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3-D shape recovery is an ill-posed inverse problem which must be solved by using a priori constraints[1]. We use symmetry and planarity constraints to recover 3-D shapes from a single image. Once we assume that the object to be reconstructed is symmetric, all that is left to be done is to estimate the plane of symmetry and establish the symmetry correspondence between the various parts of the object. The edge map of the image of an object would serve as a good representation of its 2-D shape and establishing symmetry correspondence would now mean identifying pair of symmetric curves in the edge map. The first step towards this objective is to estimate the vanishing points in the image. The vanishing points define the symmetry planes up to a scale factor. In this work, we have assumed that we know the vanishing points. A pair of curves can be reconstructed in 3-D if we know the symmetry correspondence and the plane of symmetry[1]. In order to be able to match curves, we should first extract some meaningful curves, where the word meaningful implies that the curve should make sense to a human observer. Connected components obtained after canny edge detection are broken down, based on gradient orientation, to get small curve pieces which can be then combined to form meaningful curves. Figure 1, shows these short curve pieces obtained for the image of a furniture. In order to obtain longer pieces of curves, we find the shortest paths between all pairs of short

Figure 1: Different pieces of curves are represented by different colors

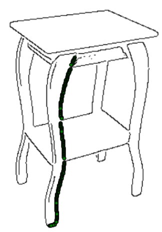

Figure 2: A long curve extracted by the shortest path algorithm

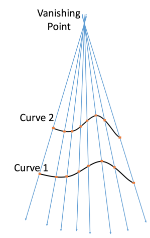

Figure 3: Polygonal approximation for the match metric

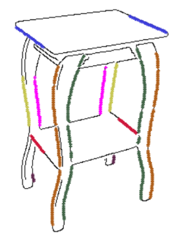

Figure 4: Correspondences found, matches have the same color

pieces of curves with a cost function that penalizes spatial separation and large turning angles (see Figure 2). In the next step, we find the optimal curve matches that minimize the number of planes required to fit the final 3-D reconstruction while simultaneously ensuring that a substantial portion of the object is reconstructed. To achieve this, we first assign a score to each pair of curves based on their shape similarity and planarity. To evaluate the planarity score, we reconstruct the two curves in 3-D, assuming they are corresponding, and then fit planes using RANSAC. The candidate planes that need to be considered to fit the final reconstruction are also discovered in this process. The shape similarity is evaluated by computing the polygonal approximation of the curves by sampling the curve using rays from the vanishing point (as shown in Figure 3) and connecting these points (shown in orange) by straight line segments. Comparing the turning angles at each of the sampled points will serve as good shape match metric. Once these scores are evaluated, we solve a constrained optimization problem which can be defined as shown below.

$$\begin{aligned} \arg\min_{\boldsymbol{X}} \quad & \boldsymbol{X^T C} + w f_p(\boldsymbol{X}) \\ \text{subject to} \quad & \boldsymbol{X^T K} \geq p \\ & \boldsymbol{DX} \leq 1 \end{aligned}$$

$\boldsymbol{X}$ is a vector with binary components which decide whether a curve combination is included in the solution, and if it is included then which candidate plane it is assigned to. We not only wan to minimize the number of planes required to fit the final reconstruction but also ensure that the chosen planes are a good fit. Hence, it involves choosing the correspondences and deciding which candidate plane it goes with. $\boldsymbol{C}$ is a vector whose components are a combination of shape match cost and planarity cost (measure of individual curve planarity) of curves involved in the correspondence, as well as the distance of the 3-D reconstructed curves from the candidate planes. $f_p(\boldsymbol{X})$ is a function that gives the number of planes used by the correspondences and w is the plane weight that gets added to the total cost for each plane chosen, thereby discouraging the algorithm from choosing too many planes. The first constraint requires the optimization framework to reconstruct a substantial portion of the object depending on the parameter p and the second constraint ($\boldsymbol{D}$ is a matrix and hence multiple individual constraints are used to represent this overall constraint) ensures that each piece of the object has no more than one match. This optimization problem can be converted to a binary integer program which can then be solved using the Gurobi optimization framework[2]. Figure 4, shows one of the results obtained. Symmetry and planarity in many ways represent the simplicity of an object and by applying these constraints we are attempting to reconstruct a simple 3-D shape that can explain the image.

References

[1] Pizlo, Z., Li, Y., Sawada, T. and Steinman, R.M., 2014, *Making a machine that sees like us*, Oxford University Press.

[2] Gurobi Optimization, Inc., 2015, *Gurobi Optimizer Reference Manual*, http://www.gurobi.com.