2001

Estimation Of Reference Crop Evapotranspiration Using Fuzzy State Models

Lameck O. Odhiambo
University of Nebraska-Lincoln, lodhiambo2@unl.edu

R. E. Yoder
University of Tennessee, Knoxville, ryoder2@unl.edu

D. C. Yoder
University of Tennessee, Knoxville

Estimation of Reference Crop Evapotranspiration Using Fuzzy State Models

L. O. Odhiambo, R. E. Yoder, D. C. Yoder

Abstract. *Daily evapotranspiration (ET) rates are needed for irrigation scheduling. Owing to the difficulty of obtaining accurate field measurements, ET rates are commonly estimated from weather parameters. A few empirical or semi–empirical methods have been developed for assessing daily reference crop ET, which is converted to actual crop ET using crop coefficients. The FAO Penman–Monteith method, which is now accepted as the standard method for the computation of daily reference ET, is sophisticated. It requires several input parameters, some of which have no actual measurements but are estimated from measured weather parameters. In this study, we examined the suitability of fuzzy logic for estimating daily reference ET with simpler and fewer parameters. Two fuzzy evapotranspiration models, using two or three input parameters, were developed and applied to estimate grass ET. Independent weather parameters from sites representing arid and humid climates were used to test the models. The fuzzy estimated ET values were compared with direct ET measurements from grass–covered weighing lysimeters, and with ET estimations obtained using the FAO Penman–Monteith and the Hargreaves–Samani equations. The estimated ET values from a fuzzy model using three input parameters (S_{yx} = 0.54 mm, r^2 = 0.90) were found to be comparable to ET values estimated with the FAO Penman–Monteith equation (S_{yx} = 0.50 mm, r^2 = 0.91) and were more accurate than those obtained by the Hargreaves–Samani equation (S_{yx} = 0.66 mm, r^2 = 0.53). These results show that fuzzy evapotranspiration models with simpler and fewer input parameters can yield accurate estimation of ET.*

Keywords. *Reference crop evapotranspiration, Fuzzy evapotranspiration model, Lysimeters, Penman–Monteith equation, Hargreaves–Samani equation.*

Evapotranspiration (ET) is the process by which water is transferred from the earth's surface to the atmosphere by evaporation from the soil, water, and wet plant surfaces, and by transpiration through plants. It is driven by the available energy (net irradiance), and is limited by the rate of energy exchange between the surface and the overlaying atmosphere (sensible and latent heat fluxes), the available soil water, and the ability of the plant to conduct water from the soil, to the leaf, and then to the bulk atmosphere (Hatfield and Fuchs, 1990). The water transfer occurs from a constantly changing surface, e.g., the plant canopy may not completely cover the soil and increases as the canopy develops, and the soil surface changes from a completely wet (free water) to a completely dry (air dry) soil surface. The net irradiance and sensible and latent heat fluxes also have temporal variation.

Accurate measurements of daily ET rates are needed for irrigation scheduling. Owing to the difficulty of obtaining accurate field measurements, ET is commonly estimated from weather parameters. A few empirical or semi–empirical methods have been developed for estimating daily reference ET from weather parameters (Jensen et al., 1990; Hatfield and Fuchs, 1990). The reference crop ET is converted to actual crop ET using crop coefficients. Where sufficient data are available, the FAO Penman–Monteith method (Allen et al., 1998) is now accepted as the standard method for the definition and computation of the daily reference evapotranspiration, i.e., evapotranspiration from a grass reference crop (a cool season grass) with specific characteristics. The FAO Penman–Monteith equation is a sophisticated expression (eq. 2) and requires several input parameters, some of which have no actual measurements, but are estimated from measured weather parameters (table 1). The simpler Hargreaves–Samani equation (Hargreaves and Samani, 1985), which requires only measured maximum and minimum temperatures in addition to estimated extraterrestrial solar irradiance, has been recommended for general use (Hargreaves, 1994). However, there is still no consensus on the most appropriate method to use for estimating ET on a daily scale using simpler and fewer input data. Hence, further research is required on reliable, robust, and widely applicable approaches to estimate ET on a daily scale and/or shorter periods.

Article was submitted for review in May 2000; approved for publication by the Soil & Water Division of ASAE in December 2000. Presented at the 2000 ASAE Annual Meeting as Paper No. 00–2031.

The authors are **Lameck O. Odhiambo**, Post–Doctoral Research Associate; **Ronald E. Yoder**, ***ASAE Member Engineer***, Professor and Head; and **Daniel C. Yoder,** ***ASAE Member Engineer***, Associate Professor, Department of Agricultural and Biosystems Engineering, University of Tennessee, Knoxville, Tennessee. **Corresponding author:** Ronald E. Yoder, Dept. of Agricultural and Biosystems Engineering, University of Tennessee, P.O. Box 1071, Knoxville, TN 37901–1071; phone 865–974–7266; fax 865–974–4514; e–mail: ryoder@utk.edu.

Table 1. Contrast between the measured data and required input parameters for the FAO Penman–Monteith equation and Model II.

Method	Measured Data	Input Parameter	Parameter Estimation
FAO Penman Monteith Equation	Incoming solar irradiance (RS)	Net irradiance at the crop surface (R_n)	R_n is estimated as the difference between incoming and outgoing irradiance of both short (R_{ns}) and long (R_{nl}) wavelengths. Calculation of R_{nl} requires estimation of clear sky solar irradiance (R_{so}) and extraterrestrial irradiance (R_a).
	Soil heat flux (G)	Soil heat flux (G)	For a one–day interval, $G \approx 0$. For longer periods, G must be estimated from soil heat capacity, air temperature, and effective soil depth.
	Relative humidity (RH)	Actual vapor pressure (e_a), Saturation vapor pressure (e_s), Slope of saturation vapor curve (Δ)	The e_a is estimated from RH_{min} and RH_{max}. The a_e can also be estimated from dew point temperature. Estimation of e_s and Δ requires T_{min}, T_{max}, and T_{mean}.
	Wind speed at 2 m height (u_2)	Wind speed at 2–m height (u_2)	u_2 is needed in the calculation of aerodynamic and canopy resistance constants.
	Air temperature (T_{min} and T_{max})	Minimum air temperature (T_{min}) and maximum air temperature (T_{max}) for 24–h period	Mean air temperature (T_{mean})
	Elevation	Psychrometric constant (γ)	Involves calculation of atmospheric pressure (P), and needs latent heat of vaporization (λ), specific heat at constant pressure (c_p), and ratio of molecular weight of water vapor/dry air (ε).
	Latitude (L)	Latitude (L)	L is needed in the calculation of R_a.
	Day of year (J)	Day of year (J)	J is needed in the calculation of R_a.
Fuzzy Model II	Incoming solar irradiance (RS)	Incoming solar irradiance (RS)	None
	Relative humidity (RH_{min} and RH_{max})	Mean relative humidity (RH)	Mean relative humidity (RH_{mean})
	Day wind speed at 2–m height (U_d)	Day time wind speed at 2–m height (U_d)	None

In this study, we examined the suitability of using a fuzzy logic approach to estimate daily ET with simpler and fewer number of parameters. The objective was to achieve an accurate estimation of daily ET using either two or three simple measurable parameters. Two fuzzy evapotranspiration models were developed and applied to estimate reference grass ET, one using two weather parameters, and the other using three. Independent weather parameters from sites representing arid and humid climates were used to test the models. The fuzzy estimated ET values were compared with direct ET measurements from grass–covered weighing lysimeters and with ET estimations from the FAO Penman–Monteith and the Hargreaves–Samani equations.

Fuzzy Sets Theory and Estimation

Fuzzy logic was introduced by Zadeh (1965) and has been successfully applied in expert systems, regression, and other data analysis methodologies (Kaufmann and Gupta, 1991; Terano et al., 1992). The concept of fuzzy logic and estimation has been used in various types of systems. Postlethwaite (1989) developed an estimator based on fuzzy logic to estimate the specific growth rate of baker's yeast for control of fermentation in batch–fed fermentation processes. Tao et al. (1994) developed an estimator based on fuzzy IF–THEN rules for multidimensional multitarget tracking with multisensor data taken in a cluttered environment. The estimator based on the IF–THEN rules consisted of Gaussian membership functions, a "*minimum* operator" to evaluate the conjunction AND, and centroid defuzzification. Saruwatari and Yomota (1995) developed a fuzzy based forecasting system to estimate irrigation water requirement on paddy fields. The system was formulated by using the fuzzy theory based on analysis of the logic of water management, which was composed from the experience and knowledge of irrigation administrators.

Shabani et al. (1996) presented an approach to an electrical power system state estimation based on the application of fuzzy logic. Significant improvements in state estimates were achieved by using a hybrid estimator incorporating fuzzy logic concepts. Chuang et al. (1997) used a fuzzy estimator to estimate the relationship between perspective projection and kinematics in a problem of controlling a robot to track a randomly moving object using visual servoing techniques. Tay and Tan (1997) developed a fuzzy system as a parameter estimator for nonlinear dynamic functions. In the studies cited, results of simulations were better using a fuzzy estimator than when using a linear model.

Ribeiro and Yoder (1997) used fuzzy logic concepts to develop a fuzzy evapotranspiration estimator for an automated irrigation control system. They used triangular membership functions and the centroid defuzzification method. The rules were formulated based on the existing knowledge about ET, as well as on the relationships between each input and ET obtained in a regression analysis. They used two weather parameters (solar irradiance and relative humidity) as inputs to the estimator, and obtained a squared correlation coefficient (r^2) of 0.68 between fuzzy estimated ET and lysimeter measured ET. Their estimator was later optimized by incorporating an adaptive neural network and yielded an r^2 of 0.74. This estimator was designed to estimate ET for a limited range of climatic conditions found at Crossville, Tennessee, U.S.A.

Fuzzy Evapotranspiration Model

A fuzzy ET model uses a fuzzy inference system to process the input weather parameters to output evapotranspiration (ET). Such a model consists of four functional components: a fuzzifier that transforms real numerical input data into fuzzy sets (a process known as fuzzification), a set of control rules (rule base) governing the relationships between input and output parameters, an inference engine that performs the fuzzy reasoning based on the control rules, and a defuzzifier that transforms the fuzzy output into real numerical numbers (a process known as defuzzification). A complete description of the fuzzy inference process can be found in several references, including Kaufmann and Gupta (1991), Jang and Sun (1995), and Tsoukalas and Uhrig (1997).

Two fuzzy ET model structures were developed, one using two input parameters, and the other using three input parameters. The input parameters included measured incoming solar irradiance (RS) in MJ m^{-2} d^{-1}, percent relative humidity (RH) computed as $(RH_{min} + RH_{max})/2$, and average daytime wind speed (U_d) in m s^{-1}. The second model had two intermediate parameters, i.e., equivalent evaporation (EV) representing the available energy for vaporization, and an atmospheric factor (C) representing the capacity of the atmosphere to absorb water vapor. The input and output data spaces were selected to include a wide variety of climates between latitudes 60° N and 60° S (table 2). For ease of reference in the text, the two fuzzy model structures are denoted Model I and Model II.

Model I

This model used two input weather parameters, solar irradiance (RS) and relative humidity (RH), to estimate ET (fig. 1). In this model, an integrated effect of daytime wind speed (U_d) and air temperature (T) on RH is assumed, i.e., the changes in T and U_d are reflected in RH. Fuzzification was achieved by categorizing the input and output data space for each parameter (i.e., RS, RH, and ET) into the five fuzzy sets shown table 3, and the degree of membership of data points in the respective fuzzy sets was determined by the Gaussian distribution curve (fig. 3). The control rules for estimating ET were based on known relationships between RS, RH, and ET. These were expressed in linguistic terms by IF–THEN statements. For example:

Rule 1:1 If RS is VERY LOW and RH is MEDIUM, then ET is VERY LOW,

Rule 1:2 If RS is MEDIUM and RH is MEDIUM, then ET is LOW,

Rule 1:3 If RS is HIGH and RH is LOW, then ET is HIGH, etc.

The IF part of the rule statement is referred to as the antecedent, and the THEN part is referred to as the consequent.

Table 2. Input and output data space used in the fuzzy evapotranspiration models.

Input/Output Parameters	Minimum	Maximum	Units
Solar irradiance (RS)	2	37	MJ m^{-2} d^{-1}
Relative humidity (RH)	20	100	%
Wind speed (U_d)	0	10	m s^{-1}
Evapotranspiration (ET)	0	12	mm d^{-1}
Equivalent evaporation (EV)	0	12	mm d^{-1}
Atmospheric factor (C)	0.5	1.5	

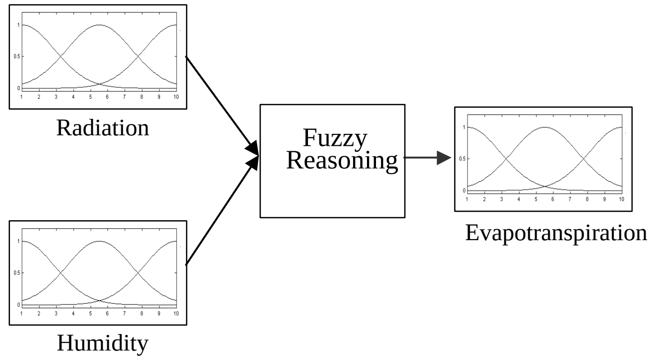

Figure 1. Structure of fuzzy Model I using two input parameters.

Model II

This model used three input weather parameters, RS, RH, and U_d, to estimate ET (fig. 2), and is based on the basic physics of heat and vapor transfer. It assumes that ET is driven primarily by the energy available for use in the vaporization process. Solar irradiance (RS) is the main energy source, so the vaporization process increases with increasing RS and is only limited by the capacity of the atmosphere to absorb water vapor. When the atmospheric limitation is removed, ET is assumed equal to net solar irradiance expressed in equivalent evaporation (EV) in mm d^{-1}. This can be calculated as:

$$EV = 1/\lambda \times (1 - \alpha)RS \text{ [MJ m}^{-2}\text{ d}^{-1}] \quad (1)$$

where λ is latent heat of vaporization and α is albedo for the reference crop ($\alpha = 0.23$).

The capacity of the atmosphere to take up water vapor depends on the relative humidity of the air and wind speed. Relative humidity and wind speed are therefore responsible for an atmospheric factor (C) representing the capacity of the atmosphere to absorb water vapor. In a manner similar to Model I, fuzzification in Model II was achieved by categorizing the data space for each input, intermediate, and output parameter (RS, RH, Ud, EV, and C) into five fuzzy sets (table 3). The degree of membership of data points in the respective fuzzy sets was determined by the Gaussian distribution curve (fig. 3). Model II consisted of two sets of control rules for estimating the two intermediate parameters, EV and C. The EV values were estimated from RS, and the C values from RH and U_d. The control rules for estimating the EV values were expressed as follows:

Rule 2:1 If RS is LOW, then EV is LOW,

Rule 2:2 If RS is MEDIUM, then EV is MEDIUM,

Rule 2:3 If RS is VERY HIGH, then EV is VERY HIGH, etc.

Similarly, the control rules for estimating the C values were expressed as follows:

Rule 3:1 If RH is VERY LOW and U_d is LOW, then C is MEDIUM,

Table 3. Fuzzy sets for input and output variable space used in the fuzzy evapotranspiration models.

Fuzzy Sets	Abbreviation
Very low	VL
Low	LO
Medium	ME
High	HI
Very high	VH

Rule 3:2 If RH is MEDIUM and U_d is low, then C is LOW,
Rule 3:3 If RH is VERY HIGH and U_d is MEDIUM, then C is VERY LOW, etc.

In Model I, ET was obtained through simple control rules and fuzzy reasoning. In Model II, the intermediate parameters, EV and C were obtained through the control rules and fuzzy reasoning, and ET was obtained by the algebraic product T–norm operation, i.e., ET = EV × C. The Mamdani fuzzy inference method (Mamdani and Assilian, 1975; Tsoukalas and Uhrig, 1997) was employed to perform the fuzzy reasoning. A *minimum* operator was used to evaluate the conjunction AND by taking the minimum of the quantified fuzzy sets, and a *truncating* operator was used to evaluate the consequent THEN part by truncating the output fuzzy set at the level of the firing strength of the rule. The truncated output fuzzy sets for all the fired rules were aggregated into a single fuzzy set. The aggregate output fuzzy set encompasses a range of output values, and was defuzzified in order to resolve a single crisp output value from the set. There are a number of methods for defuzzification. The choice of defuzzification method may have significant impact on the speed and accuracy of a fuzzy controller (Tsoukalas and Uhrig, 1997). The centroid method was selected to obtain the representative real non–fuzzy value for the output.

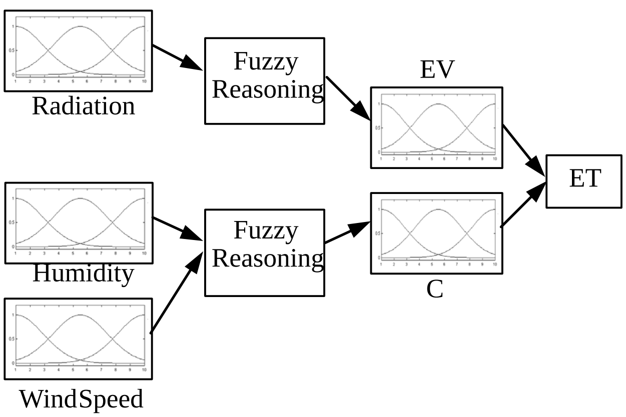

Figure 2. Structure of fuzzy Model II using three input parameters.

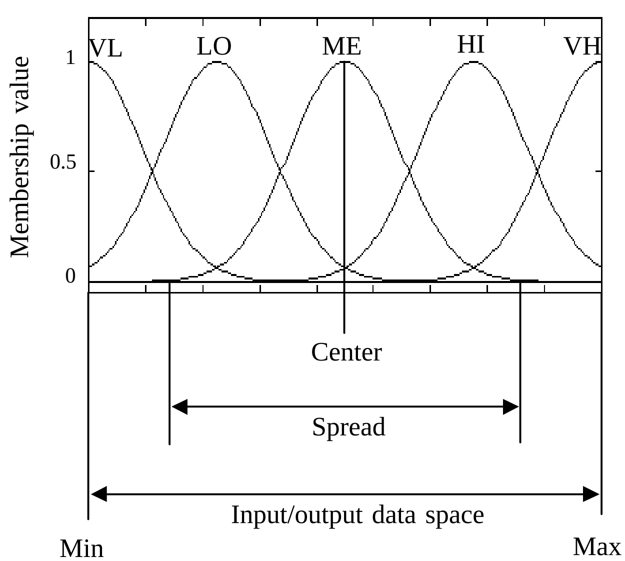

Figure 3. Gaussian distribution curve membership functions for input and output variables.

Although the control rules were based on known relationships between input and output variables, it was very difficult to know the exact consequent fuzzy set for all the conditions. The sampled input–output pairs were used to help identify some of the consequent fuzzy sets, and trial–and–error methods were used to adjust the consequent fuzzy sets one step up or down until the model output best fitted the sample data. For example, the consequent fuzzy set for Rule 1:2 was adjusted by trying "ET is VERY LOW," "ET is LOW," and "ET is MEDIUM" to determine which one gave the best fit with the sample data. The procedure was repeated for all the consequent fuzzy sets of the control rules. The centers and spreads for both the antecedent and consequent fuzzy sets were fixed such that the five fuzzy sets were evenly distributed over the data space, as shown in figure 3. The fuzzy control rules for both models (Model I and II) were adjusted using a part of the 1997 weather data collected at Crossville, Tennessee. The final rules derived for Model I are summarized in table 4, and for Model II are summarized in tables 5 and 6.

Materials and Methods

The two fuzzy ET models were used to estimate grass evapotranspiration (ET) using independent climatic data sets from three lysimeter sites representing arid and humid climates. Arid locations are classified as those locations at which the mean daily relative humidity is less than 60%, and humid locations are classified as those locations at which the mean daily relative humidity is greater than or equal to 60%(Jensen et al., 1990). A description of the lysimeter sites, climates, and locations evaluated are presented in table 7,

Table 4. Fuzzy control rules for evapotranspiration estimation using fuzzy evapotranspiration Model I.

RH \ RS	VL	LO	ME	HI	VH
VL	VL	LO	ME	HI	VH
LO	VL	LO	ME	HI	HI
ME	VL	LO	LO	ME	ME
HI	VL	VL	LO	ME	ME
VH	VL	VL	LO	LO	LO

Table 5. Fuzzy control rules for atmospheric factor used in the fuzzy evapotranspiration Model II.

WS \ RH	VL	LO	ME	HI	VH
VL	ME	LO	VL	VL	VL
LO	ME	ME	VL	VL	VL
ME	HI	ME	LO	LO	VL
HI	HI	HI	ME	LO	VL
VH	VH	HI	ME	ME	LO

Table 6. Fuzzy control rules for equivalent evaporation used in the fuzzy evapotranspiration Model II.

RS	VL	LO	ME	HI	VH
EV	VL	LO	ME	HI	VH

Table 7. Description of location and climates of lysimeter sites evaluated.

Site/Date	Lat.	Alt. (m)	RS (MJ m^2 d^{-1})	RH (%)	U_d (m s^{-1})	T (°C)
Crossville, Tenn.						
(Jul. – Sep. 1997)	35° 55′ N	573	19.9	79.5	1.0	21.0
(May – Jun. 1994)	35° 55′ N	573	22.3	78.1	1.1	17.5
Paraipaba, Brazil						
(Mar. – May 1998)	3° 29′ S	30	19.0	84.8	3.2	27.6
Bushland, Texas						
(May – Sep. 1998/99)	35° 11′ N	1170	23.5	59.4	4.2	22.2

where RS, RH, U_d, and T are the average incoming solar irradiance, relative humidity, daytime wind speed, and air temperature, respectively, for the periods evaluated (indicated in table 7). Based on the above conditions, the Bushland, Texas, site was classified as arid, while the sites at Crossville, Tennessee, and Paraipaba, Ceara, Brazil, were classified as humid. The fuzzy estimated ET values were compared with direct ET measurements from reference grass lysimeters, and ET estimations from the FAO Penman–Monteith (Allen et al., 1998) and the Hargreaves–Samani (Hargreaves and Samani, 1985) equations. The comparisons were based on daily ET values calculated from the daily mean of the relevant climatic parameters.

The instrumentation set up for data acquisition at the sites consisted of weighing–type lysimeters, which were used to directly measure ET, and automated weather stations. A summary of the characteristics of the lysimeter facilities at each site is presented in table 8. Daily ET was determined as the difference between lysimeter mass losses (from evapotranspiration) and lysimeter mass gains (from irrigation, precipitation, or dew), divided by lysimeter area. Solar irradiance, relative humidity, wind speed, and air temperature were measured at adjacent weather stations. Data from the lysimeter and the weather station were recorded with a data logger and transferred to a personal computer. Raw lysimeter ET data for well–watered grass along with supporting climatic data were obtained by personal communication with investigators working at the sites.

Daily ET based on the FAO Penman–Monteith method was computed as follows:

$$ET = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T+273} u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.34 u_2)} \qquad (2)$$

Table 8. Summary of the characteristics of the lysimeter facilities at the evaluated sites.

	Crossville, Tennessee	Paraipaba, Ceara, Brazil	Bushland, Texas
Type of lysimeter	Weighing	Weighing	Weighing
Type of scale system	Lever load cell[a]	Floor stand scale	Lever load cell[a]
Soil profile	Monolith	Reconstructed	Monolith
Wall material	Steel	Steel	Steel
Surface area (m^2)	4.0	2.25	9.0
Soil depth (m)	1.8	1.0	2.3
Drainage type	Free drainage	Free drainage	Free drainage
Sensitivity (ET mm)	0.05	0.18	0.05

[a] Counterbalance lever load cell.

where
- ET = estimated grass evapotranspiration (mm d^{-1})
- Δ = slope of the saturated vapor pressure curve (kPa $^oC^{-1}$)
- R_n = net irradiance at the crop surface (MJ m^{-2} d^{-1})
- G = soil heat flux density in MJ m^{-2} d^{-1} (positive when heat flux is toward the surface)
- γ = psychrometric constant (kPa $^oC^{-1}$)
- e_s = saturation vapor pressure (kPa)
- e_a = actual vapor pressure in kPa (derived from maximum and minimum relative humidity)
- u_2 = wind speed at 2–m height (m s^{-1})
- T = mean daily air temperature at 2–m height in °C ($T = (T_{max} + T_{min})/2$).

The slope of the saturation vapor pressure (Δ) was calculated using mean air temperature (T), and the saturation vapor pressure was computed as the mean between the saturation vapor pressure at the daily maximum and minimum air temperatures. Measured incoming solar irradiance (RS) was used to calculate the net shortwave irradiance (R_{ns}) as:

$$R_{ns} = (1-\alpha)RS \qquad (3)$$

and the net longwave irradiance (R_{nl}) as:

$$R_{nl} = \sigma\left[T_{max,K}{}^4 + T_{min,K}{}^4\right] / 2 (0.34 - 0.14\sqrt{e_a})(1.35 \times RS/R_{so} - 0.35) \qquad (4)$$

R_{ns} and R_{nl} were used to determine the net irradiance (R_n) for equation 2 as:

$$R_n = R_{ns} - R_{nl} \qquad (5)$$

The term α is albedo ($\alpha = 0.23$ for grass reference crop), σ is the Stefan–Boltzmann constant (4.90310^{-9} MJ K^{-4} m^{-2} d^{-1}), $T_{max,K}$ and $T_{min,K}$ are the maximum and minimum absolute temperatures during the 24–hour period, and R_{so} is the calculated clear–sky irradiance in MJ m^{-2} d^{-1} (Allen et al., 1998).

The Hargreaves–Samani equation for determining daily ET was expressed as follows:

$$ET = 0.0023 \times RA \times (T^oC + 17.8) \times TD^{0.50} \qquad (6)$$

where
- RA = extraterrestrial irradiance expressed in equivalent water evaporation (mm d^{-1})
- TD = $(T_{max} - T_{min})$
- T °C = $(T_{max} + T_{min})/2$

T_{max} and T_{min} are maximum and minimum temperatures in °C. The values of RA for different months and latitudes are given in Hargreaves (1994).

Results and Discussion

Graphical comparisons of fuzzy estimated ET values with lysimeter–measured ET values show that both fuzzy models are able to capture the trends in daily ET (figs. 4 through 7). As can be seen, Model I ET values gave a good fit to the lysimeter–measured ET at Crossville (figs. 4 and 5), but underestimated ET at both the Paraipaba (fig. 6) and Bushland (fig. 7) stations. An overview of the plots shows that Model II ET values gave a good fit with lysimeter–measured ET at all the stations evaluated.

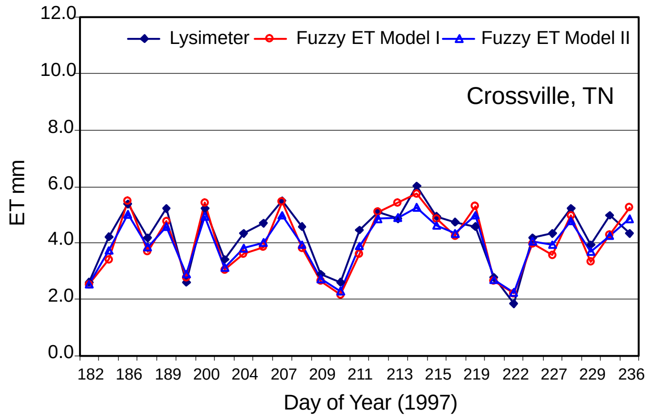

Figure 4. Graphical comparisons of fuzzy estimated ET and lysimeter–measured ET at Crossville, Tennessee (1997 data).

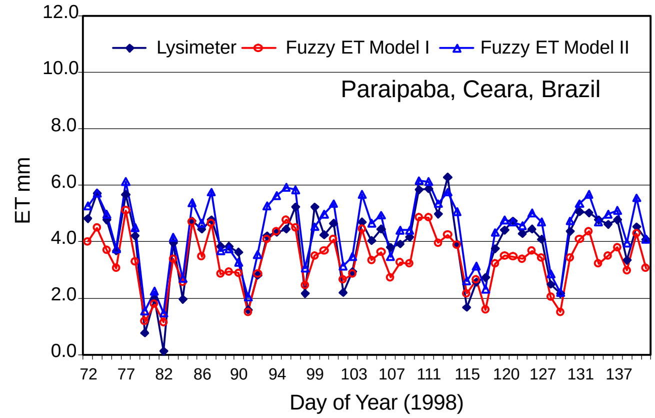

Figure 6. Graphical comparisons of fuzzy estimated ET and lysimeter–measured ET at Paraipaba, Ceara, Brazil (1998 data).

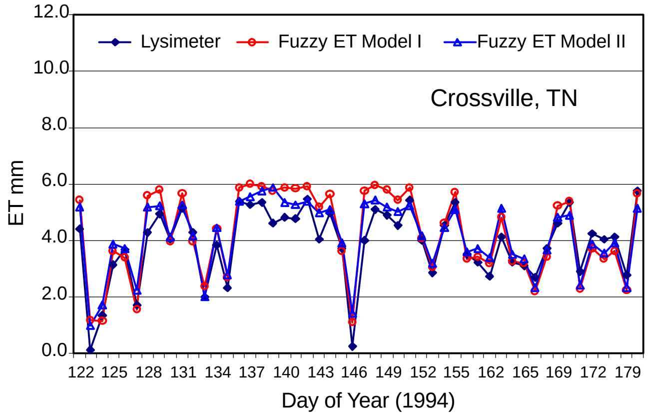

Figure 5. Graphical comparisons of fuzzy estimated ET and lysimeter–measured ET at Crossville, Tennessee (1994 data).

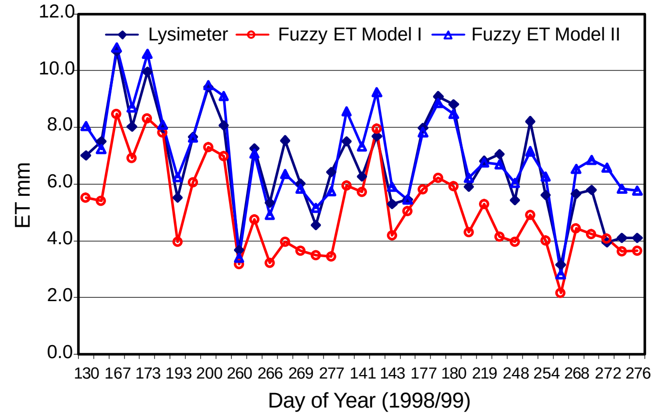

Figure 7. Graphical comparisons of fuzzy estimated ET and lysimeter–measured ET at Bushland, Texas (1998/99 data).

Statistical analyses of daily ET were done to evaluate the accuracy of the fuzzy ET models relative to lysimeter–measured ET. The statistical parameters used were the standard error of the estimate (S_{yx}) and the squared correlation coefficient (r^2). These statistical parameters were also used to compare the results of the fuzzy ET models with the ET estimations from the FAO Penman–Monteith and the Hargreaves–Samani equations (table 9).

The standard error of the estimate (S_{yx}) represents a rough estimate of the average amount of estimation error, that is, the average amount by which the estimation method will either overestimate or underestimate the true ET given by lysimeter measurements. The results show that the estimation error (S_{yx}) varied from 0.25 to 0.97 mm for Model I, 0.22 to 0.78 mm for Model II, 0.25 to 0.66 mm for the FAO Penman–Monteith equation, and 0.27 to 0.87 for the Hargreaves–Samani equation. The S_{yx} values when data at all the sites are combined were 0.73 for Model I, 0.54 for Model II, 0.50 for the FAO Penman–Monteith equation, and 0.66 for the Hargreaves–Samani equation. The results show that the estimation error for Model II and the FAO Penman–Monteith equation were low and consistently comparable at all the sites evaluated. Analysis of the estimated mean ET values, using error bars at 5% positive and negative potential error, indicates that the ET estimates of Model II and the FAO Penman–Monteith equation were not significantly different from the lysimeter–measured ET at all the sites evaluated. On the other hand, the ET estimates of Model I and the Hargreaves–Samani equation were significantly different from the lysimeter–measured ET at all sites except at the Crossville site for Model I.

The squared correlation coefficient (r^2) provides an additional measure of predictive accuracy of a model. The r^2 value can be interpreted as the strength of the straight–line relationship between the estimated ET and the lysimeter–measured ET. A high r^2 value (close to 1) indicates a perfect fit. The r^2 values obtained ranged from 0.72 to 0.89 for Model I, 0.80 to 0.90 for Model II, 0.81 to 0.88 for the Penman–Monteith equation, and 0.31 to 0.66 for the Hargreaves–Samani equation. Figures 8 through 11 show the fitted ET estimates versus lysimeter–measured ET when data at all the sites are combined. The results show that Model II and the FAO Penman–Monteith equation had the highest overall predictive accuracy and that their r^2 values were consistently similar. The range of r^2 values obtained with Model II is also comparable to the range of r^2 values obtained for the FAO Penman–Monteith equation in other evaluation studies of ET estimation methods (Jensen, et al., 1990).

The overall assessment indicates that Model II estimated ET as precisely as the FAO Penman–Monteith equation at all the sites evaluated, while Model I performed well only at the Crossville site. Both Models I and II performed better than the Hargreaves–Samani equation. Model I appear to be site–specific and works well only within a range of U_d and T

Table 9. Statistical analyses of daily estimated ET using fuzzy models and the Penman–Monteith and Hargreaves–Samani equations.

Location/Date	No. of Data	Parameter	Fuzzy Model I	Fuzzy Model II	[a]FAO PM Equation	[b]H–S Model
Crossville, Tennessee (1997)	29	Standard Error (S_{yx}) mm	0.25	0.22	0.25	0.44
		r^2	0.89	0.90	0.81	0.31
Crossville, Tennessee (1994)	50	Standard Error (S_{yx}) mm	0.57	0.46	0.35	0.72
		r^2	0.86	0.87	0.88	0.31
Paraipaba, Ceara, Brazil (1998)	60	Standard Error (S_{yx}) mm	0.56	0.44	0.44	0.27
		r^2	0.81	0.87	0.88	0.66
Bushland, Texas (1998/99)	37	Standard Error (S_{yx}) mm	0.97	0.78	0.66	0.87
		r^2	0.72	0.80	0.87	0.61
All locations combined	176	Standard Error (S_{yx}) mm	0.73	0.54	0.50	0.66
		r^2	0.73	0.90	0.91	0.53

[a] FAO Penman–Monteith Equation.
[b] Hargreaves–Samani Equation.

similar to the average conditions under which it is developed. Model I control rules can be adjusted to obtain good results for other conditions, but again the results are not transferable. Model II is more broad–based and gave good results comparable to the FAO Penman–Monteith equation under varied climatic conditions. The performance of the fuzzy models has not been tested under extreme winter conditions. However, Model II should work well in tropical conditions where temperatures are generally moderate to high, but it may need tuning (adjustment of control rules) during very cold periods. The main advantage of Model II over the FAO Penman–Monteith method is that it requires fewer and simpler parameters to achieve equivalent prediction accuracy. The contrast between the measured data and required input parameters for the FAO Penman–Monteith equation and Model II is given in table 1.

Summary and Conclusions

The objective of the study was to achieve an accurate estimation of daily ET using simpler and fewer parameters. Two fuzzy evapotranspiration models, using two or three weather parameters, were developed and applied to estimate grass ET. Independent weather parameters from sites representing arid and humid climates were used to test the models. The fuzzy estimated ET values were compared with direct ET measurements from grass lysimeters and ET estimates with the FAO Penman–Monteith and Hargreaves–Samani equations. The results show that a fuzzy model with two input parameters is site–specific, and a fuzzy model with three input parameters is broad–based. Both models performed better than the Hargreaves–Samani equation in estimating daily ET. The fuzzy model with three input parameters achieved accurate daily ET estimation comparable to the FAO Penman–Monteith equation at all the sites evaluated. The main advantage of Model II over the FAO Penman–Monteith method is that it requires simpler and fewer parameters to achieve equivalent prediction accuracy.

In further work, the authors are optimizing fuzzy ET Model II through neural training with input–output examples. This will provide a systematic way of tuning the membership functions, and extracting the fuzzy rules to make them more transferable from one site to another.

Acknowledgements

Special appreciation is extended to the Yoder Charitable Foundation for sponsoring this project. The authors are grateful to The Tennessee Agricultural Experiment Station, Crossville, Tennessee; T. A. Howell of USDA–ARS, Bushland, Texas; and F. R. de Miranda of Embrapa and the Curu Valley Experimental Station, Paraipaba, Ceara, Brazil, for providing the lysimeter data and supporting climatic data for the study.

References

Allen, R. G., L. S. Pereira, D. Raes, and M. Smith. 1998. Crop evapotranspiration: Guidelines for computing crop water requirements. Irrigation and Drainage Paper No. 56. Rome, Italy: FAO.

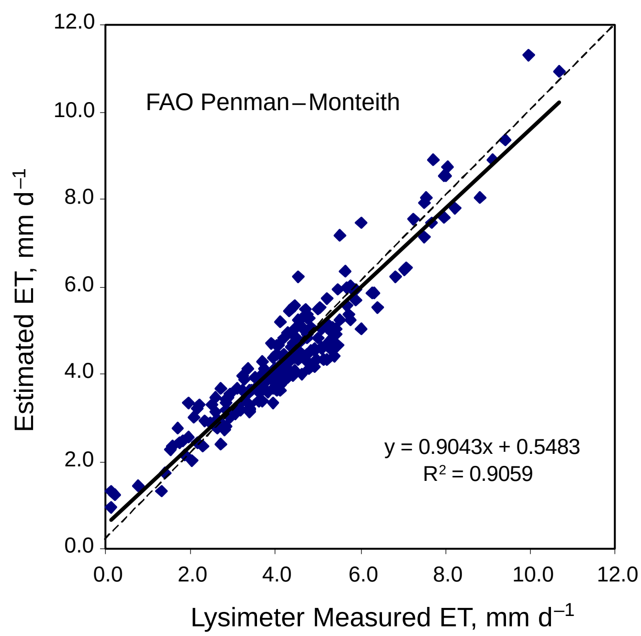

Figure 8. Estimates of ET by FAO Penman–Monteith equation versus daily lysimeter ET at the three locations (Crossville, Paraipaba, and Bushland).

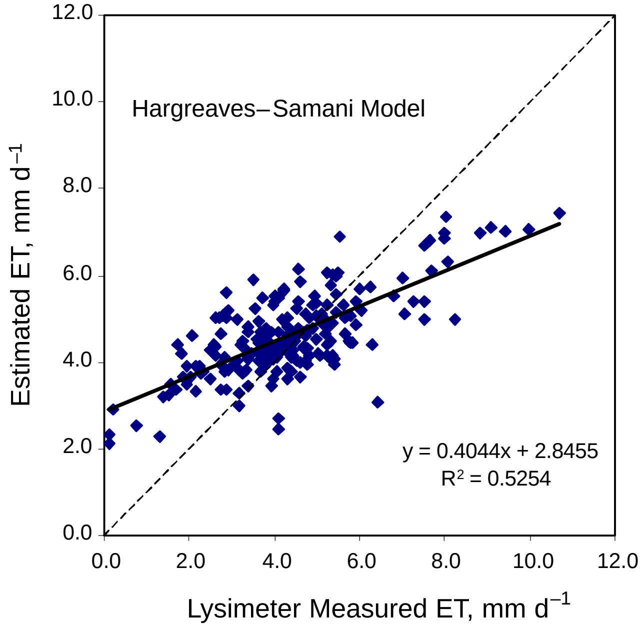

Figure 9. Estimates of ET by Hargreaves–Samani equation versus daily lysimeter ET at the three locations (Crossville, Paraipaba, and Bushland).

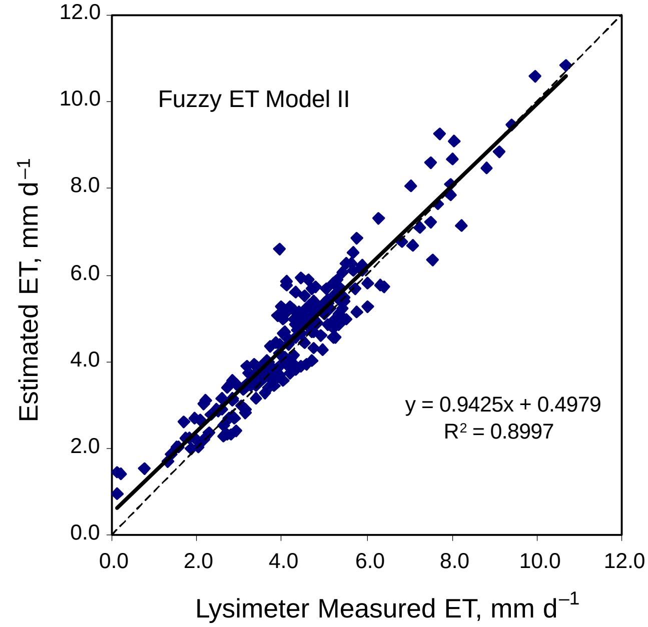

Figure 11. Estimates of ET by fuzzy ET Model II versus daily lysimeter ET at the three locations (Crossville, Paraipaba, and Bushland).

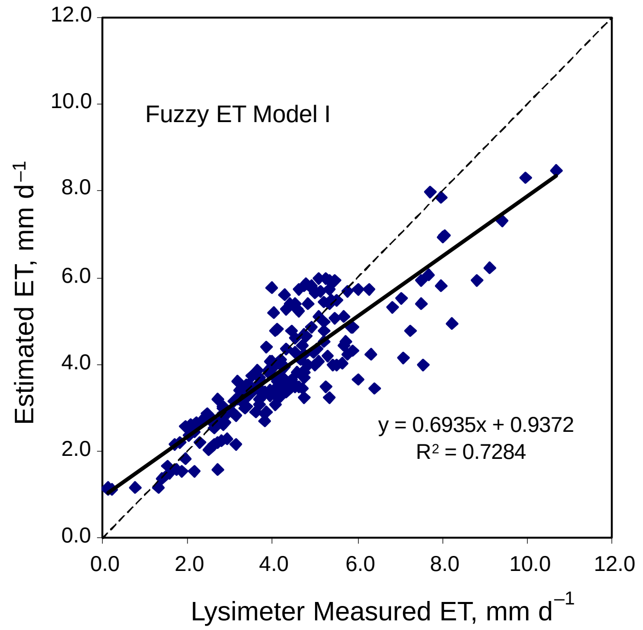

Figure 10. Estimates of ET by fuzzy ET Model I versus daily lysimeter ET at the three locations (Crossville, Paraipaba, and Bushland).

Chuang, D. M., S. C. Wu, and M. K. Hor. 1997. Adaptive fuzzy visual servoing in robot control. In *Proc. of IEEE International Conference on Robotics and Automation*, 1: 811–816. Piscataway, N.J.: IEEE.

Hargreaves, G. H. 1994. Defining and using reference evapotranspiration. *J. of Irrigation and Drainage Engineering* 120(6): 1132–1139. New York, N.Y.: ASCE

Hargreaves, G. H., and Z. A. Samani. 1985. Reference crop evapotranspiration from temperature. *Applied Eng. in Agr.* 1(2): 96–99. St. Joseph, Mich.: ASAE.

Hatfield, J. L., and M. Fuchs. 1990. Evapotranspiration models. In *Management of Farm Irrigation Systems*, ch. 3: 33–59. G. J. Hoffman, T. A. Howell, and K. H. Solomon, eds. St. Joseph, Mich.: ASAE.

Jang, J.–S. R., and C.–T Sun. 1995. Neuro–fuzzy modeling and control. *Proceedings of the IEEE* 83(3): 378–406.

Jensen, M. E, R. D. Burman, and R. G. Allen. 1990. *Evapotranspiration and Irrigation Water Requirements.* Manual and Reports on Engineering Practice No. 70. New York, N.Y.: ASCE.

Kaufmann, A., and M. M. Gupta. 1991. *An Introduction to Fuzzy Arithmetic.* New York, N.Y.: Van Nostrand Reinhold.

Mamdani, E., and S. Assilian. 1975. An experiment in linguistic synthesis with a fuzzy logic controller. *International J. of Man–Machine Studies* 7(1): 1–13. New York, N.Y.: Academic Press.

Postlethwaite, B. E. 1989. A fuzzy state estimator for fed–batch fermentation. *Chem. Eng. Res. Des*. 67(3): 267–272. London, U.K.: Transactions of the Institution of Chemical Engineers.

Ribeiro, R. S. F., and R. E. Yoder. 1997. An automated fuzzy irrigation control system. In *Proc. 18*th *Annual Irrigation Exposition and Technical Conference*, 171–178. 2–4 November. Opryland Hotel Convention Center, Nashville, Tenn.

Saruwatari, N., and A. Yomota. 1995. Forecasting system of irrigation water on paddy field by fuzzy theory. *J. of Agricultural Water Management* 28(2): 163–178. Amsterdam, The Netherlands: Elsevier.

Shabani, F., N. R. Prasad, and H. A. Smolleck. 1996. State estimation with aid of fuzzy logic. *Electric Power Systems Research* 39(1): 55–60. Amsterdam, The Netherlands: Elsevier.

Tay, T. T., and S. W. Tan. 1997. Fuzzy system as parameter estimator on nonlinear dynamic functions. *Trans. on Systems, Man and Cybernetics: Part B. Cybernetics* 27(2): 313–326. Piscataway, N.J.: IEEE.

Terano, T., K. Asai, and M. Sugeno. 1992. *Fuzzy Systems Theory and Its Applications*. Boston, Mass.: Academic Press.

Tao C. W., J. S. Taur, H. C. Kuo, J. C. Wu, and W. E. Thompson. 1994. An estimator based on fuzzy if–then rules for the multisensor multidimentional multitarget tracking problem. In *IEEE International Conference on Fuzzy Systems* 3: 1543–1548. Piscataway, N.J.: IEEE.

Tsoukalas, L. H., and R. E. Uhrig. 1997. *Fuzzy and Neural Approaches in Engineering*. New York, N.Y.: John Wiley and Sons, Inc.