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Activation of Operational Thinking During Arithmetic Practice Hinders Learning And Transfer

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Many children in the U.S. initially come to understand the equal sign operationally, as a symbol meaning "add up the numbers" rather than relationally, as an indication that the two sides of an equation share a common value. According to a change-resistance account (McNeil & Alibali, 2005b), children's operational ways of thinking are never erased, and when activated, can interfere with mathematics learning and performance, even in educated adults. To test this theory, undergraduates practiced unfamiliar multiplication facts (e.g., 17-times table) in one of three conditions that differed in terms of how the equal sign was represented in the problems. In the operational words condition, the equal sign was replaced by operational words (e.g., "multiplies to"). In the relational words condition, the equal sign was replaced by relational words (e.g., "is equivalent to"). In the control condition, the equal sign was used in all problems. The hypothesis was that undergraduates' fluency with practiced facts and transfer problems would be hindered in the operational words condition compared to the other conditions. Results supported this hypothesis, indicating that the activation of operational thinking is indeed detrimental to learning and transfer, even in educated adults.

Prior knowledge in a domain is typically beneficial for solving problems in that domain. However, when the context activates prior domain knowledge that is unhelpful for and/or incompatible with the target problem, it can fail to support—or even interfere with—performance (Bilalić, McLeod, & Gobet, 2008a, 2008b; Lippman, 1994; Lovett & Anderson, 1996; Kotovsky, Hayes, & Simon, 1985; McNeil, Weinberg et al., 2010; Ricks & Wiley, 2014; Wiley, 1998). For example, use of mnemonic letters in algebraic expressions (e.g., c stands for the price of a cake, b stands for the price of a brownie) can activate children's prior knowledge of letters as abbreviations, which hinders their ability to use letters as variables (McNeil, Weinberg et al., 2010). Similarly, presenting the equal sign within the context of an arithmetic problem (e.g., 15 + 13 = 28) can activate overly narrow ways of thinking about the equal sign, which hinders children's ability to learn from a lesson on the equal sign (McNeil, 2008). In the present study, we tested the idea that the activation of old, overly narrow ways of thinking can hinder learning, even after more generalizable ways of thinking have been acquired. Specifically, we tested the hypothesis that activating narrow ways of thinking about the equal sign during arithmetic facts practice reduces the benefits of that practice, even in educated adults.

Children in the United States tend to interpret the equal sign in a narrow way. Instead of viewing it relationally as a symbol indicating that two quantities share a common value and are, thus, interchangeable within a mathematical context, they tend to view it operationally as a signal to add up all the numbers and put the total in 'the blank' (Alibali, 1999; Baroody & Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980; Falkner, Levi, & Carpenter, 1999; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Li, Ding, Capraro, & Capraro, 2008; Kieran, 1981; Loehr, Fyfe, & Rittle-Johnson, 2014; McNeil, 2005, 2008; Perry, 1991; Powell & Fuchs, 2010; Renwick, 1932; Weaver, 1973). This is worrisome because this operational way of thinking does not generalize beyond simple arithmetic, and a relational understanding of the equal sign is critical for success in algebra (Falkner et al., 1999; Jacobs et al., 2007; Kieran, 1992; Knuth, Stephens, McNeil, & Alibali, 2006; National Research Council, 2001; Steinberg, Sleeman, & Ktorza, 1990). Indeed, Booth, Barbieri, Eyer, & Pare-Blagoev (2014) have shown that errors involving the equal sign in Algebra 1 negatively predict students' year-end algebraic competence, even after controlling for the rates of other kinds of errors, such as those involving negation and fractions.

A growing body of evidence indicates that children's operational ways of thinking about the equal sign, and equations more generally, are attributable, in part, to prior knowledge constructed from their experience with arithmetic in elementary school (e.g., Baroody & Ginsburg, 1983; McNeil & Alibali, 2005b; McNeil, Rittle-Johnson, Hattikudur, & Petersen, 2010; McNeil et al., 2012; Seo & Ginsburg, 2003; Sherman & Bisanz, 2009). Children are typically taught arithmetic in an overly narrow way with little reference to the equal sign as an indicator of mathematical equivalence. Problems

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are nearly always presented in the standard "operations = answer" format, with operations on the left and the answer on the right (McNeil et al., 2006; Seo & Ginsburg, 2003). Moreover, equality is typically expressed only with the equal sign or equal bar, rather than described in a fashion that more explicitly conveys the equivalence relation between the two sides of an equation. These narrow experiences lead children to construct narrow, operational ways of thinking about the equal sign, which are not easily overcome.

According to a "change resistance account" of children's difficulties with equations (McNeil, 2014; McNeil & Alibali, 2005b), acquisition and entrenchment of these operational ways of thinking serve as a barrier to constructing a formal, relational understanding of equations (cf. Knuth et al., 2006; Steinberg et al., 1990). Instead of coming to understand the "=" in equations relationally, as a symbol that the two sides share a common value, children may become entrenched in the idea of the equal sign, and equations more generally, as commands to perform arithmetic operations. These operational ways of thinking include the idea that "=" expresses a unidirectional process in which numbers combine to produce a result. While children may understand that "2 + 2 = 4," they will not agree that "4 = 2 + 2," claiming that these statements are incorrect or nonsensical (Baroody & Ginsburg, 1983; Behr et al., 1980; Kieran, 1980). Similarly, children tend to solve problems with operations on both sides of the equal sign (e.g., 1 + 2 + 3 = 4 + __) incorrectly, with solution strategies that reflect the use of operational ways of thinking, such as adding up all of the numbers in the problem (Add All, e.g., 1 + 2 + 3 = 4 + <u>10</u>; McNeil & Alibali, 2005b). Of note, children from countries whose early mathematics education does not promote the acquisition of operational ways of thinking do not typically produce these 'operational errors' (Li et al., 2008). Further, some studies have shown that younger children—children who may not have yet solidified operational ways of thinking about the equal sign—are more successful at solving problems with operations on both sides of the equal sign than their older peers (McNeil, 2007).

There is a growing body of evidence that the negative effects of extracting and overgeneralizing operational ways of thinking persists into adulthood, even in individuals who have also acquired a correct, relational understanding of equations. Under time pressure, American university students use the same Add All strategy used by children to solve problems with operations on both sides of the equal sign (Chesney, McNeil, Brockmole, & Kelly, 2013), particularly after completing tasks designed to activate operational ways of thinking (McNeil & Alibali 2005b, McNeil, Rittle-Johnson et al., 2010). That these errors result from early acquisition of operational ways of thinking is further supported by data from university students who completed elementary school in Asian countries, where educational practices do not typically result in acquisition of these operational ways of thinking. Such participants rarely use the Add All strategy to solve problems with operations on both sides of the equal sign, even after completing tasks intended to activate operational ways of thinking (McNeil, Rittle-Johnson et al., 2010).

Although operational ways of thinking appear to influence educated adults' math performance when activated in the moment, there is no research to date that investigates whether the activation of operational ways of thinking influences adults' *learning*. Studies with children have indicated that arithmetic practice that highlights relational thinking about the equal sign helps children learn correct strategies for solving problems with operations on both sides of the equal sign (Chesney, McNeil, Petersen, & Dunwiddie, 2012; Chesney et al., 2014; McNeil et al., 2012; McNeil et al., 2011), and that arithmetic practice that activates operational ways of thinking hinders transfer of arithmetic knowledge (Chesney et al., 2012). These findings are predicted by the change-resistance account and underscore that bidirectional relations are not necessarily implied when one interprets the symbol "=" operationally (e.g., if 2 × 2 = 4 means "when you see '2 × 2 =,' you should write '4' at the end," then it does *not* follow that when you see '4 =,' you can write '2 × 2' at the end). In contrast, a relational interpretation of "=" makes more explicit the bidirectional nature of the link between operands and their total (e.g., if 2 × 2 = 4 means "2 × 2 is the same as 4," then it follows that 4 is the same as 2 × 2).

Educated adults are different from children in that their default way of thinking about the equal sign is relational (McNeil & Alibali, 2005a). Thus, in adults, contexts invoking either relational or operational ways of thinking would serve to activate knowledge the learner already possesses, rather than to highlight concepts the learner has yet to assimilate. Yet, mere activation of operational ways of thinking could impede learning, if operational thinking is activated *instead of* relational thinking. Indeed, research suggests that operational and relational thinking tend not to be activated at the same moment. A recent study by Chesney, McNeil, Brockmole, and Kelly (2013) found that in adults, solving problems with operations on both sides of the equal sign correctly is positively predicted by the extent to which participants make *relational eye movements*, looking back and forth between the expressions on the two sides of the equal sign as if relating them. In contrast, these eye movements negatively predicted use of incorrect, operational strategies like adding all the numbers. It seems that at any particular time, operational thinking and relational thinking are negatively associated with each other. Thus, activating operational ways of thinking may effectively distract participants from correct relational thinking about the equal sign and the concept of math equivalence.

Following this logic, we predicted that increasing the activation of operational ways of thinking during math practice should reduce the benefits of that practice, both in terms of fluency with the target arithmetic facts, and in terms of transfer to related facts and to higher-level problems involving the facts. This is because increasing the salience of operational ways of thinking during arithmetic practice should strengthen unidirectional, rather than bidirectional mental connections between operands and their values (e.g. operational ways of thinking support 2 × 2 → 4, but not 4 → 2 × 2, whereas relational thinking supports both) and, thus yield weaker links between totals and their operands relative to practice where the relational meaning is highlighted. In the current study, we considered the strengthening of such connections that comes through study and practice to be *learning*. Such bidirectional links between totals and operands are thought to facilitate the use of inversion and related math concepts, such as the complement principle (if a + b = c, then c – b = a; if a × b = c, then c / b = a; see Bryant, Christie, & Rendu, 1999). These concepts are highly relational (Schneider & Stern, 2009). They depend on the principle that when two values are in an equivalence relation, that relation is maintained if the same transformation is applied to both values. Not only does x = y, but also, y = x, and x × z = y × z. As such, if x × z = w then x × z / z = w / z, and x = w / z, and w / z = x. All these facts are entailed by the equivalence relation symbolized here by "=." However, none are entailed by an operational command such as "when you see x × z, write w." As such, practice that highlights operational rather than relational thinking about the equal sign also has the potential to impact fluency with math facts that logically follow from the studied fact (4 = 2 × 2 implies 4 / 2 = 2) (see Chi & Ceci, 1987 for a discussion of the interplay of content knowledge and memory structure). Taken together, these ideas suggest that people who practice arithmetic when operational ways of thinking are activated may more weakly develop these mental connections and, thus, should make fewer gains in computational fluency with both the practiced facts and the compliments of those facts at posttest than peers practicing the same sets when those operational ways of thinking are not activated.

In the current study, we used written words to activate operational ways of thinking about the equal sign during arithmetic practice and examined the effects on participants' learning of multiplication tables, including transfer to division compliments and algebra problems involving the practiced facts. We included two control groups: one that used written words to activate relational thinking about the equal sign during practice and one that only used the standard equal sign in the problems. We predicted that participants in the operational words condition would benefit less from the practice than would participants in the other two conditions and, thus, would exhibit poorer gains in computational fluency on practiced and transfer problems.

METHOD

PARTICIPANTS

The participants were 70 students at a highly selective university in the Midwest. Each participated in a total of six sessions, and received either extra credit or $30. We excluded four participants for failure to complete all the tasks or experimenter error. Another four participants received their primary education in Asia, where, as discussed above, education practices do not typical lead students to extract operational ways of thinking from their early experience with arithmetic, and were thus excluded from the analyses. All other participants received their elementary education in the U.S, and/or U.S. provinces, protectorates or territories. The final sample included 62 students (36 female, 26 male; 49 white, 5 Asian, 1 African American, 2 Hispanic, and 5 other; *M* age = 20 years). Among those who self-reported standardized test scores (SAT and/or ACT), the mean ACT Math score was 33.1 ($n = 27$), the mean SAT Math score was 731.1 ($n = 37$), and the mean SAT Verbal score was 717.1 ($n = 38$).

MATERIALS AND APPARATUS

Unless otherwise noted training and testing stimuli were presented using Superlab 4 software (Cedrus Corporation, 2007) on Apple® iMac 5.1 computers running OS10.6. Each computer had a 17" LCD display with a resolution of 1440 x 900 pixels and a refresh rate of 60 Hz. These screen dimensions subtended approximately 34 deg x 22 deg of visual angle with participants seated ~60 cm from the screen. Degrees of visual angle are only approximate as no restraints were used to restrict head motion. Verbal responses were recorded by a SONY® MP3 IC recorder.

DESIGN

The design was a pretest–intervention–posttest randomized experiment. Participants were randomly assigned to practice unfamiliar multiplication facts in one of three conditions that differed in terms of how the equal sign was represented in the problems. To ensure that the results were not dependent on the particular times tables practiced, half of the participants in each group were randomly assigned to study the 19-times table and the other half were randomly assign to instead study the 17-times table (cf. Haverty, 1999). All participants were also assigned to study the 18-times table (which was tested at posttest) and the 15-times table. The 15-times table was included as a distracter to help reduce the possibility that cross-talk between the participants might lead them to realize that they were in different conditions. Posttest questions involved the 18-times table (which all participants practiced), the 17- and 19-times tables (which were each practiced by half the participants), and the 16-times table (which none of the participants practiced). We did not use the 15-times table in posttest, as we expected that the participants would already be quite familiar with the 15-times table, given that it involves a multiple of 5 and that 15-minute units are commonly employed in timekeeping. Thus, at posttest, participants had practice with half the times tables involved in the posttest, though which tables were practiced varied between groups. In the *operational words condition*, the equal sign was replaced in 23 of the 30 practiced problems by operational words (e.g., 8 "makes," 8 "multiplies to," and 7 "equals"). In the *relational words condition*, the equal sign was replaced in 23 of the 30 practiced problems by relational words (e.g., 8 "is the same amount as," 8 "is equivalent to," and 7 "is equal to"). Operational and relational words were chosen to match those used in prior studies with children (Chesney et al., 2012), with the exception that the operational phrase "multiplies to" was used rather than the addition specific "adds up to". In the *control condition*, the equal sign was used in all problems. This yielded six groups in a 3 *(word condition)* x 2 (*number condition*) design, with 9–11 participants in each group. The times table tested at posttest (16, 17, 18, or 19) was a within-subjects factor.

We hypothesized that the activation of operational ways of thinking would reduce the benefits of practice, particularly in regards to the participants' ability to make use of the studied information in transfer tasks, such as algebra and division problems. It was important to include a relational words condition to ensure that effects could be attributed to the operational words, rather than to the novelty of seeing words in place of the equal sign. It was important to include an equal sign only control condition to ensure that effects could be attributed to the negative effects of activating operational ways of thinking, rather than to beneficial effects of activating relational thinking.

PROCEDURE

Participants were randomly assigned to groups (as above). Each participant completed six sessions over a two-week period: a pretest session, four practice sessions, and a posttest session. Each session took ~30 min. Participants completed no more than one session per day. The mean time from first to last session (inclusive) was 10.9 days (*SD* = 0.9, range = 10–14).

Session 1: Pretest

In the first session, participants first completed an algebra pretest and then a multiplication pretest.

Algebra. Participants completed 36 algebra problems (e.g., 18 (z + 2) = 54), involving division by, division to, or multiplication of 17, 18, and 19 (12 problems each). Problems appeared on the computer screen, one at a time, centered over a text box. Problems and text boxes remained on the screen until participants entered their answers into the box.

Multiplication. Participants were asked to solve 80 multiplication facts corresponding to the 12, 13, 14, 15, 16, 17, 18, and 19 times tables (these values multiplied by 1–10). On each trial, participants first saw a letter or pair of letters, which they were asked to recite aloud. They then hit a key to bring up a problem. This problem was presented without any equality symbol (e.g. "18 x 5"). Participants were asked to hit a key as soon as they knew the answer. On this key strike, the problem would disappear, and a text box would appear into which they could enter their solution. After solving each problem, they were asked to say how they got the answer aloud.

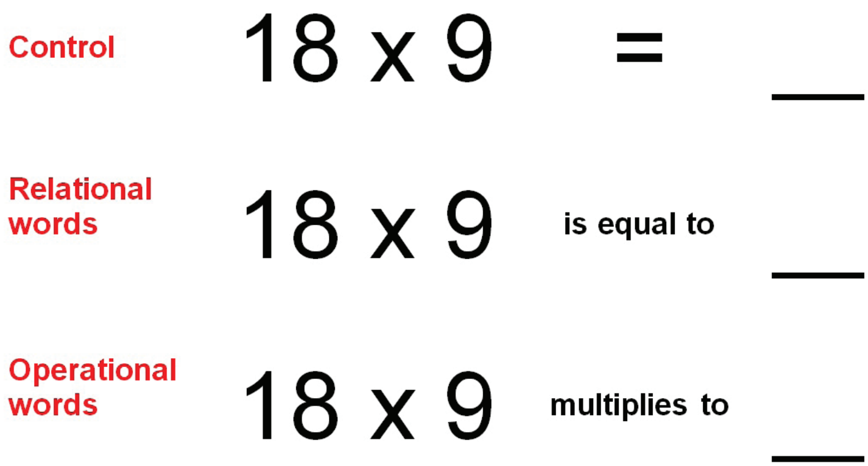
Sessions 2–5: Practice Dependent on Assigned Condition

In the first 3 practice sessions (sessions 2–4), participants began by studying a novel multiplication table (Session 2: 17 or 19, depending on their assigned number condition; Session 3: 18, Session 4: 15). While they studied, a multiplication table including all ten of the novel table's completed math-facts was presented on a computer screen. The equal sign was replaced with words as described in the design section for the participants in the Operational words and Relational words conditions, while Control participants always saw the equal sign (see Figure 1). The table remained on the screen until participants indicated that they had finished studying or five minutes had passed, whichever came first. Participants then practiced solving problems from this novel table. The computer presented problems one at a time on a computer screen. These problems did not include solutions, but did include the equal sign or the alternative words as appropriate to the participants' condition. Participants had up to five seconds to enter an answer, and then were shown the completed equation. Problems were presented several times, in random order. In session 2, they saw 6 repetitions of each problem in their assigned table (60 total). In session 3, they saw 6 repetitions of each 18 table problem, 6 of each 17 or 19 table problem, and then 3 each of problems from both tables intermingled (180 total). In session 4, they saw 6 repetitions of each 15 table problem, 6 repetitions of each 18 table problem, 6 of each 17 or 19 table problem, and then 3 each of problems from all 3 tables intermingled (270 total). At the end of sessions 2–4, flashcards of the novel practiced table were given to the participants, so they could practice at home in between sessions. The flashcards used the same word or equal sign format to represent equality as had been used in the table they studied at the start of the session. In the last practice session, session 5, no new table was added, and participants rather practiced problems from each of their 3 studied tables (6 each, intermingled, 180 total).

Session 6: Posttest

In the last session, participants completed an assessment of their understanding of the equal sign (not relevant to the

Figure 1.
Example of how the practice problems looked in the three word conditions.



current study; no significant differences across conditions were found), followed by an algebra posttest, a multiplication posttest, and a division transfer test. The algebra problems, problem formats, and methods were identical to those of the pretest. The multiplication problems were limited to the 16-, 17-, 18-, and 19-times tables but were otherwise identical to the pretest. On the division problems, participants were asked to solve 40 division facts analogous to the 16-, 17-, 18-, and 19-times tables (these values times 1–10 divided by 1–10, such as 54/3, or these values times 1–10 divided by these values, such as 72/18). Presentation formats and methods were identical to those of the multiplication problems.

RESULTS

We sought to determine if the benefit of practice differed across the practice groups. Accuracy was near ceiling at pretest. Therefore, analyses focused on reaction time: faster reaction time indicated better computational fluency. Mean RTs were determined for each participant for each kind of problem. In addition, for assessments that were given at pretest and posttest, we calculated the pre-to-post improvement in reaction time for a more direct measure of the benefit of practice.

PRETEST

Multiplication

Accuracy on the pretest multiplication problems was high, with participants solving 94.7% of the problems correctly on average. Upon inspection of the data, we noticed that the reaction times appeared to be faster for even times tables (12: M = 3807 ms, SD = 1919 ms; 14: M = 5758 ms, SD = 3240 ms; 16: M = 6200 ms, SD = 3593 ms; 18: M = 6559 ms, SD = 4588 ms) than for odd times tables (13: M = 5357 ms, SD = 2820 ms; 15: M = 4718 ms, SD = 2462 ms; 17: M = 6629 ms, SD = 3890 ms; and 19: M = 7201 ms, SD = 5181 ms). Thus, we included parity (Even or Odd) as a within-subjects factor and had four exemplars of increasing size within each parity condition (Even exemplars: size 1 = 12, size 2 = 14, size 3 = 16, size 4 = 18; Odd exemplars: size 1 = 13, size 2 = 15, size 3 = 17, size 4 = 19). We conducted a 3 (word condition: control, relational words, operational words) x 2 (number condition: 17 or 19) x 2 (parity: even or odd table) x 4 (size: 1, 2, 3, or 4) mixed ANOVA with word condition and number condition as the between-subjects factors, parity and size as the within-subjects factors, and average reaction time (in ms) as the dependent variable. Results showed no significant main effects or interactions for word condition or number condition (all p's > .10), suggesting that random assignment did a good job of equating the conditions at pretest. However, there was a main effect of parity, with reaction times faster for even versus odd times tables, $F(1, 56) = 16.18$, $p < .001$, *partial eta squared* = .22. There was also a main effect of size, $F(3, 168) = 24.94$, $p < .001$, *partial eta squared* = .31, with reaction time increasing with size, $F_{linear}(1, 56) = 33.53$, $p < .001$, *partial eta squared* = .37. These main effects were qualified by a significant parity by size interaction, $F(3, 168) = 12.85$, $p < .001$, *partial eta squared* = .19. Inspection of the data indicated that the 15-times table was driving the interaction. Reaction time for the 15-times table was lower than would be expected given its status as an odd exemplar of size 2.

Algebra

Accuracy on the pretest algebra problems was high, with participants solving 87.1% of the problems correctly on average. We conducted a 3 (word condition: control, relational words, operational words) x 2 (number condition: 17 or 19) x 3 (exemplar: 17, 18, or 19) mixed ANOVA with word condition and number condition as the between-subjects factors, exemplar as the within-subjects factor, and average reaction time (in ms) as the dependent variable. Results showed no significant main effects or interactions for word condition or number condition (all *p*'s > .10), suggesting that random assignment did a good job of equating the conditions at pretest. However, there was a main effect of exemplar, $F(2, 112) = 3.27$, $p = .04$, *partial eta squared* = .06, with reaction times fastest for algebra problems involving the 17-times table and slowest for algebra problems involving the 19-times table, $F_{linear}(1, 56) = 7.63$, $p = .008$, *partial eta squared* = .12.

POSTTEST

Multiplication Task

Accuracy on the posttest multiplication problems was high, with participants solving 94.5% of the problems correctly on average. Given that accuracy was near ceiling to begin with and did not significantly improve between pretest and posttest ($p = .484$), analyses focused on gains in reaction time. We considered improvements in reaction time from pre-to-post to be a metric of learning, as such increases in computational fluency suggest strengthened mental connections between multiplication problems and their solutions. To examine how the word condition impacted the benefit of practice on fluency with multiplication facts, we conducted a 3 (word condition: control, relational words, operational words) x 2 (number condition: 17 or 19) x 2 (parity: even or odd table) x 2 (fact type: practiced or not practiced) mixed ANOVA with word condition and number condition as the between-subjects factors, parity and fact type as the within-subjects factors, and average improvement in reaction time from pretest to posttest (in ms) as the dependent variable. Note that the 18-times table is even and practiced, the 16-times table is even and not practiced, and the 17- and 19-times tables are both odd and practiced or not practiced by different halves of the participant sample. Also note that higher values of the dependent variable indicate greater improvements in participants' performance from pretest to posttest.

Results showed a main effect of parity, with pre-to-post improvements greater for odd versus even times tables, $F(1, 56) = 14.59$, $p < .001$, *partial eta squared* = .21. There was also a main effect of fact type, with pre-to-post improvements greater for practiced versus not practiced facts, $F(1, 56) = 62.48$, $p < .001$, *partial eta squared* = .53. Most importantly, the only other effect that was statistically significant was the interaction between word condition and fact type, $F(2, 56) = 3.17$, $p = .049$, *partial eta squared* = .10. As shown in Figure 2, the difference in pre-to-post improvement for practiced versus unpracticed facts (i.e., the difference between the blue and red bars in Figure 2) was significantly smaller in the operational words condition ($M = 1885$ ms, $SD = 1877$ ms) than in the other two conditions ($M = 3471$ ms, $SD = 3318$ ms), $t(60) = 2.06$, $p = .043$, $BF_{01} = .119$, but it did not differ significantly in the relational words and control conditions (relational words $M = 2753$ ms, $SD = 2921$ ms, control $M = 4122$ ms, $SD = 3585$ ms), $t(38) = 1.32$, $p = .196$, $BF_{01} = .410$. (Note: Bayes factors are estimated via a method described by Jarosz and Wiley [2014]).

Division Task

The division task was intended as a test of transfer. As such, division problems were not included on the pretest to avoid sensitizing the participants to the problems. Accuracy on the division problems was high, with participants solving 94.4% of the problems correctly on average. Thus, the analyses of the division problems also focused on participants' reaction time, with faster reaction times (lower values of the dependent variable) indicating better performance. We conducted a 3 (word condition: control, relational words, operational words) x 2 (number condition: 17 or 19) x 2 (parity: even or odd table) x 2 (fact type: practiced or not practiced) mixed ANOVA with word condition and number condition as the between-subjects factors, parity and fact type as the within-subjects

Figure 2.
Pre-to-post improvement in reaction time as a function of word condition and fact type. Higher bars denote greater improvement. Error bars show standard error.

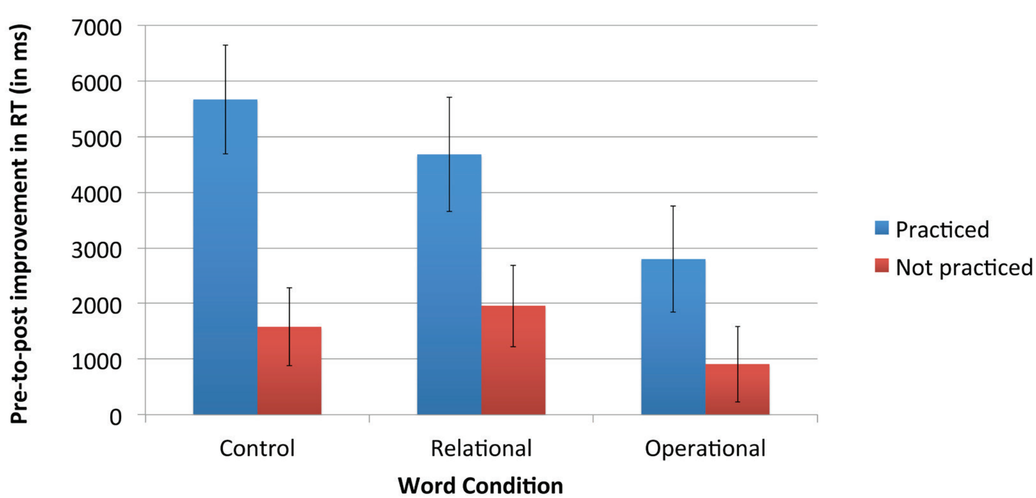
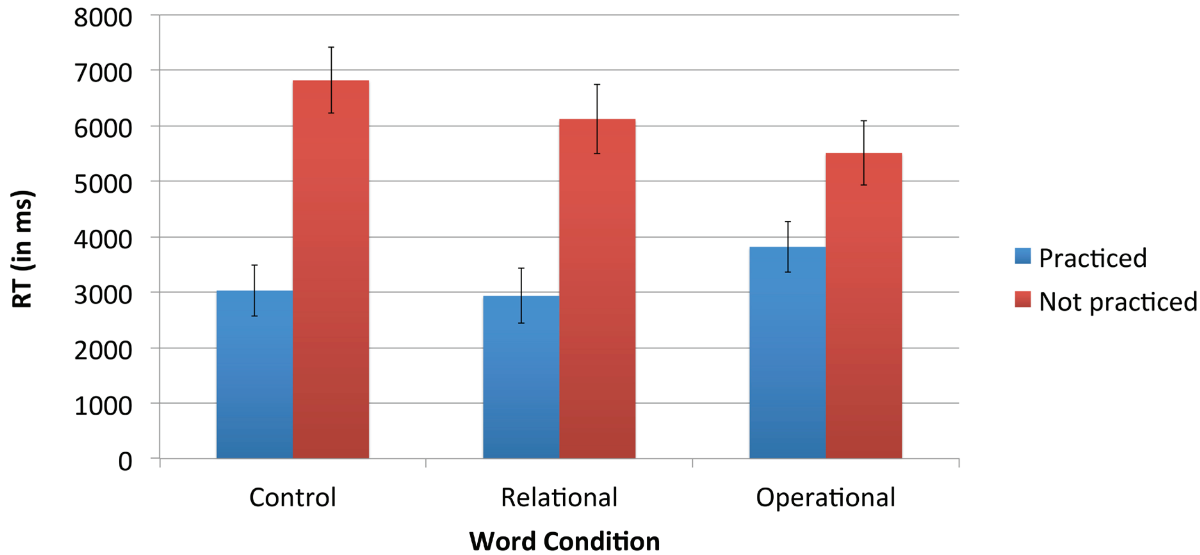


Figure 3.

Reaction times for solving division problems as a function of word condition and fact type. Lower bars denote better performance. Error bars show standard error.



factors, and average reaction time (in ms) as the dependent variable. We also ran a similar ANCOVA, with mean reaction time to pretest multiplication problems based on the 16, 17, 18, and 19 tables as a covariate in order to account for individual differences between pre-intervention ability, and conclusions were the same as those reported below.

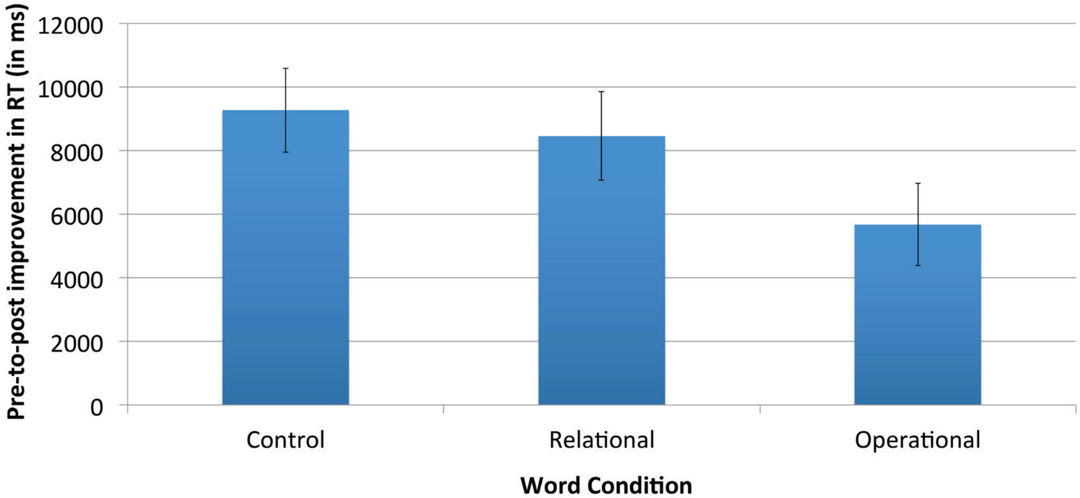
In contrast to the results with the multiplication problems, there was not a significant effect of parity, $F(1, 56) = 0.99$, $p = .33$, *partial eta squared* = .02. However, there was a main effect of fact type, with reaction time faster for division problems transformed from practiced versus unpracticed facts, $F(1, 56) = 90.66$, $p < .001$, *partial eta squared* = .62. As with the multiplication problems, the only other effect that was statistically significant was the interaction between word condition and fact type, $F(2, 56) = 4.42$, $p = .016$, *partial eta squared* = .14. As shown in Figure 3, the difference in reaction time for division problems transformed from practiced versus unpracticed facts (i.e., the difference between the red and blue bars in Figure 3) was significantly smaller in the operational words condition ($M = 1690$ ms, $SD = 2250$ ms) than in the other two conditions ($M = 3517$ ms, $SD = 2434$), $t(60) = 2.90$, $p = .005$, $BF_{01} = .017$ but it did not differ significantly in the relational words and control conditions (relational words $M = 3230$ ms, $SD = 2481$ ms, control $M = 3776$ ms, $SD = 2422$ ms), $t(38) = 0.70$, $p = .48$, $BF_{01} = .772$.

Algebra Task

Accuracy on the posttest algebra problems was high, with participants solving 88.4% of the problems correctly on average. Given that accuracy was high at pretest and did not significantly improve between pretest and posttest ($p = .469$), analyses again focused on gains in reaction time. To examine how the word condition affected transfer of the practiced multiplication facts to solving algebra problems involving the facts, we conducted a 3 (word condition: control, relational words, operational words) x 2 (number condition: 17 or 19) x 3 (fact type: odd practiced, even practiced, or odd not practiced) mixed ANOVA with word condition and number condition as the between-subjects factors, fact type as the within-subjects factors, and average improvement in reaction time from pretest to posttest (in ms) as the dependent variable.

Results showed a main effect of fact type, $F(2, 112) = 13.89$, $p < .001$, *partial eta squared* = .20, with pre-to-post improvement greatest for problems involving the two practiced times tables (odd practiced: $M = 7742$ ms, $SD = 6156$ ms; even practiced: $M = 7694$ ms, $SD = 7086$ ms) and lowest for problems involving the not practiced table ($M = 4327$ ms, $SD = 7889$ ms). There was also a marginal interaction between fact type and number condition, $F(2, 112) = 2.80$, $p = .07$, *partial eta squared* = .05, with the difference between practiced and unpracticed facts greater in those who practiced the 19-times table versus the 17-times table. Although differences across word conditions were not as strong as they were with the multiplication and division problems, the pre-to-post improvement on algebra problems involving the practiced odd times table (17 or 19 depending on number condition) in the operational word condition ($M = 5673$ ms, $SD = 4133$ ms) was less than the improvement in the other two conditions ($M = 8881$ ms, $SD = 6807$ ms), $t(60) = 2.01$, $p = .049$, $BF_{01} = .132$, and there was no difference between the relational words ($M = 8453$ ms, $SD = 5900$ ms) and control ($M = 9269$ ms, $SD = 7661$ ms) conditions, $t(38) = .374$, $p = .71$, $BF_{01} = .929$ (see Figure 4).

Figure 4.
Pre-to-post improvement in reaction time on algebra problems involving facts from the practiced odd times table (17 or 19 depending on number condition) as a function of word condition. Higher bars denote greater improvement. Error bars show standard error.



DISCUSSION

The present experiment provides the first evidence that deliberately activating operational ways of thinking during arithmetic practice impairs adults' learning from that practice. Educated adults randomly assigned to practice multiplication facts in which some of the equal signs were replaced by operational words showed poorer gains in computational fluency than both adults who practiced multiplication facts with equal signs, and adults who practiced multiplication facts in which some of the equal signs were replaced by relational words. These lower gains were exhibited not only in reaction times on the practiced problems, but also in reaction times for close transfer tasks, including division compliments and algebra problems involving the practiced facts. These results replicate and extend prior findings (Chesney et al., 2013; McNeil & Alibali, 2005b; McNeil, Rittle-Johnson et al., 2010) suggesting that educated adults are still negatively affected by the operational ways of thinking about equations that they constructed during elementary school, even though they have learned and succeeded in algebra. Results also lend further support to the change resistance account (Chesney et al., 2013; McNeil, 2014; McNeil & Alibali, 2005b; McNeil, Rittle-Johnson et al., 2010). Children's construction and overgeneralization of operational ways of thinking about equations can have long-term negative effects on individuals' performance on higher-level mathematics tasks, particularly those that invoke relational notions of mathematics.

POTENTIAL MECHANISMS

Our results support the hypothesis that activating operational ways of thinking about the equal sign can impair not only adults' learning of those facts, but also their transfer of multiplication knowledge to division tasks. However, the specific mechanisms involved in these effects remain unclear. The explanation that we have been advancing in this paper is that the operational words activate operational rather than relational ways of thinking about the equal sign, and thus fail to highlight the interchangeability of the two sides of the practiced equations. The interchangeable nature of two sides of an equation is intrinsic to relational thinking about the equal sign (Jones, Inglis, Gilmore, & Dowens, 2012). If consideration of this interchangeability is activated during practice, as is the case if the equal sign is considered relationally, then it may strengthen the bi-directional connections between math facts and their total values. The ability to make use of math fact knowledge in novel situations should be supported by such interconnections. As discussed in the introduction, the derivation of novel related math facts, such as mathematical compliments, depends on the understanding of a bidirectional equivalence relation. The statement "A × B is the same value as C" implies "C / B is the same value as A." However, the command "If you see A × B, write C" does not. Indeed, recent research (Chesney et al., 2014) has demonstrated that children who have a better understanding of equivalence relations are also better able to make use of solving strategies that rely on these interrelations (e.g. decomposition strategies: 9 + 3 = 10 + 2 = 12). Thus, participants in the operational words condition may have demonstrated poorer learning and transfer in the present study because these connections were less strongly formed, or not formed at all. In contrast, the relational words and traditional equal sign presumably activated relational thinking, at least for our highly educated sample, thereby strengthening these bidirectional connections and, in turn, aiding recall and transfer. This result is consistent with past studies showing that different arithmetic problem formats can activate different prior knowledge and lead to differences in computational fluency and solving strategies (e.g., Campbell & Albert, 2009).

One possible alternative hypothesis is that the use of operational words during practice reduced gains not because they activated prior concepts of the equal sign, but merely because they stood in contrast to the participants' default relational interpretation of the equal sign. That is, while relational words may be consistent with participants' default relational thinking about the equal sign, operational words would be inconsistent with this thinking and, thus, be distracting to the task at hand. The current paradigm cannot distinguish between the possibilities that the operational words were detrimental to learning because they invoked previously learned, incorrect operational ways of thinking, or merely because they were non-relational. However, we note that prior research with adults suggests that invoking operational ways of thinking is specifically detrimental in those who have previously learned operational ways of thinking (McNeil & Alibali, 2005a; McNeil, Rittle-Johnson et al., 2010). Regardless, in either case, the use of operational words during problem solving—words commonly used during math instruction (Hamann & Ashcraft, 1986)—would lead to poorer learning outcomes.

LIMITATIONS AND FUTURE DIRECTIONS

One limitation of the current design is that all of the participants were quite well educated and high in math ability at the start of the study. They were admitted into and attended an elite university, and their self-reported ACT and SAT scores were quite high. As such, these participants would be expected to have a firm grasp of both arithmetic and algebra and likely understand that the equal sign is a relational symbol expressing mathematical equivalence (McNeil & Alibali, 2005a). Indeed, given that the participants in the control condition received no less benefit from practice than those in the relational word condition (see Figures 2, 3, and 4), we can conclude that our control participants defaulted to a relational understanding of the equal sign in the absence of other cues. This is consistent with previous work on educated adults' default interpretation of the equal sign (McNeil & Alibali, 2005a). However, the default interpretation may be different in a less educated group of adults. Such individuals might be more likely to spontaneously activate operational ways of thinking during practice, with detrimental effects on their learning similar to what was seen for participants in the current study's operational words condition. As a consequence, in individuals with less math ability, the effect of using operational vs. relational words in practice might also manifest an added benefit in the relational words condition instead of only reduced gains in the operational words condition. Indeed, such a benefit for relational words has been shown in children who practiced math facts in a similar paradigm (Chesney et al., 2012). Future work is needed to determine if this would also be the case with less educated adults (e.g., a community college sample). If so, it is possible that educators may be able to take advantage of these simple verbal cues to aid the learning of their adult students.

Critics may also question whether it is truly detrimental for children to construct operational ways of thinking of equations, given that the highly educated students in our sample were able to become highly educated and to gain acceptance to an elite university despite having constructed them during elementary school. However, this study was not designed to establish the educational attainment of students that do and do not construct operational ways of thinking about equations, but rather to demonstrate how operational ways of thinking can continue to interfere with mathematical thought, even in a select subset of students who have succeeded in spite of them. If even these high achieving students can be hindered by the operational ways of thinking that were constructed in elementary school, one has to wonder about the detrimental effects these ways of thinking have on more typical learners or learners from disadvantaged groups. We, therefore, agree with calls from mathematics educators to determine what changes can be implemented in US elementary mathematics curricula to foster the development of relational thinking in elementary school (Baroody & Ginsburg, 1983; Carpenter, Levi, Franke, & Zeringue, 2005; Jacobs et al., 2007; McNeil, 2008; Schliemann, Carraher, & Brizuela, 2007; Stephens, Blanton, Knuth, Isler, & Gardiner, in press). Fortunately, there is mounting evidence that even minor differences in curricula, such as teaching the equal sign in concert with inequality symbols (Hattikudur & Alibali, 2010), altering the timing of practice and conceptual instruction (Loehr et al., 2014), presenting problems in concrete form (Sherman & Bisanz, 2009), writing addition facts in non-traditional, c = a + b formats (McNeil et al., 2011), and practicing addition facts organized by equivalent sums (McNeil et al., 2012) can increase this relational thinking.

CONCLUSIONS

The current findings support the position that prior knowledge can negatively impact not only problem solving, but also the ability to learn from practice. Indeed, minor changes in the surface forms of problems can help or hinder learning depending on the prior knowledge those surface forms invoke. Here we found that the operational ways of thinking constructed back in elementary school can have negative effects on mathematical performance that extend far beyond elementary school. It appears that when these operational ways of thinking are activated, they can impair learning even in highly educated adults who possess a clear understanding of the relational nature of the equal sign. Once these operational ways of thinking are acquired in elementary school, it appears that individuals never truly unlearn them (McNeil, Rittle-Johnson et al., 2010). Rather, they exist in tandem with the later learned relational thinking, and can be activated to the learners' detriment. Even adults with sufficient mastery over higher-level mathematics to garner them entry into elite universities still show negative effects of these early (mis)understandings (Chesney et al., 2012; McNeil & Alibali, 2005b; McNeil, Rittle-Johnson et al., 2010). Altering elementary school curricula so as to support the early acquisition of relational rather than operational ways of thinking about the equal sign could thus have long-term benefits, as future students would not need to overcome the impediment these operational ways of thinking present to their mathematical performance.

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