{A_i}\frac{1-q_i^A}{1-p_i^A-q_i^A}$$

Also, since $P_S^{A_i} < 1$,

$$c_S^{B_i}P_S^{A_i}\frac{1}{p_i^B+q_i^B} < c_S^{B_i}\frac{1}{p_i^B+q_i^B}, c_T^{B_i}P_S^{A_i}\frac{p_i^B}{p_i^B+q_i^B} < c_T^{B_i}\frac{p_i^B}{p_i^B+q_i^B}$$

$$\therefore C(Case\ 1) < C(Case\ 2)$$

Therefore, it is always better to cover the patient as early as possible.

Appendix F Proof of Theorem 3

Theorem 3 can be proved by comparing two cases with the same number of considered episodes in total. In the first case, the first n episodes are covered without limited LOS. In the second case, the first $(n+1)$ episodes are covered without limited LOS.

Consider a total number of N episodes, and two cases. In the first case, the first n episodes are covered without limited LOS. In the second case, the first $(n+1)$ episodes are covered without limited LOS.

For the first n episodes, the cost for the two cases will be the same, since same episodes are covered. For the last $(N-n)$ episodes, the cost for case 1 will be

$$C(Case\ 1) = P_E^{A_{n+1}}\left[c_T^{A_{n+1}} + c_S^{B_{n+1}}\frac{1}{p_{n+1}^B + q_{n+1}^B} + c_T^{B_{i+1}}\frac{p_{n+1}^B}{p_{n+1}^B + q_{n+1}^B}\right.$$

$$\left. + \sum_{i=n+2}^{N}\left(\prod_{j=n+1}^{i}\frac{p_j^B}{p_j^B + q_j^B}\right)\left(c_T^{A_i} + c_S^{B_j}\frac{1}{p_i^B + q_i^B} + c_T^{B_j}\frac{p_i^B}{p_i^B + q_i^B}\right)\right]$$

While for case 2, the cost will be

$$C(Case\ 2) = P_E^{A_{n+1}}\left\{c_S^{A_{n+1}}\frac{1}{p_{n+1}^A + q_{n+1}^A}\right.$$

$$+ P_S^{A_{n+1}}\left[c_S^{B_{n+1}}\frac{1}{p_{n+1}^B + q_{n+1}^B} + c_T^{B_{i+1}}\frac{p_{n+1}^B}{p_{n+1}^B + q_{n+1}^B}\right.$$

$$\left.\left. + \sum_{i=n+2}^{N}\left(\prod_{j=n+1}^{i}\frac{p_j^B}{p_j^B + q_j^B}\right)\left(c_T^{A_i} + c_S^{B_j}\frac{1}{p_i^B + q_i^B} + c_T^{B_j}\frac{p_i^B}{p_i^B + q_i^B}\right)\right]\right\}$$

Therefore, when the decremented cost of transition overwhelms the incremental cost of staying, condition

$$c_T^{A_{n+1}} + \left(1 - P_S^{A_{n+1}}\right) \sum_{i=n+2}^{N} \left(\prod_{j=n+1}^{i} \frac{p_j^B}{p_j^B + q_j^B} \right) c_T^{A_i}$$

$$> c_S^{A_{n+1}} \frac{1}{p_{n+1}^A + q_{n+1}^A}$$

$$- \left(1 - P_S^{A_{n+1}}\right) \left[c_S^{B_{n+1}} \frac{1}{p_{n+1}^B + q_{n+1}^B} + c_T^{B_{i+1}} \frac{p_{n+1}^B}{p_{n+1}^B + q_{n+1}^B} \right.$$

$$\left. + \sum_{i=n+2}^{N} \left(\prod_{j=n+1}^{i} \frac{p_j^B}{p_j^B + q_j^B} \right) \left(c_S^{B_j} \frac{1}{p_i^B + q_i^B} + c_T^{B_j} \frac{p_i^B}{p_i^B + q_i^B} \right) \right]$$

is satisfied, $C(Case\ 1) > C(Case\ 2)$, and it is better to cover one more episode. Generally, when the cost of forced transition is low, covering one more episode will increase the cost of staying more significantly than decrease the cost of transition, and the insurer will choose not to cover the additional episode. Vice versa.