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Rigorous bounds on the effective moduli of heterogeneous media with small-scale instabilities

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ABSTRACT

We review the theoretical bounds on the effective properties of linear elastic heterogeneous solids in the presence of constituents having nonpositive-definite elastic moduli (so-called negative-stiffness phases) which arise from small-scale instabilities. We show that for statically stable bodies the classical displacement-based variation principles hold but that the dual variation principle for traction boundary problems does not apply. We further show that the classical Voigt upper bound on the linear elastic moduli in multiphase inhomogeneous bodies and composites applies and that it imposes a stability condition: overall stability requires that the effective moduli do not surpass the Voigt upper bound. This particularly implies that, although the geometric constraints among constituents in a composite can stabilize negative-stiffness phases, the stabilization is insufficient to allow for extreme overall static elastic moduli (exceeding those of the constituents) in any (anisotropic) linear elastic medium. Stronger bounds on the effective elastic moduli of isotropic composites can be obtained from the Hashin–Shtrikman variation inequalities, which are also shown to hold in the presence of negative stiffness. Finally, through a multiscale computational study we show that, although the linear elastic moduli can never reach extreme values due to negative-stiffness phases, the viscoelastic moduli can indeed assume extreme values when including small-scale instabilities in heterogeneous media.