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Amplification and Reversal of Knudsen Force by Thermoelectric Heating

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Abstract. We show that the Knudsen thermal force generated by a thermally-induced flow over a heated beam near a colder wall could be amplified significantly by thermoelectric heating. Bidirectional actuation is achieved by switching the polarity of the thermoelectric device bias voltage. The measurements of the resulting thermal forces at different rarefaction regimes, realized by changing geometry and gas pressure, are done using torsional microbalance. The repulsive or attractive forces between a thermoelectrically heated or cooled plate and a substrate are shown to be up to an order of magnitude larger than for previously studied configurations and heating methods due to favorable coupling of two thermal gradients. The amplification and reversal of the Knudsen force is confirmed by numerical solution of the Boltzmann-ESBGK kinetic model equation. Because of the favorable scaling with decreasing system size, the Knudsen force with thermoelectric heating offers a novel actuation and sensing mechanism for nano/microsystems.

INTRODUCTION

Thermally-induced rarefied gas flows are manifested in various unexpected physical phenomena such as the thermal transpiration in a capillary from cold to warm reservoirs and thermal levitation of liquid droplets in Leidenfrost ratchets[1]. Heated objects immersed in a sufficiently rarefied gas experience the so-called Knudsen thermal force. For instance, a heated beam near a colder substrate would be repulsed from it, if the length scale of temperature gradient is comparable to the molecular mean free path in the surrounding gas. Such conditions arise in high vacuum for macroscale objects or for submicron-sized structures at atmospheric pressures. Also referred to as the radiometric force, the phenomenon has been known since late 19th century when Crookes introduced a radiometer[2], a set of thin vanes on a spindle which rotate when light shines on them. Initially misattributed to radiation pressure, Maxwell[3] was the first to explain the effect by consideration of non-equilibrium thermal stresses in the surrounding gas. In 1960-70s Knudsen thermal force effect was extensively studied experimentally because of its interference with vacuum microbalance measurements[4].

Recently, there has been a renewed interest in the Knudsen thermal force due to potential applications for actuation and sensing in nano/microsystems[5]. The gas-solid surface molecular momentum exchange is intrinsically a surface phenomena and thus the relative magnitude of Knudsen force scales favorably with decreasing size. Although to date there are no practical micro devices exploiting this effect, a few recent studies demonstrated feasibility of generating Knudsen force at the microscale. Passian et al [6] measured deflection due to the Knudsen thermal force of silicon microcantilevers under focused laser heating. Knudsen force effect, although undesirable, also occurs in heated atomic force microscopy (AFM) measurements[7].

EXPERIMENTAL SETUP

In this paper, we demonstrate significant enhancement of magnitude and reversal of the Knudsen force direction, from repulsive to attractive. We use a thermoelectrically heated plate (Peltier device) and measure the resulting force by a torsional microbalance at different ambient gas pressures. The numerical modeling based on the solution of the Boltzmann kinetic model equation for the experimental conditions show that an interplay between two thermal gradients, within the heated plate and between the plate and the substrate, leads to a significant increase of the Knudsen force magnitude as compared to other heating methods.
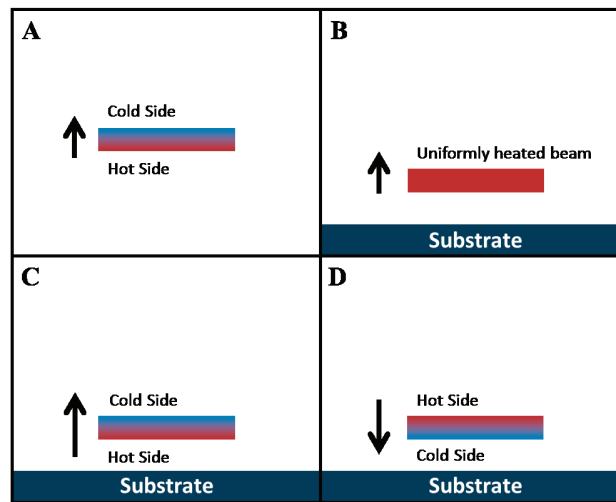
FIGURE 1. Different methods of producing Knudsen force: A) Unequally heated plate far from solid boundaries as in Crooke's radiometer; B) Uniformly heated plate close to a colder substrate; C) Unequally heated plate with hottest side facing towards the substrate referred to as positive thermal gradient; D) Unequally heated plate with hottest side pointing away from the substrate referred to as negative thermal gradient.

Seen in Fig. 1 are different methods for producing the Knudsen thermal force on a flat plate. Fig. 1 shows an unequally heated beam immersed in a quiescent ambient and placed far from solid boundaries. This configuration is similar to a single radiometer vane[8]. The uniformly heated beam near a substrate as shown in Fig. 1B closely resembles the setup in Passian's heated microcantilever experiments[6]. Note that this configuration would be typical for both resistive or laser heating of silicon microstructures because of temperature uniformity due to the small Biot number at the micro scale. Thermoelectrically heated beam near a substrate under two different voltage bias conditions is shown schematically in Fig. 1 C and D. The case in Fig. 1C has the hot side of the beam facing the substrate and is referred here as positive thermal gradient where as D with the colder side facing the substrate being referred to as negative thermal gradient. We measure the force and model the thermally-induced flow for configurations C and D and compare it to published data for cases A and B.

The experiment employed TEC1-12706 40x40 mm Peltier plate device with a thickness of 3.6 mm. Force was measured using a micro-Newton torsional balance similar to that in [9]. The balance incorporates top and bottom C-flex pivot bearings for motion control, and a Schaevitz HR-050 linear variable differential transformer for deflection measurement[10]. It is mounted in a 4.1 m^3 vacuum chamber where the pressure is measured by a Baratron capacitive pressure gauge. Calibration is done via an electrostatic fin assembly which produced forces, with repeatability determined as standard deviation over average, within 3.6% at 9 μN and less than 1.8 % at 32 μN as measured by a Fisher Scientific model XA-200 precision analytic scale. The Peltier device was mounted inside of the vacuum chamber parallel to an aluminum reaction plate attached to the arm of the torsion pendulum. The gap distance between the Peltier device was measured using a precision translation stage.

The Peltier device was tested at both polarities to create both positive and negative thermal gradients while the deflection history of the pendulum arm was measured. An example of deflection data under positive and negative thermal gradients can be seen in Fig. 3. The temperature history of the Peltier device was recorded using two K-type thermocouples. When the current was reversed, the sides of the faces of the device achieved approximately equal temperatures on the opposite faces under different polarities as can be seen in 2. It was found that under positive and negative polarities the magnitude of the thermal gradient was the same at a value of approximately 16 K. The temperature history was similar when tested at pressures ranging from 1 to 100 Pa, underlying that the radiative heat losses were dominant. It was found that the temperature history under a positive and negative temperature gradiant experienced nearly identical trends and magnitudes. An example of the temperature history can be seen in 2 and is representative of both positive and negative polarity. Temperature measurements of the substrate revealed a consistent heating of approximately 3 K. A period of time of about 15 minutes was required for the Peltier device to reach a steady temperature. The force measurement were taken at the highest magnitude of deflection to account for this.
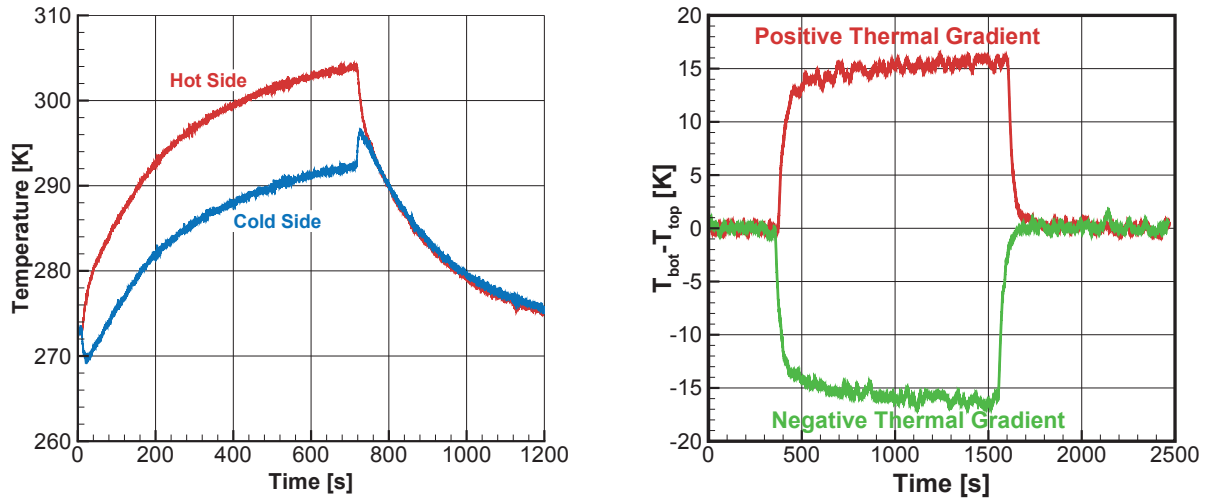
FIGURE 2. Left: Temperature history for both sides of Peltier device. Measurements taken at 10 Pascals. Right: Temperature difference between the top and bottom sides of the Peltier device for positive and negative polarities. Measurements taken at 10 Pascals.
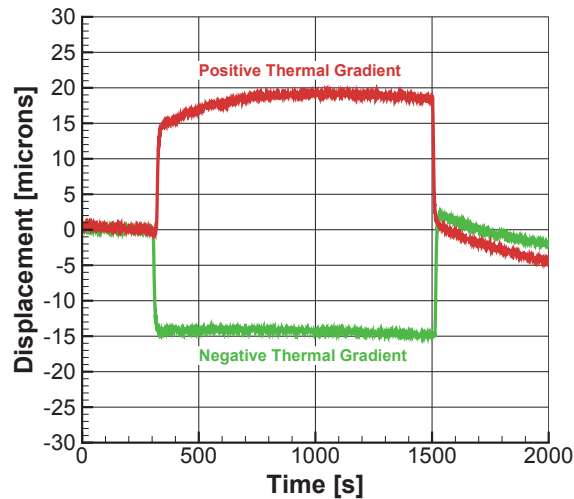

FIGURE 3. Deflection of beam for positive and negative thermal gradients. Measurements taken at 10 Pascals.

EXPERIMENTAL RESULTS

An important dimensionless quantity used to compare flow phenomenom is the Knudsen Number and is used in a way analogous to Reynolds number in continuum fluid dynamics. It is a good measure of the rarefaction of a gas. Knudsen number is simply the ratio of the mean free path λ of a particle to a characteristic length L. For simple hard sphere molecules it can be expressed as

$$Kn = \frac{\lambda}{\text{Ł}} = \frac{kT}{\sqrt{2}\pi d_{ref}^2 P (\frac{T_{ref}}{T})^{\omega-0.5}} \tag{1}$$

Where k is the Boltzmann Constant, d_{ref} is the reference hard shell molecular diameter of the gas at a specific reference temperature T_{ref}, P is the pressure of the gas, T is the temperature of the beam and ω is viscosity index of the gas. Knudsen numbers in this experiment ranged from the continuum regime to the rarefied regime.

The measured Knudsen force for positive and negative thermal gradients at multiple gap distances are plotted in Fig. 4 and Fig. 5 as a function of ambient pressure. Negative Knudsen force is observed when the thermoelectric device is in the negative thermal gradient configuration as is shown schematically in Fig. 1D. Note that the maximum magnitude of the force for both heating methods is observed at a pressure range from about 5 to 15 Pa, which corresponds to the Knudsen numbers in the transitional regime, from Kn=0.51 to 0.17, respectively. The peak magnitude of the force for direct heating (repulsive) is about 55 μN. The observed peak attractive force for a 2-mm gap under reverse thermoelectric bias was about 29 μN. For comparison, the peak radiometric force as in case Fig. 1A for the same area and temperature difference ($T_{bot} - T_{hot} = 15$ K would be 4.2 μN (see Fig. 7 of [11]). The observed force on the thermoelectrically heated beam is also much larger than that for a uniform heating case as in Fig. 1B. Using Equation (4) from [12] with a temperature difference between the beam and the substrate of 35 K, the force on the uniformly heated beam would be about 32 μN at a pressure of 10 Pa and a gap distance of 1mm.

More experimental data is plotted in Fig. 6 and Fig. 7 as a function of Pressure. Force reversal is present with a reversal of the thermal gradient. Additionally there is an increase in force magnitude compared to the uniform heating seen in [12]. As can be see with increasing gap distance there is a decrease in the maximum force measurement and the Knudsen number at which that occurs. For small gap distances of 1.0mm as seen in Fig. 6 the force readings initially follow the upward slope of equivalent uniform heating case but ultimately have a higher local maximum. With increasing gap size the thermoelectric cases begins to diverge from the equivalent uniform heating cases as seen in Fig. 7.
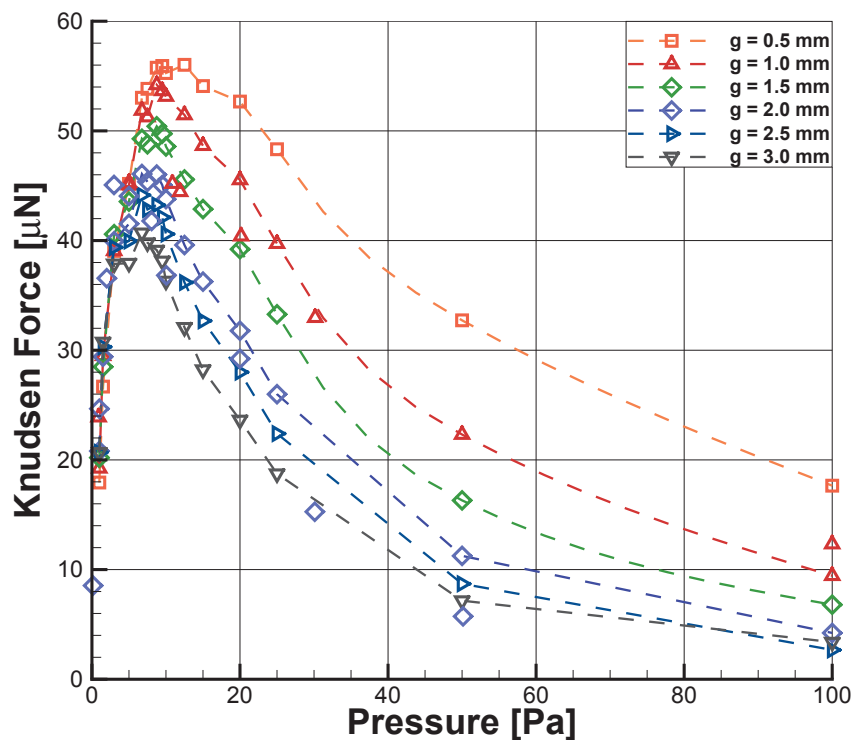

FIGURE 4. Experimental measurements of Knudsen force at different gap lengths with positive thermal gradient.
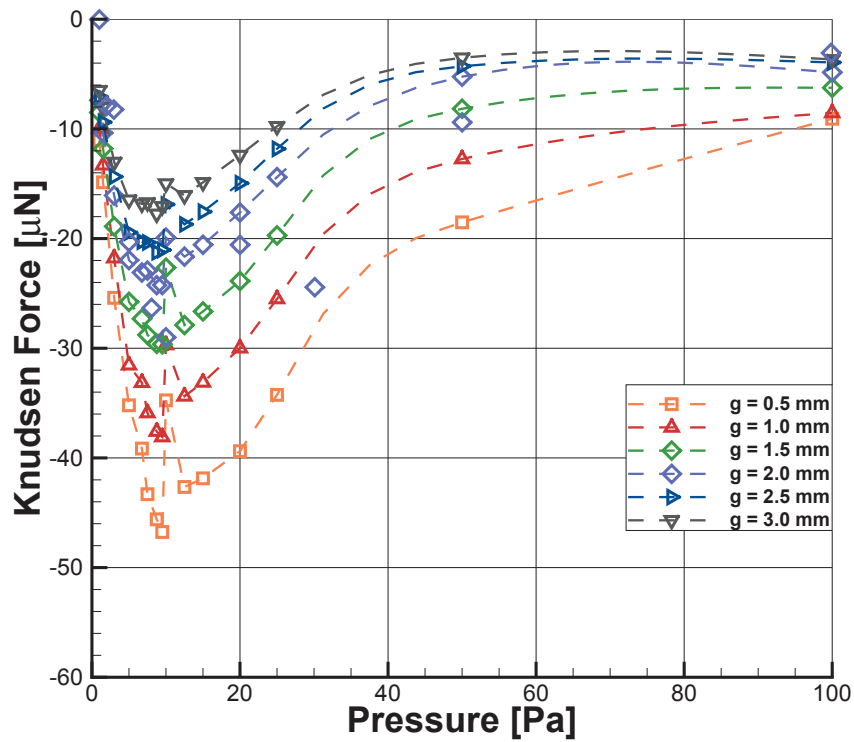
FIGURE 5. Experimental measurements of Knudsen force at different gap lengths with a negative thermal gradients.
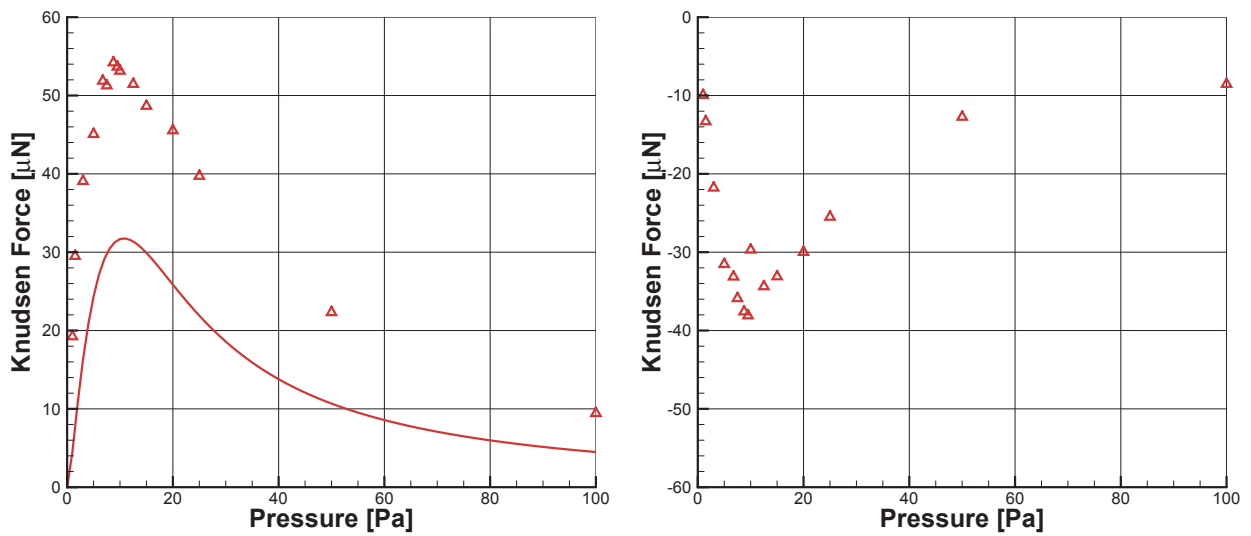

FIGURE 6. Experimental measurements of Knudsen force at a gap distance of 1.0 mm under thermoelectric heating. Left: Positive thermal gradient(markers) with solid curve representing theoretical uniform heating from [12] calculated with a temperature difference of 35 k. Right: Negative thermal gradient.
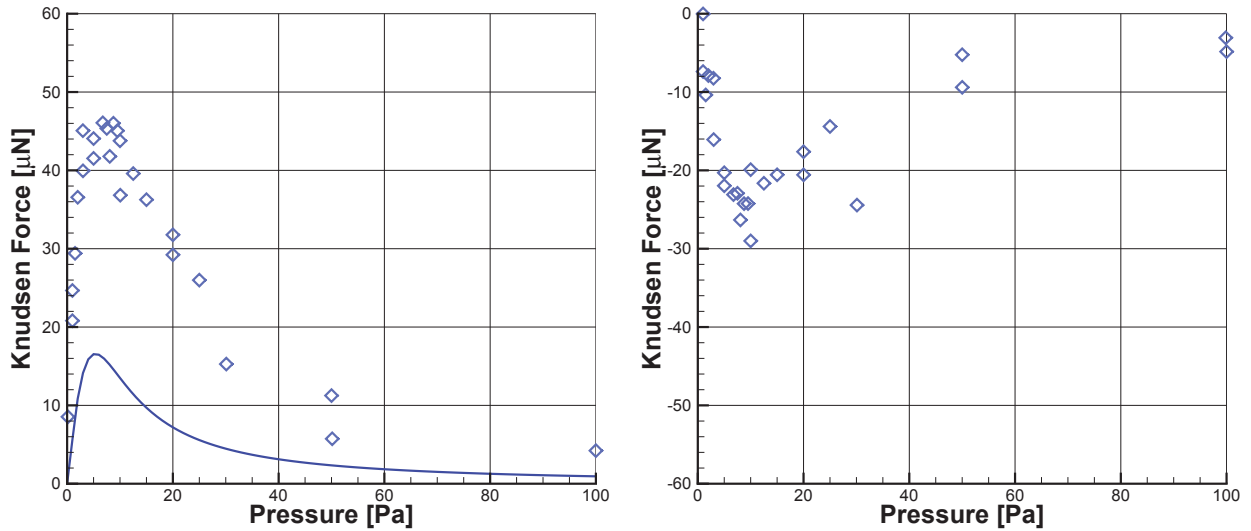
FIGURE 7. Experimental measurements of Knudsen force at a gap distance of 2.0 mm thermoelectric heating. Left: Positive thermal gradient(markers) with solid curve representing theoretical uniform heating from [12] calculated with a temperature difference of 35 k. Right: Negative thermal gradient.

NUMERICAL SIMULATION

To elucidate the observed amplification and reversal of the Knudsen force under thermoelectric heating near a substrate, we perform numerical simulations of the resulting thermally-induced rarefied flows.

Simulations are based on numerical solution of the Boltzmann kinetic equation for the velocity function $f = f(t, x, y, u, v, w)$. Here we solve the 2D quasi-steady kinetic equation with the ellipsoidal-statistical Bhatnagnar-Gross-Krook (ES-BGK) approximation given by:

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} = \nu(f - f_\gamma) \tag{2}$$

where ν is the collision frequency and f_γ is the Gaussian distribution function.

The Knudsen force on the beam is calculated by integrating the normal stress, $P_{yy} = mn \iiint (v')^2 f\, du dv dw$, along the width of the cantilever and a shear stress, $P_{xy} = mn \iiint u'v' f\, du dv dw$, along its thickness. Here m and n are molecular mass and number density, and $u' = u - \bar{u}$ and $v' = v - \bar{v}$ are the molecular thermal velocity components.

The beam geometry in the simulations corresponds to a 2D cross-section of the Peltier device at a distance of 1 mm from the substrate. We used $T_{wall} = T_{ambient} = 300K$ in the simulations. The hot side of the beam is assumed to be 330K. Only the half of the domain is computed to save the computational cost using the symmetry boundary condition at the center. A 100x100 spatial mesh with a 14x14x14 velocity mesh was used to achieve convergence in Knudsen force magnitude within 1%.

The computed temperature fields and streamlines are shown in Fig. 8, 9 and 10 for different heating methods. The arrows on the figures are proportional to the produced Knudsen force magnitude. Vortices are observed at the corners of the beams, with their size increasing with the Knudsen number. Note that the vortex direction is reversed with switching the direction of the heat flux through the beam. The magnitude of Knudsen force increases greatly for Peltier thermoelectric heating cases as compared with the uniform heating. It is explained by the pressure profiles along the beam on both surfaces shown in Fig. 8, 9 and 10.

It is believed that Knudsen force is a combination of momentum transfer between the plates and colliding molecules and shear effects on the edges when exposed to a thermal gradient. When the beam was exposed to the negative thermal gradient in this experiment the low temperature side facing the wall is still at a higher temperature than the substrate. Thus there is a momentum transfer between molecules and the plates that creates a force in the direction away from the substrate. However in this configuration there are also the shear effects that produce a force directed towards the
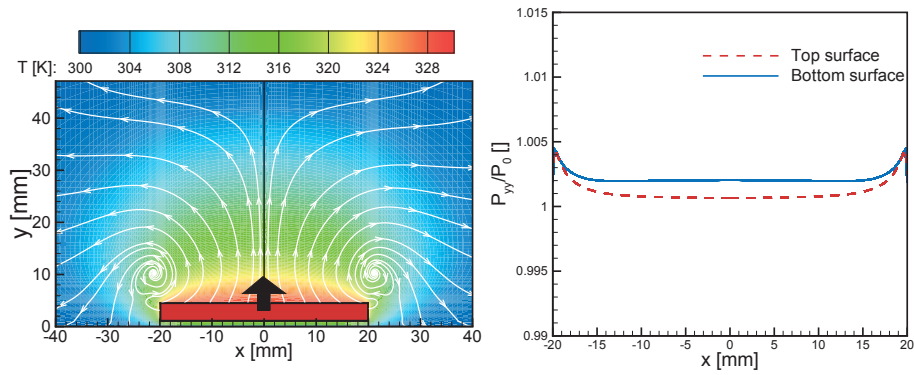
FIGURE 8. Uniform heating at a Knudsen number of Kn=0.55. Left: Computed temperature fields and streamlines. Right: Normal stress distribution of top and bottom of beam.
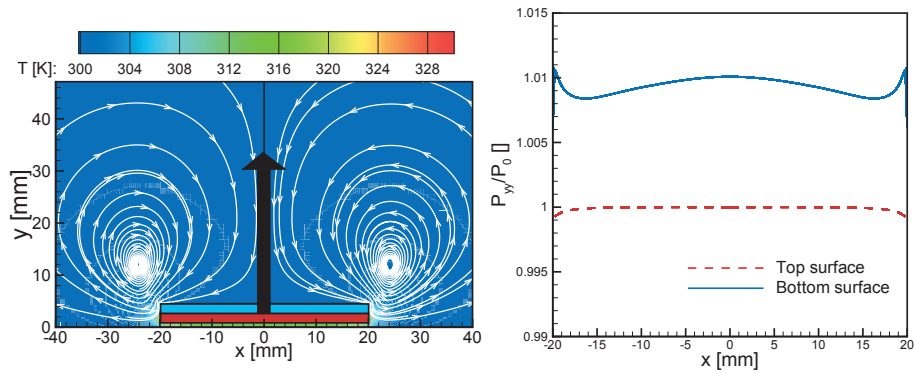

FIGURE 9. Thermoelectrically heated - positive thermal gradient at a Knudsen number of Kn=0.55. Left: Computed temperature fields and streamlines. Right: Normal stress distribution of top and bottom of beam.
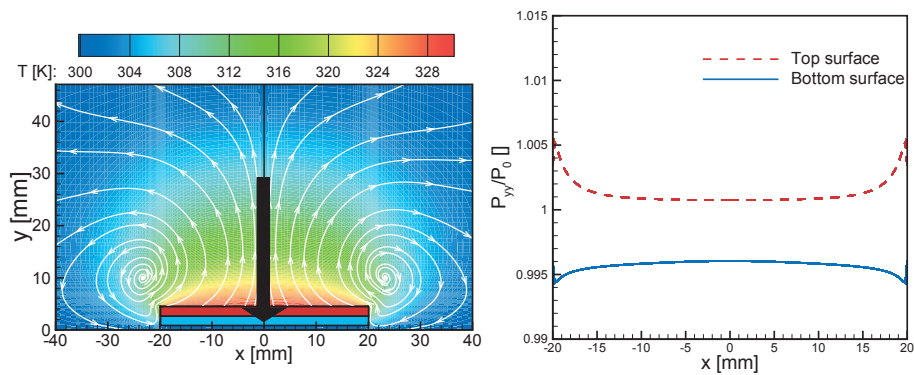

FIGURE 10. Thermoelectrically heated - negative thermal gradient at a Knudsen number of Kn=0.55. Left: Computed temperature fields and streamlines. Right: Normal stress distribution of top and bottom of beam.

substrate. These forces are in opposite directions but the shearing effects are thought to be more powerful. In a positive thermal gradient the shearing effects and momentum transfer act in the same direction and thus produce a force that has a higher magnitude of than the negative thermal gradient.

CONCLUSIONS

In summary, both the experimental measurements and modeling show large amplification of the Knudsen force generated on an unequally heated structure near a substrate. A bidirectional actuation with both attractive and repulsive forces is realized for the same geometric configuration by reversing polarity of thermoelectric element. The thermoelectrically actuation is well controllable with a favorable short response time as compared to other heating methods. The force magnitude increases with reduced characteristic size and offers a novel actuation and sensing mechanism for micro/nanosystems.

ACKNOWLEDGMENTS

REFERENCES

1. A. Würger, *Physical Review Letters* **107**, 164502 (2011).
2. W. Crookes, *Philosophical transactions of the Royal society of London* **164**, 501–527 (1874).
3. J. C. Maxwell, *Philosophical Transactions of the royal society of London* **170**, 231–256 (1879).
4. S. K. Loyalka, *The Journal of Chemical Physics* **66**, 4935 (1977).
5. A. Ketsdever, N. Gimelshein, S. Gimelshein, and N. Selden, *Vacuum* (2012).
6. A. Passian, R. J. Warmack, T. L. Ferrell, and T. Thundat, *Physical Review Letters* **90**, 124503–1–4 (2003).
7. B. A. Nelson, and W. P. King, *Applied Scanning Probe Methods IV* pp. 251–275 (2006).
8. S. Taguchi, and K. Aoki, *Journal of Fluid Mechanics* **694**, 191–224 (2012).
9. A. Ketsdever, B. D'Souza, and R. Lee, *Journal of Propulsion and Power* **24**, 1376 (2008).
10. A. Cofer, S. Heister, and A. Alexeenko, "Improved Design and Characterization of MicroNewton Torsional Balance Thrust Stand," in *49th AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, 2013, pp. AIAA Paper 2013–3856.
11. B. M. Cornella, A. D. Ketsdever, N. E. Gimelshein, and S. F. Gimelshein, *Journal of Propulsion and Power* **28**, 831–839 (2012).
12. J. Nabeth, S. Chigullapalli, and A. A. Alexeenko, *Physical Review E* **83**, 066306 (2011).