

Evaluation of Small City Traffic Signal Network

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INTRODUCTION

Small cities have minor traffic congestion which occurs on major arterials. Primarily because of the importance of the central business district (CBD), the greatest traffic delays result from the morning, noon and evening peak flows to, from and through the city center. In addition, the points within any size urban area of highest vehicle delay are at the signalized intersections. It follows then that the places where significant improvements could be achieved are in the network of traffic signals located downtown.

One constraint in what small cities can accomplish in the way reducing CBD congestion is and will continue to be financial. As a result, physical changes in the street system are not considered until every effort is expended in improving the efficiency of the existing system. In some instances improvements can be achieved through the removal or other adjustments in the regulation of parking. The TOPICS program is aimed at optimal use of the existing street system.

Because of the financial problems of small urban areas and the installation costs of coordinated traffic control systems, it becomes essential to fully evaluate the impact and economic consequences of such an installation. This was the purpose of the research reported here. In recent years the flexible coordinated traffic signal system has been acclaimed as the ultimate in moving traffic efficiently. This is the signal system with a brain; no longer is the traffic engineer tied to a pre-timed three dial local controller. He can now have the system in effect that meets the instantaneous demand most efficiently through vehicle detection, computerization and interconnection. This is good, but can this system be used effectively and economically in the small to medium-sized city? Do the demands vary enough in the small city

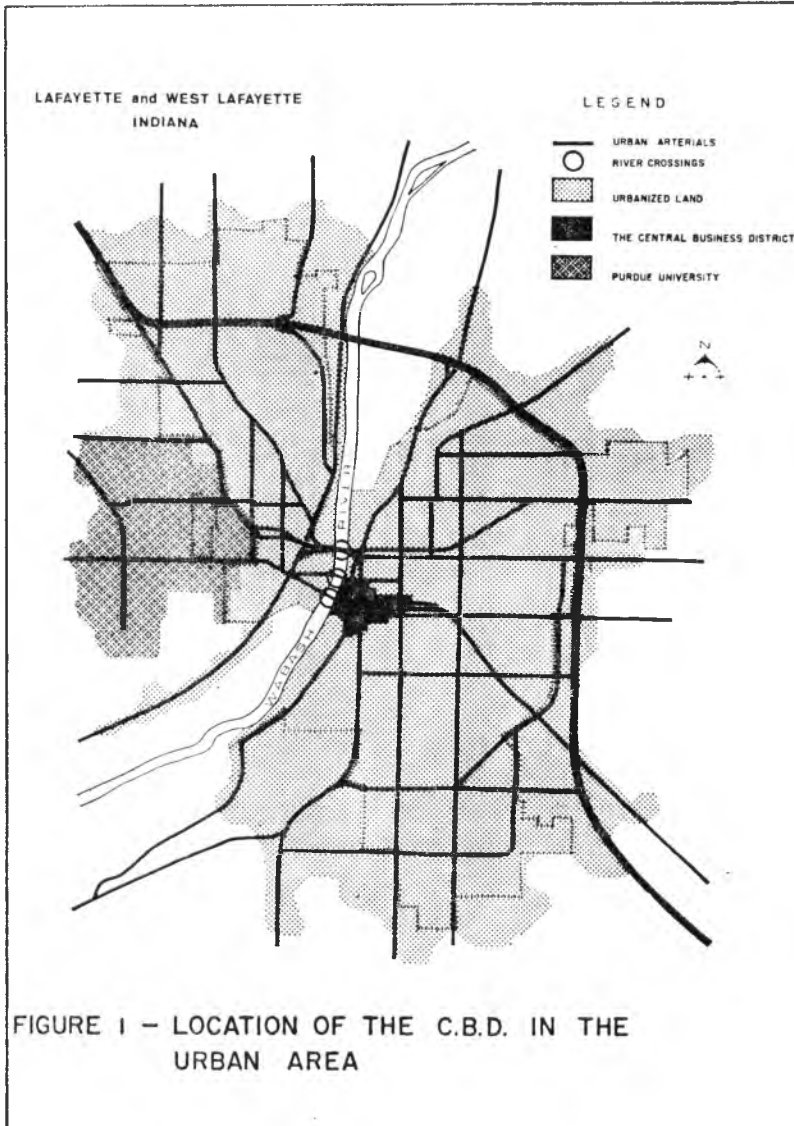
to warrant the flexibility of this new system? Do the benefits derived justify the additional expense of the flexible system? Would a less expensive system get the job done at the same efficiency? These are questions that must be answered before the policy makers can make a reasonable decision.

The purpose of this research was to determine the advantages resulting from the installation of a coordinated traffic signal control system in the CBD area of Lafayette, Indiana. See Figure 1. The Wabash River cuts through the area and only three river crossings are provided which requires that all inter-city traffic must cross the river at one of these three locations. This tends to "dump" a majority of the region's traffic in the Lafayette CBD. Many of the residents of the area reside in one city and work in the other. Work trips combined with the normal traffic and through traffic traversing the area by way of several state or federal routes which pass through the CBD, create a great deal of congestion at peak times. The CBD arterial streets are taxed to and sometimes beyond capacity causing travel times and delays to mount. Because of this, the peaks are quite high and demands on the street system vary quite extensively in the course of one day. Another factor that tends to increase congestion is the presence of very short blocks in the CBD. This part of the city was developed during the nineteenth century when establishment of blocks of short length (approximately 340 feet in this case) was common practice. Congestion is intensified during peak times when traffic at cross streets is blocked by excessively long queues.

PROCEDURE

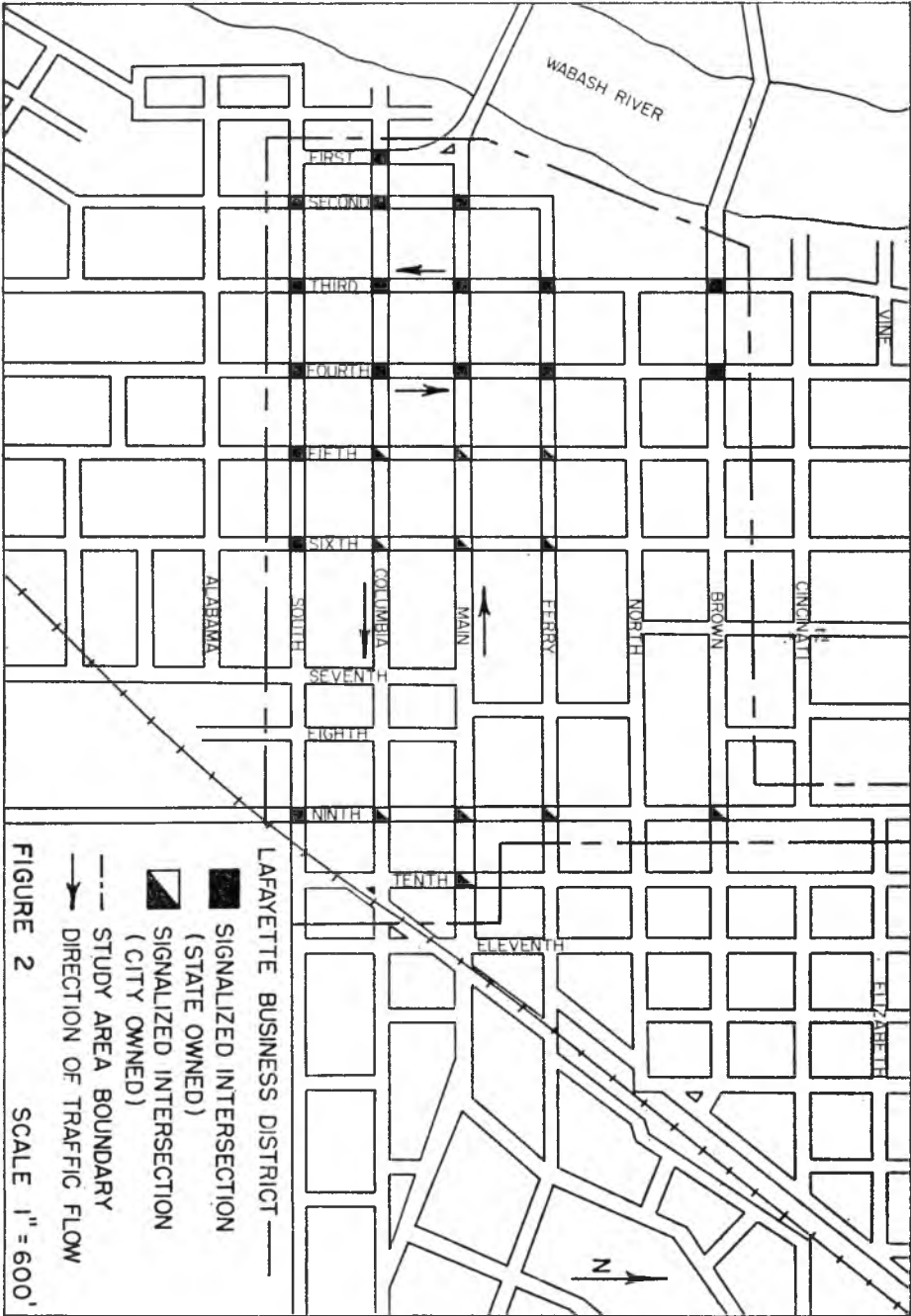
A before and after study was to be performed as a means of evaluating the coordinated signal system. The flexible coordinated system was to be compared with less costly coordinated and uncoordinated pre-timed systems. A good indicator of a system's relative value is reflected in the time required to traverse a given route through the system under varying conditions of volume. If travel time data are collected for each mode of operation for the same route and the same volumes certain conclusions as to the relative worth of each system under all conditions can be drawn. Travel time runs were to be conducted under the same conditions for each of these four modes of signal operation:

1. Uncoordinated random signal settings
2. Coordinated simultaneous signal settings
3. Coordinated single alternate signal settings
4. Flexible coordinated signal settings



An analysis will be performed using these data to determine which of these systems operates most efficiently at different volume levels. Finally benefits realized by the flexible system in monetary terms will be determined.

The study area is shown in Figure 2. The east-west arterials are two one-way couplets; Main and Columbia Streets which pass through



the heart of the CBD and Union and Salem Streets which lie on its fringe. Signal operation on Union and Salem Streets is not effected by the new installation. The study area was naturally limited by the extent of the new system; therefore, Main and Columbia Streets were the obvious choice on which to conduct the travel time runs. The study area includes the CBD and its fringe area encompassing a total of 30 signalized intersections, 14 being city owned and 16 on state roads, therefore state owned (Figure 2).

The north-south arterials are the one-way couplet of Third and Fourth Streets running through the heart of the CBD, and Ninth Street, a two-way street on the fringe. Third and Fourth Streets were chosen to make runs, in order to check the effect of the systems on cross traffic. Runs were not made in this direction for modes 1, 2, and 3 at the time this project was started in 1963. The before condition could not be reproduced in 1970; therefore, there is no check on cross traffic for these modes. However, runs were made in the north-south direction for the flexible coordinated mode. This information can be used as the before data at a later time if improvements are made to the system.

TRAFFIC SIGNAL SYSTEMS

Random

The signal system being utilized prior to the installation of the new system employed a random mode of operation, wherein each individual intersection was controlled independently by its own local controller with no attempt being made to coordinate operations. With this type of operation, signal timing is tailored to the conditions at the individual intersections. Independent operation may be desirable for isolated locations where the timing of the signal does not effect adjacent signals, but in general, it is undesirable in highly signalized areas such as the CBD. Random signal timing is characterized by numerous stops and delays, and low overall travel speeds. This is the basic condition and is referred to as mode 1 throughout the study.

Simultaneous

The second system (mode 2) utilizes a simultaneous mode of operation. With this method of signal timing, all traffic signals in the system change color indications at the same point in time, with all indications along a given route being the same with the cross streets showing the opposite color. It requires that all signals have the same cycle length and also requires that the same split be used at all loca-

tions. Simultaneous operation is characterized by high speed between stops, but low over-all speeds. It is also thought to be more efficient at high volumes (near capacity) than other modes.

Alternate

The third system has every other signal showing the same indication. As with system 2, it is fully coordinated but pretimed and not capable of responding to varying traffic demand. This system requires a common cycle length and a common split and is designated as mode 3.

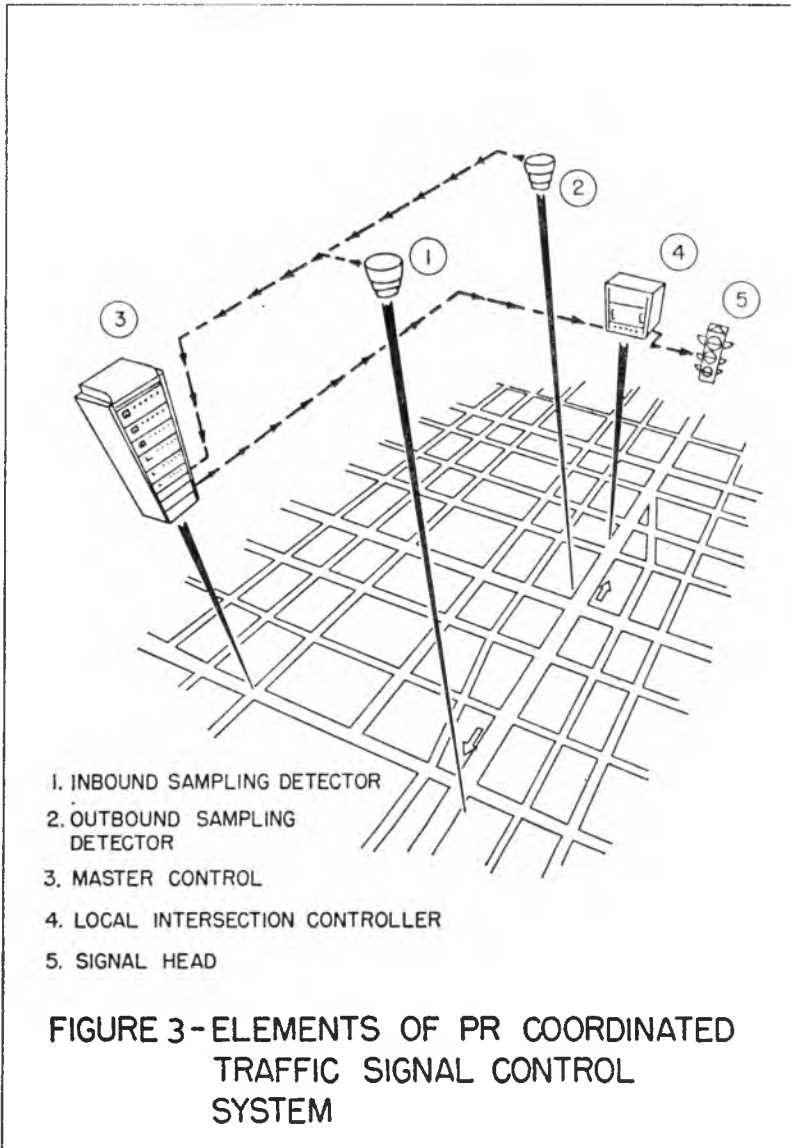
Flexible

The fourth system, the object of this study, and the system in operation today in Lafayette, is a flexible coordinated system. It was installed in 1963 and utilizes Electromatic PR Coordinated Traffic Control equipment. This equipment is quite flexible in operation, but as all equipment of this type its efficiency is dependent to a great extent upon the skill of the initial programmer and the data available to him.

The system consists of four basic components: detection, master control, local control, and interconnection. A schematic diagram of the operation is shown in Figure 3. The obvious advantage that this setup has over the other systems is its ability to monitor traffic conditions and to adjust to the conditions present on the street system at any given time. Sampling detectors at representative locations in the signal system area continuously provide the master controller with information about traffic performance.

Various combinations of six-cycle lengths, five offsets, and three-cycle splits can be specified for anticipated conditions of traffic flow. They are put into effect automatically as operating instructions to the local controllers by the master control in accordance with measured changes in traffic demand.

Proper location of sampling detectors is essential for a vehicle responsive system such as this. Detectors should be located with reference to the major traffic flow with which they are associated. A detector for outbound traffic measurement should be located where it will show a definite outbound movement (1). When possible it is desirable to have duplicate sampling points to compensate for the effect of circuit failure, street repairs, accidents, or other interruptions at any one sampling location. Detectors should be located on free flowing streets far enough from the system to give the master control ample time to adjust for varying levels of detection. Free flowing streets are required since a false indication of light traffic could be interpreted on a street that experienced much congestion and slow moving or



stopped traffic. The Lafayette system employs radar detectors at the locations shown in Figure 4.

DATA COLLECTION

The data consist of travel time runs over the range of volumes within the system for each of the four modes of signal operation, and

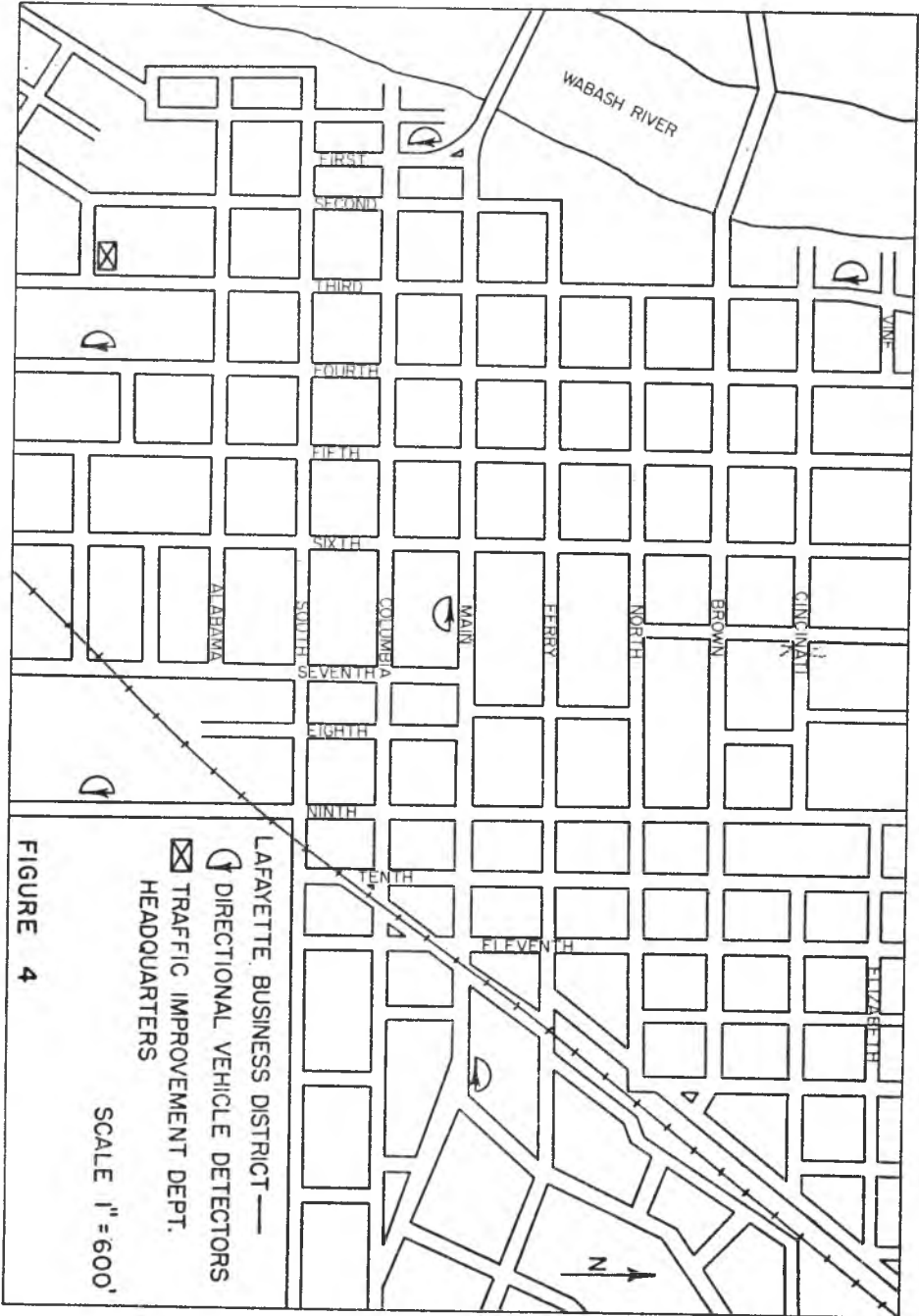
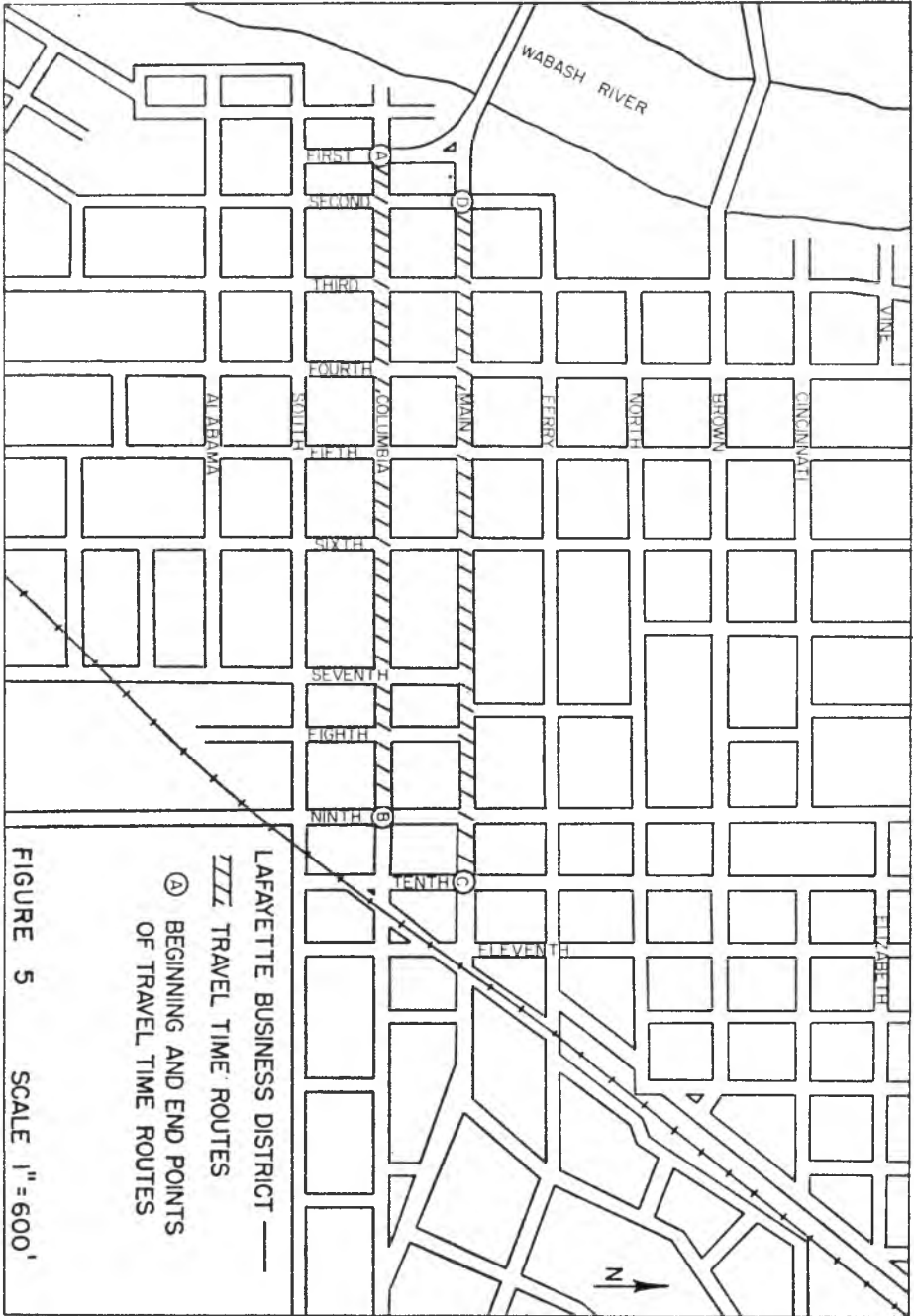


FIGURE 4

SCALE 1" = 600'



volumes collected simultaneous to the travel time. Spot speeds were also taken at various points within the system.

The east-west route chosen for the runs is shown in Figure 5. The route consists of Main Street (westbound) and Columbia Street (eastbound). These streets were used because they pass through the study area and they are the most heavily traveled arterials of the area.

The "average car" method was used in the collection of travel time data since for a given sample size this method was found to produce better statistical results than other methods when used on heavily traveled multi-lane streets (5). At least 50 runs were made on each segment of the route, as this was found to be the sample size required to achieve five percent accuracy in determining travel times for multi-lane congested streets (Table 1). Times at which runs were to be made were determined in such a way that all classes over the entire range of volumes would be represented in the final sample. This included peak times, off-peak times, and times on weekdays and weekends. The runs were also conducted at approximately the same times for each mode so that travel conditions would be made as nearly the same for all modes as possible, so as not to bias the results with runs made under different conditions for different modes.

TABLE 1.

Samples Needed for Determining Mean Over-All Speeds on Selected Test Sections Within Different Limits for 95 Percent Degree of Confidence			
<i>Test Section</i>	<i>License-Check Studies</i>	<i>Test-Car Runs</i>	
	<i>Sample Size for 5 Percent Accuracy</i>	<i>Number Needed for Accuracy of</i>	
		<i>5 Percent</i>	<i>10 Percent</i>
Signalized Urban Streets			
1. Two-lane, uncongested	32	30	8
2. Two-lane, congested	36	40	10
3. Multi-lane, uncongested	80	18	5
4. Multi-lane, congested	102	50	13
Rural Sections			
5. Two-lane, 1130 VPH	25	25	6
6. Two-lane, 1440 VPH	41	42	11
7. Four-lane, uncongested	30	—	—

Source: Reference No. 5

Note: Five percent accuracy refers to values being within five percent of the true mean speed.

Continuous manual volume counts for one-minute intervals were recorded simultaneous to travel-time runs on each route so that volumes and travel times for the individual segments of the routes could be correlated. Also pneumatic tube vehicle counters were employed at the same location so that a check on volumes could be obtained. Synchronized watches were used at all times by all members of the data collection team.

Spot speeds were also checked at various locations throughout the system for the flexible mode. These checks were made along lengthy unsignalized sections of the system to determine the average running speed of the traffic. A radar meter was employed to take a sample of at least 100 speeds during off-peak conditions. For each of the runs, data were collected and recorded as volume, total travel time, delay time, running time, stops per run and average time per stop.

DATA ANALYSIS

Analysis of Variance

An analysis of variance (ANOVA) is based upon a separation of the variation of all the observations into parts, each of which measures variability attributable to some specific source (2). The purpose of this sample variance breakdown is a comparison of the means of each population of the analysis.

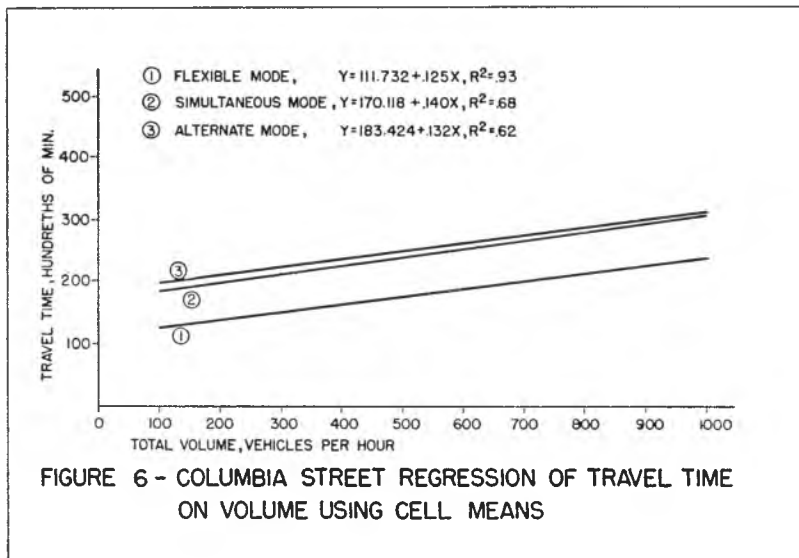
For the results obtained from the ANOVA to be meaningful certain basic assumptions should be met. The F-test (the test performed to determine significance of main effects and interactions) performed in the ANOVA assumes homogeneous variances among the several populations; therefore, variances of the variables listed above in "Data Collection" were tested for homogeneity. This check was done on a CDC 6500 computer utilizing the Datasum program (3). Datasum is a library program that summarizes data and computes various statistics from the data. Two homogeneity of variance tests (Bartlett's test and the Foster-Burr test) are also performed.

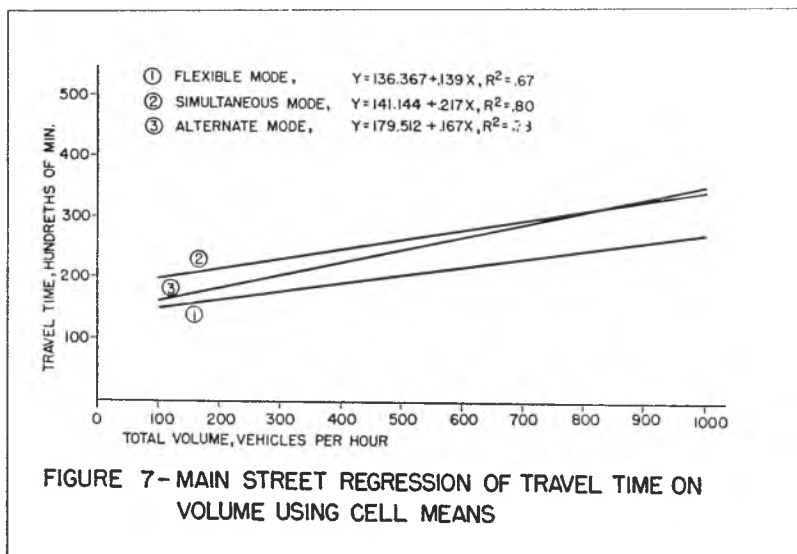
Using the chi-square values resulting from Bartlett's test, three of the six variables were shown to have heterogeneous variances. Several data transformations were used with no significant improvement in the test result shown.

Inspection of the individual cell variances showed the variances of cells within the random mode to be much more variable than the remaining data. This, combined with the presence of empty cells at the higher volume levels and undesirable effects upon the initial ANOVA, resulted in the decision to exclude the random mode from further con-

sideration in the analysis. This will not adversely affect the final outcome since a random mode of operation utilized in the CBD would undoubtedly yield the highest motorist costs. All the variables displayed homogeneous variances at the level $\alpha = 0.001$ except delay time and time per stop; however, of these two variables, only delay time is used in the economic analysis. In spite of this lack of variance homogeneity for the variable delay time, the result of its ANOVA is considered to be reliable. Due to the robust nature of the ANOVA it is capable of withstanding quite a degree of heteroscedasticity (4). The ANOVA was performed using the Purdue University library program Unequal (6).

An insignificant interaction term was exhibited in all cases, which shows that modes retain their relative rank across the full range of volumes. At no point within the range of volumes does either the simultaneous or the alternate mode give better results than the flexible mode. A graphical representation of insignificant interaction is shown in Figures 6 and 7. These graphs were obtained by plotting the results of a simple linear regression on the cell means for the variables travel time and volume. The same result could have been obtained using any one of the five variables in the analysis. Interaction in the ANOVA would be depicted as intersection points on the lines of the graphs. The intersection near the upper limit on the Main Street graph is not strong enough to produce a significant effect.





The fact that no significant interaction exists makes it possible that the tests for differences in means be made using grand means for each mode and not individual cell means. Since there is an unequal number of observations in each cell, a method that compensates for unequal cell sizes must be used to test the means. The method developed by Henry Scheffe (7) is well suited to this problem. All pairs of means for the independent variables (delay time, running time, and stops) were tested by this method. This procedure showed the flexible means for each variable to be significantly lower than the alternate and simultaneous means at the level $\alpha = 0.05$ in every case (Table 2).

Economic Analysis

The initial step of the economic analysis is identification of the costs involved. Here the concern is the total cost of making a run through the signal system, so that a comparison of the costs for the different modes of operation may be obtained.

The operation of a vehicle incurs two basic costs, these being the cost of operating the vehicle and the value placed upon the motorist's time consumed while operating the vehicle. The operating cost of the vehicle can be further divided into fixed and variable costs. Fixed costs remain constant whether the vehicle is used or left idle; examples are depreciation and insurance. Variable costs include expenses for gasoline, tires, and motor oil. These costs are dependent upon the amount of vehicle use, speed of travel, type of road surface, and other factors.

TABLE 2.
Results of Scheffe Test for Differences in Means^a

Street	Variable	Hypothesis Tested	Difference in Means	Test Value @ $\alpha = 0.05$	Result ^b
Columbia	Delay Time $s^2 = 1121.84$	$\bar{Y}_A = \bar{Y}_S$	3.52	8.20	I
		$\bar{Y}_S = \bar{Y}_F$	65.07	8.11	S
		$\bar{Y}_A = \bar{Y}_F$	61.55	7.27	S
	Running Time $s^2 = 361.64$	$\bar{Y}_A = \bar{Y}_S$	6.73	4.66	S
		$\bar{Y}_S = \bar{Y}_F$	7.04	4.57	S
		$\bar{Y}_A = \bar{Y}_F$	14.77	4.13	S
	Stops $s^2 = 1.04$	$\bar{Y}_A = \bar{Y}_S$	0.73	0.25	S
		$\bar{Y}_S = \bar{Y}_F$	1.19	0.25	S
		$\bar{Y}_A = \bar{Y}_F$	1.92	0.22	S
Main	Delay Time $s^2 = 1998.06$	$\bar{Y}_A = \bar{Y}_S$	5.75	10.77	I
		$\bar{Y}_S = \bar{Y}_F$	30.07	7.78	S
		$\bar{Y}_A = \bar{Y}_F$	35.82	10.89	S
	Running Time $s^2 = 659.10$	$\bar{Y}_A = \bar{Y}_S$	2.85	6.19	I
		$\bar{Y}_S = \bar{Y}_F$	28.44	5.62	S
		$\bar{Y}_A = \bar{Y}_F$	25.59	6.26	S
	Stops $s^2 = 1.01$	$\bar{Y}_A = \bar{Y}_S$	0.43	0.25	S
		$\bar{Y}_S = \bar{Y}_F$	1.41	0.22	S
		$\bar{Y}_A = \bar{Y}_F$	1.84	0.25	S

^a Subscript code: S, simultaneous mode; A, alternate mode; F, flexible mode.

^b I, insignificant differences in means; S, significant differences in means.

It is apparent that the total vehicle cost for making a run through the signal system consists of the cost of operating a vehicle at a given speed, the additional cost incurred by making stops, the added cost of delays, and the value placed upon the motorist's time.

Since the variables in the statistical analysis were shown to have insignificant interaction terms, and the Scheffe test shows significant differences in the means, the grand mean for each variable for the different modes was used to develop an economic equation for computing the cost per run for the different signal modes and travel time routes (Table 3). Using the independent variables; running time, delay time and stops, the following equation was developed:

$$CR_{ij} = RT_{ij}(A_1 + A_4) + DT_{ij}(A_2 + A_4) + S_{ij}(A_3)$$

where:

- CR_{ij} Total cost to motorist per run through signal system on route i and mode j .
- RT_{ij} Average running time per run required for route i and mode j .
- DT_{ij} Average delay time experienced for route i and mode j .
- S_{ij} Average number of stops made per run for route i and mode j .
- A_1 Operating cost for standard vehicle running on high type pavement at 20 mph.
- A_2 Idling cost of standard vehicle during delay.
- A_3 Excess cost incurred by bringing standard vehicle to stop from 20 mph, then accelerating back to 20 mph.
- A_4 Value placed on average motorist's time.

TABLE 3.

Over-All Means Obtained From the Statistical Analysis^a

MAIN STREET

<i>Signal Mode</i>	<i>Delay Time</i>	<i>Running Time</i>	<i>Stops Per Run</i>
Simultaneous	90.79	205.75	2.95
Alternate	96.54	202.90	3.38
Flexible	60.72	177.31	1.54
Standard Error	44.60	25.60	1.02

COLUMBIA STREET

<i>Signal Mode</i>	<i>Delay Time</i>	<i>Running Time</i>	<i>Stops Per Run</i>
Simultaneous	82.93	175.46	2.38
Alternate	79.41	182.19	3.11
Flexible	17.86	168.42	1.19
Standard Error	33.40	19.00	1.00

^a The units applicable to table values are one hundredths of a minute; to obtain time in seconds these values must be multiplied by 0.60. Stops per run have the appropriate units as written.

By checking spot speeds within the system, the average running speed was found to be approximately 20 miles per hour, therefore, this speed was used for determining operating costs. Values determined by Winfrey (8) were used for the constants A_1 , A_2 , and A_3 . Winfrey bases these costs upon an average vehicle that most closely represents the entire range of vehicles using the street system. This vehicle is thought to be representative of the range of passenger cars in use today.

Substitution of the appropriate values into the economic equation yielded the cost per run for the different signal modes and travel time routes. This cost was then expanded in each case to an annual cost by applying the associated yearly volumes for the years 1963 through 1970. Average volumes were known for the years of 1963 and 1970. In order to obtain volumes for the intervening years a linear growth was assumed, with no attempt being made to determine a growth rate factor since the time period was short and the growth was relatively small. The results of these calculations shown in Table 4 show the flexible mode produces significantly lower costs to the road-user than the simultaneous or alternate modes. The simultaneous mode is also shown to be an improvement over the alternate mode. Therefore, in order to be conservative the economic evaluation was done relative to the simultaneous system.

The money invested in the installation of the flexible system (approximately 200,000 dollars) could conceivably have been invested in business or deposited in savings; in both cases a profit in the form of interest paid on the investment would have most likely resulted. Therefore, each year's savings cannot be used for analytical purposes until an appropriate interest rate has been applied, and the worth of each year's saving is established for the base year of 1963. Here, in order to be conservative and to compensate for the effects of inflation an interest rate of ten percent was used. This procedure reveals the flexible system to have realized benefits in terms of operating and delay costs valued at approximately 790,000 dollars over the simultaneous system during the years it has been in operation (Table 5).

TABLE 4.

Annual Cost to Users for Different Signal Systems

<i>Year</i>	<i>Street</i>	<i>Modes of Operation</i>		
		<i>Simultaneous</i>	<i>Alternate</i>	<i>Flexible</i>
1963	Main	\$325,520	\$332,040	\$254,280
	Columbia	225,790	233,140	165,380
	Total	551,310	565,180	419,660
1964	Main	327,050	333,600	255,470
	Columbia	227,910	235,340	166,940
	Total	554,960	568,940	422,410
1965	Main	328,560	335,150	256,660
	Columbia	230,040	237,530	168,500
	Total	558,600	572,680	425,160
1966	Main	330,090	336,710	257,850
	Columbia	232,110	239,670	170,000
	Total	562,200	576,380	427,850
1967	Main	331,610	338,260	259,050
	Columbia	234,170	241,800	171,520
	Total	565,780	580,060	430,570
1968	Main	333,140	339,810	260,230
	Columbia	236,240	243,940	173,040
	Total	569,380	583,750	433,270
1969	Main	334,660	341,370	261,420
	Columbia	238,310	246,070	174,550
	Total	572,970	587,440	435,970
1970	Main	335,890	342,630	262,390
	Columbia	240,380	248,210	176,070
	Total	576,270	590,840	438,460

TABLE 5.
1963 Present Worth of Savings Attributable to the Flexible Traffic Signal System

Year	Total Cost Per Year		N	PWF @ 10		1963 PW of Savings	Accumulated Savings
	Simultaneous	Flexible		Savings ^a	Percent		
1963	\$551,310	\$419,660		1.0000		\$131,650	\$131,650
1964	554,960	422,410	1	.9091		120,500	252,150
1965	558,660	425,160	2	.8264		110,280	362,430
1966	562,200	427,850	3	.7513		100,940	463,370
1967	565,780	430,570	4	.6830		92,350	555,720
1968	569,380	433,270	5	.6209		84,510	640,230
1969	572,970	435,970	6	.5645		77,340	717,570
1970	576,270	438,460	7	.5132		70,720	788,290

^a Savings represents the difference in operating costs between the simultaneous and flexible systems.

CONCLUSIONS

The results of the economic analysis show quite vividly that the flexible coordinated traffic signal system was a good investment for the City of Lafayette. The analysis of the data would possibly have been more realistic if the random mode of operation had been allowed to remain in the analysis, since this was the basic condition. Conceivably the random system could have been brought up to the performance standards of the simultaneous system. Therefore the simultaneous system's performance can be thought of as being the ultimate that could have been obtained with no capital expenditure. This makes the result of the analysis more meaningful and shows again the initial investment has proven to be very sound indeed. Although a statistical analysis could not be performed using the random data, a look at the means of the variables for this mode reveals that this system probably operated at a higher cost to the user than the simultaneous system; therefore, greater savings would have been realized over the basic condition than those reported.

Since a check of the effect of the various modes on cross traffic was not possible it must be assumed that cross traffic was not adversely affected by the operation of the flexible mode. This research reports only savings produced on Main and Columbia Streets, so if the flexible mode could do no better than produce results comparable to the simultaneous mode in the crossing direction the savings reported would remain valid. Investigation of travel time runs done in the crossing direction for the flexible mode reveals no serious problem. Also a check on the average running speed shows cross traffic has not suffered unduly from the flexible system. A comparison of the effect of the modes on cross traffic would undoubtedly reveal additional savings attributable to the flexible system.

A conservative approach has been taken in this research in order to avoid unfairly biasing the result in favor of the flexible signal system. When compared to its original cost the savings attributable to this system seem staggering; however, if the combined savings for the total system were known it would probably be much greater. Other factors that tend to make the analysis conservative are the ten percent interest rate used in the economic analysis, and the fact that only 12 hour volumes were used in calculating the costs. The number of vehicles per day using the system outside the study time were small in comparison to volumes used, but this would certainly have added to the savings. The following are the major findings of this research;

1. The flexible system is shown to be a significant improvement over the simultaneous coordinated and alternate coordinated systems for the full range of volumes (400-1100 veh/hr) tested. This system produces lower running times, delay times, and number of stops per run than either of the other systems under comparison.
2. At the present time the flexible system is saving approximately 140,000 dollars per year in delays and operating costs over the next best system. This figure will undoubtedly increase in the future since volumes are ever increasing.
3. The accumulated savings attributable to the flexible system for the period of time from 1963 through 1970 amounts to approximately 790,000 dollars. This is the 1963 present worth of the savings with a ten percent rate of return applied.
4. Based upon an installation cost of about 200,000 dollars the flexible system has realized a net savings of 590,000 dollars (1963-1971).

1963 present worth of savings at ten percent rate of return	\$790,000
Cost of installation	200,000

Net savings attributable to the flexible system	\$590,000

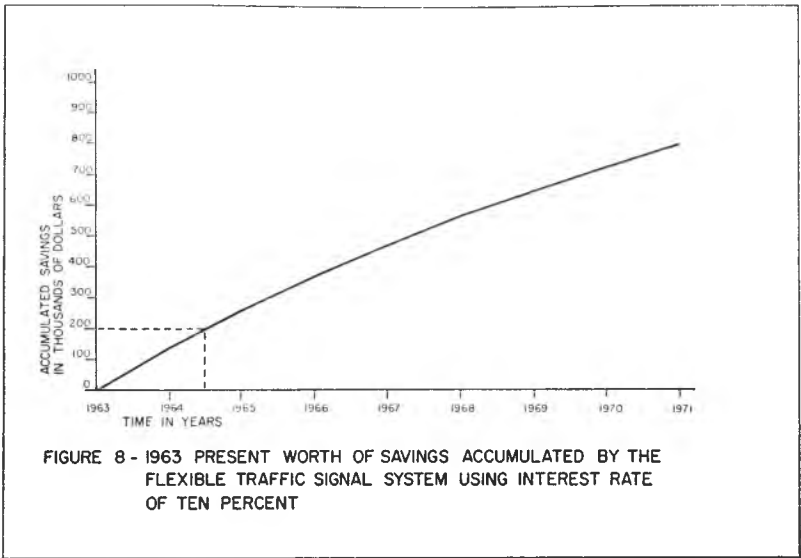


FIGURE 8 - 1963 PRESENT WORTH OF SAVINGS ACCUMULATED BY THE FLEXIBLE TRAFFIC SIGNAL SYSTEM USING INTEREST RATE OF TEN PERCENT

5. The flexible system effectively paid for itself in delay and operating cost savings in less than two years from its installation date (Figure 8).

ACKNOWLEDGMENT

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