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Fractals at elastic–plastic transitions in metals, soils, and rocks

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ABSTRACT

A range of studies indicate that a fractal growth of plastic domains is characteristic of elastic–plastic transitions in metallic, soil-like, and rock-like materials where elastic moduli and/or coefficients of friction, cohesion, and dilatation [1-5]. More specifically, all these material parameters are taken as nonfractal random fields in 2D or 3D, with weak noise-to-signal ratios, in a statistical continuum models. Statistical analysis is used to assess the anisotropy of those shear bands. All the macroscopic responses display smooth transitions but, as the randomness vanishes, they turn into a sharp response of an idealized homogeneous material. Notably, increasing hardening modulus and friction makes the transition more rapid, randomness in cohesion has a stronger effect than randomness in friction, but dilatation has practically no influence. Adapting the concept of scaling functions (first introduced for metals), we link the elastic–plastic transition in random Mohr–Coulomb media to phase transitions in condensed matter physics: the fully plastic state is a critical point and, with three order parameters (the “reduced Mohr–Coulomb stress,” “reduced plastic volume fraction,” and “reduced fractal dimension”), three scaling functions are introduced to unify the responses of different materials. The critical exponents are universal regardless of randomness in various constitutive properties and their random noise levels.

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