Journal of Problem Solving
SPECIAL ISSUE

Solving Classical Insight Problems Without Aha! Experience: 9 Dot, 8 Coin, and Matchstick Arithmetic Problems

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Keywords:
insight, problem solving, constraints, Aha! experience, 9 dot, 8 coin, matchstick

Insightful problem solving is a vital part of human thinking, yet very difficult to grasp. Traditionally, insight has been investigated by using a set of established "insight tasks," assuming that insight has taken place if these problems are solved. Instead of assuming that insight takes place during every solution of the 9 Dot, 8 Coin, and Matchstick Arithmetic Problems, this study explored the likelihood that solutions evoked the "Aha! experience," which is often regarded as the defining characteristic of insight. It was predicted that the rates of self-reported Aha! experiences might vary based on the necessary degree of constraint relaxation. The main assumption was that the likelihood of experiencing an Aha! would decrease with increasing numbers of constraints that must be relaxed, because several steps are needed to achieve a representational change and solve the problem, and thus, the main feature of suddenness of a solution might be lacking. The results supported this prediction, and demonstrated that in many cases participants do solve these classical insight problems without any Aha! experience. These results show the importance of obtaining insight ratings from participants to determine whether any given problem is solved with insight or not.

Most people know the phenomenon that after some struggle with a difficult problem, the solution simply appears out of nowhere. Such a moment of sudden comprehension is known as insight. Insight is thought to follow from restructuring or representational change processes (Duncker, 1945; Ohlsson, 1984). This means the problem solver sets up an initial mental representation of the problem that is biased because of unnecessary, false assumptions about the task. The resulting problem space is over-constrained (Ohlsson, 1992) or ill-defined (Kaplan & Simon, 1990). This problem representation needs to be changed in a proper way to increase the likelihood of solving the problem. On a phenomenological level of description, the moment of representational change is often reported being accompanied by a strong emotional response, the "Aha! experience" (Bühler, 1907; Gick & Lockhart, 1995; Kaplan & Simon, 1990).

After 100 years of problem solving research, the cognitive mechanisms underlying insight are still not fully understood. There are various conceptual and experimental reasons for that, including the repeated usage of a limited set of tasks assumed to be "insight problems," combined with the fact that there is no clear behavioral nor neural marker indicating that insight has taken place (Dietrich & Kanso, 2010), and the difficulty of discerning if a problem has been solved via insightful problem solving or more analytical processes (Metcalfe & Wiebe, 1987). This relates to another long-standing discussion about whether insight problems are something special or not (Davidson, 1995). In the past, some researchers have assumed that when "insight problems" are solved, then insight has taken place (Ash, Cushen, & Wiley, 2009; Weisberg, 1992). This reasoning is problematic and becomes circular if, as Öllinger and Knoblich point out, "Insight problems are problems that require insight, and insight occurs when insight problems are solved" (2009, p. 3). There have been attempts at classifying and defining the properties of insight problems (e.g., Weisberg, 1995), but there is no clear agreement on which types of problems will always trigger insight, because "there is no particular class of insight problems that necessarily requires a representational change; each problem can be solved without insight if the initial problem representation is adequate and the appropriate heuristics are available" (Öllinger, Jones, & Knoblich, 2014, p. 267).

More recently, researchers interested in detecting possible neural correlates of insight have begun to rely on the phenomenological dimension of insight, the Aha! experience (e.g., Aziz-Zadeh, Kaplan, & Iacoboni, 2009; Bowden, Jung-Beeman, Fleck, & Kounios, 2005; Sandkühler & Bhattacharya, 2008) that sets it apart from more analytical modes of problem solving. The Aha! experience is generally described as very pleasant, connected with emotional arousal and with

http://dx.doi.org/10.7771/1932-6246.1183

a strong certainty that the solution is correct (Sternberg & Davidson, 1995). Most phenomenological approaches use subjective reports of Aha! experiences to differentiate insightful ("with Aha!") from noninsightful ("without Aha!") solving events, assuming that "the presence or absence of insight resides in the solver's solution rather than in the problem" (Bowden & Jung-Beeman, 2007, p. 88). From these observations, it is clear that the occurrence of insight should not be assumed, and can be assessed by directly asking problem solvers about it. Although often used in insight research, no prior work has asked participants to give ratings of insight on three widely used classical insight problems: The 9 Dot Problem (Maier, 1930), the 8 Coin Problem (Ormerod, MacGregor, & Chronicle, 2002), and one Matchstick Arithmetic Problem (e.g., Knoblich, Ohlsson, Haider, & Rhenius, 1999). Thus, collecting subjective Aha! ratings on these problems in order to verify their status as insight problems was a goal of the present study.

SUBJECTIVE MEASURES OF INSIGHT

The present study collected self-reports about participants' Aha! experiences using the multi-dimensional definition by Jung-Beeman and colleagues (2004) to test whether insight ratings are predicted by the degree of representational change that might be required for each of these problems. Empirical studies have successfully implemented subjective ratings of insight for several types of problems. This has been mainly the case for the domain of compound remote associates (CRA problems, Bowden & Jung-Beeman, 2003). Based on participants' self-reports of insight (using a definition that included suddenness, obviousness, and confidence), it was possible to detect differences between insight and noninsight solutions with regard to neural activity as measured by EEG and fMRI (Jung-Beeman et al., 2004; Subramaniam, Kounios, Parrish, & Jung-Beeman, 2009). Another study (Kounios et al., 2006) was even able to show that the neural activity differed already in a preparatory interval before the problem was presented, predicting whether it would later be solved with or without insight. Kounios and Beeman (2014) provide an overview on this work, but see also Dietrich and Kanso (2010) for a review. Ellis and colleagues combined post-hoc insight ratings of how suddenly the solution came to mind with eye tracking while participants were trying to solve 60 anagram problems (Ellis, Glaholt, & Reingold, 2011). Participants were presented with an array of five scrambled letters, four of which could be combined into a solution word, and the fifth letter was a distractor. They found that insight trials had shorter response times and different patterns of fixation than noninsight trials, showing that the subjective differentiation made by participants was actually reflected in differences in behavioral measures. This shows the usefulness of self-reports. However, they found increasingly shorter viewing times on the distractor item already several seconds prior to the response and interpreted this as evidence for a gradual accumulation of solution knowledge even before participants were aware of the solution. Importantly, this was found for both insight and noninsight trials, suggesting a dissociation between the subjective experience of "suddenness" of the solution and the actual problem solving process. This questions the reliability of self-reports. Note that their definition of insight focused solely on suddenness in contrast to the multi-dimensional definition of insight as used by Jung-Beeman and colleagues (2004) and also in the present study.

Cushen and Wiley (2012) used a single-trial design with only one task, the Triangle of Circles, in a large study where they obtained post-hoc ratings of insight (based on just two aspects of solution: suddenness and surprise), but also repeated ratings on problem features during the problem solving process as a measure of representational change (Ash & Wiley, 2008). In concurrence with Ellis and colleagues, they found a disconnect between self-reports of insight and the actual solution patterns. As one possible explanation for this finding, they point out the difficulties in providing clear descriptions of insight or Aha! that participants can use as the basis for their self-reports. A large variety of different instructions can be found in the literature, and this may be responsible for different findings across studies. This important point will be discussed in more detail at the end of the present paper.

PROBLEM ANALYSIS BASED ON REPRESENTATIONAL CHANGE

The theoretical framework for the present study is Öllinger's extended representational change theory (eRCT, Öllinger et al., 2014), which is based on and elaborates Ohlsson's RCT (Knoblich et al., 1999; Ohlsson, 1992). Some of the key assumptions of the eRCT are sketched in the following. First, as already discussed, any problem can be solved with or without insight. Second, in line with Ohlsson, insight is assumed to be caused by a representational change. A representational change leads to modified prior knowledge and a modified search space, so that after a representational change, heuristics are needed to guide the search process again. Third, and this is a new aspect, insight problem solving is conceptualized as a dynamic search process that might also include recursive steps, that is, repeated instances of search, impasse, and representational change. One possibility to achieve a representational change is through the hypothetical process of constraint relaxation (Isaak & Just, 1995; Knoblich et al., 1999; Ohlsson, 1992). Self-imposed constraints that prevent a solution (for example, assumptions about rules that do not apply and that were never explicitly stated) need to be overcome or relaxed before the problem can be solved.

The present study predicted that the rates of self-reported Aha! experiences might vary based on the necessary degree of constraint relaxation. The main assumption was that the likelihood of experiencing an Aha! would decrease with increasing numbers of constraints that must be relaxed, because several steps are needed to achieve a representational change and solve the problem and thus, the main feature of suddenness of a solution might be lacking. In the following, each of the three problems used will be discussed in light of the potential constraints that they might impose on the problem solver.

Matchstick Arithmetic Problem. Knoblich and colleagues (1999; see also Knoblich, Ohlsson, & Raney, 2001; and Öllinger, Jones, & Knoblich, 2008) created a taxonomy of different types of Matchstick Arithmetic. Problem difficulty was theoretically inferred from the degree of necessary constraint relaxation, and then empirically verified (Knoblich et al., 1999). A matchstick task from the operator problem type (CR2, see Öllinger et al., 2008), requiring an intermediate degree of constraint relaxation, was selected for this study. To solve this problem, the problem solver has to overcome one main constraint, namely the assumption that operators in equations should remain constant. A second constraint is posed by requiring the problem solver to break up the chunk that is formed by the operator (the "=" sign), using chunk decomposition (instead of moving a single matchstick that is not a part of a perceptual group).

8 Coin Problem (Ormerod et al., 2002). A study by Öllinger and colleagues (Öllinger, Jones, Faber, & Knoblich, 2013) was able to disentangle the different sources of difficulty in the 8 Coin Problem by systematically varying perceptual aspects of the original problem. They found that the main source of difficulty was caused by a self-imposed 2-D constraint, but that even after a cue was given that showed coins stacked on top of each other (relaxing the 2-D constraint), an additional source of difficulty remained that originated from the tight perceptual groupings of the coins. Thus, decomposing the tight chunks in order to overcome these perceptual groupings represented a second constraint that was shown to hinder the solution.

9 Dot Problem (Maier, 1930). It has been shown by Kershaw and Ohlsson (2004) that the 9 Dot Problem entails several sources of difficulty. Even after relaxing the boundary constraint (i.e., realizing that the lines have to be drawn across the borders of the imaginary square), several obstacles remain before one can reach solution. In fact, many solvers are not able to reach a solution for the 9 Dot Problem, even with explicit hints (Chronicle, Ormerod, & MacGregor, 2001), because the new search space is too large. Just recently, Öllinger and colleagues were able to identify the necessity to restrict the new, larger search space after the relaxation of the boundary constraint as key to solution (Öllinger et al., 2014). In particular, realizing the necessity of so-called "non-dot turns" seems to constitute an additional constraint for the problem solver (Kershaw & Ohlsson, 2004). In addition, the number of non-dot turns required for a solution influenced solution rates, too, with lower solution rates for problems with two or more turns compared to problems with a single turn (Kershaw & Ohlsson, 2004). There is a further constraint, namely realizing that one of the dots (the "apex dot") must be crossed with lines repeatedly (Öllinger et al., 2014). The ability to engage in mental look-ahead to consider or imagine the placement of the 4 lines also seems to constrain solution success (Chein, Weisberg, Streeter, & Kwok, 2010; MacGregor, Ormerod, & Chronicle, 2001). As a result, there are at least three constraints that may need to be relaxed in solving this problem.

HYPOTHESES

Assuming that problems with more constraints are less likely to trigger sudden Aha! experiences, differing rates of reported Aha! experiences are expected among the three problems. Specifically, we predict lower rates of reported Aha! experiences for the 9 Dot Problem because it entails more constraints that need to be relaxed than the two other problems. The Matchstick Arithmetic Problem requires overcoming the constraint that operators are not to be manipulated, and decomposing the operator. The 8 Coin Problem requires overcoming the 2-D constraint and decomposing the tight perceptual grouping. Finally, the 9 Dot Problem requires the solver to overcome the boundary constraint, to make non-dot turns, to use an apex dot, and to use mental-lookahead to restrict the much too large search space after the boundary constraint has been relaxed. Consequently, if the Aha! experience is tightly bound to constraint relaxation, we expect higher rates of reported Aha! experiences for the Matchstick Problem and for the 8 Coin Problem than for the 9 Dot Problem.

METHOD

PARTICIPANTS

Healthy volunteers (N = 45, 14 males), most of them students (age M = 24; range: 20–33), were recruited through announcements at the University of Munich. After giving informed consent, each participant was tested individually. None of them had any neurological diseases and all had normal or corrected-to-normal acuity.

MATERIALS

9 Dot Problem. This classical task (Maier, 1930) consists of 9 dots that must be connected with 4 continuous lines, without lifting the pen. In the correct solution, the lines extend through the "barriers" of the virtual square (see Appendix for problem solutions). Participants were presented with the dots depicted on a sheet of paper (as seen in Figure 1) and drew the lines directly on the sheet where space was provided for three separate drawing attempts.

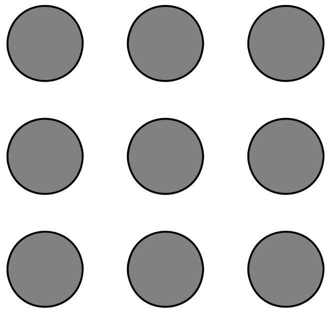

Figure 1
Initial configuration of the 9 Dot Problem.

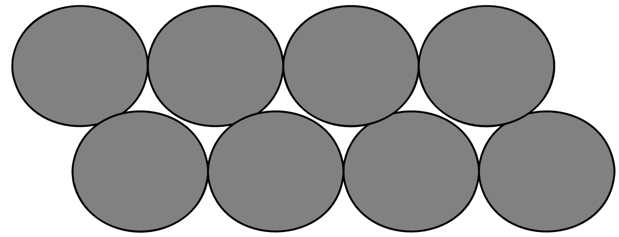

Figure 2
Initial configuration of the 8 Coin Problem.

8 Coin Problem. The 8 Coin Problem was designed by Ormerod and colleagues (2002). The problem solver is required to move two coins in order to find a constellation where each coin touches exactly three other coins. The solution consists of forming two separate clusters of three coins each, and resting a fourth coin on top of each cluster. The eight 20-cent coins were spread out on a table and participants were allowed to touch the coins and move them around during the problem solving process. No hints were given. The initial arrangement of the coins (depicted in Figure 2) was always visible on a sheet of paper so that they could get back to the start configuration at any time.

Matchstick Arithmetic Problem. This problem was taken from a group of previously used matchstick arithmetic tasks (Knoblich et al., 1999): VIII = VI - II (operator type problem, CR2). The Roman numerals were constructed out of matchsticks, and participants were asked to transform the incorrect arithmetic statement into a correct one by moving only one single matchstick. The given task requires moving a stick from the equal sign and putting it onto the minus sign: VIII - VI = II. Again, the initial arithmetic statement (see Figure 3) was depicted on a sheet of paper, with wooden matchsticks provided below. Participants were asked to move the matchsticks, but could restart at any time.

PROCEDURE

Individually, each participant was seated in a quiet room and was provided by the experimenter with a clear description of insightful ("with Aha!") and noninsightful ("without Aha!") solution experiences. To ensure comparability across studies, we used wording adapted from Jung-Beeman and colleagues (2004) (translated to German):

> We would like to know whether you experienced a feeling of insight when you solved the problem. A feeling of insight is a kind of "Aha!" characterized by suddenness and obviousness. Like an enlightenment. You are relatively confident that your solution is correct without having to check it. In contrast, you experienced no Aha! if the solution occurs to you slowly and stepwise. As an example, imagine a light bulb that is switched on all at once in contrast to slowly dimming it up. We ask for your subjective rating whether it felt like an Aha! experience or not, there is no right or wrong answer. Just follow your intuition.

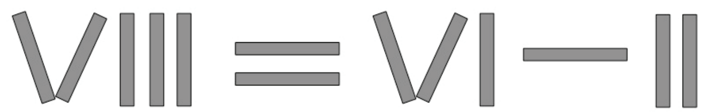

Figure 3
Initial configuration of the Matchstick Arithmetic Problem.

Participants were told that they would need to sort their solutions on a trial-by-trial basis into these two categories. They were then asked to solve the three insight problems within an upper time limit of 7 minutes each. Each problem was separately instructed with a written instruction, and participants were asked to indicate if they were already familiar with a problem. Problem order was balanced across participants. Matchsticks, 20-cent coins, paper, and pencil were provided. The experimenter measured the solution time with a watch, from a separate adjacent room. Upon producing a correct solution within the time limit, the solution was shown to the experimenter, who confirmed its correctness. When the proposed solution was incorrect, the participant was told so and could resume solving. The experimenter then asked participants whether they had experienced an Aha! or not. If they failed to produce a correct solution within the upper time limit, the experimenter interrupted the participant and, without telling the solution, moved on and presented the next problem.

RESULTS

Because the problems were already familiar to participants in 8.9% of all 135 trials (135 = 45 participants x 3 problems), these trials had to be discarded. Solutions were further

Table 1

Solution rates (frequencies) for the three problems. For unsolved trials, no Aha! could be reported. Note that the percentages for each column sum to 100%.

	Matchstick	8 Coin	9 Dot	Σ
Solved With Aha!	19 (42.2%)	8 (17.8%)	1 (2.2%)	28 (20.7%)
Solved Without Aha!	18 (40.0%)	4 (8.9%)	4 (8.9%)	26 (19.3%)
Unsolved	6 (13.4%)	31 (68.9%)	32 (71.1%)	69 (51.1%)
Familiar	2 (4.4%)	2 (4.4%)	8 (17.8%)	12 (8.9%)

divided into solved with Aha! and solved without Aha! categories. The distributions of solutions to the four categories (Solved with Aha!, Solved without Aha!, Unsolved, and Familiar) are shown in Table 1.

An Intraclass Correlation Coefficient (ICC) was computed to examine whether the three problems resulted in similar patterns of frequencies across the three solution categories. This analysis yielded a low and negative ICC of -.046, indicating disagreement between the three problems. This means, the patterns of the solution score differed, as can also be seen in Figure 4 (next page).

Cochran's Q Test for two related samples was used to further explore the disagreements in solution rates and Aha! rates as shown in Figure 4. These pairwise comparisons showed that the Matchstick solution rate (82.2%) was significantly higher than the solution rates of 8 Coin (26.7%) ($Q(1) = 22.5$, $p < 0.01$) and 9 Dot (11.1%) ($Q(1) = 24$, $p < 0.01$), but that solution rates did not differ between the 8 Coin and 9 Dot ($Q(1) = 2.3$, $p = 0.13$).

Across all problem types, participants reported that they had experienced an Aha! in only 51.9% of all correctly solved trials. First, the proportion of Aha! experiences out of total opportunities were analyzed because this led to more cells with 5 or more observations. Cochran's Q Test for 2 related samples indicated that the rate of reported Aha! experiences for Matchsticks was significantly higher than the rate for 8 Coin ($Q(1) = 5.8$, $p < 0.05$) as well as for 9 Dot ($Q(1) = 16.2$, $p < 0.01$). Also, the Aha! rate for 8 Coin was significantly higher than the one for 9 Dot ($Q(1) = 5.4$, $p < 0.05$).

The proportion of Aha! experiences out of correct solutions was also examined, even though nearly all cells had fewer than 5 observations due to extremely low solving rates for 9 Dot and 8 Coin. This may be a more theoretically informative analysis because its results are independent of the greatly differing solution rates of the three problems, and it also takes into account that Aha! experiences could only be reported in solved trials. Looking at the data this way, the 9 Dot Problem received the lowest ratio with only 20% of solutions accompanied by an Aha! The Matchsticks Problem led to an Aha! in 51.4% of solutions, and the 8 Coin in 66.7% of solutions. However, Cochran's Q Test for 2 related samples, which was computed for the same three comparisons as previously, was never significant, Matchstick vs. 8 Coin with ($Q(1) = 0.2$, $p = 0.66$), Matchstick vs. 9 Dot with ($Q(1) = 0.3$, $p = 0.56$), and 8 Coin vs. 9 Dot with ($Q(1) = 1.0$, $p = 0.32$).

DISCUSSION

The present study examined solution rates and self-reported Aha! ratings for three classical insight problems. First, the solution rates were found to differ across the three problems, and these solution rates can be compared to those found in previous studies. Due to its notorious difficulty, researchers only rarely implement the 9 Dot Problem without hints, as was done in this study, but one such study reports solution rates of 0% after participants were allowed 10 trials of drawing different solution attempts in a booklet (MacGregor et al., 2001), but given no solution feedback. Another study (Chein et al., 2010) reports very similar solution rates to the one found here, namely 10% (five participants from a $n = 51$ sample).

For the 8 Coin Problem, comparing the present results to Ormerod's original study (2002), there is a strikingly large difference in solution rates: They found 0% during a time interval of four minutes. After these four minutes, participants were given two hints, which raised solution rates to 92%. There are several possible explanations for this difference: First, it should be noted that Ormerod and colleagues collected only pilot data of 12 participants using this original coin configuration. Second, it is not clear whether participants had previously been informed that they would receive the hints. If so, this could have influenced solution rates. Third, the test material was different: Ormerod used hexagonal metal tokens (possibly based in British 20- and 50-pence coins) in order to facilitate the evaluation of the number of contacts between coins, whereas this study used 20-cent Euro coins, which are circular. This may have increased the difficulty of chunk decomposition because the hexagonal tokens create a more "tight" perceptual grouping than the circular coins. Fourth, the time limits differed (four minutes in the original study before a hint was given; seven minutes here). That time is a crucial factor is further supported by the comparison with another study on the 8 Coin Problem, where an even higher solution rate, 40%, was reported (Öllinger et al., 2013) and participants were allowed as many

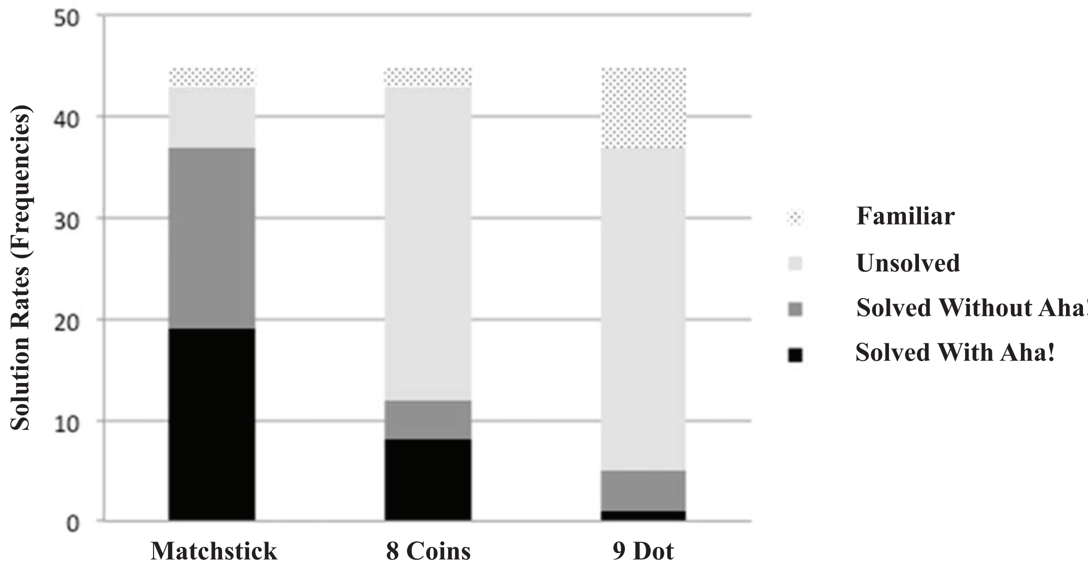

Figure 4
Number of participants (out of 45) who solved the problem with or without reporting an Aha! experience, failed to solve, or were already familiar with the problem.

solution attempts as they wished. That both the present study and Öllinger's study tested larger samples (45 and 28, respectively) than the original study (12 participants), might be an additional reason for the differing solution rates.

For the Matchstick Problem, it was rather difficult to find studies using exactly the same problem configuration, but comparable problems from the operator group (CR2) led to solution rates around 60% after six minutes (Öllinger et al., 2008). Another study reports solution rates for CR2 problems of about 68% after five minutes (data from block 1 of Experiment 2 in Knoblich et al., 1999). Note that in contrast to the present design, both studies presented the Matchstick Problems on a computer screen and had participants solve them "in the head." Another difference was that both studies included sets of several problems, and prior work has suggested that the order of Matchstick Problems can cause fixation effects (Öllinger et al., 2008). Together with the lower time limit, this might account for the lower solution rates in prior work.

Second, although the problems used here have all been considered to be "insight problems," participants reported that they had only experienced an Aha! approximately half the time that problems were solved correctly. Compared to other studies using trial-wise insight ratings, the present finding is quite typical. Other studies, using CRA problems, report numbers between 50% and 60% (Kounios et al., 2008; Subramaniam et al., 2009; Sandkühler & Bhattacharya, 2008; Wegbreit, Suzuki, Grabowecky, Kounios, & Beeman, 2012). Fedor, Szathmáry, and Öllinger (2015) found for Katona's Five Square Problem (Katona, 1940) that 74% of all solvers reported an Aha! experience. The present finding that only half of all solutions were classified as Aha! solutions casts doubt on the research practice of simply assuming that insight has taken place if an "insight problem" is solved. In particular, only 1 out of 5 participants who had solved the 9 Dot Problem reported an Aha! experience. Of course, this makes this problem seem especially problematic when it is used as a typical "insight problem" without any concurrent measurements of participants' actual solving experience. Further, the fact that some participants received feedback about their solution accuracy before reaching a correct solution could have contributed to the low overall Aha! rates if we see this feedback as a sort of hint, because it has been shown that providing hints causes more incremental and less insight-like patterns, as measured by feeling-of-warmth ratings (Cushen & Wiley, 2012; Davidson, 1995).

The main question that was tested by the present study was whether the rates of self-reported Aha! experiences in three classical insight problems could be predicted based on the necessary degree of constraint relaxation. As expected, differing rates of Aha! (based on total solving opportunities) were found across the three problems: The Matchstick Problem triggered the highest Aha! rates, followed by the 8 Coin Problem, and then the 9 Dot Problem with the lowest rate. Note that if proportions of solutions are analysed instead of proportions of total problem attempts, there is a small change in the overall pattern such that the rate for 8 Coin appears higher and more similar to the rate for Matchsticks. However, the main result of the 9 Dot Problem triggering a much lower

Aha! rate than the other two problems stays the same across both analyses. This is consistent with the hypothesis that solutions for which fewer constraints need to be relaxed feel more sudden and thus more like an "Aha!" than multi-step solutions with several constraints that must be overcome.

An alternative to considering the number of constraints is considering the type of constraints that need to be relaxed, the consequences that follow from relaxation, and the order in which the constraints are relaxed. These factors influence whether the search space for a given problem gets enlarged (making a solution less likely) or restricted (facilitating a solution). Starting with the 8 Coin Problem, assume that the first constraint to be relaxed would be the 2-D constraint. When it is relaxed, it increases the search space for solution. However, as soon as the solver is able to overcome the perceptual grouping and considers breaking up the adjacent coins, then there are only two coins that can be moved to break up the starting configuration into two equal groups of three coins. This means relaxing the second constraint (perceptual grouping) in the 8 Coin Problem restricts the search space again and thus might quickly lead to a solution. Of course the order in which these two constraints are relaxed could theoretically also be reversed: The problem solver could realize first that it is necessary to break up the tight perceptual grouping and form two separate groups of three coins each by taking the two "inner" coins and then, while moving them around, relax the 2-D constraint. For this sequence of constraint relaxation, which appears less likely than the first, breaking up the grouping might already feel like a restriction of the search space (since the target coins are identified), but this does not necessarily lead to a solution. Only if the problem solver proceeds with relaxing the second remaining constraint (2-D), and realizes that coins must be stacked on top of each other, does the search space finally narrow. This solution process appears less swift than the prior one and thus might be less likely to be accompanied by Aha! experiences, because the criterion of suddenness is lacking. This could account for the small proportion of solutions without Aha! for the 8 Coin Problem.

Similarly, with the Matchstick Problem, there are two constraints that need to be relaxed. The solver needs to overcome the assumption that operators are not to be manipulated and also recognize that the perceptual grouping of the operator (the equal sign) needs to be decomposed. Because these constraints are both directly related to the operator, relaxing one constraint might relate to simultaneously relaxing the other, quickly restricting the search space, and thus lead suddenly to a solution. If the two constraints are instead relaxed sequentially, the most likely sequence would be to first relax the operator constraint and then decompose the grouping. However, the reverse order seems also plausible: First, participants might realize that they must break up chunks such as the V (which enlarges the search space since not only the individual matchsticks that make up numerals such as II and III can be used, but also the ones belonging to the V), and later they might realize that the operator must be manipulated (again, closing the search space). This sequence appears as likely as the first, but less likely to trigger a feeling of suddenness and Aha! This reasoning could account for the larger proportion of solutions without Aha! for the Matchstick Problem than the 8 Coin.

In contrast, the solution process for the 9 Dot Problem seems to be qualitatively very different. When the first constraint is relaxed (the boundary constraint), it enlarges the search space. Realizing that one needs to make non-dot turns, however, does not restrict the enlarged search space because they could, theoretically, be made at any location outside the area of the given nine dots (Öllinger et al., 2014). Thus, relaxing the second constraint in the 9 Dot Problem might not lead as quickly to a solution as in the other problems because there still remain several other constraints such as realizing that one needs to use an apex dot, or finding the correct number of non-dot turns. Because there are multiple constraints that need to be relaxed, and because achieving each of these partial insights is likely to occur over an extended span of time rather than all at once, solutions to the 9 Dot Problem are less likely to trigger a feeling of suddenness and Aha! regardless of the order they are relaxed in. This could account for the large number of solutions that were not accompanied by an Aha! experience for this problem.

Of course, depending on their prior knowledge, problem solvers might not encounter all constraints that have been identified in the above task analysis. In addition, the order in which constraints are likely to be relaxed for each problem as discussed above are hypothetical, as the present study did not collect any trace data that might be used to confirm these assumptions. Ideally, future research should employ converging methods including think-aloud or eye-tracking methods that might be used to track solution processes, in addition to obtaining self-reports about Aha! experiences, so that the connection between solution processes and the Aha! experience can be made more directly.

This relates to some other conceptual limitations of the present study. For example, it remains unclear whether a representational change actually took place or whether an impasse occurred during solving. Thus, the results of this study cannot speak to the question of possible differences in the problem solving process between participants reporting Aha! and participants not reporting it. Several methods have been used in the past to try to capture problem solving processes. In addition to using think-aloud protocols (Ash, Jee, & Wiley, 2012; Cranford & Moss, 2012; Fleck & Weisberg, 2004; Newell & Simon, 1972), recording eye movements can also offer valuable insights into the solution process (Knoblich et al., 2001; Ellis et al., 2011). Another alternative is to conduct a move analysis (Öllinger et al., 2013; Ormerod et al., 2002) where the experimenter records the start and end point of each move while participants are solving problems with external objects. Examining the frequency of first moves can be used to infer possible biases in the initial problem representation. Similarly, recording when participants first start stacking coins on top of each other, going outside of the perimeter of the box, and touching or pointing to the operators in the matchstick problem can be used to infer when participants relax constraints. A detailed move analysis can also be conducted online, with a computer program recording participants' drag-and-drop mouse movements and reaction times (Fedor, Szathmáry, & Öllinger, 2015). Finally, obtaining ratings of problem features at multiple time points (Ash & Wiley, 2008; Cushen & Wiley, 2012) can also be used to track changes in problem representations during the solution process.

Collecting subjective measures of Aha! experiences has the advantage that it offers researchers the possibility to compare insight with noninsight solutions instead of just assuming that problems are being solved with insight. This was first successfully done by Jung-Beeman and colleagues (2004) with their set of CRA problems, but other promising new approaches to insight research, such as Rebus puzzles (MacGregor & Cunningham, 2008) or magic tricks (Danek, Fraps, von Müller, Grothe, & Öllinger, 2013, 2014b), offer the same possibility because a large set of tasks can be created. In particular, the domain of magic tricks seems ideally suited to investigate insight because the typical constraints encountered by problem solvers are well-known and systematically manipulated by the magician, triggering an initial problem representation that is misleading. When participants gain insight into a magic trick, that is, discover how the trick works, they report strong feelings of Aha! (Danek, Fraps, von Müller, Grothe, & Öllinger, 2014a). Further, Danek and colleagues found that solutions to magic tricks that occurred with insight were reached earlier, received higher confidence ratings, and were also more likely to be true than noninsight solutions (2014b).

Across studies, variations in experimenters' instructions for insight ratings may be responsible for different findings. The simple dichotomic insight/noninsight rating used here is probably not sufficient to fully capture the complex phenomenology of the Aha! experience, which has been shown to consist of at least 4 different components, namely "suddenness," "surprise," 'happiness," and "certainty" (Danek et al., 2014a). Further components, like the feeling of relief upon obtaining the solution or heightened motivation for the next task, were suggested when participants were asked to provide qualitative self-reports instead of ratings ("For you, how does an Aha! moment feel like? Please describe it in your own words!", Danek et al., 2014a, p. 5). Processing fluency has also been suggested as an underlying mechanism (Topolinski & Reber, 2010). Only some of these components are reflected in the instruction that was used in the present study. This limitation also applies to many other studies using self-reports of insight, with some of them assessing only the component of suddenness (Aziz-Zadeh et al., 2009; Ellis et al., 2011). Further research is needed to disentangle these different aspects of the Aha! experience and their relations to the solution process.

Obviously, the optimal method for determining the occurrence of insight has not been found yet. As one step toward this aim, instead of using predefined insight problems and assuming the occurrence of insight if the problem is solved, researchers need converging subjective or behavioral measures that can help to take into account participants' actual problem solving experiences.

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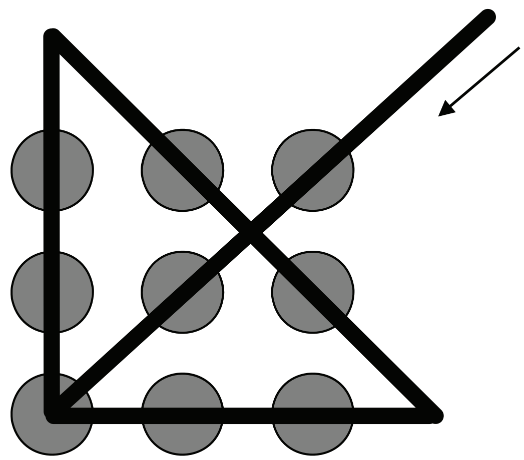
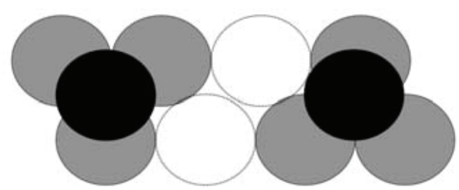
APPENDIX

A. Solution for the 9 Dot Problem.



B. Solution for the 8 Coin Problem.



C. Solution for the Matchstick Arithmetic Problem.