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Nonlinear modelling of experimental rock failure data

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Abstract: In this paper we investigate nonlinear models for rock failure data. The complexities of estimation and analysis using sample experimental data are studied when fitting a power-law type model. Both the high frequency of sampling of observations during the experiment, as well as the nature of the model with high levels of correlation of the parameter estimators result in some challenging issues in the modelling.

Keywords: Nonlinear regression; Least squares estimation; Likelihood; Power-law model.

1 Introduction

We consider methods for modelling experimental data collected in the lab concerning rock failure. Accelerating rates of foreshocks are often observed precursory to natural hazards such as earthquakes and volcanic eruptions. Similarly, rock failure in laboratory experiments is preceded by accelerating strain rates. In this work we investigate the usage of a damage mechanics model proposed by Main (2000) for the analysis of strain and strain rate data during the tertiary phase of brittle creep. The model studied consists of 3 parameters, of which we focus on the failure time and on the power-law exponent. When examining the likelihood function and the Fisher Information we find that there is substantial correlation between these parameters.

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2 Model

Main (2000) develops a damage mechanics model to explain the time-dependent, trimodal behaviour of brittle creep. We consider modelling the strain using a relationship of the form

$$\Omega = \Omega_I \left(1 + \frac{t}{m\tau_1}\right)^m + \Omega_{III} \left(1 - \frac{t}{t_f}\right)^{-v},$$

where for time t , Ω is the strain. The parameters are Ω_I , Ω_{III} , m , τ_1 , v and t_f . There is also interest in modelling strain rate $\dot{\Omega}$, the derivative of Ω with respect to time t . Here we focus on the accelerating crack growth which is associated with the second term on the right-hand-side of the above equation as it illustrates the complexities of modelling the data, i.e. a strain model of the form

$$\Omega = \omega \left(1 - \frac{t}{t_f}\right)^{-v}, \quad (1)$$

where t_f represents the time of failure which is of particular interest, while the exponent parameter v relates to the curvature of the strain relationship.

3 Estimation

We assume for the moment that the strain observations are subject to iid experimental errors with variance σ^2 , and the expectation at time t has the form given in (1) which leads to a nonlinear model (Bates and Watts, 1988; Fahrmeir et al., 2013).

We apply a nonlinear least squares estimation procedure to fit the model with parameters v and ω for given t_f to the strain data (Heap et al. (2009) gives details of lab experiments). This seems to be an effective numerical approach for parameter estimation. Models were fitted for failure times t_f over a suitable range of values and the best model selected; this corresponded to $t_f = 138.864$. Table 1 presents the estimates of the fitted model with this particular failure time value.

TABLE 1. Estimation of v and ω from strain for $t_f = 138.864$.

Parameter	Estimate	Std. Error
v	2.147×10^{-2}	4.104×10^{-5}
ω	1.741	1.143×10^{-4}
σ	0.001628	

The fitted model and residuals are given in Figure 1. Looking at the left panel we can see that the fitted model appears to represent the experimental

data well, but the residuals show some concerning departures from the assumed model. Firstly there are multiple small waves with a length of about ten minutes, and secondly there are two irregular large waves, which indicate a more severe discrepancy between the fitted values and the data. A partial autocorrelation analysis of the residuals suggests that residuals are autocorrelated, which may possibly be due to the nature of the experiment. Therefore, we expect the estimates to be reliable, but the SEs may be misleading.

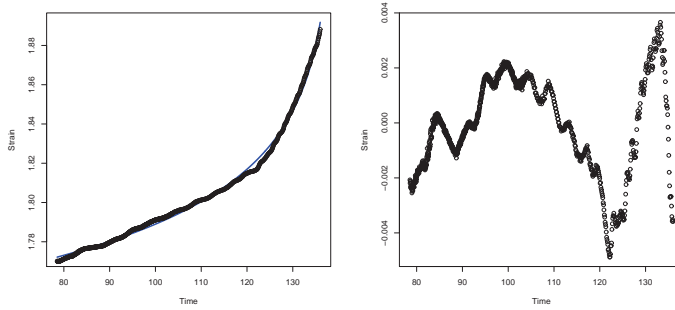


FIGURE 1. Fitted regression line with strain observations (left) and residuals (right) for the estimation from the experimental lab data.

The estimated parameters (obtained by minimising RSS over a range of t_f values, with $\text{argmin}(t_f) = 143.37$) are given in Table 2 for the strain rate data, and the fitted model is shown in Figure 2, with the residuals. The estimates for the strain and the strain rate produce very similar fits when viewed on the strain scale. However, the SEs for the strain rate would appear to be more reliable. On investigating the differences between the parameters estimated by the two models, the t_f over which the RSS was minimised is not very well determined as the RSS does not vary greatly over a range of t_f -models. Also, the parameter estimators are highly correlated so changes in t_f lead to changes in the other two parameters. Nevertheless, the strain and strain rate models provide a useful representation of the experimental data.

TABLE 2. Estimation of v and ω' (the constant multiplicative parameter from $\hat{\Omega}$ model) from strain rate for $t_f = 143.37$.

Parameter	Estimate	Std. Error
v	0.25634	0.05680
ω'	0.13246	0.04688
σ	0.002479	

4 Nonlinear modelling

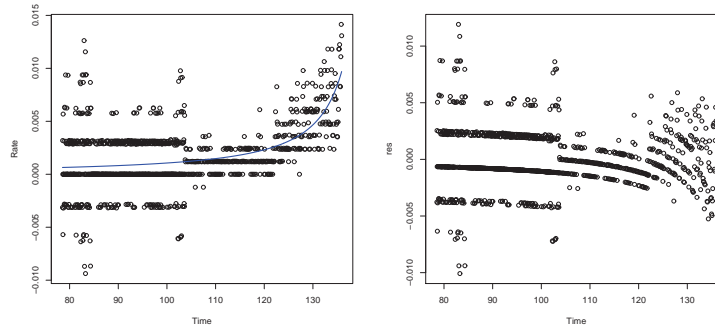


FIGURE 2. Fitted regression line with strain rate observations (left) and residuals (right) for the estimation from the experimental lab data.

4 Discussion

In this paper we have considered the application of nonlinear models for analysis of experimental rock failure data during the tertiary phase of brittle creep. These 3-parameter models have been used to explore different features of the data and highlight some of the challenges of analysing such data. In future work, where lab experiments are repeated under identical conditions, it is expected that the use of replication will enable the features we have seen to be investigated more fully, and provide further insights into the processes related to natural hazards such as earthquakes and volcanic eruptions.

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