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A State-Space Modeling Approach and Subspace Identification Method for Predictive Control of Multi-Zone Buildings with Mixed-Mode Cooling

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ABSTRACT

The paper presents a control-oriented modeling approach for multi-zone buildings with mixed-mode (MM) cooling that incorporates their mode switching behavior. A forward state-space representation with time-varying system matrices is presented and used for establishing a detailed prediction model of a multi-zone MM building. The linear time-variant state-space (LTV-SS) model, which is considered as a true representation of the building, is used for developing data-driven linear time-invariant state-space models based on the subspace identification algorithm. The simplified black-box model can successfully capture the switching behavior of the MM building with the RMSE of 0.64 ℃.

1. INTRODUCTION

Mixed-mode (MM) cooling is a hybrid approach for space conditioning which employs free cooling (using natural ventilation) and mechanical systems to minimize building energy use and maintain occupant thermal comfort (Brager et al., 2007). However, to fully exploit the benefits of this hybrid system, the switching between modes should be intelligent and optimized. Hence, advanced control strategies, such as model predictive control (MPC), are required. MPC is particularly suitable for slow response dynamic systems and requires real-time solution and implementation of optimal control sequences, within a future time horizon, with the most up-to-date information on inputs and environmental disturbances for the dynamic system model. Thus, obtaining a model that provides reliable predictions and can be implemented in real controllers is crucial for achieving robust performance.

Modeling complexity is a major challenge for multi-zone MM buildings due to the coupling between thermal and airflow dynamics. (Hu and Karava, 2014a). Also, MM buildings typically confront abrupt changes of system dynamics due to the mode switch between natural ventilation and mechanical cooling. Thus, the models usually inherit the nonlinearity of natural ventilation and they should be able to capture the system switching behavior. The most sophisticated available models can be found in whole-building energy and air flow simulation tools such as EnergyPlus (May-Ostendorp et al., 2011; Tanner and Henze, 2014). These tools provide detailed modeling of a wide range of building features including mixed-mode cooling and can also integrate different system simulations. However, it is rather difficult to directly use these models for predictive control strategies as they are far too complex and their execution times can become intolerable (Hu and Karava, 2014a). Also, such models do not offer flexibility in the management of uncertainty as they require the solution of stochastic differential equations for description of a system to be identified (Brohus et al., 2012). This problem can be alleviated by developing simplified models that maintain the important dynamics which are relevant for control purposes. Although a number of examples of MPC using simplified building or plant models exist in the literature (Ma et al., 2009; Gyalistras and Gwerder, 2010; Gwerder et al., 2013), such approaches may not be appropriate for any building system.

To address this challenge, the research presented in this paper aims to develop a control-oriented modeling approach for model-predictive control of multi-zone buildings with mixed-mode cooling. Methodologically, the present study extends previous work on MPC for MM buildings based on (a) physical (white-box) (Hu and Karava, 2014a; May-

Ostendorp et al., 2011; Coffey, 2011) models by establishing a linear state-space representation with varying coefficient matrices enabling the formulation of forward models; (b) data-driven models based on gray-box approach that require an airflow network (Hu and Karava, 2014b) by formulating single and hybrid linear time-invariant state-space models based on subspace identification (4SID) algorithm. Rather than using multiple models for different operation modes, thus requiring extra computation resource for model selection (Spindler, 2004), the simplified model presented in this study can capture the relevant system dynamics with the building operating in different modes. Furthermore, this modeling approach is more adaptive as it can be fine-tuned efficiently during the actual operation.

2. FORWARD MODELING APPROACH FOR MIXED-MODE BUILDINGS

2.1 Linear Time-Variant State-Space Model

The thermal dynamics of interior building zones are typically predicted by applying the heat balance method (Pederson et al., 1998) which explicitly models the heat transfer rate to the interior and exterior surfaces and to the zone air based on energy conservation. The conventional thermal network approach discretizes the building into zones, which are modeled using a network of nodes with interconnecting paths through which heat flows by convection, conduction and radiation. Heat gains due to solar radiation and internal sources are lumped in the thermal nodes while heat storages in thermal mass are represented by capacitances. A heat balance is performed at each node to determine the node temperature and heat flow between connected nodes. This results into a set of coupled ordinary differential and algebraic equations that can be solved simultaneously:

$$C_i \frac{dT_i}{dt} = \sum_{k=1}^{n} \frac{T_{k,i} - T_i}{R_{k,i}} + Q_{gain} + Q_{aux} + Q_{IHG} + Q_{ij} \tag{1}$$

where, R is the resistance associated with convection, conduction, and radiation, C is the thermal capacitance of the structure and air, Q_{gain} is the solar heat gain for the surfaces heat balance, Q_{aux} is the auxiliary heat supply for the air heat balance. The internal heat gain Q_{IHG} is split into a radiative and convective part (ASHRAE, 2009) included in the energy balance for the surface and air node respectively. The heat transfer associated with airflow Q_{ij}, such as the air exchange between zones, the infiltration or natural ventilation, can be computed from:

$$Q_{ij} = \mu \cdot \dot{m}_{ij} \, c_p \left(T_j - T_i \right) \tag{2}$$

where, μ is the upwind operator used in order to account for the influence of the upwind control volume j to i with $\mu = 1.0$ if the flow is from volume j to i and $\mu = 0$ otherwise; c_p is the specific heat of air, T_i and T_j is the air temperature in zone i and j respectively; $\dot{m}_{ij}$ is the air exchange flow rate. The multi-zone airflow network method (Hensen, 1990) that represents building spaces by homogeneous nodes that are linked with other nodes through openings between spaces, is adopted to calculate the flow rate $\dot{m}_{ij}$. This method can predict overall ventilation flow rates for the entire building and individual flow rates through openings, caused by pressure differences due to wind and buoyancy forces, or mechanical systems along with air exchanges between zones. For buildings optimally designed for natural ventilation there is strong coupling between heat and air flows. To account for this effect in the present study, the thermal and the airflow network models can be coupled using the “Onion” method (Hensen, 1990).

Rearranging the terms in Equation (1), the thermal dynamic system for a building with mixed-mode cooling can be formulated using the state-space representation as follows:

$$\begin{aligned} \dot{x} &= Ax + Bu + f\left(x, u, \dot{m} \right) \\ y &= Cx + Du \end{aligned} \tag{3}$$

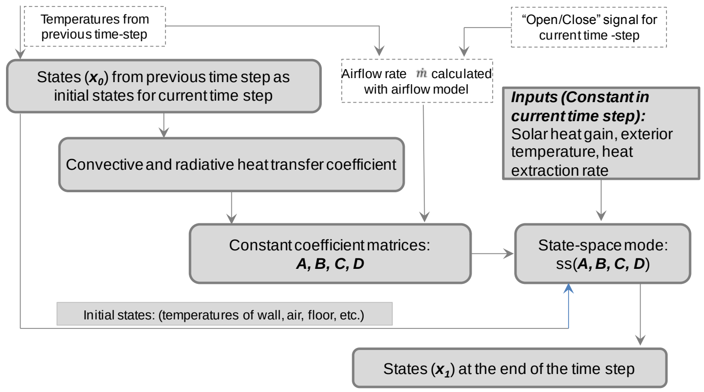


Figure 1: Flowchart for the formulation of the linear time variant state-space model

in which, *A, B, C, D* are coefficient matrices and the size of *A* matrix decides the system order. *x* is the state vector that represents the temperature of each node. *u* is the input vector (e.g. outside temperature, solar heat gain, auxiliary heat supply or extraction rate) and *y* is the output vector (e.g. zone air temperature, wall surface temperature). This forward state-space model is nonlinear due to the term $f(x, u, \dot{m})$ that represents the heat transfer associated with the airflow and it can be linearized and discretized in time as follows:

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k \\ y_k &= C x_k + D u_k \end{aligned} \tag{4}$$

In this linear time-variant state-space model (LTV-SS), instead of finding the air mass flow rate $\dot{m}$ using zone temperatures (e.g. state variables) at the current time-step, the airflow rate can be calculated using zone temperatures from the previous time-step. In this way, $\dot{m}$ becomes known in the current time step. Furthermore, the heat transfer coefficients of convection and radiation are calculated using temperatures from the previous time step so they can be seen as constant in the current time step. The flow chart for the formulation of the LTV-SS model is illustrated in Figure 1. This modeling approach was validated with experimental data collected in a two-zone test-building under four operation modes (Hu and Karava, 2014b).

2.2 LTV-SS Model for Multi-zone Building with Mixed-Mode Cooling

The LTV-SS formulation presented in the previous section is used to develop a model for an institutional building (located in Montreal, Canada) with mixed-mode cooling (Figure 2). The natural ventilation design concept of the building includes: (a) inlet grilles with motorized dampers (opening area about 1.4 m^2) located at the end of the corridors in the southeast and northwest façade of each floor, and (b) five three-storey atria that are separated with a floor slab and connected with grilles (4 m^2 area) equipped with motorized dampers to enhance buoyancy-driven flow. The atrium is located on the southwest façade (facing 35° west of south) of the building extending from the second to the sixteenth floor and each of the five three-storey atria has dimensions of 9 m × 12 m × 12 m high, with motorized roller shades on all glass surfaces. The building has high levels of thermal mass in the form of exposed concrete floor slabs in the atrium (0.1 m thick) and the corridors (0.4 m thick), which are located adjacent to the inlet grilles on the southeast and northwest ends and extended all the way to the atrium. The mechanical cooling in the atrium zone is assumed to be a variable air volume (VAV) system. The VAV system has a cooling supply temperature of 13 °C with maximum and minimum flow rate of 1.0 m^3/s and 0.2 m^3/s respectively. When heating is required in the zone, the VAV system supplies reheated air with minimum flow rate. The damper for air supply and the valve for hot water supply for the reheat coil are assumed to be controlled by a PID controller (one minute sampling time step). The south-west facing atrium façade is assumed to have roller shade (with total transmittance of 6.4% and total absorptance of 47.1%) controlled with heuristic rules: the atrium façade is fully shaded when the incident beam radiation is higher than 400 W/m^2, otherwise, the façade is not shaded. Thus, the solar gain on internal surfaces is affected by the blind control.

For simplicity, this study focuses on a generic section of the building with an atrium connected to six corridors as shown in Figure 2. Each corridor has one exterior façade where the inlet grilles are installed. The corridors have dimensions of $30\text{ m} \times 1.8\text{ m} \times 3\text{ m}$ and act as long air "duct" for delivery of outside air into the atrium zone. The

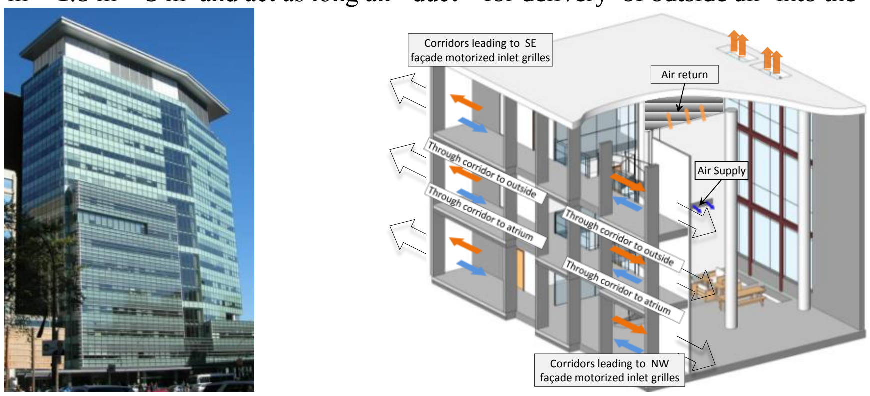


Figure 2: Outside view of the building and its mixed-mode cooling concept

total floor area of the atrium is 108 m^2 with a height of 11.6 m. To establish a model considering the potential temperature stratification, the atrium is divided into three stacked zones and each of them is connected with two corridors through large openings using the Multiple Opening Model (Walton and Dols, 2010). Thus, with the zoning method, the building section has nine simulation zones. Detailed information on the airflow network model is presented in Hu and Karava (2014a).

A 280^{th}-order LTV-SS model is established for the investigated building section. The input vector $u \in \mathbb{R}^{52}$ includes controlled inputs such as the heat extraction rate (provided by the HVAC system or natural ventilation) and uncontrolled inputs (disturbances), which are the exterior temperature (T_{ext}), solar gain on internal and external surfaces (S_{ij}), and the internal heat gain (Q_{IHG}). Details of the inputs, outputs and state variables are shown in Table 1. The matrix $C \in \mathbb{R}^{280\times280}$ is an identity matrix so that the output vector $y \in \mathbb{R}^{280\times1}$ is identical to the state vector $x \in \mathbb{R}^{280\times1}$. D is a zero matrix. The matrices $A \in \mathbb{R}^{280\times280}$ (state matrix) and $B \in \mathbb{R}^{280\times52}$ (input matrix) can be found from the balance equations for each thermal node. Both matrices are time-variant as their elements associated with the airflow rate, convection and radiant heat transfer coefficients vary with time.

Table 1: States, inputs, and outputs used in the forward state-space model

States (x):	Inputs (u):	Outputs (y):
$x = [T_i\ , T_j, T_k]^T$ ▪ i: zone index ▪ j: surface index ▪ k: mass node index	$u = [T_{ext}, S_{ij}, Q_{IHG}, Q]^T$ ▪ T_{ext}: exterior air temperature; ▪ S_j: solar radiation on surfaces j; ▪ Q_{IHG}: internal heat gain; ▪ Q: heat extraction rate;	$y = [T_i\ , T_j, T_k]^T$ ▪ i: zone index ▪ j: surface index ▪ k: mass node index
○ *In natural ventilation mode, there is no HVAC cooling/heating supply;* ○ *In mechanical mode, consider HVAC cooling/heating supply (calculated with zone temperature, VAV discharge temperature and flow rate);* ○ *Internal heat gain is assumed to be zero;* ○ *Solar gain on internal surfaces calculated based on heuristic blind control;*		

3. SUBSPACE IDENTIFICATION

3.1 Subspace Identification Algorithm

For real-time MPC implementation in MM buildings, an essential requirement is the efficiency of the prediction model. The developed 280^{th}-order LTV-SS model for the investigated institutional MM building is a high-order model and it requires the calculation of the mass flow rate through the motorized grilles and between interconnected zones with an airflow model. Hence, the model needs to be simplified for use in actual predictive controllers. For this purpose, the 4SID algorithm is adopted which uses the following state-space structure:

$$x_{k+1} = Ax_k + Bu_k + Ke_k \tag{5}$$
$$y_k = Cx_k + Du_k + e_k$$

in which, e_k is zero mean Gaussian white noise, $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the input vector, $y_k \in \mathbb{R}^l$ is the output vector, and *A*, *B*, *C*, *D*, *K* are system matrices. The objective of the algorithm is to determine the system order n and to find the system matrices. A detailed description of the method can be found in Overschee and Moor (1999) and Prívara et al. (2013). Among the different options, the N4SID algorithm is selected, for its availability in Matlab®. The white noise e_k is assumed to be zero, since we are using simulation results and sufficient input parameters, rather than actual measurements, for the identification. In this study, the detailed 280th-order LTV-SS model is viewed as a true representation of the building and the simulation results from it become the dataset for model training and calibration.

3.2 Training Dataset

The selection of training data which are the inputs and outputs of the system under investigation is important for the particular identification. The inputs should include all the possible system excitations, while the outputs can be selected according to the application of the identified model. For the identification problem in this study, the model inputs include the weather disturbance and heating/cooling supply to the zone by the mixed-mode system. The weather disturbance includes the outdoor temperature T_{ext}, wind speed W_{spd} and direction W_{dir}, and transmitted solar radiation in corridors $Q_{tr,cor}$ and atrium $Q_{tr,at}$ (the blind in the atrium is ON when the incident beam radiation on the façade is higher than 400 W/m^2). A unique feature of the identification is the mode switch between free cooling (using natural ventilation) and mechanical cooling, which results in different system dynamics. The switching behavior is indicated by the input $W_{spd,IO}$ based on the fact that when the building is in natural cooling mode, windows are open so that the outdoor wind speed has significant impact on the zone temperature inside the building; when the building is in mechanical cooling mode, windows are closed and thus the wind effect becomes negligible. The $W_{spd,IO}$ is the same as the real-time wind speed W_{spd} when windows open and becomes zero when windows close. The VAV box discharge air temperature and the supply flow rate are also inputs for the model identification accounting for the heat extraction from the zone when the mechanical cooling in ON.

The model is used for the development of predictive control strategies to achieve a trade-off between reduction of energy consumption and thermal comfort maintenance evaluated using the operative temperature index. Therefore, for the performance metrics evaluation, information for the air and surrounding surfaces temperature is required and thus these two parameters (air and weighted-average mean surface temperature) in the control-targeted zone (atrium) become the model outputs. Information on the system inputs and outputs is provided in Table 2. Note that the VAV box is controlled with a PID controller which samples every minute, hence, the simulation time step-size of the detailed model should be one minute. Directly using the one-minute time step simulation results (three-month

Table 2: Training inputs and outputs for the SID model

Notation	**Scaling range**	**Description**
Inputs:		
T_{ext}	[0, 40]	Outdoor temperature, °C
$Q_{tr,at}$	[0, 8000]	Transmitted solar radiation into atrium, affect by the blind control (threshold, 400 W/m^2), W
$Q_{tr,cor}$	[0, 8000]	Transmitted solar radiation into corridors, W
W_{spd}	[0, 10]	Wind speed, m/s
W_{dir}	[0, 360]	Wind direction, degrees
T_{sup}	[0, 40]	VAV discharge air temperature, °C
V_{sup}	[0, 1]	VAV supply flow rate, m^3/s
$W_{spd,IO}$	[0, 10]	Wind speed indicator for close/open window: when window close, it is zero, otherwise, it equals to real wind speed W_{spd}, m/s
Outputs:		

T_{at}	[0, 40]	Atrium zone temperature, ℃
T_s	[0, 40]	Area-weighted atrium zone surface temperature, ℃

simulation) would require significant computational effort. Thus, hourly averaged data are used for the identification. Furthermore, instead of directly using the selected parameter values, all the inputs and outputs are normalized considering an approximate range [0, 1] to avoid scaling issues (Spindler, 2004).

3.3 Signal Excitation

The system identification needs high quality training data that would cover a sufficiently large range in the frequency domain so that the identified model can predict the system dynamics under different excitation signals (Overschee and Moor, 1999). However, when the objective is to find a model suitable for control, it is not necessary for the training data to cover the entire frequency domain, but rather some control-relevant selection of frequencies. The data are generated based on prior knowledge of the time constants of the system. Let τ_H, τ_L represent the slowest and the fastest systems time constants, then the required frequency spectrum to be covered by the generated signal is:

$$\frac{1}{\beta\tau_H} \le \omega \le \frac{\alpha}{\tau_L} \tag{6}$$

where α defines the ratio of closed-and-open loop responses and the β defines the settling time. Their typical values are α =2 and β = 3, which corresponds to 95% of settling time (Braun et al., 2002). The time constant of the studied building can be found with the system matrix A in the full-scale model (Ruscio, 2009; Spindler, 2004):

$$T_i = -\frac{1}{\lambda_i(A)} \tag{7}$$

in which, $\lambda_i(A)$ denotes the eigenvalues of A matrix. For the dynamic system under consideration, the maximum and minimum time constant τ_H and τ_L are 90 and 2 hours which indicate the response time of temperature inside the massive corridor floor and the air zone temperature. Thus, with Equation 6, the required frequency spectrum to be covered by the generated signal is [1.0288e-6 Hz, 2.7778e-4 Hz]. Note that for buildings with mixed-mode cooling, it is critical to accurately predict the system dynamics under both the free cooling and mechanical cooling mode. Thus, a three-month (May to July) simulation period with the mixed-mode building controlled using a standard heuristic (rule-based) strategy (T_{ext} ∈ [15 ℃, 25 ℃], W_{spd} < 7.5 m/s) was implemented for generating training data. When mechanical cooling is required, night set back control is applied (set point temperature range from 13 ℃ to 30 ℃). The frequency spectrum covered in the three-month mode switch sequence was analyzed with the fast Fourier transfer (FFT) method (Duhamel and Vetterli, 1990) and the results are shown in Figure 3. It is observed that the mode switch sequence can cover the required frequency spectrum so that the simulation results with the heuristic control sequence have good quality for training the simplified model of the mixed-mode building.

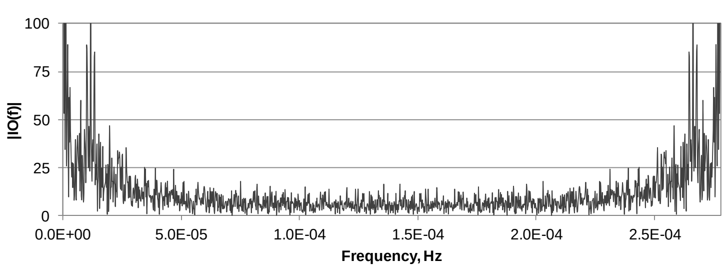


Figure 3: Frequency spectrum of the input data sequence (window open/close)

3.4 Selection of Model Order and Hankel Matrix Size

In this modeling representation, the states lose their physical meaning and an appropriate order needs to be determined to obtain the best fit with the training data. A common approach for the order selection suggests 2nd-3rd order dynamics per output temperature (Prívara et al. 2013), which leads to 4th-6th order dynamics based on the two system outputs considered herein. However, since the white noise of the system is set to be zero, a higher order state is required to reach better identification results. Moreover, there is mode switch which leads to significant change of the system dynamics in different time steps, so that the state order should be large to capture the switching dynamics.

After testing the fitting error for 10th, 20th, 30th, and 40th orders, a 30th order model appeared to have the best overall performance.
The size of Hankel matrices which is required in the training process and defines how far into the past/future the "measured" dataset is searched for training an appropriate model. Bigger size may lead to better results. However, there is a trade-off between computational difficulties and the matrices size. The study investigated the performance of models trained with different Hankel matrices size, i.e. 5, 10, 15, 30, 50, and 75. The model trained from the matrices size of 50 turns out to have the best performance.

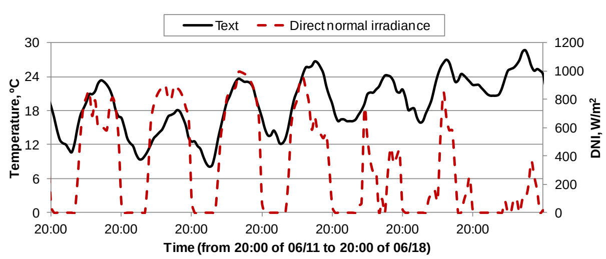
4. SYSTEM IDENTIFICATION RESULTS

4.1 Single-Model

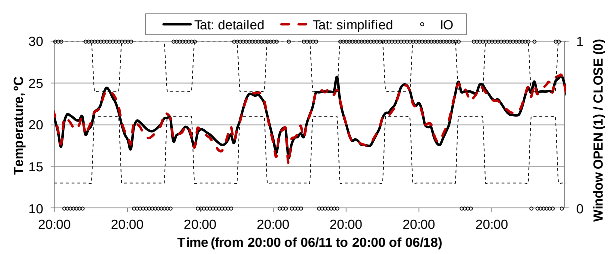
The system identification method described above is used to develop a simplified prediction model for mixed-mode buildings. A four-month (May to August) simulation period with the forward LTV-SS model is considered. A heuristic strategy is used for the mixed-mode cooling in order to prepare the datasets for training (May to July) and calibration (August). The TMY3 weather data for Montreal, Canada were used as the weather inputs in the simulation. Figure 4(b) presents results for the atrium air temperature obtained with the detailed and the simplified prediction model for a week during the training period. The sequence of mode switch between mechanical cooling and free cooling is denoted by *IO*, with the value 1 indicating that the motorized grilles are open, thus free cooling is ON and the value 0 means that the grilles close, thereby the mechanical cooling is ON. It can be seen that the simplified model results match well with those obtained with the detailed LTV-SS model, though small discrepancies are observed at the moments of mode switch due to the abrupt change of the system dynamics. However, overall good prediction accuracy is obtained as shown in Table 3. For the atrium air temperature prediction, the RMSE for the zone air and weighted-average surface temperature between the simulation and identification results is equal to 0.57 ℃ and 0.32 ℃ respectively. Figure 4(d) presents a comparison (frequency distribution for the error range) between the detailed and simplified model for the entire training period. It can be seen that 91% and 98% of the simplified model prediction error is in the range of [-1.0 ℃, 1.0 ℃] for the atrium zone and the surrounding surface temperature. The simplified model is subsequently used for the prediction of the building thermal behavior for a period of 7 days in August (calibration set) with results (Figure 5) showing a good prediction accuracy with the RMSE for the atrium zone air and weighted-average surface temperature equal to 0.64 ℃ a 0.44 ℃. The results show better model performance compared to that reported (1.21 ℃ in sunspace zone) in previous studies for mixed-mode buildings with high solar gains (Spindler, 2004).

4.2 Hybrid Model

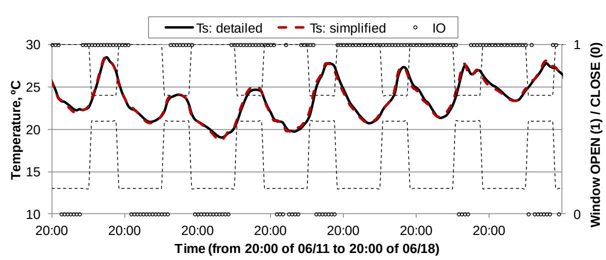
The present study investigated the feasibility of a hybrid modeling prediction approach using different simplified models for the two operation modes under investigation. The training data was firstly prepared by running the



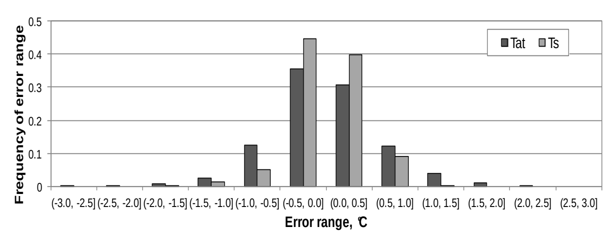
(a) Weather conditions



(b) Atrium zone air temperature

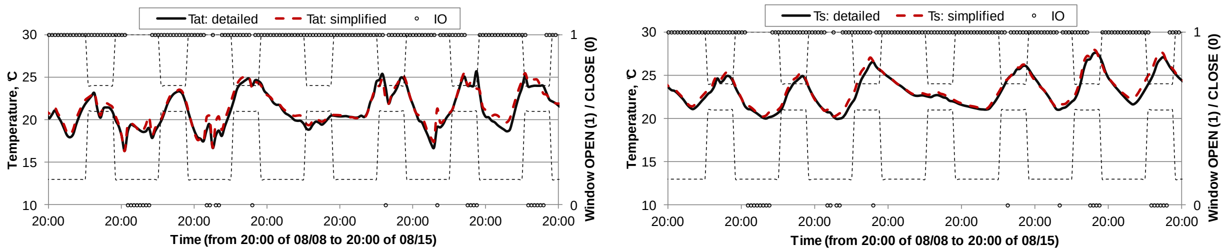


(c) Area-weighted surface temperature in the atrium zone



(d) Frequency distribution of error range

Figure 4: Comparison between the detailed and simplified model: training set



(a) Atrium zone air temperature (b) Area-weighted surface temperature in the atrium zone

Figure 5: Comparison between the detailed and simplified model: calibration set

Table 3: Identification results for the 4SID single and hybrid model

Model		Temp.	Training		Calibration	
			RMSE	R^2	RMSE	R^2
Single-model		T_{at}	0.57	0.96	0.64	0.94
		T_s	0.32	0.99	0.44	0.97
Hybrid-model	CLOSE	T_{at}	0.23	0.99	0.35	0.93
		T_s	0.19	0.99	0.32	0.92
	OPEN	T_{at}	0.22	0.99	0.34	0.99
		T_s	0.30	0.99	0.88	0.94

detailed LTV-SS model for each operation mode. Since each mode is considered separately, there is no mode switch indicator used as an input for the training. The identified models are then applied together for predicting the dynamic behavior of the switched system. An important aspect of the hybrid prediction approach is the state initialization during the mode (and model) switching. For instance, the switch dynamic system has two subsystems 1 and 2:

System-1, representing mode 1, has the form:

$$\begin{aligned} x_{k+1}^1 &= A_1 x_k^1 + B_1 u_k^1 \\ y_k^1 &= C_1 x_k^1 + D_1 u_k^1 \end{aligned} \tag{8}$$

System-2, representing mode 2, has the form:

$$\begin{aligned} x_{k+1}^2 &= A_2 x_k^2 + B_2 u_k^2 \\ y_k^2 &= C_2 x_k^2 + D_2 u_k^2 \end{aligned} \tag{9}$$

At the moment of switching from subsystem 1 to 2, the state vector x^2 has to be initialized – e.g. setup initial states for the dynamic development. Different from previous modeling work on hybrid systems, in which the states are physical variables and the ending states from previous operation mode can be adopted as initial states for the following operation mode (Petridis and Kehagias, 1998), the states of each simplified model (for each mode) in this study have lost their physical meaning, thus the ending states from the previous mode cannot be directly inherited as initial states for the upcoming operation mode. To address this issue, a warm-up simulation is used for each mode switch to find the initial state for the upcoming operation mode (Figure 6a). Note that there are initial states x_0^2 generated from the N4SID model training process of each operation model. The warm-up simulation uses the simplified model corresponding to the coming operation mode (Mode 2) and applies its trained initial states x_0^2 as the warm-up initial states. The model inputs of previous operation mode u_{k-1} are used as the inputs for the warm-up simulation. The ending states x_{k-1}^2 from the warm-up simulation then become the initial states for the upcoming operation mode.

Based on the identification results for the hybrid-model shown in Table 3, good prediction accuracy is obtained with each individual model for its corresponding operation mode with RMSE smaller compared to the single-model.

However, the hybrid prediction model does not show advantage over the single model as there are considerable errors at the moments of mode switch (indicated by the dashed arrow in Figure 6b) as the states cannot be precisely

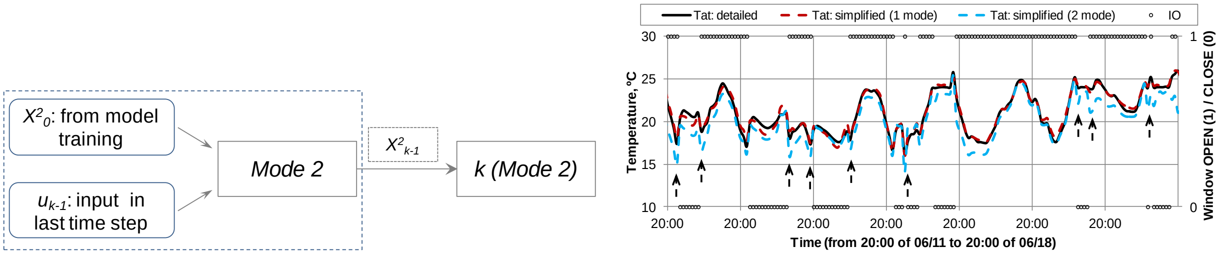


Figure 6: (a) Warm-up simulation for state initialization (Left); (b) comparison between the detailed and the hybrid simplified model (Right)

initialized using the warm-up simulation approach. The warm-up simulation using the inputs from the previous mode but applying the prediction model for the upcoming mode would result in inconsistencies in the values for T_{at} and T_s between the warm-up simulation results and the real outputs. Although the hybrid-model did not show better performance compared to the single-model, it can be seen as an initial attempt towards developing hybrid black-box models for buildings with mode switching that may lead to significant improvements in future research. The approach presented herein could result in superior overall performance for building energy systems with slower dynamics, compared to air systems, such as hydronic heating and cooling while such endeavors are currently under investigation.

4.3 Pseudo-Random Mode Switch

The identified model (single-model) will be used for developing MPC strategies for MM buildings. Since the MPC algorithm needs to search the optimal mode switching from multiple candidate sequences, the decisions are strongly dependent on the prediction of the overall energy cost and thermal comfort when these candidate mode switches are operated. Hence, the simplified model should be able to accurately capture this behavior. For this purpose, a pseudo-random model switching sequence is generated for validating the simplified model. The frequency spectrum shown in Figure 7(a) indicates that the sequence covers a variety of frequencies and contains equal power within a fixed bandwidth at any center frequency. It can be seen as a white noise signal and thus has high randomness (Duhamel and Vetterli, 1990). The fact that the model accurately captures the system dynamics with the random inputs and mode switch is a verification of its quality (Prívara et al. 2011). Both the full-scale LTV-SS model and the simplified model are used to predict the system behavior with the results plotted in Figure 7(b) indicating that the training data for the model identification provides sufficient excitation.

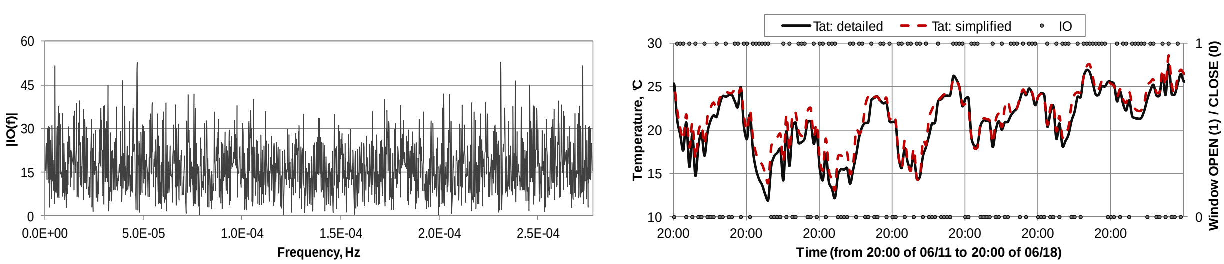


Figure 7: Comparison between the detailed and simplified model: (a) frequency spectrum of pseudo-random sequence; (b) air temperature comparison

5. CONCLUSIONS

This paper presented a control-oriented model for a complex building with MM cooling based on the sub-space identification method. The developed linear time-invariant (LTI) model was then validated with random control inputs. The main findings of the study can be summarized as follows:

- The N4SID identification was successfully applied to reduce the 280th-order LTV-SS model to 30th-order LTI-SS model. With an indicator denoting the mode switch between natural ventilation and mechanical

cooling, the simplified model can successfully capture the switching behavior of the MM building, with the RMSE for the atrium zone air and weighted-average surface temperature equal to 0.64 ℃ a 0.44 ℃.
- The hybrid model identification method though did not show advantage compared to the single-model, it provides an initial framework for future model identification for building energy systems with switched dynamics.

The presented system identification approach based on simulated input and output training data can be adopted to develop simplified models for MM buildings using measured data.

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