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## Mixed lubrication analysis of vane sliding surface in rotary compressor mechanisms

### - Influences of friction of vane sliding surface on lubricating condition between vane top and rolling piston -

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### ABSTRACT

To investigate the influences of friction of the vane sliding surface between the vane and vane slot on the lubricating conditions between the vane top and the rolling piston, we perform a numerical analysis of rolling piston and vane motion, considering mixed lubrication of the vane sliding surface between the vane and vane slot. Analysis results indicate that the risk of scuffing is high between the vane top and piston due to increased PV value near crank angle  $\psi = 180^\circ$ , at which the lubrication is poor when the contact friction forces of the vane sliding surface increase. Moreover, the vane separated from the piston may collide with the piston at crank angle  $\psi = 180^\circ$ , and excessive impact may occur between the vane top and the piston due to increased contact friction forces of the vane sliding surface.

### 1. INTRODUCTION

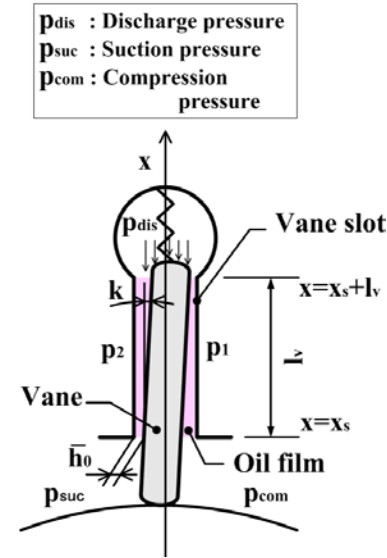
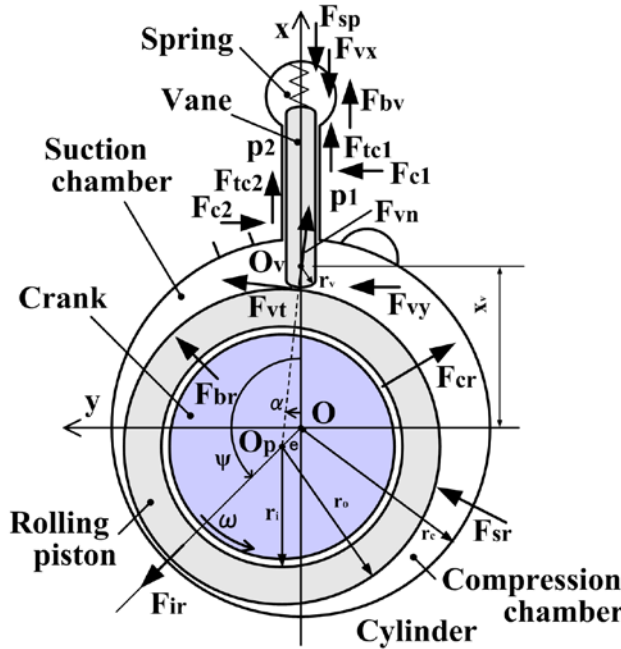
In the compression mechanisms of a rotary compressor for air conditioners, a vane and a rolling piston separate the suction and compression chambers. The rolling piston is driven by a crank eccentrically revolving on the shaft axis while rotating on its own axis. The vane undergoes reciprocating motion, and its top usually comes into contact with the rolling piston by the spring force and discharge pressure. The lubricating condition between the rolling piston and the vane top is critical because the contact pressure is high owing to line contact of the vane top. To prevent damage such as scuffing or abnormal wear, it is important to clarify the lubricating characteristics between the vane top and the rolling piston. Because the normal force acting on the piston at vane contact is greatly influenced by friction of the vane sliding surface between the vane and the vane slot, mixed lubrication analysis of the vane sliding surface between the vane and the vane slot is required to accurately obtain the normal force acting on the piston at vane contact.

To investigate the lubricating characteristics between the vane top and the rolling piston, we perform a numerical analysis of the motion of the rolling piston and the motion of the vane considering mixed lubrication of the vane sliding surface between the vane and vane slot. The cases of different friction coefficients of the vane sliding surface at solid contact between the vane and vane slot were calculated, and the results were compared. The results elucidate the influences of friction of the vane sliding surface between the vane and vane slot on the lubricating characteristics between the vane top and the rolling piston.

### 2. GOVERNING EQUATIONS

In the compression mechanism of air conditioner rotary compressors, the compression chamber is composed of a rolling piston, a vane, and a cylinder. The rolling piston, which is driven by the crank, rotates eccentrically, decreasing the volume of the compression chamber and increasing the pressure of the refrigerant.

This analysis solves, as a coupled problem the equation of vane motion and the equations of equilibrium of forces and moments, the equations of equilibrium of forces for the rolling piston, the rotational equation of motion of the rolling piston, the modified Reynolds equation, the elastic contact equation of the vane sliding surface between the vane and vane slot, and the Reynolds equation of the sliding section between the rolling piston and the crank area.



**Figure 1:** Static coordinate system with origin  $O$  at cylinder center **Figure 2:** Coordinate system for calculation of friction forces of vane sliding surface

### 2.1 Motion of vane and rolling piston in the $x$ - and $y$ -directions

Figure 1 shows the static coordinate system with origin  $O$  at the cylinder center. As that figure shows, the equation of motion of the vane and the equations of equilibrium of forces and moments are expressed as follows<sup>(1, 2)</sup>:

$$m_v \ddot{x}_v = F_{vnx} + F_{vtx} + F_{to1} + F_{to2} + F_{tc1} + F_{tc2} + F_{bv} - F_{vx} - F_s \quad (1)$$

$$0 = F_{vny} + F_{vty} + F_{o1} - F_{o2} + F_{c1} - F_{c2} + F_{vy} \quad (2)$$

$$0 = M_{vn} + M_{vt} + M_{o1} + M_{o2} + M_{c1} + M_{c2} \quad (3)$$

The equation of motion of the rolling piston and the equations of equilibrium of forces are expressed as follows<sup>(3, 4)</sup>:

$$0 = F_{crx} + F_{srx} + F_{brx} - F_{irx} - F_{vnx} - F_{vtx} \quad (4)$$

$$0 = -F_{cry} + F_{sry} + F_{bry} + F_{iry} + F_{vny} - F_{vty} \quad (5)$$

Here, the subscripts  $x$  and  $y$  indicate the  $x$ - and  $y$ -direction components, respectively.

It is assumed that the solid contact region between the rolling piston and vane is in a boundary lubrication zone. Therefore, the friction force acting on the piston at vane contact  $F_{vt}$  is calculated by Coulomb's law as follows:

$$F_{vt} = \gamma_1 \mu_p F_{vn} \quad (6)$$

$$\gamma_1 = \text{sgn}(V_{pv}) \quad (7)$$

$$V_{pv} = r_o \omega_p + (r_o + r_v) \dot{\alpha} \quad (8)$$

The equations and the explanations for calculation of the friction forces on the vane sliding surface are described as follows. Figure 2 shows the coordinate system for calculation of friction force on the vane sliding surface. The vane always inclines in the clockwise direction (Fig. 2) because of the effect of the compression pressure  $p_{com}$ <sup>(1)</sup>. Therefore, solid contact between the vane and vane slot occurs at the lower end on the suction side and at the upper

end on the discharge side. Assuming that the vane sliding surface between the vane and vane slot is in a mixed lubrication zone, we perform a mixed lubrication analysis to calculate the friction forces on the vane sliding surface between the vane and vane slot. The influence of side leaks is negligibly small because the vane width is long enough as compared with the length of effectual oil film on the vane sliding surface near the lower end of the suction side and near the upper end of the discharge side between the vane and vane slot. Therefore, the vane sliding surface is treated as a surface having infinite width along the perpendicular of the plane in Fig. 2. The oil film reactive forces of the vane sliding surface on the discharge side and the suction side between the vane and vane slot are calculated by the modified Reynolds equations, as follows<sup>(5)</sup>:

$$\frac{\partial}{\partial x} \left( \Phi_x \frac{\bar{h}^3}{\eta} \frac{\partial p}{\partial x} \right) = 6U \frac{\partial h_r}{\partial x} + 6U\sigma \frac{\partial \Phi_s}{\partial x} + 12 \frac{\partial h_r}{\partial t} \quad (9)$$

The vane sliding surface is treated as a surface having longitudinal surface roughness. The direction parameter of the surface roughness  $\chi$  used here is 3.0.

The local oil film thickness of vane sliding surface  $h_r$  is expressed as follows:

$$h_r = \frac{\bar{h}}{2} \left\{ 1 + \operatorname{erf} \left( \frac{\bar{h}}{\sqrt{2}\sigma} \right) \right\} + \frac{\sigma}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{\bar{h}}{\sigma} \right)^2 \right) \quad (10)$$

The average oil film thicknesses on the discharge side and the suction side between the vane and vane slot are

$$\bar{h}_1 = c_v - \bar{h}_0 - k(x - x_s) \quad (11)$$

$$\bar{h}_2 = \bar{h}_0 + k(x - x_s) \quad (12)$$

For calculating the contact forces of the vane sliding surface between the vane and vane slot, Patir and Cheng's approximate expression, based on Greenwood and Tripp's theory, is used<sup>(6,7)</sup>:

$$p_c = \begin{cases} k_c E' \times 4.4086 \times 10^{-5} \left( 4 - \frac{\bar{h}}{\sigma} \right)^{6.804} & (\bar{h} < 4\sigma) \\ 0 & (\bar{h} \geq 4\sigma) \end{cases} \quad (13)$$

The oil film reactive forces  $F_{o1}$ ,  $F_{o2}$  and contact forces  $F_{c1}$ ,  $F_{c2}$  are calculated by integration of oil film pressure  $p$  and of contact pressure  $p_c$ .

The contact friction forces on the vane sliding surface between the vane and vane slot  $F_{tc1}$ ,  $F_{tc2}$  are expressed as follows:

$$F_{tc1} = \gamma_2 \mu_v w_c \int p_c(\bar{h}_1) dx \quad (14)$$

$$F_{tc2} = \gamma_2 \mu_v w_c \int p_c(\bar{h}_2) dx \quad (15)$$

$$\gamma_2 = \operatorname{sgn}(-\dot{x}_v) \quad (16)$$

The fluid friction forces on the vane sliding surface on the discharge side and on the suction side between vane and vane slot  $F_{fo1}$ ,  $F_{fo2}$  are assumed to obey Newton's law of viscosity.

$$F_{fo1} = w_c \int \eta \dot{x}_v / \bar{h}_1 dx \quad (17)$$

$$F_{fo2} = w_c \int \eta \dot{x}_v / \bar{h}_2 dx \quad (18)$$

## 2.2 Motion of the rolling piston in the rotation direction

Figure 3 shows a moving coordinate system with origin  $O_p$  at the crank center. The moving coordinate system eccentrically rotates the circumference of the cylinder center. A moving coordinate system is used for the calculation of angular velocity of piston rotation  $\omega_p$ . As shown in Fig. 3(a), the rotational equation of motion of a rolling piston about the piston center is expressed as follows<sup>(3,8)</sup>:

$$I \dot{\omega}_p = M_r - M_s - r_o F_{vt} \quad (19)$$

Here, the viscous friction moment due to refrigerant gas between the piston and cylinder is negligibly small, and therefore not considered further in this analysis.

The friction moments  $M_s$  on both end surfaces of the rolling piston are expressed as follows<sup>(9)</sup>:

$$M_s = \frac{2\pi\eta\omega_p(r_o^4 - r_i^4)}{c_s} \quad (20)$$

The equations for the calculation of the friction force and moment due to oil film viscosity between the piston and crank are explained as follows. Figure 3(b) shows an analysis model of oil film between the piston and crank. The direction and the value of the oil film reactive force between the piston and crank  $F_{cr}$  in Fig. 3(b) are obtained by Eqs. (4)–(5). The sliding section between the rolling piston and crank is regarded as a hydrodynamic journal bearing. The oil film pressure between the piston and crank is calculated by the Reynolds equation<sup>(8,10)</sup>. Because the width diameter ratio ( $w_s / (2*r_i)$ , where  $w_s$  is the width of the crank) is less than 4.0, the Reynolds equation of finite width bearing is used, as follows:

$$\frac{\partial}{r_i^2 \partial \theta} \left( \frac{h_r^3}{\eta} \frac{\partial p_r}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \frac{h_r^3}{\eta} \frac{\partial p_r}{\partial z} \right) = 6\omega_c \frac{\partial h_r}{\partial \theta} + 12 \frac{\partial h_r}{\partial t} \quad (21)$$

$$h_r = c_r \times (1 + \varepsilon_c \cos(\theta - \varphi)) \quad (22)$$

Using boundary conditions such that the ambient pressure at both ends equals the discharge pressure  $p_{dis}$ , oil film pressure between the piston and crank is calculated from Eq. (21). Although the oil film might break under negative pressure, the following calculation indicates that the oil film pressure is positive over the entire bearing area owing to the high discharge pressure. Thus, the frictions moment  $M_r$  due to oil film viscosity between piston and crank are calculated by integration over the whole bearing area, as follows:

$$M_r = \iint \left( \frac{\eta\omega_c r_i}{h_r} + \frac{h_r}{2r_i} \frac{\partial p_r}{\partial \theta} \right) r_i^2 d\theta dz \quad (23)$$

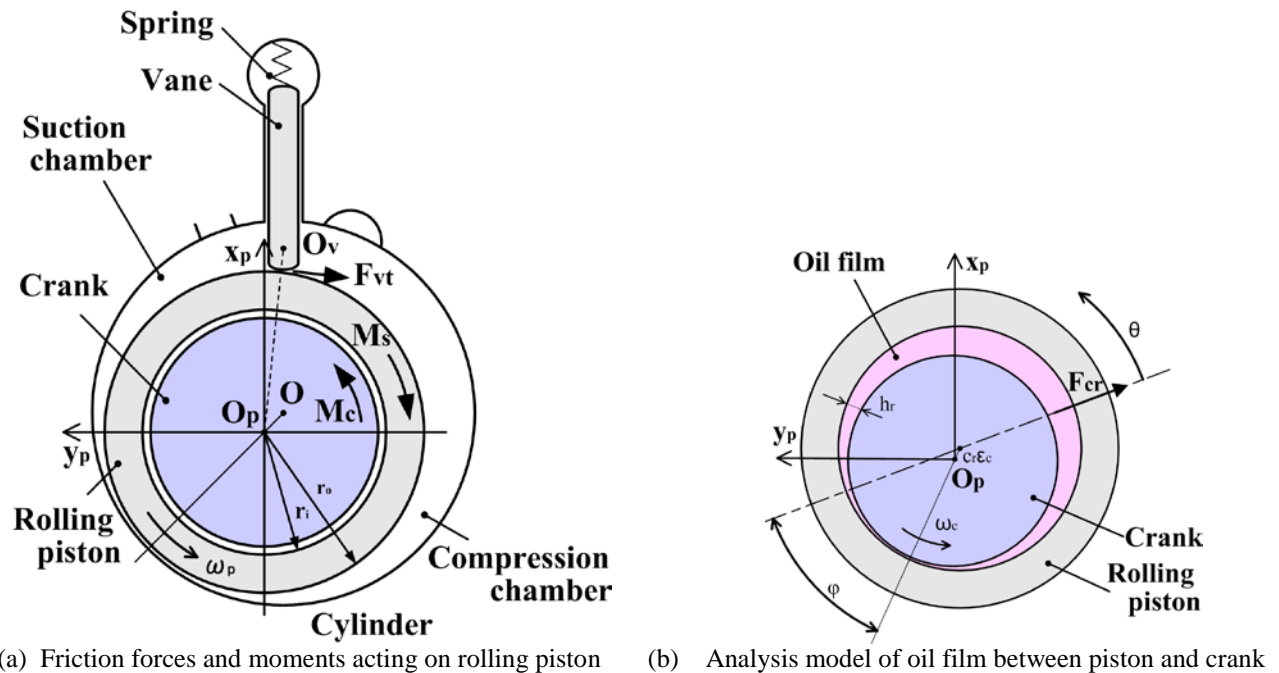


Figure 3: Moving coordinate system with origin  $O_p$  at crank center

### 3. ANALYSIS PROCEDURE AND CONDITIONS

#### 3.1 Analysis Procedure

Eqs. (1)–(5), (9), (13), (19), and (21) are solved as a coupled problem, and the average oil film thickness at the lower end of vane slot  $\bar{h}_0$ , the inclination of the vane  $k$ , the eccentricity of the crank with respect to piston center  $\varepsilon_c$ , the angle from vane centerline to minimum oil film position  $\varphi$ , the angular velocity of piston rotation  $\omega_p$ , and the normal force acting on the piston at vane contact  $F_{vn}$  are numerically calculated by the Newton–Raphson method. The time differentials of  $\bar{h}_0$  and  $k$  used in the calculation of the squeeze terms in the modified Reynolds equations, and the time differentials of  $\varepsilon_c$  and  $\varphi$ , used in the calculation of the squeeze terms in the Reynolds equations, are approximately calculated by the following equation:

$$\begin{pmatrix} \dot{\bar{h}}_0 \\ \dot{k} \\ \dot{\varepsilon}_c \\ \dot{\varphi} \end{pmatrix} = \frac{1}{\Delta t} \begin{pmatrix} \bar{h}_{0t+\Delta t} - \bar{h}_{0t} \\ k_{t+\Delta t} - k_t \\ \varepsilon_{ct+\Delta t} - \varepsilon_{ct} \\ \varphi_{t+\Delta t} - \varphi_t \end{pmatrix} \quad (24)$$

Moreover, the force variations and the moments acting on the rolling piston and vane are considered. Consequently, the coupled problem is solved recursively along the time axis.

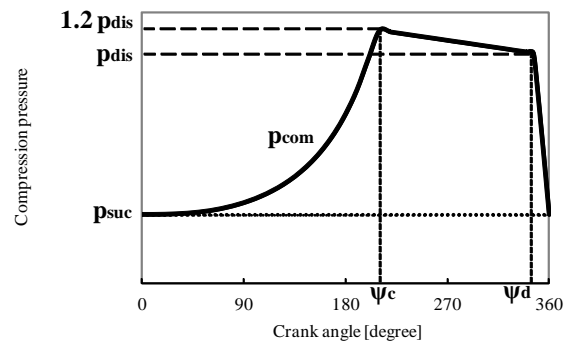
### 3.2 Analysis conditions

The analysis conditions are shown in Table 1. The cylinder radius is 36.5 mm. The rolling piston outer radius is 29.7 mm. The crank eccentricity is 6.8 mm. The calculations were executed for the case of different friction coefficient of vane sliding surface at solid contact between the vane and vane slot  $\mu_v$ . Moreover, the friction coefficient between the vane and piston  $\mu_p$  is 0.12.

Figure 4 shows the relationship between the crank angle  $\psi$  and the compression pressure  $p_{com}$ , which was used as the analysis condition. It is assumed that the refrigerant discharge starts when the compression pressure  $p_{com}$  increases to 1.2-fold the discharge pressure  $p_{dis}$ .<sup>(9)</sup>

**Table 1:** Analysis conditions

Radius of cylinder, $r_c$ [mm]	36.5
Outer radius of piston, $r_o$ [mm]	29.7
Inner radius of piston, $r_i$ [mm]	20.0
Eccentricity of crank, $e$ [mm]	6.8
Oil viscosity, $\eta$ [ $10^{-3}$ Pa · s]	2.8
Rotating Frequency, $f_o$ [Hz]	60
Discharge pressure / Suction pressure, $p_{dis} / p_{suc}$ [MPa]	4.25/1.27 ( $\Delta p=2.98$ )
Friction coefficient at solid contact between vane and vane slot, $\mu_v$	0.0, 0.10, 0.20, 0.30, 0.40
Friction coefficient between piston and vane, $\mu_p$	0.12

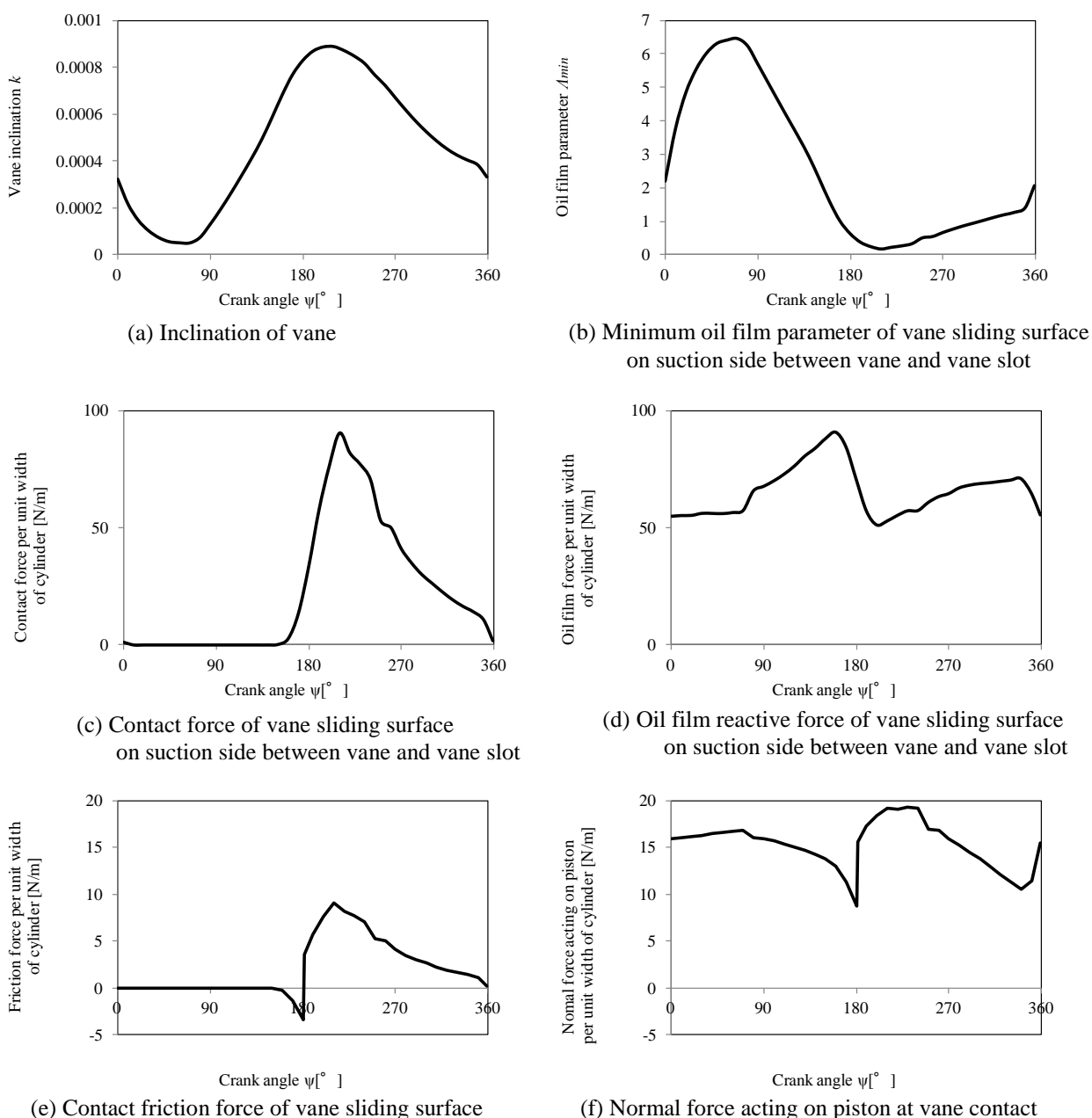


**Figure 4:** Pressure variation of refrigerant in compression chamber during one cycle

## 4. RESULTS AND DISCUSSION

### 4.1 Lubricating characteristics of vane sliding surface

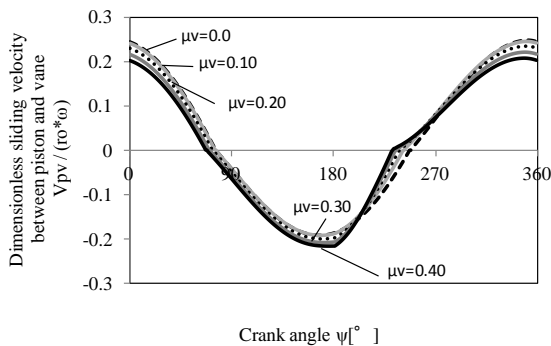
Figure 5 shows the analysis results when the discharge pressure  $p_{dis}$  is 4.25 MPa, the suction pressure  $p_{suc}$  is 1.27 MPa, the rotating frequency  $f_o$  is 60 Hz, and the friction coefficient at solid contact between vane and vane slot  $\mu_v$  is 0.1. The horizontal axis is the crank angle  $\psi$ . Figures 5(a) and (b) respectively show variations of the vane inclination  $k$  and the minimum oil film parameter of the vane sliding surface on the suction side between the vane and vane slot  $A_{min}$  ( $= h_{min}/\sigma$ ) through one crank revolution. Figures 5(c) and (d) respectively show variations of the contact force of vane sliding surface per unit width of cylinder  $F_{c2} / w_c$  and oil film reactive force of vane sliding surface per unit width of cylinder  $F_{o2} / w_c$  on the suction side between the vane and vane slot through one crank revolution. Figures 5(e) and (f) respectively show variations of the contact friction force of vane sliding surface per unit width of cylinder between the vane and vane slot and the normal force acting on the piston at vane contact per unit width of cylinder  $F_{vn} / w_c$  through one crank revolution.



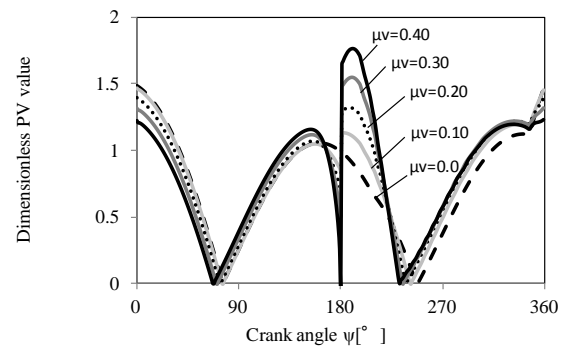
**Figure 5:** Lubrication characteristics of vane sliding surface  
 ( $p_{dis}/p_{suc} = 4.25 / 1.27$  MPa ( $\Delta p = 2.98$  MPa),  $f_0 = 60$  Hz,  $\mu_v = 0.10$ )

As shown in Fig. 5(a), the vane always inclines in the clockwise direction (Fig. 2) because the vane inclination  $k$  is always larger than  $0^\circ$ . Therefore, it can be seen from the vane geometry (Fig. 2) that  $\bar{h}_0$  becomes the minimum oil film thickness  $h_{min}$  on the suction side between the vane and vane slot. As shown in Fig. 5(b),  $\lambda_{min} (= h_{min}/\sigma)$  increases between  $\psi = 0^\circ$  and  $\psi = 180^\circ$  because the oil film pressure on the suction side is raised by the wedge film effect. Because the wedge film effect disappears when the vane motion reverses at  $\psi = 180^\circ$ ,  $\lambda_{min}$  decreases from  $\psi = 180^\circ$ . Therefore, the oil film reactive force on the suction side between the vane and vane slot decreases near  $\psi = 180^\circ$ , as shown in Fig. 5(d), and the contact forces on the suction side between the vane and vane slot increases rapidly near  $\psi = 180^\circ$  as shown in Fig. 5(c).

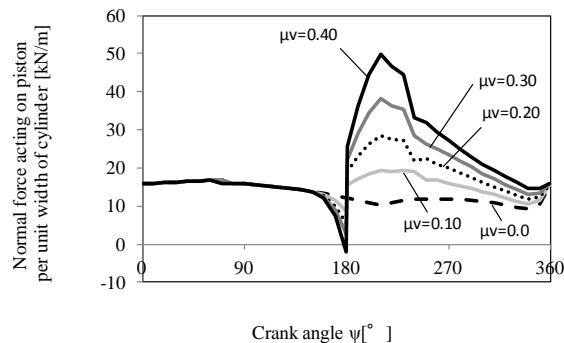
Under these analysis conditions, solid contact does not occur on the discharge side between the vane and vane slot. Therefore, the contact friction force of the vane sliding surface is based on the contact force on the suction side between the vane and vane slot. As shown in Fig. 5(e), it is found that the contact friction force of the vane sliding surface is negative from  $\psi = 0^\circ$  to  $\psi = 180^\circ$  and is positive from  $\psi = 180^\circ$  to  $\psi = 360^\circ$ . This is because the contact friction force of the vane sliding surface acts in opposite direction from vane movement. From  $\psi = 0^\circ$  to  $\psi = 180^\circ$ , the contact friction force of the vane sliding surface acts on the vane to separate the vane and the piston, and its value is maximum at  $\psi = 180^\circ$ . Therefore, the normal force acting on the piston at vane contact is minimized at  $\psi = 180^\circ$ , as shown in Fig. 5(f).



(a) Relative sliding velocity between vane and rolling piston



(b) PV value between vane top and rolling piston



(c) Normal force acting on piston at vane contact

**Figure 6:** Influences of friction of vane sliding surface ( $p_{dis}/p_{suc} = 4.25/1.27$  MPa ( $\Delta p = 2.98$  MPa),  $f_0 = 60$  Hz)

#### 4.2 Influences of friction of vane sliding surface between vane and vane slot

Figure 6 shows the analysis results when the friction coefficient of the vane sliding surface at solid contact between vane and vane slot  $\mu_v$  is 0.0, 0.10, 0.20, 0.30, and 0.40.

Figure 6(a) shows the relative sliding velocity between the piston and the vane through one revolution of the crank. It is found that the sliding velocity decreases for all crank angles when the friction force of the vane sliding surface increases. This is because friction force at vane contact  $F_{vt}$ , which controls the rotating motion of the piston to prevent slip at the vane top, increases when the normal force acting on the piston at vane contact  $F_{vn}$  increases owing to an increase in  $\mu_v$ .

Figure 6(b) shows variation of the PV value between the piston and vane in a dimensionless form. Here, the dimensionless PV value is defined as the ratio of the PV value to that of the  $\mu_v = 0.0$ , crank angle  $\psi = 180^\circ$  case. The PV value is calculated from  $F_{vn}$  and the relative sliding velocity  $V_{pv}$ . It can be seen that the PV value near  $\psi = 180^\circ$  increases when the contact friction force of the vane sliding surface increases. This is because  $F_{vn}$  near  $\psi = 180^\circ$  increases when  $\mu_v$  increases. Near  $\psi = 180^\circ$  is an area where the risk of scuffing is high owing to high PV value



through one revolution of the crank. Consequently, it is found that the risk of scuffing between the piston and vane top increases when the contact friction force of the vane sliding surface increases.

Figure 6(c) shows variation of the normal force acting on a piston at vane contact per unit width of cylinder  $F_{vn}/w_c$ . The value of  $F_{vn}$  becomes negative near the crank angle  $\psi = 180^\circ$  when the friction coefficient  $\mu_v$  is 0.40. It is found that the vane separates from the piston at  $\psi = 180^\circ$  because of the increase of the friction forces of the vane sliding surface. Because  $F_{vn}$  increases rapidly after  $\psi = 180^\circ$ , the vane separated from the piston collides with the piston and an excessive impact occurs between the vane top and the piston. Thus, the risk of surface damage at the vane top becomes high. The analysis results reveal the necessity of decreasing the friction force of vane sliding surface to prevent separation of the vane and the piston.

## 5. CONCLUSION

The numerical analysis of the motion of the rolling piston and the motion of the vane considering mixed lubrication of the vane sliding surface has been performed to investigate the influences of the contact friction of the vane sliding surface on the lubricating condition between the vane top and rolling piston in a rotary compressor. The following results were obtained.

1. The risk of scuffing is high between the vane top and piston because of increased PV value near crank angle  $\psi=180^\circ$ , at which the lubricating condition is severe when the contact friction forces of the vane sliding surface increases.
2. The vane separated from the piston collides with the piston and an excessive impact occurs between the vane top and the piston because of the increased the contact friction forces of the vane sliding surface. Thus, the risk of surface damage at the vane top becomes high.
3. Consequently, to maintain good vane top surface condition, it is important to guarantee and to maintain the sufficient lubrication of the vane sliding surface.

## NOMENCLATURE

$c_r$	Radial clearance between piston and crank
$c_s$	Clearance between piston end and bearing end
$c_v$	Clearance between vane and vane slot
$d_v$	Vane height
$E'$	Equivalent elastic modulus
$e$	Crank eccentricity
$F_{br}$	Fluid friction force on piston end faces attributable to revolution
$F_{bv}$	Fluid friction force on vane end faces
$F_{c1}, F_{c2}$	Contact forces of vane sliding surface between vane and vane slot
$F_{cr}$	Oil film reactive force between piston and crank
$F_{ir}$	Centrifugal force attributable to revolution
$F_{o1}, F_{o2}$	Oil film reactive forces of vane sliding surface between vane and vane slot
$F_s$	Friction forces on both end surfaces of piston
$F_{sp}$	Spring force
$F_{sr}$	Gas pressure-difference force between compression chamber and suction chamber
$F_{tc1}, F_{tc2}$	Contact friction forces of vane sliding surface between vane and vane slot
$F_{to1}, F_{to2}$	Fluid friction forces of vane sliding surface between vane and vane slot
$F_{vn}$	Normal force acting on piston at vane contact
$F_{vt}$	Friction force acting on piston at vane contact

$F_{vx}, F_{vy}$	Forces acting on vane due to refrigerant pressure difference
$f_0$	Rotating frequency
$\bar{h}$	Average oil film thickness of vane sliding surface
$\bar{h}_0$	Average oil film thickness at lower end of vane slot on the suction side at $x = x_s$ in Fig. 6
$\bar{h}_1, \bar{h}_2$	Average oil film thicknesses on the discharge side and suction side between vane and vane slot
$h_T$	Local oil film thickness of vane sliding surface
$h_{min}$	Minimum average oil film thickness on the suction side between vane and vane slot
$h_r$	Oil film thickness between piston and crank
$I$	Mass moment of inertia of a piston
$k$	Vane inclination
$k_c$	Constant in force-compliance relationship
$l_v$	Vane slot length
$M_{c1}, M_{c2}$	Moments of contact force of vane sliding surface between vane and vane slot
$M_{o1}, M_{o2}$	Moments of oil film reactive forces of vane sliding surface between vane and vane slot
$M_r$	Friction moment due to oil film viscosity between piston and crank
$M_s$	Fluid friction moment on piston end faces due to revolution
$M_{vn}$	Moment of normal force acting on vane at piston contact
$M_{vt}$	Friction moment acting on vane at piston contact
$m_v$	Vane mass
$O$	Cylinder center
$O_p$	Crank center
$O_v$	Center of vane top arc
$O-x, y$	Static coordinates with origin at cylinder center
$O_p-x_p, y_p$	Moving coordinates with origin at crank center
$p$	Oil film pressure of vane sliding surface between vane and vane slot
$p_1, p_2$	Oil film pressure between vane and vane slot corresponding to discharge side and suction side
$p_c$	Contact pressure between vane and vane slot
$p_{com}$	Compression pressure
$p_{dis}$	Discharge pressure
$p_{suc}$	Suction pressure
$p_r$	Oil film pressure between piston and crank
$r_c$	Cylinder radius
$r_i$	Inner piston radius
$r_o$	Outer piston radius
$r_v$	Vane top radius
$t$	Time
$U$	Vane sliding velocity
$V_{pv}$	Relative sliding velocity between piston and vane
$x_v$	Displacement of vane along the $x$ -direction
$x_s$	Length from cylinder center to lower end of vane slot along the $x$ -direction
$w_c$	Cylinder width
$\alpha$	Attitude angle of crank center with respect to center of vane top arc
$\gamma_1$	Constant with value $-1, 0, \text{ or } 1$ ( $-1$ when $V_{pv} < 0$ , $0$ when $V_{pv} = 0$ , and $1$ when $V_{pv} > 0$ )
$\gamma_2$	Constant with value $-1, 0, \text{ or } 1$ ( $-1$ when $-x_v < 0$ , $0$ when $-x_v = 0$ , and $1$ when $-x_v > 0$ )
$\varepsilon_c$	Piston eccentricity with respect to crank center
$\eta$	Oil viscosity

$\theta$	Rotational angle of crank from crank centerline in the direction of $F_{cr}$
$A_{min}$	Minimum oil film parameter of vane sliding surface between vane and vane slot
$\mu_v$	Friction coefficient of vane sliding surface at solid contact between vane and vane slot
$\mu_p$	Friction coefficient between piston and vane
$\sigma$	Standard deviations of composite roughness
$\Phi_x$	Pressure flow factor
$\Phi_s$	Shear flow factor
$\varphi$	Angle from crank centerline in the direction of $F_{cr}$ to minimum oil film position
$\psi$	Crank angle
$\psi_c$	Starting angle of refrigerant discharge
$\psi_d$	Angle of discharge valve
$\omega$	Angular velocity of crank
$\omega_c$	Relative angular velocity between crank and piston
$\omega_p$	Angular velocity of piston rotation on the moving coordinate system

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