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RESEARCH ARTICLE

An Investigation of Starting Point Preferences in Human Performance on Traveling Salesman Problems

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Previous studies have shown that people start traveling sales problem tours significantly more often from boundary than from interior nodes. There are a number of possible reasons for such a tendency: first, it may arise as a direct result of the processes involved in tour construction; second, boundary points may be perceptually more salient than interior points, and selected for that reason; and third, starting from the boundary may make the task easier or be more likely to result in a better tour than starting from the interior. The present research investigated each of these possibilities by analyzing start point frequencies in previously unpublished data and by conducting an experiment. The analysis of start points provided some slight but contradictory support for the hypothesis that start selections result from the process of tour construction, but no evidence for the perceptual salience explanation. The experiment required participants to start tours either from a boundary or from an interior point, to test whether there was an effect on the quality of tour construction. No evidence was found that starting point affected either the length of tours or the time required to produce them. However, there was some indication that starting from a central location may be more likely to result in crossed arcs.

INTRODUCTION

In the past two decades there has been substantial progress in the study of human performance on combinatorial optimization problems, in particular the two-dimensional traveling salesman problem (TSP), which requires finding the shortest closed path through a set of points in the plane. A recent review identified at least six distinct theoretical positions and cited some 60 articles, many of which reported empirical findings (MacGregor & Chu, 2010). It has also been observed that the theoretical relevance of a number of empirical findings has not been fully explored (MacGregor, 2013). One such finding is that when free to begin a TSP tour from any point, people show a powerful predisposition to select a boundary point (i.e. a point on the perimeter of the point set, also referred to as the *convex hull*). One example of this preference was reported in MacGregor, Ormerod and Chronicle (2000), where 71 of 99 participants (72%) started a tour of a 48-node point set from a boundary point, compared with 29% expected by chance. More recently, MacGregor (2012) found that boundary starts were selected in 71% of 277 tours compared to an expected 50%.

While there appears to be a preference for certain starting points, a question remains whether this is a significant component of the solution process or an essentially arbitrary decision imposed by having to produce a serial solution, by drawing it or tracing it on a touch screen. For example, the pyramid model does not proceed in a serial fashion but rather uses a top-down clustering procedure that, at the final level, produces a complete solution (Graham, Joshi, & Pizlo, 2000). This underlines the possibility that human solutions are generated by parallel processes that take place in advance of the step-by-step physical solution, in which case selection of a starting point may not have any intrinsic role in the actual solution process. However, Best (2005) has presented evidence against this interpretation, which suggests that when human tours are collected solvers do not reproduce an already completed solution but that they work interactively with the problem in a serial fashion. Some models incorporate both elements, hypothesizing a fast global process that produces an initial outline that then guides local decisions that serially produce a detailed solution (Best, 2005; Kong & Schunn, 2007a, 2007b; MacGregor, Ormerod, & Chronicle, 2000).

If selection of a starting point is an intrinsic aspect of generating a TSP tour, then any complete model has to incorporate features that reflect this characteristic of human solutions. The present article represents a preliminary exploration of this issue, by addressing possible explanations for the observed human preference for starting tours from the boundary of a point set.

There are at least three plausible reasons why people may be more likely to start tours from boundary nodes. One is that boundary nodes are selected as a result of the cognitive processes that underlie tour production. For example, some theories propose that the process of tour construction begins with establishing an initial contour that surrounds or passes

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through the point set (Best, 2005; Kong & Schunn, 2007a, 2007b; MacGregor, Ormerod, & Chronicle, 2000). It has been proposed that points that lie on or near this initial contour are likely to be selected as start points (MacGregor, 2012). If so, then if an initial contour lies on, or near, the convex hull, then so will the starting point. A second reason for their predominance as starting points is that boundary nodes may be more salient than interior nodes and are selected simply because they stand out. A third reason is that starting from a boundary point may be more effective than starting from an interior point, resulting in shorter tours or faster tour construction. This might occur if, for example, a boundary start reduces the cognitive load in completing the task, or decreases the likelihood of making decisions that predetermine a poorer outcome than necessary (such as creating crossed arcs). The present article addresses each of these initial contour, relative salience and effectiveness explanations for the predominance of boundary starts. It should be emphasized that the investigations reported here are initial and exploratory, and hopefully future research will build on the present results to provide more definitive conclusions.

INITIAL CONTOUR EXPLANATION

Several theories have proposed that an initial contour around or through the set of nodes is formed that then guides local serial decisions to produce a completed tour. For the sequential convex hull (SCH) model, for example, this initial contour is the convex hull itself (MacGregor, Ormerod, & Chronicle, 2000). A second theoretical approach has proposed that the initial contour is a modified convex hull drawn through the centroids of clusters of nodes lying on the boundary of the point set and then elaborated (Best, 2005). A third suggestion is that the contour consists of a spline curve through the centroids of clusters distributed throughout the point set (Kong & Schunn, 2007a, 2007b).

While all three theories either imply or explicitly state that a tour is constructed serially from a starting point, the procedure for starting-point selection has not necessarily been expressed as part of a model's specification. Nevertheless, it is possible to deduce some characteristics of starting points from the published descriptions.

For the SCH model the initial contour is the convex hull, and if a starting point is selected that lies on the initial contour (MacGregor, 2012), then all start points will be hull points.

The description of the Kong and Schunn model refers to a *current point* that is set to be the starting point, following which points are grouped into clusters whose centroids become *reference points* (Kong & Schunn, 2007b). The current point (starting point) and reference points are then connected to form a closed spline curve, from which we can conclude that the starting point necessarily lies on the spline curve.

Best's model consists of four main stages, the first being the clustering of points based on proximity. Next, a global plan is constructed, initially based on the convex hull through the centroids of boundary clusters. Stage 2 is therefore similar to the initial stage of the SCH model, except it uses boundary clusters instead of individual boundary points. However, stage 2 will omit any interior clusters from the global plan, and so at stage 3 unincorporated clusters are inserted between pairs of boundary clusters, using a cheapest insertion criterion. Stage 3 is therefore similar to the second stage of the SCH model, except using clusters rather than individual points. The result of the first three stages, according to Best, is that clusters are placed in serial order and the final stage processes clusters in this order. At this last stage, local decisions are made to select the order for connecting nodes within clusters, starting with the "current cluster" and proceeding in the order in which clusters have been placed. While it is clear that the starting point must therefore be selected from the current cluster, how this is determined and at what stage of the process appears not to be described in the model's published description. Nevertheless, there appear to be two reasonable possibilities. One is that a starting cluster is determined at stage 2, in which case it will be on the "convex hull of clusters" (2005, p. 259). The second is that it is determined at stage 3, in which case all clusters would be eligible. In either case, the starting node is selected from within the starting cluster. In the former case (Best Model version A), because the starting cluster lies near the convex hull, the starting point must also lie relatively close to the convex hull, although not necessarily on it. In the latter case (Best Model version B), the starting point may be located within any cluster.

If TSP solvers select start points on the initial contour (or close to it), then the approaches predict starting points (i) on the hull (SCH), (ii) close to, but not necessarily on, the hull (Best Model A), and (iii) distributed throughout the point set, depending on the locations of clusters (Best Model B, Kong & Schunn model).

Recently, MacGregor (2012) reported findings where a significant majority of tours started from a point on the hull, which was interpreted as consistent with the SCH model. However, because the stimuli comprised 10 points only, there may have been no salient clusters beyond the individual nodes themselves. Because of this, the results may not have provided a fair test of the other two approaches, which emphasize the role of clusters as well as initial contours. Using stimuli with a salient cluster near the boundary or near the center would avoid this limitation, and provide for a better comparison of the approaches. Figure 1 provides a schematic illustration of the differences between the theoretical approaches using stimuli having relatively large clusters placed either near the boundary or near the center.

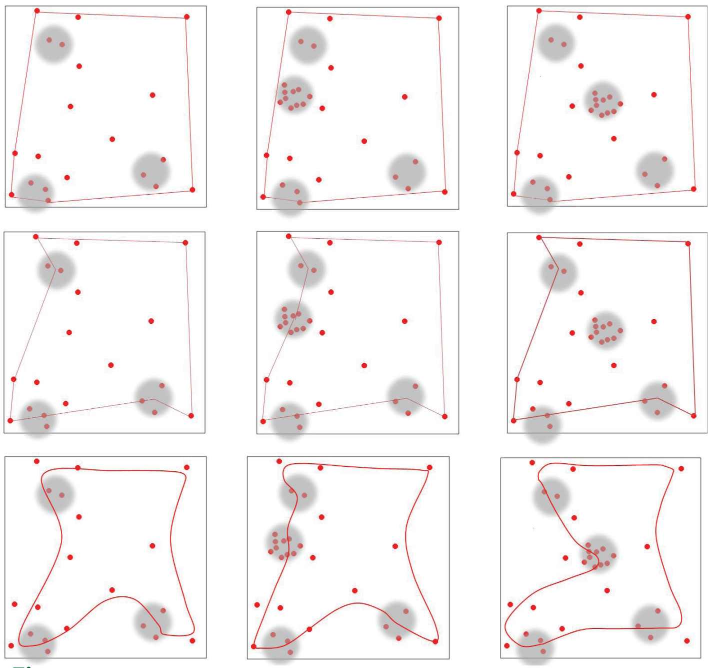

Figure 1
The nodes in the figures in the left, center and right columns represent no cluster, boundary cluster and center cluster conditions, respectively, the shaded circles represent possible clusters, while the lines in the figures in the top, center and bottom rows illustrate schematic initial contours consistent with the SCH, Best Model A, and Best Model B/Kong and Shunn models, respectively.

The figure shows three different point sets across the columns with the same three repeated in the rows. The point set in the first column contains no large, salient clusters, the point set in the center column has a large cluster close to a boundary, while the point set in the third column has exactly the same cluster located centrally (possible clusters are indicated by shading). Lines illustrate potential initial contours for the SCH model (top row), Best's model version A (center row) and the Best model version B and Kong and Schunn model (bottom row).

The initial contour for the SCH model passes through each of the boundary points in order, and therefore is as shown for each of the three point sets. For the other models, the initial contour will depend on the perceptual clusters that an individual forms, and those suggested in the figure are illustrative only. Nevertheless, it is possible to formulate some general differences between the approaches.

If indeed participants tend to select starting points from those lying on or near the initial contour then the SCH approach suggests no effect on starting selections of adding a cluster off-boundary or near-center: boundary starting points should remain the preference whether or not there are salient clusters, and regardless of the locations of those clusters. Under this model, the expectation would be that the relative frequency of boundary starts would be unchanged by adding clusters near the boundary or near the center.

Conversely, if the initial contour conforms to Best's (2005) model (version A), then adding a salient cluster near the boundary could influence starting point selection, because the initial contour will be drawn inside by the cluster and away from the convex hull. Some or all of the nodes in the large cluster become potential starting points and, because they are non-boundary points, they will reduce the relative frequency of boundary starts. Adding a cluster to the center should have no similar effect as the initial contour will remain close to the perimeter of the point set. The expectation is therefore that adding a cluster near the boundary may reduce the relative frequency of boundary starts whereas adding a cluster to the center is less likely to do so.

Finally, if the Kong and Schunn model is correct, and the initial contour is a spline curve that passes through all salient clusters, then adding a large cluster close to the boundary will have a similar effect to that of the Best model version A, while adding a cluster to the center will ensure that the spline curve passes through the center and may attract the selection of at least some starting points to this location. The same applies to the Best model B version. In both cases, the indication is that adding a salient cluster near the boundary or near the center may attract starting points to those locations and reduce the relative frequency of boundary starts.

SALIENCY EXPLANATION

A different explanation for the predominance of hull starts is that boundary points are relatively more salient than interior points. According to Koch and Ullman's (1985) definition of saliency, points in the visual scene are salient to the extent that they differ from their neighbors. With random dot arrays most of the visual features by which points can differ—size, shape, orientation, hue, brightness, and so—are absent. Among the few that remain are the location of points and the proximity of neighbors. Each of these may be considered as separate dimensions, each with polar opposites of potential salience.

In terms of location of neighbors, boundary points differ from other points in having neighbors on one side only, representing one pole of the location dimension. At the opposite pole, a central point differs in being surrounded by an approximately equal density of neighbors. In terms of proximity of neighbors, at one pole, points within clusters differ from others by having a number of close neighbors. At the opposite pole, isolated points may stand out by having no close neighbors.

The two dimensions of potential saliency are illustrated in Figure 2. Although they are represented orthogonally, they are not necessarily independent. For example, hull points may also tend to be isolated points. The figure also shows a number of potential outcomes, under the assumptions that the factors influence saliency and that saliency influences

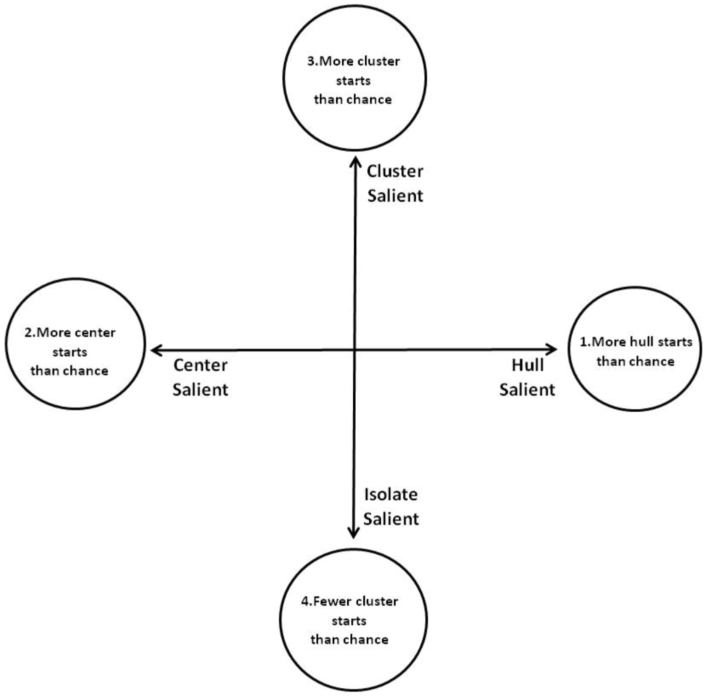

Figure 2
Potential dimensions of saliency and predicted outcomes for starting point selection.

start selections. Considering first the horizontal dimension, the pole to the right proposes that hull points are salient, in which case hypothesis 1 is that hull points will be selected as starts more frequently than expected by chance. At the opposite pole, points may be salient because of their central location, in which case we might expect that center points will be selected as starts at above chance levels (H2). The two hypotheses are not necessarily mutually exclusive and both hull and center points could be selected at above chance levels.

Considering next the vertical axis, if clusters are salient, then more cluster points may be selected as starts than expected by chance (H3). At the other extreme, if isolated points are salient, then isolated points may be selected more frequently. Again the two effects could theoretically co-exist, but in the present case, "isolated" points have been defined as points falling outside of a large cluster. For this reason, the hypothesis is framed in terms of the selection of fewer cluster points rather than more isolated points (H4).

EFFECTIVENESS EXPLANATION

It is possible that starting from a boundary location may be more likely to lead to shorter tours than starting from other locations. This might occur if doing so expedited the decision process, or helped avoid choices, such as creating crossings, that constrain tours to being suboptimal. Solvers may be sensitive to this potential advantage of boundary starts and select them accordingly.

Both the initial contour and saliency explanations were tested using previously unreported starting data from an experiment described in MacGregor (2013). A new experiment was conducted to examine the effectiveness explanation.

ANALYSIS OF STARTING DATA

MacGregor (2013) reported an experiment where 19 participants constructed tours of 15 point sets. The point sets were created by first randomly generating a 20-node array, subject to the constraint that there was sufficient unoccupied space to comfortably locate a 10-node cluster near a boundary and near the center. This represented a no-cluster stimulus. Next, a 10-node cluster was randomly generated and located close to the boundary, to create a boundary cluster stimulus. A center cluster stimulus was created by placing the same cluster centrally. An example of one such stimulus triplet was shown in Figure 1. This procedure was iterated five times to create the 15 stimuli, five each in no cluster, boundary cluster, and center cluster conditions. Participants were free to start tours from any point, providing data on the relative frequency of boundary versus interior starts and cluster versus no-cluster starts.

RESULTS AND DISCUSSION

Overall, participants chose starting points on the boundary for 160 of the 285 tours (56%), compared with 28% expected based on the relative frequency of boundary points. Significance was tested by comparing the mean number of boundary starts across the 15 problems with the mean number expected by chance, using a one-sample t-test. The resulting t-value was 4.11 (df = 18), $p < 0.001$. The finding supports previous reports of a significant preference for boundary starts. At the same time, the proportion of boundary starts was substantially lower than previously observed (56% compared with approximately 70%) which could be due to the presence of the clusters. Also, there may be considerable individual differences. In the present case, three participants chose hull starts on all 15 of their tours, while three did so on three or fewer.

Initial Contour Explanation

As described above, the initial contour explanation for the high incidence of boundary starts proposes that participants select points that lie on (or near) a contour that forms early in the tour construction process. According to the sequential convex hull model, adding a cluster of nodes near the boundary or at the center of the array should not affect starting point selection, and boundary points should remain preferred. If Best's (2005) proposal, Model A version, is correct, then adding a cluster near the boundary, but not the center, may attract starting selections away from the boundary. In contrast, if either the Best model (version B) or the Kong and Schunn approach (2007a) is correct, then adding a salient cluster either near the hull or at the center may attract starting selections. In other words, under the SCH approach the expectation is no reduction in boundary starts by adding boundary or center clusters. Under the Best model (version A), boundary starts may be reduced by adding a boundary cluster but not a center cluster. Both the Best (version B) and Kong and Schunn models suggest that boundary starts may be reduced by the addition of either type of cluster. To test these different possibilities, the proportion of boundary starting points was compared for each condition, but before doing so an adjustment was made for differences in the availability of different start locations. This arose because the no cluster conditions had 20 nodes, while the two cluster conditions had 30.

The proportion of boundary points available as starting points differed among the three conditions, at 37% for the no cluster condition and 23% for each of the cluster conditions. To allow for this, the expected proportion of boundary starts was subtracted from the observed proportion for each participant in each of the three conditions. The resulting proportions of boundary starts above chance were 28%, 34% and 20%, for the no cluster, boundary cluster, and center cluster conditions, respectively. Differences between conditions were compared using a repeated measures ANOVA, which indicated no overall significant differences, $F(2, 36) = 2.64$, MSe =0 .04, $p = 0.09$, $eta^2 = .13$. Nevertheless, although there was no overall significance, a priori contrasts were examined between specific pairs of conditions, to test theoretical predictions.

The SCH model suggests that there should be no difference in the frequency of boundary starts between the no-cluster condition and either of the cluster conditions in relative frequency of boundary starts. No significant differences were observed between the no-cluster and boundary cluster conditions, $t(18) = 1.05$, $p = .92$, $d = .49$, between the no-cluster and center cluster conditions, $t(18) = 1.30$, $p = .64$, d = .61, or between the boundary cluster and center cluster conditions, $t(18) = 2.16$, $p = .13$,d = 1.02 (all comparisons Bonferroni corrected). Best's model (version A) suggests that fewer boundary starts may occur in the boundary cluster condition than in the other two conditions. As indicated above, no difference was found between the boundary cluster and no cluster conditions, or between the boundary cluster condition and the center cluster condition and, although not significant, both differences were in the opposite direction.

Both the Best model (version B) and the Kong and Schunn model allow for the possibility that there will be relatively fewer boundary starts in both cluster conditions than in the no cluster condition, which was not observed in either of the above comparisons. Nevertheless, the fact that the center cluster condition had fewest boundary starts, even though non-significant in the comparisons, may reflect some support for these interpretations. Additional research will be necessary to investigate whether locating clusters at a variety of interior positions does indeed have the effect of attracting starting choices away from the boundary.

While the results may be more consistent with the SCH model than the others insofar as the inclusion of a large cluster had no significant effect on starting preferences, they were inconsistent in another respect. The fact that a substantial proportion of non-boundary nodes were selected as start points (44%) is not consistent with the SCH explanation, whereas it is consistent with the other approaches. In this respect, the results may be seen as more supportive of the other models than of the SCH. At the same time, the results may indicate that the initial contour explanation in general is at fault, and that my previous suggestion (MacGregor, 2013), that nodes lying on or near the contour are selected as starting points is incorrect. If so, then starting point selection in general, and the preference for hull starts in particular, must be determined by other factors.

Saliency Explanation

In the introduction, a definition of saliency was provided, based on Koch and Ullman (1985), from which two dimensions of saliency were derived. These were presented in Figure 2, together with four related hypotheses. The hypotheses rest on the assumptions that: hull nodes, center nodes, cluster nodes and isolated nodes are salient; and that salient nodes are more likely to be selected as start points. The hypotheses proposed that there will be more hull starts than chance (H1), more center starts than chance (H2), more cluster starts than chance (H3) and fewer cluster starts than chance (H4). While the third and fourth hypotheses are mutually exclusive, either could be interpreted as being consistent with a saliency explanation. Overall, it is possible for any three of the four effects to be observed.

As shown above, H1 was confirmed by the data, with more tours starting from hull points than expected by chance. Hypothesis 2 was tested by counting how many tours started from a center point, where "center point" was defined as the point closest to either the center of mass or the centroid of the point set. Overall, 23 of 285 tours (8%) started from a center point, compared with 4% expected based on their relative frequency. Significance was tested by comparing the mean number of center starts across the 15 problems with the mean number expected by chance, using a one-sample *t*-test. The *t*-value was 1.20 (df = 18), $p = .24$, $d = .57$. The finding fails to support a preference for center starts, although one participant did select a center start on 10 of 15 tours.

Hypotheses 3 and 4 were tested by counting the number of start points selected from the 10-node clusters in the two cluster conditions. A larger number than expected by chance would support H3, while a smaller number than expected

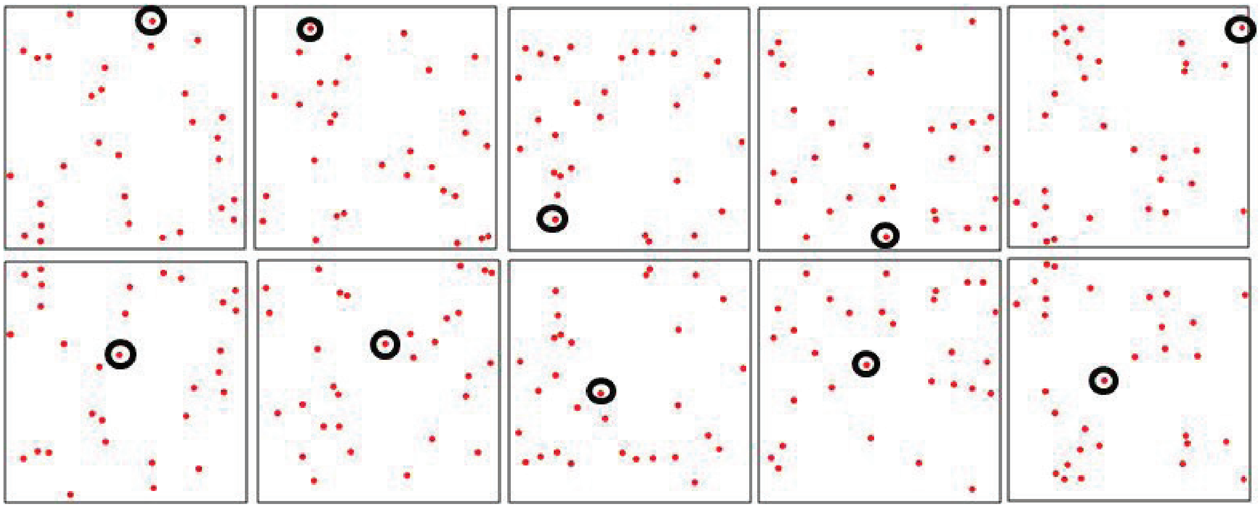

Figure 3
The upper row shows the boundary start stimuli of the experiment, the lower the corresponding center start stimuli. The required starting points are circled.

would support H4. For the two cluster conditions overall, the frequency of cluster starts was 31% compared with a chance expectation of 33% (i.e. for each condition 10 of the 30 points were in the large clusters). The observed frequency did not differ significantly from chance. For the two conditions separately, the frequencies of cluster starts were 24% and 37% for boundary and center cluster conditions, respectively, compared with a chance expectation of 33% in both cases. In each case, one sample *t*-tests compared the observed mean number of cluster starts with the mean number expected if selections were made at random. The results were $t(18) = -1.56$, $p = .14$, $d = .74$, and $t(18) = 0.44$, $p = .67$, $d = .21$, for boundary cluster and center cluster conditions, respectively, neither differing significantly from chance. The results did not offer strong evidence that people either select or avoid large clusters when choosing a starting node. Overall, the results were consistent with only one of the four salience hypotheses. Since this was the already known result, that hull nodes are selected more frequently as starts, the present data offered no additional support for a salience explanation.

The final explanation for starting preferences considered here, the effectiveness explanation, was examined experimentally, as described below.

EXPERIMENT

The primary purpose of the experiment was to determine whether the location of the node from which a tour starts—interior or boundary—influences the quality of performance. To create as extreme a contrast as possible, and because central points are the most frequently selected interior starting points (MacGregor, 2012), the experiment compared central starts with boundary starts.

METHOD

Participants

Participants were twenty volunteers recruited from the campus community at the University of Victoria. They were each given a $20 cafeteria voucher for participating.

Stimuli

The stimuli were ten 30-node point sets. Each set was displayed within an on-screen area of 125x134mm, using nodes of radius 3mm. Initially, five sets were generated randomly, and then each was used to create a matched twin by flipping the coordinates about the horizontal axis. Members of each pair were therefore structurally identical but not visually identical. One member of each pair was designated as a boundary start stimulus, the other as a center start stimulus.

For each center start stimulus the point closest to the center of mass of the 30 nodes was selected as the start point. Because central points may be closer to their nearest neighbor on average than boundary points, each boundary starting point was selected as the point on the convex hull whose distance to its nearest neighbor was as similar as possible to that of the corresponding center start stimulus. The means (and standard deviations) of the nearest neighbor distances were 68.68 (16.36) pixels and 68.64 (16.09) pixels for the center start and boundary start stimuli, respectively. Start points were marked with a surrounding ring. Figure 3 illustrates the 10 stimuli.

Procedure

Participants were tested individually. Each problem was presented on screen and participants completed a tour by pointing and clicking. Participants were instructed to start each tour from the designated point and to try to find the shortest tour connecting all the points. An example was provided. There was no fixed time limit, although participants were told that each example should require only "a few minutes" to complete.

Half of the participants received the five center start stimuli first followed by the boundary start stimuli, and half completed the tasks in the reverse order.

RESULTS AND DISCUSSION

For each stimulus set the shortest (optimal) tour was found and participants' tours were expressed in terms of the percentage length above the optimal (PAO) by subtracting the optimal length from each tour and dividing by the optimal length (x100). Solution times for each tour were also obtained, and the number of tours containing crossed arcs was counted.

The mean PAO (and standard deviation) was 4.22% (3.20%) for boundary and 3.82% (2.46%) for center start tours. The corresponding results for solution times were 94.33s (44.84s) and 95.22s (46.35s), respectively. Paired sample t-tests indicated no significant differences between boundary and center start conditions for either PAO, $t(19) = 0.77$, $p = .54$, $d = .34$, or for time, $t(19) = 0.18$, $p = .86$, $d = .08$. In addition, the differences between boundary and center start performances were tested for each of the five stimulus pairs separately and no significant differences emerged (all p-levels > .05) for either PAO or solution times.

For each participant the number of tours with crossed arcs was counted for the boundary and center start conditions. For boundary starts, there were a total of two tours with crossings, yielding a mean (standard deviation) across the 20 participants of 0.10 (0.31). The corresponding results for center starts were seven tours with crossings, and a mean (standard deviation) of 0.35 (0.59). A paired samples t-test indicated that there was no significant difference between the means, although in this case the p-value did not reject the null hypothesis by as wide a margin as with the other dependent variables, $t(19) = 1.75$, $p < .10$, $d = .78$.

GENERAL DISCUSSION

Previous studies have shown that people start TSP tours significantly more often from a boundary than from an interior node. There are a number of possible reasons for such a tendency: first, it may arise as a direct result of the processes involved in tour construction; second, boundary points may be perceptually more salient than interior points, and selected for that reason; third, starting from the boundary may make the task easier or be more likely to result in a better tour than starting from the interior. The present research investigated each of these possibilities.

One aspect of the process of tour construction that may influence starting point selection is the formation of an initial contour that several theories hypothesize. The SCH model proposes that tour construction begins by connecting adjacent nodes on the convex hull. Best's (2005) model incorporates a modification of this process, in which the initial contour is drawn through the centroids of clusters close to the convex hull. A third theory holds that the initial contour is a spline curve passing through the centroids of all clusters (Kong and Schunn, 2007a, 2007b). Each of these positions potentially could explain a predominance of boundary starts. To test them, unpublished starting data from MacGregor (2013) were analyzed to examine the effects of a prominent cluster of nodes placed either near the boundary or near the center of point sets. The expectation under the SCH model is that the added clusters should have no effect on the frequency of boundary starts, which would continue to be the sole choice. Under Best's model (version A) hull starts might be reduced, but in the boundary cluster condition only. A reduction of hull starts in both cluster conditions would be consistent with both the Kong and Schunn model and Best model (version B). The results indicated no significant effect of adding a cluster either to the boundary or center of point sets. However, the fact that a substantial proportion of start points (44%) were interior nodes rather than boundary nodes is consistent with these models and inconsistent with the SCH model. Also, fewer hull starts were observed here than in previous research, at 56% compared with 70% or more, which may have been due to the presence of the large clusters. Overall, however, the analyses of the initial contour explanation were inconclusive. This may signify that the initial contour explanation is incorrect, and that there is some other reason for the preference for boundary starts.

The same data provided an opportunity to test whether a preference for hull starts may be due to their perceptual salience. Following Koch and Ullman (1985), several potential reasons for a point's salience were identified: being on the boundary, being at the center, being in a cluster, and being isolated from other points. However, the findings failed to provide any significant evidence for the salience hypothesis, beyond the initial finding of a preference for hull starts.

Finally, an experiment was conducted to test whether starting from the boundary resulted in superior performance over starting from an interior location. The results showed no evidence of significant differences in performance, either in the relative lengths of tours or in the time required to complete them and failed to support a hypothesis that the observed tendency for participants to select boundary nodes as starting points arises because it results in superior performance. At the same time, the results indicated a borderline significant association between center starts and a higher incidence of crossings. This may indicate that participants prefer to start from a hull node because doing so helps to eliminate crossings. The possibility would be supportive of other theoretical approaches (van Rooij, Stege, & Schactman, 2003; Vickers, Lee, Dry, & Hughes, 2003) and may merit further investigation.

LIMITATIONS

The present research represents a first attempt to address the issues that it raises, and should be viewed as exploratory only. One limitation is that both the initial contour and saliency explanations were tested using existing data rather than data from experiments specifically designed to test them. Another is that, although the effectiveness explanation was tested using an experiment designed for the purpose, constraining participants to start from specified nodes may have interfered with their performance and influenced the results. I am grateful to Matthew Dry for this suggestion.

CONCLUSIONS

The investigation failed to provide persuasive evidence for any of the three proposed explanations for the observed preference for human participants to start tours from nodes on the boundary of point sets. It is possible that one or more of the explanations is correct but the present data were insufficient to discern it. It is also possible that the observed effect is due to another factor that remains to be identified and investigated in future research.

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REFERENCES

Best, B. J. (2005). A model of fast human performance on a computationally hard problem. *Proceedings of the 27th Annual Conference of the Cognitive Science Society* (pp. 256–261). Wheat Ridge, CO: Cognitive Science Society.

Dry, M., Preiss, K., and Wagemans, J. (2012). Clustering, randomness, and regularity: Spatial distributions and human performance on the traveling salesperson problem and minimum spanning tree problem. *The Journal of Problem Solving*, *4*(1), 1–17. http://dx.doi.org/10.7771/1932-6246.1117

Graham, S. M., Joshi, A., & Pizlo, Z. (2000). The traveling salesman problem: a hierarchical model. *Memory & Cognition*, *28*(7), 1191–1204. http://dx.doi.org/10.3758/BF03211820

Koch, C., & Ullman, S. (1985). Shifts in selective visual attention: Towards the underlying neural circuitry. *Human Neurobiology*, *4*, 219–227.

Kong, X., & Schunn, C. D. (2007a). Global vs. local information processing in visual/spatial problem solving: The case of traveling salesman problem. *Cognitive Systems Research*, *8*(3), 192–207. http://dx.doi.org/10.1016/j.cogsys.2007.06.002

Kong, X., & Schunn, C. D. (2007b). Information seeking in complex problem solving. *Proceedings of ICCM-Eighth International Conference on Cognitive Modeling* (pp. 261–266). Oxford, UK: Taylor & Francis.

MacGregor, J. N. (2012). Indentations and starting points in traveling sales tour problems: implications for theory. *The Journal of Problem Solving*, *5*(1), 2–17. http://dx.doi.org/10.7771/1932-6246.1140

MacGregor, J. N. (2013). Effects of cluster location on human performance on the traveling salesperson problem. *The Journal of Problem Solving*, *5*(2), 33–41. http://dx.doi.org/10.7771/1932-6246.1151

MacGregor, J. N., and Chu, Y (2011). Human performance on the traveling salesman and related problems: A review. *The Journal of Problem Solving*, *3*(2), 1–29. http://dx.doi.org/10.7771/1932-6246.1090

MacGregor, J. N. & Ormerod, T. (1996). Human performance on the traveling salesman problem. *Perception & Psychophysics*, *58*(4), 527–539. http://dx.doi.org/10.3758/BF03213088

MacGregor, J. N., Ormerod, T. C., & Chronicle, E. P. (2000). A model of human performance on the travelling salesperson problem. *Memory & Cognition*, *28*(7), 1183–1190. http://dx.doi.org/10.3758/BF03211819

van Rooij, I., Stege, U., & Schactman, A. (2003). Convex hull and tour crossings in the Euclidean traveling salesperson problem: Implications for human performance studies. *Memory & Cognition*, *31*(2), 215–220. http://dx.doi.org/10.3758/BF03194380

Vickers, D., Lee, M.D., Dry, M., & Hughes, P. (2003). The roles of the convex hull and number of intersections in performance on visually presented travelling salesperson problems. *Memory & Cognition*, *31*(7), 1094–1104. http://dx.doi.org/10.3758/BF03196130